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ULTIMATE CALCULATION METHOD FOR CELESTIAL NAVIGATION

—A New and Accurate Method of Finding a Ship's Position at Sea—

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ABSTRACT

This paper describes the new method for Celestial Navigation to find a ship's position immediately and accurately using personal computers. Authors call it "Ultimate Calculation Method for Celestial Navigation". It has theoretically been known for a long time. However, authors developed the system for personal computers which draws a huge globe on the screen of PC(Personal Computer) using the orthographic projection, displays the intersection of the circle of position around the viewpoint to the screen center and decides the latitude and longitude of ship's position precisely. This method calculates the position within a difference of less than 0.01′(0.6") in both latitude and longitude. It is not necessary to input the estimated position as is necessary for the Intercept method.

Keywords: navigation, ship, position, calculation, celestial body

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1. ALGORITHM OF ULTIMATE CALCULATION METHOD FOR CELESTIAL NAVIGATION

1.1 Principle of Ultimate Calculation Method for Celestial Navigation

First, rotate a line which links the geographical position (GP) of a celestial body with the center of the earth as axis, and make a cone with its vertical angle zenith distance. The curve which contacts with the earth's surface becomes a circle. The GP becomes the centre of the circle, and its radius is the zenith distance. It is called the circle of position(COP). The ship's position is supposed to be somewhere on this circle.

The circle is drawn on the globe with the GP of a celestial body as the centre and the zenith distance as the radius of the circle. This is the COP. The same procedure is followed for other celestial bodies. The intersection of COPs is determined as the ship's position.

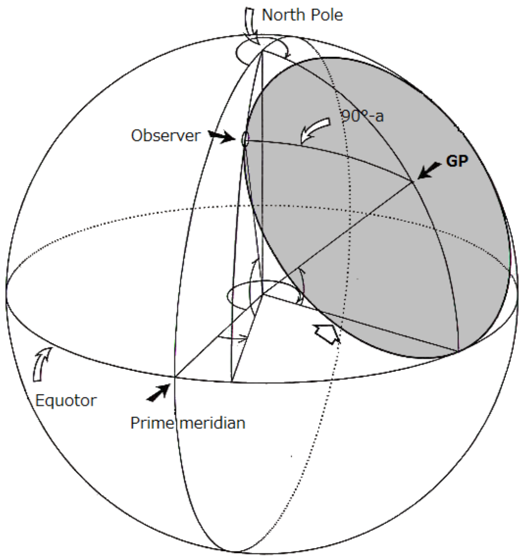

Fig.1 Geographical Position (GP) of a celestial body

A huge globe is needed for this method to specify the GP of celestial bodies by using more than two times observations, to draw the circles with GP as the centre and co-altitude as the radius, and to find the intersection of COPs, which is the ship's position.

In Fig. 2, S_1 and S_2 are the GPs of two celestial bodies, respectively, and P_1 and P_2 are the intersections of the COPs; one of the two intersections can be determined as the ship's position.

In this method, orthographic projection is used to draw a globe.

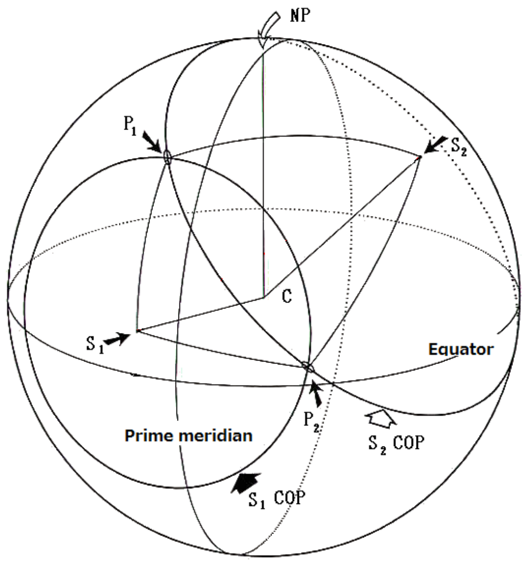

Fig.2 Intersections (P) of Circle of Positions

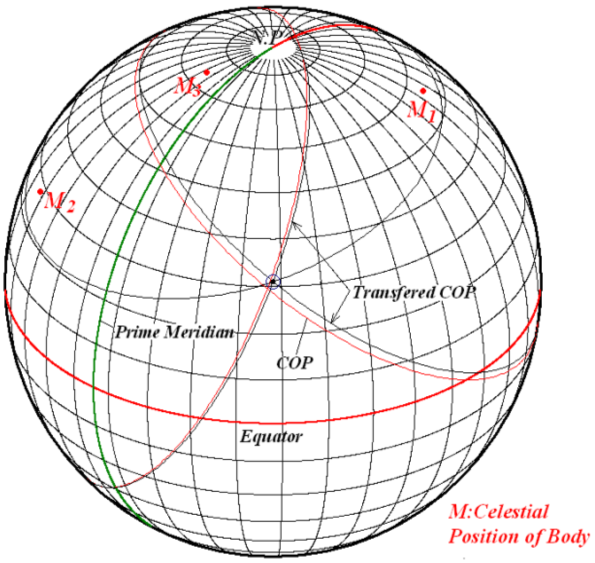

Fig. 3 Celestial Observation and Transferred COP

What should be the globe magnification?

The globe can be magnified to 1,000 times because the grid space of the latitudes is larger than that of the longitudes near the Arctic or the Antarctic.

1.2 Algorithm of Ultimate Calculation Method for Celestial Navigation

(1) COP formula

The Greenwich hour angle (GHA) and declination at the time of observation are needed to fix the GP of the celestial body onto the globe. GHA is equal to the longitude, and declination is equal to the latitude.

In the following formula, ℓ_E, L_E represent the latitude and longitude of the screen centre, respectively; ℓ_P, L_P represent the latitude and longitude on COP, respectively; h_G represents GHA; d represents declination; a_t represents the observed true altitude; and Dia represents the semi-diameter of the globe. The COP formula is used when the globe is drawn as well.

The semi-diameter depends on the resolution of the computer.

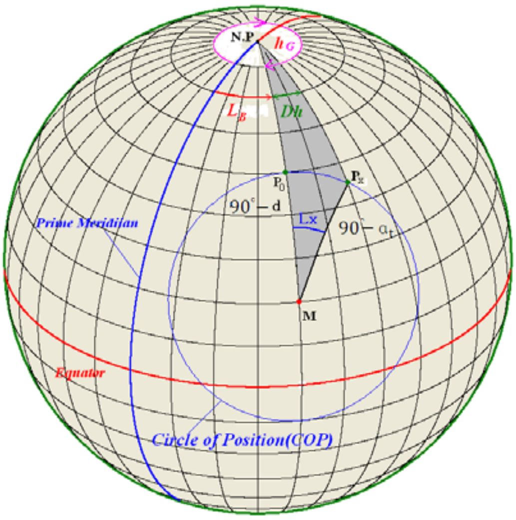

Fig.4 Celestial Triangle and COP

In Fig. 4, M represents the GP of the celestial body. P_0 represents the point on the (southern) meridian. P_x represents a given point.

$M \sim P_0$, $M \sim P_x$ represent the co-altitude, and ***N.P*** (North pole)$\sim$M represent the co-declination.

In the shaded celestial triangle（△M, N.P, Px）if the value $\angle L_x$ is given, the co-altitude for P_x, ***Xp***, is calculated using the following formula:

$$X_P = \cos^{-1}(\sin d \cdot \sin a_t + \cos d \cdot \cos a_t \cdot \cos L_X) \qquad \text{----------} \ (1.2)$$

Therefore, the latitude for COP , ℓ_P ,is fixed as follows;

$$\ell_P = 90^\circ - X_P \qquad \text{----------} \ (1.3)$$

$\angle D_h$ is fixed as follows;

$$\sin a_t = \cos Xp \cdot \sin d + \sin Xp \cdot \cos d \cdot \cos Dh \qquad \text{----------} \ (1.4)$$

As the East longitude is eastward of the prime meridian and the West longitude is westward of the prime meridian, the longitude for the celestial body, L_B, is calculated as follows:

$$L_B = 360° - h_G \qquad \text{----------(1.5)}$$

If the value of L_B is greater than 180°, minus 360°.

L_P (longitude on COP) can be fixed by deforming(1.4) , and adding L_B.

$$L_P = \cos^{-1}\left(\frac{\sin a_t - \cos X_P \cdot \sin d}{\sin X_P \cdot \cos d}\right) + L_B \qquad \text{--------------- (1.6)}$$

(2) Transformation to the plane coordinate

Now the coordinates for COP are calculated. They are transformed to the plane coordinate to draw the globe on the screen.

To transform these values to the plane coordinate, solve the formula in the following condition. North Latitude is shown as＋, South Latitude as－, East Longitude as＋, West Longitude as－, coordinates of screen centre as

(ℓ_E, L_E) , and

$L = L_p - L_E$.

The result is calculated with the coordinates of the screen centre as (0,0), right side of the X-axis as ＋, and lower side of the Y-axis as ＋.

$$X = Dia \cdot \sin(L) \cdot \cos(\ell_P)$$
$$Y = -Dia(\sin(\ell_P) \cdot \cos(\ell_E) - \sin(\ell_E) \cdot \cos(\ell_P) \cdot \cos(L)) \qquad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ (1.7)$$

This formula can be found if the viewpoint of coordinate expression by external prospection projection oblique aspect is changed from the center of the earth to infinity.

The globe should be rotated up and down, right to left on the screen. The amount of rotation must be changed from 1' to 30°.

Programming techniques are needed to move the intersection to the centre on the screen within several seconds just by clicking it.

The COP can be displayed after calculating L_X as above from P_0 in the clock wise direction to the original point P_0 by a multiple of 360 repeatedly, thereby linking each points.

(3) Shadow processing

If X, Y are on the rear of the globe, the grids for the latitude and longitude, COP, and shore lines are not displayed when the following condition is concluded;

$$X^2 + Y^2 > Dia^2 \qquad \cdot\ \cdot\ \cdot\ \cdot\ (1.8)$$

If the angular distance (Dx) between the coordinates of the screen centre and some point is more than 90˚, the point is on the rear of the globe.

In the actual calculation, the following formula is used.

$$0 \geq (\sin(\ell_x)\cdot\cos(\ell_E) - \sin(\ell_E)\cdot\cos(\ell_x)\cdot\cos(L)) \quad \cdot\cdot\cdot\cdot\cdot \ (1.9)$$

Under these conditions, ℓ_E represents the latitude of the screen centre, ℓ_x , the latitude of some point, and L, the difference in longitude between the two points.

The globe drawn in this book uses 61,000 items of coastline data from the National Oceanic and Atmospheric Administration (NOAA), United States Department of Commerce.

On the globe, geographical figures including the Arctic and Antarctic circles, and COP are displayed on the screen.

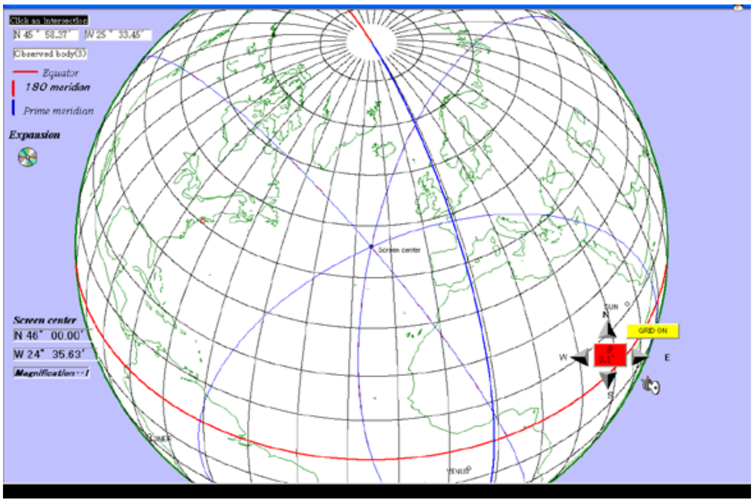

Fig. 5 The globe shown on the screen

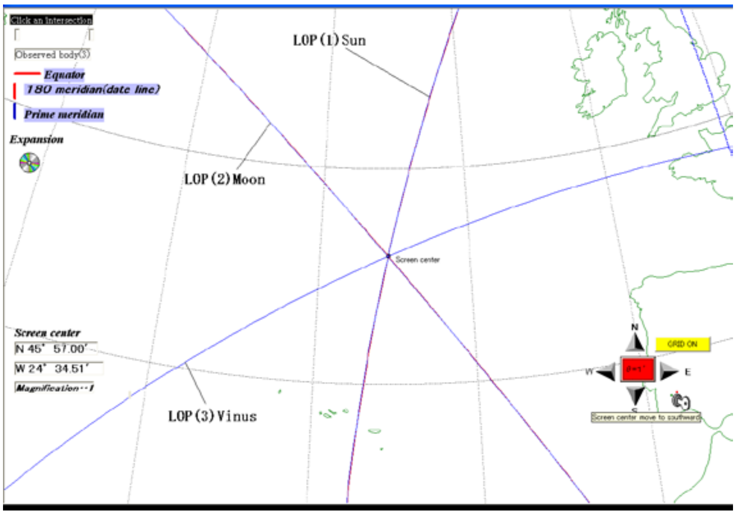

Fig.6 Extended figure of Fig.5 to 4 times

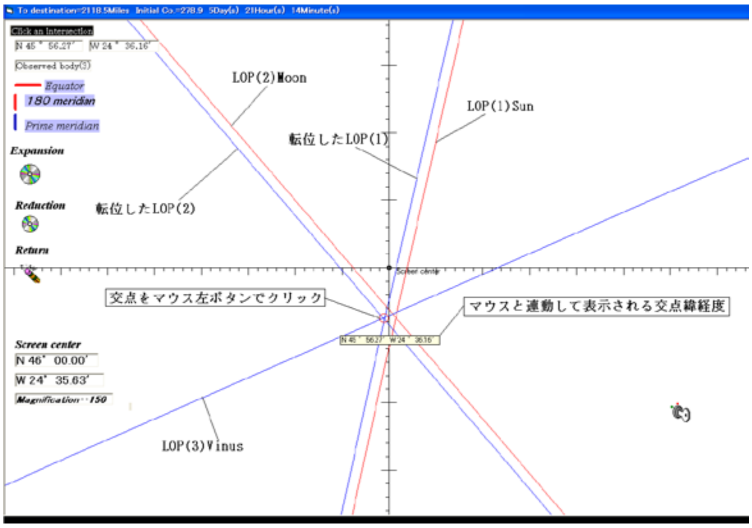

Fig.7 Extended figure of Fig.5 to 150 times

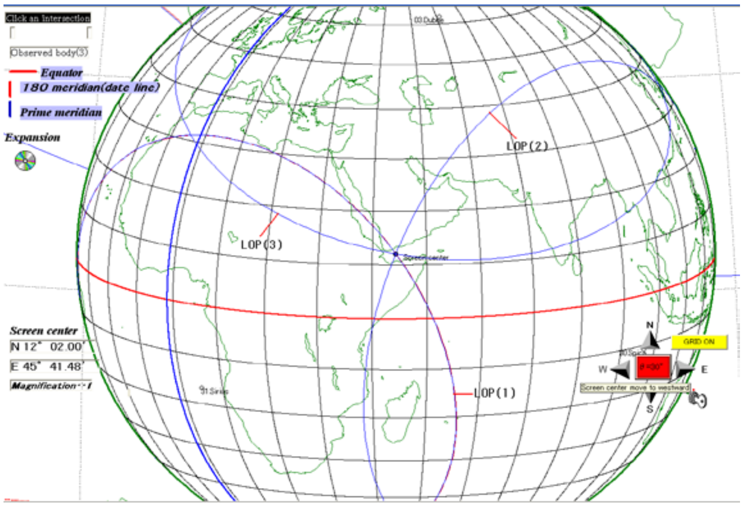

Fig.8 Three COPs on the globe

Fig. 8 shows the figure which was verified using the example from the book 'Celestial Navigation' written by Susumu Sakai, published by Kaibundo Publishing Co., in 1961. Fig. 9 is a 400-times magnified version of Fig. 8. The grid interval for the latitude and the longitude is 1' each.

The result obtained using the method described in 'Celestial Navigation' is a little different from that obtained using the ultimate calculation method described above; however, it can be said that it is calculated correctly as the solution with a manual calculation using the nautical almanac.

The result obtained using the method described in 'Celestial Navigation' is N12˚04.0′E45˚40.0′.

The result obtained using the Ultimate Calculation method is N12˚03.87′E45˚40.33′.

The method described in 'Celestial Navigation' yields the result of N12˚03.87′ E45˚40.30′, which was calculated using GHA, declination, and the true altitude in an example of the book. There is a difference of 0.18′ with the accurate computation. This is the limitation of a manual calculation using nautical almanac.

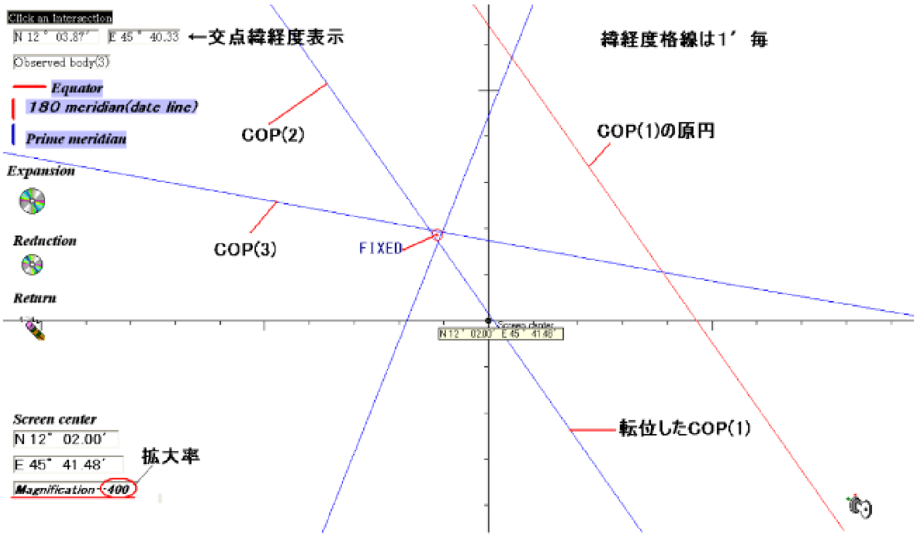

Fig. 9 400x magnification of Fig. 8

(4) The decision of the true position out of the two intersections obtained by two celestial observations

Two intersections are obtained by two celestial observations, and we need to decide which point of the two is the true position.

In order to determine the true position, we observe the azimuth angle for the celestial body and judge the correct position from the two.

2. THE PRECISION OF ULTIMATE CALCULATION METHOD

This method calculates the given data by simultaneous observation within a difference of about less than 0.01′(0.6") in both latitude and longitude and displays the intersection precisely.

Some examples are as follows;

[Example 1]

There are two intersections obtained using two celestial observations as follows;

	Observation 1	**Observation2**
*Greenwich hour angle(*hg*)	100.000000°	200.000000°
*Declination (*d*)	+30.000000°	−10.000000°
*True altitude (*a*)	11°06′22.6522"	29°21′56.7222"

From the data obtained by simultaneous observation, the exact solution for the intersection is,

1) N50˚00′00.000",
   E150˚00′00.000"
2) S39˚34′39.113",
   W140˚01′09.405"

Here, the term “exact solution” means the analytic calculation without using the estimated position. The verification for the result is as follows:

Fig.10 Intersection(1) on the globe

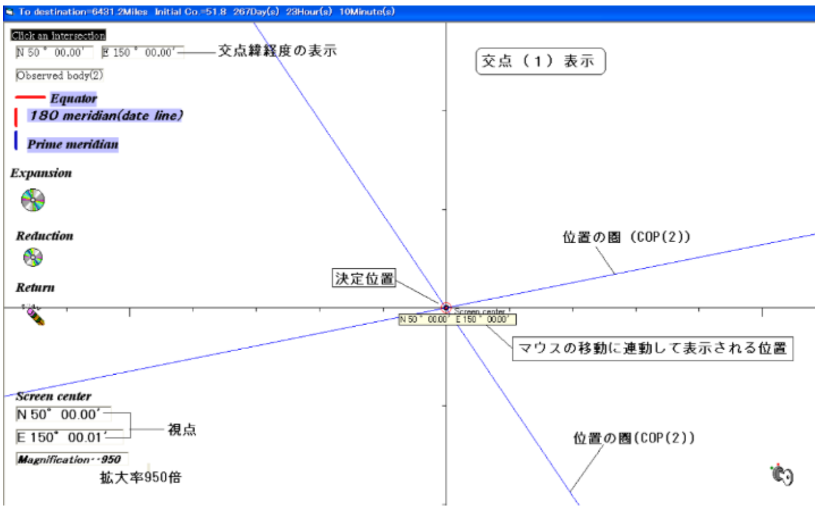

Fig.11 Magnified version of Fig.10

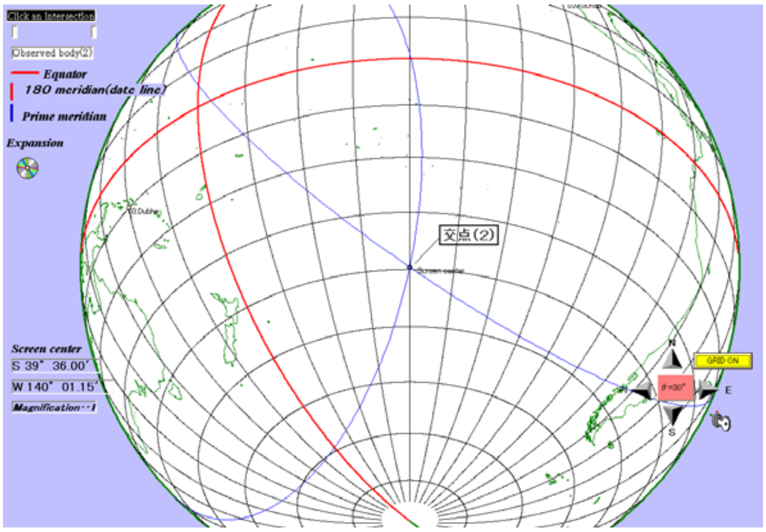

Fig.12 Intersection(2) on the globe

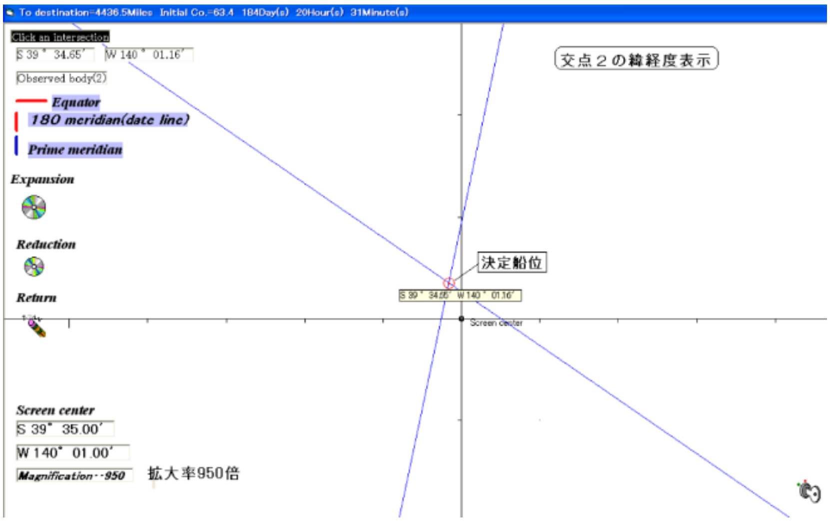

Fig.13 Magnified version of Fig.12

- The exact solution for intersection(1);
  N50˚00′00.000", E150˚00′00.000"
- Displayed latitude and longitude;
  N50˚00′00.000", E150˚00′00.000"

<Difference> 0.0" , 0.0"

- The exact solution for intersection(2)
  S39˚34′39.113", W140˚01′09.405"
- Displayed latitude and longitude
  S39˚34′39.0", W140˚01′09.6"

<Difference> 0.1" , 0.2"

[Example 2]

Calculate the intersection of COP by simultaneous observation for three celestial bodies.

Observations 1 , 2 and 3

■ Hg

20.000000° 320.000000° 0.000000°

■ *d*

+35.00000° +45.000000°-20.000000°

■ *a*

63.355492° 51.826331° 49.701413°

From the data, the result is as follows;

● The exact solution for the intersection

N20°00′00.000", E5°00′00.000"

● Displayed latitude and longitude

N20°00′00.0", E50°00′00.0"

There is no difference between the value of the exact solution and the calculated result. Here, exact solution means the analytic calculation using the least square method, without using the estimated position.

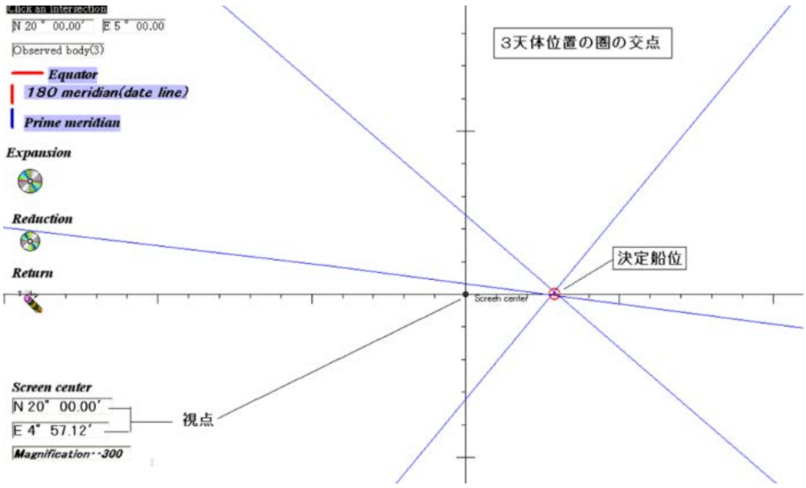

Fig.14 Intersection of COPs for three celestial bodies

◆ **Comprehension Evaluation for Precision of Position Fixing**

The error of position fixing is within 0.01′(0.6″) when the observation was carried out at a latitude of less than 80° by simultaneous observation, and when if the altitude was more than 89° (high altitude observation), the error of position fixing was only 0.01′ in latitude and 0.05 in longitude when the observation was carried out at a latitude of 85° and the actual distance was 81 m.

The error of position fixing was about 0.5′ for both latitude and longitude when observations were carried out with a run which has some intervals between observations and needs a long distance transposition. If the altitude is more than 70° (high altitude observation), the error of position fixing is less than 1′ by observation with run.

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