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Terada, Daisuke

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# **ONBOARD EVALUATION OF THE TRANSVERSE STABILITY FOR OFFICERS BASED ON NONLINEAR TIME SERIES MODELING**

**Daisuke TERADA** \*

## **ABSTRACT**

In this study, the convenience method for the onboard evaluation of the transverse stability is proposed based on an exponential autoregressive modeling procedure, which is a kind of nonlinear time series analysis. The verification of the proposed method is implemented by using results of model experiments concerning the parametric roll resonance and onboard data under the ordinary navigation. It can be confirmed that transverse stability can evaluate by monitoring the behavior of characteristic roots on the exponential autoregressive model.

**Keywords:** AIC (Akaike Information Criterion), navigation support system, nonlinear roll motion, exponential autoregressive modeling

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\* Researcher, National Research Institute of Fisheries Engineering, Fisheries Research Agency. 7620-7 Hasaki, Kamisu, Ibaraki, 314-0408, Japan. Email : dterada@affrc.go.jp

## 1. INTRODUCTION

For officers, the transverse stability is very important factor to keep the safe navigation of the operating ship. In general, officers confirm the static transverse stability by using a loading calculator. However, in actual navigation, the transverse stability changes due to the influence of waves. In extraordinary circumstances, the ship occurs of a serious accident such as the capsizing. Therefore, in order to protect the accident, we consider that officers should keep monitoring the roll motion, which is directly related with the transverse stability during the navigation. If we can always monitor the roll motion, then we can understand the state of the motion appropriately by using the knowledge of the time series analysis. Note that there is many studies such as have been published in the past STAB conference and the ISSW (e.g. Umeda et al. 2007; Kawahara et al. 2009; Francescutto and Umeda 2010; Umeda and Yamamura 2010 and so on) from the viewpoint of naval architecture concerning this issue.

In a little past, we showed the relationship between the dynamical system of the linear or the nonlinear roll motion and the time series model (Terada and Matsuda 2011). That is, the dynamical system of the linear roll motion can be approximated by the stationary autoregressive model, and the dynamical system of the nonlinear roll motion can be approximated by the time-varying autoregressive model. Thus, if we can get the time series data of the roll motion, we can evaluate the transverse stability directly for any data intervals by using the results of the stationary or the time-varying autoregressive modeling. However, using of these methods is difficult for the onboard evaluation of the transverse stability, because the calculation cost is too large.

To solve this problem, we attempt to apply an exponential autoregressive modeling procedure (Haggan and Ozaki, 1981). In this procedure, the time-varying autoregressive coefficient is approximated by the exponential function, and the estimation calculation of coefficients is implemented by the least squares method. As well as the time-varying autoregressive modeling procedure, in the exponential autoregressive modeling procedure a characteristic root calculated from a characteristic equation of the model is also very important to evaluate the transverse stability. That is, if all characteristic roots lie inside of the unit circle, then the system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as the nonlinear for the restoring force. Therefore, since officers can understand the dynamics of the roll motion under navigation in detail, it is considered that the proposed method is useful as the way to motivate the safe navigation in officers.

To confirm the effectiveness of the proposed method, we analyzed the data of the steady state and the parametric roll resonance. The obtained findings are reported.

## 2. NONLINEAR STOCHASTIC DYNAMICAL SYSTEM

Consider the following nonlinear stochastic dynamical system concerning the roll motion:

$$\ddot{x}(t) + f(\dot{x}(t)) + g(x(t)) = u(t) \quad (1)$$

where  $x(t)$  indicates a roll angle, the notation  $(\cdot)$  and  $(\ddot{\cdot})$  indicate the 1<sup>st</sup> and the 2<sup>nd</sup> order differential operator with time,  $f(\cdot)$  indicates the nonlinear mapping function concerning the damping force,  $g(\cdot)$  indicates the nonlinear mapping function concerning the restoring force and  $u(t)$  indicates an external disturbance that is treated with the random variable, respectively. Note that  $u(t)$  has the finite variance, but is not white noise sequence. And Equation 1 can be written by the following vector form:

$$\dot{\mathbf{x}}_t = \mathbf{f}(\mathbf{x}_t) + \mathbf{u}_t \quad (2)$$

where, as the notation  $(T)$  means the transpose,

$$\begin{aligned} \mathbf{x}_t &= [\dot{x}(t), x(t)]^T, \\ \mathbf{f}(\mathbf{x}_t) &= [-f(x(t)) - g(x(t)), \dot{x}(t)]^T, \\ \mathbf{u}_t &= [u(t), 0]^T. \end{aligned}$$

According to the locally linearization method (Ozaki 1986), Equation 2 can be discretized as follows:

$$\mathbf{x}_n = \text{EXP}[\mathbf{K}_{n-1}\Delta t] \cdot \mathbf{x}_{n-1} + \mathbf{B}_{n-1}\mathbf{u}_n \quad (3)$$

where,

$$\begin{aligned} \mathbf{x}_n &= [\dot{x}_n, x_n]^T, \quad \mathbf{K}_n = \frac{1}{\Delta t} \text{LOG}(\mathbf{A}_n), \\ \mathbf{A}_n &= \mathbf{I} + \mathbf{J}_n^{-1} \{ \text{EXP}(\mathbf{J}_n \Delta t) - \mathbf{I} \} \mathbf{F}_n, \\ \text{LOG}(\mathbf{A}_n) &= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} (\mathbf{A}_n - \mathbf{I})^k, \\ \mathbf{J}_n &= \frac{\partial \mathbf{f}(\mathbf{x}_n)}{\partial \mathbf{x}_n}, \quad \mathbf{F}_n \mathbf{x}_n = \begin{pmatrix} -f(\dot{x}_n) & -g(x_n) \\ \dot{x}_n & 0 \end{pmatrix}, \end{aligned}$$

$\Delta t$  indicates a discrete interval and  $\mathbf{B}_{n-1}\mathbf{u}_n$  is a two-dimensional colored noise sequence, which is obtained by the stochastic integral.

### 3. TIME SERIES MODEL

In Equation 3, since the term of the noise is not the white noise sequence, it is necessary to transform the colored noise sequence into the white noise sequence in order to deal with the problem stochastically. As the way to do the whitening, Yamanouchi (1956) showed the way to use the discrete autoregressive process. That is, in Equation 3, let

$$e_n \circ \mathbf{B}_{n-1} \mathbf{u}_n. \quad (4)$$

Then this can be approximated by the following  $m$ -th order discrete autoregressive process.

$$e_n = \sum_{i=1}^m \mathbf{D}_i e_{n-i} + \mathbf{w}_n, \quad (e_n = \mathbf{w}_n \text{ for } i = 0), \quad (5)$$

where  $\mathbf{w}_n$  is a  $2 \times 2$  Gaussian white noise sequence with  $N(0, \text{diag}(\sigma_1^2, \sigma_2^2))$  and  $\mathbf{D}_n$  indicates a  $2 \times 2$  autoregressive coefficient matrix. On the other hands, the following relation is evident.

$$\begin{aligned} e_n &= \mathbf{x}_n - \mathbf{A}_{n-1} \mathbf{x}_{n-1} \\ e_{n-1} &= \mathbf{x}_{n-1} - \mathbf{A}_{n-2} \mathbf{x}_{n-2} \\ &\vdots \\ e_{n-m} &= \mathbf{x}_{n-m} - \mathbf{A}_{n-m-1} \mathbf{x}_{n-m-1} \end{aligned} \quad (6)$$

Therefore, by substituting Equations 6 into Equation 5, we can obtain the following two dimensional  $(m+1)$ -th order time-varying autoregressive model.

$$\mathbf{x}_n = \sum_{i=1}^{m+1} \mathbf{C}_i \mathbf{x}_{n-i} + \mathbf{w}_n. \quad (7)$$

Here  $\mathbf{C}_i$  ( $i=1, \dots, m+1$ ) is the time-varying autoregressive coefficient matrix, which is expressed as follows:

$$\mathbf{C}_1 = \mathbf{D}_1 + \mathbf{A}_{n-1}, \mathbf{C}_2 = \mathbf{D}_2 - \mathbf{D}_1 \mathbf{A}_{n-2}, \dots, \mathbf{C}_m = \mathbf{D}_m - \mathbf{D}_{m-1} \mathbf{A}_{n-m}, \mathbf{C}_{m+1} = -\mathbf{D}_m \mathbf{A}_{n-m-1}.$$

Moreover, by using the following relation

$$\dot{\mathbf{x}}_n @ \frac{1}{\Delta t} (\mathbf{x}_n - \mathbf{x}_{n-1}) \quad (8)$$

Equation 7 can be approximated by the following the  $M$ -th (  $\geq m+1$  ) order scalar time-varying autoregressive model

$$x_n = \sum_{i=1}^M a_{n,i} x_{n-i} + w_n, \quad (9)$$

where  $a_{n,i}$  indicates time-varying autoregressive coefficients,  $w_n$  is the Gaussian white noise sequence with  $N(0, \sigma_2^2)$ . Now, since  $a_{n,i}$  is time-varying autoregressive coefficients, suppose that the following relation

$$\sum_{i=1}^M \left\{ f_i + \rho_i \exp\left\{ \frac{\gamma}{2} x_{n-1}^2 \right\} \right\} a_{n,i} \quad (10)$$

where  $\varphi_i$  is a linear term of autoregressive coefficients,  $\pi_i$  is a time-varying term of autoregressive coefficients and  $\gamma$  is a scaling parameter. Thus, Equation 9 can be written as follows:

$$x_n = \sum_{i=1}^M \left\{ f_i + \rho_i \exp\left\{ \frac{\gamma}{2} x_{n-1}^2 \right\} \right\} x_{n-i} + w_n. \quad (11)$$

This time series model, which is called an exponential autoregressive (Exp AR) model, is firstly introduced by Ozaki and Oda (1978). And then characteristics are investigated by Haggan and Ozaki (1981). According to Haggan and Ozaki (1981), consider the following characteristic equations of Equation 11:

$$\begin{cases} \lambda^M - f_1 \lambda^{M-1} - \dots - f_{M-1} \lambda - f_M = 0: & \text{As } x_{n-1} = 0 \\ \lambda^M - (f_1 + \rho_1) \lambda^{M-1} - \dots - (f_{M-1} + \rho_{M-1}) \lambda - (f_M + \rho_M) = 0: & \text{As } x_{n-1} = \pm \infty \end{cases} \quad (12)$$

Note that the Exp AR model is one class of the radial basis function (RBF) approximation model in the neural network approach. As the study of the prediction of time series for roll motion, there is Ueno and Han (2013).

If all roots of these equations lie inside of the unit circle, then the nonlinear stochastic dynamical system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be

evaluated as the nonlinear for the restoring force.

#### 4. FITTING OF THE TIME SERIES MODEL

As to the estimation of the model order  $M$  and the coefficients  $\{\gamma, (\varphi_i, \pi_i; i = 1, \dots, M)\}$  in the Exp AR model, for simplicity, by fixing the parameter  $\gamma$  at one of a grid of values, we estimated the model order  $M$  and the corresponding  $\varphi_i, \pi_i$  parameters as well as Haggan and Ozaki (1981). As  $N$  is the total number of observations, after fixing  $\gamma = \gamma_0$ , the Exp AR model for  $n = M+1, \dots, N; i = 1, \dots, M$  can be written as follows:

$$x_n = \sum_{i=1}^M \left\{ \hat{f}_i + \rho_i \exp[-g_0 x_{n-1}^2] \right\} x_{n-i} + w_n. \quad (13)$$

So the matrix form of Equation 13 can be written  $\mathbf{X}^{(n)} = \mathbf{H}\beta + w$ , where,  $n = N - M, \dots, N$ ,

$$\begin{aligned} \mathbf{X}^{(n)} &= (x_n, x_{n-1}, \dots, x_{n-(N-M-1)})^T, \\ \mathbf{Y}^{(n)} &= (\exp[-g_0 x_n^2] x_n, \exp[-g_0 x_{n-1}^2] x_{n-1}, \dots, \exp[-g_0 x_{n-(N-M-1)}^2] x_{n-(N-M-1)})^T, \\ \mathbf{H} &= (\mathbf{X}^{(n-1)}, \mathbf{Y}^{(n-1)}, \mathbf{X}^{(n-2)}, \mathbf{Y}^{(n-2)}, \dots, \mathbf{X}^{(n-i)}, \mathbf{Y}^{(n-i)}), \\ b &= (\hat{f}_1, \rho_1, \hat{f}_2, \rho_2, \dots, \hat{f}_i, \rho_i)^T, \\ w &= (w_n, w_{n-1}, \dots, w_{M+1})^T, \end{aligned}$$

so that the normal equations for  $\beta$  become  $\mathbf{X}^{(n)} = \mathbf{H}\beta$ . Hence  $\beta$  can be found from

$$\hat{\beta} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{X}^{(n)}. \quad (14)$$

The model order  $M$  of the fitted model is selected by using the Akaike Information Criterion (AIC) for nonlinear time series (Ozaki and Oda 1978)

$$\text{AIC}(M) = (N - M) \log \hat{S}_{2,M}^2 + 2(2M + 1) \quad (15)$$

where

$$\hat{S}_{2,M}^2 = \frac{(\hat{w}_N^2 + \hat{w}_{N-1}^2 + \dots + \hat{w}_M^2)}{N - M} \quad (16)$$

is the least squares estimate of the residual variance of the model.

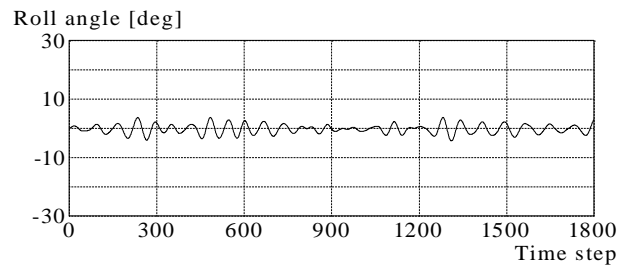
## 5. VERIFICATION

### 5.1 Used time series data

To verify the proposed procedure, we analysed the data of the steady state and the parametric roll resonance. As to the data of the steady state under the ordinary navigation, we used the roll angle data of the research vessel “Taka-maru”, which belongs the NRIFE. Table 1 shows the principal particulars of Taka-maru. The data as shown in Fig.1 was measured at sampling interval 0.1 [s] when the ship was running 4 knots in beam waves. From this figure, it can be seen that the maximum value of the absolute value of the roll angle is about 5 degrees and the motion is steady and stable. However, since the size of the ship is small, we felt that the ship shakes intensely. Actually, some of the crew for measurement of data became seasickness.

**Table 1** Principal particulars of the Taka-maru

Items	Ship
Length between perpendiculars: L	25.0 m
Breadth: B	5.2 m
Mean draft: T	2.0 m
Block coefficient: $C_b$	0.442
Metacentric height: GM	0.52 m
Natural roll period: $T_\phi$	5.95 s



**Fig. 1** Time series of the roll angle measured in onboard

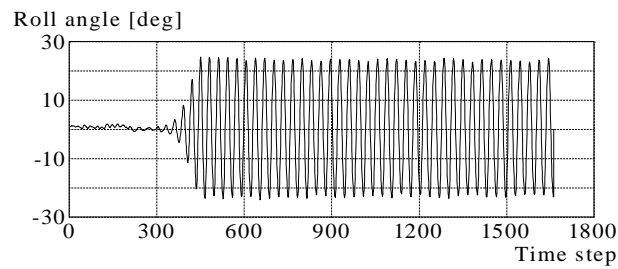
As to the data of the parametric roll resonance, we used results of model experiments concerning the Post-Panamax Container ship implemented by Hashimoto et al (2005). Table 2 shows the principal particulars of the sample ship. The data as shown in Figs.2 ~ 3 was measured at sampling interval 0.1 [s], when the ship was running under the condition of causing the parametric roll resonance in head seas. Fig. 2 shows the result in regular waves and Fig. 3 shows the result in irregular waves. From the Fig. 2 in shown the result of regular waves, it can be seen that the amplitude becomes large rapidly after about the 300<sup>th</sup>, and becomes the steady state in roll angle 25 [degrees]. From the Fig. 3 in shown the result of irregular waves, it can be seen that



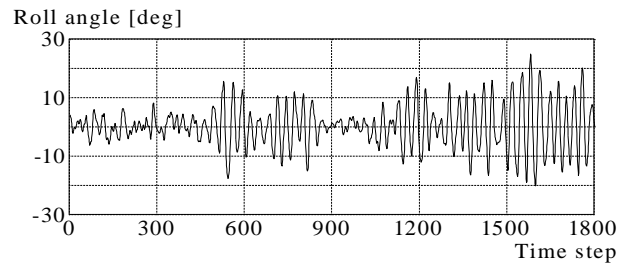
the absolute value of amplitude exceeds 10 degrees in about the 500<sup>th</sup> ~ 900<sup>th</sup> and in about 1100<sup>th</sup> ~ 1800<sup>th</sup>.

**Table 2** Principal particulars of the sample ship by using the study of Hashimoto et al (2005)

Items	Ship	Model
L	238.8m	2.838m
B	42.8m	0.428m
T	14.0m	0.14m
$C_b$	0.630	0.630
GM	1.08m	0.0106m
$T_\phi$	30.3 s	3.20 s



**Fig. 2** Time series of the roll angle measured in model experiments:  
The result of regular waves



**Fig. 3** Time series of the roll angle measured in model experiments:  
The result of irregular waves

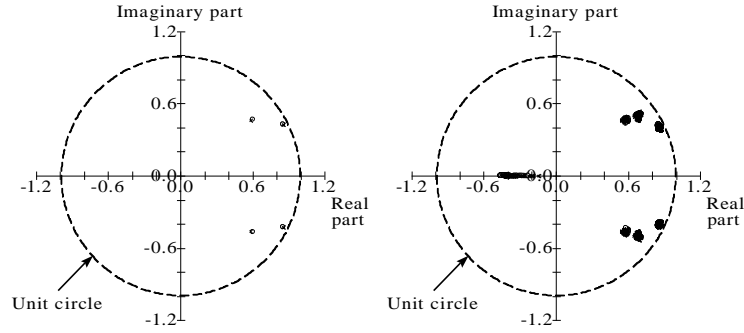
## 5.2 Results and discussions

The analysis for the data of ( $N=300$ ) set based on the Exp AR modelling was recursively performed for 300 samples. Figs. 4 ~ 6 show the arrangement of characteristic roots calculated by using the Equation 12. Fig. 4 shows results concerning the onboard data as shown in Fig. 1, Fig. 5 shows results concerning the experimental data in regular waves as shown in Fig. 2, and Fig. 6 shows results concerning the experimental data in irregular waves as shown in Fig. 3, respectively. In these figures, the left hand side shows the result of first 300 samples and the right hand

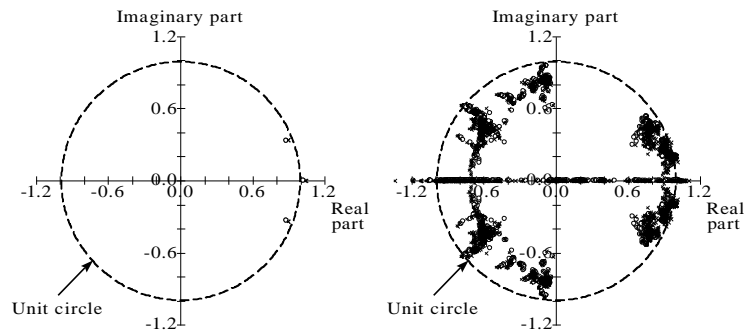
side shows all results of the data of ( $N-300$ ) set, respectively. And the symbol “○” indicates the characteristic root in the case of  $x_{n-1} = 0$  and the symbol “×” indicates the characteristic root in the case of  $x_{n-1} = \pm\infty$ , respectively. As mentioned before, all characteristic roots lie inside of the unit circle, then the system is stationary and stable. Moreover, when the real part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluate as the nonlinear for the damping force. And also, when the imaginary part of the characteristic root changes from positive/negative to negative/positive, the dynamical system for the roll motion can be evaluated as the nonlinear for the restoring force. In Fig. 4, it can be seen that all characteristic roots lie inside of the unit circle and the arrangement of symbols “○” and “×” are almost same. Therefore, we can judge that the onboard data as shown in Fig. 1 is the stationary and stable. In Fig. 5, it can be seen that all characteristic roots do not lie inside of the unit circle. As to the result of first 300 samples, one pare is outside of the unit circle. Therefore, we can judge that the experimental data as shown in Fig. 2 is the unstable from the analysis of the data of first 300 samples only, although the amplitude of the roll angle is very small. In this case, officers must devise thoroughgoing measures to prevent the large amplitude roll motion based on the applied ship operation. In the right hand side of Fig. 6, it can be seen that all characteristic roots do not lie inside of the unit circle. Thus, we can judge that the experimental data of the data of ( $N-300$ ) set in irregular waves as shown in Fig. 3 is the non-stationary and unstable. However, as to the result of first 300 samples as shown in the left hand side of Fig. 6, all characteristic roots lie inside of the unit circle, this case is the stationary and stable, although a few nonlinearity with respect to the restoring force because bath characteristic roots indicated with “○” and “×” are different concerning imaginary part. It means that officers must always pay attention to the roll motion in order to prevent the large amplitude roll motion in irregular waves. From these results, we can conclude that officers can keep the safe navigation in the meaning of preventing the large amplitude roll motion by monitoring it based on the Exp AR modelling.

## 6. CONCLUSIONS

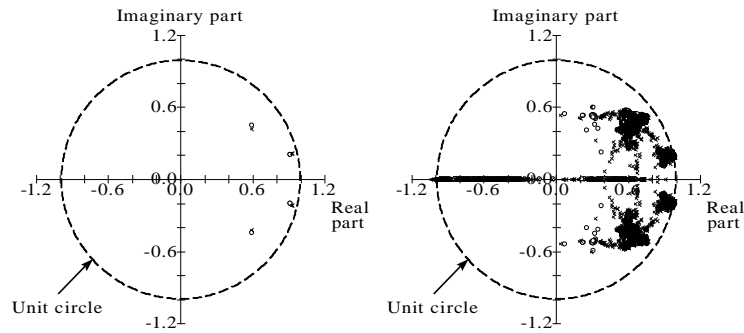
In this study, we propose the onboard evaluation method of the transverse stability for officers based on the exponential autoregressive modeling procedure. To confirm the effectiveness of the proposed method, we analyzed the data of the steady state and the parametric roll resonance. As the result, we can confirm that the characteristics of the dynamical system for the roll motion can understand by monitoring the behavior of the characteristic root of the characteristic equation in the exponential autoregressive model. Therefore, we conclude that the proposed method can be used as the navigation support system to protect the serious accident.



**Fig. 4** Characteristic roots calculated by using Equation 12 concerning the onboard data as shown in Fig. 1: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of the data of  $(N-300)$  set.



**Fig. 5** Characteristic roots calculated by using Equation 12 concerning the onboard data as shown in Fig. 2: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of the data of  $(N-300)$  set.



**Fig. 6** Characteristic roots calculated by using Equation 12 concerning the onboard data as shown in Fig. 3: The left hand side shows the result of the data of first 300 samples; The right hand side shows all results of the data of  $(N-300)$  set.

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