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Tamura, Hideaki

Matsubayashi, Yoichi

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Hideaki Tamura
Yoichi Matsubayashi

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GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

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Hideaki Tamura

Graduate School of Economics

Kobe University

Yoichi Matsubayashi

Graduate School of Economics

Kobe University

Abstract

This paper develops a new method for solving both equity premium and risk free rate puzzles based on the standard utility function. The method for solving the equity premium puzzle in accordance with Mehra and Prescott (1985) needs to be simultaneously consistent with the method for solving the risk-free rate puzzle presented by Weil (1989). That is, the reasonable estimated values for the degree of relative risk aversion in the former solution and for the subjective discount rate in the latter solution need to plausibly fall within experiential bounds. This study indicates that a consistent solution is possible for the equity premium and risk-free rate puzzles even when there is a standard constant relative risk aversion (CRRA) type utility function. This solution is possible by formularizing the Euler equation for consumption, considering the precautionary saving effect.

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Correspondence to:

Yoichi Matsubayashi

Graduate School of Economics, Kobe University

Rokkodai, Nada-ku, Kobe, 657-8501, Japan

TEL +81-78-803-6852

E-mail myoichi@econ.kobe-u.ac.jp

1. Introduction

A central issue in the verification of the permanent income hypothesis since Hansen and Singleton (1982) has been the estimation of parameters in the Euler equation for consumption (the subjective discount rate and degree of relative risk aversion) based on the consumption-based capital asset pricing model (C-CAPM). However, uncertainty has arisen in the estimation of the degree of relative risk aversion using U.S. data that depends on the estimation period, making it difficult to have confidence in these analysts.

Hansen and Singleton (1983, 1984) reported on the negative degree of relative risk aversion when using monthly U.S. data. In addition, results often have been dismissed due to the over-identification restriction of J-statistics. Another issue is the equity premium puzzle shown by Mehra and Prescott (1985), who demonstrated that the general equilibrium model using stock return, rate of return on treasury bills, and sample annual consumption growth rate from 1890 to 1979 maximizes the restrictions imposed on the stock price index and average rate of return on treasury bills. Mankiw and Zeldes (1991) addressed this problem using the relational expression derived by applying the Taylor approximation to the Euler equation, but they also reported that the degree of relative risk aversion calculated using Mehra and Prescott's (1985) sample is 26.3. In particular, they reported an unusually high value of 89 when taking the post-war sample from 1948 to 1988.

Weil (1989) pursued the values of the risk premium and risk-free rate for each combination of degree of relative risk aversion (γ) and elasticity of inter-temporal substitution ($1/\rho$) in the non-independent and identically distributed (i.i.d) dividend growth process using Kreps–Porteus type preferences. Specifying a constant relative risk aversion (CRRA)-type preference ($\rho = \gamma$), he derived a risk premium of 6.37%. Although that specification approximates the actual level wherein $\beta = 0.98$, $\gamma = 20$, and $1/\rho = 0.05$, the 15.01% risk-free rate at that time was high. He also derived a risk premium of 5.72% and a risk-free rate of 0.85%, which are close to actual levels, wherein $\beta = 0.95$, $\gamma = 45$, and $1/\rho = 0.1$, but noted this result depends on a high value for γ. This inconsistency is referred to as the risk-free rate puzzle.

According to the retrospective review by Mehra and Prescott (2003), the existing solution for the equity premium puzzle based on traditional theory is largely split among the alternative preference structure (the time non-separable model that does not assume separability in relation to time and habit formation), the model that incorporates idiosyncratic and uninsurable income risk and models incorporating a disaster state and survivorship bias. Solutions not based on traditional theory are affected by borrowing constraints, liquidity premiums, taxes and regulation.

Of the above solutions, the solution for both the equity premium and the risk-free rate puzzles in accordance with Mehra and Prescott (2003) is limited to the alternative preference structure (the time non-separable model that does not assume separability in relation to time and habit formation). An evaluation is as follows.

First, the model that does not assume separability in relation to time mitigates the severe (reciprocal) relationship between the degree of relative risk aversion and inter-temporal elasticity of substitution, which comes into existence under the traditional CRRA-type utility function, and also introduces the Kreps–Porteus type utility function that has been partitioned respectively, as is exemplified in Epstein and Zin (1989, 1991). This model expresses the inter-temporal elasticity of substitution as the weighted average of the marginal rate of substitution (MRS) for consumption and market portfolio. This builds the CAPMthat generalizes both the C-CAPM and Sharp–Lintner CAPM. However, the inadequacy of any special device to influence an estimation of the degree of relative risk aversion implies it will not solve the equity premium puzzle, although it is believed that the risk-free rate puzzle can be solved by making the inter-temporal elasticity of substitution and degree of relative risk aversion independent of each other.

Next, habit formation determines the utility of current consumption relative to past consumption that is considered to have become habitual or consumption when benchmarked to neighbours. Constantinides (1990) defines utility as the difference between current and past consumption (habit) with a lag, indicating that the equity premium puzzle can be solved when the weighting of past consumption (habit) is increased, even when the coefficient for the degree of relative risk aversion is low according to the internal habit model that calculates a high degree of effective risk aversion. In addition, Campbell and Cochrane (1995) presented a model that was consistent for both consumption and asset market data incorporating the possibility of an economic downturn using the state variable that risk aversion changes in a non-linear way. Abel (1990) presented a model that can solve the equity premium puzzle while also avoiding the risk-free rate puzzle modelled in accordance to the proportion of consumption (habit). These habit formation models can solve the equity premium puzzle, anticipating a consumer with an extreme dislike of consumption risk even when the degree of risk aversion is small. In addition, it is believed that the dislike of consumption risk boosts demand for safe assets and reduces the risk-free interest rate and will be used to solve the risk-free rate puzzle. Although these habit formation models can solve both the equity premium and risk-free rate puzzles given limited assumptions, all are based on methods that complicate the standard CRRA-type utility function. Thus, they can hardly be referred to as generic solutions.

The Euler equation for consumption that uses the standard CRRA-type utility function has featured only two explanatory variables till date: the consumption growth rate and return on assets. On the other hand, future expected marginal utility rises under a theoretical model of precautionary saving due to the increased income uncertainty when $U''' > 0$, and the maximisation of household first-order condition of expected utility. However, this suggests that an increase in income uncertainty influences the MRS for the inter-temporal indifference curve assumed for estimating the utility function parameters. That is, the MRS for an indifference curve will always change during optimization of multiple periods when there is a relative change in income uncertainty between the present and future due to the change in relative status of a utility function that considers present and future uncertainty. Thereafter, if estimate employs only the consumption growth rate and return on assets without correction of that influence, there will be an incorporated bias for the estimated value of the parameters themselves, distorting the estimate for the degree of relative risk aversion.

The Euler equation for consumption that is expanded to include income uncertainty as an explanatory variable needs to be formularized to estimate parameters with a greater degree of information to correct for this precautionary saving effect and thereby estimate parameters correctly. This study formularizes the Euler equation for consumption (the coefficient of variation (CV) model) with three explanatory variables (consumption growth rate, return on assets, and growth rate of income CV). Using the Mankiw and Zeldes (1991) method, the estimated degree of relative risk aversion can be explained using the two covariances between the return on assets and consumption growth rate and the return on assets and growth rate of the income CV while clarifying the features by comparing results of the analysis using the Euler equation for consumption (the “normal model”) that uses the two traditional explanatory variables of the consumption growth rate and return on assets using U.S. data.

This paper is organized as follows. Section 2 identifies the features of the CV model through analysis of a model that uses the same Taylor expansion as Mankiw and Zeldes (1991) while also formularizing the Euler equation for consumption under income uncertainty. Section 3 calculates the degree of relative risk aversion and subjective discount rate for the normal model and the CV model using the U.S. monthly real consumption growth rate, real return on assets, and the income uncertainty index for February 1978 to December 2010. Section 4 considers the correction effect for the degree of relative risk aversion under the CV model from the perspective of a two-period model. Section 5 presents our conclusions.

２． Model

２．１ Euler equation for consumption under income uncertainty

This section specifies the method for setting an optimal consumption model under income uncertainty using the marginal utility that is influenced by the income CV under the standard CRRA-type utility function to derive the Euler equation for consumption that includes the income CV.

The individual's expected marginal utility function under income uncertainty can be expressed as follows from equation (A.6) in Appendix A:

$$U^{*}{}'(C_t) = C_t^{-\gamma}\left[1 + 0.5(\gamma + \gamma^2)CV_t^2\right]. \tag{1}$$

However, C_t represents an individual's real consumption for period t, CV_t^2 represents the square value of the CV for consumption at period t: $CV_t^2 = (h_t / C_t)^2$ where h_t represents the standard deviation of consumption at period t due to income uncertainty and γ is a parameter representing a constant degree of relative risk aversion.

From equation (1), the individual's expected marginal utility under income uncertainty is the expected marginal utility of the normal model multiplied by $1 + 0.5(\gamma + \gamma^2)CV_t^2$. The expected marginal utility rises in proportion to the square value of CV_t when the CV for consumption increases.[1]

[1] This analysis is a modification referred to in Skinner (1988).

The multi-period optimal consumption model that uses the individual's expected utility function (equation (A.5) in Appendix A) under income uncertainty is set as follows:

$$\max \quad E_t[\sum_{i=0}^{\infty}\beta^i U^*(C_{t+i})], \tag{2}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} q_{jt}A_{jt+1} + C_t = \sum_{j=1}^{N}(q_{jt}+d_{jt})A_{jt} + Y_t. \tag{3}$$

However, β is the subjective discount rate $(0<\beta<1)$, q_{jt} is the price of asset j at period t (j=1,2,…,N), d_{jt} is the dividend from asset j at period t (j=1,2,…,N), A_{jt} is the quantity of asset j held at period t, Y_t is the non-asset income for period t and $E_t[\bullet]$ is the conditional expectation operator based on information available at time t.

There are N assets in the economy, and individuals select cash flow for each asset and consumption that maximizes the present discounted value of their expected utility derived through consumption from the present (t = 0) to the future.

Solving the above optimization problem results in the following first-order condition for maximization:

$$E_t[\beta \frac{U^{*}{'}(C_{t+1})}{U^{*}{'}(C_t)}(\frac{q_{jt+1}+d_{jt+1}}{q_{jt}})]-1=0. \tag{4}$$

Here, the rate of return r_{jt+1} on asset j is defined as $r_{jt+1}=(q_{jt+1}+d_{jt+1})/q_{jt}-1$, so $(q_{jt+1}+d_{jt+1})/q_{jt}$ in equation (4) can be substituted for $(1+r_{jt+1})$. Therefore, by replacing this then substituting equation (1) into equation (4), the individual's Euler equation for consumption under income uncertainty when there is a CRRA-type utility function can be expressed as follows:

$$E_t[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{1+0.5(\gamma+\gamma^2)CV_{t+1}^2}{1+0.5(\gamma+\gamma^2)CV_t^2}(1+r_{jt+1})]-1=0 \qquad \text{(j=1,2,…,N)} . \tag{5}$$

Transforming $1+0.5(\gamma+\gamma^2)CV_t^2$ in the expected marginal utility of equation (1) using a linear approximation of the exponential function for the Taylor expansion formula results in $1+0.5(\gamma+\gamma^2)CV_t^2 \cong \exp[0.5(\gamma+\gamma^2)CV_t^2]$. Therefore, the middle term of equation (5) can be transformed as follows:

$$\frac{1+0.5(\gamma+\gamma^2)CV_{t+1}^2}{1+0.5(\gamma+\gamma^2)CV_t^2} \cong \left(\frac{\exp(CV_{t+1}^2)}{\exp(CV_t^2)}\right)^{0.5(\gamma+\gamma^2)}$$

Applying the transformed middle term to equation (5) leads to the following for the Euler equation for consumption. The transformation also adds the growth rate of the exponential of the squared value of the CV for consumption as an explanatory variable while the coefficient for the degree of relative risk aversion $0.5(\gamma+\gamma^2)$ is applied as an index of that growth rate of the CV for consumption.

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\exp(CV_{t+1}^2)}{\exp(CV_t^2)}\right)^{0.5(\gamma+\gamma^2)}(1+r_{jt+1})=1\,. \tag{6}$$

This is the Euler equation for consumption under income uncertainty formularized with the three explanatory variables (consumption growth rate, return on assets and the growth rate of the CV for consumption).

２．２　Analyzing the model for determining the degree of relative risk aversion and subjective discount rate

Mankiw and Zeldes (1991) applied the Taylor expansion of the two variable functions to the Euler equation with two variables from equation (7) as explanatory variables (consumption growth rate and return on assets). This led to the relational expression among the equity premium, degree of relative risk aversion and covariance between return on assets and consumption growth rate when constantly abbreviated as expressed as equation (8).

$$E[(1+r^i)(1+g^C)^{-\gamma}]=1+\rho\,, \tag{7}$$

$$E[r_i]-\bar{r} \cong \gamma Cov(r^i,g^C)\,. \tag{8}$$

However, $g^C = (C_{t+1}/C_t) - 1$ and the subscript for time have been abbreviated. In addition, r^i represents the rate of return on risk asset i, and $\bar{r}$ represents the rate of return on the risk-free asset. $E[r^i] - E[\bar{r}]$ represents the equity premium, and ρ represents the time preference rate (equivalent to $(1/\beta) - 1$).

In accordance with equation (8), the degree of relative risk aversion for the normal model is defined as the equity premium divided by the covariance between the return on assets and the consumption growth rate. That is,

$$\gamma^*_{NM} \cong (E[r_i] - \bar{r})/\sigma_{ic}, \qquad (9)$$
$$\sigma_{ic} = Cov(r^i, g^C).$$

In addition, the relational expression for the risk-free asset that can be used in the results of equation (8) can be expressed as follows:

$$\bar{r} \cong \rho + \gamma E[g^C] - 0.5(\gamma + \gamma^2)Var(g^C). \qquad (10)$$

Substituting $\rho = (1/\beta) - 1$ into equation (10), and solving for the subjective discount rate β results in the following equation:

$$\beta \cong 1/(1 + E[\bar{r}] - \gamma E[g^C] + 0.5(\gamma + \gamma^2)\sigma_c^2), \qquad (11)$$
$$\sigma_c^2 = Var(g^C).$$

The subjective discount rate for the normal model is obtained by substituting γ^*_{NM} as determined from equation (9) and substituting the average, variance and covariance for each variable into equation (11).

On the other hand, the CV model formularized in the preceding paragraph uses three explanatory variables (consumption growth rate, return on assets and the growth rate of income CV). This can be expressed in the same way as equations (7) and (8) for the normal model as follows from equation (B.6) in Appendix B:

$$E[(1+r^i)(1+g^C)^{-\gamma}(1+g^{eCVSQ})^{0.5(\gamma+\gamma^2)}] = 1+\rho, \tag{12}$$

$$E[r^i] - \bar{r} \cong \gamma Cov(r^i, g^C) - 0.5(\gamma+\gamma^2)Cov(r^i, g^{eCVSQ}). \tag{13}$$

However, $g^{eCVSQ} = (\exp(CV_{t+1}^2)/\exp(CV_t^2)) - 1$, and the subscript for time has been abbreviated.

In the normal model shown in equation (8), the degree of relative risk aversion was explained only through the covariance between the return on assets and consumption growth rate. The degree of relative risk aversion when extended to the CV model is explained by both the covariance between the return on assets and the consumption growth rate and the covariance between return on assets and the growth rate of income CV.

Equation (13) can be rewritten as the function for degree of relative risk aversion γ, resulting in the following:

$$Cov(r^i, g^C)\gamma \cong E[r^i] - \bar{r} + 0.5Cov(r^i, g^{eCVSQ})(\gamma+\gamma^2). \tag{14}$$

In addition, applying $Cov(r^i, g^{eCVSQ}) = 0$ to equation (14) results in the following equation, which is the same as the equation for determining the degree of relative risk aversion for the normal model (equation (8)):

$$Cov(r^i, g^C)\gamma \cong E[r^i] - \bar{r}. \tag{15}$$

Under normal market conditions when the equity premium exceeds zero, the covariance between return on assets and the consumption growth rate is positive and the covariance between return on assets and the growth rate of the income CV is positive or negative, the relationship for determining the degree of relative risk aversion under the normal model and the CV model based on equation (14) and equation (15) is shown in Figure 1.

【 Figure 1 】

Figure 1 determines the degree of relative risk aversion for the normal model by the level of γ^*_{NM} that results in the straight line with the gradient of $Cov(r^i, g^C)$ on the left-hand side of equation (15) being consistent with the equity premium of $E[r^i] - \bar{r}$. On the other hand, in the CV model, when the covariance between the return on assets and growth rate of income CV is negative, the right-hand side of equation (14) will resemble (a) with a curve that shifts to the right of segment $E[r^i] - \bar{r}$ due to the monotonic increase function of $\gamma + \gamma^2$ when $\gamma > 0$. Thus, the degree of relative risk aversion for the CV model will be determined by the level of $\gamma^*_{CV\_a}$ that intersects with the straight line with the gradient of $Cov(r^i, g^C)$ from the left-hand side of equation (14). In addition, when the covariance between return on assets and the growth rate of the income CV is positive, the right-hand side of equation (14) will resemble (b) with a curve that shifts to the right from segment $E[r^i] - \bar{r}$. Therefore, it will be determined by the level of $\gamma^*_{CV\_b}$ that intersects with the straight line with the gradient of $Cov(r^i, g^C)$ from the left side of equation (14).

The equity premium puzzle notes that the covariance between return on assets and consumption growth rate obtained from actual data is extremely low relative to the equity premium. Therefore, the degree of relative risk aversion γ^*_{NM} derived from equation (15) is an extremely large value exceeding what is considered normal (within 10), or the theoretical model only partially explains the equity premium within what is considered normal for the degree of relative risk aversion (within 10).

On the other hand, under the CV model using equation (14), the estimated result can be greatly improved over the normal model, even when covariance between return on assets and the consumption growth rate is extremely small because the degree of relative risk aversion can be determined with the $\gamma^*_{CV\_a}$ being much lower than γ^*_{NM} when covariance between return on assets and growth rate of the income CV is a large negative value.

Solving equation (14) for γ means the determinant level for the degree of relative risk aversion for the CV model can be defined as follows:

$$\gamma^*_{CV} \cong \frac{2(\sigma_{ic} - 0.5\sigma_{iv}) \pm \sqrt{4(\sigma_{ic} - 0.5\sigma_{iv})^2 - 8\sigma_{iv}(E[r_i] - \bar{r})}}{2\sigma_{iv}} \tag{16}$$

$\sigma_{ic} = Cov(r^i, g^C)$, $\sigma_{iv} = Cov(r^i, g^{eCVSQ})$.

The condition for solving the degree of relative risk aversion under a positive risk premium by the discriminant $D = 4(\sigma_{ic} - 0.5\sigma_{iv})^2 - 8\sigma_{iv}(E[r_i] - \bar{r})$ from equation (16) is $\sigma_{iv} = Cov(r^i, g^{eCVSQ}) \leq 0$. Return on assets and the income uncertainty index must conditionally have zero or negative covariance. That is, a requirement for solving the CV model is choosing the appropriate choice of income uncertainty index—i.e. income uncertainty falls when return on assets rises and rises when return on assets falls.

Substituting $\rho = (1/\beta) - 1$ into equation (B.5) in Appendix B and solving for the subjective discount rate β results in the following equation:

$$\begin{aligned}\beta \cong 1/(1 + E[\bar{r}] - \gamma E[g^C] + 0.5(\gamma + \gamma^2)\sigma_c^2 + 0.5(\gamma + \gamma^2)E[g^{eCVSQ}] \\ + 0.25(\gamma + \gamma^2)\{0.5(\gamma + \gamma^2) - 1\}\sigma_v^2 - 0.5\gamma(\gamma + \gamma^2)\sigma_{cv}) \qquad (17) \\ \sigma_c^2 = Var(g^C),\ \sigma_v^2 = Var(g^{eCVSQ}),\ \sigma_{cv} = Cov(g^C, g^{eCVSQ}).\end{aligned}$$

Substituting the γ_{CV}^* determined from equation (16) and the average, variance and covariance for each variable into equation (17), we obtain the subjective discount rate for the CV model.

3. Empirical analysis using the U.S. income uncertainty index and concurrent solution of the equity premium and risk-free rate puzzles

3.1 Data and processing methods

For calculating the degree of relative risk aversion and subjective discount rate we used monthly data for the real consumption growth rate, real return on assets (for equities, the S&P 500 Index©; for U.S. government bonds, the secondary market rate for 90-day Treasury bills) and the income uncertainty index from February 1978 to December 2010. Real consumption uses the aggregate amount of nondurable goods and services per person.

The following four indices are used for the income uncertainty index data.

- Labour Share (spline conversion to monthly) (CV1).
- Reciprocal of Unit Profit (spline conversion to monthly) (CV2).
- Unemployment rate UNRATE (CV3).
- The reciprocal of the University of Michigan consumer confidence index UMCSENT (CV4).

Labour Share and Unit Profit are not released monthly, so quarterly data are converted to monthly data through non-linear interpolation using the cubic spline function.

It is best to use the figures for each income uncertainty index adjusted for the average value and standard deviation benchmarked to the CV for consumption. However, no continuous long-term U.S. data exist for the CV for consumption. We have used original data that has not been adjusted for the average value and standard deviation. For the observation period, Figure 2 demonstrates the movement of CV1–CV4 indexed to 100 at the beginning of the period.

【 Figure 2 】

Figure 2 shows that CV4 moves almost three months to more than six months ahead of CV3 and at about the same time or much earlier than CV1 and CV2. In addition, the extent of movement is much smaller.

3.2 Calculations of the degree of relative risk aversion and subjective discount rate

3.2.1 Trends in Risk premium

The average value for the risk premium is needed to calculate the degree of relative risk aversion, and a period displaying a positive average value must be chosen to achieve a stable result. Figure 3 illustrates the movement in average value of the risk premium calculated for each month of the observation period. We set February 1978 as the first month and set the end month as each month after the next month of the first month up until December 2010. Figure 3 illustrates that the average value of the risk premium was steadily positive for each period when the end month was on or after January 1985.

【 Figure 3 】

3.2.2 Calculations

The calculation of the degree of relative risk aversion considers the observation period and calculates the covariance for each variable and the average of risk premium for each period. To obtain the degree of relative risk aversion, the results are substituted into equation (9) for the normal model and into equation (16) for the CV model. Table 1 presents the descriptive statistics for each variable used in this calculation.

【 Table 1 】

In addition, the calculation of the subjective discount rate considers the observation period and calculates the average, the variance and the covariance for each variable for each period. [Remark 2] Together with the results from calculating the degree of relative risk aversion for the normal model, these calculations are substituted into equation (11) to obtain the subjective discount rate of the normal model. In a similar fashion, together with the results of calculating the degree of relative risk aversion for the CV model, these are substituted into equation (17) to obtain the subjective discount rate of the CV model. Table 2 presents the descriptive statistics for each variable used in this calculation.

【 Table 2 】

Figures 4–7 illustrate the results of calculating the degree of relative risk aversion and subjective discount rate for the observation period for each income uncertainty index (CV1–CV4). The movement in the result of the degree of relative risk aversion and subjective discount rate calculated for the normal model and CV model (considering the case that the average values for the income uncertainty index from CV1 to CV4 are 0.1, 0.3[Editor1] and 0.5) are illustrated in the graph.

【 Figure 4 】

【 Figure 5 】

【 Figure 6 】

【 Figure 7 】

These graphs illustrate the results of the solution for each period wherein the equity premium has a stable positive value for the end months of January 1988 and beyond. Average values of the solution for each period are shown below the graph.

Comparing Figures 4–7, the graph for the CV model is not shown in Figure 6, which uses CV3 because $\sigma_{iv} = Cov(r^i, g^{eCVSQ}) > 0$, and the real root condition for the solution in equation (16) is not met. Therefore, CV3is noncompliant for the income uncertainty index. As is evident in Figure 2, since there is a change three to six months after the other income uncertainty indices (CV1, 2 and 4), the lag in this timing can be attributed to the collapse of the normally anticipated negative correlation.

The degree of relative risk aversion for the CV model is lower than the normal model due to the upper graph in Figures 4, 5 and 7 and the average value of the solution. In addition, an even lower degree of relative risk aversion is obtained if the average values of the income uncertainty index are increased to 0.1, 0.3 and 0.5. This indicates the CV model that considers the negative covariance relationship for the return on assets and the income uncertainty index can solve the equity premium puzzle that points to an abnormal value for the degree of relative risk aversion.

In addition, the subjective discount rate for the normal model repeatedly rises and falls substantially and holds the level beyond 1 after August 2001 (bottom graphs in Figures 4, 5 and 7) and the average value of the solution, whereas the subjective discount rate for the CV model has levelled at slightly below 1. In addition, it has fallen due to a rise of 0.1, 0.3 and 0.5 in the average values of the income uncertainty index. This levelling is pronounced for CV4 in Figure 7. This indicates the CV model that considers the negative covariance for the return on assets and income uncertainty index can improve the risk-free rate puzzle that indicatesan abnormal value for the subjective discount rate.

Comparing Figures 4, 5 and 7 and examining the relative merits of income uncertainty indices indicate that the degree of relative risk aversion and the subjective discount rates are more suitable values for CV4 than CV1 and CV2, and the best as an income uncertainty index. This is attributed to the negative covariance of the return on assets and income uncertainty index in Table 1 being the largest for CV4.

4 Explanation of the CV model using the two period model based on Mehra and Prescott (1985) data

Aggregating the utility function in equation (2) as i=0, 1 for just the two periods, and replacing the total utility with Z results in the following equation:

$$Z = U^*(C_t) + \beta U^*(C_{t+1}). \tag{18}$$

The Euler equation, equation (6), represents the first-order condition of expected utility maximization derived from the multi-period optimal consumption model. That condition is the MRS of the inter-temporal indifference curve $-dC_{t+1}/dC_t$ which is the same as the gradient of the budget constraint line $(1+r_{t+1})$. To obtain this MRS, replace the complete differential of equation (18) with zero to arrive at $dZ = U^*'(C_t)dC_t + \beta U^*'(C_{t+1})dC_{t+1} = 0$. Doing so generates the following equation:

$$-\left.\frac{dC_{t+1}}{dC_t}\right|_{dZ=0} = \frac{U^*'(C_t)}{\beta U^*'(C_{t+1})} = \frac{C_t^{-\gamma}[1+0.5(\gamma+\gamma^2)CV_t^2]}{\beta C_{t+1}^{-\gamma}[1+0.5(\gamma+\gamma^2)CV_{t+1}^2]}. \tag{19}$$

Transforming the CV item for equation (19) using the Taylor expansion formula for the same index function as in Section 2 means the MRS for the indifference curve under the two-period model for the CV model can be represented as follows:[2]

$$-\left.\frac{dC_{t+1}}{dC_t}\right|_{dZ=0} = \frac{1}{\beta}\left(\frac{C_t}{C_{t+1}}\right)^{-\gamma}\left(\frac{\exp(CV_t^2)}{\exp(CV_{t+1}^2)}\right)^{0.5(\gamma+\gamma^2)} \tag{20}$$

On the other hand, the MRS for the indifference curve under the two-period model for the normal model can be represented by the following equation that eliminated the CV item for equation (20) by substituting $\exp(0)=1$ for the numerator and the denominator in equation (20) for the CV for consumption CV_t to remain zero.

$$-\left.\frac{dC_{t+1}}{dC_t}\right|_{dZ=0} = \frac{1}{\beta}\left(\frac{C_t}{C_{t+1}}\right)^{-\gamma} \quad . \tag{21}$$

[2] When the MRS for equation (20) is the same as the gradient for the budget constraint line, it can be transformed to derive the consumption Euler equation under income uncertainty（equation (6)).

Now, in the case that income uncertainty rises from period t to period t+1 so that $CV_{t+1} > CV_t$, the MRS for the indifference curve under the normal model will not change due to equation (21). However, in the CV model under equation (20), the third item on the right side, which is the growth rate for the consumption CV $\exp(CV_t^2)/\exp(CV_{t+1}^2)$, will be below 1. Therefore, the decline in the MRS produces a change in the focus on future consumption for the entire indifference curve. On the other hand, if income uncertainty falls from period t to period t+1 so that $CV_t > CV_{t+1}$, the MRS for the indifference curve under the normal model will not change due to equation (21). However, in the CV model under equation (20), the third item on the right side, which is the growth rate for the consumption CV $\exp(CV_t^2)/\exp(CV_{t+1}^2)$, will exceed 1. Therefore, the rise in the MRS prompts a change in the focus on present consumption for the entire indifference curve.

Figure 8 shows the correction effect using the two-period model in the estimated value of the degree of relative risk aversion due to the CV model based on the Mehra and Prescott (1985) data used to indicate the U.S. equity premium puzzle.

【 Figure 8 】

In Figure 8, the solid line represents the budget constraint that corresponds to the maximum value (1.40649) and the minimum value (0.72354) for the annualized rate of the return on stocks from 1949 to 1979. The dotted line rising rightward shows the extent of the maximum value (1.04080) and minimum value (0.99650) for the annualized consumption growth rate (cons) fixed at the corresponding subjective equilibrium point. Thus, the magnification of the standard deviation for the stock return relative to the standard deviation for the consumption growth rate is 0.13555/0.01227=11.05x. The extent of change in the consumption growth rate is very small in comparison to the large change in the budget constraint line. The indifference curve for the normal model under this constraint is illustrated by the extremely large convexity in the dotted line in Figure 8. The degree of relative risk aversion for the corresponding CRRA-type utility function is also extremely large.

On the other hand, under the CV model that considers the negative covariance relationship for return on assets and income uncertainty index, a rise in return on assets coincides with a decline in future income uncertainty under household sentiment. Therefore, the increase in the MRS shifts the focus on present consumption for the entire indifference curve. The indifference curve indicating a rise in MRS is not constrained by the budget constraint corresponding to a minimum value (0.72354) for return on stocks. Because it is possible to gently approach something like the solid line in Figure 8, there is no increase in convexity of the indifference curve even when the consumption growth rate fluctuates only slightly. The degree of relative risk aversion for the CRRA-type utility function can be estimated within the normally anticipated realm.

Return on assets and income uncertainty generally trend towards a negative correlation. When an excessive return degree of relative risk aversion is estimated because the fluctuation in consumption is small relative to the fluctuation in return on assets, applying the CV model improves estimation results.

5 Conclusions

A typical attempt to solve the equity premium puzzle uses a non-CRRA alternative preference structure (the time non-separable model that does not assume separability in relation to time and habit formation). However, the generalization and specialization of these utility functions have not yielded generic solutions to the puzzle. Thus, from the viewpoint of parsimony, it would be preferable if the equity premium puzzle could be explained using a simple CRRA utility function.

To this point, data indicates that our model is useful in solving the equity premium puzzle and risk-free rate puzzle with the CRRA utility function. Moreover, it could act as one of the generic solution because there is no need to assume particular household behaviours such as habit formation.

This study performed two tasks to solve the puzzle. First, in addition to the return on assets when deciding the optimal consumption path over multiple periods, we assumed that households accounted for income uncertainty. Similar to the extensive acceptance of precautionary savings arising from income uncertainty, our assumptions have already been accepted by several researchers. Second, we formulated the effect of precautionary savings and withdrawal of these savings that arise from the change in income uncertainty by incorporating them into the Euler equation. Doing so enabled us to estimate the CRRA utility function parameters for households, simultaneously accounting for both the profitability of financial assets and income uncertainty.

Until now, dynamic optimization of consumption across periods and precautionary savings have been discussed as separate topics. We believe that unifying them within one Euler equation is essential to the development of consumption theory and to stable estimations of generalized method of moments (GMM). This unification forges a close and compatible relationship between theory and reality and emerges as a principal way to solve the stagnation seen in applying consumption theories such as C-CAPM, which has been affected by this puzzle over the past 30 years.

Appendix A: Derivation of the individual's expected marginal utility function under income uncertainty

First, we take C_t to represent real consumption for an individual's period t, and take $U(C_t)$ to represent the instantaneous utility function that has additive separability for that point in time. Next, we represent the level of wavering in consumption due to income uncertainty as standard deviation h_t, and assume an uncertain situation in which case there is a 50% probability that individual consumption will increase by only h_t and a 50% probability it will decrease by only h_t. The utility that takes account of the uncertainty in the individual consumption level C_t at this time is as follows.

$$U^*(C_t) = 0.5U(C_t - h_t) + 0.5U(C_t + h_t) \tag{A.1}$$

Here, taking the extent of reduction in the utility level for the consumption level C_t due to the income uncertainty as $\rho(C_t, h_t)$, $\rho(C_t, h_t) = U(C_t) - U^*(C_t)$ and equation (A.1) result in the following equation.

$$U(C_t) - \rho(C_t, h_t) = 0.5U(C_t - h_t) + 0.5U(C_t + h_t) \tag{A.2}$$

Taking the Taylor expansion up to the second order term of the $U(C_t - h_t)$ and $U(C_t + h_t)$ on the right hand side of equation (A.2) and substituting them respectively into equation (A.2) once again means that $\rho(C_t, h_t)$ can be expressed as follows.

$$\rho(C_t, h_t) = -0.5U''(C_t)h_t^2 \tag{A.3}$$

Now, we specify the utility function as the following constant degree of relative risk aversion (CRRA) type.

$$\begin{aligned} U(C_t) &= C_t^{1-\gamma}/(1-\gamma), \quad && \gamma \neq 1, \\ &= \ln(C_t), && \gamma = 1. \end{aligned}$$

However, γ is a parameter representing a constant degree of relative risk aversion, and $1/\gamma$ represents the inter-temporal elasticity of substitution. The second derivative of the CRRA type utility function is $-\gamma / C_t^{\gamma+1}$, so substituting this into equation (A.3) means that $\rho(C_t, h_t)$ can be expressed as follows.

$$\rho(C_t, h_t) = 0.5\gamma C_t^{1-\gamma}(h_t / C_t)^2 \tag{A.4}$$

However, $(h_t / C_t)^2$ represents the square value of the coefficient of variation for consumption at period t, so hereinafter we express this as CV_t^2.

Substituting the CRRA type utility function and equation (A.4) into $U^*(C_t) = U(C_t) - \rho(C_t, h_t)$ means the individual expected utility function under income uncertainty can be expressed as follows.

$$U^*(C_t) = C_t^{1-\gamma} / (1-\gamma)\left[1 - 0.5(\gamma - \gamma^2)CV_t^2\right] \tag{A.5}$$

Taking the derivative of equation (A.5), the individual's expected marginal utility function under income uncertainty can be expressed as follows.[3]

$$U^{*\prime}(C_t) = C_t^{-\gamma}\left[1 + 0.5(\gamma + \gamma^2)CV_t^2\right] \tag{A.6}$$

Appendix B: The Taylor expansion for the three variable's Euler equation of the coefficient of variation model

From equation (12) , target equation for the Taylor expansion can be expressed as follows.

$$f(r^i, g^C, g^{eCVSQ}) = (1 + r^i)(1 + g^C)^{-\gamma}(1 + g^{eCVSQ)})^{0.5(\gamma + \gamma^2)} \tag{B.1}$$

On equation (B.1), by taking the Taylor expansion of the three variable functions up to the second order with $r^i = g^C = g^{eCVSQ} = 0$ or thereabouts and substituting the derived derivatives from a

3 $\partial CV_t / \partial C_t = -h_t / C_t^2$ is used in the development from equation (A.5) to equation (A.6).

separate calculation into it results in the following approximation equation.

$$f(r^i, g^C, g^{eCVSQ}) = 1 + r^i - \gamma g^C - \gamma r^i g^C + 0.5(\gamma + \gamma^2)(g^C)^2$$
$$+ 0.5(\gamma + \gamma^2) g^{eCVSQ} + 0.25(\gamma + \gamma^2)[0.5(\gamma + \gamma^2) - 1](g^{eCVSQ})^2$$
$$- 0.5\gamma(\gamma + \gamma^2) g^C g^{eCVSQ} + 0.5(\gamma + \gamma^2) r^i g^{eCVSQ} \tag{B.2}$$

Substituting the result of equation (B.2) into equation (12) leads to the following.

$$E[r^i] - \gamma E[g^C] - \gamma\{E[r^i]E[g^C] + Cov(r^i, g^C)\} + 0.5(\gamma + \gamma^2)\{(E[g^C])^2 + Var(g^C)\}$$
$$+ 0.5(\gamma + \gamma^2) E[g^{eCVSQ}]$$
$$+ 0.25(\gamma + \gamma^2)\{0.5(\gamma + \gamma^2) - 1\}\{(E[g^{eCVSQ}])^2 + Var(g^{eCVSQ})\}$$
$$- 0.5\gamma(\gamma + \gamma^2)\{E[g^C]E[g^{eCVSQ}] + Cov(g^C, g^{eCVSQ})\})$$
$$+ 0.5(\gamma + \gamma^2)\{E[r^i]E[g^{eCVSQ}] + Cov(r^i, g^{eCVSQ})\} \cong \rho \tag{B.3}$$

Ignoring each item $E[r^i]E[g^C]$, $(E[g^C])^2$, $(E[g^{eCVSQ}])^2$, $E[g^C]E[g^{eCVSQ}]$, $E[r^i]E[g^{eCVSQ}]$ with a comparatively small value, replacing with 0, and solving for $E[r^i]$ results in the following equation.

$$E[r^i] \cong \rho + \gamma E[g^C] + \gamma Cov(r^i, g^C) - 0.5(\gamma + \gamma^2) Var(g^C)$$
$$- 0.5(\gamma + \gamma^2) E[g^{eCVSQ}] - 0.25(\gamma + \gamma^2)\{0.5(\gamma + \gamma^2) - 1\} Var(g^{eCVSQ})$$
$$+ 0.5\gamma(\gamma + \gamma^2) Cov(g^C, g^{eCVSQ}) - 0.5(\gamma + \gamma^2) Cov(r^i, g^{eCVSQ}) \tag{B.4}$$

The rate of return on the risk free asset is determined to have no relationship to the consumption growth rate and the growth rate of income coefficient of variation, so substituting $Cov(r^i, g^C) = 0$ and $Cov(r^i, g^{eCVSQ}) = 0$ into equation (B.4) results in the following equation.

$$\bar{r} \cong \rho + \gamma E[g^C] - 0.5(\gamma + \gamma^2) Var(g^C) - 0.5(\gamma + \gamma^2) E[g^{eCVSQ}]$$
$$- 0.25(\gamma + \gamma^2)\{0.5(\gamma + \gamma^2) - 1\} Var(g^{eCVSQ})$$
$$+ 0.5\gamma(\gamma + \gamma^2) Cov(g^C, g^{eCVSQ}) \tag{B.5}$$

Subtracting equation (B.5) from equation (B.4) leads to the following equation relating to the equity premium, the degree of relative risk aversion, the covariance between the return on assets and the consumption growth rate, as well as the growth rate of income coefficient of variation.

$$E[r^i] - \bar{r} \cong \gamma Cov(r^i, g^C) - 0.5(\gamma + \gamma^2) Cov(r^i, g^{eCVSQ}) \qquad \text{(B.6)}$$

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Table 1

Descriptive statistics for the calculation of the degree of relative risk aversion

		σ ic	σ iv (0.1)	σ iv (0.3)	σ iv (0.5)	Equity Premiun
CV1	max	0.0000236	−0.0000001	−0.0000008	−0.0000022	0.0058050
	min	0.0000029	−0.0000003	−0.0000023	−0.0000064	0.0018320
	mean	0.0000139	−0.0000002	−0.0000015	−0.0000040	0.0036489
	s.d.	0.0000060	0.0000000	0.0000003	0.0000009	0.0009072
CV2	max	0.0000236	−0.0000015	−0.0000137	−0.0000383	0.0058050
	min	0.0000029	−0.0000052	−0.0000472	−0.0001326	0.0018320
	mean	0.0000139	−0.0000030	−0.0000274	−0.0000769	0.0036489
	s.d.	0.0000060	0.0000011	0.0000096	0.0000275	0.0009072
CV3	max	0.0000236	0.0000063	0.0000560	0.0001554	0.0058050
	min	0.0000029	0.0000000	−0.0000002	−0.0000012	0.0018320
	mean	0.0000139	0.0000034	0.0000306	0.0000849	0.0036489
	s.d.	0.0000060	0.0000012	0.0000111	0.0000308	0.0009072
CV4	max	0.0000236	−0.0000094	−0.0000858	−0.0002428	0.0058050
	min	0.0000029	−0.0000175	−0.0001597	−0.0004525	0.0018320
	mean	0.0000139	−0.0000127	−0.0001154	−0.0003266	0.0036489
	s.d.	0.0000060	0.0000023	0.0000211	0.0000602	0.0009072

Notes:
CV1 : Labor Share (spline conversion to monthly)
CV2 : Reciprocal for Unit Profit (spline conversion to monthly)
CV3 : Unemployment rate UNRATE
CV4 : Reciprocal of the University of Michigan consumer confidence index UMCSENT

Table 2

Descriptive statistics for the calculation of the subjective discount rate

		E[r]	E[gc]	σ c^2	E[gecv] (0.1)	E[gecv] (0.3)	E[gecv] (0.5)
CV1	max	0.0029470	0.0017634	0.0000182	0.0000014	0.0000125	0.0000352
	min	0.0017890	0.0013067	0.0000116	−0.0000048	−0.0000429	−0.0001185
	mean	0.0024434	0.0015304	0.0000140	−0.0000022	−0.0000199	−0.0000545
	s.d.	0.0003133	0.0000994	0.0000019	0.0000015	0.0000139	0.0000385
CV2	max	0.0029470	0.0017634	0.0000182	0.0000850	0.0007814	0.0022632
	min	0.0017890	0.0013067	0.0000116	−0.0000136	−0.0001096	−0.0002353
	mean	0.0024434	0.0015304	0.0000140	0.0000105	0.0001118	0.0004043
	s.d.	0.0003133	0.0000994	0.0000019	0.0000201	0.0001810	0.0005026
CV3	max	0.0029470	0.0017634	0.0000182	0.0000441	0.0004100	0.0012343
	min	0.0017890	0.0013067	0.0000116	−0.0000333	−0.0002676	−0.0005820
	mean	0.0024434	0.0015304	0.0000140	−0.0000071	−0.0000434	−0.0000088
	s.d.	0.0003133	0.0000994	0.0000019	0.0000188	0.0001668	0.0004624
CV4	max	0.0029470	0.0017634	0.0000182	0.0000482	0.0005126	0.0018613
	min	0.0017890	0.0013067	0.0000116	−0.0000196	−0.0000925	0.0000807
	mean	0.0024434	0.0015304	0.0000140	−0.0000016	0.0000498	0.0004978
	s.d.	0.0003133	0.0000994	0.0000019	0.0000128	0.0001167	0.0003378

		σ v^2 (0.1)	σ v^2 (0.3)	σ v^2 (0.5)	σ vc (0.1)	σ vc (0.3)	σ vc (0.5)
CV1	max	0.0000000	0.0000002	0.0000018	0.0000000	−0.0000001	−0.0000002
	min	0.0000000	0.0000002	0.0000013	0.0000000	−0.0000003	−0.0000009
	mean	0.0000000	0.0000002	0.0000015	0.0000000	−0.0000001	−0.0000004
	s.d.	0.0000000	0.0000000	0.0000001	0.0000000	0.0000001	0.0000002
CV2	max	0.0000006	0.0000497	0.0003830	−0.0000002	−0.0000021	−0.0000058
	min	0.0000003	0.0000256	0.0001984	−0.0000006	−0.0000054	−0.0000152
	mean	0.0000005	0.0000380	0.0002932	−0.0000003	−0.0000031	−0.0000087
	s.d.	0.0000001	0.0000080	0.0000608	0.0000001	0.0000008	0.0000023
CV3	max	0.0000009	0.0000708	0.0005452	−0.0000002	−0.0000018	−0.0000050
	min	0.0000004	0.0000328	0.0002529	−0.0000005	−0.0000047	−0.0000130
	mean	0.0000006	0.0000453	0.0003490	−0.0000003	−0.0000027	−0.0000076
	s.d.	0.0000001	0.0000101	0.0000774	0.0000001	0.0000006	0.0000016
CV4	max	0.0000025	0.0002024	0.0015847	−0.0000003	−0.0000029	−0.0000084
	min	0.0000014	0.0001132	0.0008800	−0.0000009	−0.0000080	−0.0000227
	mean	0.0000018	0.0001454	0.0011335	−0.0000005	−0.0000050	−0.0000141
	s.d.	0.0000003	0.0000259	0.0002036	0.0000001	0.0000012	0.0000035

Notes:
CV1 : Labor Share (spline conversion to monthly)
CV2 : Reciprocal for Unit Profit (spline conversion to monthly)
CV3 : Unemployment rate UNRATE
CV4 : Reciprocal of the University of Michigan consumer confidence index UMCSENT

Figure 1

Determination of the degree of relative risk aversion

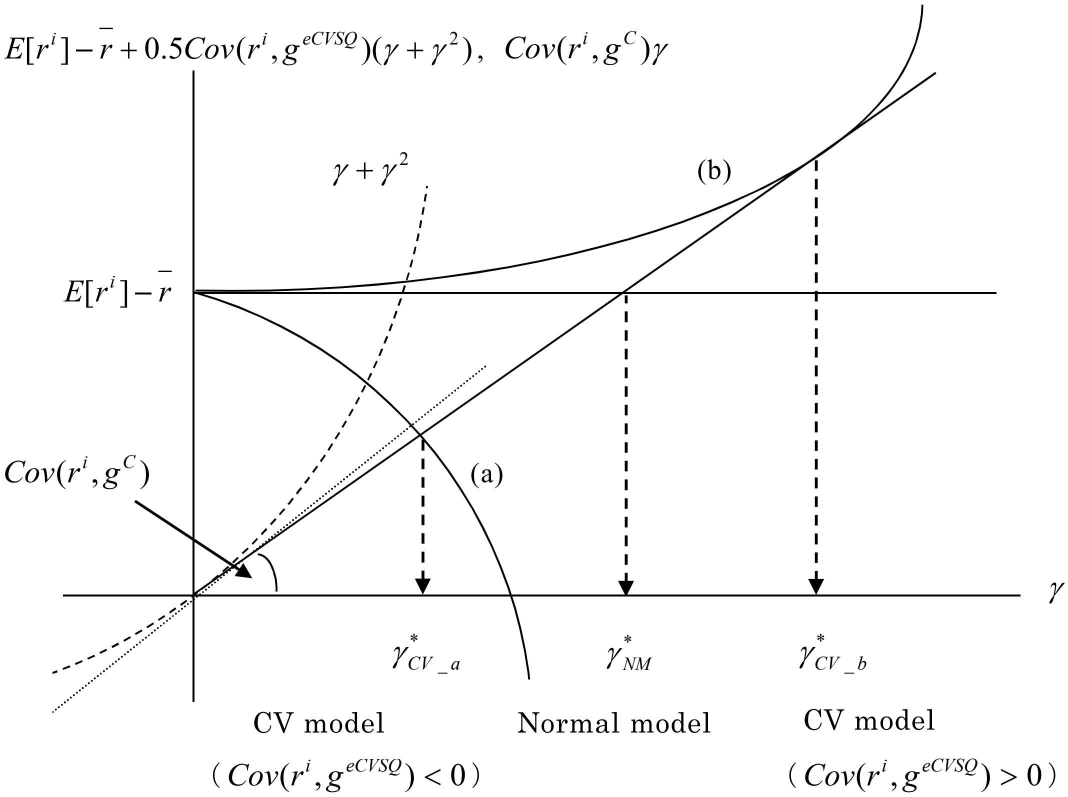


Figure 2

Movement of the US income uncertainty index（CV1-4, Feb-1978=100）

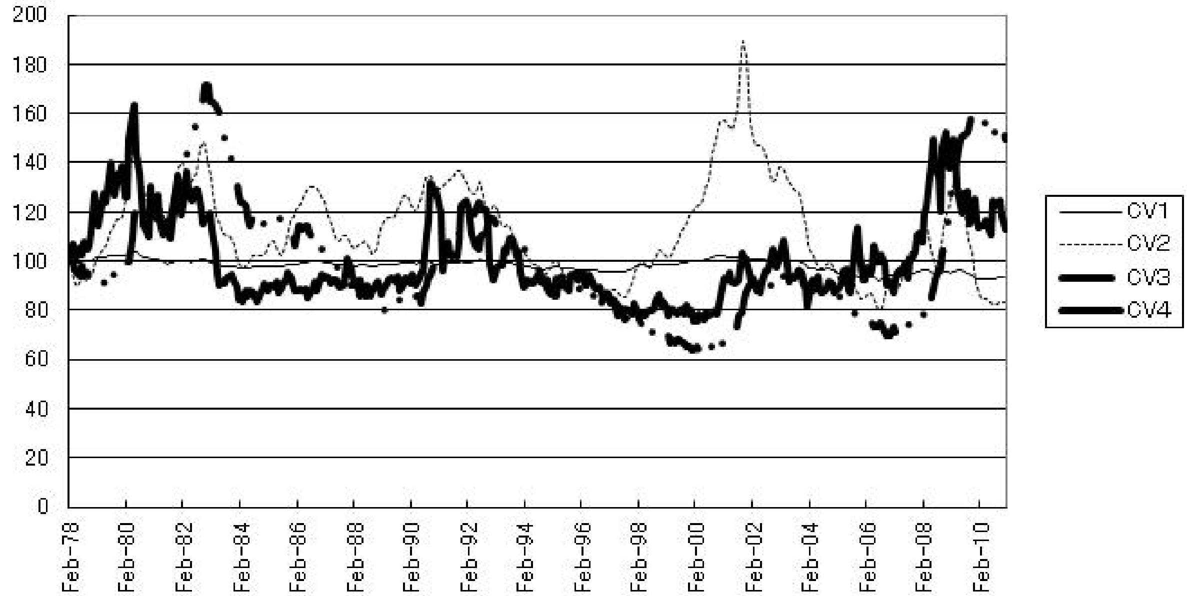


Figure 3

Movement in the average value of the risk premium

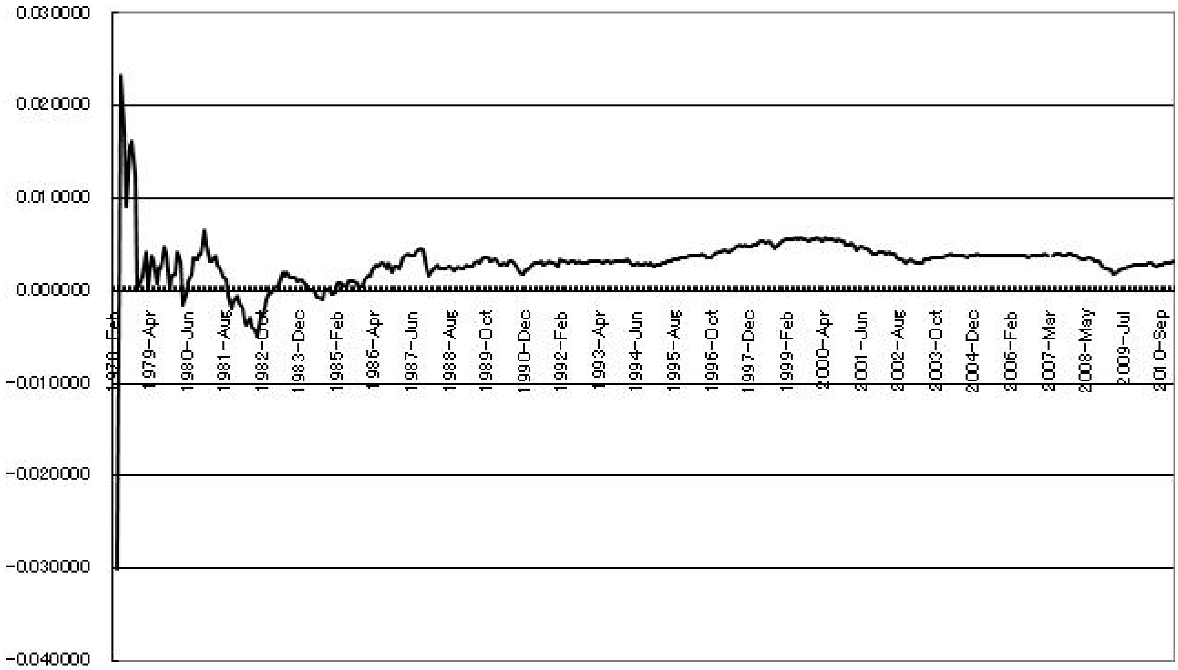
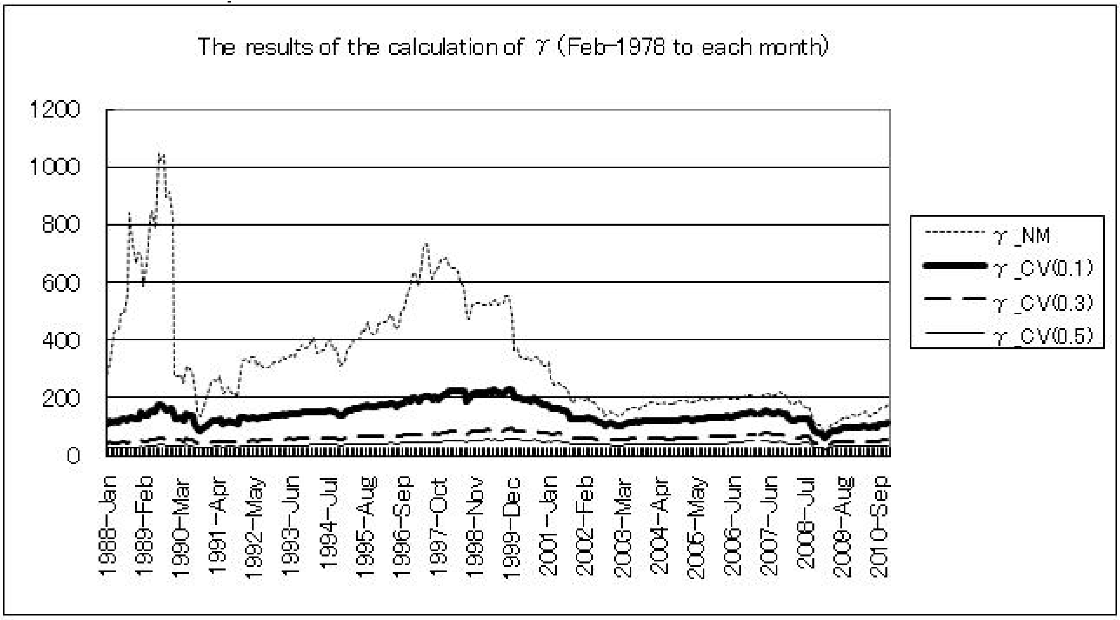


Figure 4

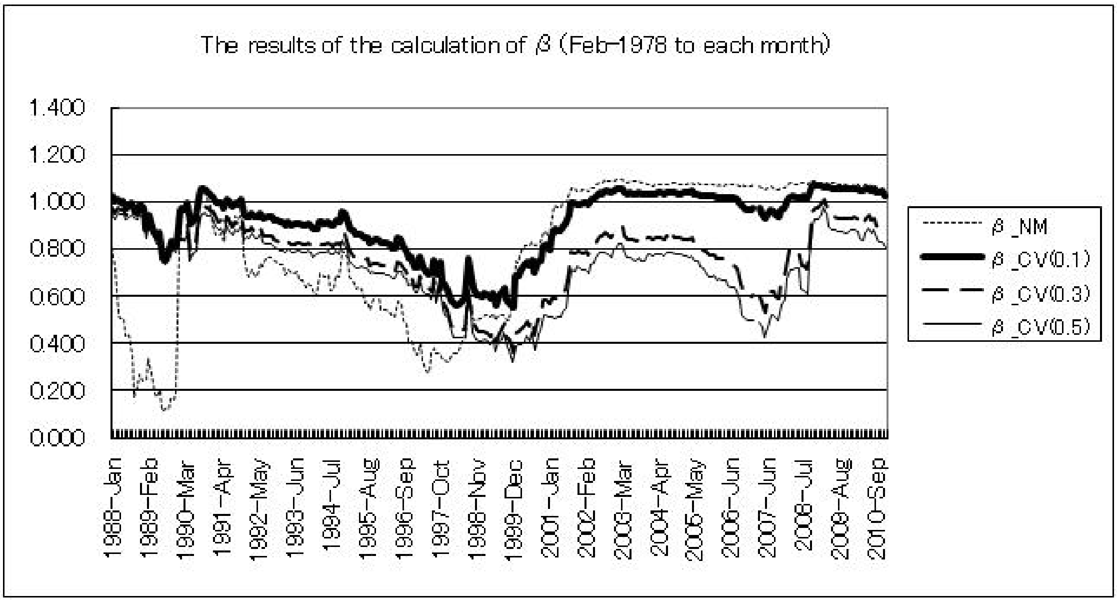
The results of the calculation of the degree of relative risk aversion and subjective discount rate

(Income uncertainty index: Labor Share CV1)



Average value of the solution

γ_NM	γ_CV(0.1)	γ_CV(0.3)	γ_CV(0.5)
333.259	144.966	61.990	39.175



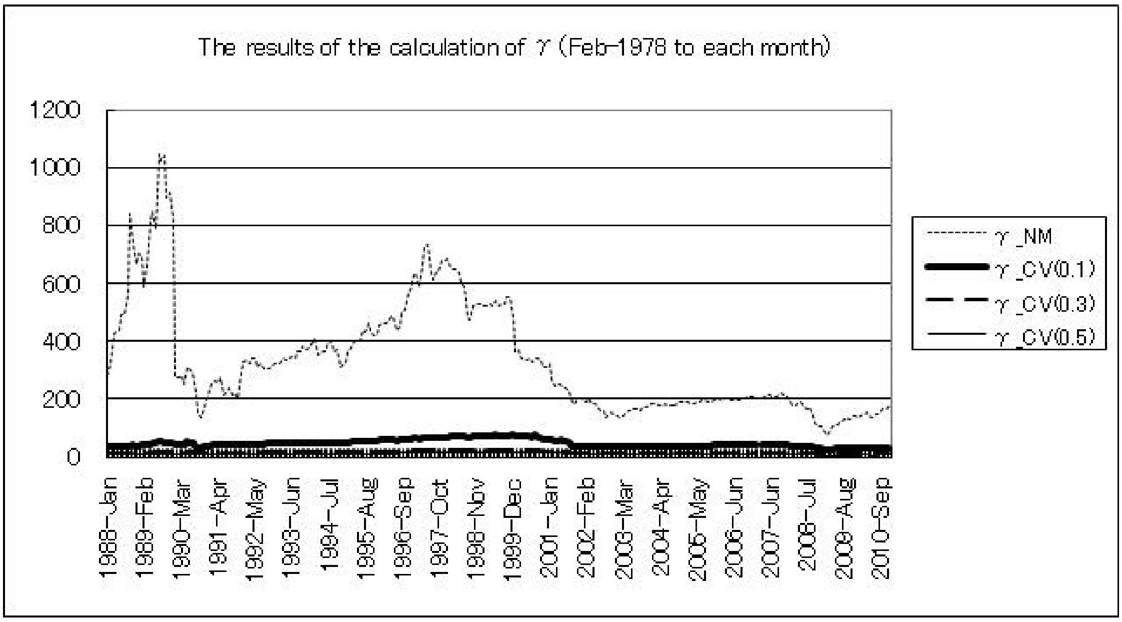
Average value of the solution

β_NM	β_CV(0.1)	β_CV(0.3)	β_CV(0.5)
0.800	0.914	0.766	0.714

Figure 5

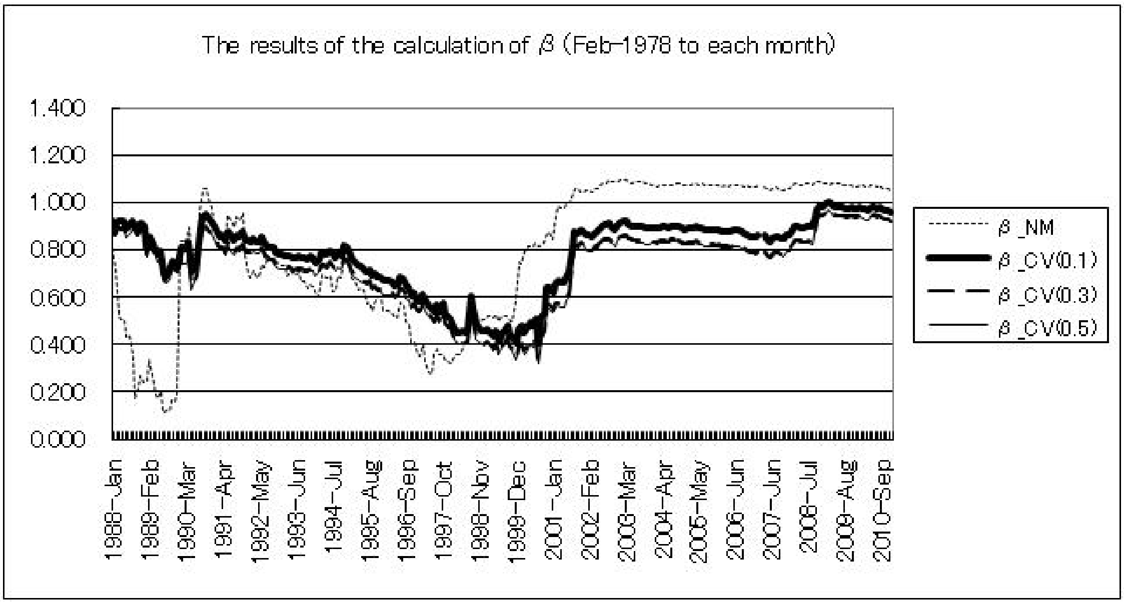
The results of the calculation of the degree of relative risk aversion and subjective discount rate

(Income uncertainty index: The reciprocal of Unit Profit CV2)



Average value of the solution

γ _NM	γ _CV(0.1)	γ _CV(0.3)	γ _CV(0.5)
333.259	46.688	16.176	9.600



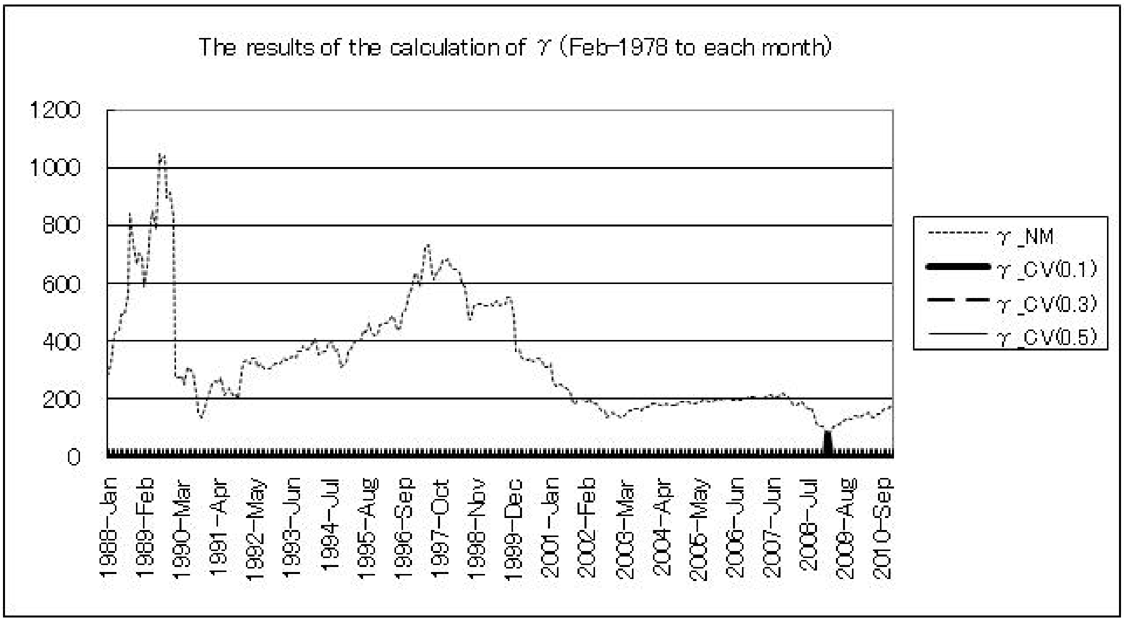
Average value of the solution

β _NM	β _CV(0.1)	β _CV(0.3)	β _CV(0.5)
0.800	0.785	0.734	0.725

Figure 6

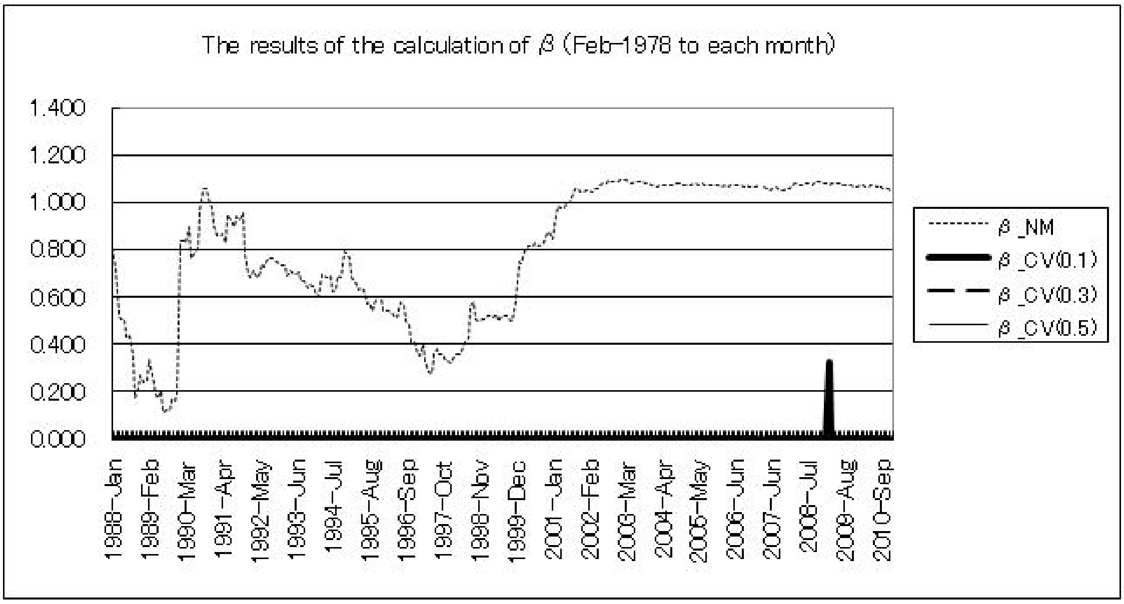
The results of the calculation of the degree of relative risk aversion and subjective discount rate

(Income uncertainty index: The Unemployment Rate UNRATE CV3)



Average value of the solution

γ _NM	γ _CV(0.1)	γ _CV(0.3)	γ _CV(0.5)
333.259	#NUM!	#NUM!	#NUM!

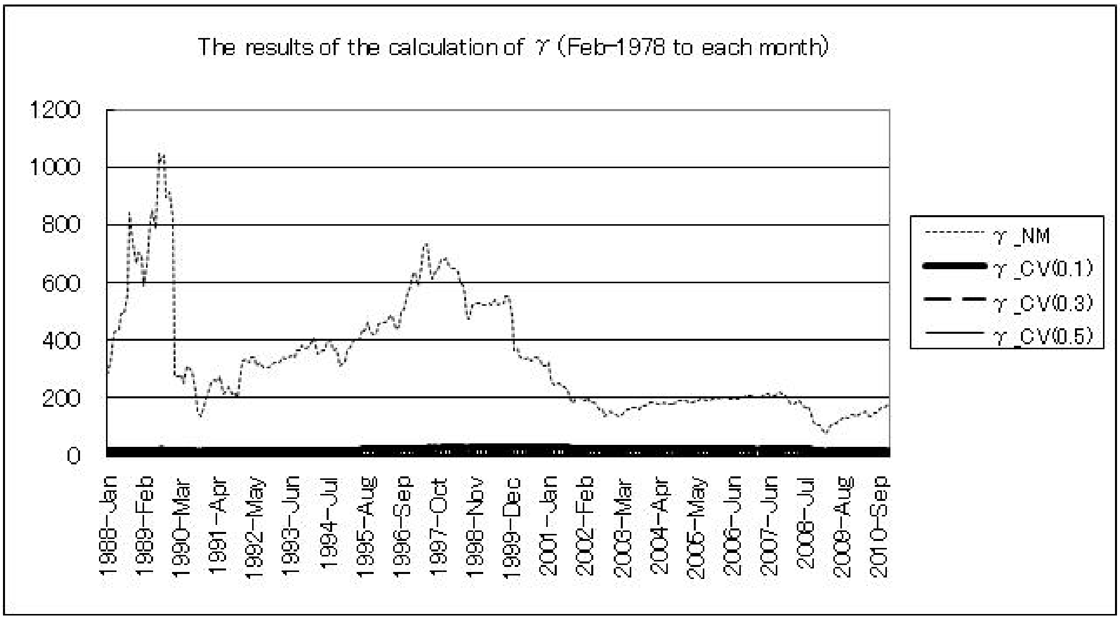


Average value of the solution

β _NM	β _CV(0.1)	β _CV(0.3)	β _CV(0.5)
0.800	#NUM!	#NUM!	#NUM!

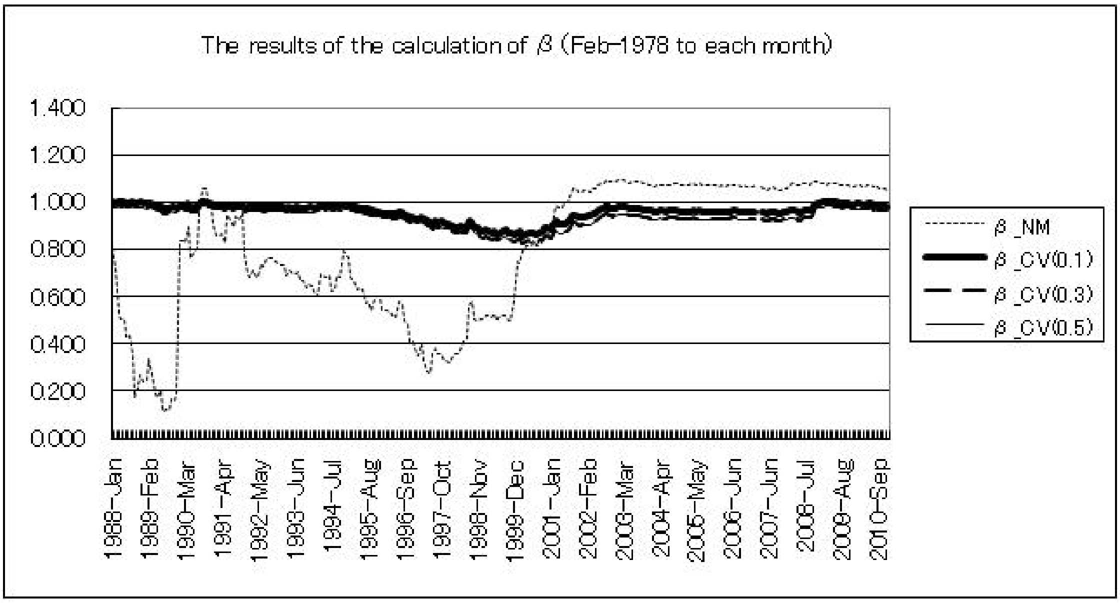
Figure 7

The results of the calculation of the degree of relative risk aversion and subjective discount rate (Income uncertainty index: The reciprocal of the University of Michigan consumer confidence index UMCSENT CV4)



Average value of the solution

γ_NM	γ_CV(0.1)	γ_CV(0.3)	γ_CV(0.5)
333.259	22.657	7.444	4.270



Average value of the solution

β_NM	β_CV(0.1)	β_CV(0.3)	β_CV(0.5)
0.800	0.957	0.935	0.931

Figure 8

The correction effect in the estimated value of the degree of relative risk aversion