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# Robert Torrens and the Classical Theory of Growth

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# Robert Torrens and the Classical Theory of Growth[†]

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This paper reconstructs Torrens's theory of economic growth. Actual wage is determined by the capital-population ratio between the maximum and the minimum wage. In the process of growth, the maximum wage keeps being lowered by the decreasing marginal productivity in agriculture, while the minimum wage remains constant. Based on these notions of wages, Torrens's economic dynamics are described as follows. In the early stage of growth, capital increases faster than population so that the actual wage rises above the minimum. Thereafter, the economy grows with a tendency for the population to increase faster than the capital while limiting the actual wage below the decreasing maximum until it enters a stationary state and the actual wage and profit rate are reduced to their minimum. This theory has been attributed to Ricardo by some scholars, but Torrens proposed a more ingenious theory than Ricardo's.

## 1. Introduction

In the recent issue of the *Cambridge Journal of Economics*, Rosell (2016) proposed a valuable reconstruction of the theory of distribution presented by Robert Torrens (1780?–1864) in his text entitled *On the Means of Improving the Conditions of the Labouring Classes*, which is appended to the final edition of *External Corn Trade* (Torrens, 1829).[1] The main conclusions in Rosell's reconstruction indicate the following: Torrens's approach to distribution considers that the wage rate is set as a result of competition among capitalists or among workers, differing from the neoclassical economist's way of identifying the real wage rate as the marginal products of labour, and an original conception of a surplus of labour to explain profit, based on the classical notions of necessary labour and labour commanded, without resorting to the Ricardian labour theory of value or the Marxist theory of value. His approach thus does not require fixing one of two distribution variables exogenously in order to determine the allocation of the net produce between the capitalist's and worker's income—a way which differs from both Ricardo's way of requiring the wage rate given exogenously and Sraffa's way of requiring the profit rate given exogenously.

The 'statics model' of distribution reformulated by Rosell (2016, p. 15/18) can be clearly found in the 1829 text. However, most discussions in the text were developed within the *dynamic* view. In particular, Torrens's dynamic theory of economic growth based on three notions of wage—the maximum wage, minimum wage and actual wage—is worthy of reconstruction because it explicitly formed a more ingenious theory than that which has been attributed to David Ricardo (1772–1823) by Hicks and Hollander (1977) and Casarosa (1982).

This paper reconstructs the economic growth theory in the 1829 text and clarifies Torrens's vision of the dynamics of a closed economy. Torrens's view on economic dynamics can be explicitly explained by the growth model, which incorporates the wage satisfying dynamic equilibrium between the growth rates of population and of capital, the minimum wage fixed exogenously and the maximum wage depending on the fertility of the lands under cultivation. Torrens repeatedly emphasised that the maximum wage rate in terms of corn continues falling from the necessity of resorting to inferior lands in the process of capital accumulation, but Rosell omits this point from his considerations.

Torrens's dynamic approach requires the classical theory of differential rent, the core of 'the principles of rent' (Torrens, 1829, p. 466), so that *diminishing* returns in agricultural production must be assumed. Torrens did consider distribution issues in connection with both the theory of rent and the law of diminishing returns, but such connections are never involved in

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[1] O'Brien (1966) points out that Torrens's letter to Wilmot-Horton dated 23 March 1826 was in fact the draft of the 1829 Appendix. The Appendix, according to Robbins (1958, p. 273) was reprinted with minor alterations in four publications: a tract entitled *Colonel Torrens on the Wages of Labour, and on the Means of Improving the Condition of the Working Classes. A New Edition, Corrected* (1832), which is, according to De Vivo (2006, xxiii), 'a reprint (with minor alterations) of the Appendix to the 1829 edition of *External Corn Trade*', *On Wages and Combination* (Torrens, 1834), *A Letter to Russell* (Torrens, 1837) and *A Letter to Ashley* (Torrens, 1844).

Rosell's 'static model', in which returns to scale are assumed to be constant.

It should be also noted that in 1829, unlike before, Torrens argued a long-run tendency in population to increase faster than capital. The argument has an important role in clarifying his vision of economic dynamics. Such a long-term analysis is not involved in Rosell's model, in which the system is assumed to be limited to a single period. Also, as Torrens claimed to Nassau William Senior (1790–1864), the credit of being the first to prove the tendency, the change of his position was never trivial for him.

This paper is organised as follows. Section 2 introduces Torrens's concept of wages, based on which Torrens's model of economic growth is set up in Section 3. Section 4 clarifies his vision on economic dynamics using the reconstructed model and presents a more theoretical understanding of the maximum wage depending on the land productivity and a more theoretical explanation of his correction concerning the growth rates of population and of capital. Section 5 presents some concluding remarks.

## 2. Wages

### *2.1 Actual wage*

The term 'wages' is defined as the 'articles [necessaries and comforts] of wealth which the labourer receives, in exchange for his labour'. It is 'real wages' or 'commodity wages' that 'consist of the quantity of necessaries and comforts which the labourer receives' while it is 'nominal wages' or 'money wages' that are 'the sum of money in which he is paid'. An increase (a decrease) in 'the quantity of necessaries and comforts which the labourer received' means a raise (fall) in real wages (Torrens, 1829, p. 454). All the discussions in the 1829 Appendix are explained with the wage in terms of a commodity, 'corn', that is, the real wage in terms of corn.

The real level of 'actual wages'[2] is determined by 'the ratio between population and capital, or, more correctly, between the quantity of labour and the amount of capital' in the labour market (p. 468).[3] There, however, are the limits beyond which the actual wage rate cannot raise and below which it cannot fall.

### *2.2 Maximum wage*

The limit beyond which the actual wage rate cannot rise is the 'maximum' wage rate. What Torrens calls 'the maximum of wages' is defined as 'that quantity of the products of industry which remains, after replacing the advances, not consisting of wages, and paying the capitalist the lowest rate of profit which will induce him to continue the work of production' (p. 461). His numerical example in terms of corn illustrates that the amount of 'maximum wages' received by 100 workers (200 quarters) is the amount of output (428 quarters) minus the amount of non-wages fund (200 quarters) minus the amount of minimum profits (28 quarters) which is calculated as the amount of capital (400 quarters), composed of wages fund and non-wages fund, times the lowest profit rate (7%):

$$\underbrace{2\text{qrs.}\times \overbrace{100\text{ men}}^{\text{Workers}}}_{\text{Maxmum wages}} = \underbrace{428\text{qrs.}}_{\text{Output}} - \underbrace{200\text{qrs.}}_{\text{Non wages fund}} - \underbrace{\overbrace{(2\text{qrs.}\times 100\text{ men} + 200\text{qrs.})}^{\text{Capital}} \times 7\%}_{\text{Minimum profits}}.$$

The maximum wage per capita thus can be calculated by:

$$2\text{qrs. per capita} = \frac{428\text{qrs.}/100\text{ men}}{1+7\%} - \frac{200\text{qrs.}}{100\text{ men}},$$

that is,

$$\text{Maxmum wage rate} = \frac{\text{Output per capita}}{1+\text{Minimum profit rate}} - \text{Non wages fund per capita.}$$

The maximum wage rate depends on the following three circumstances: 'the quality of the soil, the skill with which labour is applied, and the degree of freedom which is allowed to trade' (p. 462). Torrens states that the maximum wage rate 'may be raised, either by the cultivation of land of a better quality, or

[2] The minimum wage is equal to what Torrens had called 'the market price of labour' in the first edition of *External Corn Trade* (Torrens, 1815).

[3] Considering that capital composts of not only wages fund but also of non-wages fund, Torrens later changed the phrase 'the amount of capital' into 'the quantity of the ingredients of capital destined for its maintenance' (1834, p. 22).

by improvements in the effective powers of industry; and that this maximum will be lowered, either by resorting to poorer soils, or by a falling off in the effective powers of industry' (p. 458). Our work on the dynamics of a *closed* economy requires analyses of the effects of land productivity and labour skill upon the economy.

According to Torrens, in an industrial country such as England, which is densely populated, the maximum wage rate tends to be depressed 'by the necessity of resorting to inferior soils' for supplying food to large numbers of people (p. 457). Hence, in order to 'raise the maximum of wages to the highest point', England should carry on 'a perfectly free trade with thinly-peopled countries, in which none but soils of first-rate quality are under tillage' (p. 459). That is, improving the degree of freedom allowed to international trade has the same effect as that of increasing the land productivity upon the maximum wage.[4] Torrens was dissatisfied with the government policies:

> England having acquired in manufacturing industry an efficacy unexampled in the history of the world, and having the new countries of the American continent open to her commerce, is placed in that precise situation in which the maximum of wages may be elevated to the highest attainable point. But the vast, the incalculable advantages of this situation are counteracted by the restrictive system [of Corn Laws], which excludes cheap raw produce from our markets. (pp. 459–60)

*2.3 Minimum wage*

The limit below which the actual wage rate cannot permanently fall is the 'minimum' wage rate. What Torrens calls 'the minimum of wages' is defined as 'a quantity of the necessaries and conveniences of life sufficient to preserve the labourer in working condition, and to induce him to keep up the race of labourers'.[5] The quantity and variety of these commodities is exogenously determined by 'climate and custom', or 'natural causes' and 'moral causes' (pp. 460–61):

> A labourer in Hindostan may continue to work with perfect vigour, while receiving a supply of clothing which would be insufficient to preserve a labourer in Russia from perishing. Even in countries situated in the same climate, different habits of living will often occasion variations in the minimum of wages, as considerable as those which are produced by natural causes. The labourer in Ireland will rear a family under circumstances which would not only deter an English workman from marriage, but would force him on the parish for personal support. (p. 460)[6]

Note that 'moral causes', unlike 'natural causes', can be artificially altered. Torrens argues that the improvement of the habits of living in Ireland 'would raise the minimum of wages in that country to an equality with their minimum in England' (p. 460). Torrens thus acknowledges the possible alterations in the minimum wage, but he insists that the minimum 'may, in any given time and place, be regarded as very nearly stationary' because the alterations in it 'cannot be suddenly effected':

> So far as this minimum depends upon climate, it is unchangeable; and even so far as it is determined by the habits of living, and [the established scale of comfort],[7] it can be effected only by those circumstances of prosperity or decay, and by those moral causes of instruction and civilization, which are ever gradual in their operation. (p. 461)

The minimum wage, from the definition, does not necessarily mean a physical subsistence level. Torrens, however, mentions it as 'the extreme minimum, below which the merely animal wants of the labouring population cannot be supplied' (p. 470). The 'extreme minimum' of wages would consist in a quantity of the

[4] According to O'Brien (1966, p. 338), in Torrens's 1826 letter to Wilmot-Horton which is considered to be the draft of the 1829 Appendix, emigration, instead of free trade, had been proposed as one of the means of raising the maximum wage.

[5] The minimum wage is the same as what Torrens called 'the natural price of labour' in the first edition of *External Corn Trade*. In the Appendix, Torrens reuses some passages of the first edition with slight alternation and the phrase 'the natural price of labour' is replaced with 'the minimum of wages'.

[6] The similar sentence appeared in the first edition of *External Corn Trade* (Torrens, 1815, p. 63), which Ricardo cited in the *Principles of Political Economy and Taxation* (Ricardo, 1819, p. 97)

[7] There was originally the phrase 'the prudential check which may exist with respect to marriage'. However, the prudential check should mean limiting the birthrate under a fixed minimum wage. Therefore, Torrens (1834, p. 12–13) later corrected the text as shown with the square bracket. In a same way, the phrase 'a prudential check to marriage' (1829, p. 460) in a different part was later replaced with 'a taste for the comforts of life' (1834, p. 12). Torrens's notion of 'scale of comfort' is considered to stem from the Malthusian idea of habit formation.

necessaries 'sufficient for the support of animal life' (p. 467) but in no quantity of the conveniences. Torrens's discussion of economic dynamics was developed in the case of the 'extreme minimum' wage exogenously fixed at the physical subsistence level.

## 3. Reconstructing Torrens's theory of economic growth

### *3.1 Assumptions*

The model is built on the following suppositions.

*Supposition 1. The system is a closed economy.*

Torrens himself describes the dynamics of a closed economy.

*Supposition 2. Corn is the only commodity produced in the system.*

This is the assumption made by Torrens for the sake of simplification.

*Supposition 3. Labour and land are the only inputs to production, but 'seed, material, and machinery' are not used in the production. Thus, capital consists of the wages fund for employing labour.*[8]

This is assumed in order to avoid complication of the analysis.

*Supposition 4. The maximum wage depends on the land productivity (diminishing returns in agricultural production) and the labour skill, the minimum wage is fixed at the physical subsistence level, and the actual wage is determined by the ratio of capital to labour between both wages.*

Based on Supposition 1, the effect of trade liberalisation on the maximum wage must be exempted from the consideration. As Section 2 mentioned, Torrens's discussion was developed in the case of the 'extreme minimum' wage exogenously fixed at the physical subsistence level. The determination of the actual wage rate follows the wages fund theory: 'the one and only cause which can determine where, between the maximum and minimum, the [actual] wages of … labourers, shall be fixed, is, the proportion which the farmer's capital bears to the quantity of labour' (p. 462).[9]

*Supposition 5. There are diminishing returns to labour in agriculture.*[10]

This assumption is necessary not only for his concept of maximum wage (see above) but also for 'the principles of rent' (see below).

*Supposition 6. The marginal product of labour, or the labour product in the marginal land, is divided wages as income of workers and profits as income of capitalists. Rent as income of landowners is that surplus of total production which remains once workers and capitalists have obtained their income.*

This assumption is based on the theory of differential rent, which was developed in detail in Chapter 8, Part 1, of the 1829 edition of *External Corn Trade*.

*Supposition 7. Landowners and workers consume all their income, while capitalists save out of their profit income and invest it to employ labour only if the rate of profit is higher than the minimum. The growth rate of capital is an increasing function of the difference between the profit rate and the minimum profit rate.*

The assumption of capital accumulation can be obviously driven from Torrens's statement that 'the lowest rate of profit … will induce him [capitalist] to continue the work of production'.[11]

*Supposition 8. The term 'population' is used with the same meaning as the number of workers, namely, 'labour population'. The growth rate of population is an increasing function of the difference between the actual and the minimum wage rate.*

The assumption is based on the Malthusian law of

[8] This differs from Rosell's assumption of the inputs of corn and seed to production. He, however, removes the uses of material and machinery in the production from his suppositions (see Torrens, 1829, p. 464). Our simplification as well as Rosell's should be allowed in a single homogeneous corn growth model.

[9] Hollander (1968) and O'Brien (2004, pp. 131–35) reconstructed the wages fund model incorporating non-wages in Torrens's *Wages and Combination*.

[10] This differs from Rosell's constant returns assumption.

[11] In Chapter 2 of *Wages and Combination*, Torres, in illustrating the case in which the farmer obtains 24% of profits above the minimum rate, 7%, states, 'Increasing profits always occasion a more rapid accumulation of capital' (1834, pp. 33–36).

population.

*3.2 The model*

Let us now reconstruct a simple model of Torrens's economic dynamics, following the above assumptions.

*Production function.* Denoting the amount of land under cultivation by $L$, and an index of diminishing returns by $a$ with $0 < a < 1$, let the physical output of corn as the products of land be simply $xL^a$, where $x$ means as follows. Denoting the quantity of labour employed per unit of land under cultivation by $l$ with $l > 0$, let $x$ define as $x = Al^a$, where $A$ is a constant coefficient with $A > 0$. Assuming that $l$ is exogenously given, from such a definition of $x$, we can obtain the following production function of corn as the products of labour combined with land:

$$Y = AN^a. \tag{1}$$

where $Y$ is the physical output of corn and $N$ is the quantity of labour bestowed to land.[12]

Let us now consider the case in which the improvement of labour skill enables a smaller number of workers to produce the same quantity of corn as before while the amount and quality of land remains the same: that is, the case in which $l$ is exogenously reduced while $x$ and $a$ remain constant. Then $A$ rises. The raised $A$ thus indicates the improvement of labour skill when the land productivity remains the same.

*Actual wage rage.* From Supposition 3, capital (denoted $K$) consists of the wage fund for employing $N$ units of labour under a rate of actual wage (denoted $w$): $K = wN$. The actual wage rate thus is determined by the capital-population ratio, following the wages fund theory (Supposition 4):

$$w = \frac{K}{N}. \tag{2}$$

*Profit rate.* From Supposition 6, the marginal products of $N$ units of labour, $N \times dY/dN$, must be divided as workers' wages and capitalists' profits: $N(dY/dN) = wN + rK$. Considering $dY/dN = aAN^{a-1}$ and $K = wN$, we obtain

$$r = \frac{aAN^{a-1}}{w} - 1. \tag{3}$$

*Rent.* Since, following the theory of differential rent, rent (denoted $R$) can be defined as that surplus of total production which remains once workers and capitalists have obtained their income, $Y - (wN + rK) = Y - N(dY/dN)$. Considering $dY/dN = aAN^{a-1}$, we obtain

$$R = (1 - a)AN^a. \tag{4}$$

Torrens argues that it is necessary and clear, 'from the principles of rent', that 'the high rent' must 'be paid for all the more fertile soils' with resorting to the inferior lands (pp. 466–67), which is easily shown by $dR/dN = a(1 - a)AN^{a-1} > 0$.

*Capital accumulation.* From Supposition 7, the growth rate of capital is an increasing function of the difference between the profit rate and the minimum profit rate (denoted $r_{\min}$). If the minimum is given as $r_{\min} > 0$, denoting a reaction coefficient by $b$, with $0 < b < 1$, the capital accumulation function can be given as the following specified equation:

$$k = b(r - r_{\min}), \tag{5}$$

where $k$ is the growth rate of capital: $k = \dot{K}/K$.[13]

*Population growth.* From Supposition 8, the growth rate of population is an increasing function of the difference between the actual and the minimum wage rate (denoted $w_{\min}$). If the minimum is given as $w_{\min} > 0$, denoting a reaction coefficient by $c$, with $0 < c < 1$, the population growth can be given as the following specified equation:

[12] Substituting $x = Al^a$ with $Y = xL^a$, it follows that $Y = A(lL)^a$. Further substituting $N = lL$, we can obtain equation (1).

[13] A dot over a variable denotes a derivative with respect to time.

$$n = c(w - w_{\min}), \tag{6}$$

where $n$ is the growth rate of population: $n = \dot{N}/N$.

In addition, the system is assumed to satisfy the conditions that allow the economy to grow but not to grow indefinitely (see Casarosa, 1982, p. 230):

$$\lim_{N\to 0} AN^a > w_{\min}(1 + r_{\min}),$$

and

$$\lim_{N\to \infty} AN^a < w_{\min}(1 + r_{\min}).$$

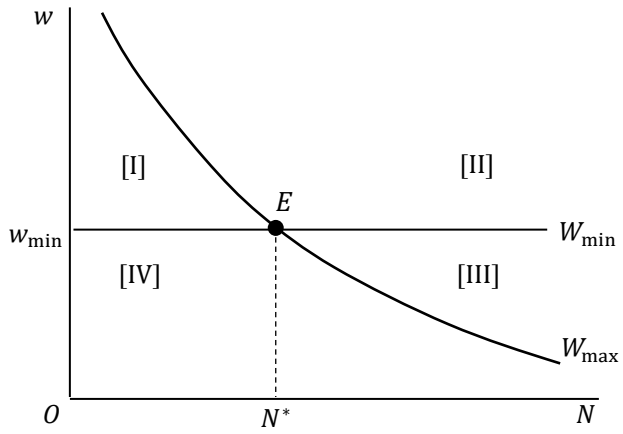
If the initial values of $K$ and $N$ are exogenously given, the system, composed of equations (1)–(6), is determined.

## 4. Torrens vision on economic dynamics

### *4.1 The maximum wage curve and the minimum wage line*

In the $(N, w)$ plane, the minimum wage is simply described as a horizontal line, as Figure 1 shows. We may call it the minimum wage line, $W_{\min}$.

**Figure 1.**
**The maximum wage curve and the minimum wage line**




On the other hand, on some suppositions, we have to redefine the maximum wage which we explained in Subsection 2.2. From Supposition 3, non-wages funds, such as seed, material and machinery, are not inputted to production. From Supposition 6, we also must consider the labour products in the no-rent land—namely, the marginal products of labour, instead of the output per capita. Therefore, the maximum wage can be redefined as

$$\text{Maximum wage rate} = \frac{\text{Marginal products of labour}}{1 + \text{Minimum profit rate}}$$

From equation (3), we obtain

$$w = \frac{aAN^{a-1}}{1 + r}. \tag{7}$$

Since a worker is paid the maximum wages when a capitalist only obtains the minimum rate of profits, $w_{\max} = w|_{r=r_{\min}}$, we can obtain

$$w_{\max} = \frac{aAN^{a-1}}{1 + r_{\min}}. \tag{8}$$

This is described as a convex curve in Figure 1 (see Appendix A). We may call it the maximum wage curve, $W_{\max}$. It is evident from equation (8) that the land productivity lowers the maximum wage in the process of economic growth and that the exogenous improvement of labour skill raises the maximum.

In Figure 1, the maximum wage curve and the minimum wage line intersect at a point, $E$, and divide the space into four regions [I]–[VI]. Torrens discusses the motions of actual wage between the maximum and the minimum wages, or in other words, the movements of the system in Region [I] which exist between the maximum wage carve and the minimum wage line (see Appendix B). The system starting from a point in Region [I], according to Torrens (1829, p. 457), cannot morally shoot up through the maximum wage curve, while it is impossible that the system cannot move below the minimum wage line (see Subsection 4.2).

From equations (3), (5) and (7), we obtain the following dynamic relationship:

$$w \gtreqless w_{\max} \Longrightarrow k \lesseqgtr 0.$$

Further, from (6), we obtain

$$w \gtreqless w_{\min} \Longrightarrow n \lesseqgtr 0.$$

Therefore, both capital and population increase if the system exists in Region [I]: that is, $k > 0$ and $n > 0$ if $w \in (w_{\min}, w_{\max})$. Torrens states,

> When, in the progress of wealth and population, wages and profits have fallen to their minimum, and when the next quality of land to be taken in cannot be made to yield a reproduction sufficient to pay these minimum wages and to replace advances with minimum profits, then that which is saved from revenue to be added to capital cannot be employed at home … (p. 469)

'At this point', Torrens says, 'population remain[s] stationary' (p. 469); more strictly, both population and capital remain stationary in the domestic market.[14] The maximum wage carve and the minimum wage line intersect at point $E$:

$$(N^*, w^*) = \left( \left[ \frac{aA}{w_{\min}(1 + r_{\min})} \right]^{\frac{1}{1-a}}, w_{\min} \right).$$

As they intersect when $aAN^{a-1}/(1 + r_{\min}) = w_{\min}$, it follows that both the rates of wage and of profit are reduced to their minimum at the stationary state point. The existence of such a point also is ensured by the continuity of the two loci ($W_{\max}$ and $W_{\min}$) and the following assumption: $\lim_{N\to 0} AN^a > w_{\min}(1 + r_{\min})$ and $\lim_{N\to\infty} AN^a < w_{\min}(1 + r_{\min})$ (see Casarosa, 1982, p. 231).

Torrens's notions of the maximum and the minimum wages are close to what Hicks and Hollander (1977), in their reconstruction of the Ricardian growth model, call 'the roof' and 'the flour', respectively. It is evident that there is the idea of the limit below which the wage cannot fall in both Torrens's and Ricardo's texts. However, it should be noted that Torrens obviously presents the concept of the maximum wage, while Ricardo, in his text, never clearly proposes what Hicks and Hollander call 'the roof', which is a necessary tool for their rational reconstruction.[15] In sum, Torrens gives a clearer explanation of the upper limit of wage than Ricardo.

*4.2 The dynamic equilibrium wage curve*

Since both capital and population increase in Region [I], the motions of actual wage depend on the difference between the growth rates of capital and population. Taking the natural log of both sides of equation (4), and differentiating with respect to time, we obtain the following relationship:

$$k \gtreqless n \Longrightarrow \omega \gtreqless 0,$$

where $\omega$ is the increase rate of $w$: $\omega = \dot{w}/w$. Torrens obviously discusses the possibility of economic growth that capital and population go on increasing in the same ratio:

> But though *labour and capital should go on increasing in the same proportion*, and though *they should constantly preserve the same ratio to each other*, yet the necessity of resorting to inferior soils might gradually reduce the maximum of wages until it coincided with the extreme minimum, below which labour cannot be sustained. At this point, the supply of labour could be no further increased … (1829, pp. 468–69; emphasis added).[16]

From equations (5) and (6), the balance between the growth rates of capital and of population is

$$b(r - r_{\min}) = c(w - w_{\min}). \tag{9}$$

Substituting in equation (3), we can obtain the dynamic equilibrium wage rate (denoted $\widehat{w}$) as follows.

$$\widehat{w} = \frac{cw_{\min}}{2c} - \frac{b(1 + r_{\min})}{2c} + \frac{\sqrt{[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1}}}{2c}. \tag{10}$$

This is described as a convex curve between the maximum wage curve and the minimum wage line, as Figure 2 shows (see Appendix C). We may call it the dynamic equilibrium wage curve, $\widehat{W}$.

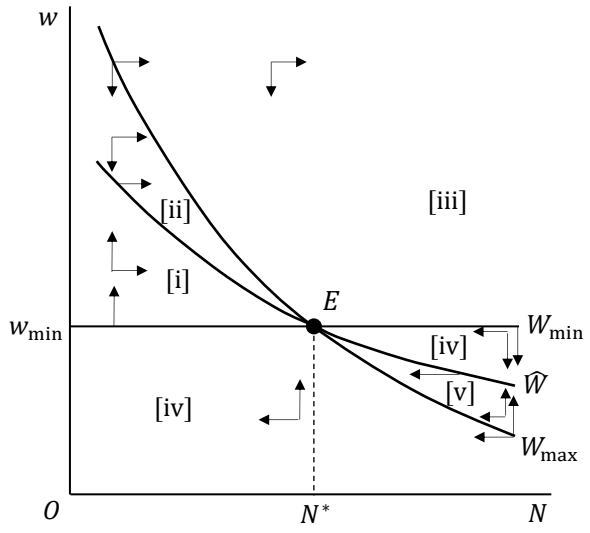
We can now describe the phase diagram for the dynamics of $w$ and $N$ in Figure 2. In this figure, the maximum wage curve ($W_{\max}$), the dynamic equilibrium curve ($\widehat{W}$) and the minimum wage line ($W_{\min}$) divide the space into six regions. The arrows show the directions of motion in each region (see Appendix D).

[14] Torrens mentions the possibility that the surplus capital 'will be invested in foreign loans and foreign adventures' (1829, p. 469). Further research requires the analyses of his theories of international trade and systematic colonisation, which are out of the scope of the present work.

[15] Hicks and Hollander (1977, p. 353), in setting up the Ricardian model, state, 'We shall allow ourselves… to describe the model in terms that, admittedly, are not Ricardo's, but that we think will quickly make it more intelligible to the modern economist'.

[16] O'Brien argues that Torrens here 'rejects a *simple* Wage-Fund analysis' (1966, p. 337, original emphasis). However, his argument, as Vint properly criticises, 'is incomplete' because 'Torrens is here making use of a long run analysis and not the short run wages fund doctrine' (1994, p. 105).

**Figure 2.**
**Phase diagram: the dynamics of wage and population**




In his view of economic dynamics, Torrens discusses the system starting from a point in Region [i]. In the progress of capital and population, the economy necessarily enters Region [ii]. Once arriving at the region, the economy grows until it reaches a stationary state point, $E$, while both the wage and the profit rate keep falling to their minimum level.

It is Casarosa (1982) who first strictly incorporated the idea of 'the dynamic equilibrium path of the wage rate' (p. 234) into the Ricardian growth model built by Hicks and Hollander. However, Casarosa gave no textual evidence of Ricardian dynamic equilibrium growth, and we cannot find Ricardo's own obvious statement of such a growth in his text. On the other hand, Torrens clearly mentioned the possibility of such growth of an economy. His view on economic dynamics therefore can be given a more appropriate explanation by applying the Ricardian growth model which Hicks, Hollander and Casarosa rationally reconstructed.

### *4.3 The tendency in population to increase faster than capital*

Torrens's early perspective had indicated that population tends to increase faster than the means of subsistence so that the wage keeps decreasing. For example, in the 1817 paper on the Poor Laws, Torrens stated that in a country such as the America of his time where 'an abundance of new and fertile land' remain, 'capital and subsistence may be made to increase as fast as man can multiply his kind', but in a country, such as England here 'all the good and well-situated lands of a country have already been appropriated and occupied… capital and subsistence can no longer be *kept up to the population*' (Torrens, 1817, pp. 517–18, original emphasis). [17] If capital is continuously accumulated as a ratio less than that of population growth, the economy tends to be reduced to 'the starving point' (p. 511) or 'the extreme limit of subsistence' (p. 512), in which all the people must be confined 'to the business of providing for merely animal wants' (p. 513).

Now note that capital increases faster than population in Region [i], while population increases faster than capital in Region [ii] in our model. Torrens's *early* view can be explained in that the economy has a tendency to move in Region [ii], or at least, the system departing from a point in Region [i] has a tendency to immediately enter Region [ii].

In 1829, Torrens revised his previous view and insisted 'that, in almost every society, the tendency is not to increase population faster than capital; but, on the contrary, to increase capital more rapidly than population' (1829, pp. 474–75).

'A few weeks' (Blaug, 1958, p. 113) after the publication of Torrens's text, Senior's letter to Thomas Robert Malthus (1766–1834) dated 15 March was brought out, criticising:

> You would still say, that, in the absence of disturbing causes, population has a tendency to increase faster than food… I should still say, that, in the absence of disturbing causes, food

[17] Consistent with Malthus (1798), Torrens (1817, p. 523) states that 'population… must have a tendency to increase in a geometrical ratio'. However, he also argues, 'If we were to extend tillage over inferior tracts, or to heighten the cultivation of our fertile lands, each additional quantity of capital applied to the soil, would yield a less proportional return; and while the generative powers (unless reduced by the deterioration of the human constitution) remained unimpaired, the productive powers which supply subsistence would be perpetually decreasing' (pp. 523–24). According to Blaug (1958, p. 104), 'Malthus never emphasized the law of diminishing returns in the *Essay on Population*', but the 'Ricardian economists… were quick to relate the arithmetic ratio explicitly to the assumption of diminishing returns in agriculture'. In such a sense, Torrens was one of the Ricardian economists.

has a tendency to increase faster than population… (Senior, 1829, p. 58)

Torrens later claimed his anticipation of Senior's criticism of Malthus:

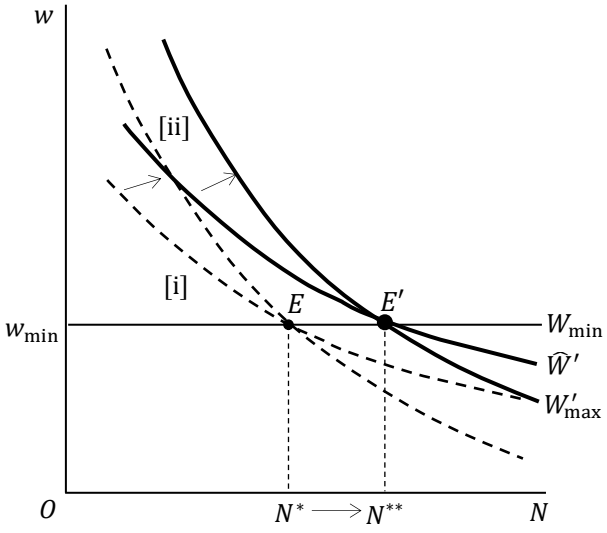
It was maintained that there is a tendency in population to increase faster than the capital which gives employment to labour; and that this tendency keeps down wages to the starving point. I believe I may venture to claim the credit of being the first to refute this doctrine… In the appendix to the last edition of my *Essay on the Corn Trade*, I proved that there was no tendency in population to increase faster than capital; but that, on the contrary, the tendency is for capital to increase faster than population. This correct and consolatory view of the question was subsequently taken, I believe, in a correspondence which was published, between Mr. Malthus and Mr. Senior… The doctrine, therefore, that population has a tendency to increase faster than capital, may be considered as exploded. (Torrens, 1833, pp. 43–44)

However, Torrens's criticism of Malthus was not appropriate. Malthus (1798, pp. 9, 11) only stated that population, 'when unchecked', has a tendency to increase faster than the means of subsistence, while Torrens (1829, pp. 476–77) considers that there is not such a tendency because, 'in every civilized community' or 'in every country of Europe except Ireland', the people's '*power* "to increase and multiply"… is checked and controuled [sic] by the prevailing efficacy' of the 'instruction' of 'prudence and precaution'.

However, even though Torrens's criticism was never proper, his new position has an importance in the analysis of his dynamic motions of the system. The 'only possible means of eradicating pauperism', Torrens emphasised in 1817, 'is by keeping down the population to a level with capital and subsistence'. One of ways of doing so, which he had devised, was 'a prudential or moral restraint for preventing the birth of superfluous numbers' (1817, p. 518).[18] At the time, he believed that a 'moral instruction' (p. 519) for limiting birth had not prevailed among the working classes.

Torrens, in 1829, however, acknowledged that the working classes sufficiently acquire prudential habits so that they can keep the growth rate of population not to exceed that of capital. That is, the system, which departed from a point in Region [i], tends to remain in the same region by the diffusion of prudential habits among workers. In this region, capital increases faster than population so that wages enjoyed by workers continue to increase. Torrens states, 'The fact that the condition of the labouring classed has improved with the progress of wealth and civilization, demonstrates that population has not a tendency to increase faster than capital' (1829, p. 473). Torrens (1829, pp. 473–74) here cites Adam Smith's mention that a nation's conditions improved with an increase in skill as a result of the division of labour (Smith, 1776, pp. 23–24). An improvement of labour skill and thus an exogenous raise in $A$ enlarges the area of Region [i], in which capital increases faster than population, as Figure 3 shows (see Appendix 5). Torrens might have had a theoretical perspective of the positive effects of improved skill on the working classes.

**Figure 3.**
**The effect of the improvement of labour skill**




## 5. Concluding remarks

In his 1829 text, Torrens put forward not only the *statics* theories of wage and profit reformulated by Rosell, but also the *dynamics* theory of economic growth based on the notions of the maximum, minimum and actual wage. His view on economic dynamics can be explicitly explained by the growth model, which incorporates a dynamic equilibrium

[18] The other way Torrens (1817, p. 518) thought of was a self-supporting emigration for 'removing' the superfluous numbers who had already come into the world.

between the growth rates of population and of capital. Such a theory has been attributed to Ricardo by Hicks, Hollander and Casarosa, but Torrens presented a more ingenious theory than Ricardo's.

The actual wage rate is determined by the capital-population ratio between the maximum and the minimum wage rate. In the accumulation of capital and the increase in population, the maximum wage rate keeps being lowered by the decreasing marginal productivity of labour in agriculture, while the minimum wage rate is exogenously fixed and thus is constant through time. Based on these notions of wages, Torrens's economic dynamics is *generally* described as follows.

In the early stage of economic growth, capital increases faster than population so that the actual wage rate keeps rising above the minimum. Thereafter, once arriving at the dynamic equilibrium wage path, in which population and capital grow at the same ratio, the economy sets about growing with a tendency for population to increase faster than capital so that the actual wage rate keeps falling below the decreasing maximum and profit rate simultaneously goes on falling, until the system converges at the stationary state where the both the actual wage and the profit rate equals their minimum.

In Torrens's 1829 view, workers in every civilised country, sufficiently acquire prudential habits so that they can keep the growth rate of population not to exceed that of capital. Such habits, with the cooperation of a temporary effect of the improved labour skill, enable the actual system to grow while maintaining the condition in which capital increases faster than population and workers enjoy higher wages.

In concluding this work, we should briefly treat Torrens's late positon of the relationship between population and capital. In 1857, he re-published his early pamphlet entitled *The Economists Refuted* (Torrens 1857). The re-published pamphlet has not only some trivial corrections and the deletion of Chapter 6 on the 'observations on the expediency of making peace with France' which appeared in the 1808 edition, but also great revisions to Chapter 4. In particular, it should be noted that the following sentence was added:

> [F]rom the relative proportions according to which population and capital have, in all old countries, been hitherto found to increase, the supply of labour has such a tendency to exceed the demand for it, that the labouring classes, even when there is no extraordinary stagnation or revulsion in the channels of industry, are commonly reduced to a degree of distress… (Torrens, 1857, p. 35)

Considering that an improvement of labour skill has no permanent effect in the enlargement of Region [i], in 1857, Torrens might have been about to consider that the system had already gone out the region and continued progressing in Region [ii], in which population increases faster than capital. At any rate, this should be observed in future research.

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## Appendix A

In the $(N, w)$ plane, the maximum wage is described as the convex curve by satisfying

$$\frac{dw_{\max}}{dN} = -\frac{a(1-a)AN^{a-2}}{1+r_{\min}} < 0,$$

and

$$\frac{d^2w_{\max}}{dN^2} = \frac{a(1-a)(2-a)AN^{a-3}}{1+r_{\min}} > 0.$$

## Appendix B

As a necessary and sufficient condition for the system not to shoot up through the maximum wage curve, the elasticity of the labour population with respect to the maximum wage rate is equal to or strictly more than minus one (see Dome, 1993, pp. 32–3):

$$\frac{dw_{\max}}{dN}\frac{N}{w_{\max}} \geq -1.$$

The elasticity of the labour population with respect to the maximum wage rate is

$$\frac{dw_{\max}}{dN}\frac{N}{w_{\max}} = -\frac{a(1-a)AN^{a-2}}{1+r_{\min}}\frac{N}{aAN^{a-1}/(1+r_{\min})}$$

$$= -(1-a).$$

From $-(1-a) \geq -1$, the condition must be $a \geq 0$. The diminishing returns assumption, $0 < a < 1$, satisfies it and allows the system, as Torrens describes, to move between the maximum and the minimum wages and to converge at the stationary state point.

## Appendix C

In the $(N, w)$ plane, the dynamic equilibrium wage is described as the convex curve by satisfying

$$\frac{d\hat{w}}{dN} = -\frac{a(1-a)bAN^{a-2}}{\sqrt{[cw_{\min} - b(1+r_{\min})]^2 + 4abcAN^{a-1}}} < 0,$$

and

$$\frac{d^2\hat{w}}{dN^2} = \left[(2-a)\{cw_{\min} - b(1+r_{\min})\}^2 N^{a-3} + 2a(3-a)bcAN^{2(a-2)}\right] \times \frac{a(1-a)bA}{[\{cw_{\min} - b(1+r_{\min})\}^2 + 4abcAN^{a-1}]^{\frac{3}{2}}} > 0.$$

On the other hand, since the maximum wage curve and the minimum wage curve intersect when

$$w_{\min} = \frac{aAN^{a-1}}{1+r_{\min}},$$

it follows that

$$\frac{aAN^{a-1}}{w_{\min}} - (1 + r_{\min}) = 0.$$

Multiplying both sides of this equation by $4bcw_{\min}$ $(> 0)$, we obtain

$$4bcw_{\min}\left[\frac{aAN^{a-1}}{w_{\min}} - (1 + r_{\min})\right] = 0.$$

Adding $(cw_{\min})^2 + 2bcw_{\min}(1 + r_{\min})$ $(> 0)$ to the left side, we obtain the following inequality.

$$(cw_{\min})^2 + 2bcw_{\min}(1 + r_{\min}) + 4bcw_{\min}\left[\frac{aAN^{a-1}}{w_{\min}} - (1 + r_{\min})\right] > 0,$$

$$(cw_{\min})^2 - 2bcw_{\min}(1 + r_{\min}) + 4abcAN^{a-1} > 0.$$

Further, adding $[b(1 + r_{\min})]^2$ $(> 0)$ to both sides, we obtain the following inequality

$$(cw_{\min})^2 - 2bcw_{\min}(1 + r_{\min}) + [b(1 + r_{\min})]^2 + 4abcAN^{a-1} > [b(1 + r_{\min})]^2,$$

$$[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1} > [b(1 + r_{\min})]^2,$$

$$\frac{1}{1 + r_{\min}} > \frac{b}{\sqrt{[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1}}}.$$

Multiplying both sides by $a(1 - a)AN^{a-2}$ $(> 0)$, we obtain

$$\frac{a(1 - a)AN^{a-2}}{1 + r_{\min}} > \frac{a(1 - a)bAN^{a-2}}{\sqrt{[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1}}},$$

$$\frac{dw_{\max}}{dN} < \frac{d\widehat{w}}{dN} (< 0).$$

Therefore, considering that the dynamic equilibrium curve intersects the maximum wage curve at the point $(N^*, w^*)$ and that $d^2w_{\max}/dN^2 > 0$ and $d^2\widehat{w}/dN^2 > 0$, we can find that in the range $N \in (0, N^*)$, the dynamic equilibrium curve exists below the maximum wage curve, while in the range $N \in (N^*, +\infty)$, the former is above the latter.

**Appendix D**

Let us consider the case of the actual wage rate above (below) the dynamic equilibrium wage:

$$w \gtrless \frac{cw_{\min}}{2c} - \frac{b(1 + r_{\min})}{2c} + \frac{\sqrt{[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1}}}{2c}.$$

From this, we obtain

$$c(w - w_{\min}) \gtrless b\left(\frac{aAN^{a-1}}{w} - 1 - r_{\min}\right),$$

$$c(w - w_{\min}) \gtrless b(r - r_{\min}),$$

$$n \gtrless k.$$

That is, $n \gtrless k$ if $w \gtrless \widetilde{w}$. This means that population (capital) increases faster than capital (population) in the system which exists above (below) the dynamic equilibrium curve. Considering $\omega \lessgtr 0$ if $n \gtrless k$, we can find that the actual wage rate rises when the system starts from the lower regions, [i], [v] and [vi], than the dynamic equilibrium curve while it falls when the system from the upper region, [ii], [iii] and [iv]. It is evident that the actual wage rate remains stationary when the system is in the dynamic equilibrium curve.

On the other hand, it is obvious from equation (6) that population decreases when the system starts from the upper regions, [i], [ii] and [iii], above the minimum wage line—that is, if the actual wage rate is above the minimum—while it decreases when the system is from the lower regions, [iv], [v] and [vi]—that is, if the actual wage rate below the minimum.

Therefore, the motion in each region is shown as the arrows in Figure 2.

**Appendix E**

The following relations ensure that an exogenous rise in labour productivity, as described in Figure 4, enlarges region II, where capital increases faster than population.

$$\frac{\partial w_{\max}}{\partial A} = \frac{aN^{a-1}}{1 + r_{\min}} > 0,$$

$$\frac{\partial \widehat{w}}{\partial A} = \frac{abN^{a-1}}{\sqrt{[cw_{\min} - b(1 + r_{\min})]^2 + 4abcAN^{a-1}}} > 0,$$

$$\frac{\partial w^*}{\partial A} = 0,$$

$$\frac{\partial N^*}{\partial A} = \frac{a}{w_{\min}(1 - a)(1 + r_{\min})}\left[\frac{aA}{w_{\min}(1 + r_{\min})}\right]^{\frac{a}{1-a}} > 0.$$