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Cheap Talk with an Exit Option: A Model of Exit and Voice*

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Abstract

The paper presents a formal model of the exit and voice framework proposed by Hirschman [20]. More specifically, we modify Crawford and Sobel's [10] cheap talk model such that the sender of a cheap talk message has an exit option. We demonstrate that the presence of the exit option may increase the informativeness of cheap talk and improve welfare if the exit option is relatively attractive to the sender and relatively unattractive to the receiver. Moreover, it is verified that perfect information transmission can be approximated in the limit. The results suggest that the exit reinforces the voice in that the credibility of exit increases the informativeness of the voice.

Keywords: Exit, Voice, Cheap Talk, Informativeness, Credibility

Journal of Economic Literature Classification Numbers: D23, D82, J5, L22

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1 Introduction

Since the publication of the book *Exit, Voice, and Loyalty* by Hirschman [20], the exit-voice perspective has been widely adopted by studies in the field of political science, and it has also been extended to various studies on relationships and organizations, such as employer-employee (or union) relationships, buyer-seller relationships, hierarchies, public services, political parties, families, and adolescent development (See Hirschman [20] and [21]).

Broadly speaking, the exit and the voice are alternative means of dealing with problems that arise within an ongoing relationship or organization. For example, consider an employer-employee relationship.[1] Suppose an employee finds himself/herself in undesirable situations regarding the conditions of employment, compensation packages, and rules at the workplace. In this situation, the employee usually has two options. One is to quit the job; this is the exit option. The other is to express his dissatisfaction directly to the employer; this is the voice option. Hirschman insists that the voice as well as the exit option is important for the sustainability of relationships and organizations—a concept that has been hitherto neglected in economics.

With regard to the workings of the exit and voice options in a real economy, how the exit interacts with the voice is a point of considerable interest, which is the main point of Hirschman's discussion. From one perspective, the exit works as a complement to the voice. Indeed, in Palgrave's dictionary [21], Hirschman briefly points out, "[t]he availability and threat of exit on the part of important customer or group of members may powerfully reinforce their voice."[2] However, it is not very clear why and how the exit can reinforce the voice. The present paper aims to clarify this by analyzing a formal model of exit and voice.

In this paper, the exit is regarded as a decision to terminate an ongoing relationship, whereas the voice is interpreted as an activity involving sending a costless message that serves for the improvement of the relationship. In other words, we identify the voice with "cheap talk" for transmitting useful information.[3] Among others, the model by Crawford

[1] See Freeman [13].

[2] In his 1970 book [20], Hirschman seems to emphasize on a substitute aspect between exit and voice. However, in 1987 Palgrave's dictionary [21], he turned to insist that a complementarity aspect of exit and voice is also important.

[3] As we see later, Banerjee and Somanathan [4] also identify voice as an activity of sending a cheap talk

and Sobel [10] (hereinafter referred to as CS) is the most successful one describing cheap talk with private information. We employ the CS model as the basis of the environment that we consider in the present paper and extend it to the situation in which an exit option is available.

The CS model has two players. One player possesses private information about the current state of the relationship, which is randomly drawn. In order to transmit the information, he sends a costless message to his partner, and the latter responds with a decision affecting both the players' payoffs. In CS, the latter is called the Receiver (R) and the former is called the Sender (S). CS shows that the incongruence between both players' preferences restricts the informativeness of cheap talk; in particular, they demonstrate that perfect information transmission via cheap talk is impossible as an equilibrium behavior unless the players' preferences completely coincide.

In the present paper, we assume that S has an exit option after he observes R's decision. When S exercises the exit option, both players obtain their exit payoffs independent of the action chosen by R.

The key feature of our results is the difference between the players' payoff when S chooses to stay and an optimal action is chosen (maximum stay payoff) and one when she chooses to exit (exit payoff). Consider the case where R's difference is large and S's difference is small but positive. In this case, R's optimal action may induce S's exit as it differs from S's and cannot attain S's maximum stay payoff. Since R's large difference gives R a strong incentive to prevent S from exercising the exit option, R chooses an action close to the S's optimal action, instead of her own. Expecting R's response, S has a strong incentive to transmit more accurate information via cheap talk as more accurate information directly enhances his payoff. It then follows that the presence of S's exit option increases the informativeness of cheap talk, which in turn may increase not only S's payoff but also that of R. Moreover, we show that as S's difference approaches 0, the information transmission via cheap talk in the most informative equilibrium becomes almost perfect. In other words, the exit reinforces the voice in that the presence of the exit increases the informativeness of the voice. This is the main finding of this paper.

CS also shows that on the most efficient equilibrium, the more congruent both players' preferences are, the more informative is the cheap talk. In other words, when the exit

message.

option is not available, the informativeness of cheap talk is determined mainly by the degree of incongruence between the players' preferences. However, when the exit option is available, another determinant of the informativeness comes into play: *the credibility of exit*. A smaller difference between S's maximum stay payoff and exit payoff makes his choice of the exit option more credible, which in turn enables a more informative cheap talk transmission on the equilibrium. Thus, we show that informative cheap talk transmission can be carried out even if S's preference is not exactly similar to that of R. Our main claim—the exit reinforces the voice in that the credibility of exit increases the informativeness of the voice—is consistent with Hirschman's [21].

Furthermore, the credibility of exit serves for the informative communication only if R is sufficiently averse to S's exit. Indeed, we present an example in which R's equilibrium payoff decreases as the exit outcome becomes more attractive to R. R's aversion to exit is necessary for the credibility of exit to have an impact on the informative communication.

The rest of the paper is organized as follows. In Section 2, we give an overview of the related literature. In Section 3, we present a formal model of exit and voice. In Section 4, we analyze a specific model (called uniform-quadratic environment with constant difference) and present the main claim of this paper. In Section 5, we present a sufficient condition for the main claim to hold in a general model. Finally, in Section 6, we summarize the results.

2 Related Literature

To the author's knowledge, the exit-voice perspective has seldom been analyzed in any formal model in economics despite the vast citations.[4] Banerjee and Somanathan's study [4] is one exception in that it presents a game-theoretical model of voice. Like us, they consider the voice as an activity of sending a cheap talk message. However, their model differs from ours in some respects. First, they do not consider the exit option, and therefore, they do not investigate the interplay between the exit and voice, which the present paper focuses on. However, they consider the collective aspect of voice formation, which is abstracted out from our model. In this regard, the present paper can be considered as a complement to their paper. Gehlbach [14] presents a formal model of exit and voice. In his model, the voice is considered as some costly activity of gathering the members' various opinions,

[4] For efforts in the field of political science, see, for example, the survey by Dowding et al. [12].

consolidating them, and bargaining with the leader of the organization. However, in his model, there is no asymmetric information, and therefore, the voice does not play the role of an information transmitter. Although his model sheds the light on one aspect of voice, in this paper, we mainly analyze the role of voice as an information transmitter.

Apart from the exit-voice perspective, the CS model *per se* has attracted considerable attention and has been extended to various directions.[5] However, the exit option's effect on cheap talk has rarely been analyzed. An exception, Matthews [26], deals with a cheap talk game with a congress and a president (the receiver and sender, respectively) with veto power, which is similar to the exit option in our model. In particular, the timing of the events in his model is approximately the same as that in ours. However, there is a large difference with respect to what private information pertains to. In Matthews, private information concerns the sender's preference, while in our model, it pertains to the current state of the relationship. One may consider such a difference to be small, but it leads to very different outcomes: in Matthews, the informativeness of cheap talk is constrained with a strict upper bound, independent of the exit payoff. However, in our model, we show that an equilibrium can be close to that with perfect information transmission to any degree. In other words, Matthews does not emphasize that the existence of the exit increases the informativeness of cheap talk, which is the main claim of the present paper.

Chiba and Leong [8] also consider the exit option's effect on cheap talk. However, in their model, it is *not the sender, but the receiver* who has an exit option. Although they also show that the presence of the exit option can facilitate the information transmission, the logic is completely different from ours. In their model, the change of the sender's incentive that occurs when the exit option is really exercised brings about the improvement of communication. On the other hand, in our model, the credibility of exit is important; the presence of the exit option *per se* serves for the improvement of communication even when it is *not* exercised on the equilibrium path.

The present paper is also related to the literature on delegation. Among others, Dessein [11] shows that the manager (the receiver) can realize more efficient outcome by delegating her decision right to her subordinate (the sender) who has a specialized information about the current state.[6] However, it also has been recognized that delegating the formal decision

[5] For example, see Krishna and Morgan [24], Battaglini [5], and Chen et al. [7], among others.

[6] As for the delegation, see also Holmström [22], Melumad and Shibano [27], and Alonso and Matouschek [2].

right to the subordinate might be very difficult because the manager often has a temptation to overrule the subordinate's decision (Baker et al. [15] and Alonso and Matouschek [3]). Along the line of this argument, Gibbons et al. [16] propose three ways of committing to not overruling the subordinate's decision: (1) selling the firm (Grossman and Hart [18] and Hart and Moore [19]), (2) intentionally being ignorant of the current state (Aghion and Tirole [1]), and (3) utilizing the reputation mechanism (Baker et al. [15] and Alonso and Matouschek [3]). Our results imply that the delegation outcome can be approximately realized by a simple contract that makes the sender's maximum stay payoff very close to his exit payoff (Remark 1). In other words, the present paper insists that the credibility of exit is another way to successful delegation.

There is a relatively large amount of the literature on the effect of the exit option in the environments where the receiver can commit to delegation or message-contingent mechanism, while she cannot in this paper.

Delegation models with an exit option has been analyzed under the titles of "closed rule" in political economics (Gilligan and Krehbiel [17] and Krishna and Morgan [23]) and "veto-delegation" in organizational economics (Dessein [11], Marino [25], and Mylovanov [28]). Note that all the paper listed above deal with the receiver's exit option, not the sender's. Gilligan and Krehbiel [17] and Krishna and Morgan [23] show that the closed rule (delegation with receiver's exit option) may be superior to the open rule (communication) in legislative process. Dessein [11] shows that veto-delegation (delegation with receiver's exit option) is inferior to pure delegation, whereas Marino [25] obtains the opposite result. The difference of their results mainly pertains to the difference of default decisions they assume. Mylovanov [28] demonstrates that veto-delegation can replicate the optimal outcome realized by message-dependent mechanisms if the default decision can be appropriately chosen.

In addition, Compte and Jehiel [9] and Bester and Krähmer [6] analyze mechanism design problems when an exit option is available. Since the environments described in their paper are somehow different from ours, the logic of our model does not apply in their models. Compte and Jehiel [9] show that the presence of the exit option makes it more difficult to implement an efficient outcome. This is contrary to the conclusion of our paper: the existence of the exit option leads to more efficient outcome via more informative cheap talk. Bester and Krähmer [6] derive the condition for the first best outcome to be realized by incomplete contracts including Sender's exit option. In their setting, however, the receiver's stay payoff does not depend upon the state.

Shimizu [29] is a companion paper which analyzes the effect of sender's exit option on information transmission in a model with a discrete state space. Shimizu demonstrates that it is possible in a finite states setting that the informative communication requires rather a moderate size of incongruence than a small size of that.

3 Setup

There are two players, namely, the sender (S, male) and the receiver (R, female). At the beginning of the game, the current state of the relationship between S and R, $t \in T$ is randomly chosen according to a probability distribution $F(t)$. A realized state is observed by S but not by R. On the basis of this observation, S chooses a message $m \in M$ to be sent to R. This message is cheap talk in that it is payoff-irrelevant. After R receives S's message, R chooses an action $a \in A$ relevant to both players' payoffs.

Up to this point, all elements are the same as in CS. Now, we introduce the concept of exit. After observing R's action, S chooses whether to stay or exit. If S chooses to stay, S's and R's payoffs are given by $y^S(t, a)$ and $y^R(t, a)$, respectively. If S chooses to exit, S's and R's payoffs are $U^S(t)$ and $U^R(t)$, respectively.[7]

We assume that $T = M = [0, 1]$ and $A = \mathbb{R}$.[8] $F(t)$ has a continuous density $f(t)$, where $f(t) > 0$ for any $t \in T$. We assume that for $i = S, R$, $\partial y^i(t, a)/\partial a$ is defined and continuously differentiable, and

$$\begin{aligned}
&\forall t\ \exists a \text{ such that } \frac{\partial y^i(t,a)}{\partial a} = 0,\\
&\forall t, \forall a,\ \frac{\partial^2 y^i(t,a)}{\partial a^2} < 0,\\
&\forall t, \forall a,\ \frac{\partial^2 y^i(t,a)}{\partial a \partial t} > 0.
\end{aligned}$$

[7]Even in the situation where exit payoffs are dependent of a, if we normalize payoffs such that

$$\begin{aligned}
&\hat{y}^i(t,a) = y^i(t,a) - U^i(t,a),\\
&\hat{U}^i = 0,
\end{aligned}$$

and $\hat{y}^i$ and $\hat{U}^i$ satisfy all the conditions we will assume below, our main results would hold.

[8]$M = [0, 1]$ is an innocuous assumption as long as we consider the situations in which rich messages are available.

The 1st and the 2nd lines imply that, given any t, $y^i(t,\cdot)$ is single-peaked, whereas the 3rd line refers to the single crossing property. Furthermore, we assume that $U^i(t)$ is continuously differentiable in t for $i = S, R$.

We consider a perfect Bayesian equilibrium as an equilibrium concept. We also focus only on the class of equilibria with pure strategies. A pure strategy perfect Bayesian equilibrium is defined by $(\mu, p, \alpha, \epsilon)$, where

- $\mu : T \to M$: S's message strategy,
- $p : M \times T \to [0,1]$: R's posterior belief density function over T on the observation of m,
- $\alpha : M \to A$: R's action choice strategy, and
- $\epsilon : T \times A \to \{0,1\}$: S's exit strategy. To be more precise, $\epsilon = 1$ refers to exit and $\epsilon = 0$ refers to stay.

The equilibrium conditions are as follows:

- $\mu(t)$ must be an optimal message for type t of S given R's strategy and S's exit strategy, i.e.,

$$\mu(t) \in \arg\max_{m \in M} \left\{\epsilon(t,\alpha(m))U^S(t) + (1-\epsilon(t,\alpha(m)))y^S(t,\alpha(m))\right\}, \quad \forall t \in T.$$

- R's posterior belief must be updated by adhering as much as possible to the Bayesian approach, i.e.,

$$\int_{t' \in T} f(t')\mathbb{I}\{\mu(t') = m\}dt' > 0 \Rightarrow p(m,t) = \frac{f(t)\mathbb{I}\{\mu(t) = m\}}{\int_{t' \in T} f(t')\mathbb{I}\{\mu(t') = m\}dt'},$$

  where $\mathbb{I}$ is the identity function.

- $\alpha(m)$ must be an optimal action for R given R's posterior belief and S's exit strategy, i.e.,

$$\alpha(m) \in \arg\max_{a \in A} \int_{t \in T} \left\{\epsilon(t,a)U^R(t) + (1-\epsilon(t,a))y^R(t,a)\right\} p(m,t)dt, \quad \forall m \in M.$$

- $\epsilon(t,a)$ must be an optimal exit choice for type t of S given a realized action a, i.e.,

$$\forall t \in T, \forall a \in A, \begin{cases} y^S(t,a) < U^S(t) & \Rightarrow \epsilon(t,a) = 1, \\ y^S(t,a) > U^S(t) & \Rightarrow \epsilon(t,a) = 0. \end{cases}$$

We state that an action a is *induced on the equilibrium path* if there exists $t \in T$ such that $a = \alpha \circ \mu(t)$ and $\epsilon(t, a) = 0$. For ease of exposition, given an interval $\tau \subseteq T$ where $\inf \tau = \underline{t}$ and $\sup \tau = \bar{t}$, the posterior density function based on the observation that $t \in \tau$ is denote by f_τ; that is

$$f_\tau(t) = \begin{cases} \frac{f(t)}{F(\bar{t})-F(\underline{t})} & \text{if } t \in \tau, \\ 0 & \text{otherwise.} \end{cases}$$

4 Uniform-Quadratic Environment with Constant Difference

In this section, we employ a more specific model, which we call *a uniform-quadratic environment with constant difference*. In this model, $F(t)$ is a uniform distribution function on $[0, 1]$ and S's and R's stay payoffs are respectively expressed as

$$y^S(t, a) = Y^S - (t + b - a)^2,$$
$$y^R(t, a) = Y^R - (t - a)^2,$$

for some $b > 0$. Here, b is called *a bias* that represents a degree of incongruence between S's and R's optimal actions.[9] A uniform-quadratic environment was originally analyzed in Section 4 of CS. Furthermore, we assume that both players' exit payoffs are independent of t. We define the difference between i's maximum stay payoff and exit payoff by $D^i = Y^i - U^i$ for $i = S, R$.

4.1 Preliminary Results: Environment without an Exit Option

We first revisit CS's results in an environment without an exit option. If the exit option is not available, perfect information transmission via cheap talk does not occur. This is mainly because S has an incentive to send a upwardly biased message to R.

To be more precise, CS shows that in any equilibrium, there are finite intervals partitioning T and S informs R about which interval a true state is lying in via cheap talk. The necessary and sufficient condition for the existence of the equilibrium with N intervals is

$$b < \left\langle \frac{1}{2N(N-1)} \right\rangle, \tag{1}$$

[9] For interpretations of biases in the real world, see the discussion in Dessein [11], among others.

where $\langle \cdot \rangle$ is the operator such that

$$\left\langle \frac{x}{y} \right\rangle = \begin{cases} \frac{x}{y}, & \text{if } y \neq 0, \\ \infty, & \text{if } y = 0, \ x \neq 0. \end{cases}$$

In other words, regarding N as the informativeness of the cheap talk, the informativeness is determined by the bias b. The smaller b is, the more intervals the equilibrium has. Henceforth, we only focus on the most informative equilibrium or the equilibrium with the most intervals.[10] Indeed, CS shows that the equilibrium with the most intervals is Pareto superior to any other equilibrium with fewer intervals.

4.2 Environment with an Exit Option: Case of Large Bias

Hereafter, we consider the environment in which an exit option is available for S. In this situation, when D^R is so large that R has a strong incentive to avoid S's exit, the introduction of S's exit option drastically changes the nature of equilibria. To see this, consider the case in which $b \geq \frac{1}{2}$. (1) implies that if the exit option is not available, there exists only a babbling equilibrium. Even if the exit option is available, when S's difference D^S is sufficiently large, the existence of the exit option has no effect on equilibrium behavior since it is anticipated that S would never choose the exit option.

However, when S's exit becomes more credible, there can be equilibria in which some positive amount of information is transmitted via cheap talk. Consider the extreme case in which $D^S = 0$. In this case, S has no incentive to exercise the exit option if and only if R chooses S's optimal action $t + b$ at every state. Since it is anticipated that R would do the best to avoid S's exit, S has an incentive to perfectly informs R of the true state. In other words, perfect information transmission can be done via cheap talk.

To a less extreme degree, consider the case in which $D^S > 0$. Even in this case, there exists some informative equilibria as long as D^R is sufficiently large and D^S is sufficiently small. Indeed, we can construct such an equilibrium as follows: first, we recursively define a decreasing sequence $\{s_n\}_{n=0}^N$ and an associated action sequence $\{a_n\}_{n=1}^N$. We set $s_0 = 1$. For $n \geq 0$,

1. if $s_n = 0$, we stop the recursive process and denote n by N;

[10] Che et al. [7] present a condition that selects the most informative equilibrium in uniform-quadratic models.

2. if $s_n > 0$, then we pick the larger solution a' of the equation $(s_n + b - a')^2 = D^S$ and name it as a_{n+1}. Furthermore, we pick up the smaller solution s' of the equation $(s' + b - a_{n+1})^2 = D^S$, and we define $s_{n+1} = \max\{s', 0\}$.[11]

Next, we specify equilibrium strategies such that

- the state space $[0, 1]$ is partitioned into $[s_N, s_{N-1}], (s_{N-1}, s_{N-1}], \ldots, (s_1, s_0]$,
- S informs R of which interval the realized state is lying on via cheap talk messages,
- when R is informed that the state is lying on the nth interval from the right, she chooses an action a_n, and
- when S is indifferent between exit and stay, he stays.

For each interval except $[s_N, s_{N-1}]$, the associated action is the only one that can deter S's exit when the state is lying on that interval. Therefore, R has no incentive to deviate from the equilibrium strategy when D^R is sufficiently large.[12]

For the last interval from the right, $[s_N, s_{N-1}]$, the associated action a_N is the one that makes S indifferent between exit and stay at the right end of the interval, s_{N-1}. This means that if R would choose a smaller action than a_N, S would exit around the right end. Since R has no incentive to choose a larger action than a_N, R has no incentive to choose an action other than a_N when she is informed that the state is lying on the last interval.

Furthermore, when D^S is sufficiently small,[13] it is verified that $N \geq 2$. In other words, some positive amount of information can be transmitted via cheap talk. While it is never exercised on the equilibrium path, the existence of the exit option can induce more informative communication.

[11] We can derive more explicit forms as follows:

$$N\text{: a natural number satisfying } \frac{1}{2N} \leq \sqrt{D^S} < \frac{1}{2(N-1)},$$

$$s_n = \begin{cases} 1 - 2n\sqrt{D^S} & n = 0, \ldots, N-1, \\ 0 & n = N, \end{cases}$$

$$a_n = 1 + b - (2n-1)\sqrt{D^S} \quad n = 1, \ldots, N.$$

[12] More precisely, it is the case when $\sqrt{D^R} \geq \sqrt{D^S} + b$.

[13] More precisely, it is the case when $D^S < \frac{1}{4}$.

4.3 Characterization of No-Exit Equilibria

In this subsection, we focus on the equilibrium in which the exit option is never exercised on the equilibrium path. We assume that $D^S > 0$. We call it *no-exit equilibrium (NEE).* The following result asserts that any NEE is characterized by a partition of the state space consisting of finite number of intervals.[14]

Lemma 1 In any equilibrium, there are only finite actions induced on the equilibrium path.

All proofs are relegated to the the Appendix A. Lemma 1 implies that any NEE $(\mu, p, \alpha, \epsilon)$ is characterized by a partition $\{\tau_n\}_{n=1,\ldots,N}$ of $[0,1]$ such that

- N is finite;
- τ_n is an interval for $n = 1, \ldots, N$; and
- there exist $\{t_n\}_{n=0,\ldots,N}$, $\{m_n\}_{n=1,\ldots,N}$, $\{a_n\}_{n=1,\ldots,N}$ such that
  - $\inf \tau_n = t_{n-1}$ and $\sup \tau_n = t_n$ for $n = 1, \ldots, N$,
  - $0 = t_0 < t_1 < \cdots < t_N = 1$,[15]
  - $\mu(t) = m_n$ for any $t \in \tau_n$, $m_n \neq m_{n'}$ for $n \neq n'$, and therefore, $p(m_n, t) = f_{\tau_n}(t)$ for $n = 1, \ldots n$,
  - $\alpha(m_n) = a_n$ for $n = 1, \ldots, N$, and
  - $\epsilon(t, a_n) = 0$ for $t \in \tau_n$ and $n = 1, \ldots, N$.

Below, we derive the equilibrium condition for NEE with N intervals. The following result shows that any interval of NEE is classified into three categories.

Lemma 2 Fix an NEE and an interval $\hat{\tau}$ from the equilibrium partition $\{\tau_n\}_{n=1,\ldots,N}$. Let $\inf \hat{\tau} = \underline{t}$, $\sup \hat{\tau} = \bar{t}$, and $\hat{a} = \alpha \circ \mu(t)$ for $t \in \hat{\tau}$. Then, $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$. Moreover, $\hat{\tau}$ belongs to any one of the following categories:

Interval $\mathcal{N}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

[14] Generally, there can be an equilibrium with infinite number of intervals. This is because no length constraint is necessary for intervals in which S is induced to choose the exit option at any state (Interval $\mathcal{E}$ in Subsection 4.4).

[15] We denote a increasing (decreasing) sequence of thresholds by $\{t_n\}$ ($\{s_n\}$, respectively).

- $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$,
- $\hat{a} = \frac{\underline{t}+\bar{t}}{2}$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) > U^S$.

Interval $\mathcal{A}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $2\sqrt{D^S} > \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) = U^S$.

Interval $\mathcal{F}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} = 2\sqrt{D^S}$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$, and
- $y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S$.

Furthermore, in any interval, the receiver has an incentive to choose $\hat{a}$ if $\sqrt{D^R} \geq \sqrt{D^S} + b$.

Interval $\mathcal{N}$ is a non-accommodating interval in the sense that R can choose her optimal action without inducing S's exit. Interval $\mathcal{A}$ is an accommodating interval in that the constraint for no exit is binding at the right end of the interval and R has to choose an action more favorable for S than R's optimal action. Interval $\mathcal{F}$ is a fully accommodating interval in that the constraint for no exit is binding at both ends of the interval and any other action than $\hat{a}$ necessarily induces S to exercise the exit option at some states. This lemma also asserts that any NEE interval cannot be longer than $2\sqrt{D^S}$, for otherwise S would exercise the exit option at some states no matter which action R chooses.

The important fact is that on the boundary state of two adjoining intervals, S must be indifferent between sending messages corresponding to the intervals. This implies that possible equilibrium configurations of intervals are restricted. For example, an interval $\mathcal{F}$ cannot be directly connected to an interval $\mathcal{N}$, for otherwise S will have a strict incentive to choose the action corresponding to the interval $\mathcal{N}$ at any state sufficiently close to the boundary point. By exhausting all possibilities, we can derive the equilibrium condition for S. The following is the formal statement:

Lemma 3 Given any NEE with N intervals,

(i) for $N = 1$, the equilibrium condition for the sender is $\sqrt{D^S} \geq \frac{1}{2N}$, and

(ii) for $N \geq 2$, a configuration of intervals is any one of the following five patterns:

(I) $\mathcal{N}, \ldots, \mathcal{N}$,

(II) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}$,

(III) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$,[16]

(IV) $\mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$, or

(V) $\mathcal{F}, \ldots, \mathcal{F}$.

The equilibrium condition for the sender in each case is the following:

(I) $b < \frac{1}{2N(N-1)}$ and $\sqrt{D^S} > \frac{1}{2N} + Nb$.

(II) $\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b$ and $\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb$.

(III) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ and $\frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N} b$ for some $i = 2, \ldots, N-1$.

(IV) $\sqrt{D^S} < \frac{1}{2(N-1)}$ and $\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N} b$.

(V) $\sqrt{D^S} = \frac{1}{2N}$.

These conditions are illustrated in Figure 1. Given R's response, the equilibrium conditions consists of the indifference condition for S and the length constraints on each type of intervals. The details are as follows:

(I): The 1st condition is the indifference condition. The 2nd condition is the length constraint on $\mathcal{N}$.

(II): The 1st condition is the indifference condition. The 2nd conditions is the length constraints on $\mathcal{A}$.[17]

[16] This case occurs only if $N \geq 3$.

[17] It is verified that the length constraints on $\mathcal{N}$ are satisfied whenever the other conditions are satisfied.

(III): The 1st condition is a mixture of the indifference condition and the length constraint on $\mathcal{F}$. The 2nd condition is the length constraint on $\mathcal{A}$.[18]

(IV): The 1st condition is a mixture of the indifference condition and the length constraint on $\mathcal{F}$. The 2nd condition is the length constraint on $\mathcal{A}$.

(V): The condition is the length constraint on $\mathcal{F}$.

<Figures 1 should be inserted>

Combined with these lemmas, we derive the equilibrium condition for NEE.

Theorem 1 Suppose $\sqrt{D^R} \geq \sqrt{D^S} + b$. Then, an NEE with N intervals exists if and only if both (1) and (2) hold:

(1) Any one of (1-1)–(1-3) holds:[19]

  (1-1) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$,

  (1-2) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \ldots, N$, or

  (1-3) $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(2) $\sqrt{D^S} \geq \frac{1}{2N}$.

This theorem asserts that an NEE with sufficiently large number of intervals exists if and only if b is sufficiently small and/or D^S is sufficiently small as long as D^R is sufficiently large. Identifying the equilibrium number of intervals with *the informativeness of cheap talk*, we can interpret this result as follows: in our model, there are two determinants of the informativeness of cheap talk. One is the smallness of b, which refers to the degree of incongruence between S's and R's preferences. This is extensively discussed by CS and in other literature. The other is the smallness of D^S, which is newly found. We interpret the smallness of D^S as a degree of *S's credibility of exit*. In other words, the smaller D^S is, the more credible S's threat of exit is and the more informative information cheap talk can convey. Henceforth, we call such an NEE *an NEE driven by the credibility of exit*.[20]

Furthermore, the previous theorem implies the following important facts:

[18] Similarly, it is verified that the other conditions implies the length constraint on $\mathcal{N}$.

[19] For the definition of the operator $\langle \cdot \rangle$, see Section 4.1.

[20] To be more precise, an NEE driven by the credibility of exit is defined as an NEE that does not include $\mathcal{N}$ as an equilibrium interval, or NEE with a configuration of intervals (IV) or (V) in Lemma 3.

Corollary 1 Suppose that $\sqrt{D^R} > b$. Then, as U^S approaches Y^S (equivalently, D^S approaches 0), there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ converges pointwise to $t + b$.

This corollary implies that approximately perfect information transmission is possible via cheap talk in the limit. The corollary also implies that the equilibrium actions converge to the optimal action for S at any state. In other words, our result implies that even if a commitment to delegation is impossible, the credibility of the exit option can bring about a similar outcome.

Corollary 2 Suppose that $\sqrt{D^R} > b$ and $b < \frac{1}{2\sqrt{3}}$. Then, if U^S is sufficiently close to Y^S (equivalently, D^S is sufficiently close to 0), there exists an equilibrium in the environment with the exit, in which S's and R's ex ante payoffs are both larger than those in any equilibrium in the environment without the exit.[21]

This corollary implies that the presence of S's exit option increases the ex ante payoff of R as well as S. Therefore, giving S an exit option is Pareto-improving.

Remark 1 Corollaries 1 and 2 imply that, as U^S approaches Y^S, an equilibrium outcome nearly seems to be one that occurs when R would delegate her decision right to S, and it is desirable even for R as long as b is sufficiently small. We can also show that an almost delegation outcome can be carried out by the following simple contract. Let us consider the situation in which players' maximum stay payoffs are determined by the splitting of the joint surplus Y. Before a state is realized, R proposes a contract that specifies an allocation of Y between S's share Y^S and R's share Y^R. This contract does not depend upon the message. If $Y > U^S + U^R + b^2$, an allocation with $Y^S = U^S + \varepsilon$ and $Y^R = Y - Y^S$ for sufficiently small ε leads to an almost delegation outcome because it satisfies the premise of Theorem 1 for a large N.

4.4 Characterization of General Equilibria

In Appendix B, we derive the conditions for equilibria including those other than NEEs in the uniform-quadratic environment with constant difference. Below we draw a rough sketch.

[21] Dessein [11] shows that, when $b < \frac{1}{2\sqrt{3}}$, R had better delegate her decision right to S if possible.

Given any equilibrium interval $\hat{\tau}$ where inf $\hat{\tau} = \underline{t}$, sup $\hat{\tau} = \bar{t}$, and $\hat{a} = \alpha \circ \mu(t)$ for $t \in \hat{\tau}$. Then, $\hat{\tau}$ belongs to either Intervals $\mathcal{N}, \mathcal{A}, \mathcal{F}$ or any one of the following categories:[22]

- **Interval $\mathcal{R}$:**
  * $\bar{t} - \underline{t} > -b + \sqrt{D^S} + \sqrt{D^R}$,
  * $\hat{a} = \underline{t} + \sqrt{D^R}$,
  * $y^S(\underline{t}, \hat{a}) > U^S$
  * $y^S(\bar{t}, \hat{a}) < U^S$, and
  * S exercises the exit option on the right side of the interval.

- **Interval $\mathcal{B}$:**
  * $\bar{t} - \underline{t} > 2\sqrt{D^S}$,
  * $\hat{a} \in [\underline{t} + b + \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}]$,
  * $y^S(\underline{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \underline{t} + b + \sqrt{D^S}$,
  * $y^S(\bar{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \bar{t} + b - \sqrt{D^S}$, and
  * S exercises the exit option around either end of the interval; in particular, S does so around both ends of the interval when $\hat{a} \in (\underline{t} + b + \sqrt{D^S}, \bar{t} + b - \sqrt{D^S})$.

- **Interval $\mathcal{E}$:**
  * there is no constraint for the length of interval,
  * $\hat{a} \in (-\infty, \underline{t} + b - \sqrt{D^S}] \cup [\bar{t} + b + \sqrt{D^S}, \infty)$,
  * $y^S(\underline{t}, \hat{a}) \leq U^S$ where the equality holds only $\hat{a} = \underline{t} + b - \sqrt{D^S}$,
  * $y^S(\bar{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \bar{t} + b + \sqrt{D^S}$,and
  * S almost always exercises the exit option in this interval.

[22] In taking all kinds of equilibria into consideration, the length constraint on Interval $\mathcal{A}$ should be modified as follows:

$$2\sqrt{D^S} - 2b \leq \bar{t} - \underline{t} < \min\{2\sqrt{D^S}, -b + \sqrt{D^S} + \sqrt{D^R}\}.$$

Note that the condition $\sqrt{D^R} \geq \sqrt{D^S} + b$ which appears in Lemma 2 implies that

$$2\sqrt{D^S} \leq -b + \sqrt{D^S} + \sqrt{D^R}.$$

In any type of the intervals listed here, S necessarily exercises the exit option at some states. For example, when $\sqrt{D^R} \leq b - \sqrt{D^S}$ (Case 8), R intentionally induces S to exercise the exit option by choosing $\hat{a} \in (-\infty, \underline{t} + b - \sqrt{D^S}] \cup [\bar{t} + b + \sqrt{D^S}, \infty)$ such that S's stay payoff becomes sufficiently small compared to his exit payoff.

Also, in Appendix B, we completely enumerate the possible equilibrium intervals and the possible connections of them, which enable us to check whether a given configuration of intervals constitutes an equilibrium. Using these, we investigate equilibria when D^R is not so large and demonstrate that R's aversion to S's exit is necessary for the credibility of exit to serve for the informative communication.

First, when $\sqrt{D^R} < \sqrt{D^S} + b$, possible NEE configurations are restricted as follows:

- $\mathcal{N} \ldots \mathcal{N}$
- $\mathcal{N} \ldots \mathcal{N}\mathcal{A}$
- $\mathcal{A}$

This means that the condition $\sqrt{D^R} \geq \sqrt{D^S} + b$, which is the sufficient condition for Theorem 1, is also necessary for the existence of informative NEE driven by the credibility of exit.

Moreover, we demonstrate that an increase in R's exit payoff can lead to a *decrease* in her equilibrium payoff. Consider the case in which $b = \sqrt{D^S} = \frac{1}{4}$, and $Y^R = 1$. In this case, as the value of U^R changes, the equilibrium outcome changes as follows:

- When $U^R \leq \frac{3}{4}$ (Case 2), a configuration $\mathcal{F}\mathcal{F}$ constitutes an equilibrium. It gives R the ex ante expected equilibrium payoff $V^R = \frac{11}{12}$.
- When $\frac{3}{4} < U^R < 1$ (Case 4), only a configuration $\mathcal{R}$ constitutes an equilibrium.[23] It gives R the ex ante expected equilibrium payoff V^R as follows:

$$V^R = U^R - \int_0^{\sqrt{D^R}} (t - \sqrt{D^R})^2 dt + D^R \sqrt{D^R}.$$

V^R is illustrated in Figure 3. It is obvious that it has a downward jump at $U^R = \frac{3}{4}$. This means that an increase in U^R reduces her equilibrium payoff.

[23]Since $1 > 2\sqrt{D^S}$, a configuration $\mathcal{A}$ cannot constitute an equilibrium.

<Figures 3 should be inserted>

Let us see what happens in more detail. When $U^R \leq \frac{3}{4}$, the following strategy profile constitutes an equilibrium:

- the state space is partitioned into $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1]$,
- S informs R of which interval the realized state is lying on,
- when R is informed that $t \in [0, \frac{1}{2})$, she chooses $a = \frac{1}{2}$, and
- when R is informed that $t \in [\frac{1}{2}, 1]$, she chooses $a = 1$.

In this equilibrium, R attempts to deter S from exercising the exit option as she is averse to exit. Fix the strategy profile and suppose that U^R becomes more than $\frac{3}{4}$. R then does no longer do her best to stop S's exit. To be more concrete, her best response is

$$\begin{cases} \frac{1}{4} & \text{if she is informed that } t \in [0, 1/2), \\ \frac{3}{4} & \text{if she is informed that } t \in [1/2, 1]. \end{cases}$$

This response induces S to choose the exit option on the right side of each interval. Given this response, S is no longer indifferent between sending messages corresponding to those two intervals on the boundary state $t = \frac{1}{2}$. For, if she sends the message indicating $t \in [0, \frac{1}{2})$, then

$$y^S\left(\frac{1}{2}, \frac{1}{4}\right) = Y^S - \frac{1}{4} < Y^S - \frac{1}{16} = U^S,$$

and therefore, S exercises the exit option and obtains a payoff of U^S. On the other hand, if she sends the message indicating $t \in [\frac{1}{2}, 1]$, then

$$y^S\left(\frac{1}{2}, \frac{3}{4}\right) = Y^S > Y^S - \frac{1}{16} = U^S,$$

and therefore, she chooses to stay and obtains a payoff of Y^S, which is strictly larger than U^S. It then follows that such an information transmission cannot sustain as an equilibrium outcome. On the whole, an increase in R's exit payoff deteriorates the informativeness of equilibrium communication, which in turn reduces her equilibrium payoff.

This is not an irregular outcome. Indeed, we can show that a similar outcome would happen whenever $\sqrt{D^S} \leq b$ and $\sqrt{D^S} \leq \frac{1}{4}$. All in all, we conclude that R's exit aversion is necessary for the credibility of exit to serve for the informative communication.

5 General Model

In this section, we extend the results obtained in the uniform-quadratic environment with constant difference to more broad environments. More precisely, we derive a sufficient condition for the existence of NEE driven by the credibility of exit in the general setting.

Since $y^i(t,\cdot)$ is single-peaked, we can identify a unique maximizer of $y^i(t,\cdot)$ for any t. It is denoted by $\sigma^i(t)$. We posit the following assumption:

Assumption 1 Any one of the following conditions hold:

(a) $\sigma^S(0) > \sigma^R(0)$.

(b) $\sigma^R(1) > \sigma^S(1)$.

When (a) holds, define $b = \sigma^S(0) - \sigma^R(0)$, and when (b) holds, $b = \sigma^R(1) - \sigma^S(1)$.

Roughly speaking, Assumption 1 requires that players' optimal actions differ in either end of the state space in some direction. This assumption holds in the models in which players' stay payoffs have a form of quadratic loss function with constant bias.

Let $Y^S(t) = y^S(t, \sigma^S(t))$ be S's maximum stay payoff at state t. Let us denote the difference between S's maximum stay payoff and exit payoff at state t by $D^S(t) = Y^S(t) - U^S(t)$. Whenever $D^S(t) > 0$, we can uniquely define $\gamma_-(t)$ and $\gamma_+(t)$ such that

$$\gamma_-(t) < \gamma_+(t),$$
$$y^S(t, \gamma_-(t)) = y^S(t, \gamma_+(t)) = 0.$$

In other words, $\gamma_-(t)$ and $\gamma_+(t)$ are the actions that makes S indifferent between stay and exit at state t. On the basis of the assumptions on y^S and U^S, it is verified that γ_+ and γ_- are continuously differentiable and Lipschitz continuous in t, and

$$\gamma_-(t) < \sigma^S(t) < \gamma_+(t)$$

holds for any t. We posit the following assumption:

Assumption 2 $\gamma_-(t)$ and $\gamma_+(t)$ are strictly increasing in t.

This assumption is met in the uniform-quadratic environment with constant difference.

Lastly, we focus on the situation in which $U^R(t)$ is sufficiently small such that she attempts to do her best to deter S's exit.

Assumption 3 Let $\hat{A}$ be the convex hull of $[\gamma_-(0), \gamma_+(1)] \cup [\sigma^R(0), \sigma^R(1)]$. Then, under Assumption 1,

- if (a) is required in Assumption 1, then for any t the following must hold

$$U^R(t) < \min_{t' \in T} \; y^R(t, \gamma_-(t')) - \max_{t' \in T, a \in \hat{A}} \left\{ \frac{\partial y^R(t', a)}{\partial a} \right\} \times \max \left\{ \max_{t' \in T} \left\{ \frac{d\gamma_-(t')}{dt} \right\}, \; \max_{t' \in T} \left\{ \frac{d\gamma_+(t')}{dt} \right\} \right\},$$

  or

- if (b) is required in Assumption 1, then for any t the following must hold

$$U^R(t) < \min_{t' \in T} \; y^R(t, \gamma_+(t')) - \max_{t' \in T, a \in \hat{A}} \left\{ \frac{\partial y^R(t', a)}{\partial a} \right\} \times \max \left\{ \max_{t' \in T} \left\{ \frac{d\gamma_-(t')}{dt} \right\}, \; \max_{t' \in T} \left\{ \frac{d\gamma_+(t')}{dt} \right\} \right\}.$$

We can show that under these assumptions, if $\gamma_+(t) - \gamma_-(t)$ is sufficiently small, there exists an NEE with many intervals.

Theorem 2 Suppose that Assumptions 1–3 hold and $D^S(t) > 0$ for any t. We define[24]

$$\underline{\delta} = \inf_{t > t'} \frac{\sigma^S(t) - \sigma^S(t')}{t - t'},$$
$$\overline{\delta} = \sup_{t > t'} \frac{\sigma^R(t) - \sigma^R(t')}{t - t'},$$
$$\underline{N} = \frac{\underline{\delta} + 2\overline{\delta}}{2b} + 1.$$

Then, for any natural number $N' \geq \underline{N}$, there exists an NEE with N' or more intervals if the condition $\gamma_+(t) - \gamma_-(t) \leq \overline{\gamma}$ holds for any t where

$$\overline{\gamma} = \frac{\underline{\delta}}{2(N' - 1)}.$$

Moreover, the length of each interval can be made less than $\frac{1}{N'-1}$.

In Appendix C, we discuss the conditions of Theorem 2 in more details.

The theorem is proved by constructing an equilibrium strategy profile in a similar way as done in Section 4.2. According to whether (a) or (b) holds in Assumption 1, we determine the direction of construction of intervals so that there is no incentive for R's deviation on the last interval. For example, if (a) holds, we construct a decreasing sequence of the

[24]It is verified that $\underline{\delta} > 0$ and $\overline{\delta} < \infty$.

thresholds, similarly as in Section 4.2. Assumption 2 guarantees that the recursive process is well done.[25] The upper bound on $(\gamma_+ - \gamma_-)$ guarantees that the length of each interval can be made so short that there are sufficiently many intervals.[26]

If S's difference $D^S(t)$ is independent of t, we can reduce $\gamma_+(t) - \gamma_-(t)$ to any degree by letting D^S approach to 0; this implies that perfect information transmission via cheap talk can be approximated.

Corollary 3 Suppose that Assumptions 1 and 3 hold. If $D^S = D^S(t)$ for any t, then there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ converges pointwise to $\sigma^S(t)$ as D^S approaches 0.

Remark 2 Dessein [11] considers the situation in which R can commit to delegation and analyzes when delegation dominates communication in the viewpoint of R's payoff. To do so, he puts more restriction to the environment than the present paper does; to be more concrete, he assumes that both players' payoffs are symmetric and the difference between players' optimal actions is independent of t. His Proposition 3 states that delegation dominates communication whenever the difference is sufficiently small. Combined with our Corollary 3, it then follows that, in Dessein's environment, the presence of the exit option is Pareto-improving whenever $|\sigma^S(t) - \sigma^R(t)|$ is sufficiently small.

6 Concluding Remarks

This paper investigates the interplay between exit and voice by analyzing a modified version of Crawford and Sobel's [10] model in which the sender has an exit option after the receiver makes a decision. The key feature of our results is the difference between players' maximum stay payoff and exit payoff. We find that in the case where the receiver's difference is large and the sender's difference is small but positive, the latter's exit is so credible that the former makes a decision that is desirable to the latter so as to prevent him from exercising the exit option; through this, accurate information can be transmitted via cheap talk on the equilibrium. In other words, it is shown that the informativeness of cheap talk is determined by not only the degree of incongruence between both players' preferences but also the credibility of the sender's exit, which is measured by the smallness of the

[25] For the case that violates Assumption 2, see the example in C.3.

[26] For the case without such an upper bound, see the example in C.4.

sender's difference. Furthermore, the perfect information transmission via cheap talk can be approximated in the limit. To the author's knowledge, these results are unprecedented in the literature on cheap talk with private information.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Suppose, to the contrary, that there exists an equilibrium with infinite actions induced on the equilibrium path. Then, there must exist actions a_1, a_2, and a_3 induced on the equilibrium path such that $a_1 < a_2 < a_3$ and $a_3 - a_1 < \min\{\sqrt{D^S}, b\}$. Taking S's incentive into consideration, the following must hold:

$$\{t|\alpha \circ \mu(t) = a_2\} \subseteq (\max\{a_1 - b, 0\}, a_3 - b) \neq \emptyset.$$

Since for any $t < a_3 - b$,

$$\sqrt{D^S} > a_3 - a_1 > t + b - a_1 > t + b - a_2,$$

$\epsilon(t, a_1) = \epsilon(t, a_2) = 0$ must hold for any t such that $\alpha \circ \mu(t) = a_2$. On the other hand, for any $t < a_3 - b$,

$$a_2 > a_1 > a_3 - b > t.$$

This means

$$-\int_{\{t|\alpha\circ\mu(t)=a_2\}} (t - a_1)^2 dt > -\int_{\{t|\alpha\circ\mu(t)=a_2\}} (t - a_2)^2 dt.$$

It follows that R would deviate to choosing a_1 after receiving a message inducing a_2. This is a contradiction. ∎

A.2 Proof of Lemma 2

First of all, if $t < \hat{a} - b - \sqrt{D^S}$ or $t > \hat{a} - b + \sqrt{D^S}$, then $\epsilon(t, \hat{a}) = 1$. This implies that the length of any NEE interval must be less than or equal to $2\sqrt{D^S}$. Henceforth, we focus on the case in which $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$.

Let us denote R's expected equilibrium payoff on choosing a conditional on the assumption that $t \in \hat{\tau}$ by $\tilde{V}^R(a)$ and, for ease of exposition, define $W(a) = (\bar{t} - \underline{t})(\tilde{V}^R(a) - U^R)$.

Then, we obtain the following expression:

$$W(a) =$$

$$\begin{cases} 0 & \text{if } a \in A_1 = (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2 = (\underline{t} + b - \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}), \\ (\bar{t} - \underline{t})D^R - \int_{\underline{t}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_3 = [\bar{t} + b - \sqrt{D^S}, \underline{t} + b + \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4 = (\underline{t} + b + \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}), \\ 0 & \text{if } a \in A_5 = [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

In any NEE, the optimal action must lie on A_3, for otherwise some type of S would choose an exit option. The unique local maximizer on A_3, denoted by a^*, is

$$a^* = \begin{cases} \frac{\underline{t}+\bar{t}}{2} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \bar{t} + b - \sqrt{D^S} & \text{if } \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b. \end{cases}$$

Note that $a^* = \hat{a}$ in any case. When $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$, it is verified that $y^S(\underline{t}, a^*) > 0$ and $y^S(\bar{t}, a^*) > 0$ hold. In the case of $\bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$ it is verified $y^S(\bar{t}, a^*) = 0$ and

$$y^S(\underline{t}, a^*) \begin{cases} > U^S & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S}, \\ = U^S & \text{if } \bar{t} - \underline{t} = 2\sqrt{D^S}, \end{cases}$$

hold.

In both cases, it is verified that $\sqrt{D^R} \geq \sqrt{D^S} + b$ implies that $W(a^*) \geq 0$ and W has no local maximum on A_2 and A_4. It follows that a^* is a global optimal action. ∎

A.3 Proof of Lemma 3

Condition (i) is immediately proved from Lemma 2. Throughout the proof, we consider $N \geq 2$. On the boundary point of two adjoining intervals, S must be indifferent between sending actions corresponding to the intervals. By Lemma 2, they must be any one of the following cases:

- $\mathcal{NN}$,
- $\mathcal{NA}$,
- $\mathcal{AF}$, or
- $\mathcal{FF}$.

This implies that possible configurations of intervals of NEE is restricted to (I)–(V).

Consider (I). In this type of equilibrium, by the analysis in CS (see Section 4.1),

$$t_n = nt_1 + 2n(n-1)b, \quad n = 0, \dots, N,$$

where

$$t_1 = \frac{1 - 2N(N-1)b}{N}.$$

The equilibrium condition is

$$\begin{aligned} &t_1 - t_0 > 0, \\ &t_N - t_{N-1} < 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} b &< \left\langle \frac{1}{2N(N-1)} \right\rangle, \\ \sqrt{D^S} &> \frac{1}{2N} + Nb. \end{aligned}$$

Consider (II). In this type of equilibrium,

$$\begin{aligned} t_n &= \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \dots, N-1, \\ 1, & n = N, \end{cases} \\ a_n &= \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \dots, N-1, \\ 1 - \sqrt{D^S} + b, & n = N. \end{cases} \end{aligned}$$

The equilibrium condition is

$$\begin{aligned} &y^S(t_{N-1}, a_{N-1}) = y^S(t_{N-1}, a_N), \\ &t_1 - t_0 > 0, \\ &t_{N-1} - t_{N-2} < 2\sqrt{D^S} - 2b, \\ &t_N - t_{N-1} > 0, \\ &2\sqrt{D^S} > t_N - t_{N-1} \geq 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} &\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b, \\ &\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb, \end{aligned}$$

where

$$t_1 = \frac{2 - 2\sqrt{D^S} - 2(2N^2 - 4N + 1)b}{2N - 1}.$$

Consider (III). Given any $i = 2, \ldots, N-1$, consider the following configuration:

$$\underbrace{\mathcal{N}, \ldots, \mathcal{N}}_{i-1 \text{ times}}, \mathcal{A}, \underbrace{\mathcal{F}, \ldots, \mathcal{F}}_{N-i \text{ times}}.$$

In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \ldots, i-1, \\ 1 - 2(N-n)\sqrt{D^S}, & n = i, \ldots, N, \end{cases}$$
$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \ldots, i-1, \\ t_n - \sqrt{D^S} + b, & n = i, \ldots, N. \end{cases}$$

The equilibrium condition is

$$\begin{aligned} & y^S(t_{i-1}, a_{i-1}) = y^S(t_{i-1}, a_i), \\ & t_1 - t_0 > 0, \\ & t_{i-1} - t_{i-2} < 2\sqrt{D^S} - 2b, \\ & t_i - t_{i-1} > 0, \\ & 2\sqrt{D^S} > t_i - t_{i-1} \geq 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} & \sqrt{D^S} < \frac{1 - (2i^2 - 4i + 1)b}{2N - 2i + 1}, \\ & \frac{1}{2N} + \frac{(i-1)^2}{N}b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N}b, \end{aligned}$$

where

$$t_1 = \frac{2 - 2(2N - 2i + 1)\sqrt{D^S} - 2(2i^2 - 4i + 1)b}{2i - 1}.$$

Consider (IV). In this type of equilibrium,

$$t_n = \begin{cases} 0, & n = 0, \\ 1 - 2(N-n)\sqrt{D^S}, & n = 1, \ldots, N. \end{cases}$$

The equilibrium condition is

$$t_1 - t_0 > 0,$$
$$2\sqrt{D^S} > t_1 - t_0 \geq 2\sqrt{D^S} - 2b.$$

Then, we obtain

$$\sqrt{D^S} < \frac{1}{2(N-1)},$$
$$\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b.$$

The derivation of the equilibrium condition for (V) is immediate. Since in this type of equilibrium,

$$t_n = 2\sqrt{D^S}n, \quad n = 0, \ldots, N,$$

it must hold that $t_N = 1$, or equivalently,

$$\sqrt{D^S} = \frac{1}{2N}.$$

∎

A.4 Proof of Corollary 1

It is obtained directly from Theorem 1 and on the basis of the fact that each interval has a length of $2\sqrt{D^S}$ or less (Lemma 2). ∎

A.5 Proof of Corollary 2

By Corollary 1, it is obvious that the sequence of S's ex ante equilibrium payoffs V^S converges to Y^S as $D^S \to 0$. Similarly, as for R's ex ante equilibrium payoff V^R,

$$V^R - V^S = Y^R - Y^S + b\int_0^1 (2t + b - 2\alpha \circ \mu(t))^2 dt \to Y^R - Y^S - b^2$$

as $D^S \to 0$. Then, V^R converges to $Y^R - b^2$. On the other hand, according to CS, S's and R's largest equilibrium ex ante payoffs in the environment without the exit are as follows:

$$\hat{V}^S = Y^S - \frac{4N^2(N^2+2)b^2 + 1}{12N^2},$$
$$\hat{V}^R = Y^R - \frac{4N^2(N^2-1)b^2 + 1}{12N^2},$$

respectively, where N is the largest natural number satisfying (1). By a direct calculation, if $b < \frac{1}{2\sqrt{3}}$, then

$$\hat{V}^S < Y^S,$$
$$\hat{V}^R < Y^R - b^2.$$

This completes the proof. ∎

A.6 Proof of Theorem 2

We suppose that the presupposition of Theorem 2 and for any $t > 0$, $\gamma_+(t) - \gamma_-(t) \leq \overline{\gamma}$ hold in any lemmas appearing in this proof. Further, in this proof, we suppose that $\sigma^S(0) > \sigma^R(0)$ and $b = \sigma^S(0) - \sigma^R(0)$. When $\sigma^R(1) > \sigma^S(1)$, we can prove the proposition by reversing all the variables in the following proof at the center of point 1/2.

First, we prove the following lemma:

Lemma 4 Given any $\ell > 0$ and suppose $\gamma_+(t) - \gamma_-(t) \leq \frac{\delta\ell}{2}$ for any t. Then, for any $\tilde{t} \geq \ell$, there exists $\hat{t}$ such that $\hat{t} \in (\tilde{t} - \ell, \tilde{t})$, $\gamma_+(\hat{t}) = \gamma_-(\tilde{t})$, and $y^S(t, \gamma_-(\tilde{t})) > U^S(t)$ for any $t \in (\hat{t}, \tilde{t})$.

Proof:
For any t, since $\gamma_+(t) - \gamma_-(t) \leq \frac{\delta\ell}{2}$,

$$\gamma_-(t) > \sigma^S(t) - \frac{\delta\ell}{2},$$
$$\gamma_+(t) < \sigma^S(t) + \frac{\delta\ell}{2}$$

hold. Therefore,

$$\gamma_-(\tilde{t}) - \gamma_+(\tilde{t} - \ell) > \sigma^S(\tilde{t}) - \sigma^S(\tilde{t} - \ell) - \underline{\delta}\ell \geq 0,$$

where the last inequality implied by the definition of $\underline{\delta}$. Then, since γ_+ is continuous in t and $\gamma_-(\tilde{t}) < \gamma_+(\tilde{t})$, we can define $\hat{t} = \max\{t|\gamma_+(t) = \gamma_-(\tilde{t})$ such that $\hat{t} \in (\tilde{t} - \ell, \tilde{t})$. Furthermore, for any $t \in (\hat{t}, \tilde{t})$,

$$\gamma_-(t) < \gamma_-(\tilde{t}) < \gamma_+(t),$$

where the first inequality is implied by Assumption 2. Then, it follows that $y^S(t, \gamma_-(\tilde{t})) > U^S(t)$. ∎

Next, we prove the following lemma:

Lemma 5 There exists $\kappa > 0$ such that $t > t'$ and $\gamma_-(t) = \gamma_+(t')$ imply $t - t' \geq \kappa$.

Proof:
Suppose $t > t'$ and $\gamma_-(t) = \gamma_+(t')$. The assumption that $D^S(t) > 0$ for any t implies that $\gamma_+(t) - \gamma_-(t) > 0$ for any t. Then, $\min_t \{\gamma_+(t) - \gamma_-(t)\}$ exists and is strictly positive. Therefore,

$$\gamma_-(t) = \gamma_+(t') \geq \gamma_-(t') + \min_t \{\gamma_+(t) - \gamma_-(t)\}$$

holds. On the other hand, by the Lipschitz continuity of γ_-, we obtain

$$t - t' \geq \frac{\gamma_-(t) - \gamma_-(t')}{\ell},$$

where $\ell > 0$ is Lipschitz constant. Finally,

$$t - t' \geq \frac{\min_t \{\gamma_+(t) - \gamma_-(t)\}}{\ell}$$

holds, which implies that we obtain the lemma by setting $\kappa = \frac{\min_t \{\gamma_+(t) - \gamma_-(t)\}}{\ell}$. ∎

By using these lemmas, we recursively define a decreasing sequence $\{s_n\}_{n=0}^N$ in T as follows: first, we define $s_0 = 1$. For $n \geq 0$,

1. if $s_n = 0$, we stop the recursive process and denote n by N;
2. if $s_n > 0$ and there exists $s' \in T$ such that $\gamma_+(s') = \gamma_-(s_n)$, we define $s_{n+1} = s'$; and
3. if $s_n > 0$ and there exists no $s' \in T$ such that $\gamma_+(s') = \gamma_-(s_n)$, then we define $s_{n+1} = 0$.

Lemma 4 implies that $\{s_n\}_{n=0}^N$ is a strictly decreasing sequence, and Lemma 5 implies that N is necessarily finite and $s_N = 0$. Furthermore, by setting $\ell = \frac{1}{N'-1}$, Lemma 4 directly implies $s_{n-1} - s_n < \frac{1}{N'-1}$ for any n, and therefore, $N \geq N'$.

The construction of $\{s_n\}$ and Lemma 4 directly imply the following lemma (the proof is omitted):

Lemma 6

$\forall n = 1, \dots, N,\ \forall t \in [s_n, s_{n-1}],\ y^S(t, \gamma_-(s_{n-1})) \geq U^S(t)$,

$\forall n = 1, \dots, N-1,\ \forall \hat{a} \neq \gamma_-(s_{n-1}),\ \exists \hat{s} \in (s_n, s_{n-1})$ such that $y^S(t, \hat{a}) < U^S(t)\ \forall t \in [s_n, \hat{s})$ or $\forall t \in (\hat{s}, s_{n-1}]$.

Furthermore, we obtain the following result:

Lemma 7 $\gamma_-(s_{N-1}) \geq \sigma^R(s_{N-1})$.

Proof:
Since $\gamma_+(t) - \gamma_-(t) \leq \overline{\gamma} = \frac{\underline{\delta}}{2(N'-1)}$ holds for any t,

$$\gamma_-(s_{N-1}) > \sigma^S(s_{N-1}) - \frac{\underline{\delta}}{2(N'-1)}$$

holds. Meanwhile, by Assumption 1 and Lemma 4,

$$\sigma^S(s_{N'-1}) - \sigma^R(s_{N-1}) \geq \sigma^S(s_N) - \sigma^R(s_N) - \overline{\delta}(s_{N-1} - s_N) > b - \frac{\overline{\delta}}{N'-1}$$

holds. Then, we obtain

$$\begin{aligned}\gamma_-(s_{N-1}) &> \sigma^S(s_{N-1}) - \frac{\underline{\delta}}{2(N'-1)} \\ &> \sigma^R(s_{N-1}) + b - \frac{\underline{\delta}}{2(N'-1)} - \frac{\overline{\delta}}{N'-1} \\ &\geq \sigma^R(s_{N-1}).\end{aligned}$$

This completes the proof. ∎

Let us return to the proof of Theorem 2. We define $\{a_n\}_{n=1}^N$ as follows:

$$a_n = \gamma_-(t_{n-1}).$$

Then, we construct a candidate for an equilibrium, $(\mu, p, \alpha, \epsilon)$, as follows:

$$\begin{aligned}&\{\tau_n\}_{n=1,\ldots,N} \text{ is a partition of } [0,1], \\ &\inf \tau_n = s_n \text{ and } \sup \tau_n = s_{n-1}, \quad n = 1, \ldots, N, \\ &\mu(t) = m_n, \quad \text{if } t \in \tau_n, \\ &p(m_n, t) = f_{\tau_n}(t), \\ &\alpha(m_n) = a_n, \\ &\epsilon(t,a) = 0, \quad \text{iff } y^S(t,a) \geq U^S(t).\end{aligned}$$

From Lemmas 6 and 7, Assumption 3 ensures that R has no incentive to deviate from α. Moreover, it is verified that S in $t \in \tau_n$ has no incentive to send message $m_{\tilde{n}}$ for $\tilde{n} \neq n$. Then, $(\mu, p, \alpha, \epsilon)$ constitutes an equilibrium. ∎

A.7 Proof of Corollary 3

It is verified that $\gamma_+(t) - \gamma_-(t) \to 0$ as $D^S \to 0$. Then, it is sufficient to show that Assumption 2 holds. We prove only (a) (we can similarly prove (b)). Suppose, to the contrary, that there exists $t > t'$ such that $\gamma_-(t) \leq \gamma_-(t')$. Then, we obtain

$$\begin{aligned} D^S &= y^S(t', \sigma^S(t')) - U^S(t) \\ &= \int_{\gamma_-(t')}^{\sigma^S(t')} \frac{\partial y^S(t', a)}{\partial a} da \\ &< \int_{\gamma_-(t')}^{\sigma^S(t')} \frac{\partial y^S(t, a)}{\partial a} da \\ &\leq \int_{\gamma_-(t)}^{\sigma^S(t)} \frac{\partial y^S(t, a)}{\partial a} da \\ &= y^S(t, \sigma^S(t)) - U^S(t) \\ &= D^S. \end{aligned}$$

Note that the fourth inequality holds since $\gamma_-(t) \leq \gamma_-(t')$, $\sigma^S(t) > \sigma^S(t')$, and $\frac{\partial y^S(t,a)}{\partial a} \geq 0$ for any $a \leq \sigma^S(t)$. This is a contradiction. ∎

B Characterization of General Equilibria

In this section, we derive the conditions for equilibria including those other than no-exit equilibria in the uniform-quadratic environment with constant difference.

We assume $D^S > 0$ and $D^R > 0$ throughout this appendix. First of all, S's optimal exit strategy is as follows:

$$\epsilon(t, a) = \begin{cases} 0 & \text{if } a - b - \sqrt{D^S} \leq t \leq a - b + \sqrt{D^S}, \\ 1 & \text{otherwise.} \end{cases}$$

Next, we consider R's best response. Fix an interval τ where $\inf \tau = \underline{t}$ and $\sup \tau = \bar{t}$. Suppose R is informed via cheap S's message that a realized state t is lying on the interval τ. Denote R's expected equilibrium payoff on choosing a conditional on the belief that $t \in \tau$ by $\tilde{V}^R(a)$ and, for ease of exposition, define $W(a) = (\bar{t} - \underline{t})(\tilde{V}^R(a) - U^R)$, which is written as follows:

Case 1: $\bar{t} - \underline{t} \le 2\sqrt{D^S}$

$W(a) =$

$$\begin{cases} 0 & \text{if } a \in A_1^1 := (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2^1 := [\underline{t} + b - \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}], \\ (\bar{t} - \underline{t})D^R - \int_{\underline{t}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_3^1 := [\bar{t} + b - \sqrt{D^S}, \underline{t} + b + \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4^1 := [\underline{t} + b + \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}], \\ 0 & \text{if } a \in A_5^1 := [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

Case 2 : $\bar{t} - \underline{t} > 2\sqrt{D^S}$

$W(a) =$

$$\begin{cases} 0 & \text{if } a \in A_1^2 := (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2^2 := [\underline{t} + b - \sqrt{D^S}, \underline{t} + b + \sqrt{D^S}], \\ 2\sqrt{D^S}D^R - \int_{a-b-\sqrt{D^S}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_3^2 := [\underline{t} + b + \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4^2 := [\bar{t} + b - \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}], \\ 0 & \text{if } a \in A_5^2 := [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

By tedious calculation, we obtain the following result:

Lemma 8 We define

$$a_j^i = \arg\max_{a \in A_j^i} W(a).$$

Then, $a_1^1 = A_1^1$, $a_5^1 = A_5^1$, $a_1^2 = A_1^2$, $a_3^2 = A_3^2$, $a_5^2 = A_5^2$. And as for a_2^1;

- If $b < \sqrt{D^S} - 2\sqrt{D^R}$, then

$$a_2^1 = \{\underline{t} + b - \sqrt{D^S}\}.$$

- If $b = \sqrt{D^S} - 2\sqrt{D^R}$, then

$$a_2^1 = \begin{cases} \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} < -b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}, \underline{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \ge -b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $\sqrt{D^S} - 2\sqrt{D^R} < b < \sqrt{D^S} + \sqrt{D^R}$, then

$$a_2^1 = \begin{cases} \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \le -b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} \ge -b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $b \geq \sqrt{D^S} + \sqrt{D^R}$, then

$$a_2^1 = \{\underline{t} + b - \sqrt{D^S}\}.$$

As for a_3^1;

$$a_3^1 = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} \leq 2\sqrt{D^S} - 2b, \\ \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b. \end{cases}$$

As for a_4^1;

- If $b + \sqrt{D^S} \leq \sqrt{D^R}$, then

$$a_4^1 = \begin{cases} \{\underline{t} + b + \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \leq b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\bar{t} - \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} \geq b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $\sqrt{D^R} < b + \sqrt{D^S} < 2\sqrt{D^R}$, then

$$a_4^1 = \begin{cases} \{\bar{t} + b + \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} < \frac{3(b+\sqrt{D^S}) - \sqrt{12D^R - 3(b+\sqrt{D^S})^2}}{2}, \\ \{\bar{t} + b + \sqrt{D^S}, \underline{t} + b + \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} = \frac{3(b+\sqrt{D^S}) - \sqrt{12D^R - 3(b+\sqrt{D^S})^2}}{2}, \\ \{\underline{t} + b + \sqrt{D^S}\} & \text{if } \frac{3(b+\sqrt{D^S}) - \sqrt{12D^R - 3(b+\sqrt{D^S})^2}}{2} \\ & \quad < \bar{t} - \underline{t} \leq b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\bar{t} - \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} \geq b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $b + \sqrt{D^S} = 2\sqrt{D^R}$, then

$$a_4^1 = \begin{cases} \{\bar{t} + b + \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} < \frac{3}{2}(b + \sqrt{D^S}), \\ \{\bar{t} + b + \sqrt{D^S}, \bar{t} - \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} \geq \frac{3}{2}(b + \sqrt{D^S}). \end{cases}$$

- If $b + \sqrt{D^S} > 2\sqrt{D^R}$, then

$$a_4^1 = \{\bar{t} + b + \sqrt{D^S}\}.$$

As for a_2^2;

- If $\sqrt{D^S} < \sqrt{D^R}$, then

$$a_2^2 = \begin{cases} \{\underline{t} + b + \sqrt{D^S}\} & \text{if } b \leq -\sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } -\sqrt{D^S} + \sqrt{D^R} \leq b \leq \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + b - \sqrt{D^S}\} & \text{if } b \geq \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $\sqrt{D^R} \leq \sqrt{D^S} \leq 2\sqrt{D^R}$, then

$$a_2^2 = \begin{cases} \{\underline{t} + \sqrt{D^R}\} & \text{if } b \leq \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + b - \sqrt{D^S}\} & \text{if } b \geq \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

- If $\sqrt{D^S} > 2\sqrt{D^R}$, then

$$a_2^2 = \begin{cases} \{\underline{t} + b - \sqrt{D^S}\} & \text{if } b < \sqrt{D^S} - 2\sqrt{D^R}, \\ \{\underline{t} + b - \sqrt{D^S}, \underline{t} + \sqrt{D^R}\} & \text{if } b = \sqrt{D^S} - 2\sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } \sqrt{D^S} - 2\sqrt{D^R} < b \leq \sqrt{D^S} + \sqrt{D^S}, \\ \{\underline{t} + b - \sqrt{D^S}\} & \text{if } b \geq \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

As for a_4^2;

- If $\sqrt{D^S} \leq \sqrt{D^R}$, then

$$a_4^2 = \begin{cases} \{\bar{t} + b - \sqrt{D^S}\} & \text{if } b < \sqrt{-\frac{D^S}{3} + D^R}, \\ \{\bar{t} + b - \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}\} & \text{if } b = \sqrt{-\frac{D^S}{3} + D^R}, \\ \{\bar{t} + b + \sqrt{D^S}\} & \text{if } b > \sqrt{-\frac{D^S}{3} + D^R}. \end{cases}$$

- If $\sqrt{D^R} < \sqrt{D^S} < \frac{3}{2}\sqrt{D^R}$, then

$$a_4^2 = \begin{cases} \{\bar{t} - \sqrt{D^R}\} & \text{if } b \leq \sqrt{D^S} - \sqrt{D^R}, \\ \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \sqrt{D^S} - \sqrt{D^R} \leq b < \sqrt{-\frac{D^S}{3} + D^R}, \\ \{\bar{t} + b - \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}\} & \text{if } b = \sqrt{-\frac{D^S}{3} + D^R}, \\ \{\bar{t} + b + \sqrt{D^S}\} & \text{if } b > \sqrt{-\frac{D^S}{3} + D^R}. \end{cases}$$

- If $\frac{3}{2}\sqrt{D^R} \leq \sqrt{D^S} < 2\sqrt{D^R}$, then

$$a_4^2 = \begin{cases} \{\bar{t} - \sqrt{D^R}\} & \text{if } b < -\sqrt{D^S} + 2\sqrt{D^R}, \\ \{\bar{t} - \sqrt{D^R}, \bar{t} + b + \sqrt{D^S}\} & \text{if } b = -\sqrt{D^S} + 2\sqrt{D^R}, \\ \{\bar{t} + b + \sqrt{D^S}\} & \text{if } b > -\sqrt{D^S} + 2\sqrt{D^R}. \end{cases}$$

- If $\sqrt{D^S} \geq 2\sqrt{D^R}$, then $a_4^2 = \{\bar{t} + b + \sqrt{D^S}\}$.

In order to characterize R's best response, the following lemma is necessary:

Lemma 9 Suppose $b < \sqrt{D^S} - \sqrt{D^R}$ and $-b + \sqrt{D^S} + \sqrt{D^R} < \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$. Then, there exists $\hat{T}$ such that $\hat{T} \in (-b + \sqrt{D^S} + \sqrt{D^R}, 2\sqrt{D^S} - 2b)$ and

$$W\left(\frac{\bar{t} + \underline{t}}{2}\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} W(\underline{t} + \sqrt{D^R}) \;\Leftrightarrow\; \bar{t} - \underline{t} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \hat{T}.$$

Moreover, such $\hat{T}$ is unique.

Now we are in a position to completely characterize R's best response.

Proposition 1 Let

$$A_E := A_1^1 \cup A_5^1 = A_1^2 \cup A_5^2.$$

Then, Receiver's best response a^* is as follows:

Case 1: $b < \sqrt{D^S}$ and $b \leq -\sqrt{D^S} + \sqrt{D^R}$

$$a^* = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \{\bar{t} + b - \sqrt{D^S}\} & \text{if } 2\sqrt{D^S} - 2b \leq \bar{t} - \underline{t} \leq 2\sqrt{D^S}, \\ A_3^2 & \text{if } \bar{t} - \underline{t} > 2\sqrt{D^S}. \end{cases}$$

Case 2: $\sqrt{D^S} \leq b \leq -\sqrt{D^S} + \sqrt{D^R}$

$$a^* = \begin{cases} \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \leq 2\sqrt{D^S}, \\ A_3^2 & \text{if } \bar{t} - \underline{t} > 2\sqrt{D^S}. \end{cases}$$

Case 3: $-\sqrt{D^S} + \sqrt{D^R} < b < \sqrt{D^S}$ and $b \geq \sqrt{D^S} - \sqrt{D^R}$

$$a^* = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \{\bar{t} + b - \sqrt{D^S}\} & \text{if } 2\sqrt{D^S} - 2b \leq \bar{t} - \underline{t} \leq -b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} > -b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

Case 4: $-\sqrt{D^S} + \sqrt{D^R} < b < \sqrt{D^S} + \sqrt{D^R}$ and $b \geq \sqrt{D^S}$

$$a^* = \begin{cases} \{\bar{t} + b - \sqrt{D^S}\} & \text{if } \bar{t} - \underline{t} \leq -b + \sqrt{D^S} + \sqrt{D^R}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} > -b + \sqrt{D^S} + \sqrt{D^R}. \end{cases}$$

Case 5: $\sqrt{D^S} - 2\sqrt{D^R} < b < \sqrt{D^S} - \sqrt{D^R}$

$$a^* = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} < \hat{T}, \\ \left\{\frac{\bar{t}+\underline{t}}{2}, \underline{t} + \sqrt{D^R}\right\} & \text{if } \bar{t} - \underline{t} = \hat{T}, \\ \{\underline{t} + \sqrt{D^R}\} & \text{if } \bar{t} - \underline{t} > \hat{T}. \end{cases}$$

Case 6: $b = \sqrt{D^S} - 2\sqrt{D^R}$

$$a^* = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} < \sqrt{12D^R}, \\ \left\{\frac{\bar{t}+\underline{t}}{2}, \underline{t} + \sqrt{D^R}\right\} \cup A_E & \text{if } \bar{t} - \underline{t} = \sqrt{12D^R}, \\ \{\underline{t} + \sqrt{D^R}\} \cup A_E & \text{if } \bar{t} - \underline{t} > \sqrt{12D^R}. \end{cases}$$

Case 7: $b < \sqrt{D^S} - 2\sqrt{D^R}$

$$a^* = \begin{cases} \left\{\frac{\bar{t}+\underline{t}}{2}\right\} & \text{if } \bar{t} - \underline{t} < \sqrt{12D^R}, \\ \left\{\frac{\bar{t}+\underline{t}}{2}\right\} \cup A_E & \text{if } \bar{t} - \underline{t} = \sqrt{12D^R}, \\ A_E & \text{if } \bar{t} - \underline{t} > \sqrt{12D^R}. \end{cases}$$

Case 8: $b \geq \sqrt{D^S} + \sqrt{D^R}$

$$a^* = A_E.$$

These cases are illustrated in Figure 2.

<Figures 2 should be inserted>

Based on R's response, candidates for equilibrium intervals can be classified into the following the six categories (where $\hat{a} = \alpha \circ \mu(t)$ for $t \in \tau$):

- **Interval $\mathcal{N}$:**
  - $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$,
  - $\hat{a} = \frac{\bar{t}+\underline{t}}{2}$,
  - $y^S(\underline{t}, \hat{a}) > U^S$,
  - $y^S(\bar{t}, \hat{a}) > U^S$, and
  - S never exercises the exit option in this interval.

- **Interval $\mathcal{A}$:**
  - $2\sqrt{D^S} - 2b \leq \bar{t} - \underline{t} < \min\{2\sqrt{D^S}, -b + \sqrt{D^S} + \sqrt{D^R}\}$,
  - $\hat{a} = \bar{t} + b - \sqrt{D^S}$,
  - $y^S(\underline{t}, \hat{a}) > U^S$,
  - $y^S(\bar{t}, \hat{a}) = U^S$, and
  - S never exercises the exit option in this interval.
- **Interval $\mathcal{F}$:**
  - $\bar{t} - \underline{t} = 2\sqrt{D^S}$,
  - $\hat{a} = \bar{t} + b - \sqrt{D^S}$,
  - $y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S$, and
  - S never exercises the exit option in this interval.
- **Interval $\mathcal{R}$:**
  - $\bar{t} - \underline{t} > -b + \sqrt{D^S} + \sqrt{D^R}$,
  - $\hat{a} = \underline{t} + \sqrt{D^R}$,
  - $y^S(\underline{t}, \hat{a}) > U^S$
  - $y^S(\bar{t}, \hat{a}) < U^S$, and
  - S exercises the exit option on the right side of the interval.
- **Interval $\mathcal{B}$:**
  - $\bar{t} - \underline{t} > 2\sqrt{D^S}$,
  - $\hat{a} \in A_3^2$,
  - $y^S(\underline{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \underline{t} + b + \sqrt{D^S}$,
  - $y^S(\bar{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \bar{t} + b - \sqrt{D^S}$, and
  - S exercises the exit option around either end of the interval. Particularly, S does so around both ends of the interval when $\hat{a} \in \text{int } A_3^2$.
- **Interval $\mathcal{E}$:**

* there is no constraint on the length of interval,
* $\hat{a} \in A_E$,
* $y^S(\underline{t}, \hat{a}) \leq U^S$ where the equality holds only $\hat{a} = \underline{t} + b - \sqrt{D^S}$,
* $y^S(\bar{t}, \hat{a}) \leq U^S$ where the equality holds only if $\hat{a} = \bar{t} + b + \sqrt{D^S}$,and
* S almost always exercises the exit option in this interval.

Using this classification of intervals, we can completely enumerate possible intervals and possible connections of them in each case described in Proposition 1 as follows:

Case 1: Interval: $\mathcal{N}, \mathcal{A}, \mathcal{F}, \mathcal{B}$

Connection: $\mathcal{N}\mathcal{N}, \mathcal{N}\mathcal{A}, \mathcal{A}\mathcal{F}, \mathcal{A}\mathcal{B}, \mathcal{F}\mathcal{F}, \mathcal{F}\mathcal{B}, \mathcal{B}\mathcal{F}, \mathcal{B}\mathcal{B}$

Case 2: Interval: $\mathcal{A}, \mathcal{F}, \mathcal{B}$

Connection: $\mathcal{A}\mathcal{F}, \mathcal{A}\mathcal{B}, \mathcal{F}\mathcal{F}, \mathcal{F}\mathcal{B}, \mathcal{B}\mathcal{F}, \mathcal{B}\mathcal{B}$

Case 3: Interval: $\mathcal{N}, \mathcal{A}, \mathcal{R}$

Connection: $\mathcal{N}\mathcal{N}, \mathcal{N}\mathcal{A}, \mathcal{N}\mathcal{R}$

Case 4: Interval: $\mathcal{A}, \mathcal{R}$

Connection: no possible connection

Case 5: Interval: $\mathcal{N}, \mathcal{R}$

Connection: $\mathcal{N}\mathcal{N}, \mathcal{N}\mathcal{R}$

Additional Constraint on $\mathcal{N}$: $\bar{t} - \underline{t} \leq \hat{T}$

Additional Constraint on $\mathcal{R}$: $\bar{t} - \underline{t} \geq \hat{T}$

Case 6: Interval: $\mathcal{N}, \mathcal{R}, \mathcal{E}$

Connection: $\mathcal{N}\mathcal{N}, \mathcal{N}\mathcal{R}, \mathcal{R}\mathcal{E}, \mathcal{E}\mathcal{E}$

Additional Constraint on $\mathcal{N}$: $\bar{t} - \underline{t} \leq \sqrt{12D^R}$

Additional Constraint on $\mathcal{R}$: $\bar{t} - \underline{t} \geq \sqrt{12D^R}$

Additional Constraint on $\mathcal{E}$: $\bar{t} - \underline{t} \geq \sqrt{12D^R}$

Case 7: Interval: $\mathcal{N}, \mathcal{E}$

Connection: $\mathcal{E}\mathcal{E}$

Additional Constraint on $\mathcal{N}$**:** $\bar{t} - \underline{t} \leq \sqrt{12D^R}$

Additional Constraint on $\mathcal{E}$**:** $\bar{t} - \underline{t} \geq \sqrt{12D^R}$

Case 8: Interval: $\mathcal{E}$

Connection: $\mathcal{E}\mathcal{E}$

C On the Conditions of Theorem 2

In this appendix, we mainly discuss the conditions of Theorem 2 and the generality of our results. In all the examples bellow, we assume that both players' exit payoffs are constant and Assumption 3 holds.

C.1 Variable Bias

Assumption 1 is much less strict than the constant bias assumption made by the ordinary uniform-quadratic model. Consider the following example.[27]

We assume that

$$y^S = Y^S - (t-a)^2,$$
$$y^R = Y^R - (ct-b-a)^2,$$

where $b > 0$ and $c > 0$. Furthermore, $F(t)$ is uniform. Then, since

$$\sigma^S(t) = t,$$
$$\sigma^R(t) = ct - b,$$

it is verified that Assumption 1 is met. Note that when $c > 1 + b$, we obtain

$$\sigma^S(0) > \sigma^R(0),$$
$$\sigma^S(1) < \sigma^R(1).$$

In other words, the sign of the incongruence between S's and R's preferences is reversed.

[27] This case is a variant of that shown in Melumad and Shibano [27].

We define N such that

$$\frac{1}{2(N-1)} > \sqrt{D^S} \geq \frac{1}{2N}. \tag{2}$$

Then, as D^S approaches 0, N increases to infinity.

We define $t_0 = 0$, and

$$\begin{aligned} t_n &= 1 - 2(N-n)\sqrt{D^S}, \quad n = 1, \ldots, N, \\ a_n &= 1 - (2N - 2n + 1)\sqrt{D^S}, \quad n = 1, \ldots, N. \end{aligned}$$

We construct a candidate for an equilibrium as follows:

$$\begin{aligned} &\{\tau_n\}_{n=1,\ldots,N} \text{ is a partition of } [0,1], \\ &\inf \tau_n = t_{n-1} \text{ and } \sup \tau_n = t_n, \quad n = 1, \ldots, N, \\ &\mu(t) = m_n, \quad t \in \tau_n, \\ &p(m_n, t) = f_{\tau_n}(t), \\ &\alpha(m_n) = a_n, \\ &\epsilon(t, a) = 0, \quad \text{iff } y^S(t, a) \geq U^S. \end{aligned}$$

Since $y^S(t, \alpha \circ \mu(t)) \geq U^S$, the exit option is never chosen on the equilibrium path. However, for $n = 2, \ldots, N$, since $y^S(t_{n-1}, a_n) = y^S(t_n, a_n) = U^S$, if R chooses $\tilde{a} \neq a_n$, some types of S belonging to τ_n would choose the exit option. Therefore, R with a sufficiently small U^R has no incentive to deviate from the equilibrium action a_n.

Consider R's incentive after receiving a signal m_1. Since $y^S(t_1, a_1) = U^S$ and $a_1 > \sigma^S(t_1)$, if R chooses $\tilde{a} < a_1$, some types of S close to t_1 would choose the exit option. Therefore, R with sufficiently small U^R has no incentive to choose $\tilde{a} < a_1$. However, R's expected payoff function *when the exit option is never chosen* is single-peaked and the maximum is attained at

$$a^* = c\mathbb{E}[t | t \in \tau_1] - b.$$

Suppose

$$N \geq \frac{c}{b}.$$

Then, by (2),

$$\begin{aligned}
a^* &= c\mathbb{E}[t|t \in \tau_1] - b \\
&< c\left[1 - 2(N-1)\sqrt{D^S}\right] - b \\
&\leq c\left(1 - \frac{N-1}{N}\right) - b \\
&\leq 0 \\
&< 1 - (2N-1)\sqrt{D^S} \\
&= a_1.
\end{aligned}$$

This implies that a deviation $\tilde{a} > a_1$ is never beneficial for R with a sufficiently large D^R. Thus, it is evident that the above candidate indeed constitutes an equilibrium.

C.2 Variable Sender's Differences

Even for the environments where S's difference is not constant, Theorem 2 gives us a sufficient condition for the existence of NEE with many intervals. Consider the following example.

We assume that

$$\begin{aligned}
y^S &= Y^S(t) - (t + b - a)^2, \\
y^R &= Y^R(t) - (t - a)^2,
\end{aligned}$$

where $b > 0$. Note that Assumption 1 is met. We obtain

$$\begin{aligned}
\gamma_+(t) &= t + b + \sqrt{D^S(t)}, \\
\gamma_-(t) &= t + b - \sqrt{D^S(t)}.
\end{aligned}$$

Then, $\gamma_+(t) - \gamma_-(t) < \overline{\gamma}$ holds for any t if and only if

$$\sqrt{D^S(t)} \leq \frac{1}{4(N-1)} \quad \forall t.$$

On the other hand, Assumption 2, that is, $\frac{d\gamma_-(t)}{dt} > 0$, holds if and only if

$$\sqrt{D^S(t)} > \frac{1}{2}\frac{dD^S(t)}{dt} \quad \forall t.$$

If these conditions hold, Theorem 2 guarantees the existence of NEE with N or more intervals.

C.3 Necessity of Assumption 2

Assumption 2 is crucial for the construction of NEE driven by the credibility of exit. Consider the following example.

We assume that

$$y^S(t,a) = d^2 t + \varepsilon - (t+b-a)^2,$$
$$y^R(t,a) = Y^R(t) - (t-a)^2,$$

where $b > 0$, $\frac{1}{N-1} > d > \frac{2}{2N-1}$ for some natural number N, and ε is a sufficiently small positive real number. This model satisfies Assumption 1. However,

$$\gamma_-(t) = t + b - \sqrt{d^2 t + \varepsilon}.$$

Then, Assumption 2 is violated. In this case, we cannot indeed construct an NEE driven by the credibility of exit. However, there exists the following equilibrium with N intervals characterized by $\{t_n\}_{n=0}^N$ and $\{a_n\}_{n=1}^N$(as long as $Y^R(t)$ is sufficiently large for any t):

$$t_N = 1,$$
$$t_{n-1} = t_n - 2\sqrt{d^2 t_n + \varepsilon} + d^2 \quad n = N, \dots, 2,$$
$$t_0 = 0,$$
$$a_1 \text{ is some action satisfying } y^S(t, a_1) < 0 \quad \forall t,$$
$$a_n = t_n + b - \sqrt{d^2 t_n + \varepsilon} \quad n = 2, \dots, N.$$

where ε is sufficiently small such that

$$t_1 > 0,$$
$$2\sqrt{d^2 t_1 + \varepsilon} < d^2.$$

It is verified that such ε indeed exists.

On the equilibrium path, S chooses an exit option when $t \in [0, t_1)$. In order to avoid this, $a \in [b - \sqrt{\varepsilon}, b + \sqrt{\varepsilon}]$ must be chosen, but this action induces some types of S belonging to $[t_1, t_2]$ to deviate from the equilibrium strategy.

C.4 Necessity of the Upper Bound on $(\gamma_+ - \gamma_-)$

The upper bound on $(\gamma_+ - \gamma_-)$ is also a crucial condition for Theorem 2. Consider the following example.[28]

We assume that

$$\begin{aligned} y^S &= (1+t)\sqrt{a} - \frac{1}{\sqrt{1+4b}}a, \\ y^R &= (1+t)\sqrt{a} - a, \end{aligned}$$

where $b < 0$. In this example, Assumptions 1 and 2 are satisfied. However, it is verified that as long as $\min_t \max_a y^S(t,a) > U^S$,

$$\gamma_+(1) - \gamma_-(1) > \sqrt{2}(1+4b)$$

holds. Then, the presupposition of Theorem 2 is not satisfied for sufficiently large N.

In this example, if $y^S(\hat{t}, \hat{a}) = U^S$, then for $t < \hat{t}$, $y^S(t, \hat{a}) < U^S$. This implies that S receives a positive payoff in any boundary point, except at $t = 1$. Therefore, there is no NEE driven by the credibility of exit.

[28] This case is a special case shown in Marino [25].

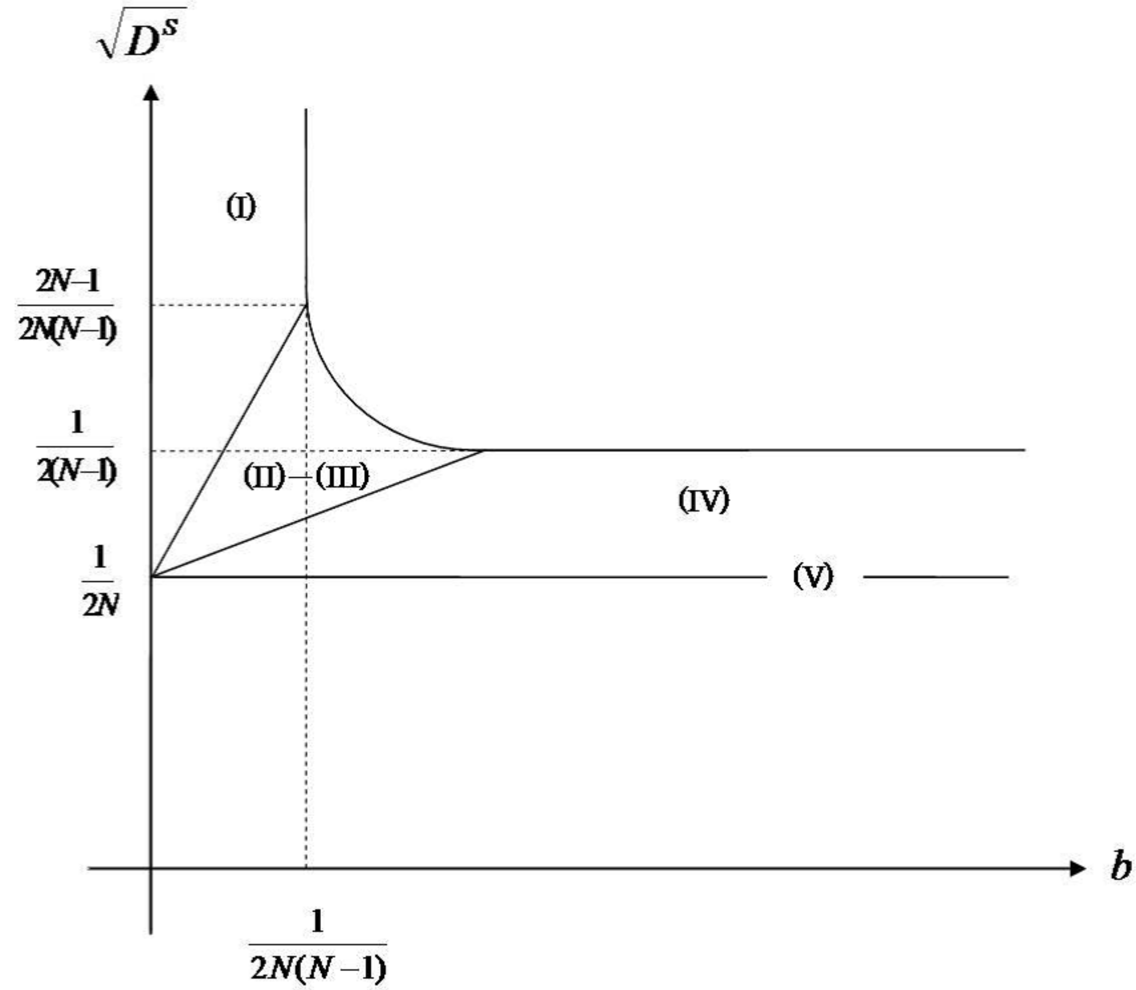

Figure 1: Equilibrium condition of NEE with N intervals ($N \geq 2$)

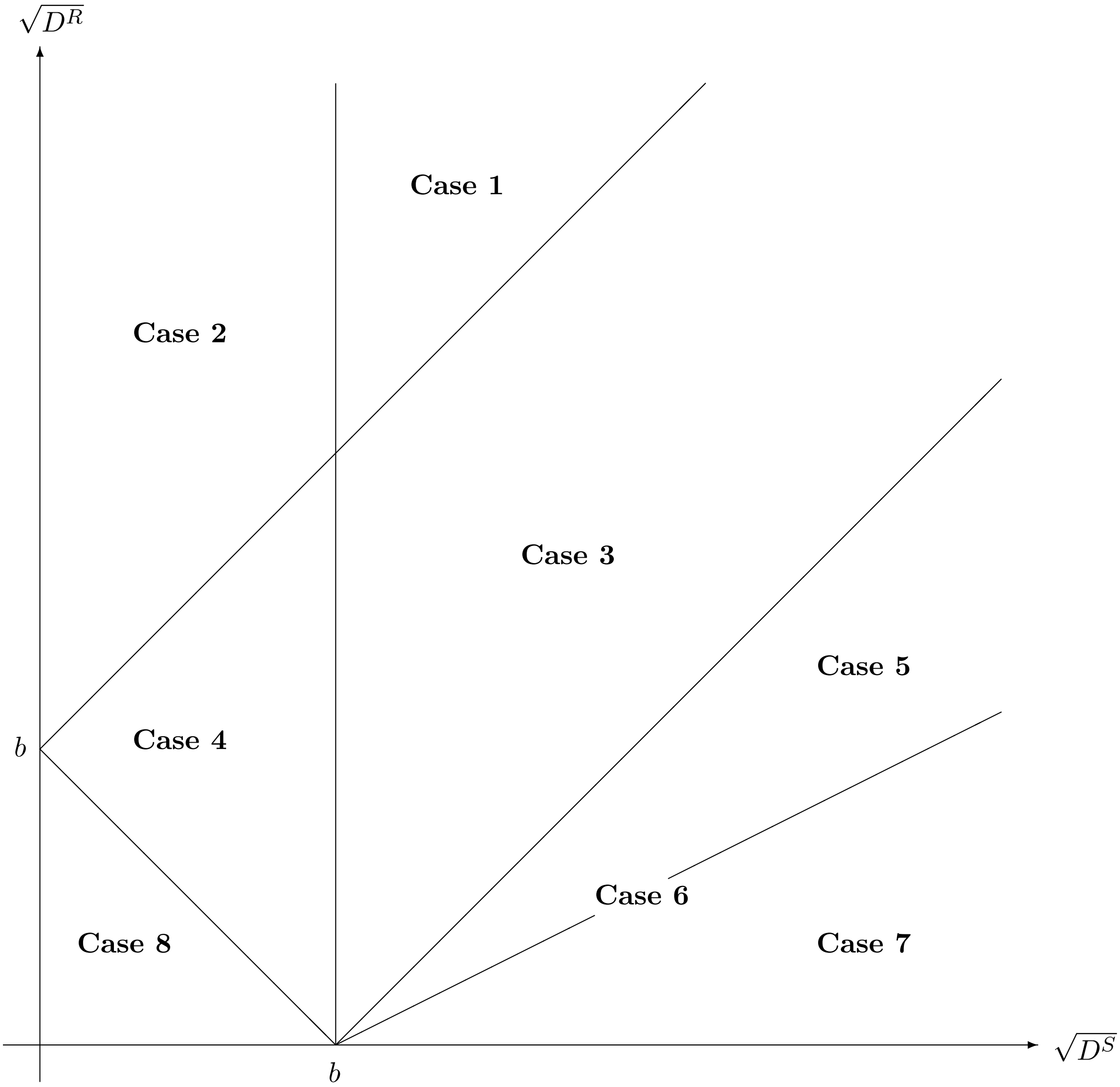

Figure 2: R's best response patterns

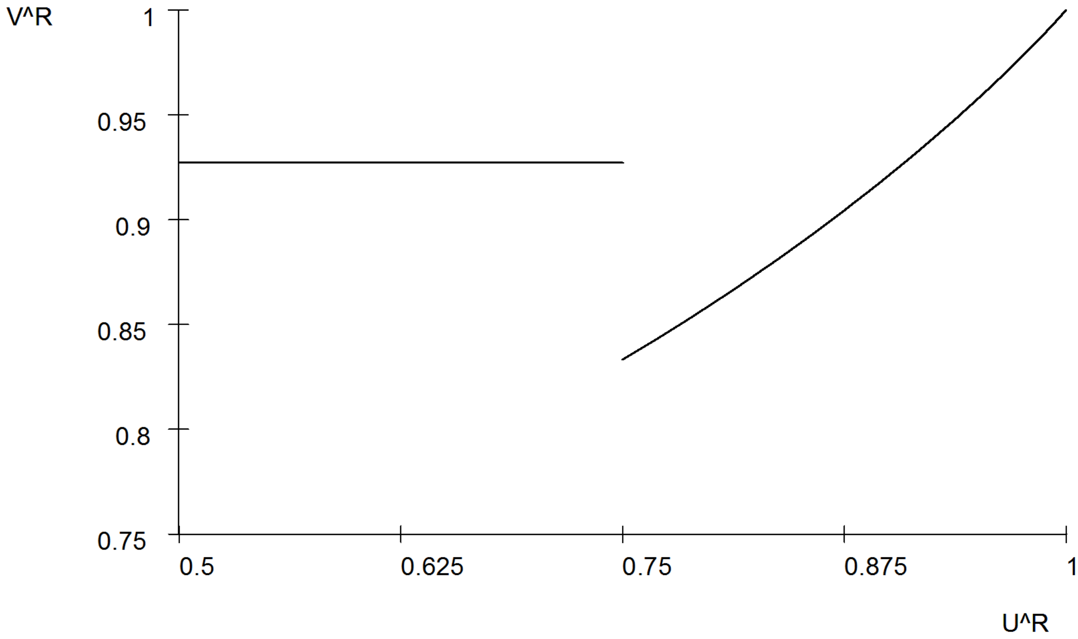

Figure 3: S's ex ante expected equilibrium payoff