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Abstract

This paper investigates the optimal degree of privatization for a public firm in a homogeneous mixed oligopoly. I show that full privatization is optimal when a public firm has a severe productivity disadvantage or competes with many private firms. The optimal degree of partial privatization is increasing in the degree of productivity disadvantage and the number of private firms. I further show that partial privatization can be optimal for a public firm even when full privatization would completely remove any productivity disadvantages.

Keywords: quantity-setting competition, partial privatization, mixed oligopoly
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1 Introduction

The privatization of public firms is a significant issue in most countries. The standard framework for understanding this issue is a mixed oligopoly model since public firms often compete with private firms in these countries.[1] In an early study on mixed oligopoly, De Fraja and Delbono (1989) show that when public firms compete with many private firms, they should maximize profits rather than welfare in order to improve overall social welfare. This result implies that full privatization is socially preferable when the market is sufficiently competitive. However, they do not consider the possibility of partial privatization, in which public firms respect both social welfare and their own profits. Matsumura (1998) explicitly considers the possibility of partial privatization and shows that partial privatization is optimal in a mixed duopoly when a public firm is as productive as its private competitor. The literature now contains many analyses on the optimal degree of privatization, following Matsumura (1998).

The most relevant studies to this article, in addition to Matsumura (1998), are Matsumura and Kanda (2005) and Fujiwara (2007).[2] Matsumura and Kanda (2005) generalize Matsumura (1998) to a mixed oligopoly setting with more than two private firms and show that at least partial privatization is socially preferable in the short run. Fujiwara (2007) investigates a differentiated mixed oligopoly and shows that partial privatization is socially preferable in the short run.[3] However, the literature lacks general analyses on the relationships between firms' productivity, market concentration, and the optimal degree of privatization. I investigate these relationships in a homogeneous mixed oligopoly and show that full privatization is optimal when public firms have severe productivity disadvantages or compete with many private firms and that the optimal degree of partial privatization is increasing in the degree of productivity disadvantage and the number of private firms.

The results in this study, similar to Matsumura (1998), imply that a degree of privatization is socially preferable even when privatization does not improve the productivity of public firms but that full privatization is not socially preferable if public firms are as productive as private firms. In other words, full privatization is socially preferable only when a public firm has some productivity disadvantages. The relationship between productivity and privatization is very important since most people suspect that X-inefficiency arises in public firms and expect that privatizing such firms removes the X-inefficiency and consequently improves social welfare.[4] I explicitly consider the productivity improvements derived from privatization, and I show that even if full privatization makes public firms as productive as private firms, full privatization is not always socially preferable. Partial privatization is more preferable when the marginal productivity improvements from additional privatization are negligible when the firm is close

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[1]The earliest papers on mixed markets include Merrill and Schneider (1966), Harris and Wiens (1980), and Beato and Mas-Colell (1984). See De Fraja and Delbono (1990) and Nett (1993) for a survey of this literature.

[2]Other studies concerning the optimal degree of privatization include Ishibashi and Kaneko (2008), Ohnishi (2010), and Wang and Chen (2011).

[3]The main goal in Matsumura and Kanda (2005) and Fujiwara (2007) is to investigate the optimal degree of privatization in the long run, when private firms can enter and exit the mixed market, which is not the aim of this article.

[4]Some existing studies, such as De Fraja (1993), Schmidt (1996), Hart, Shleifer, and Vishny (1997), Corneo and Rob (2003), Matsumura and Matsushima (2004), Ishibashi and Matsumura (2006), and Miyazawa (2008), investigate productivity or cost differences endogenously, but these studies do not consider the relationship between productivity improvements and the degree of privatization.

to full privatization, when the number of private firms is sufficiently small, and when the productivity disadvantage itself is minor.

Finally, I present a uniform and intuitive explanation for these results. Privatization has three effects on the equilibrium social welfare in mixed oligopoly. Because privatization reduces the concern for social welfare and therefore the output of public firms, it can increase the deadweight loss in the oligopolistic market. At the same time, however, privatization can mitigate inefficient substitutions from private production to public production, and because it can mitigate the X-inefficiency of public firms, privatization can also improve their productivity. A higher degree of privatization is socially preferable when the latter two effects are significant and the former is not, which is the case when there are many private competitors.

The remainder of this article is organized as follows. In Section 2, I formulate the model. Next, in Section 3, I investigate the optimal degree of privatization. Finally, in Section 4, I present concluding remarks.

2 Model

Consider the market for a homogeneous commodity. The inverse demand function is given by $p(q)$, where $p \geq 0$ is the price of the commodity and $q \geq 0$ is its total quantity. $n + 1$ firms compete in this market. Firm 0 is owned by either the state, private investors, or a combination of both, whereas firms 1 to n are owned only by private investors. Let $\alpha \in [0, 1]$ be the state's share in firm 0. α also represents the degree of privatization. I refer to firm 0 as a public firm when $\alpha > 0$ and as a fully privatized firm when $\alpha = 0$ and to firm j, $j \in \{1, \ldots, n\}$, as a private firm.

Given the ownership structures of these firms, each firm chooses its output quantity $x_i \geq 0$ simultaneously. Firm i's cost function is given by $c_i(x_i)$, and its profit is written as $\Pi_i(\mathbf{x}) = p(X) - c_i(x_i)$, where $\mathbf{x}$ is an output quantity vector and $X = \sum_{i=0}^{n} x_i$, $i \in \{0, \ldots, n\}$. For now, suppose that the ownership structure of a firm does not affect its cost.[5]

The payoffs of firm 0 and firm j, U_0 and U_j, are given by

$$U_0 = \alpha W(\mathbf{x}) + (1 - \alpha)\Pi_0(\mathbf{x}), \qquad U_j = \Pi_j(\mathbf{x}),$$

where $W(\mathbf{x})$ is the social welfare. That is,

$$W(\mathbf{x}) = \int_0^X p(q)dq - pX + \sum_{i=0}^{n} \Pi_i(\mathbf{x}) = \int_0^X p(q)dq - \sum_{i=0}^{n} c_i(x_i).$$

Firm 0 maximizes the weighted sum of social welfare and its own profit in proportion to the state's share of ownership.[6]

Assumption 1 $p(q)$ *is twice differentiable and satisfies* $p'(q) < 0$ *for any* (p, q) *such that* $p > 0$ *and* $q \geq 0$.

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[5]At the end of the next section, I investigate the case in which the state's share of the firm. α, affects the public firm's cost.

[6]The assumption of proportionality is just for simplicity. See Matsumura (1998) and subsequent papers for more details.

Assumption 2 c_i *is twice differentiable and is strictly increasing in* x_i *for any* $x_i \geq 0$.

The first-order conditions for the firms' maximization problems are

$$(1 - \alpha)p'x_0 + p - c_0' = 0 \tag{1}$$

$$p'x_j + p - c_j' = 0. \tag{2}$$

Assumption 3 *The relevant second-order conditions for* (1) *and* (2) *are satisfied.*

Let $R_0(X_{-0}; \alpha)$ and $R_j(X_{-j})$ be the reaction functions for firm 0 and firm j, respectively, so that,

$$R_0(X_{-0}; \alpha) = \arg \max_{x_0 \geq 0} U_0(x_i, X_{-i}; \alpha)$$

and

$$R_j(X_{-j}) = \arg \max_{x_j \geq 0} U_j(x_j, X_{-j}),$$

where $X_{-i} = X - x_i$.

Let x_i^E be the equilibrium quantity of output for firm i. Then, the equilibrium social welfare is written as

$$W^E \equiv \int_0^{X^E} p(q)dq - \sum_{i=0}^n c_i(x_0^E), \tag{3}$$

where $X^E = \sum_{i=0}^n x_i^E$.

Additionally, assume the following.

Assumption 4 *For any* α, (i) $c_0''(x_0^E) \geq (1 - \alpha)p'(X^E)$ *when* $x_0^E > 0$, *and* (ii) $c_j''(x_j^E) > p'(X^E)$ *and* $p''(X^E)x_j^E + p'(X^E) < 0$ *for any* j *such that* $x_j^E > 0$.

This assumption guarantees that $-1 \leq R_0'(X_{-0}^E; \alpha)$ and $-1 < R_j'(X_{-j}^E) < 0$, where $R_0'(X_{-0}; \alpha) = \frac{\partial R_0(X_{-0}; \alpha)}{\partial X_{-0}}$. Assumptions 3 and 4 are satisfied for the standard set of assumptions that $p''(q) \leq 0$ for any (p, q) such that $p > 0$ and $q \geq 0$ and that $c_i''(x_i) \geq 0$ for any x_i.

3 Results

I investigate the optimal level of α and thus the optimal degree of privatization in this section. Let $A^* = \arg \max_{\alpha \in [0,1]} W^E$ be the set of optima. Further, let α^* be the optimum if A^* is a singleton. First, consider the optimality of privatization. Suppose that $x_i^E > 0$ for any i. Then, the effect of privatization on social welfare is captured by differentiating (3) as follows.

$$\frac{dW^E}{d\alpha} = \frac{dX^E}{d\alpha}p(X^E) - \sum_{i=0}^n \left(\frac{dx_i^E}{d\alpha} c_i'(x_i^E) \right) \tag{4}$$

$$= \sum_{i=0}^n \left\{ \frac{dx_i^E}{d\alpha} \left(p - c_i'(x_i^E) \right) \right\} \tag{5}$$

$$= -p'(X^E) \left[(1 - \alpha)\frac{dx_0^E}{d\alpha}x_0^E + \sum_{j=1}^n \left\{ \frac{dx_j^E}{d\alpha}x_j^E \right\} \right], \tag{6}$$

where the derivation from (5) to (6) follows from the first-order conditions, (1) and (2). The following results about the effect on the equilibrium quantities hold.

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Lemma 1 *Suppose that $x_i^E > 0$ for any i. Then, $\frac{dx_0^E}{d\alpha} > 0$, $\frac{dx_j^E}{d\alpha} > 0$ for any j, and $\frac{dX^E}{d\alpha} > 0$.*

The explanation of these results is similar to those in Matsumura (1998) and Matsumura and Kanda (2005). An increase in α increases the incentive for firm 0 to increase output to improve social welfare. Because of the private firms' strategic substitutability, that is, $R_j{}' < 0$, it follows that the increase in α increases x_0^E and decreases x_j^E. In addition, because of the private firms' lower substitutability, that is, $R_j{}' > -1$, the above effects result in an increase in X^E.

When $\alpha = 1$, following from (6), $\frac{dW^E}{d\alpha}$ is rewritten as

$$\frac{dW^E}{d\alpha} = -p'(X^E) \sum_{j=1}^{n} \left\{ \frac{dx_j^E}{d\alpha} x_j^E \right\}. \tag{7}$$

Let $x_i^E(0)$ and $x_i^E(1)$ be the equilibrium quantities when $\alpha = 0$ and when $\alpha = 1$, respectively. Then, the following result holds.

Proposition 1 (privatization of state-owned firms) *Suppose $x_0^E(0) > 0$ and $x_j^E(1) > 0$ for some j. Then, $1 \notin A^*$ holds irrelevant to $\{c_i\}_{i=0}^{n}$ and n.*

This proposition states that public firms should be at least partially privatized if they compete with some private firms in a homogeneous commodity market. This result is a generalization of the results in Matsumura (1998) and Matsumura and Kanda (2005). Matsumura (1998), which is a seminal study in this literature, shows that public firms should be at least partially privatized when the market is a duopoly, that is, when there is one private firm. Matsumura and Kanda (2005) find a similar result in the oligopolistic setting where a public firm competes with more than two private firms with an identical cost function. In this proposition, I show that a similar result holds even when private firms are heterogeneous.

The intuition is as follows.[7] When $\alpha = 1$, the public firm is a welfare maximizer and consequently sets its quantity so that marginal cost equals price, following from (1). This behavior of the public firm is clearly socially preferable if there is no strategic interaction between public and private firms, but this is no longer the case if strategic interaction matters. The private firms, which maximize their profits, set their quantities so that their marginal costs are equal to their marginal revenues, which are less than the price, following from (2). It follows that a slight decrease in x_0 and a subsequent increase in x_j results in an efficient product substitution and consequently improves the social welfare. When the public and private firms decide their quantities simultaneously, the benevolent public firm overproduces since it fails to take the strategic effect of its choice on the private firms' decisions into account. The result shows that at least partially privatizing the public firm is socially preferable because the public firm behaves less aggressively, as follows from Lemma 1.[8]

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[7]See also Matsumura (1998) and Matsumura and Kanda (2005) for an intuitive explanation of this result.

[8]The fact that the public firm decides to overproduce in a simultaneous quantity-setting competition in a mixed market is well known in this literature. For example, Beato and Mas-Colell (1984) show that the equilibrium quantity in a sequential quantity-setting competition where the public firm is the Stackelberg leader is lower than that in a simultaneous quantity-setting competition and that the equilibrium social welfare of the former competition is higher than that of the latter one in a mixed duopoly.

Next, consider the optimal degree of privatization. When $\alpha = 0$, following from (6), $\frac{dW^E}{d\alpha}$ is rewritten as

$$\frac{dW^E}{d\alpha} = -p'(X^E) \sum_{i=0}^{n} \left\{ \frac{dx_i^E}{d\alpha} x_i^E \right\}.$$ (8)

Then, the following result holds.

Proposition 2 (partial privatization of efficient public firms) *Suppose $x_0^E(1) > 0$. Then, $0 \notin A^*$ holds irrelevant to $\{c_i\}_{i=0}^{n}$ and n if $x_0^E(0) \geq \max_{j \in \{1,...,n\}} x_j^E(0)$.*

This proposition states that full privatization is not socially preferable if the public firm can have the largest market share when it is fully privatized. Since the market share is determined by relative productivity, as is implied in (1) and (2), the proposition implies that full privatization is not socially preferable if the public firm is at least as productive as the most productive private firm. This result is a generalization of the result in Matsumura (1998), who shows that full privatization is not socially preferable when the market is a duopoly and the public firm is as productive as the private firm, or, equivalently, when a fully privatized public firm produces as much output as the private firm does in equilibrium. In this proposition, I show a similar result when the market is an oligopoly and the public firm produces as much output as the largest private firm does in equilibrium. This proposition clearly implies that a similar result holds in the case of identical private firms, that is, $0 \notin A^*$ if $x_0^E(1) > 0$ and $c_0'(x) = c_j'(x)$ for any $x \geq 0$ for any j.

The intuition for this result is as follows. When Assumption 4 holds, the private firms' reactions to the other firms' quantity setting are less substitutive, that is, $R_j' > -1$, and consequently the equilibrium aggregate quantity is increasing in the equilibrium quantity of the public firm.[9] Moreover, the equivalence of the equilibrium quantities of the fully privatized firm and a private firm implies that these firms produce the commodity at an equivalent margin, $p - c_i'(x_i)$. Therefore, a slight increase in x_0 from $x_0^E(0)$ increases both the consumer surplus and the producer surplus.

Now, I investigate the relationships among the cost functions, the number of private firms, and the optimal degree of privatization. In order to present meaningful results, I employ the following specification.[10]

Specification 1 *The inverse demand function is $p(q) = a - bq$, and the cost functions are $c_0(x) = \frac{1}{2}(k + \Delta)x^2$ and $c_j(x) = \frac{1}{2}kx$ for all j, where $a > 0$, $b > 0$, $k > 0$, and $\Delta \geq 0$.*

This specification that the inverse demand function is linear and the cost functions are quadratic is fairly standard in this literature. In addition, for later discussion, I employ a specification similar to that of De Fraja and Delbono (1989).[11] $\Delta \geq 0$ represents the public firm's degree of

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[9]These results are explicitly shown in the proof of Lemma 1.

[10]The properties of c_i and n change the equilibrium quantities depending on the relationships between p', p'' (if differentiable), c_i', and c_i'' (if differentiable) in general. Investigation in a general setting is very complicated and does not produce any fruitful results.

[11]As noted by De Fraja and Delbono (1989), specifying a quadratic cost function is useful since it precludes the possibility that no private firm produces any output. I can present a similar result below in the text under another specification with constant marginal costs.

productivity disadvantage. As is shown in Proposition 2, if the public firm had a productivity advantage, that is, $\Delta < 0$, then $0 \notin A^*$ would hold regardless of the parameters.

Then, the following results hold.

Proposition 3 (productivity, number of private firms, and privatization) *Given Specification 1, (i) A^* is a singleton, (ii) there exists a finite $\bar{\Delta} > 0$ (a finite $\bar{n} \geq 1$) such that $\alpha^* = \bar{\alpha} \in (0, 1)$ if $\Delta < \bar{\Delta}$ ($n < \bar{n}$) and $\alpha^* = 0$ if $\Delta \geq \bar{\Delta}$ ($n \geq \bar{n}$), (iii) $\bar{\alpha}$ is decreasing in Δ and n, and (iv) $\bar{\Delta}$ ($\bar{n}$) is decreasing in n (Δ).*

This proposition states that a higher degree of privatization is socially preferable for public firms with severe productivity disadvantages and for those with many private competitors. This proposition fills a gap in the current literature. De Fraja and Delbono (1989) show that a public firm should maximize profits, in other words, should be fully privatized when the number of private firms is sufficiently large, which is similar to this proposition. However, they do not consider the possibility of partial privatization and productivity differences between public and private firms. This proposition fills the gap between full nationalization and full privatization in the analyses of privatization in a mixed oligopoly. In fact, when productivity is symmetric among public and private firms, as in De Fraja and Delbono (1989), full privatization is not socially optimal if partial privatization is possible.

Matsumura (1998) shows that full privatization is socially preferable when the productivity disadvantage is sufficiently high.[12] However, he does not state the tendency of the optimal degree of privatization, nor does he provide any implications about the relationship between the number of private firms and optimal degree of privatization. Fujiwara (2007) shows that the optimal degree of privatization is increasing in the number of private firms in a differentiated mixed oligopoly. However, since he does not consider productivity differences, he does not provide any implications about the relationship between the productivity disadvantage and the optimal degree of privatization.

In addition to filling the theoretical gaps in the literature explained above, I provide a uniform and intuitive explanation for these results is as follows. Privatization, or a decrease in α, has two opposite effects on social welfare due to the drop in the public firm's output. First, it mitigates inefficient production substitutions from private firms to the public firm, but second, it aggravates the deadweight loss in the oligopolistic market. On the one hand, when the productivity disadvantage is severe, the first effect is significant and a high degree of privatization is socially preferable. On the other hand, when the number of private firms is large, the second effect is not significant and again, a high degree of privatization is socially preferable.

The results in Proposition 3 imply the following results.

Corollary 1 *When $\Delta = 0$, $\alpha^* \in (0, 1)$ and $\lim_{n \to \infty} \alpha^* = 0$ for any level of the parameters.*

Matsumura (1998) shows the optimality of partial privatization when the public firm is as productive as the private firms. This result is the simplest generalization of his result, and it also implies that the finding of De Fraja and Delbono (1989) which implies possible optimality of full privatization of public firms is due to the disregard of partial privatization.

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[12]Matsumura (1998) uses cost functions with constant marginal costs in his explanation.

I have not considered the possibility that the privatization could improve the public firm's productivity thus far. However, most people suspect that X-inefficiency arises in public firms and that privatizing the firms removes the X-inefficiency in the public firm and consequently improves social welfare. To take the possibility that privatization improves productivity into account, I employ the following specification instead of Specification 1.

Specification 2 *The inverse demand function is $p(q) = a - bq$ and the cost functions are $c_0(x) = \frac{1}{2}\{k + \mu\alpha + (\Delta - \mu)\alpha^2\}x^2$ and $c_j(x) = \frac{1}{2}kx$ for all j, where $a > 0$, $b > 0$, $k > 0$, $\Delta > 0$ and $\mu \in [0, \Delta]$.*

The terms in the brace in $c_0(x)$ are the degree of productivity of the public firm, which simplifies to $k + \Delta$ when $\alpha = 1$ and to k when $\alpha = 0$. This specification represents the assumption that full privatization completely removes the X-inefficiency of the public firm denoted by Δ. It also represents another assumption that the level of productivity improvement effect, which is represented by $-\{\mu + 2(\Delta - \mu)\alpha\}$, is decreasing (if $\mu < \Delta$) or constant (if $\mu = \Delta$) in the degree of privatization, or in other words, is increasing in α. μ is the level of the productivity improvement effect when $\alpha = 0$. Since I am interested in the optimality of full privatization, the level of μ is significant, as is shown below.

Then, I present the following result.

Proposition 4 (productivity improvement and privatization) *Given Specification 2, full privatization is not socially optimal when $b > \frac{(n+4)\mu + \sqrt{k\mu + (n+4)^2\mu^2}}{4}$.*

This proposition states that full privatization is not socially preferable even when privatization improves the productivity of public firms if the number of private competitors n is sufficiently small and the productivity improvement effect almost disappears as the firm approaches full privatization, or in other words, if μ is sufficiently small. Furthermore, full privatization is never socially preferable when the productivity improvement effect completely disappears, that is, $\mu = 0$. Finally, it implies that full privatization is not socially preferable when the level of X-inefficiency Δ is small if the level of the productivity improvement effect is constant, that is, $\mu = \Delta$.

The intuition is as follows. Under Specification 2, privatization has a third effect on social welfare in addition to the first two in the explanation of Proposition 3, because it improves productivity. Full privatization is not socially preferable when μ is sufficiently small and consequently this third effect almost disappears, as is already stated, when n is sufficiently small and consequently the second effect of aggravating deadweight loss is large, and when $\mu = \Delta$ and Δ are sufficiently small and consequently the first effect of mitigating inefficient product substitution is not significant.

4 Concluding Remarks

I investigate the optimal degree of privatization and its relationship with productivity and market concentration in a homogeneous mixed oligopoly to fill gaps in the literature. Then, I show that full privatization is optimal when a public firm has a severe productivity disadvantage or competes with many private firms and that the optimal degree of partial privatization is increasing in the degree of productivity disadvantage and the number of private firms.

The relationship between productivity and privatization is very important because most people suspect that X-inefficiency arises in public firms and expect that the privatization of the firms removes the X-inefficiency and consequently improves social welfare. I explicitly consider the productivity improvements derived from privatization and show that even if full privatization completely removes the X-inefficiency in public firms and consequently makes fully privatized firms as productive as private firms, full privatization is not socially preferable when the productivity improvement effect almost disappears as the firm approaches full privatization, when the number of private firms is sufficiently small, and when the productivity disadvantage itself is minor.

I present a uniform and intuitive explanation for these results. Privatization has three effects on the equilibrium social welfare in a mixed oligopoly. The output of public firms falls because they are less concerned with social welfare, which aggravates the deadweight loss in the oligopolistic market. At the same time, however, this drop in output mitigates inefficient product substitutions from private firms to public firms. Furthermore, privatization can mitigate X-inefficiency and consequently improve the productivity of public firms. A higher degree of privatization is socially preferable when the latter two effects are significant and the former effect is not significant, which is the case when there are many private competitors.

Needless to say, the results in this article are restrictive because I only investigate the equilibrium outcome in a homogeneous market in the short run. I also neglect the existence of foreign firms in the same market. Investigations in more general settings are objectives for future research.

Appendix A

A.1 Proof of Lemma 1

Following from (1),

$$\frac{\partial R_0}{\partial \alpha} = \frac{p'x_0}{(1-\alpha)p''x_0 + (2-\alpha)p' - c_0''} > 0, \tag{A1}$$

where $x_0 = R_0$, for $R_0 > 0$. The sign of this inequality follows from Assumptions 1 and 3. Then, the two below equations follow from (1) and (2).

$$\frac{dx_0^E}{d\alpha} = \frac{\partial R_0}{\partial \alpha} + R_0' \left(\frac{dX^E}{d\alpha} - \frac{dx_0^E}{d\alpha} \right)$$

$$\frac{dx_j^E}{d\alpha} = R_j' \left(\frac{dX^E}{d\alpha} - \frac{dx_j^E}{d\alpha} \right)$$

Solving these equations gives

$$\frac{dX^E}{d\alpha} = \frac{1}{1+R_0'}\frac{\partial R_0}{\partial \alpha} \bigg/ \left(1 - \sum_{i=0}^{n}\frac{R_i'}{1+R_i'}\right) > 0$$

$$\frac{dx_0}{d\alpha} = \frac{1}{1+R_0'}\frac{\partial R_0}{\partial \alpha}\left(1 - \sum_{j=1}^{n}\frac{R_j'}{1+R_j'}\right) \bigg/ \left(1 - \sum_{i=0}^{n}\frac{R_i'}{1+R_i'}\right) > 0$$

$$\frac{dx_j}{d\alpha} = \frac{1}{1+R_0'}\frac{\partial R_0}{\partial \alpha}\frac{R_j'}{1+R_j'} \bigg/ \left(1 - \sum_{i=0}^{n}\frac{R_i'}{1+R_i'}\right) < 0,$$

in which the signs of inequalities follow from Assumption 4 and (A1).

Proof of Proposition 1

When $x_0^E(0) > 0$, $x_0^E(1) > 0$. Without loss of generality, I can neglect private firms such $x_j^E(1) = 0$, that is, I can focus on the competition among the public firm and private firms such that $x_j^E(1) > 0$. Then, from Lemma 1 and (7), $\frac{dW^E}{d\alpha} < 0$.

Proof of Proposition 2

Suppose $\max_{j\in\{1,\dots,n\}} x_j^E(0) > 0$. Focusing on the competition among the public firm and private firms such that $x_j^E(0) > 0$ gives equation (8). Then, following from Lemma 1 and $x_0^E \geq \max_{j\in\{1,\dots,n\}} x_j^E(0)$,

$$\frac{dW^E}{d\alpha} > -p'(X^E)\sum_{i=0}^{n}\left\{\frac{dx_i^E}{d\alpha}x_0^E(0)\right\} = -p'(X^E)\frac{dX^E}{d\alpha}x_0^E(0) > 0.$$

Suppose $\max_{j\in\{1,\dots,n\}} x_j^E(0) = 0$. Then, $x_j^E(\alpha) = 0$ for all j and for any α. When this is the case, $\alpha = 1$ is the unique optimum since I suppose $x_0^E(1) > 0$.

Proof of Proposition 3

Given Specification 1,

$$x_0^E = \frac{a(b+k)}{G}, \quad x_1^E = \frac{a\{(1-\alpha)b + k + \Delta\}}{G},$$

where

$$G = \{(1-\alpha)b + k + \Delta\}(b+k) + b(b+k) + nb\{(1-\alpha)b + k + \Delta\}.$$

Let $\bar{\alpha}$ be a solution of $\frac{dW^E}{d\alpha} = 0$. Then, there is a unique solution

$$\bar{\alpha} = \frac{(b+k)^2 - nb\Delta}{(b+k)^2 + nbk}. \tag{A2}$$

10

Note that α is in $[0, 1]$ and that $\bar{\alpha} < 1$ holds. Further note that $\alpha = 1$ is not the optimum, as is shown in Proposition 1. Then, the optimal level of α is $\max\{\bar{\alpha}, 0\}$, which is unique (claim (i)).

$$\bar{\alpha} \gtreqqless 0 \Leftrightarrow \Delta \lesseqqgtr \frac{(b+k)^2}{nb} \equiv \bar{\Delta}$$

and

$$\bar{\alpha} \gtreqqless 0 \Leftrightarrow n \lesseqqgtr \frac{(b+k)^2}{b\Delta} \equiv \bar{n}$$

(claim (ii)). Furthermore, $\frac{d\bar{\alpha}}{d\Delta} < 0$ and $\frac{d\bar{\alpha}}{dn} < 0$ (claim (iii)), and $\frac{d\bar{\Delta}}{dn} < 0$ and $\frac{d\bar{n}}{d\Delta} < 0$ (claim (iv)).

Corollary 1

When $\Delta = 0$, $\bar{\alpha} = \frac{(b+k)^2}{(b+k)^2+nbk} \in (0, 1)$, following from (A2). Then, $\lim_{n \to \infty} \bar{\alpha} = 1$.

Proof of Proposition 4

Given Specification 2,

$$\frac{dW^E(0)}{d\alpha} > 0 \Leftrightarrow b > \frac{(n+4)\mu + \sqrt{k\mu + (n+4)^2\mu^2}}{4}.$$

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