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(Citation)
神戸大学経済学研究科 Discussion Paper,1216

(Issue Date)
2012-08-17

(Resource Type)
technical report

(Version)
Version of Record

(URL)
https://hdl.handle.net/20.500.14094/81004238

Three Essays on Robustness and Asymmetries in Central Bank Forecasting

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Robust control and asymmetries in central bank forecasting

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August 17, 2012

Abstract

This paper introduces asymmetric central bank forecasting into the standard New Keynesian model within the context of robust control theory. Asymmetric forecasting expresses policymakers' reservations about economic forecasts, and the degree of their reservations is reflected as an asymmetric preference whose existence warrants laying aside the assumption that policymakers' base decisions primarily on rational expectations.

This study concludes that monetary policy becomes more aggressive because of policymakers' reservations about forecasts stemming from asymmetry, and preference for policies robust enough to overcome unanticipated situations. In addition, adopted policies will likely amplify economic fluctuations and significantly reduce social welfare.

Keywords: robust control, asymmetric forecasting, bounded rationality

JEL classification: E50, E52, E58

1 Introduction

The importance of considering deviations from rational expectations is receiving growing recognition in monetary policy analysis. The customary view is that policymakers, following rational expectations, forecast the economy and implement an optimal plan with confidence in their constructed model. Unfortunately, the customary view implies that outcomes are vulnerable to uncertainties that policymakers failed to consider.

Hansen and Sargent (2008) posit a robust control approach to address the fact that the rational expectations model overlooks uncertainties, and therefore adopted policies need

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to be more robust. In their approach, monetary policymakers have reservations about their model and organize an optimal plan for the uncertain economic environment.

To further incorporate deviations from assumptions about rational expectations, Branch (2011), Capistrán and Timmermann (2009), and others empirically and theoretically emphasize asymmetric forecasting. Expectations based on asymmetric preferences introduce additional bias in policymakers' rational expectations, and bias is amplified by forecasting asymmetry. Asymmetric forecasting suitably describes the realistic context of economic forecasting—namely, policymakers' actual forecasts are subjective and imperfect; however, they simultaneously seem to include their best rational expectations.

This paper inserts a robust control algorithm and asymmetric forecasting into the standard New Keynesian model. We focus on the difference between the standard model and our model, which incorporates uncertainty stemming from asymmetric forecasting.

This paper proceeds as follows. Section 2 explains robust control and asymmetric preference and develops the model. Section 3 presents numerical results for asymmetric preferences and policymakers' preference for robustness. Section 4 concludes the paper.

2 The model

2.1 Basic settings

Branch (2011), Capistrán and Timmermann (2009), Surico (2007), and Riboni and Ruge-Murcia (2010) suggest that central bank forecasting may become asymmetrical. In their views, asymmetric preferences generate positive and negative deviations from rational expectations about inflation and output gaps. The resulting subjective forecast for any variable is specified as follows:

$$\hat{E}_t z_{t+1} = E_t z_{t+1} + \varphi_z \sigma_{z,t}, \tag{1}$$

where φ_z is an asymmetric preference in subjective forecasting and $\sigma_{z,t}$ is a conditional variance for the variable. According to (1), asymmetric forecasting $\hat{E}_t z_{t+1}$ adds bias $\varphi_z \sigma_{z,t}$ to the rational expectation level, $E_t z_{t+1}$. In other words, the existence of asymmetric preferences introduces a deviation from rational expectations into policymakers' forecasts.

Asymmetric preferences embodied in forecasts reflect the central bank's reservations about its forecast (i.e., bankers question whether their expectations are rationally grounded). As a result, policymakers wish to include a margin of error in their evaluation of the economy. Since real policy and policymakers lack a determinant reference for the economy and form expectations about factors outside of the economic model, it is essential to consider forecasting asymmetry. In our approach, policymakers' insecurity about their forecast leads them to adopt policies that introduce uncertainty into the economy.

In addition, the central bank implements policies by considering the worst case that might arise because of uncertainties stemming from subjective forecasting. For instance, the policymakers first consider the situation in which uncertainty increases welfare loss and then adopt a policy likely to offset it. Hansen and Sargent (2008) establish this min-max algorithm and call the attempted response "robust control." Adopting this robust control algorithm, policymakers are concerned about model misspecifications generated by ill-considered uncertainties and wish to fortify their model against them.

Hansen and Sargent (2008) ascribe uncertainty to a (fictitious) malevolent agent who wants to distort the economy. As the leader in a Stackelberg game, the monetary policymaker takes the strategy of the malevolent agent as given and plans his optimal strategy. The strategy involves creating an approximating model that specifies a robust decision rule which remains in force even if a non-modeled uncertainty materializes. Applications of the robust control theory for monetary policy analysis appear in Leitemo and Söderström (2008), Tillmann (2009), Walsh (2004), and elsewhere.

The distinction of this paper is that it combines asymmetric forecasting with a robust control algorithm and incorporates them into a standard New Keynesian model. We assume that monetary policymakers recognize that a bias is induced by their reduced confidence in their forecast, and they wish to pursue the policy that produces robust results even if the bias disturbs the economy. Our contribution is to consider the scenario that emerges because monetary policymakers and policy operate without the confidence that conditions have been fully evaluated.

2.2 Robust control and asymmetric forecasting

This section combines asymmetric forecasting and robust control and incorporates them into the canonical New Keynesian model. Monetary policy obeys the min-max strategy of attempting to minimize welfare loss after uncertainty maximizes it.

We assume the following explicit specification of the New Keynesian model for the economic system:

$$\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa y_t + e_t, \tag{2}$$

$$y_t = \hat{E}_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \hat{E}_t \pi_{t+1}) + u_t, \tag{3}$$

Equation (2) corresponds to the New Keynesian Phillips curve based on profit maximization by firms in Calvo's sticky-price setting, and Equation (3) is the dynamic IS curve induced by consumption, a Euler equation that captures the agent's intertemporal optimization. Note that expectation $\hat{E}_t$ is the asymmetric forecast for the variables specified as follows:

$$\hat{E}_t\pi_{t+1} = E_t\pi_{t+1} + \varphi_\pi \sigma_{\pi,t}, \tag{4}$$
$$\hat{E}_t y_{t+1} = E_t y_{t+1} + \varphi_y \sigma_{y,t}, \tag{5}$$

where φ_π and φ_y are the central bank's asymmetric preferences, which recognize the tendency for forecasting bias. Inserting asymmetric forecasting into the system (2 and 3) yields the following relationships:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t + \beta\varphi_\pi\sigma_{\pi,t} + e_t, \tag{6}$$
$$y_t = E_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) + \left(\frac{\varphi_\pi}{\sigma}\right)\sigma_{\pi,t} + \varphi_y\sigma_{y,t} + u_t, \tag{7}$$

Equations (6) and (7) are the distorted Phillips curve and the IS equation with asymmetric forecasting biases, respectively. In addition, the budget constraint for the malevolent agent is

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1}(\sigma^2_{\pi,t+1} + \sigma^2_{y,t+1}) \leq \eta_0, \tag{8}$$

where η_0 is the supremum of the budget which represents the model misspecifications that concern policymakers.

Taking the model structure (6-8) as given, discretionary monetary policy encounters the problem of minimizing the welfare loss function. To this end, set the Lagrangian as follows:

$$\mathcal{L}_t = \left(\frac{1}{2}\right)(\pi_t^2 + \lambda y_t^2) - \left(\frac{\theta}{2}\right)(\sigma^2_{\pi,t} + \sigma^2_{y,t}) - \mu_t^\pi(\pi_t - \beta E_t\pi_{t+1} - \kappa y_t - \beta\varphi_\pi\sigma_{\pi,t} - e_t) - \mu_t^y\left(y_t - E_t y_{t+1} + \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) - \left(\frac{\varphi_\pi}{\sigma}\right)\sigma_{\pi,t} - \varphi_y\sigma_{y,t} - u_t\right). \tag{9}$$

After rearranging for the first-order condition, we obtain

$$y_t = -\frac{\kappa}{\lambda}\pi_t, \tag{10}$$
$$\sigma_{\pi,t} = \frac{\beta\varphi_\pi}{\theta}\pi_t, \tag{11}$$

The condition does not restrict the IS curve because the interest rate can adjust fully to the demand shock with elasticity σ. Pursuant to this condition, the bias for inflation forecasting $\sigma_{\pi,t}$ increases with inflation asymmetry φ_π and decreases with the inverse of the preference for robustness θ.

To derive worst-case inflation, insert conditions (10) and (11) into the distorted Phillips curve (6). We then obtain

$$\pi_t = \beta A E_t \pi_{t+1} + A e_t, \tag{12}$$

where

$$A \equiv \frac{\lambda\theta}{\lambda\theta + \kappa^2\theta - \lambda\beta^2\varphi_\pi^2}.$$

Let us guess the solution of (12) with the AR(1) cost-push shock. The solution for worst-case inflation is guessed as $\pi_t = a_1 e_t$, and its expectation $E_t\pi_{t+1} = a_1\rho e_t$ to obtain:

$$\pi_t^W = \frac{A}{1-\beta\rho A} e_t. \tag{13}$$

Next, determine the worst-case output gap through the first-order condition

$$y_t^W = -\frac{\kappa A}{\lambda(1-\beta\rho A)} e_t. \tag{14}$$

Insert (13) and (14) into the distorted IS curve and rearrange with respect to i_t, which yields the worst-case interest rate as

$$i_t^W = B e_t + \sigma u_t, \tag{15}$$

where

$$B \equiv \frac{\sigma\kappa\theta(1-\rho)A + \lambda(\theta\rho + \beta\varphi_\pi^2)A}{\lambda\theta(1-\beta\rho A)}.$$

$\rho = 0$ in (13-15) corresponds to solutions with a white noise cost-push shock.

The worst-case model defines the economy's movement when policymaker's pessimism proves to be justified. However, in reality, a useful policy to accommodate uncertainty is to consider an approximating model in which the worst-case distortion does not appear when the policy has a preference for robustness. To derive the approximating model, we cannot utilize first-order conditions as we did for deriving the worst-case model. Instead, the worst-case inflation and output gap are inserted into the IS curve without distortion to obtain the approximate output gap. Namely

$$\begin{aligned} y_t^A &= E_t y_{t+1}^W - \left(\frac{1}{\sigma}\right)(i_t^W - E_t \pi_{t+1}^W) + u_t \\ &= \frac{(\lambda - \sigma\kappa)\rho A - (1 - \beta\rho A)\lambda B}{\sigma\lambda(1 - \beta\rho A)} e_t. \end{aligned} \tag{16}$$

Similarly, inflation is approximated with the pure New Keynesian Phillips curve as

$$\pi_t^A = \frac{\lambda - \kappa^2 A}{\lambda(1 - \beta\rho A)} e_t. \tag{17}$$

Along with our models, response coefficients for the shock are substantially complicated to investigate the qualitative property about asymmetric preference and preference for robustness, so that we confirm the numerical implications for them in Section 3.

2.3 System specification

The system specification (6 and 7) solves using an iterative algorithm with Bellman's equation. We briefly explore the solution for our system based on Giordani and Söderlind (2004).

Most important, asymmetries φ_π and φ_y are included as a loading for control variables, and uncertainties $\sigma_{\pi,t}$ and $\sigma_{y,t}$ are entirely disguised by the shocks. Keeping these facts in mind, we situate backward- and forward-looking variables in the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/\sigma \\ 0 & 0 & 0 & \beta \end{pmatrix} \begin{pmatrix} u_{t+1} \\ e_{t+1} \\ E_t y_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = \begin{pmatrix} \rho^y & 0 & 0 & 0 \\ 0 & \rho^\pi & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -\kappa & 1 \end{pmatrix} \begin{pmatrix} u_t \\ e_t \\ y_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1/\sigma \\ 0 \end{pmatrix} i_t$$
$$+ \begin{pmatrix} \varphi_y & \varphi_\pi/\sigma \\ 0 & \beta\varphi_\pi \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{y,t+1} \\ \sigma_{\pi,t+1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{t+1}^u \\ \varepsilon_{t+1}^e \end{pmatrix}. \tag{18}$$

More succinctly,

$$\boldsymbol{\Psi_{t+1}} = \boldsymbol{A\Psi_t} + \boldsymbol{B^* u_t^*} + \boldsymbol{C\Gamma_{t+1}}. \tag{19}$$

Further, re-rewrite the system with a partitioned matrix for the backward- and forward-looking variables as

$$\begin{pmatrix} \boldsymbol{\xi_{t+1}} \\ \boldsymbol{z_{t+1}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A_{11}} & \boldsymbol{A_{12}} \\ \boldsymbol{A_{21}} & \boldsymbol{A_{22}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi_t} \\ \boldsymbol{z_t} \end{pmatrix} + \begin{pmatrix} \boldsymbol{B} & \boldsymbol{\Phi} \end{pmatrix} \begin{pmatrix} i_t \\ \boldsymbol{\Omega_{t+1}} \end{pmatrix} + \boldsymbol{C} \begin{pmatrix} \boldsymbol{\epsilon_{t+1}} \\ \boldsymbol{0_{2\times 1}} \end{pmatrix}, \quad (20)$$

where

$$\boldsymbol{\xi_t} \equiv \begin{pmatrix} u_t \\ e_t \end{pmatrix}, \quad \boldsymbol{z_t} \equiv \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}, \quad \boldsymbol{\Omega_{t+1}} \equiv \begin{pmatrix} \sigma_{y,t+1} \\ \sigma_{\pi,t+1} \end{pmatrix}, \quad \boldsymbol{\epsilon_{t+1}} \equiv \begin{pmatrix} \varepsilon^u_{t+1} \\ \varepsilon^e_{t+1} \end{pmatrix},$$

and

$$\boldsymbol{\Phi} \equiv \begin{pmatrix} \varphi_y & \varphi_\pi/\sigma \\ 0 & \beta\varphi_\pi \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Note that forecasting asymmetries are included in loading matrix $\boldsymbol{\Phi}$. From the system, we construct the Bellman equation as follows:

$$\boldsymbol{J_t} = \boldsymbol{r_t} + \boldsymbol{\beta E_t}\big(\boldsymbol{\Psi'_{t+1} V_{t+1} \Psi_{t+1}} + \boldsymbol{\upsilon_{t+1}}\big), \quad (21)$$

where

$$\boldsymbol{r_t} \equiv \begin{pmatrix} \boldsymbol{\xi_t} & \boldsymbol{z_t} \end{pmatrix} \begin{pmatrix} \boldsymbol{Q_{11}} & \boldsymbol{Q_{12}} \\ \boldsymbol{Q_{21}} & \boldsymbol{Q_{22}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi_t} \\ \boldsymbol{z_t} \end{pmatrix} + 2\begin{pmatrix} \boldsymbol{\xi_t} & \boldsymbol{z_t} \end{pmatrix} \begin{pmatrix} \boldsymbol{U_1} \\ \boldsymbol{U_2} \end{pmatrix} \boldsymbol{u^*_t} + \boldsymbol{u^{*\prime}_t R u^*_t}. \quad (22)$$

After guessing the solution for the expectation term as $\boldsymbol{E_t z_{t+1}} = \boldsymbol{C_{t+1} E_t \xi_t}$, the following relationship between backward- and forward-looking variables can be derived (for detail derivation, see Söderlind (2003)):

$$\boldsymbol{z_t} = \boldsymbol{D_t \xi_t} + \boldsymbol{G_t(\Phi) u^*_t}, \quad (23)$$

$$\boldsymbol{\xi_{t+1}} = \boldsymbol{\tilde{A}_t \xi_t} + \boldsymbol{\tilde{B}_t(\Phi) u^*_t} + \varepsilon_{t+1}, \quad (24)$$

where

$$\begin{aligned} \boldsymbol{D_t} &= (\boldsymbol{A_{22}} - \boldsymbol{G_{t+1} A_{12}})^{-1}(\boldsymbol{C_{t+1} A_{11}} - \boldsymbol{A_{21}}), \\ \boldsymbol{G_t(\Phi)} &= (\boldsymbol{A_{22}} - \boldsymbol{C_{t+1} A_{21}})^{-1}(\boldsymbol{C_{t+1} B^*_1(\Phi)} - \boldsymbol{B^*_2}), \\ \boldsymbol{\tilde{A}_1} &= \boldsymbol{A_{11}} + \boldsymbol{A_{12} D_t}, \\ \boldsymbol{\tilde{B}_t}(\Phi) &= \boldsymbol{B^*_1(\Phi)} + \boldsymbol{A_{12} G_t(\Phi)}. \end{aligned}$$

where $\boldsymbol{B}_1^*(\boldsymbol{\Phi})$ is the upper side of partition matrix and $\boldsymbol{C}_{t+1}$ is the solution for the guess. Insert (23 and 24) into (22) and optimize the Bellman equation to obtain the following policy function of the system:

$$\boldsymbol{u}_t^* = -\boldsymbol{F}_t(\Phi)\boldsymbol{\xi}_t. \tag{25}$$

Then forward-looking variables (23) can be expressed as

$$\boldsymbol{z}_t = (\boldsymbol{D}_t - \boldsymbol{G}_t(\Phi)\boldsymbol{F}_t(\Phi))\boldsymbol{\xi}_t. \tag{26}$$

From Equations (25) and (26), it is clear that the forecasting asymmetries directory affects the optimal policy plan and the dynamics of the system.[1] From (25), it also is evident that policy decision $F_t(\Phi)$ affects both its instrument and the worst-case shocks.

[1]Based on these relationships, the Bellman equation is iterated until $\boldsymbol{F_t}(\boldsymbol{\Phi})$ and $\boldsymbol{C}_{t+1}$ converge to fixed values.

3 Numerical results

3.1 Response coefficients for a cost-push shock

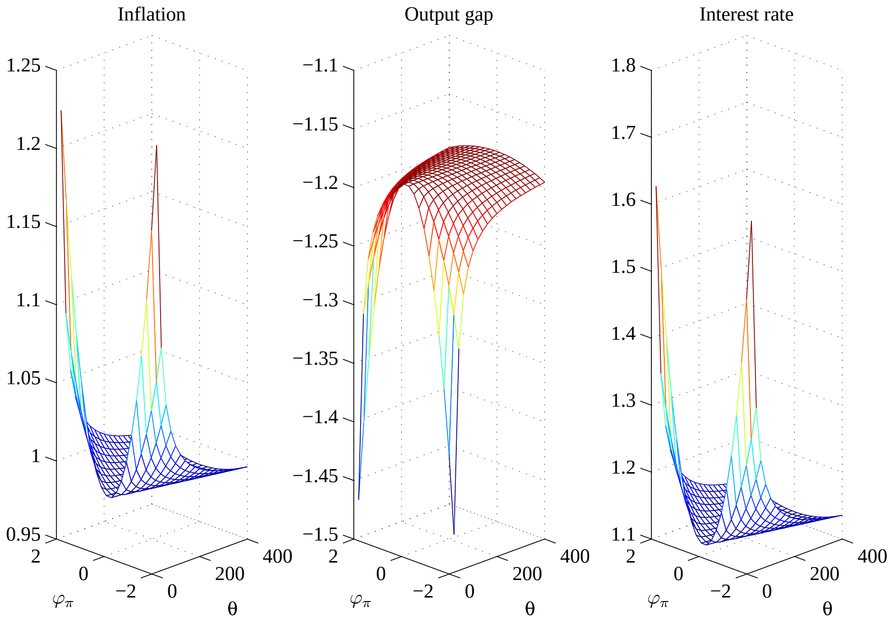

Figure 1: Response coefficients for a cost-push shock in the worst-case model

Table 1 specifies the calibrated parameters used in the analysis. These parameters are basic values taken mainly from Clarida, Galin, and Gertler (2000). The remaining parameters, which they excluded, are trade-off parameters for the central bank's targeted output gap λ and the degree of inertia in the cost-push shock ρ^{π}.[2] We set 0.25 for the output trade-off according to McCallum and Nelson (2000), Walsh (2003), and Tillmann (2009) among others, and 0.35 for the shock inertia as in Giannoni and Woodford (2003).

Taking these parameters as given, and based on relationships articulated in the worst-case model (12-14), we depict coefficients of the variables for the cost-push shock in Figure 1. Inflation and interest rates respond positively to the shock while output reacts negatively. Note that these tendencies become more vigorous when the absolute value of the inflation asymmetry increases. In contrast, the response is progressively muted as policymakers' preference for robustness diminishes (θ increasing).

Alongside the worst-case model, Figure 2 shows the reaction coefficients in the approximating model. They differ from previous results in that inflation displays a decreasing

[2] The inertia for demand shock, ρ^{y}, is also reported, although the parameter is less important because the demand shock affects only the interest rate adjustment, which is always equal to elasticity σ.

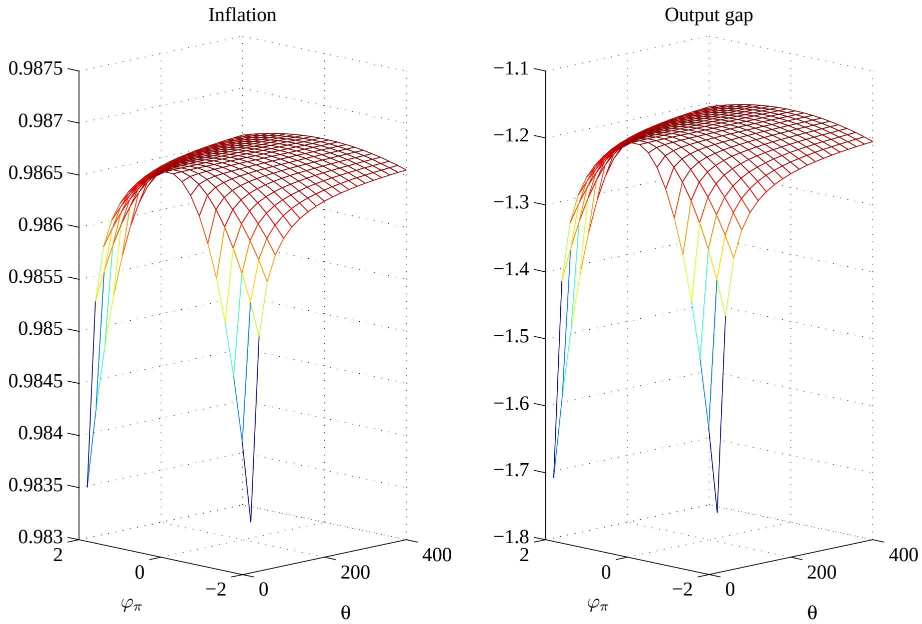

Figure 2: Response coefficients for a cost-push shock in the approximating model

responsiveness to inflation asymmetry, although sensitivity is extremely low. Accordingly, the monetary policy response in the approximating model seems governed by output gap adjustments.[3]

[3]This result is not consistent with the impulse response functions in later sections. The contradiction might arise because the guessed solution is only among many possible solutions.

3.2 Detection Error Probabilities

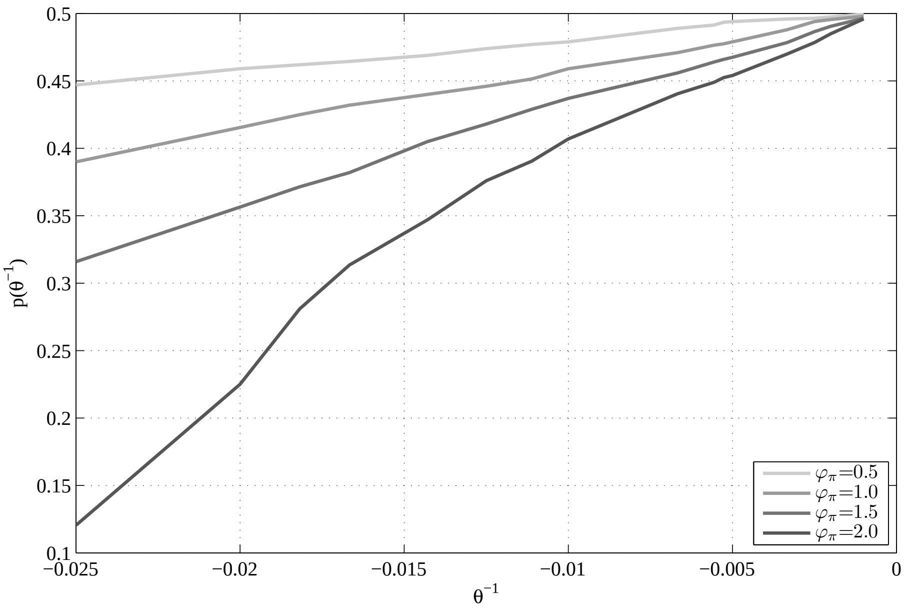

Figure 3: Detection error probabilities with various φ_π

The detection error probability determines the desired lower bound for robustness parameter θ. If planners have extremely low preferences for robustness, the min-max theory of robustly attempting to minimize welfare loss breaks down. At the breakdown point, the objective function for the inner maximization problem of the multiplier game becomes convex, and therefore the solution goes to infinity (Hansen and Sargent, 2008, Chapter 6; Giordani and Söderlind, 2004).

To avoid that misfortune, we adopt the following error detection probability:

$$p(\theta^{-1}) = \text{Prob}(\ln(L^A/L^W) < 0|Approx.) + \text{Prob}(\ln(L^A/L^W) > 0|Worst), \tag{27}$$

where $\text{Prob}(\ln(L^A/L^W) < 0|Approx.)$ is the probability that the worst-case model is chosen when the data are generated from the approximating model, and $\text{Prob}(\ln(L^A/L^W) > 0|Worst)$ is vice versa. Hansen and Sargent (2008, Chapter 9 and 14), find it reasonable to select a robustness parameter between 0.10 or 0.20 for the probability points of the detection error.

Figure 3 presents the simulated error detection probability. The figure suggests the probability is 0.10 when the preference for robustness approaches 40. Accordingly, we set 50 as the preference to preserve theoretical desirability.

3.3 Impulse response functions

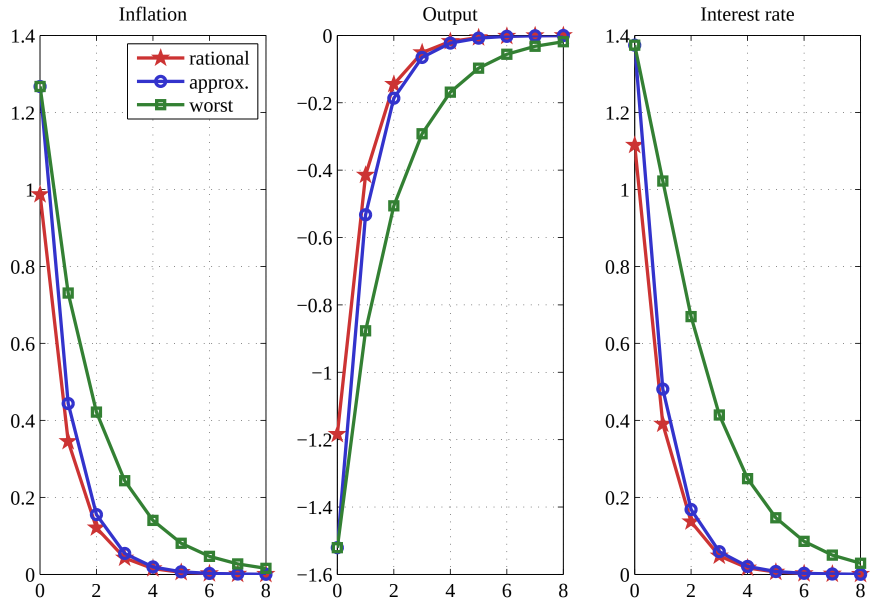

Figure 4: Impulse responses with $\varphi_\pi = 2.0$ and $\theta = 50$

Figure 4 presents impulse response functions for the rational expectations model, approximating model, and worst-case model. The asymmetry and robustness parameters are set at 2.0 and 50. From the figure, asymmetry in inflation forecasting enhances the aggressiveness of monetary policy given policymakers' preferences for robust control. In fact, the asymmetry drives a larger surge in inflation, contraction in output, and rise in interest rates. These responses imply that doubts about forecasts and fear of uncertainty inspire excessive aversion to forecasting uncertainty among central bankers; thereby, making the economy more volatile and the interest rate response more vigorous. A similar interpretation of the robustness parameter can be seen in Leitemo and Söderström (2008).

To confirm the phenomenon, Figure 5 plots impulse responses in the approximating model with various values of φ_π. From this, according to increases in φ_π, deviations from rational expectations increase and bankers' aversion to forecast uncertainty grows.

On the other hand, Figure 6 shows that the vigor of impulse responses to inflation asymmetry are fully suppressed if policymakers eschew robustness in rectifying model misspecifications (corresponding to a larger θ).

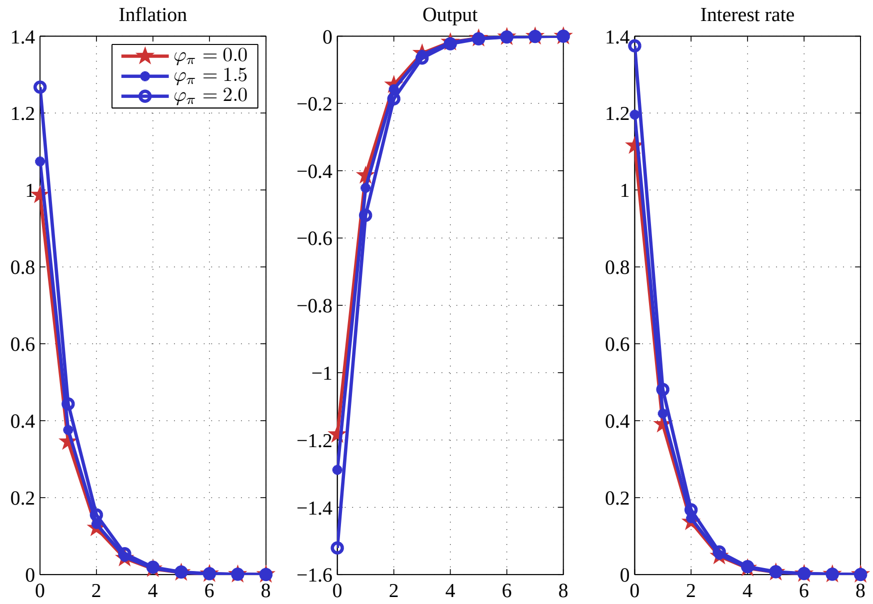

Figure 5: Impulse responses of the approximating model for various φ_π when $\theta = 50$

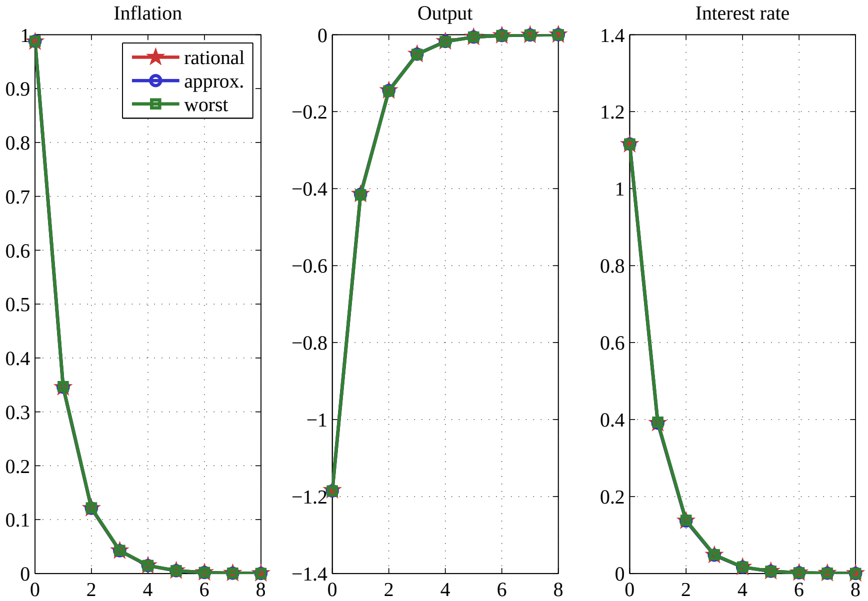

Figure 6: Impulse responses with $\varphi_\pi = 2.0$ and $\theta = 5000$

These results are consistent with Giannoni (2002), Giordani and Söderlind (2004), Hansen and Sargent (2001), and Leitemo and Söderström (2008). Policymakers' heightened preference for robustness enhances their aversion to uncertainty and results in more aggressive monetary policy.

3.4 Welfare loss

Welfare losses calculated by values of φ_π and θ are reported in Figure 7. In the figure, together with lower confidence in forecasting and heightened preference for robustness, welfare loss is monotonically increasing. In contrast, if the central bank is confident that its forecasts embody rational expectations or if it disregards deviations from the rational expectations model, the welfare loss tends to be identical with the level of rational expectations.

Table 2 reports the numerical results of the welfare loss. Recovery of the welfare loss for rational expectations level ($\varphi_\pi = 0.0$) is achieved when (inverse of willingness for) robustness θ equals 1700 in the line of $\varphi_\pi = 2.0$. This result suggests that discarding robustness is required to approach the loss for the rational expectations model when confidence in forecasting is relatively low.

All told, if policymakers deal assertively with uncertainty stemming from asymmetric

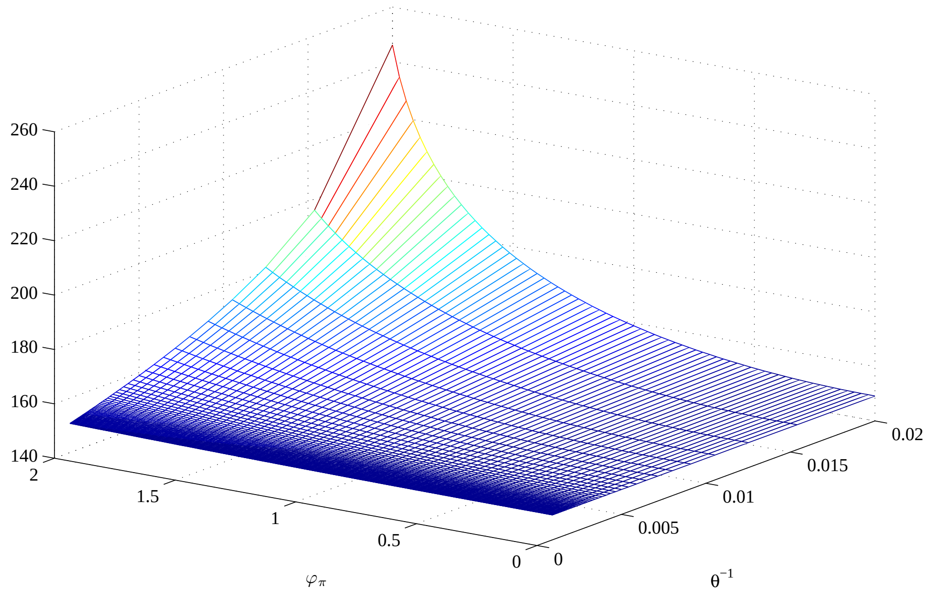

Figure 7: Welfare losses with various φ_π and θ

preferences, the economy becomes more volatile and the anticipated welfare losses are greater.

4 Conclusion

This paper has examined robust control for uncertainty generated by asymmetric forecasting of monetary policy. Beginning with the basic New Keynesian model, we assume the central bank lacks perfect confidence in economic forecasts and prefers a robust response to counteract uncertainty. The degree of the central bank's confidence or lack thereof is represented by an asymmetric preference in forecasting.

The simulation results suggest that an increase in inflation asymmetry prompts a more vigorous policy reaction to a cost-push shock because the asymmetry provokes central bankers' aversion to uncertainty. Moreover, welfare loss increases as asymmetry increases. Overall, these tendencies are exacerbated when policymakers lack confidence in forecasts and prefer robustness.

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Table 1: Calibration

β	σ	κ	λ	ρ^y	ρ^π
0.99	1.00	0.30	0.25	0.35	0.35

Table 2: Welfare losses for the approximating model

Parameters	$\theta = 50$	$\theta = 200$	$\theta = 1700$	$\theta = 5000$
$\varphi_\pi = 0.0$	149.17	149.17	149.17	149.17
$\varphi_\pi = 0.5$	151.36	149.70	149.23	149.19
$\varphi_\pi = 1.0$	158.92	151.36	149.42	149.25
$\varphi_\pi = 2.0$	246.10	158.92	150.18	149.51

Monetary policy delegation with robust control and forecasting asymmetries

Taro Ikeda *†

August 17, 2012

Abstract

This paper considers problems in delegating monetary policy to a central banker that prefers a robust optimal policy when its economic model suffers uncertainty stemming from asymmetric forecasting. It distinguishes the outcomes of delegating to conservative versus activist monetary policymakers under conditions of asymmetric forecasting. Results suggest the economy fluctuates less under guidance by a conservative central banker than under an activist banker. Accordingly, social welfare approaches the level attained under rational expectations if policy is delegated to a conservative banker.

Keywords: delegation problem for monetary policy, robust control, asymmetric forecasting

JEL classification: E52, E58

1 Introduction

The recent financial crisis illuminates numerous issues in conducting monetary policy during periods of uncertainty. In examining difficulties imposed by the crisis, numerous researchers have investigated how monetary policymakers tackle events that are not explicitly anticipated in their constructed models—in a word, uncertainty.

Hansen and Sargent (2008) incorporated uncertainty into the agent decision problem by adapting robust control techniques. In their algorithm, the agent assumes that the worst case induced by uncertainty is a given, and then it chooses the course that maximizes social welfare. Uncertainty affects the economic dynamics through the min-max decision rule. Robust control notably has been applied to monetary policy analysis by Leitemo and Söderström (2008), Tillmann (2009a, b), and Walsh (2004) among others.

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Branch (2011), Capistrán and Timmermann (2009), and Surico (2007) stressed asymmetric forecasting in considering deviations from rational expectations (RE). Ikeda (2012) combined a robust control algorithm and forecasting asymmetry and built them into the standard New Keynesian system. He found that forecasting asymmetry promotes aggressive monetary policy excessively and reduces social welfare.

Following Ikeda (2012), this paper examines the monetary policy delegation problem. Examining policy tradeoffs between stabilizing inflation and output, we discuss how the central banker's characterization as aggressive or conservative affects the New Keynesian economy.

The paper proceeds as follows. Section 2 briefly introduces the model. Section 3 reports numerical results of the economy's impulse responses and welfare gains among the models considered. Section 4 concludes the study.

2 Summary of the model

The New Keynesian Phillips curve and the dynamic IS curve based on asymmetric forecasting are

$$\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa y_t + e_t, \quad \text{and} \tag{1}$$

$$y_t = \hat{E}_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \hat{E}_t \pi_{t+1}) + u_t, \tag{2}$$

where $\hat{E}_t$ denotes the asymmetric forecast. The conventional specification for asymmetric forecasting in period $t+1$ takes the form of

$$\hat{E}_t \pi_{t+1} = E_t \pi_{t+1} + \varphi_\pi \sigma_{\pi,t} \tag{3}$$

$$\hat{E}_t y_{t+1} = E_t y_{t+1} + \varphi_y \sigma_{y,t} \tag{4}$$

The asymmetries φ_π and φ_y reflect the degree of reservations for the central bank in the forecast. If the asymmetry is large, the bank regards its forecast as less accurate and the forecast actually becomes to include a margin toward a rational expectation.

Inserting asymmetric forecasting (3-4) into the model, the standard New Keynesian system can be rewritten as:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \beta \varphi_\pi \sigma_{\pi,t} + e_t, \tag{5}$$

$$y_t = E_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1}) + \left(\frac{\varphi_\pi}{\sigma}\right)\sigma_{\pi,t} + \varphi_y \sigma_{y,t} + u_t, \tag{6}$$

Equations (5-6) denote the distorted Phillips curve and the distorted IS curve, respectively. Cost-push shock e_t and demand shock u_t follow the AR (1) process:

$$
\begin{aligned}
e_t &= \rho^\pi e_{t-1} + \varepsilon_t^\pi \quad \text{with} \quad 0 \leq \rho^\pi < 1, \quad \varepsilon_t^\pi \sim \text{i.i.d.} N(0,1) && (7)\\
u_t &= \rho^y u_{t-1} + \varepsilon_t^y \quad \text{with} \quad 0 \leq \rho^y < 1, \quad \varepsilon_t^y \sim \text{i.i.d.} N(0,1) && (8)
\end{aligned}
$$

The matrix for this system of backward-looking and forward-looking variables can be expressed as follows:[1]

$$
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \sigma^{-1} \\ 0 & 0 & 0 & \beta \end{pmatrix}
\begin{pmatrix} u_{t+1} \\ e_{t+1} \\ E_t y_{t+1} \\ E_t \pi_{t+1} \end{pmatrix}
=
\begin{pmatrix} \rho^y & 0 & 0 & 0 \\ 0 & \rho^\pi & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -\kappa & 1 \end{pmatrix}
\begin{pmatrix} u_t \\ e_t \\ y_t \\ \pi_t \end{pmatrix}
+
\begin{pmatrix} 0 \\ 0 \\ \sigma^{-1} \\ 0 \end{pmatrix} i_t
$$
$$
+ \begin{pmatrix} \varphi_y & \sigma^{-1}\varphi_\pi \\ 0 & \beta\varphi_\pi \\ 0 & 0 \\ 0 & 0 \end{pmatrix}
\begin{pmatrix} \sigma_{y,t+1} \\ \sigma_{\pi,t+1} \end{pmatrix}
+
\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}
\begin{pmatrix} \varepsilon_{t+1}^u \\ \varepsilon_{t+1}^e \end{pmatrix} \quad (9)
$$

Stacking these as follows,

$$
\boldsymbol{\Psi_{t+1}} = \boldsymbol{A\Psi_t} + \boldsymbol{B^* u_t^*} + \boldsymbol{C\Gamma_{t+1}}. \quad (10)
$$

where

$$
\boldsymbol{B^* u_t^*} =
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \sigma^{-1} \\ 0 & 0 & 0 & \beta \end{pmatrix}^{-1}
\begin{pmatrix} 0 & \varphi_y & \sigma^{-1}\varphi_\pi \\ 0 & 0 & \beta\varphi_\pi \\ \sigma^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\begin{pmatrix} i_t \\ \sigma_{y,t+1} \\ \sigma_{\pi,t+1} \end{pmatrix} \quad (11)
$$

Then, the monetary policy objective is specified as a quadratic form of

$$
E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda y_t^2), \quad (12)
$$

where λ is a tradeoff parameter between inflation and the output gap. In this sense, the central banker that confronts a small λ emphasizes stabilizing inflation (conservatism), whereas the banker confronting a large λ addresses output stabilization (activism).

In the context of optimization, the min-max decision rule adds a penalty for uncer-

[1] See Giordani and Söderlind (2004) for manipulations of the matrix.

tainty to the standard loss (12). Therefore, the problem is formulated as

$$\min\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda y_t^2 - \theta(\sigma_{\pi,t+1}^2 + \sigma_{y,t+1}^2) \right) \qquad (13)$$
$$s.t. \quad \boldsymbol{\Psi_{t+1}} = \boldsymbol{A\Psi_t} + \boldsymbol{B^* u_t^*} + \boldsymbol{C\Gamma_{t+1}}.$$

The explicit first-order conditions of the problem with a pen and pencil method are

$$y_t = -\frac{\kappa}{\lambda}\pi_t \quad \text{and} \qquad (14)$$
$$\sigma_{\pi,t} = \frac{\beta\varphi_\pi}{\theta}\pi_t. \qquad (15)$$

These conditions imply that the asymmetry in output forecasting has no effect, since conditions related to the IS equation do not affect the solution.

In pursuit of a numerical solution, formulating and solving Bellman equations yields the policy function

$$\boldsymbol{u_t^*} = -\boldsymbol{F_t}(\boldsymbol{\Phi})\boldsymbol{\xi_t}, \qquad (16)$$

where $\boldsymbol{\xi_t}$ is the bundle of shocks and $\boldsymbol{\Phi}$ denotes a loading matrix including asymmetric parameters. Also, the bundle of forward-looking variables $\boldsymbol{z_t}$ is determined as

$$\boldsymbol{z_t} = (\boldsymbol{D_t} - \boldsymbol{G_t}(\boldsymbol{\Phi})\boldsymbol{F_t}(\boldsymbol{\Phi}))\boldsymbol{\xi_t}. \qquad (17)$$

According to (16-17), the dynamics of monetary policy and the economy are directly affected by loading matrix $\boldsymbol{\Phi}$, which consists of forecasting asymmetries. With these solutions, we can derive the approximating model under which as a practical matter uncertainty does not appear (corresponding to the system of $\sigma_{\pi,t+1} = \sigma_{y,t+1} = 0$ with the worst-case solutions for inflation and output).

The next section presents numerical results centered on the tradeoff represented by differing values of λ. Conditioned on the uncertainty in asymmetric forecasting, how does the economic dynamics depend on the central banker's characteristics, and when is it desirable to delegate policymaking to a conservative or an activist central banker?

3 Numerical results

3.1 Impulse responses

We calibrate the intertemporal discount rate β equal to 0.99, the demand elasticity for interest rate σ at 1.00, and the slope of the Phillips curve κ at 0.30, as in Clarida, Gali, and Gertler (2000). Also, inertia for the cost-push shock is 0.35 from Giannoni and Woodford (2003). The (inverse of) the central bank's preference for robustness θ is set at 50 according to a detection error probability. See Hansen and Sargent (2008) for criteria.

Figure 1 shows the impulse responses under a conservative central banker ($\lambda = 0.05$) and Figure 2 under an activist banker ($\lambda = 0.20$). As per Figure 1, under a conservative central banker intent on stabilizing inflation, the impulse responses of the approximating model are close to those of the RE model. As a result, monetary policy adjusts the nominal interest rate moderately.[2] On the other hand, as in Figure 2, the activism of the banker causes vigorous movements of economy and its instrument toward a cost-push shock. The level of contraction in output is smaller than under the conservative case, but, the surge in inflation justifies the excess tightening of monetary policy. Also, note that these responses do not depend on the sign of asymmetry φ_π, as asymmetry is always squared in solutions within a linear-quadratic framework.

As Figure 3 shows, the monetary policy response varies according to the values of tradeoff parameter λ. Figure 3 confirms the evidence in Figures 1 and 2. Policy responses for a cost-push shock in the approximating model are slightly attenuated compared to responses in the RE model for lower values of λ. For larger values of λ, in contrast, the interest rate is gradually hiked higher than is called for in the RE model. When λ reaches 0.20, the interest rate is raised about 1.2% in the RE model, but 1.3% in the approximating model (with $\varphi_\pi = 2.0$). The monetary policy stance to a cost-push shock depends on the inflation asymmetry as well as the tradeoff between stabilizing inflation and the output gap.

[2] The attenuated policy response in the approximating model implies the Brainard principle in which cautiousness toward uncertainty weakens a policy's response to it.

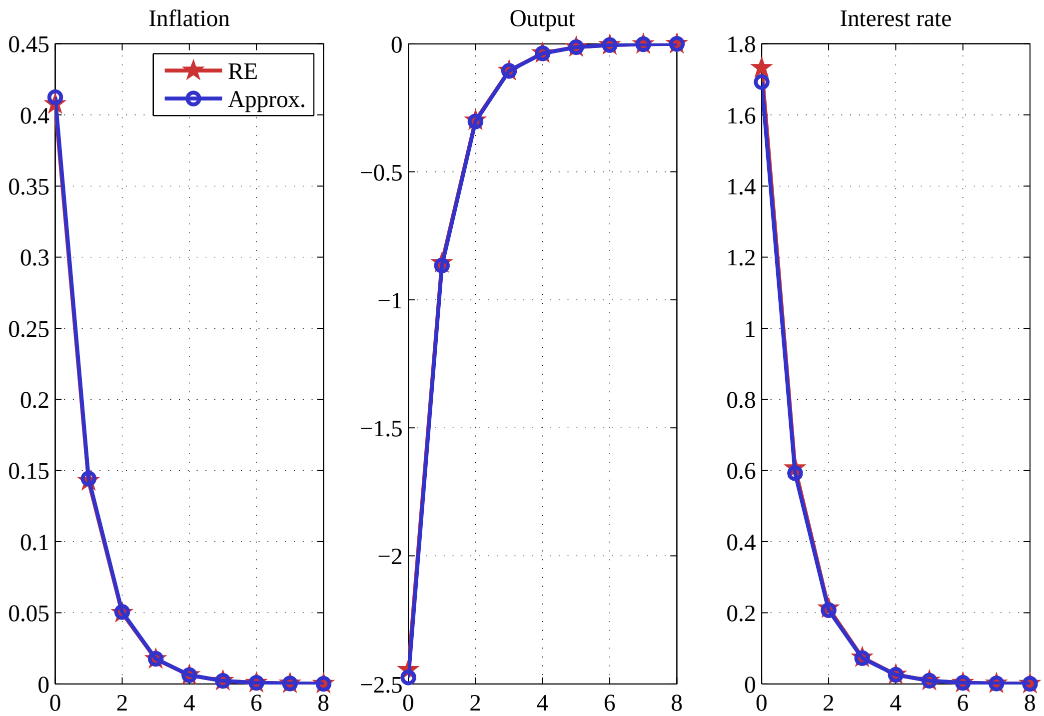

Figure 1: Impulse Responses with $\varphi_\pi = 2.0$ and $\lambda = 0.05$

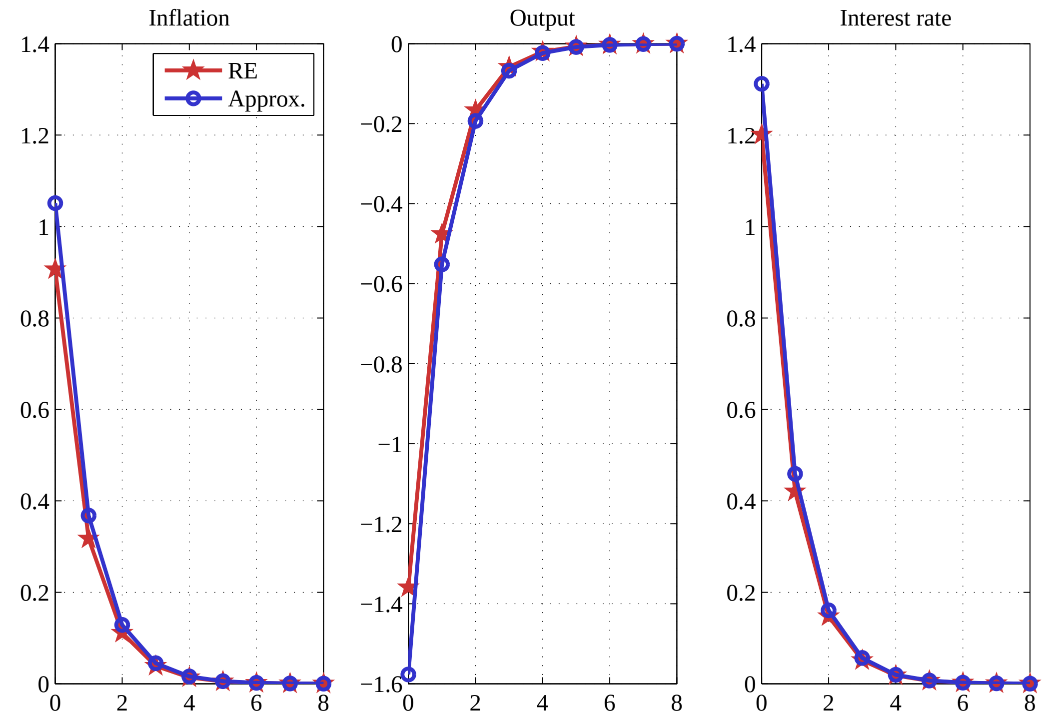

Figure 2: Impulse Responses with $\varphi_\pi = 2.0$ and $\lambda = 0.20$

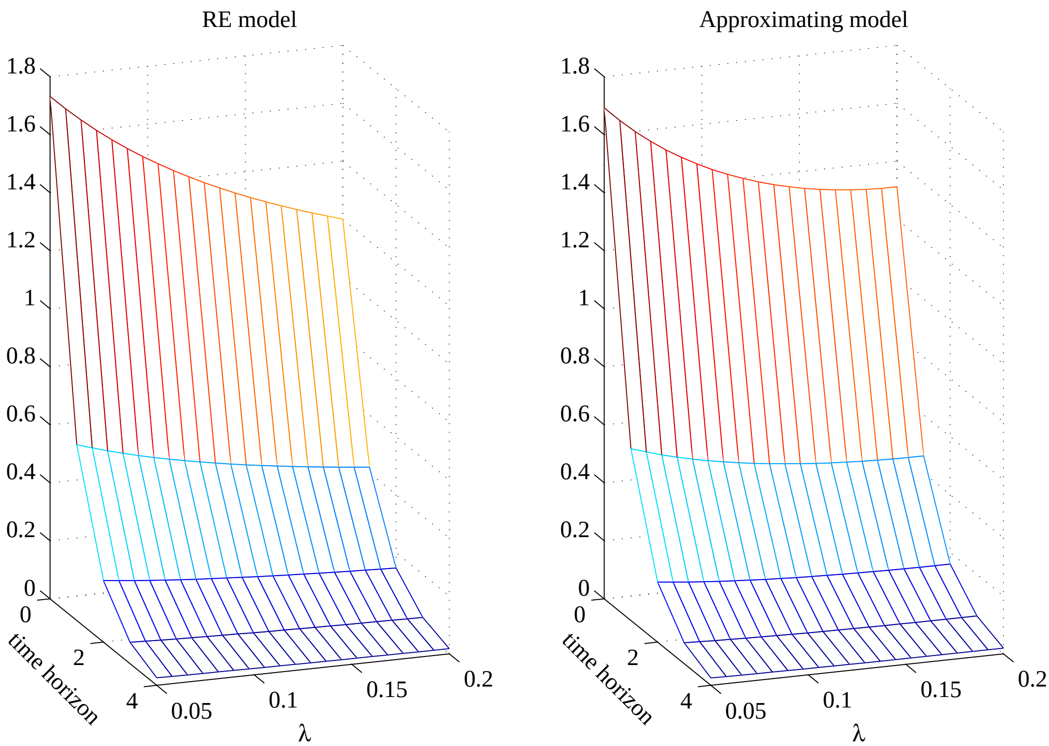

Figure 3: Interest Rate Responses with Various λ when $\varphi_\pi = 2.0$ (approximating model)

3.2 Welfare gains under delegation

Figure 4 depicts welfare gains between approximating and RE models for values of trade-off parameter. The left panel shows the welfare gains (losses) of the approximating model in the conservative case compared to the RE model. The middle panel compares the activist case to the RE case. The right panel compares the approximating model of conservatism to that of activism. The left and middle panels suggest that the central bank's preference for robustness and the existence of asymmetric preferences reduce social welfare compared to the RE model, but the degree of welfare loss is substantially less under a conservative banker. For instance, delegating policy to a conservative central banker can significantly suppress deviation from the RE model. Also, convergence to the RE model can be achieved when inflation asymmetry and preference for robustness approach zero. This tendency makes sense, for the RE model can be recovered with asymmetry and the (inverse of) robustness parameter being zero.

The right panel of Figure 4 supports the implications above. The conservative central banker enhances welfare, and this tendency is notable when inflation asymmetry and preference for robustness are increased. If the central bank has reservations about its economic forecast and a preference for robustness when its model displays forecasting uncertainty, welfare gains are substantially larger under a conservative than under an activist banker. This implication resembles Tillman (2009a), who also reports that

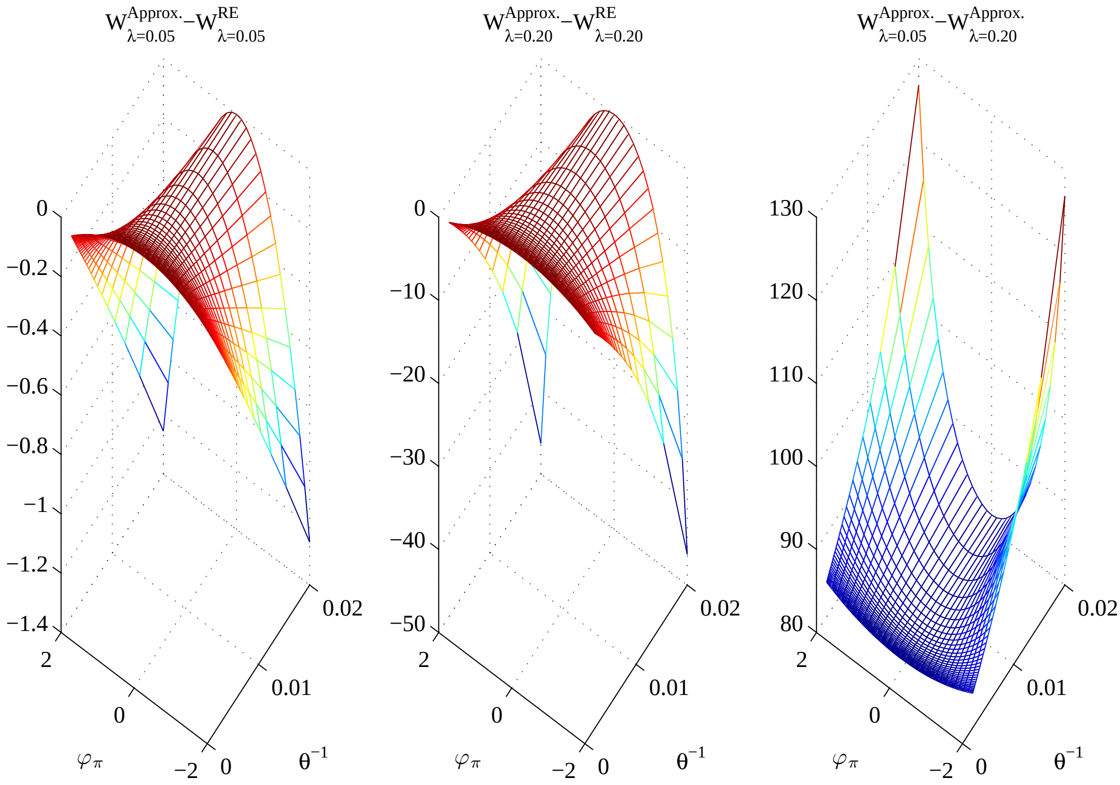

Figure 4: Comparison of Welfare Gains between Approximating and RE Models for Values of λ

Note: Social welfare is calculated as a minus of social loss. $W^{Approx.}_{\lambda=0.05}$ denotes welfare when $\lambda = 0.05$ in the approximating model and $W^{RE}_{\lambda=0.05}$ in the RE case. $W_{\lambda=0.20}$ corresponds to the case of $\lambda = 0.20$.

conservative central banking could curtail welfare losses under uncertain circumstances.

4 Conclusion

We have analyzed the problem of delegating monetary policy when the central bank has a preference for robust optimal policy under conditions of asymmetric forecasting. Numerical results suggest that social welfare toward a cost-push shock is high and close to the outcome achieved under rational expectations if monetary policy is delegated to the conservative central banker.

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Robustness for asymmetric forecasting in the presence of heterogeneity with a New Keynesian model

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August 17, 2012

Abstract

This paper investigates three types of heterogeneity stemming from asymmetric preferences (Type I), uncertainty about an economic model (Type II), and both asymmetric preferences and model uncertainty (Type III) in asymmetric forecasting by a central bank and private agents. Results suggest that asymmetric preferences amplify volatility of economic system, similar to that in homogeneous forecasts. However, if the central bank's and private agents' asymmetric preferences share the same directionality under Type III heterogeneity, social welfare under Type III is higher than that under Type I heterogeneity. In contrast, Type I heterogeneity is desirable in different signs of asymmetric preferences.

Keywords: robust control, asymmetric forecasting, heterogeneous expectations

JEL classification: E50, E52, E58

1 Introduction

Rational expectations have dominated macroeconomic analysis since 1970s, and monetary policy analysis is no exception. However, the central assumption of rational expectations that agents always possess complete information about underlying economic structures has been sharply criticized, and homogeneity in formulating expectations has not been supported by empirical studies. In response, there exist two major refinements to the expectation theory, the learning and the robust control. In the learning approach,

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economic agents learn to recognize economic laws of motion, and eventually, reach conclusions consistent with a rational expectations solution (Evans and Honkapohja, 2001).[1] On the other hand, the robust control assumes that the agents have willingness to adjust misspecifications between data-generated and approximating models (Hansen and Sargent, 2008). Agents adopt a max—min optimization principle to address uncertainty arising from model misspecifications. In a sense, robust controls incorporate agents' beliefs about the reliability of an economic model, an issue excluded from rational expectations theory.

Time is the major difference between the learning and robust control. Learning consumes time, whereas agents can acquire a decision rule based on robust controls quickly with limited data. In addition, the agents in learning pursue the rational expectations solution as a end point for a decision when learnability condition is satisfied, whereas the robust control approach regards it as one of many possible solutions within an entropy ball deduced by an algorithm.

In addition to these two refinements, asymmetric forecasting is also an important departure from the conventional rational expectations theory. Branch (2011), Capistrán and Timmermann (2009), and Surico (2007) empirically support the existence of asymmetric preferences in monetary policy. Agents with asymmetric preferences introduce asymmetries into expectations and breed asymmetric biases into expectations, either of which could be optimistic or pessimistic, depending on the particular asymmetric preference.

Understanding heterogeneity in expectations is important for injecting reality into macroeconomics and monetary policy analysis. The robust control approach is taking hold in modern macroeconomics, including monetary economics, because it imparts a degree of realism to decision making that escapes the conventional rational expectations theory. Therefore, this paper considers heterogeneous asymmetry in forecasting under a robust control algorithm to cast the analysis in a New Keynesian framework. In this analysis, the central bank and public agents proffer differing economic forecasts arising from their asymmetric preferences and uncertainties. Further, we consider that social planners wish to defend policy outcomes against uncertainties stemming from asymmetric forecasting. Heterogeneity arises from differences in forecasters' asymmetric preferences and their respective uncertainties about an economic model. We consider three types of heterogeneity: heterogeneity arising from asymmetric preferences (Type I), uncertainty about an economic model (Type II), and both asymmetric preferences and model uncer-

[1] Giannitsarou (2003), Honkapohja and Mitra (2006), and Branch and McGough (2009) analyzed heterogeneous expectations under learning and reported that system stability conditions could differ through heterogeneity.

tainty (Type III).

This paper proceeds as follows. Section 2 analyzes these three types of heterogeneity. Section 3 confirms theoretical results with numerical simulations to compare welfare outcomes. Section 4 concludes this paper.

2 Model settings

This section introduces three types of forecasting heterogeneity with asymmetry. We address them through a robust control algorithm and deduce their respective social welfare gains presented in the models. Analysis reveals that asymmetric preference is the most important factor because it disciplines differences in results by the types of heterogeneity.

2.1 Type I: heterogeneity from asymmetric preferences

First, we consider Type I heterogeneity in which asymmetric forecasting is induced solely by differences in the central bank's and private agents' asymmetric preferences.

The central bank's asymmetric forecast for any variable z_t can be expressed as

$$\hat{E}_t^{CB} z_{t+1} = E_t z_{t+1} + 2\varphi_z^{CB} \sigma_{z,t} \tag{1}$$

where φ_z^{CB} is its asymmetric preferences and $\sigma_{z,t}$ is uncertainty inferred by asymmetric forecasting.

Similarly, private agents' asymmetric forecasting is expressed as

$$\hat{E}_t^{P} z_{t+1} = E_t z_{t+1} + 2\varphi_z^{P} \sigma_{z,t}, \tag{2}$$

where φ_z^P denotes their asymmetric preferences. Then, social expectations can be expressed as an average of the bank's and private agents' forecasts as

$$\hat{E}_t z_{t+1} = \frac{1}{2}(\hat{E}_t^{CB} z_{t+1} + \hat{E}_t^{P} z_{t+1}) = E_t z_{t+1} + (\varphi_z^{CB} + \varphi_z^{P})\sigma_{z,t}. \tag{3}$$

Accordingly, the basic New Keynesian economy with asymmetric forecasting yields the following relations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \beta(\varphi_\pi^{CB} + \varphi_\pi^{P})\sigma_{\pi,t} + e_t, \tag{4}$$

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1}) \\ & + \left(\frac{\varphi_\pi^{CB} + \varphi_\pi^{P}}{\sigma}\right)\sigma_{\pi,t} + (\varphi_y^{CB} + \varphi_y^{P})\sigma_{y,t} + u_t, \end{aligned} \tag{5}$$

Equation (4) is the New Keynesian Phillips curve (NKPC) distorted by asymmetric forecasting. Equation (5) is the distorted dynamic IS (DIS) curve. The NKPC implies intertemporal optimization by monopolistic firms facing Calvo-type price rigidities. The DIS is derived from the private sector Euler equation.

Taking these distorted structures as given, social planners wish to fortify outcomes against uncertainty by establishing decision rules. To this end, we set the Lagrangian as

$$\begin{aligned}\mathcal{L}_t = &\left(\frac{1}{2}\right)(\pi_t^2 + \lambda y_t^2) - \left(\frac{\theta}{2}\right)(\sigma_{\pi,t}^2 + \sigma_{y,t}^2)\\ &-\mu_t^\pi\big(\pi_t - \beta E_t\pi_{t+1} - \kappa y_t - \beta(\varphi_\pi^{CB} + \varphi_\pi^P)\sigma_{\pi,t} - e_t\big)\\ &-\mu_t^y\Bigg(y_t - E_t y_{t+1} + \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1})\\ &\qquad - \left(\frac{\varphi_\pi^{CB} + \varphi_\pi^P}{\sigma}\right)\sigma_{\pi,t} - (\varphi_y^{CB} + \varphi_y^P)\sigma_{y,t} - u_t\Bigg).\end{aligned}$$

The optimization conditions of this problem are

$$y_t = -\frac{\kappa}{\lambda}\pi_t, \qquad \text{and} \tag{6}$$

$$\sigma_{\pi,t} = \frac{\beta(\varphi_\pi^{CB} + \varphi_\pi^P)}{\theta}\pi_t. \tag{7}$$

Substituting conditions (6) and (7) into the distorted systems and positing a cost-push shock that follows a first-order autoregressive process yields the worst-case solutions as

$$\pi_t^W = \frac{\hat{A}}{1 - \beta\rho\hat{A}}e_t, \tag{8}$$

$$y_t^W = -\frac{\kappa\hat{A}}{\lambda(1 - \beta\rho\hat{A})}e_t, \tag{9}$$

$$i_t^W = \hat{B}e_t + \sigma u_t, \tag{10}$$

where

$$\hat{A} \equiv \frac{\lambda\theta}{\lambda\theta + \kappa^2\theta - \lambda\beta^2(\varphi_\pi^{CB} + \varphi_\pi^P)^2}.$$

and

$$\hat{B} \equiv \frac{\sigma\kappa\theta(1-\rho)\hat{A} + \lambda(\theta\rho + \beta(\varphi_\pi^{CB} + \varphi_\pi^P)^2)\hat{A}}{\lambda\theta(1 - \beta\rho\hat{A})}.$$

Inserting worst-case solutions into non-distorted systems yield the following approximating model:

$$y_t^A = \frac{(\lambda - \sigma\kappa)\rho\hat{A} - (1-\beta\rho\hat{A})\lambda\hat{B}}{\sigma\lambda(1-\beta\rho\hat{A})} e_t. \tag{11}$$

Similarly, inflation is approximated with the pure New Keynesian Phillips curve as

$$\pi_t^A = \frac{\lambda - \kappa^2\hat{A}}{\lambda(1-\beta\rho\hat{A})} e_t. \tag{12}$$

With these solutions, we can restore homogeneity to asymmetric forecasting if $(\varphi_\pi^{CB} + \varphi_\pi^P)^2 = \varphi_\pi^2$ in $\hat{A}$ and $\hat{B}$. We can do so because the expression $(\varphi_\pi^{CB} + \varphi_\pi^P)^2$ is the sole difference between worst-case solutions under heterogeneous and homogeneous forecasting models.

2.2 Type II: heterogeneity from uncertainty

Next, we consider Type II heterogeneity that arises solely from uncertainty about the model. Assuming the central bank's asymmetric forecast is

$$\hat{E}_t^{CB} z_{t+1} = E_t z_{t+1} + 2\varphi_z \sigma_{z,t}^{CB}, \tag{13}$$

where $\sigma_{z,t}^{CB}$ is the uncertainty considered by the policy. The private agents' asymmetric forecast is

$$\hat{E}_t^P z_{t+1} = E_t z_{t+1} + 2\varphi_z \sigma_{z,t}^P, \tag{14}$$

where $\sigma_{z,t}^P$ denotes the uncertainty that they must resolve. Note that the asymmetric preference, φ_z, is identical for policymakers and private agents. When Type II uncertainty prevails, social expectations are formed as the average of Equations (13) and (14), as shown in the following equation:

$$\hat{E}_t z_{t+1} = \frac{1}{2}(\hat{E}_t^{CB} z_{t+1} + \hat{E}_t^P z_{t+1}) = E_t z_{t+1} + \varphi_z(\sigma_{z,t}^{CB} + \sigma_{z,t}^P). \tag{15}$$

Employing the expectation formulation of (15) yields the following economic structures:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \beta\varphi_\pi(\sigma_{\pi,t}^{CB} + \sigma_{\pi,t}^P) + e_t, \tag{16}$$

and

$$y_t = E_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) + \left(\frac{\varphi_\pi}{\sigma}\right)(\sigma^{CB}_{\pi,t} + \sigma^{P}_{\pi,t}) + (\sigma^{CB}_{y,t} + \sigma^{P}_{y,t}) + u_t, \quad (17)$$

Social planners who seek robustness set the Lagrangian as

$$\begin{aligned}\mathcal{L}_t = &\left(\frac{1}{2}\right)(\pi_t^2 + \lambda y_t^2) - \left(\frac{\theta}{2}\right)\big((\sigma^{CB}_{\pi,t})^2 + (\sigma^{P}_{\pi,t})^2 + (\sigma^{CB}_{y,t})^2 + (\sigma^{P}_{y,t})^2\big)\\ &-\mu_t^\pi\big(\pi_t - \beta E_t\pi_{t+1} - \kappa y_t - \beta\varphi_\pi(\sigma^{CB}_{\pi,t} + \sigma^{P}_{\pi,t}) - e_t\big)\\ &-\mu_t^y\Bigg(y_t - E_t y_{t+1} + \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1})\\ &\qquad - \left(\frac{\varphi_\pi}{\sigma}\right)(\sigma^{CB}_{\pi,t} + \sigma^{P}_{\pi,t}) - \varphi_y(\sigma^{CB}_{y,t} + \sigma^{P}_{y,t}) - u_t\Bigg).\end{aligned}$$

First-order conditions of the problem are

$$y_t = -\frac{\kappa}{\lambda}\pi_t, \quad (18)$$

$$\sigma^{CB}_{\pi,t} = \frac{\beta\varphi_\pi}{\theta}\pi_t, \quad (19)$$

and

$$\sigma^{P}_{\pi,t} = \frac{\beta\varphi_\pi}{\theta}\pi_t, \quad (20)$$

Conditions (19) and (20) suggest that policymaker's and private agents' uncertainties are equivalently determined. Substituting these conditions into the distorted NKPC and DIS yields

$$\pi_t^W = \frac{\bar{A}}{1 - \beta\rho\bar{A}}e_t, \quad (21)$$

$$y_t^W = -\frac{\kappa\bar{A}}{\lambda(1 - \beta\rho\bar{A})}e_t, \quad (22)$$

$$i_t^W = \bar{B}e_t + \sigma u_t. \quad (23)$$

where

$$\bar{A} \equiv \frac{\lambda\theta}{\lambda\theta + \kappa^2\theta - 2\lambda\beta^2\varphi_\pi^2},$$

and

$$\bar{B} \equiv \frac{\sigma\kappa\theta(1-\rho)\bar{A} + \lambda(\theta\rho + 2\beta\varphi_\pi^2)\bar{A}}{\lambda\theta(1-\beta\rho\bar{A})}.$$

Solutions of the approximating model are

$$y_t^A = \frac{(\lambda - \sigma\kappa)\rho\bar{A} - (1-\beta\rho\bar{A})\lambda\bar{B}}{\sigma\lambda(1-\beta\rho\bar{A})} e_t. \tag{24}$$

and

$$\pi_t^A = \frac{\lambda - \kappa^2\bar{A}}{\lambda(1-\beta\rho\bar{A})} e_t. \tag{25}$$

These solutions are exactly the same as those for the homogeneous model. Therefore, Type II heterogeneity falls from consideration.

2.3 Type III: heterogeneity from both asymmetric preferences and uncertainty

Now, we consider heterogeneity that arises from the central bank's and private agents' asymmetric preferences and model uncertainty. When Type III heterogeneity prevails, the expectation formulation of central bank is specified as

$$\hat{E}_t^{CB} z_{t+1} = E_t z_{t+1} + 2\varphi_z^{CB}\sigma_{z,t}^{CB}, \tag{26}$$

and that of private agents is

$$\hat{E}_t^{CB} z_{t+1} = E_t z_{t+1} + 2\varphi_z^{P}\sigma_{z,t}^{P}, \tag{27}$$

Therefore, the averaged social expectation is given by

$$\hat{E}_t z_{t+1} = E_t z_{t+1} + \varphi_z^{CB}\sigma_{z,t}^{CB} + \varphi_z^{P}\sigma_{z,t}^{P}. \tag{28}$$

From these, the distorted New Keynesian system becomes

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t + \beta\varphi_\pi^{CB}\sigma_{\pi,t}^{CB} + \beta\varphi_\pi^P\sigma_{\pi,t}^P + e_t, \tag{29}$$

$$\begin{aligned} y_t &= E_t y_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) \\ &\quad + \left(\frac{\varphi_\pi^{CB}}{\sigma}\right)\sigma_{\pi,t}^{CB} + \left(\frac{\varphi_\pi^P}{\sigma}\right)\sigma_{\pi,t}^P + \varphi_y^{CB}\sigma_{y,t}^{CB} + \varphi_y^P\sigma_{y,t}^P + u_t, \end{aligned} \tag{30}$$

For Type III heterogeneity, we set the Lagrangian as

$$\begin{aligned} \mathcal{L}_t &= \left(\frac{1}{2}\right)(\pi_t^2 + \lambda y_t^2) - \left(\frac{\theta}{2}\right)(\sigma_{\pi,t}^2 + \sigma_{y,t}^2) \\ &-\mu_t^\pi\left(\pi_t - \beta E_t\pi_{t+1} - \kappa y_t - \beta\varphi_\pi^{CB}\sigma_{\pi,t}^{CB} - \beta\varphi_\pi^P\sigma_{\pi,t}^P - e_t\right) \\ &-\mu_t^y\Bigg(y_t - E_t y_{t+1} + \left(\frac{1}{\sigma}\right)(i_t - E_t\pi_{t+1}) \\ &\qquad - \left(\frac{\varphi_\pi^{CB}}{\sigma}\right)\sigma_{\pi,t}^{CB} - \left(\frac{\varphi_\pi^P}{\sigma}\right)\sigma_{\pi,t}^P - \varphi_y^{CB}\sigma_{y,t}^{CB} - \varphi_y^P\sigma_{y,t}^P - u_t\Bigg). \end{aligned}$$

This optimization yields the following first-order conditions:

$$y_t = -\frac{\kappa}{\lambda}\pi_t, \tag{31}$$

$$\sigma_{\pi,t}^{CB} = \frac{\beta\varphi_\pi^{CB}}{\theta}\pi_t, \tag{32}$$

and

$$\sigma_{\pi,t}^P = \frac{\beta\varphi_\pi^P}{\theta}\pi_t, \tag{33}$$

Worst-case solutions can be derived from these conditions as:

$$\pi_t^W = \frac{\tilde{A}}{1 - \beta\rho\tilde{A}}e_t, \tag{34}$$

$$y_t^W = -\frac{\kappa\tilde{A}}{\lambda(1 - \beta\rho\tilde{A})}e_t, \tag{35}$$

and

$$i_t^W = \tilde{B}e_t + \sigma u_t, \tag{36}$$

where

$$\tilde{A} \equiv \frac{\lambda\theta}{\lambda\theta + \kappa^2\theta - \lambda\beta^2\big((\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2\big)},$$

and

$$\tilde{B} \equiv \frac{\sigma\kappa\theta(1-\rho)\tilde{A} + \lambda\Bigg(\theta\rho + \beta\big((\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2\big)\Bigg)\tilde{A}}{\lambda\theta(1-\beta\rho\tilde{A})}.$$

In addition, the approximating model's solutions are

$$y_t^A = \frac{(\lambda - \sigma\kappa)\rho\tilde{A} - (1-\beta\rho\tilde{A})\lambda\tilde{B}}{\sigma\lambda(1-\beta\rho\tilde{A})}e_t. \tag{37}$$

and

$$\pi_t^A = \frac{\lambda - \kappa^2\tilde{A}}{\lambda(1-\beta\rho\tilde{A})}e_t. \tag{38}$$

Altogether, solutions exhibit closely similar forms under all three types of heterogeneity. The only differences are in the denominator of A and the numerator of B as $(\varphi_\pi^{CB} + \varphi_\pi^P)^2$ in Type I and $(\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2$ in Type III. The corresponding parts are integrated as φ_π^2 in the homogeneous model.

2.4 Comparison of Types I and III

The difference between Type I and Type III heterogeneity is how their solutions incorporate asymmetries. To consider the difference, we define the following convolution of asymmetries in solutions for each type:

$$\Psi^I \equiv (\varphi_\pi^{CB} + \varphi_\pi^P)^2, \tag{39}$$

and

$$\Psi^{III} \equiv (\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2. \tag{40}$$

Ψ^I and Ψ^{III} generate the entire difference of consequences between Type I and Type III heterogeneity, since the social welfare is monotonously decreasing with them (Ikeda, 2012). Therefore, when Ψ^I equals Ψ^{III}, social welfare is identical under Types I and III.

This situation is conditioned as

$$\Psi^I - \Psi^{III} = 2\varphi_\pi^{CB}\varphi_\pi^P = 0. \tag{41}$$

To achieve condition (41), at least one party (the central bank or private agents) must be fully rational.

Social welfare under Type I heterogeneity exceeds Type III when

$$\Psi^I - \Psi^{III} = 2\varphi_\pi^{CB}\varphi_\pi^P < 0. \tag{42}$$

In this condition, the directionality of agents' asymmetric preferences differs from that of the central bank, suggesting that the belief for uncertainty is entirely adverse among the policymaker and private agents. The central bank's and private agents' beliefs offset each other, weakening the transmission of uncertainty into the economy through asymmetric forecasting (Equation (3)). As a result, the fluctuation stemming from model uncertainty is reduced.

In addition, when the condition

$$\Psi^I - \Psi^{III} = 2\varphi_\pi^{CB}\varphi_\pi^P > 0 \tag{43}$$

holds, Type III heterogeneity imparts greater social welfare because uniformly directional asymmetries amplify uncertainty under Type I heterogeneity.

In summary, asymmetric preferences interact when asymmetry is heterogeneous and uncertainty is homogeneous, whereas they are independent when asymmetry and uncertainty are simultaneously heterogeneous. Differences in effects on social welfare depend on whether the interaction of asymmetric preferences between agents is active.

3 Numerical comparison

This section numerically confirms the results obtained in Section 2. To this end, we calculate and compare welfare loss under Type I and III heterogeneity. Our strategy is simply to replace the solution of the standard-homogeneous model[2], φ_π^2, with Ψ^I and Ψ^{III} in calculating losses under Types I and III. For instance, under Type I heterogeneity, Ψ^I can be substituted in the homogeneous model as

$$\varphi_\pi^2 = (\varphi_\pi^{CB} + \varphi_\pi^P)^2,$$

[2]For the difference in solutions between Type I and III models and the standard model, see the construction of $\bar{A}$ and $\tilde{B}$ in the Type II model or Ikeda (2012).

since this is the only difference from the standard-homogeneous model. As a result, the simulation can be implemented following the relation

$$\varphi_\pi = \sqrt{(\varphi_\pi^{CB} + \varphi_\pi^P)^2}. \tag{44}$$

We replace the asymmetric parameter according to condition (44) in the standard model and calculate social loss.

In addition, the relation between Type III and the standard model is

$$\varphi_\pi^2 = (\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2.$$

Accordingly,

$$\varphi_\pi = \sqrt{(\varphi_\pi^{CB})^2 + (\varphi_\pi^P)^2}. \tag{45}$$

After calculating losses under both types of heterogeneity, we manipulate the welfare gain between them as

$$Welfaregain = -\big(var(\pi) + \lambda var(y) \mid \text{case I}\big) + \big(var(\pi) + \lambda var(y) \mid \text{case III}\big). \tag{46}$$

In condition (46), we define social welfare as negative social loss. For simulation, parameters are calibrated in the manner shown in Ikeda (2012). The discount rate β is 0.99, the elasticity of interest rate σ is 1.00, the slope of the NKPC κ is 0.30, the objective trade-off parameter λ is 0.25, and the inertia of the cost-push shock is 0.35. In addition, the (inverse of) willingness for robustness θ is set at 50 according to the detection error probabilities (Hansen and Sargent, 2008, chapter 9).

Figure 1 displays the simulated welfare gains. Its numerical results also support those obtained in Section 2. In the region where asymmetries bear different signs, social welfare under Type I exceeds that under Type III (the gain becomes positive). In the region where their signs are identical, welfare is higher under Type III as the interaction term of Type I asymmetries become positive. This vigorously amplifies economic fluctuations, and therefore, gains in the region are negative. These results sustain the analysis in Section 2.

4 Conclusion

This paper analyzed the heterogeneity of asymmetric forecasting with a robust control algorithm. We considered heterogeneity stemming from asymmetric preferences (Type I), uncertainty about the economic model (Type II), and both asymmetric preferences and

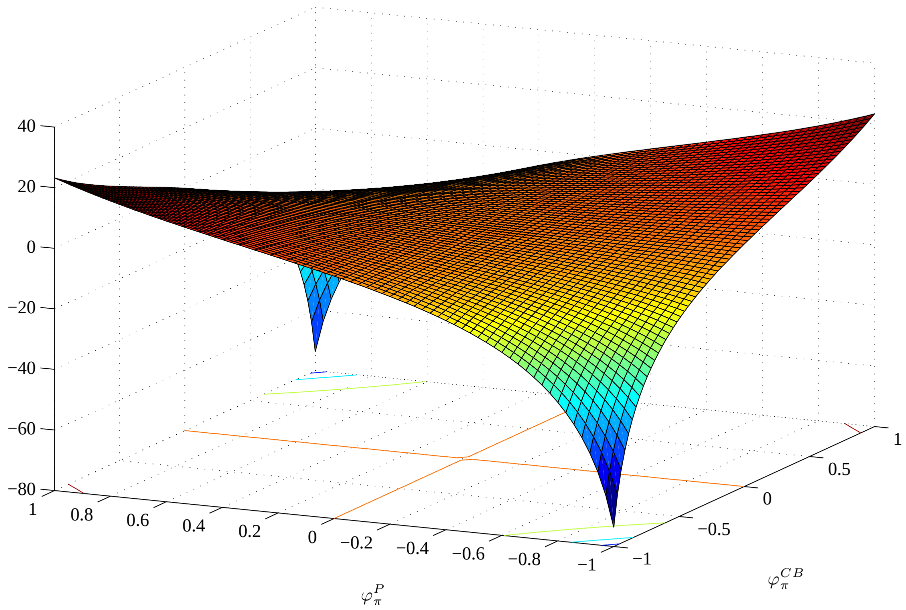

Figure 1: Welfare gains between Type I and Type III heterogeneity

model uncertainty (Type III). Significant differences from the homogeneous model appear only in Types I and III. Our results suggest that social welfare under Type I heterogeneity exceeds that under Type III when the direction of heterogeneous asymmetric preferences differs between the central bank and private agents. That is, the interaction of them turns negative in Type I, reducing the size of a transmission pass for uncertainty to the model. This in turn reduces the economic variance under Type I heterogeneity, whereas the outcome under Type III heterogeneity escape the interaction effect of asymmetric preferences.

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