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Lock! Risk-Free Arbitrage in the Japanese Racetrack Betting Market

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This paper finds that arbitrage was possible in two out of 175 Japanese thoroughbred races even after taking account of (a) the size of the minimum betting unit and (b) the negative effect of arbitrage on the odds. The guaranteed profits in these two races were 5,120 yen (about $64) and 340 yen.

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1. Introduction

The racetrack betting market shares many characteristics with the stock market, including uncertain future earnings, a large number of participants, and potential access to widely available rich information sets. Furthermore, Thaler and Ziemba (1988, p.162) point out a couple of advantages of the racetrack betting market: (1) each bet has a well-defined termination point at which its value becomes certain, and (2) quick and repeated feedback facilitates learning. On these grounds, racetrack betting markets should have a better chance of being efficient compared to the stock market or other financial markets.

Past literature, however, has found arbitrage opportunities in the racetrack betting market, where risk-free profits are guaranteed. Such an opportunity is called a "lock". To explain the "lock", let us consider a hypothetical betting market where one can buy an arbitrary amount of betting ticket without changing the odds. Suppose that *n* horses contest a race and that a successful unit bet on horse *i* to win, WIN_i, returns O_i units including the original stake. Also suppose that a unit bet on $EXACTA_{ij}$ returns O_{ij} units when horses *i* and *j* finish first and second in correct order. Then, to secure a return of one unit when horse *i* wins, one must bet either (a) $1/O_i$ units on WIN_i or (b) $1/O_{ij}$ units on $EXACTA_{ij}$ for every $j \neq i$. Consequently, one can make a risk-free profit when the following inequality is satisfied:

$$\sum_{i=1}^{n} \min\left\{\frac{1}{O_i}, \sum_{j\neq i}\frac{1}{O_{ij}}\right\} \leq 1.$$

If one can bet on $TRIFECTA_{ijk}$ (first three in order), risk-free arbitrage is possible when

$$P \equiv \sum_{i=1}^{n} \min\left\{\frac{1}{O_i}, \sum_{j\neq i}\min\left\{\frac{1}{O_{ij}}, \sum_{k\neq i,j}\frac{1}{O_{ijk}}\right\}\right\} \leq 1. \qquad (1)$$

In general, a typical pari-mutuel betting market offers a variety of bet types such as Win, Exacta, Quinella (first two in either order), and Trifecta. Since each bet type has its own pool, the odds will be different among them. Arbitrage is possible if the difference is large enough.

Nevertheless, the opportunities for the lock in pari-mutuel betting markets are considered to be extremely rare because of the following two reasons.[1] Firstly, arbitrage is self-destructive. Suppose that one buys WIN_i as a part of the arbitrage strategy. Then the return O_i decreases as a result, and it becomes more difficult to satisfy the inequality (1). Secondly, the above illustration ignores the existence of the minimum betting unit. In reality, one must buy at least one unit of the betting ticket even if an infinitesimal amount is enough for the arbitrage. Therefore the chance for arbitrage is much smaller than it seems to be at first glance.

These arguments give rise to one question: how often is the lock happening? The answer of past literature is not satisfactory. Hausch and Ziemba (1990a) construct a lock in the show pool of the 1979 Alabama Stakes at Saratoga, but they do not investigate the frequency of arbitrage opportunities. Edelman and O'Brien (2004) find that a lock is possible in 31 out of 2667 Australian thoroughbred races in early 2000. Their analysis, however, neglects the self-destructive effect of the arbitrage on the odds and the size of the minimum betting unit. Hence their result shows only the upper limit of the frequency of locks.

We shed a new light on this issue with the help of a unique sales data of Japanese thoroughbred races operated by a local municipality (National Association of Racing), which contains the sales volume of all bet types. We incorporate the size of the minimum betting unit into the analysis, and we check every possible strategy of arbitrage given the available bet types (including Quinella and Trifecta). Furthermore, we calculate the exact odds *after* execution of the arbitrage. This enables us to judge whether the arbitrage is ex-post profitable.

The main findings in the next section are as follows. Firstly, a lock is possible in two out of 175 races (1.14%). Secondly, the guaranteed profits of these races are 5,120 yen (about $64) and 340 yen (about $4.25). In light of the frequency of arbitrage opportunities, the amount of the guaranteed profit is not large enough to cover the opportunity cost of time. Thirdly, if one does not take account of the minimum betting unit and the negative effect of the arbitrage on the odds, the inequality (1) is satisfied for 35 out of 175 races (20.0%). This figure demonstrates that we must give proper consideration to these two factors in order to evaluate the true profitability of the arbitrage.

2. Data and results

We analyze published final sales data and final odds data of Japanese thoroughbred races operated by Arao city in the period of September 30 through December 23 of year 2011. There are 175 races, and the average total sales amount is 6,436,444 yen (approximately $80,000) per race. The average pool size of Win, Brackets Quinella,[2] Quinella, Exacta, and Trifecta are 118,800 yen, 193,264 yen, 480,807 yen, 740,348 yen, and 3,893,970 yen respectively. The average number of starters is 8.35.

As a preliminary step, we compute the value of P in equation (1) for every 175 races. There is no race with $P < 0.7$. There are three races with $0.7 \leq P < 0.8$, six races with $0.8 \leq P < 0.9$, and 26 races with $0.9 \leq P \leq 1$. In total, there are 35 races (20.0% of 175 races) that satisfy the inequality (1). This percentage is much higher than that of Edelman and O'Brien (2004) (1.16% of 2667 races). This might be due to the peculiarity of the Japanese local thoroughbred races.[3]

Next, we try to construct an arbitrage strategy for these 35 races with $P \leq 1$.[4] In doing so, we must take account of the minimum betting unit and the negative change of the relevant odds. To take a simple example, let us examine Table 1, the final odds tables of Race 11 at Arao on November 25, 2011.[5] Suppose that one needs to secure a payout of 10,000 yen regardless of the order of arrival. Then Table 2 shows the best strategy when the odds are fixed at the level of Table 1. Since the minimum betting unit is 100 yen (about $1.25), one needs to buy 22 units of WIN_1, one unit of WIN_3, and so on. If Horse No. 1 wins, the payout is 10,120 yen. If Horse No.2 wins, the payout is from 10,040 yen (when Horse No. 8 finishes second) to 71,490 yen (when Horse No.7 finishes second). The total cost of this strategy is 8,200 yen (82 units of betting tickets), and the minimum payout is 10,040 yen. Hence the guaranteed net profit is 1,840 yen.

In reality, however, the odds will change from Table 1 if one buys the combination of the betting tickets specified in Table 2. The odds (including the original stake) of WIN_i, O_i, is computed in the following manner in the Japanese local thoroughbred races (National Association of Racing):

$$O_i = \max\left\{0.1 \times \mathrm{INT}\left[1 + 7.38\frac{S}{S_i}\right], 1\right\}, \qquad (2)$$

where INT[x] gives the largest integer not exceeding x, S is the total amount bet in the win pool, and S_i is the amount bet on WIN_i.[6] For example, if one buys 22 additional tickets of WIN_1, O_1 decreases from 4.6 to 4.4. Table 3 shows the best strategy to secure a payout of 10,000 yen after taking account of this negative change of the odds. The total cost is now 8,500 yen (85 units of betting tickets), and the minimum payout is 10,000 yen (when Horse No. 4 wins). Hence the guaranteed net profit is reduced to 1,500 yen.

We have done the same calculation for other races, and have found that the guaranteed net profit is positive in two out of 175 races. In Race 11 on November 25, the guaranteed net profit is maximized by the betting strategy of Table 4. The total cost is 45,200 yen, and the minimum payout is 50,320 yen. Thus the guaranteed net profit is 5,120 yen. In Race 5 on October 20, the maximum of the guaranteed net profit is 340 yen.[7] These profits would be too small for an arbitrager to survive given the low frequency of arbitrage opportunities.

Finally, let us discuss the practicality of the arbitrage. Suppose that the arbitrager is free to bet once all other bettors have placed their bets. Then she can calculate back S and S_i from the public information, namely equation (2) and the odds tables (the Appendix explains the method). Therefore she can compute the amount of ex post profit of her bets in advance. As Gramm et al. (2012) and others point out, however, one can only attempt to be one of the last bettors in practice. Mori and Hisakado (2009) find that almost half of the win tickets are bought within the last ten minutes in the races of the Japan Racing Association (JRA). Since this inflow of late money might eliminate the discrepancy of the odds among different bet types, the above-mentioned strategy is not risk-free in a strict sense.[8]

3. Conclusions

This paper has investigated the true frequency of arbitrage opportunities in the Japanese thoroughbred races. It is the first attempt to take account of the following three factors simultaneously: the size of the minimum betting unit, the negative effect of the arbitrage on the odds, and the existence of various exotic bets. It has found that arbitrage is possible in two out of 175 races. The guaranteed profits in these races are 5,120 yen (about $64) and 340 yen. Given the required investment of time and effort, the frequency of arbitrage opportunities and the level of the guaranteed profits are too low for professional arbitragers. This might be the reason why the apparent inefficiency still exists in the market.

Notes

1. As for other types of betting markets, Hausch and Ziemba (1990b) demonstrate that an optimal cross-track betting yields a guaranteed profit in seven out of ten Triple Crown races between 1982 and 1985. Pope and Peel (1989), Shin (1993) and Marshall (2009) find arbitrage opportunities between bookmakers. Vlastakis et al. (2009) and Franck et al. (forthcoming) find arbitrage opportunities among bookmakers and exchange markets.
2. This is a unique bet type existing only in Japan. One bets on the brackets numbers, not the horse numbers.
3. Note that we use the data of Trifecta in calculation of P whereas Edelman and O'Brien (2004) do not.
4. It is impossible to construct a lock for the races with $P > 1$.
5. The odds table of Trifecta is available upon request.
6. The odds of $EXACTA_{ij}$ and $TRIFECTA_{ijk}$ are calculated in the same way.
7. The total cost of this arbitrage is 13,100 yen. The odds tables and the exact betting strategy are available upon request.
8. Rosenbloom (1992) describes other risks of the lock.

Appendix: Derivation of the sales amount from the odds table

Table 1 shows that the largest odds of Exacta is 1906.3 ($EXACTA_{36}$), and the second largest odds is 1429.7 ($EXACTA_{35}$). First suppose that $S_{36} = 1$, that is, only one unit is bet on $EXACTA_{36}$. Then $S_{35} \geq 2$ because S_{35} must be larger than S_{36}. If $(O_{36}, S_{36}) = (1906.3,1)$ is substituted in (2), S becomes 2582.9. If $(O_{35}, S_{35}) = (1429.7,2)$, however, S becomes 3874.3 and a contradiction occurs. Secondly, suppose that $S_{36} = 2$. Then $S_{35} \geq 3$. If $(O_{36}, S_{36}) = (1906.3,2)$, S must be 5165.9. On the other hand, if $(O_{35}, S_{35}) = (1429.7,3)$, S becomes 5811.4 and a contradiction occurs again. Thirdly, suppose that $S_{36} = 3$. Then $S_{35} \geq 4$. If $(O_{36}, S_{36}) = (1906.3,3)$, $S = 7748.8$. If $(O_{35}, S_{35}) = (1429.7,4)$, $S = 7748.5$ and it is consistent with the above calculation. Finally, if $(S, S_{36}) = (7749,3)$ is substituted in (2), $O_{36} = 1906.3$ and it is consistent with Table 1. These arguments demonstrate that S must be 7749 (or its multiples). The published sales data has confirmed that S was indeed 7749.

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Table 1. The final odds of Race 11 at Arao on November 25, 2011.

(a) Exacta

	O_{ij}		O_{ij}		O_{ij}		O_{ij}
1-1	…	2-1	11.9	3-1	714.9	4-1	5.4
1-2	11.9	2-2	…	3-2	1143.8	4-2	16.6
1-3	238.3	2-3	357.5	3-3	…	4-3	197.2
1-4	3.5	2-4	13.7	3-4	301.0	4-4	…
1-5	12.8	2-5	46.9	3-5	1429.7	4-5	16.8
1-6	197.2	2-6	336.4	3-6	1906.3	4-6	178.8
1-7	336.4	2-7	714.9	3-7	1906.3	4-7	260.0
1-8	12.6	2-8	50.2	3-8	1143.8	4-8	14.3
	O_{ij}		O_{ij}		O_{ij}		O_{ij}
5-1	30.5	6-1	519.9	7-1	714.9	8-1	28.8
5-2	84.1	6-2	635.5	7-2	1143.8	8-2	90.8
5-3	381.3	6-3	1429.7	7-3	1429.7	8-3	635.5
5-4	39.8	6-4	301.0	7-4	301.1	8-4	34.7
5-5	…	6-5	714.9	7-5	1906.3	8-5	73.4
5-6	440.0	6-6	…	7-6	817.0	8-6	381.3
5-7	817.0	6-7	1143.8	7-7	…	8-7	953.2
5-8	59.6	6-8	635.5	7-8	953.2	8-8	…

(b) Win

	1	2	3	4	5	6	7	8
O_i	4.6	1.3	133.7	4.2	29.4	80.2	120.3	37.6

Table 2. The best strategy to secure a payout of 10,000 yen when the odds are fixed at the level of Table 1.

	Odds of Table 1	Units	Payout (Yen)
WIN_1	4.6	22	10,120
WIN_3	133.7	1	13,370
WIN_4	4.2	24	10,080
WIN_5	29.4	4	11,760
WIN_6	80.2	2	16,040
WIN_7	120.3	1	12,030
WIN_8	37.6	3	11,280
$EXACTA_{21}$	11.9	9	10,710
$EXACTA_{23}$	357.5	1	35,750
$EXACTA_{24}$	13.7	8	10,960
$EXACTA_{25}$	46.9	3	14,070
$EXACTA_{26}$	336.4	1	33,640
$EXACTA_{27}$	714.9	1	71,490
$EXACTA_{28}$	50.2	2	10,040
	Total	82	

Table 3. The best strategy to secure a payout of 10,000 yen after taking account of the negative change of the odds.

	Ex-Post Odds	Units	Payout (Yen)
WIN_1	4.4	23	10,120
WIN_3	124.7	1	12,470
WIN_4	4.0	25	10,000
WIN_5	27.7	4	11,080
WIN_6	73.4	2	14,680
WIN_7	113.4	1	11,340
WIN_8	35.7	3	10,710
$EXACTA_{21}$	11.7	9	10,530
$EXACTA_{23}$	337.6	1	33,760
$EXACTA_{24}$	13.5	8	10,800
$EXACTA_{25}$	46.0	3	13,800
$EXACTA_{26}$	318.8	1	31,880
$EXACTA_{27}$	637.6	1	63,760
$EXACTA_{28}$	49.1	3	14,730
	Total	85	

Table 4. The betting strategy that maximizes the guaranteed net profit.

	Ex-Post Odds	Units	Payout (Yen)
WIN_1	3.7	136	50,320
WIN_3	104.0	5	52,000
WIN_4	3.4	148	50,320
WIN_5	23.2	22	51,040
WIN_6	63.3	8	50,640
WIN_7	91.0	6	54,600
WIN_8	29.8	17	50,660
$EXACTA_{23}$	320.5	2	64,100
$EXACTA_{24}$	12.6	40	50,400
$EXACTA_{25}$	43.1	12	51,720
$EXACTA_{26}$	303.7	2	60,740
$EXACTA_{27}$	641.0	1	64,100
$EXACTA_{28}$	46.2	11	50,820
$TRIFECTA_{213}$	319.6	2	63,920
$TRIFECTA_{214}$	23.0	22	50,600
$TRIFECTA_{215}$	73.8	7	51,660
$TRIFECTA_{216}$	370.6	2	74,120
$TRIFECTA_{217}$	544.4	1	54,440
$TRIFECTA_{218}$	69.2	8	55,360
	Total	452	