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Procurement Auctions with Uncertainty in Corruption

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Abstract

This paper considers a situation in which a corrupt government official does not commit to using the common corruption scheme called right of first refusal in a procurement auction. Under the right of first refusal, the contractors (or bidders) participate in a sequential auction, and there is no inefficiency in project allocation. However, in cases in which the scheme is not practiced, both contractors participate in a simultaneous auction, and the disadvantaged contractor bids more aggressively than the advantaged contractor. I found that such uncertainty regarding the practice of corruption schemes can lead to inefficiency, even when the corruption scheme itself is not practiced.

Keywords: Procurement Auctions; Corruption, Right of first refusal

JEL classification: C72, D44, L14

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1. Introduction

In this paper, I examine the impact of corruption on contractor bidding behavior, expected price and allocation when the implementation of corruption schemes is uncertain. Corruption is a major problem in both developed and developing countries. According to reports by the

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Department of Justice and Federal Investigation Bureau in the United States, there were 20,446 cases of stimulus fraud in the private sector between 1988 and 2007. Because corruption can increase contract prices and result in an inefficient allocation of contracts, many researchers have explored the impact of corruption on the outcomes of procurement auctions. However, the possibility that the expected corruption schemes might not come to fruition has been ignored. Practically speaking, it seems important to recognize that corruption schemes are not always guaranteed to be practiced. Because regulatory authorities consider corruption to be a serious problem, more thorough work is being done to uncover corruption, and the practice of corruption schemes may depend on the judgments of corrupt government officials' regarding how stringent the oversight process is.

Corruption is a type of implicit contract between a briber and bribee, and one of corruption's features is that it is illegal. Because it is illegal, the implementation of the contract is kept secret. Thus, other contractors are not aware of and are not involved in the corrupt scheme, though they may be aware of the potential existence of a corrupt relationship between specific contractors and government officials. For the same reason, corrupt contractors cannot take government officials to court when corruption schemes are not implemented. In other words, the corrupt government is not necessarily committing to following through with these schemes. The only way for a corrupt contractor to make implicit illegal contracts credible is to control the timing of payment at the close of auctions. However, such a bribery-payment scheme can only decrease, rather than eliminate, the risk of paying government officials to execute a plan on which they do not follow through. How the use of corruption schemes might affect the prices of the procured projects and the efficiency of project allocation remains unclear.

I consider the following situation. In a one-shot, two-stage game, the government procures a project, and it has a collusive relationship with one of its contractors via a corruption scheme.

Under the scheme, the government official can revise the bid submitted by the corrupt contractor (called the advantaged contractor in this paper) before the amount of the bids becomes public information; thus, the advantaged contractor will be awarded the contract unless the bid submitted by the opponent (called the disadvantaged contractor) is lower than advantaged contractor's costs. When the advantaged contractor wins the auction outright, the government official makes changes that allow the advantaged contractor to derive a greater surplus from the contract. The payment from the advantaged contractor to the government official is a portion of the advantaged contractor's surplus. However, such plans do not always come to fruition. The existence of collusive relationships and of possible corruption schemes is common knowledge, but whether such plans are actually executed is uncertain.

Using this scheme specification, I create a situation in which the advantaged contractor can be almost automatically awarded the project without making his own strategic moves as long as the corruption scheme is practiced. By adding the question of uncertainty to the problem of corruption schemes, I focus on the advantaged contractor, who joins the competition and chooses his bid strategically. The results indicate that inefficiency in project allocation can occur when the plan is not executed. The implications are as follows. Because the disadvantaged contractor knows about the potential for corruption, he bids more aggressively than the corrupt contractor, but it is uncertain that corrupt behavior will occur. As a result, the projects will be awarded to the contractor whose costs are higher.

This paper continues as follows. In section 2, I briefly review the past literature, summarize the major topics and findings in the field of corruption in procurement auctions, and discuss what can be done to extend prior research. In section 3, I construct a model of corruption in a sealed bid first-price procurement auction with an uncommitted government official. In section 4, I first show the bidding behavior of contractors and discuss the allocation of contracts in fair

procurement auctions as a benchmark. Next, I model a corrupt procurement auction in which the government official does not commit to the corruption scheme in order to characterize the equilibrium. However, because this is an asymmetric auction model without a closed-form solution, I derive a numerical solution to the model and comparative statics in section 5. I also present the characteristics of the bidding behavior of contractors and explain the potential inefficiency of this game.

2. Related Literature

Many of the related studies distinguish themselves from one another by using different corruption schemes and by using different definitions of corrupt contractors. In many settings, the corrupt government allows the corrupt contractor revise his or her bid after all bids are submitted. This means that a sequential auction occurs in which the corrupt or advantaged contractors bid after the honest contractors and corruption does not cause inefficiency, even though the expected prices are higher than in fair auctions (Jones and Menezes (1995), Menezes and Monteiro (2003), Lengwiler and Wolfstetter (2004), Lee (2008), Burguet and Perry (2008)). However, if there is a factor that drives honest or disadvantaged contractors to bid more aggressively than corrupt contractors, then inefficiency occurs (Burguet and Perry (2008), Lee (2008)).

The studies most closely related to the present study are Arozamena and Weinschelbaum (2005) and Burguet and Perry (2008). Burguet and Perry (2008) employ a corrupt arrangement in which the government allows the corrupt contractor to revise the bid only when the contractor fails to submit the lowest bid, whereas regular right of first refusal allows the contractor to revise his bid even when he wins the auction outright. Because the government does not help the corrupt contractor when he wins the auction outright, the contractor does not pay a bribe to the

government, which means that the corrupt contractor may enjoy a higher surplus than when he loses the auction during the first round but wins it by revising his bid. The essential factor is the size of the payment for bribery relative to that of the total possible surplus. In this case, the corrupt contractor has an incentive to join the competition from the beginning, which makes the auction game simultaneous and less inefficient.

Arozamena and Weinshelbaum (2005) use a strategy similar to mine in that at least one of the contractors does not know that a corrupt relationship exists between the government and the opponent. The honest contractor assigns positive probabilities to both the possibility that corruption will be practiced and the possibility that it will not be. This suspicion leads the honest contractor to bid more aggressively than in a usual fair auction but less aggressively than when it is certain that the corruption scheme will be executed. However, their setting does not allow the corrupt contractor to face a simultaneous game. The only focus is on the bidding behavior of honest contractors, whereas my concern is the corrupt contractor's behavior.

3. The Model

Information

Consider a situation in which a government official procures a public project. This is a two-stage game with two contractors competing for the project. Each contractor is denoted as either the advantaged contractor or the disadvantaged contractor by A or D, respectively. ($i \in \{A, D\}$). The advantaged and disadvantaged contractors are defined by whether each has an implicit contract with the government. I describe the contract in the next subsection. Contractor i has a project completion cost of c_i. Completion cost c_i is randomly distributed iid over the interval $[\underline{c}, \bar{c}]$. Let $F(c_i)$ be the probability cumulative function, and let $f(c_i)$ be the corresponding density function. A contractor's (completion) cost is private information. I assume

that $F(\cdot)$ is differentiable and that $f(\cdot)$ is continuously differentiable in $[\underline{c}, \bar{c}]$. The probability that the procurement auction is the target of a corruption scheme orchestrated by the government official and the advantaged contractor is $p > 0$. The probability is common knowledge, but the corruption scheme is implemented only after the auction is closed.

The vNM utility function and the Corruption Scheme

The vNM utility functions in this game are represented as follows. Let b_A be the advantaged bidder's bid in the procurement auction. If the corruption scheme is not practiced (recall that the occurrence of collusion is revealed to the advantaged contractor only after he submits the bid) and if the advantaged contractor submits the lowest bid, then the contract is awarded to him. In such cases, the advantaged contractor's vNM utility is $b_A - c_A$. However, if his bid is not the lowest, then the advantaged contractor's vNM utility is 0.

If the corruption scheme is not practiced and if advantaged contractor submits the lowest bid, then the government official revises his bid based on the second lowest bid (i.e., the disadvantaged contractor's bid, b_D) with the payment share $1 - \lambda$ $(0 \leq \lambda \leq 1)$ and the contract is awarded to the advantaged contractor. In this case, the advantaged contractor's vNM utility is $\lambda(b_D - c_A)$.[2] Here, it is assumed that the advantaged contractor shares cost-related information with the government officials.

The vNM function for the advantaged contractor $(i = A)$ is given by

$$\pi_A^{Fair}(c_A, b_A, b_D) = \begin{cases} b_A - c_A & \text{if } b_D > b_A \\ \dfrac{1}{2}(b_A - c_A) & \text{if } b_D = b_A \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

and

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[2] Here I assume that λ is exogenously determined. Usually, the contractor does not necessarily report to the government official its true cost c_A. In some literature, λ is determined in a bargaining game to maximize the joint surplus of the *advantaged* bidder and the government officials. However, I do not focus on this aspect.

$$\pi_A^{Corrupt}(c_A, b_A, b_D) = \begin{cases} \lambda(b_D - c_A) & \text{if } b_A \geq c_D \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

whereas that of the disadvantaged contractor (i = D) is given by

$$\pi_D^{Fair}(c_D, b_A, b_D) = \begin{cases} (b_D - c_D) & \text{if } b_A > b_D \\ \frac{1}{2}(b_D - c_D) & \text{if } b_A = b_D \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and

$$\pi_D^{Corrupt}(c_D, b_A, b_D) = \begin{cases} b_D - c_D & \text{if } c_A > b_D \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Expected Profit

The expected profit functions are as follows. The advantaged contractor's expected profit is given by

$$\Pi_A(b_A, |b_D, c_A, c_D) = p(b_A - c_A)[1 - F(\beta_D^{-1}(b_A))]$$

$$+ (1-p)\lambda \left[\int_{c_A}^{\beta_D^{-1}(b_A)} (\beta_D(x) - c_A)dx + \int_{\beta_D^{-1}(b_A)}^{\bar{c}} (\beta_D(x) - c_A)dx \right] \tag{5}$$

The first term starting with p is the expected profit when the corruption scheme is not practiced. Then, the auction is a fair ordinary first-price sealed-bid procurement auction, and the advantaged contractor wins the auction with a probability of $1 - F(\beta_D^{-1}(b_A))$. The second term is the expected profit when the planned corruption scheme is implemented. The first term in square brackets is the expected profit when the advantaged contractor successfully submits the lowest bid, and the government official revises b_A to the level of the disadvantaged contractor's bid b_D so that the advantaged contractor has the greater surplus. The second term in square brackets is the expected profit when the advantaged contractor fails to submit the lowest bid, and the government official revises it to b_D if $b_D \geq c_A$. The disadvantaged contractor's expected profit is given by

$$\Pi_D(b_D, |b_A, c_A, c_D) = p(b_D - c_D)[1 - F(\beta_D^{-1}(b_A))] + (1-p)(b_D - c_D)(1 - F(b_D)) \tag{6}$$

The first term is the expected profit when the scheme is not practiced, whereas the second term is the profit when the scheme is practiced. In such a case, the disadvantaged contractor must match the advantaged contractor's cost rather than his bid. Therefore, the probability of winning is $1-F(b_D)$. Based on these settings, I obtain the Bayesian Nash equilibrium of this game.

4. Equilibrium Analysis

In this section, I determine the equilibrium in this game and provide the implications of the factors at play. Because contractors have an identical cost distribution but exhibit different utility functions, the strategies in equilibrium are expected to be asymmetrical. Because proving the existence and uniqueness of the Bayesian Nash equilibrium is not the focus of this game, I omit the proof here and simply note the conditions for the game.[3] However, one can reference the proof presented in Lebrun for more information (1995).

The corruption scheme is practiced with the probability one (*p=0*) and zero (*p=1*) When the corruption scheme is practiced with the probability of one (*p=0*), the advantaged contractor submits any bid $b_A \in \mathbb{R}_+$ because this value is changed by the government official to b_D if $b_D \geq c_A$, which means that disadvantaged contractor chooses his strategy as if he faced his opponent's hypothetical bid function

$$\beta_A^*(c_A|p=1) = c_A \tag{7}$$

Therefore, the disadvantaged contractor's bid function in equilibrium is

$$\beta_D^*(c_D|p=1) = c_D \tag{8}$$

In this situation, efficient project allocation is achieved. The implication is that the disadvantaged contractor faces the fiercest competition possible. Eventually, the government successfully collects the information on both the advantaged and the disadvantaged contractor, although the project price achieved is not always the lowest (i.e., $price(\text{A wins}|p=1) \geq c_A$, and $price(\text{D wins}|p=1) = c_D$).

When the probability that the corruption scheme will be executed is zero (i.e., *p=1*, which indicates a fair auction), the advantaged and disadvantaged contractor have symmetric bid functions that satisfy the following first-order conditions.

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[3] The proof is available upon request.

$$1 - F\left(\beta_j^{-1}(b_i)\right) = (b_i - c_i)f\left(\beta_j^{-1}(b_i)\right)\frac{d\,\beta_j^{-1}(b_i)}{db_i} \qquad (9)$$

Again, in this case, there is no inefficiency in project allocation. The probability that the corruption scheme will be practiced is p ($0<p<1$) Given (c_A, c_D), the advantaged contractor's expected profit maximization problem is given by

$$\max_{b_A} p(b_A - c_A)[1 - F(\beta_D^{-1}(b_A))] + (1-p)\lambda \int_{\beta_D^{-1}(b_A)}^{\bar{c}} (\beta_D - c_A)dF(c) \qquad (10)$$

The disadvantaged contractor's expected profit maximization problem is given by

$$\max_{b_D} p(b_D - c_D)[1 - F(\beta_D^{-1}(b_A))] + (1-p)(b_D - c_D)\left(1 - F(b_D)\right) \qquad (11)$$

Maximization problems (6) and (7) imply the following first-order conditions.

$$1 - F\left(\beta_D^{-1}(b_A)\right) = (b_A - c_A)f\left(\beta_D^{-1}(b_A)\right)\frac{1 - F\left(\phi_D(b_A)\right)}{\left(b_A - \phi_A(b_A)\right)}\frac{d\beta_D^{-1}(b_A)}{db_A} \qquad (12)$$

and

$$p[1 - F\left(\beta_A^{-1}(b_D)\right)] + (1-p)\left(1 - F(b_D)\right)$$
$$= p\left[(b_D - c_D)f\left(\beta_A^{-1}(b_D)\right)\frac{d\,\beta_A^{-1}(b_D)}{db_D}\right] + (1-p)[(b_D - c_D)f(b_D)] \qquad (13)$$

Equation (12) indicates a marginal condition under a fair sealed-bid first-price procurement auction, which implies that the advantaged contractor bids as if he were participating in a fair procurement auction as long as there is a strictly positive probability that the corruption scheme is practiced. Remember that if $p=0$ (i.e., the corruption scheme is always implemented), the advantaged contractor's optimal choice of bid is any real number that is greater than his costs because the advantaged contractor wins the auction if the disadvantaged contractor's bid is higher than the advantaged contractor's costs. In such a case, consequently, the disadvantaged contractor needs to behave as if she were facing a sequential auction in which the advantaged contractor would submit his bid after the disadvantaged contractor submitted her bid. However, once the probability of a fair auction becomes strictly positive ($p>0$), the advantaged contractor

begins to consider the possibility that the scheme will not be implemented, and the current procurement auction becomes a fair sealed-bid first-price auction in which the advantaged contractor needs to bid simultaneously with the disadvantaged contractor. The portion of the advantaged contractor's payments to the government official has no impact on the advantaged contractor's marginal decision.

By adding two boundary conditions that are common in competitive procurement auctions, I derive the following necessary conditions of the Bayesian Nash equilibrium for this game.

Proposition 1. Given the assumptions (A1) and (A2), a pair of pure strategies (β_A^*, β_D^*) in the Bayesian Nash equilibrium for this game satisfies the following conditions.

1. $\forall i \in \{A, D\}, \phi_i(\bar{c}) = \bar{c}$

2. $\forall i \in \{A, D\}, \exists \underline{b} \in \mathbb{R}$ *such that* $\phi_i(\underline{b}) = \underline{c}$

3. *The functions* $\phi_A(\cdot) \equiv \beta_A^{-1}$ *and* $\phi_D(\cdot) \equiv \beta_D^{-1}$ *are the solutions of the system of ordinary differential equations of*

$$\phi_D'(b_A) = \frac{1 - F(\phi_D(b_A))}{(b_A - \phi_A(b_A))} \tag{14}$$

$$\phi_A'(b_D) = \frac{(1-p)\big[1 - F(b_D) - (b_D - \phi_D(b_D))f(b_D)\big] + p\big[1 - F(\phi_A(b_D))\big]}{p(b_D - \phi_D(b_D))f(\phi_A(b_D))} \tag{15}$$

That corruption schemes become more likely to be practiced indicates that the disadvantaged contractor has a higher chance of competing with the advantaged contractor in terms of cost, as that figure is lower than the advantaged contractor's bid. To match his opponent's cost, the disadvantaged contractor shifts his bidding schedule downward, and his lowest bid $\underline{b}_D$ decreases to the lowest cost $\underline{c}$. However, the advantaged contractor considers the other side of the situation. If it is more likely that the government official will implement the

corruption plan, then the advantaged contractor will experience softer price competition, and thus, he shifts the bidding schedule upward. At the same time, however, the advantaged contractor knows that it is still possible that the plan will not be implemented and that he may therefore face more aggressive bidding behavior on the part of the disadvantaged contractor under a fair auction. As a result, the advantaged bidder needs to decrease the cost of his lowest bid $\underline{b}_A$ to his lowest possible cost $\underline{c}$. Assuming these conditions and using a numerical approach developed by Bajari (2001), I conduct the computations in the next section.

5. Computations

Methodology

Because I cannot derive closed-form inverse bid functions $\phi_A(b_A)$ and $\phi_D(b_D)$, I numerically derive them using the computation algorithm developed by Bajari (2001). Bajari's (2001) algorithm allows us to approximate inverse bid functions using a flexible functional form (e.g., a high-order polynomial) and find a set of coefficients that approximately satisfy the first-order conditions and the boundary conditions.

First, define the distribution of costs to be truncated normal with support $[\underline{c}, \bar{c}]$. . The probability density function for bidder $i \in \{A, D\}$ is as follows.

$$f(c_i) = \begin{cases} \dfrac{1}{2\pi\sigma^2} \exp\left[\dfrac{-(c_i-\mu)^2}{2\sigma^2}\right] & \forall c_i \in [\underline{c}, \bar{c}] \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Next, I assume that the form of the inverse bid function for a contractor $i \in \{A, D\}$ takes the fourth order polynomial:

$$\phi_A(b_A; \underline{b}, \gamma_A) = \underline{b} + \sum_{k=0}^{4} \gamma_{A,k} (b_A - \underline{b})^k \tag{17}$$

$$\phi_D(b_D; \underline{b}, \gamma_D) = \underline{b} + \sum_{k=0}^{4} \gamma_{D,k} (b_D - \underline{b})^k \tag{18}$$

where $\gamma_A \equiv \{\gamma_{A,k}\}_{k=0}^{4}$ and $\gamma_D \equiv \{\gamma_{D,k}\}_{k=0}^{4}$. In equilibrium, the inverse bid functions $\phi_A(b_A)$ and $\phi_D(b_D)$ satisfy:

$$1 - \frac{f(\phi_D(b_A))\phi_D'(b_D)(b_A - \phi_A(b_A))}{1 - F(\phi_D(b_D))} = 0 \tag{19}$$

$$1 - \frac{f(\phi_A(b_D))\phi_A'(b_D)(b_D - \phi_D(b_D))}{(1-p)\{(1 - F(b_D)) - (b_D - \phi_D(b_D))f(b_D)\} + pF(\phi_A(b_D))} = 0. \tag{20}$$

Now, define the following functions $G_A(b_A; \underline{b}, \gamma_A, \gamma_D)$ and $G_D(b_D; \underline{b}, \gamma_A, \gamma_D)$:

$$G_A(b_A; \underline{b}, \gamma_A, \gamma_D) \equiv 1 - \frac{f\left(\phi_D(b_A; \underline{b}, \gamma_D)\right)\phi_D'(b_A; \underline{b}, \gamma_D)(b_A - \phi_A(b_A; \underline{b}, \gamma_A))}{1 - F(\phi_D(b_A; \underline{b}, \gamma_D))} \tag{21}$$

$$G_D(b_D; \underline{b}, \gamma_A, \gamma_D)$$

$$\equiv 1 - \frac{f\left(\phi_A(b_D; \underline{b}, \gamma_A)\right)\phi_A'(b_D; \underline{b}, \gamma_A)(b_D - \phi_D b_D; \underline{b}, \gamma_A))}{(1-p)\{(1 - F(b_D)) - (b_D - \phi_D(b_D; \underline{b}, \gamma_A))f(b_D)\} + pF(\phi_A(b_D; \underline{b}, \gamma_A))} \tag{22}$$

These functions correspond to the left-hand side of equations (18) and (19), where the inverse bid functions are replaced with approximated inverse bid functions. I evaluate the first-order conditions on a grid of points that are uniformly spaced on the compact interval $[\underline{b}, \bar{c}]$. Letting $ngrid$ denote the number of points in the grid, I define the m-th point in the grid $bgrid_m$, as follows

$$bgrid_m(\underline{b}) = \underline{b} + \frac{m(\bar{c} - \underline{b})}{ngrid + 1} \tag{23}$$

12

I bind the vectors of coefficients γ_A, and γ_D and the lowest possible bid $\underline{b}$, thereby allowing the approximated bid functions to satisfy the first-order conditions at the grid points and the boundary conditions simultaneously. To do so, I define the following function $H(\underline{b}, \gamma_A, \gamma_D)$.

$$
\begin{aligned}
H(\underline{b}, \gamma_A, \gamma_D) \\
\equiv \sum_{k=0}^{ngrid} \left(G_A(bgrid_k(\underline{b}); \underline{b}, \gamma_A, \gamma_D) \right)^2 + \left(G_D(bgrid_k(\underline{b}); \underline{b}, \gamma_A, \gamma_D) \right)^2 \\
+ \left(\underline{c} - \phi_A(\underline{b}, \underline{b}, \gamma_A) \right)^2 + \left(\underline{c} - \phi_D(\underline{b}, \underline{b}, \gamma_D) \right)^2 + \left(\underline{c} - \phi_A(\overline{c}, \underline{b}, \gamma_A) \right)^2 \\
+ \left(\underline{c} - \phi_D(\overline{c}, \underline{b}, \gamma_D) \right)^2.
\end{aligned}
$$

(24)

If the function $H(\underline{b}, \gamma_A, \gamma_D)$ is equal to zero, then the first order conditions are satisfied at the grid points and the boundary conditions are also satisfied. Since it is hard to find such γ_A, γ_D, and $\underline{b}$, I find their values by minimizing the value of $H(\underline{b}, \gamma_A, \gamma_D)$:

$$
\min_{\underline{b}, \gamma_A, \gamma_D} H(\underline{b}, \gamma_A, \gamma_D) \text{ subject to } \underline{c} \leq \underline{b} \leq \overline{c}
$$

(25)

To find the solutions, we can use a standard non-linear least square algorithm

Characteristics of the Equilibrium

I compute the inverse bidding functions using the algorithm introduced above. I let the cost be distributed over $[\underline{c}, \overline{c}] = [0,50]$ and the mean, and I set the standard deviation of the probability distribution to $(\mu, \sigma) = (25,10)$. Furthermore, I set the probability that the corruption scheme is not implemented at $p=0.5$. The results are reported in Figure 1. As shown, the bid function for both advantaged and disadvantaged contractors is increasing in their costs. Figure 1 also reports that both contractors bid at the same price when their technology is at the highest level (i.e., when their costs are the lowest) or the lowest level (i.e., when their costs are the highest). Except in these two cases, the disadvantaged contractor always bids more aggressively than the advantaged contractor.

The disadvantaged contractor has a chance to win the contract only when his bid profitably matches the advantaged contractor's cost, which is lower than advantaged contractor's bid. However, the advantaged contractor does not have an incentive to bid lower than the disadvantaged contractor. Suppose that the advantaged contractor bids more aggressively than the disadvantaged contractor. In such cases, he can always profitably deviate from the strategy without changing the probability of winning both an auction with a fixed outcome and an auction without a fixed outcome. Therefore, the disadvantaged contractor always bids lower than the advantaged contractor for the given cost.

Proposition 2: When the probability that the corruption scheme will be implemented increases,

1. The advantaged and disadvantaged contractors bid more aggressively than when they are in a fair auction ($p=1$) and less aggressively than when they are in a perfectly corrupt auction ($p=0$).

2. The disadvantaged contractor bids more aggressively when there is a strictly positive probability that the corruption scheme will be implemented ($0<p<1$).

Efficiency in Project Allocation

Figure 2 and Table 1 provide insight into how the possibility of corruption and an uncommitted government official can cause the contract to be allocated efficiently or inefficiently. I compare possible combinations of ex-post costs and bids for advantaged and disadvantaged contractors. In a procurement auction with an uncommitted government official, there exist two potential inefficiencies with respect to the costs associated with the advantaged contractor.

- **Type I inefficiency**: If the advantaged contractor has a lower cost than the disadvantaged contractor, then the contract may be allocated inefficiently when the corruption scheme is practiced.

- **Type II inefficiency**: If the disadvantaged contractor has a lower cost than the advantaged contractor, then the contract may be allocated inefficiently when the corruption scheme is practiced.

There are four cases that generates either type I or type II efficiency.

Case A: $\underline{c} \leq \hat{c}_D \leq \beta_D^{-1}(\hat{c}_A)$

Suppose that *advantaged* contractor has the cost of $c_A = \hat{c}_A$ and disadvantaged contractor's cost $c_D = \hat{c}_D$ is in $\underline{c} \leq \hat{c}_D \leq \beta_D^{-1}(\hat{c}_A)$. Their bids are $b_A = \beta_A(\hat{c}_A)$, and $b_D = \beta_D(\hat{c}_D)$ respectively. If the corruption scheme is practiced, *disadvantaged* bidder wins the contract since $\beta_A(\hat{c}_A) > \beta_D(\hat{c}_D)$ and if the scheme is not practiced, *disadvantaged* contractor wins the contract again since $\beta_A(\hat{c}_A) > \beta_D(\hat{c}_D)$. Since $\hat{c}_A > \hat{c}_D$, the allocation is always efficient.

Case B: $\beta_D^{*-1}(\hat{c}_A) \leq \hat{c}_D \leq \hat{c}_A$

Suppose that *advantaged* contractor has the cost of $c_A = \hat{c}_A$ and *disadvantaged* contractor's cost $c_D = \hat{c}_D$ in $\beta_D^{*-1}(\hat{c}_A) \leq \hat{c}_D \leq \hat{c}_A$. Their bids are $b_A = \beta^*_A(\hat{c}_A)$, and $b_D = \beta^*_D(\hat{c}_D)$ respectively. If the scheme is practiced, *advantaged* contractor wins since $\beta^*_D(\hat{c}_D) > \hat{c}_A$ If the corruption scheme does not occur, disadvantaged contractor wins the contract because $\beta^*_D(\hat{c}_D) > \beta^*_A(\hat{c}\_A)$. Since $\hat{c}_A > \hat{c}_D$, the contract allocation is inefficient if the corruption scheme is not practiced, but inefficient if the scheme is practiced. (Type II inefficiency)

Case C: $\hat{c}_A \leq \hat{c}_D \leq \beta_D^{*-1}(\beta_A^*(\hat{c}_A))$

Suppose that advantaged contractor has the cost of $c_A = \hat{c}_A$ and *disadvantaged* contractor's cost $c_D = \hat{c}_D$ is in $\hat{c}_A \leq \hat{c}_D \leq \beta_D^{*-1}(\beta_A^*(\hat{c}_A))$. . Their bids are $b_A = \beta^*_A(\hat{c}_A)$, and $b_D = \beta^*_D(\hat{c}_D)$ respectively. If the corruption scheme is practiced, *advantaged* contractor wins since $\beta_D^*(\hat{c}_D) > \hat{c}_A$. If the corruption scheme is not practiced, *disadvantaged* contractor wins the contract because $\beta_A^*(\hat{c}_A) > \beta_D^*(\hat{c}_D)$. Because $\hat{c}_D > \hat{c}_A$ the contract allocation is efficient if the corruption scheme is practiced, but inefficient if the corruption scheme is not practiced.

Case D: $\beta_D^{*-1}(\beta_A^*(\hat{c}_A)) < \hat{c}_D$

Suppose that advantaged contractor has the cost of $c_A = \hat{c}_A$ and *disadvantaged* contractor's cost $c_D = \hat{c}_D$ is in $\beta_D^{*-1}(\beta_A^*(\hat{c}_A)) < \hat{c}_D$. Their bids are $b_A = \beta^*{}_A(\hat{c}_A)$, and $b_D = \beta^*{}_D(\hat{c}_D)$ respectively. If the corruption scheme occurs, advantaged contractor wins because $\beta_D(\hat{c}_D) > \hat{c}_A$. If the corruption scheme does not occur, advantaged contractor wins the contract because $\beta^*{}_D(\hat{c}_D)\beta_A^*(\hat{c}_A)$. Because $\hat{c}_D > \hat{c}_A$, the contract allocation is always efficient. (Type I inefficiency)

In Type I inefficiency, inefficient contract allocation occurs not due to corruption but due to the cautionary behaviors of the *disadvantaged* contractor, and this is the key result of this study. Because the existence of a corruption scheme in this context is not something that the participants in the auction are actively aware of, the contractors may bid more aggressively or less aggressively depending on whether individual contractors have an illegal contract with government officials. Because the contractors are preparing for possible corruption, inefficient allocation can occur even when there is no artificial price manipulation by the government. Type I inefficiency can occur because of the disadvantaged contractor's behavior, which is predicated on his knowledge that a corruption scheme may be at work.

Type II inefficiency occurs in an auction that includes the right of first refusal. If the contractor with the right of refusal has the lower-level technology ex-post, then corruption can allow the contractor to bid high enough to have the opportunity to observe the opponent's bid and determine whether the opponent will be awarded the contract or not. Because the disadvantaged contractor will bid less aggressively than when he knows with certainty that a corruption scheme is in place, the advantaged contractor has more room to win the contract. Type II inefficiency occurs because the disadvantaged contractor takes cautionary measures, even when corruption does not occur.

6. Comparative Statistics

Let me return to Figure 3 and compare the two sets of curves in Figure 1. These are the bid functions for *advantaged* and *disadvantaged* contractors when the probability of corruption is $p=0.05$ and $p=0.2$. An increase in p means that the possibility of corruption is more credible. The intercepts of the curves, are the bids corresponding to the lowest possible cost $\underline{c}$. When the chances that the corruption scheme will be implemented increase (i.e., when $p=0.2$ changes to $p=0.05$), the lowest bids increase from $\underline{b}(p=0.05)$ to $\underline{b}(p=0.2)$. Recall that the bid functions increase with cost and that they are continuous. Also remember that both of the contractors' bids correspond to the highest cost. Thus, both of the contractors will bid more aggressively as the possibility that the scheme will be implemented increases. Furthermore, based on Proposition 1, the magnitude that the *advantaged* contractor lowers his bid for a given cost is less than the amount by which the *disadvantaged* contractor lowers her bid.

Expected Price

Expected price of contract in equilibrium is given by

$E[project\ cost] =$

$$
\int_0^c \left\{ p \int_0^{\beta_D^{-1}(\hat{c})} \beta_D(c)f(c)dc + (1-p) \int_0^{\beta_D^{-1}(\hat{c})} \beta_D(c)f(c)dc \right.
$$

$$
+ p \int_{\beta_D^{-1}(\hat{c})}^{\hat{c}} \beta_D(c)f(c)dc + (1-p) \int_{\beta_D^{-1}(\hat{c})}^{\hat{c}} \beta_D(c)f(c)dc \quad (26)
$$

$$
+ p \int_{\hat{c}}^{\beta_D^{-1}(\widehat{\beta_A(\hat{c})})} \beta_D(c)f(c)dc + (1-p) \int_{\hat{c}}^{\beta_D^{-1}(\widehat{\beta_A(\hat{c})})} \beta_D(c)f(c)dc
$$

$$
\left. + p \int_{\beta_D^{-1}(\beta_A(\hat{c}))}^{\bar{c}} \beta_D(c)f(c)dc + (1-p) \int_{\beta_D^{-1}(\beta_A(\hat{c}))}^{\bar{c}} \beta_D(c)f(c)dc \right\} d\hat{c}
$$

Figure 4 shows the how the relationship between the probability of that the corruption scheme is not practiced (the case of fair auction) p and the expected price of the contract. The expected cost is decreasing in the probability of fair auction. This is because that as the probability of the practice of corruption scheme increase, the prices corresponding to the possible prices become higher when the corruption scheme is practiced with higher probabilities than the case with lower probabilities, although the entire curve of bid function of lower probability lie in below the ones with lower probabilities.

Winning Probabilities

The advantaged and disadvantaged contractors' winning probabilities are given by

$$\Pr(A \ wins.; c_A = \hat{c}_A. \beta_A^*, \beta_D^*)$$

$$= \begin{cases} (1-p) + p \int_{\beta_D^{-1}(\beta_A(\hat{c}_A))}^{\bar{c}} f(c)dc & \text{if } \hat{c}_A < \underline{b} \qquad (27) \\ (1-p) \int_{\beta_D^{-1}(\hat{c}_A)}^{\beta_D^{-1}(\beta_A(\hat{c}_A))} f(c)dc + \int_{\beta_D^{-1}(\beta_A(\hat{c}_A))}^{\bar{c}} f(c)dc & \text{if } \hat{c}_A \geq \underline{b} \end{cases}$$

$$\Pr(D \ wins.; c_D = \hat{c}_D. \beta_A^*, \beta_D^*) = p \int_{\beta_A^{-1}(\beta_D(\hat{c}_D))}^{\bar{c}} f(c)dc + (1-p) \int_{\beta_D(\hat{c}_D)}^{\bar{c}} f(c)dc \qquad (28)$$

The first term of equation (27) of the advantaged contractor's probability of winning comes from the cost domain in which the advantaged contractor wins because his costs are lower than the common lowest bid among contractors. In this domain, the advantaged contractor is definitely awarded the contract regardless of his bid and the disadvantaged contractor's bid if the corruption scheme is practiced because $\underline{b}_A = \underline{b}_D > \hat{c}_A \geq \underline{c}$.

Figures 4 and 5 show the relationship between each contractor's probability of winning and the probability of a fair procurement auction. Whereas the advantaged contractor's probability of winning has a negative relationship with the probability of a fair auction, the disadvantaged contractor's probability of winning has a positive relationship with the probability

of a fair auction. This is again because $\underline{b}_A = \underline{b}_D \geq \underline{c}$. As long as there is a strictly positive probability that the corruption scheme will be implemented, the disadvantaged contractor runs the risk of losing the contract even when he has the lower cost $c_D = \underline{c}$.

Proposition 3:

1. Expected prices decrease with the probability that the corruption scheme will be executed.

2. The advantaged contractor's probability of winning increases and the disadvantaged contractor's probability of winning decreases with the probability that the corruption scheme will be executed. Moreover, the disadvantaged contractor's probability of winning does not equal one when her cost is the lowest possible.

We can interpret our findings by comparing bid function curves. When the advantaged contractor considers that the scheme is always practiced (*p=0*), he submits a bid any value, since he knows that it is going to be revised by the government. However, once the contractors perceive that the probability of a fair auction is strictly positive (*p>0*), the advantaged contractor faces the possibility that his bid will become public information without being revised by the government officials. Therefore, the advantaged contractor has to submit a meaningful bid that is greater than his cost c_A, but because there is still a positive probability that the corruption scheme will be implemented, he bids less aggressively than the disadvantaged bidder. In contrast, for the disadvantaged contractor, *p>0* indicates weaker price competition than *p=0*. Thus, the disadvantaged contractor bids less aggressively than when *p=0*. However, because he knows that there is a strictly positive probability that the scheme will be implemented (*1-p>0*), the disadvantaged contractor bids more aggressively than the advantaged contractor. As a result, the bid function curves of both contractors are located above the 45-degree line, with the disadvantaged contractor's curve located below the advantaged contractor's bid function curve.

As the probability that the corruption scheme will be implemented increases, both contractors' bidding curves shift downwards, as demonstrated by the vertical differences between the advantaged and disadvantaged contractor bid function curves ceteris paribus. This movement of bid functions can be interpreted as follows. Let me restate my findings by adding the positive probability that the (promised) corruption scheme will not be executed.

1. When the procurement auction is fair ($p=1$) or perfectly corrupt ($p=0$), project allocation is not inefficient.

2. When the probability that the corruption scheme is not practiced becomes strictly positive ($p > 0$), two possible types of inefficiencies emerge. One is achieved when the scheme is practiced, whereas the other is achieved when the scheme is not practiced.

3. As the probability that the corruption scheme will be implemented increases, the vertical differences between the advantaged and disadvantaged contractor bid function curves increases, indicating that project allocation inefficiency can occur within a wider range of costs.

Keeping in mind that regulatory agencies detect and punish corruption because it creates unfair situations, the results presented here become quite interesting. When the government is perfectly corrupt and everybody in the economy knows so, there is no inefficient allocation, and projects are delivered with the lowest cost, although the convention itself is unfair. If a government prohibits bribes from contractors to the government, whether such plans are actually executed may depend on the detection capabilities of the regulatory authorities. However, if the detection level is low, economic performance could be very low, meaning that inefficiency and higher project costs will more likely occur, even in an auction without corrupt arrangements. This provides a rationale for very stringent investigation procedures by third parties such as the

FBI, which will presumably decrease to very close to zero the probability that corruption schemes will be carried out. This means that efficient and reasonable project costs become more likely.

7. Conclusion

Although a great deal of literature has investigated the impact of variations in corruption schemes on the prices of contracts and the efficiency of contract allocation, these studies have not taken into account cases in which the probability that corruption schemes will not be practiced is strictly positive. In this paper, I introduced the idea of a government official who does not commit to implementing a corruption scheme in the context of sealed-bid, first-price procurement auction with identical type distribution and different utility functions. In such a situation, the advantaged contractor bids more aggressively than in a corrupt auction but less aggressively than in a fair auction. Alternatively, the disadvantaged contractor bids less aggressively than in a corrupt auction but more aggressively than in a fair auction. The implications of these different behaviors can be interpreted with respect to cautionary measures that can be taken in both situations. I have shown that because of differences in the certainty that corrupt plans will actually be executed; there can be two types of inefficiency in project allocation.
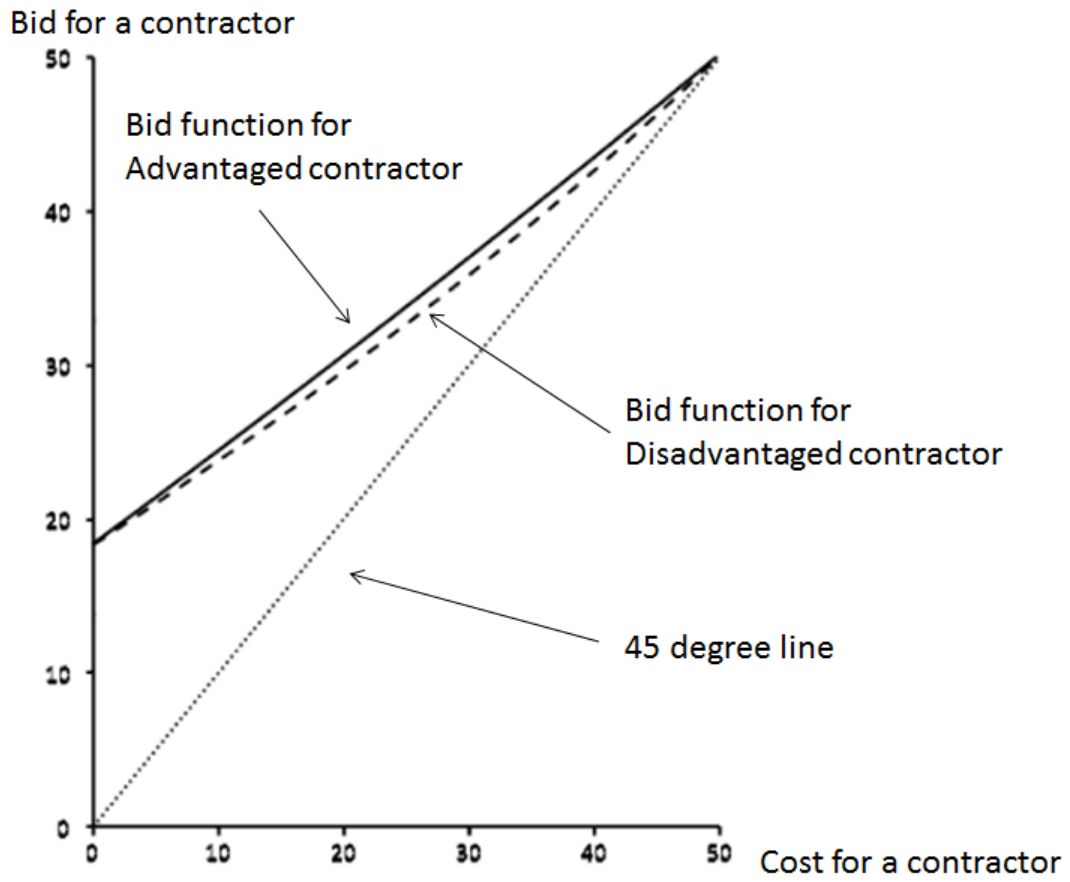
Figure 1: Advantaged (upper curve) and Disadvantaged (lower curve) Contractors' Bid
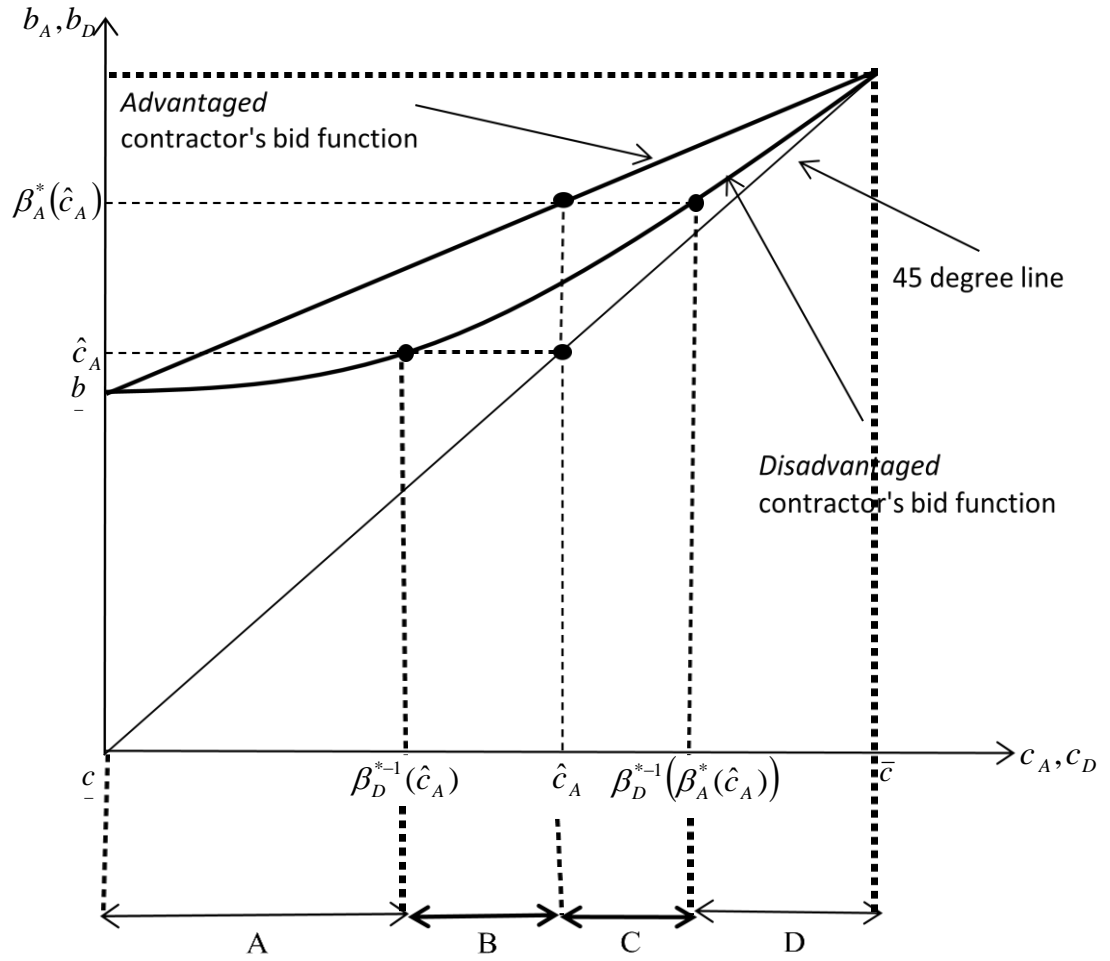Functions

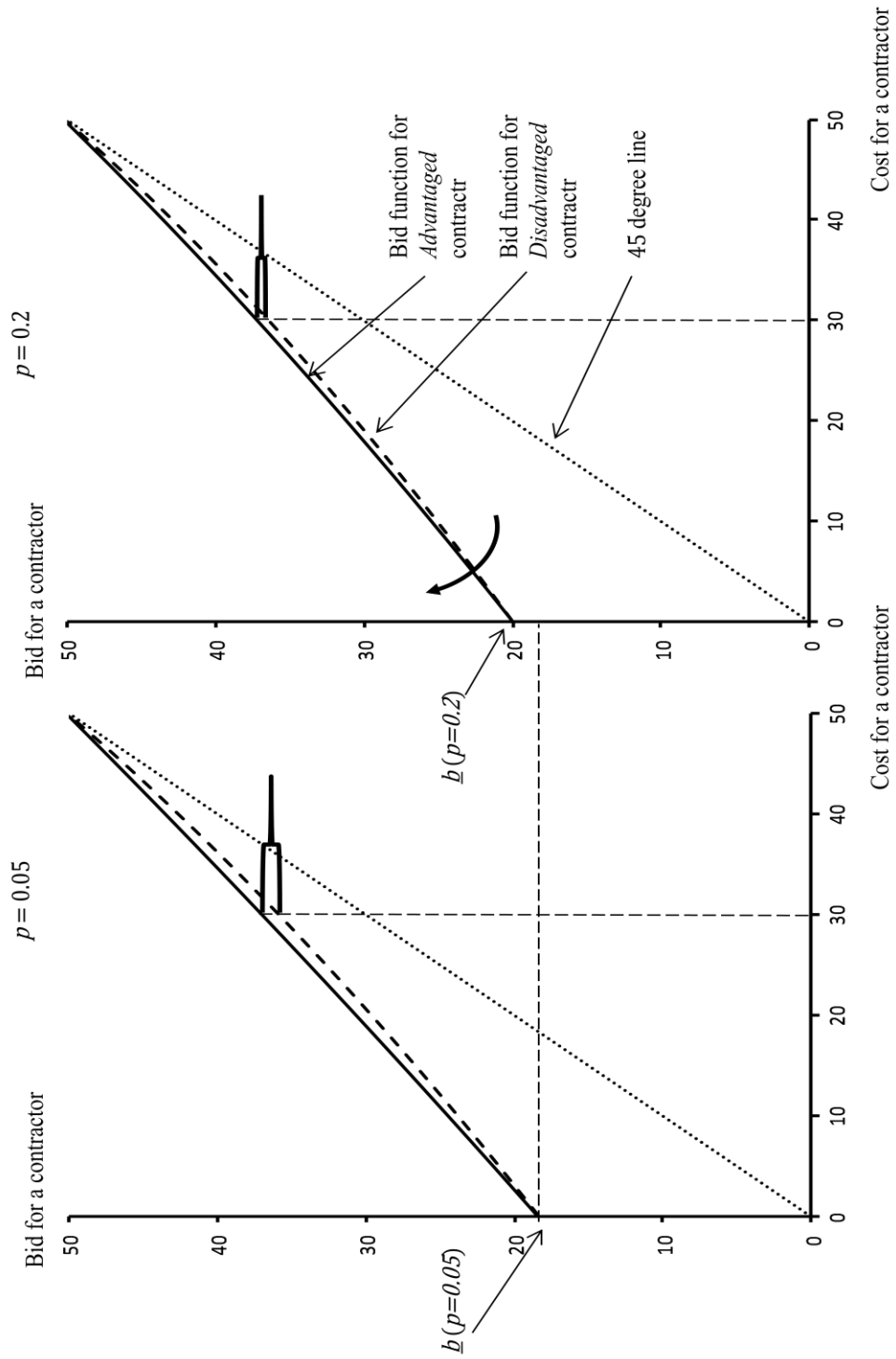Figure 2: Efficiency in Project Allocation
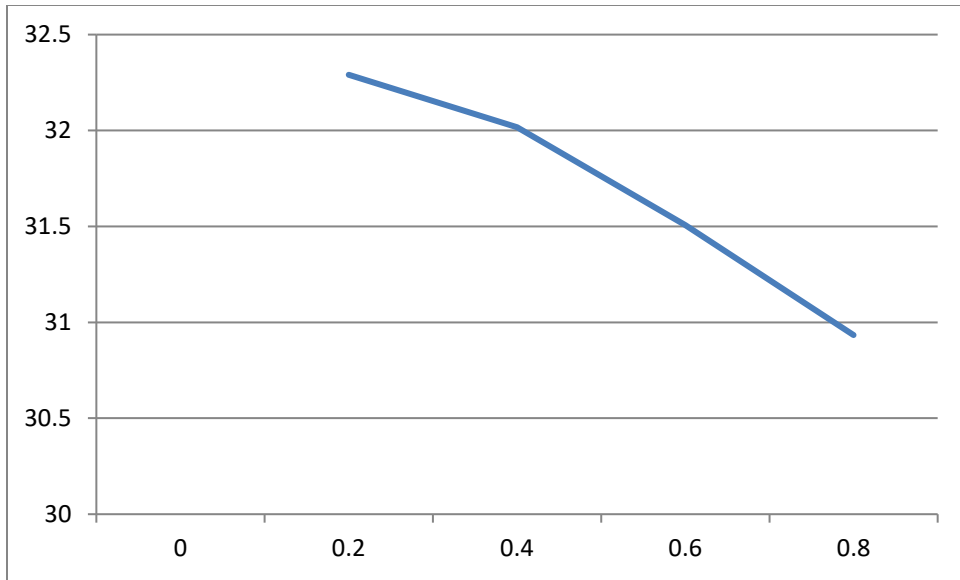
Figure 3: Comparative Statics in *p*

Figure 4: Expected Cost of the Project

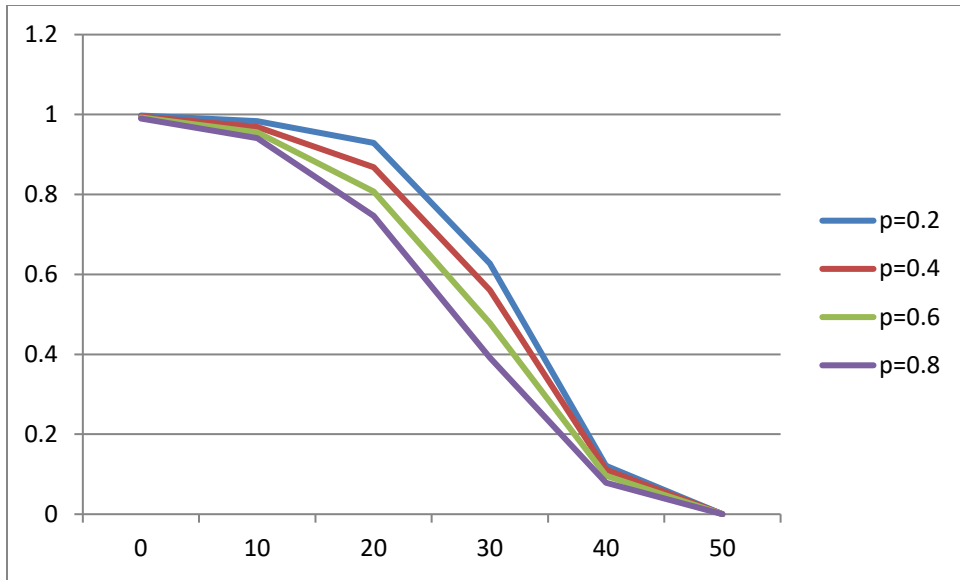(X: probability of fair auctions (p), Y: expected cost)

Figure 5: Advantaged Contractor's Probability of Winning

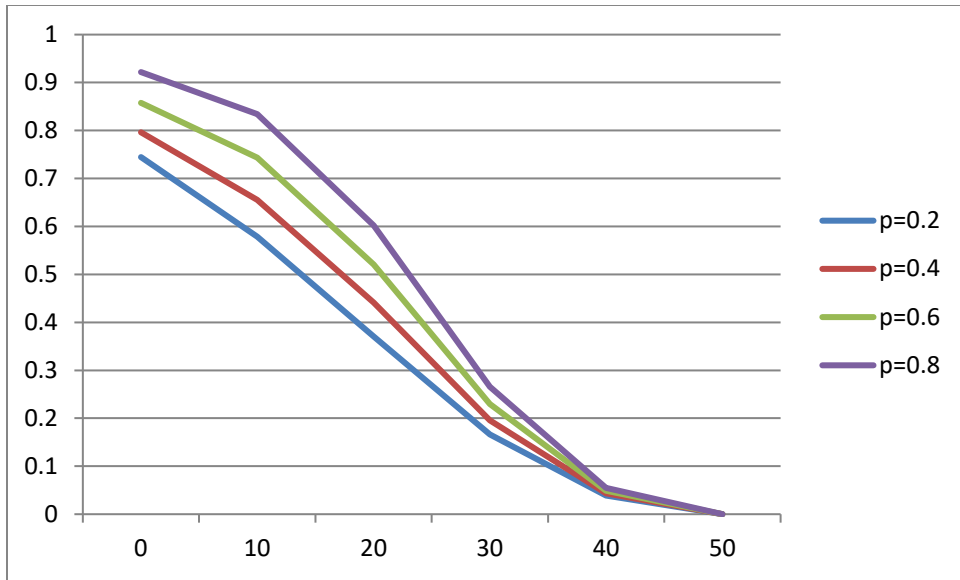(X: Cost for Advantaged Contractor, Y: Probability of Winning)

Figure 6: Disadvantaged Contractors Probability of Winning

(X: Cost for Disadvantaged Contractor, Y: Probability of Winning)

For given $\hat{c}_A \in [\underline{c},\ \bar{c}]$

Case A. $\underline{c} \le c_D \le \beta_D^{-1}(\hat{c}_A) < \hat{c}_A$ Contractor with c_D always wins and the allocation is efficient.

Case B. $\beta_D^{-1}(\hat{c}_A) < c_D < \hat{c}_A$

(Type I inefficiency)

(1) If price manipulation occurs: Contractor with $\hat{c}_A$ wins the allocation is **inefficient**.

(2) If price manipulation does not occur: Contractor with c_D wins the allocation is efficient

Case C. $\hat{c}_A < c_D < \beta_D^{-1}(\beta_A(\hat{c}_A))$

(Type II inefficiency)

(1) If price manipulation occurs: Contractor with $\hat{c}_A$ wins the allocation is efficient.

(2) If price manipulation does not occur: Contractor with c_D wins the allocation is **inefficient**

Case D. $\beta_D^{-1}(\beta_A(\hat{c}_A)) \le c_D \le \bar{c}$ Contractor with $\hat{c}_A$ always wins and the allocation is efficient.

Table 1: Efficiency in Allocation

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