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Downstream new product development and upstream process innovation

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Abstract

This study considers the role of the upstream process research and development (R&D) when downstream develops new products. We build a model in which an upstream firm conducts cost-reducing investment and two downstream firms develop new products. We assume that all products are differentiated. We show that downstream product development promotes upstream investment. We also demonstrate that downstream product development is a strategic complement if upstream R&D efficiency is high, while it is a strategic substitute if it is low. This implies that the occurrence of complementary equilibrium does not need asymmetry in the differentiated final-product markets and is in sharp contrast to the previous study.

Key words: Upstream process R&D; Product development; Complementary equilibrium

JEL classification: D43; L13; O31

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1 Introduction

In vertical structures, research and development (R&D) in the upstream plays a key role. This is because when markets expand due to introducing new products downstream, since use of inputs increases, the upstream cost-reducing R&D becomes even more important (Fontana and Guerzoni, 2008). For example, in assembly industries, such as automobiles and computers, if new products are developed downstream, it be capable of promoting the upstream investment to reduce the production cost of inputs.

We consider the role of upstream process R&D when downstream develops new products. To do so, we build a model comprising an innovative upstream firm and two downstream firms that develop new products in the case where all final products are differentiated. We show that upstream investment increases as downstream product development progresses. We also demonstrate that in a downstream product-R&D game, complementary equilibrium appears if upstream R&D efficiency is high but asymmetric equilibrium appears if it is low. Basak and Mukherjee (2018) reveal that emergence of complementary equilibrium always needs asymmetric product differentiation. However, we find that complementary equilibrium appears even without such asymmetry. This result arises from a fall in input price caused by increasing upstream investment due to downstream market expansion. Hence, our findings provide new insights into the studies of innovation and vertical structures.

Some studies also focus on upstream process R&D, however, their frameworks and purposes substantially differ from ours. Chen and Sappington (2010) examine the effects of vertical integration and separation on upstream innovation. Although Hu et al. (2020) consider upstream process R&D, their purpose is to examine the relationship between upstream cost-reducing R&D and cross-holdings among downstream.

In this study, all proofs are illustrated in the Supplementary Material.

2 Model and Results

We consider a vertically related market with an upstream firm (U) and two symmetric downstream firms (Di, $i = 1, 2$). Di uses one unit of the input to produce one unit of the final product, and it competes in Cournot fashion.[1] For simplicity, we omit other production costs for Di. The U decides the input price w and makes a take-it-or-leave-it-offer.

The U engages in R&D to reduce the constant marginal cost $c \in (0, 1)$. To create demands by introducing a new product, Di chooses whether to conduct R&D paying a fixed-cost $f > 0$ or not. Let Di be the existing product $q_{e,i}$ and its new product $q_{n,i}$. When $D1$ and $D2$ introduce new products, inverse demands are[2]

$$\begin{aligned} p_{e,i} &= 1 - q_{e,i} - \gamma(q_{n,i} + q_{e,j} + q_{n,j}), \\ p_{n,i} &= 1 - q_{n,i} - \gamma(q_{e,i} + q_{e,j} + q_{n,j}), \end{aligned} \tag{1}$$

where $p_{e,i}$ ($p_{e,j}$) is the price of the existing product of Di (Dj) and $p_{n,i}$ ($p_{n,j}$) is the price of the new product of Di (Dj), $i \neq j$ and $i, j = 1, 2$. $\gamma \in [0, 1)$ measures the degree of product substitutability among final products.

The gross profit of Di is

$$\pi_{Di}(q_{e,i}, q_{n,i}) \equiv (p_{e,i} - w)q_{e,i} + (p_{n,i} - w)q_{n,i}. \tag{2}$$

If Di innovates, its profit is $\pi_{Di}(q_{e,i}, q_{n,i}) - f$. If Di does not, its profit is $\pi_{Di}(q_{e,i}, 0)$.

The profit of U is

$$\pi_U \equiv (w - (c - x))Q - kx^2, \tag{3}$$

where x is the investment level and kx^2 is the R&D cost. $k > 0$ denotes R&D efficiency. Q is the demand for the input. $Q = \sum_i q_{e,i}$ if nobody innovates. $Q = \sum_i q_{e,i} + q_{n,j}$ if only Dj innovates. $Q = \sum_i q_{e,i} + \sum_i q_{n,i}$ if everyone innovates.

We consider the following four-stage game. In the first stage, $D1$ and $D2$ independently and simultaneously choose whether to do R&D by paying the fixed cost (I) or not (N). In the second stage, U decides the investment level. At the third stage, U charges input price. Last, downstream competes *à la* Cournot.

This timing structure corresponds to the difficulty in R&D. In general, product development requires a sunk cost, such as a long-term contract with researchers, and it takes much longer time. Hence, the downstream R&D is at the first stage. It is not needed the effort such that produces prototype and repeatedly tests its safety, so the upstream R&D is at the second stage. Downstream can flexibly adjust their production, so the quantity of final products is decided in the final stage. The solution concept is the subgame perfect Nash equilibrium.

Depending on downstream investment decisions, four regimes can arise: II, IN, NI, and NN. Using (1)–(3), we obtain the equilibrium solutions, which are reported in Appendix A.

To ensure a positive marginal cost after investment, we assume

$$k > k_0 \equiv \frac{1}{2c(1+2\gamma)}.$$

We establish the following results from Appendix A.

Proposition 1. *(i)* $x^{II} > x^{IN} = x^{NI} > x^{NN}$. *(ii)* $\partial x^r/\partial k < 0$ *and* $\partial x^r/\partial \gamma < 0$*, where* $r = II, IN, NI, NN$.

Corollary 1. *(i)* $w^{NN} > w^{IN} = w^{NI} > w^{II}$. *(ii)* $\partial w^r/\partial k > 0$ *and* $\partial w^r/\partial \gamma > 0$*, where* $r = II, IN, NI, NN$.

The logic behind part (i) of Proposition 1 is as follows. As U engages in cost-reducing investment, U invests a lot if it can sell input a lot. An innovation by Di increases the number of product varieties, so the demand for the input also expands. If $D1$ and $D2$ innovate, because the input demand is the largest among all regimes and the sales opportunity of inputs is similarly the largest, the investment level becomes the largest. Hence, when nobody innovates, the investment size becomes the least among all regimes. If only Di innovates, the investment becomes intermediate level.

Part (ii) is intuitive. The first result is natural. Although larger γ makes competition tougher, in our model, it reduces downstream market size. The latter effect is dominant, so the input demand shrinks. This impedes upstream investments.

Proposition 1 immediately yields Corollary 1. Since a larger investment corresponds to a lower input price, we obtain the ranking of the input price. Part (ii) is a natural one. The effects of γ are similar to those of part (ii) of Proposition 1.

Our model has a similar timing structure to Banerjee and Lin (2003): Downstream first invests, and after observing it, the upstream charges the price. They emphasize that the raising input price extracts benefits of downstream investment.[3] By contrast, in our study, since the market expansion due to downstream R&D promotes upstream investment, as a result, the input price falls. Corollary 1 implies that upstream R&D is very influential in vertical structures.

To illustrate equilibrium, we define two R&D benefits of Di (Chowdhury, 2005). One is *non-strategic benefit*, which is $\Phi_I \equiv \pi_{D1}^{IN} - \pi_{D1}^{NN} = \pi_{D2}^{NI} - \pi_{D2}^{NN}$. The other is *strategic benefit*, which is $\Phi_N \equiv \pi_{D1}^{II} - \pi_{D1}^{NI} = \pi_{D2}^{II} - \pi_{D2}^{IN}$. ($\Phi_N$ and Φ_I are reported in Appendix B.) Di innovates if $f < \Phi_I$, and it does not innovate if $\Phi_N < f$. Hence, $IN\&NI$ can appear if $\Phi_N < \Phi_I$; $NN\&II$ can appear if $\Phi_I < \Phi_N$. These arguments and Appendix A yield Proposition 2.

Proposition 2.

1. Suppose that $k \in (k_0,\ 1/(4\gamma))$. Then, $\Phi_I < \Phi_N$. (i) If $f < \Phi_I$, II appears, (ii) if $f > \Phi_N$, NN appears, and (iii) if $\Phi_I \leq f \leq \Phi_N$, $NN\&II$ can appear.

2. Suppose $k > 1/(4\gamma)$ or $1/(4\gamma) \leq k_0$. Then, $\Phi_N < \Phi_I$. (i) If $f < \Phi_N$, II appears, (ii) if $f > \Phi_I$, NN appears, and (iii) if $\Phi_N \leq f \leq \Phi_I$, $IN\&NI$ can appear.

When the fixed-cost f is small (large) because I (N) is the dominant strategy, II (NN)

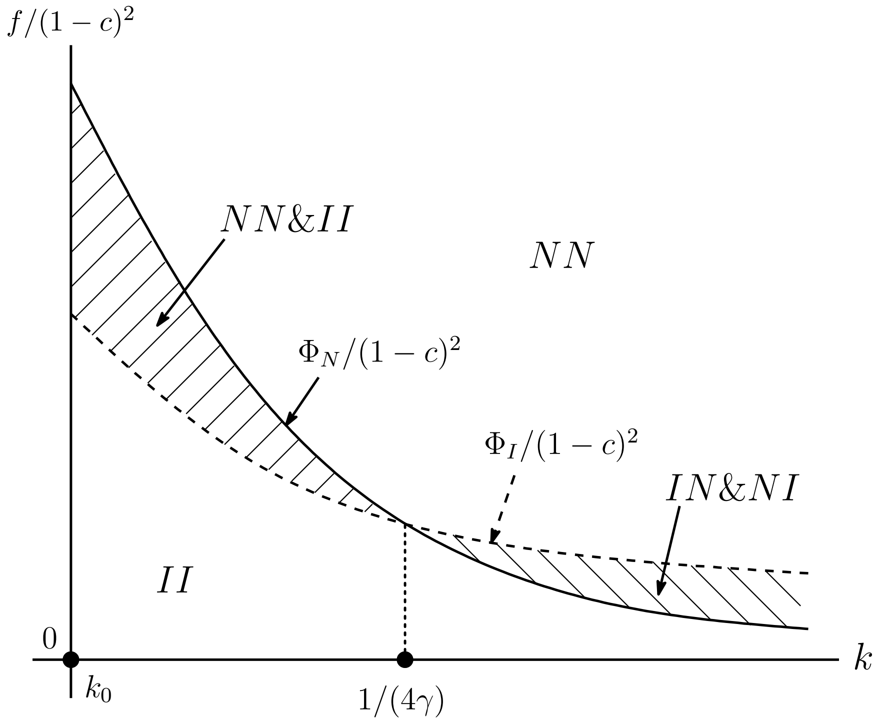

Figure 1: Equilibrium of the game ($k_0 < 1/(4\gamma)$)

appears. If f is an intermediate size, Di's strategy depends on the upstream R&D efficiency k: (i) if k is small, $NN\&II$ can appear. (ii) If k is large, $IN\&NI$ can appear (see Figure 1).

The intuition is as follows. (i) When k is small, the upstream R&D is efficient. In this case, if Di deviates from II, its markets become half. Furthermore, the input price jumps, so Di does not deviate from II. If Di deviates from NN, the number of product markets increases. As upstream R&D efficiency is high and the range of the drop in input price is larger, downstream production costs largely fall. However, this promotes rival's production and makes competition in the existing product market tougher, so the benefit of R&D can be canceled. Di does not deviate.

Basak and Mukherjee (2018) find that, in a unionized duopoly, emergence of the complementary equilibrium needs *asymmetric product differentiation* and *decentralized unions*. In contrast, we show that the complementary equilibrium appears even if there is no asymmetry in product differentiation. This implies that the upstream R&D has an important role for the downstream innovation, and therefore, gives a new insight into the previous literature.

(ii) When k is large (or $1/(4\gamma) \le k_0$), because upstream R&D is inefficient, the effects of upstream investment on the input price weakens. If Di deviates from II, its product markets become half and input price rises. However, when k is large because the input price is high and the downstream production cost is also high, and the profit loss from losing a market is small. That is, the R&D benefit is small, so Di chooses N when the rival chooses I. The deviation from NN increases product market and lowers input price. Then, although the R&D benefit is small, the input price is high because k is large. Hence, a fall in production cost through the decrease in input price becomes attractive. Di chooses I when the rival chooses N.

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Appendix A. Equilibrium Solutions

NN

$$w^{NN} = \frac{(1+c)(\gamma+2)k-1}{2(\gamma+2)k-1}; \quad x^{NN} = \frac{1-c}{2(\gamma+2)k-1}; \quad \pi_U^{NN} = \frac{(1-c)^2k}{2(\gamma+2)k-1},$$
$$q_{e,i}^{NN} = \frac{(1-c)k}{2(\gamma+2)k-1}; \quad \pi_{Di}^{NN} = \left(q_{e,i}^{NN}\right)^2, \quad \text{for } i=1,2.$$

IN (NI)

$$x^{IN} = x^{NI} = \frac{(1-c)(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)},$$
$$w^{IN} = w^{NI} = \frac{2(1+c)(2+2\gamma-\gamma^2)k-(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad \pi_U^{IN} = \frac{(1-c)^2(3-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)},$$
$$q_{e,1}^{IN} = q_{n,1}^{IN} = \frac{(1-c)(2-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad q_{e,2}^{IN} = \frac{2(1-c)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)},$$
$$\pi_{D1}^{IN} = \frac{2(1-c)^2(2-\gamma)^2(1+\gamma)k^2}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2}; \quad \pi_{D2}^{IN} = \left(q_{e,2}^{IN}\right)^2,$$

where $q_{e,2}^{NI} = q_{n,2}^{NI} = q_{e,1}^{IN} = q_{n,1}^{IN}$, $q_{e,2}^{IN} = q_{e,1}^{NI}$, $\pi_{D2}^{NI} = \pi_{D1}^{IN}$, and $\pi_{D1}^{NI} = \pi_{D2}^{IN}$.

II

$$w^{II} = \frac{(1+c)(2\gamma+1)k-1}{(4\gamma+2)k-1}; \quad x^{II} = \frac{1-c}{(4\gamma+2)k-1}; \quad \pi_U^{II} = \frac{(1-c)^2k}{(4\gamma+2)k-1},$$
$$q_{e,i}^{II} = q_{n,i}^{II} = \frac{(1-c)k}{2\left[(4\gamma+2)k-1\right]}; \quad \pi_{Di}^{II} = \frac{(1-c)^2(\gamma+1)k^2}{2\left[(4\gamma+2)k-1\right]^2},$$

Appendix B.

$$\Phi_N = \frac{(1-c)^2(1-\gamma)k^2 \begin{bmatrix} 1+4\gamma-\gamma^2+16(2+6\gamma+6\gamma^2+2\gamma^3-\gamma^4)k^2 \\ -8(2+4\gamma+3\gamma^2-\gamma^3)k \end{bmatrix}}{2[(4\gamma+2)k-1]^2\left[4(2+2\gamma-\gamma^2)k-(3-\gamma)\right]^2} > 0$$

$$\Phi_I = \frac{(1-c)^2(1-\gamma)k^2\left[8(8+8\gamma-4\gamma^4)k^2-8(2+2\gamma+\gamma^2-\gamma^3)k-2\gamma^2+5\gamma-1\right]}{[2(\gamma+2)k-1]^2\left[4(2+2\gamma-\gamma^2)k-(3-\gamma)\right]^2} > 0.$$

Notes

[1]Our main results do not alter in Bertrand competition.

[2]The other possible setting is that the existing and new products are differentiated. The formula in such case is $p_{e,i} = 1-(q_{e,i}+q_{e,j})-\gamma(q_{n,i}+q_{n,j})$ for $i \neq j$. However, our main results do not alter, so we employ a simpler form (1).

[3]Banerjee and Lin (2003) show that a fixed-price contract of input-price resolves this hold-up problem. In contrast, Takauchi and Mizuno (2019) demonstrate that the fixed-price contract can harm upstream and downstream.

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“Downstream new product development and upstream process innovation” Supplementary Material (Not for Publication)

This supplement provides all proofs and supporting results.

I Proofs

Proof of Proposition 1. (i) $x^{II} - x^{IN} = \frac{2k(1-c)(1-\gamma)}{[(4\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]} > 0$ and $x^{IN} - x^{NN} = \frac{2k(1-c)(1-\gamma)(2-\gamma)}{[2(\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]} > 0$. (ii) The partial derivative of x^r ($r = II, IN, NN$) with respect to k yields $\partial x^{II}/\partial k = -\frac{2(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial k = -\frac{4(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial k = -\frac{2(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} < 0$. The partial derivative of x^r with respect to γ yields $\partial x^{II}/\partial \gamma = -\frac{4(1-c)k}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial \gamma = -\frac{4(1-c)(\gamma^2-6\gamma+8)k}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial \gamma = -\frac{2(1-c)k}{[2(\gamma+2)k-1]^2} < 0$. □

Proof of Corollary 1. (i) $w^{NN} - w^{IN} = \frac{k(1-c)(2-\gamma)(1-\gamma)}{[2(\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]} > 0$ and $w^{IN} - w^{II} = \frac{k(1-c)(1-\gamma)}{[(4\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]} > 0$. (ii) The partial derivative of w^r with respect to k yields $\partial w^{II}/\partial k = \frac{(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial k = \frac{2(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial k = \frac{(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} > 0$. The partial derivative of w^r with respect to γ yields $\partial w^{II}/\partial \gamma = \frac{2(1-c)k}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial \gamma = \frac{2(1-c)k(4-\gamma)(2-\gamma)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial \gamma = \frac{(1-c)k}{[2(\gamma+2)k-1]^2} > 0$. □

Proof of Proposition 2. Comparing Φ_N with Φ_I, we have

$$\Phi_N - \Phi_I = \frac{(1-c)^2(1-\gamma)^2 k^2 (1-4\gamma k)\ g(k,\gamma)}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\gamma + \left(-4\gamma^2+8\gamma+8\right)k - 3\right]^2},$$

where $g(k,\gamma) \equiv 16(3\gamma^4+5\gamma^3+16\gamma^2+22\gamma+8)k^3 - 12(5\gamma^3+5\gamma^2+8\gamma+6)k^2 + 24\gamma^2 k - 3\gamma + 3$. We show that $g(k,\gamma) > 0$ and sign$\{\Phi_N - \Phi_I\}$ depends only on $1-4\gamma k$. To prove $g(k,\gamma) > 0$, it is sufficient to show that $g(k,\gamma)$ has its minimum value at $k = k_0$ and $c = 1$, and that value is positive.

First, we show that $g(k,\gamma)$ is an increasing function of k; that is, $g(k,\gamma)$ is the smallest at $k = k_0$. The first derivative $g(k,\gamma)$ with respect to k is $\partial g(k,\gamma)/\partial k = 24[2(3\gamma^4+5\gamma^3+16\gamma^2+22\gamma+8)k^2 - (5\gamma^3+5\gamma^2+8\gamma+6)k + \gamma^2]$. The $\partial g(k,\gamma)/\partial k$ is a quadratic function of k and the coefficient of k^2 is positive. Hence, by solving $\partial g(k,\gamma)/\partial k > 0$ for k, we have $k < k_1$ and $k > k_2$, where k_1 and k_2 are roots in $g(k,\gamma) = 0$ on k and $k_1 < k_2$.

As $k_0 = 1/[2c(2\gamma+1)]$ decreases with c, k_0 has its minimum value at $c = 1$. We illustrate k_1, k_2, and k_0 at $c = 1$ in Figure I.1. Using numerical calculation, we find that for $\gamma \in [0,1]$, the unique root for $k_2 - k_0|_{c=1} = 0$ is $\gamma = 1$. Hence, $\partial g(k,\gamma)/\partial k > 0$ for any $k > k_0$. Therefore,

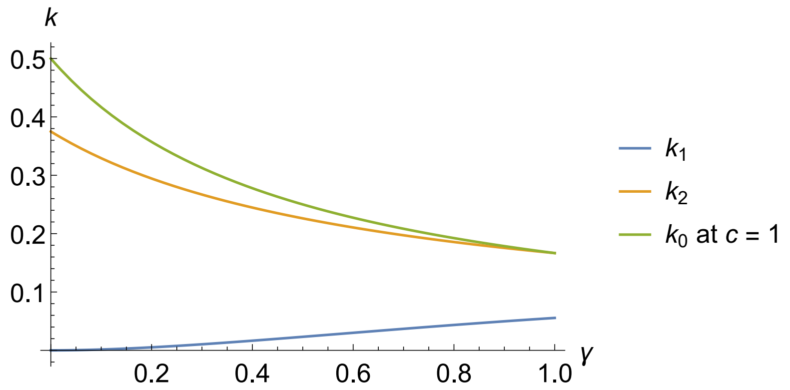

Figure I.1: k_0 at $c = 1$ and the two roots for $g(k, \gamma) = 0$

$g(k, \gamma)$ takes its minimum value at $k = k_0$.

Second, we show $\partial g(k_0, \gamma)/\partial c < 0$. Derivation yields $\partial g(k_0, \gamma)/\partial c = (\partial g(k_0, \gamma)/\partial k)(\partial k_0/\partial c) = (\partial g(k_0, \gamma)/\partial k)\left[-\frac{1}{2c^2(2\gamma+1)}\right] < 0$. The last inequality is satisfied because $\partial g(k, \gamma)/\partial k > 0$. Hence, $g(k_0, \gamma)$ is a decreasing function for c and it takes its minimum value when $c = 1$.

From the above discussion, $g(k_0, \gamma)$ has the following minimum value at $k = k_0$ and $c = 1$: $g(k_0, \gamma)\big|_{c=1} = \frac{(1-\gamma)^2(\gamma+1)}{(2\gamma+1)^3} > 0$. Because $g(k_0, \gamma)\big|_{c=1}$ is positive, $\forall k > k_0$, $g(k, \gamma) > 0$. This result implies that sign$\{\Phi_N - \Phi_I\}$ depends only on $1 - 4\gamma k$. Hence, $\Phi_N > \Phi_I$ iff $k < 1/(4\gamma)$. □

II Downstream Differentiated Bertrand

This section provides the equilibrium of the game in which downstream market competition is differentiated Bertrand. In this case, we also obtain a similar result as in the Cournot case. To identify Bertrand rivalry, we attach "ˆ" to the variables of the equilibrium solutions.

- *NN regime:* As $q_{n,i} = q_{n,j} = 0$ for $i \neq j$, the demand functions are $q_{e,i} = \frac{(1-\gamma)-p_{e,i}+\gamma p_{e,j}}{1-\gamma^2}$ and $q_{e,j} = \frac{(1-\gamma)-p_{e,j}+\gamma p_{e,i}}{1-\gamma^2}$. From these, we obtain $\hat{w}^{NN} = \frac{1-(c+1)(2-\gamma)(\gamma+1)k}{1-2(2-\gamma)(\gamma+1)k}$, $\hat{x}^{NN} = \frac{1-c}{2(2-\gamma)(\gamma+1)k-1}$, $\hat{\pi}_U^{NN} = \frac{(1-c)^2k}{2(2-\gamma)(\gamma+1)k-1}$, $\hat{p}_{e,i}^{NN} = \frac{1-(\gamma+1)k(c-2\gamma+3)}{2(\gamma^2-\gamma-2)k+1}$, and $\hat{\pi}_{Di}^{NN} = \frac{(1-c)^2(1-\gamma)(\gamma+1)k^2}{(2(\gamma^2-\gamma-2)k+1)^2}$.

- *IN (NI) regime:* Only Di innovates, so $q_{n,j} = 0$. The demand functions are $q_{e,i}=\frac{(1-\gamma)-(\gamma+1)p_{e,i}+\gamma(p_{n,i}+p_{e,j})}{(1-\gamma)(2\gamma+1)}$, $q_{n,i}=\frac{(1-\gamma)-(\gamma+1)p_{n,i}+\gamma(p_{e,i}+p_{e,j})}{(1-\gamma)(2\gamma+1)}$, and $q_{e,j}=\frac{(1-\gamma)-(\gamma+1)p_{e,j}+\gamma(p_{e,i}+p_{n,i})}{(1-\gamma)(2\gamma+1)}$. Solving the game, we obtain $\hat{w}^{IN} = \frac{2(c+1)(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$, $\hat{x}^{IN} = \frac{(1-c)(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$, $\hat{\pi}_U^{IN} = \frac{(1-c)^2(\gamma(\gamma+5)+3)k}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$, $\hat{p}_{e,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{p}_{n,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{p}_{e,j}^{IN} = \frac{-2(2\gamma+1)k(2\gamma c+c+2(1-\gamma)\gamma+3)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{\pi}_{Di}^{IN} = \frac{2(1-c)^2(1-\gamma)(2\gamma+1)(3\gamma+2)^2k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$, and $\hat{\pi}_{Dj}^{IN} = \frac{4(1-c)^2(1-\gamma)(\gamma+1)^3(2\gamma+1)k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$.

- *II regime:* When $D1$ and $D2$ develop new products, the demand functions are $q_{e,i} = \frac{(1-\gamma)-(2\gamma+1)p_{e,i}+\gamma(p_{n,i}+p_{n,j}+p_{e,j})}{(1-\gamma)(3\gamma+1)}$, $q_{e,j} = \frac{(1-\gamma)-(2\gamma+1)p_{e,j}+\gamma(p_{n,i}+p_{n,j}+p_{e,i})}{(1-\gamma)(3\gamma+1)}$,

$q_{n,i} = \frac{(1-\gamma)-(2\gamma+1)p_{n,i}+\gamma(p_{n,j}+p_{e,i}+p_{e,j})}{(1-\gamma)(3\gamma+1)}$, and $q_{n,j} = \frac{(1-\gamma)-(2\gamma+1)p_{n,j}+\gamma(p_{n,i}+p_{e,i}+p_{e,j})}{(1-\gamma)(3\gamma+1)}$. Solving the game, we obtain $\hat{w}^{II} = \frac{(c+1)(3\gamma+1)k-(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{x}^{II} = \frac{(1-c)(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{\pi}_U^{II} = \frac{(1-c)^2(\gamma+1)k}{(6\gamma+2)k-(\gamma+1)}$, $\hat{q}_{e,i}^{II} = \frac{(3\gamma+1)k(\gamma c+c-\gamma+3)-2(\gamma+1)}{4(3\gamma+1)k-2(\gamma+1)}$, and $\hat{\pi}_{Di}^{II} = \frac{(1-c)^2(1-\gamma)(\gamma+1)(3\gamma+1)k^2}{2(2(3\gamma+1)k-(\gamma+1))^2}$.

We next derive the best response. Given that Dj develops a new product, if $f < \hat{\Phi}_I$, then Di develops a new product; otherwise, it does not: $\hat{\Phi}_I \equiv (1-c)^2(\gamma-1)k^2\Big[\frac{\gamma+1}{(2(\gamma-2)(\gamma+1)k+1)^2} - \frac{2(2\gamma+1)(3\gamma+2)^2}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2}\Big]$.

Given that Dj does not develop a new product, if $f < \hat{\Phi}_N$, then Di develops a new product; otherwise, it does not: $\hat{\Phi}_N \equiv \frac{1}{2}(1-c)^2(\gamma-1)(\gamma+1)k^2\Big[\frac{8(\gamma+1)^2(2\gamma+1)}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2} - \frac{3\gamma+1}{(\gamma-2(3\gamma+1)k+1)^2}\Big]$. Figure B illustrates the equilibrium in downstream Bertrand competition.

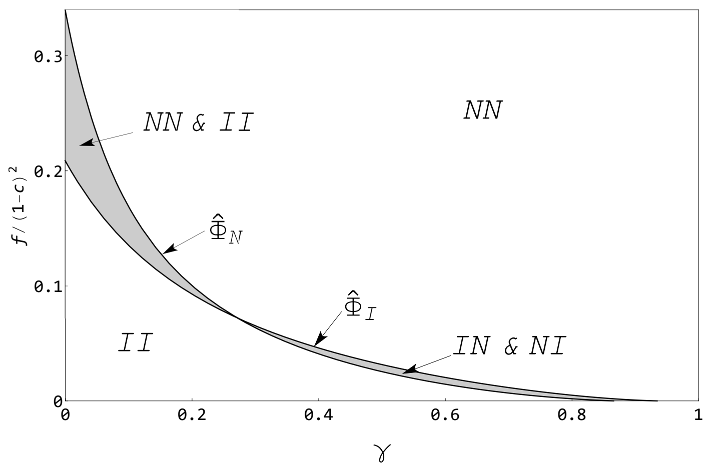

Figure B: Equilibrium in the γ-$f/(1-c)^2$ plane, where $k = 1$

III Welfare Analysis

We define consumer surplus and gross total surplus (excluding the downstream R&D cost f) as follows: $CS = \frac{1}{2}[q_{e,1}^2 + q_{e,2}^2 + q_{n,1}^2 + q_{n,2}^2] + \gamma[q_{e,1}(q_{e,2} + q_{n,1} + q_{n,2}) + q_{e,2}(q_{n,1} + q_{n,2}) + q_{n,1}q_{n,2}]$ and $TS = CS + \pi_U + \pi_{D1} + \pi_{D2}$. We have equilibrium surpluses: $CS^{II} = \frac{(c-1)^2(3\gamma+1)k^2}{2((4\gamma+2)k-1)^2}$, $TS^{II} = \frac{(c-1)^2k((13\gamma+7)k-2)}{2((4\gamma+2)k-1)^2}$, $CS^{IN} = \frac{(c-1)^2(\gamma^3-7\gamma^2+8\gamma+6)k^2}{(\gamma+(-4\gamma^2+8\gamma+8)k-3)^2}$, $TS^{IN} = \frac{(c-1)^2k((7\gamma^3-33\gamma^2+24\gamma+42)k-(\gamma-3)^2)}{(\gamma+(-4\gamma^2+8\gamma+8)k-3)^2}$, $CS^{NN} = \frac{(c-1)^2(\gamma+1)k^2}{(1-2(\gamma+2)k)^2}$, and $TS^{NN} = \frac{(c-1)^2k((3\gamma+7)k-1)}{(1-2(\gamma+2)k)^2}$. Note that $CS^{IN} = CS^{NI}$ and $TS^{IN} = TS^{NI}$.

A. Underinvestment in terms of consumer surplus

Comparing the consumer surpluses, we can show the following Result.

Result 1. *(i) Assume that "II" appears if the equilibrium regime is II and NN. Then, from the consumer surplus perspective, underinvestment downstream occurs if $f > \Phi_N$. (ii) Assume* *that "NN" appears if the equilibrium regime is II and NN. Then, from the consumer surplus perspective, underinvestment downstream occurs if* $f > \min\{\Phi_N, \Phi_I\}$.

Proof of Result 1. Case (i). From Proposition 2, the equilibrium regime is either "$IN\&NI$" or "NN" if $f > \Phi_N$; the equilibrium regime is "II" if $f < \phi_N$. Hence, to prove the first part of Result 1, we need only show that $CS^{II} > CS^{IN}(= CS^{NI}) > CS^{NN}$. This is because underinvestment occurs only if $f > \Phi_N$.

Case (ii). Applying a similar discussion, we find that underinvestment occurs only if $f > \min\{\Phi_N, \Phi_I\}$, where the equilibrium regime is also either "$IN\&NI$" or "NN". Hence, in both cases, if we show $CS^{II} > CS^{IN} > CS^{NN}$, the proof is complete.

First, we consider $\text{sign}\{CS^{II} - CS^{IN}\}$.

$$CS^{II} - CS^{IN} = \frac{(1-c)^2(1-\gamma)k^2\ \psi_1^{CS}}{2[(4\gamma+2)k-1)]^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2},$$

where $\psi_1^{CS} \equiv 8(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)k^2 - 8\gamma(2-\gamma^2)k - \gamma^2 + 2\gamma - 3$.

The $\text{sign}\{CS^{II} - CS^{IN}\}$ depends only on ψ_1^{CS}. Because ψ_1^{CS} is a quadratic function of k and the coefficient of k^2 is positive, $\psi_1^{CS} = 0$ has two roots, k_1^{CS} and k_2^{CS}. Solving $\psi_1^{CS} > 0$ for k, we obtain $k < k_1^{CS}$ or $k > k_2^{CS}$, where

$$k_1^{CS} \equiv \frac{2\gamma(2-\gamma^2)-\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)}; \quad k_2^{CS} \equiv \frac{2\gamma(2-\gamma^2)+\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)}.$$

Here, we compare k_2^{CS} with k_0. Let us consider the case $c = 1$. By using numerical calculation, we find that $\forall\gamma \in [0, 1)$, $k_0|_{c=1} > k_2^{CS}$. Because k_0 takes its minimum value at $c = 1$, $k_0 > k_2^{CS} > k_1^{CS}$ for any $c > 0$. Hence, $CS^{II} - CS^{IN} > 0$.

Next, we consider $CS^{IN} - CS^{NN}$ and apply a similar proof as the above.

$$CS^{IN} - CS^{NN} = -\frac{(c-1)^2(\gamma-1)k^2\ \psi_2^{CS}}{(1-2(\gamma+2)k)^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2},$$

where $\psi_2^{CS} \equiv 4(3\gamma^4 - 6\gamma^3 - 6\gamma^2 + 16\gamma + 8)k^2 - 4\gamma(\gamma^2 - 2\gamma + 2)k - 3 + 2\gamma$.

The $\text{sign}\{CS^{IN} - CS^{NN}\}$ depends only on ψ_2^{CS}. Because the coefficient of k^2 in ψ_2^{CS} is positive, by solving $\psi_2^{CS} > 0$ for k, we obtain $k < k_3^{CS}$ or $k > k_4^{CS}$, where

$$k_3^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)-(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)}; \quad k_4^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)+(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)}.$$

We show $k_0 > k_4^{CS}(> k_3^{CS})$. At $c = 1$, by using numerical calculation, we find that $\forall\gamma \in [0, 1)$, $k_0|_{c=1} > k_4^{CS}$. Because k_0 takes its minimum value at $c = 1$, for any $c > 0$, $k_0 > k_4^{CS} > k_3^{CS}$ holds. Therefore, $CS^{IN} - CS^{NN} > 0$. □

B. Underinvestment in terms of total surplus

Hereafter, we assume $k > \max\{1/2, k_0\}$.

To consider the best regime maximizing total surplus, we define the gross benefits of an increase in the number of downstream firms conducting R&D: $\Psi_{21}^{TS} \equiv TS^{II} - TS^{IN} = TS^{II} - TS^{NI}$, $\Psi_{10}^{TS} \equiv TS^{IN} - TS^{NN} = TS^{NI} - TS^{NN}$, and $\Psi_{20}^{TS} \equiv (TS^{II} - TS^{NN})/2$. More precisely, a rise in the number of downstream firms conducting R&D increases the total surplus if the following conditions are satisfied: $\Psi_{21}^{TS} > f$, $\Psi_{10}^{TS} > f$, or $2\Psi_{20}^{TS} > 2f$.

To provide the result for total surplus, we must compare the gross benefits of an increase in the number of downstream innovating firms. Here, we implicitly define $g^{TS}(\gamma, k)$, which has the same sign as $\Psi_{21}^{TS} - \Psi_{10}^{TS}$. That is, $k^{TS}(> \max\{1/2, k_0\})$ is the root of the following equation:

$$g^{TS}(\gamma, k) \equiv 64\gamma(9\gamma^4 + \gamma^3 - 88\gamma^2 - 106\gamma - 32)k^4 + 32(-15\gamma^4 + 20\gamma^3 + 142\gamma^2 + 112\gamma + 20)k^3 \\ +4(29\gamma^3 - 97\gamma^2 - 298\gamma - 126)k^2 - 4(\gamma^2 - 17\gamma - 26)k - 3 - \gamma = 0.$$

Note that we can depict all roots of $g^{TS}(\gamma, k) = 0$ as in Figure III.1. The blue curves are the set of pairs (γ, k) that satisfies $g^{TS}(\gamma, k) = 0$. The dashed line is $k = 1/2$. Hence, given γ, we can uniquely determine $k = k^{TS}$ as the largest root if it exists for $k > 1/2$. In the shaded area, $g^{TS}(\gamma, k) > 0$.

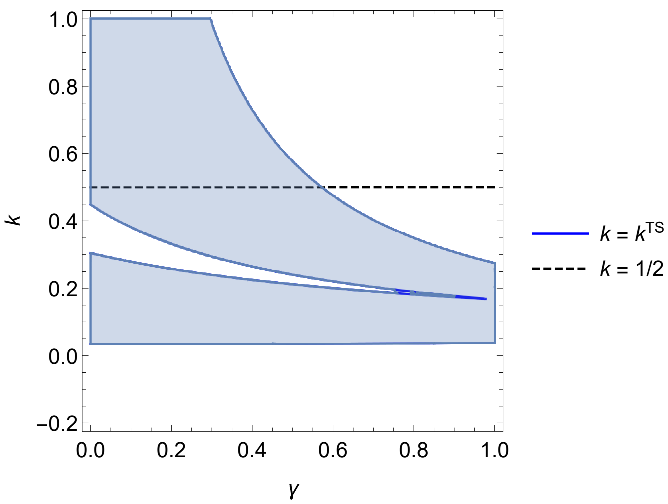

Figure III.1: Region of $g^{TS}(k, \gamma) > 0$

Then, comparing the total surpluses, we can show the condition under which underinvestment occurs.

Result 2. *We restrict the minimum value of k by $\max\{1/2, k_0\}$. (i) Assume that "NN" is realized if the equilibrium regimes are "II" and "NN". Then, from the total surplus perspective, the conditions under which underinvestment in downstream development occurs is given as* *follows:*

$$\begin{aligned}
&\Phi_I < f < \Psi_{20}^{TS} \quad \textit{if} \quad \min\{1/2, k_0\} < k \le 1/(4\gamma),\\
&\Phi_N < f < \Psi_{20}^{TS} \quad \textit{if} \quad \max\{1/2, k_0, 1/(4\gamma)\} < k \le k^{TS},\\
&\Phi_N < f < \Psi_{10}^{TS} \quad \textit{if} \quad \max\{1/2, k_0, k^{TS}, 1/(4\gamma)\} < k.
\end{aligned}$$

(ii) Assume that "II" is realized if the equilibrium regimes are "II" and "NN". Then, (ii) from the total surplus perspective, underinvestment in downstream development occurs if the following condition is satisfied:

$$\begin{aligned}
&\Phi_N < f < \Psi_{20}^{TS} \quad \textit{if} \quad \max\{1/2, k_0\} < k \le \max\{1/(4\gamma), k^{TS}\},\\
&\Phi_N < f < \Psi_{10}^{TS} \quad \textit{if} \quad \max\{1/(4\gamma), k^{TS}\} < k.
\end{aligned}$$

Proof of Result 2. In both regimes, we need to show, which regime maximizes total surplus given the downstream R&D cost f.

We consider case (i) and compare the gross benefits. First, we consider $\Psi_{21}^{TS} - \Psi_{10}^{TS}$.

$$\Psi_{21}^{TS} - \Psi_{10}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2 \; g^{TS}(k,\gamma)}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2 \left[\left(-4\gamma^2+8\gamma+8\right)k-3\gamma\right]^2}.$$

$\text{sign}\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$ depends only on $g^{TS}(\gamma,k)$. From Figure III.1, we obtain $g^{TS}(\gamma,k) > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

Next, we consider $\Psi_{21}^{TS} - \Psi_{20}^{TS}$.

$$\Psi_{21}^{TS} - \Psi_{20}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2 \; g^{TS}(\gamma,k)}{4[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2 \left[\left(-4\gamma^2+8\gamma+8\right)k-3+\gamma\right]^2}.$$

Hence, $\text{sign}\{\Psi_{21}^{TS} - \Psi_{20}^{TS}\}$ is the same as $\text{sign}\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$. Then, we obtain $\Psi_{21}^{TS} - \Psi_{10}^{TS} > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

Finally, we consider $\Psi_{20}^{TS} - \Psi_{10}^{TS}$.

$$\Psi_{20}^{TS} - \Psi_{10}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2 \; g^{TS}(\gamma,k)}{4[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2 \left[\left(-4\gamma^2+8\gamma+8\right)k-3+\gamma\right]^2}.$$

$\text{sign}\{\Psi_{20}^{TS} - \Psi_{10}^{TS}\}$ is also same as $\text{sign}\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$. Thus, $\Psi_{20}^{TS} - \Psi_{10}^{TS} > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

From these, we have the following ranking of thresholds:

$$\begin{aligned}
&\Psi_{10}^{TS} < \Psi_{20}^{TS} < \Psi_{21}^{TS} \quad \text{if} \quad \max\{1/2, k_0\} < k < k^{TS},\\
&\Psi_{21}^{TS} \le \Psi_{20}^{TS} \le \Psi_{10}^{TS} \quad \text{if} \quad k \ge k^{TS}.
\end{aligned} \tag{1}$$

Then, we establish Lemma 1.

Lemma 1. *(i) For* $\max\{1/2, k_0\} < k < k^{TS}$*, the best regime for total welfare is "II" if* $f \leq \Psi_{20}^{TS}$ *and "NN" if* $f > \Psi_{20}^{TS}$*. (ii) For* $k \geq k^{TS}$*, the best regime for total welfare is "II" if* $f \leq \Psi_{21}^{TS}$*, "IN" or "NI" if* $\Psi_{21}^{TS} < f \leq \Psi_{10}^{TS}$*, and "NN" if* $f > \Psi_{10}^{TS}$*.*

Comparing Ψ_{10}^{TS}, Ψ_{20}^{PS}, Ψ_{21}^{TS}, Φ_I, and Φ_N, we show the ranking of thresholds. First, we show that $\Phi_I < \min\{\Psi_{10}^{TS}, \Psi_{21}^{TS}\}$. The difference $\Psi_{10}^{TS} - \Phi_I$ yields

$$\Psi_{10}^{TS} - \Phi_I = \frac{(1-c)^2(1-\gamma)k^2\ g_{10,I}^{TS}}{[1-2(\gamma+2)k]^2\left[\left(-4\gamma^2+8\gamma+8\right)k-3+\gamma\right]^2},$$

where $g_{10,I}^{TS} \equiv 4(7\gamma^4 - 10\gamma^3 - 42\gamma^2 + 32\gamma + 40)k^2 - 4(4\gamma^3 - 11\gamma^2 - 6\gamma + 16)k + 2\gamma^2 - 7\gamma + 4$.

The $\text{sign}\{\Psi_{10}^{TS} - \Phi_I\}$ depends only on $g_{10,I}^{TS}$. Solving $g_{10,I}^{TS} = 0$ for k, we have two roots, $k_{10,I}^1$ and $k_{10,I}^2$, where $k_{10,I}^1 < k_{10,I}^2$. Since the coefficient of k^2 in $g_{10,I}^{TS}$ is positive, we have $g_{10,I}^{TS} > 0$ if $k < k_{10,I}^1$ or $k > k_{10,I}^2$. In addition, using numerical calculation, we can show that $k_{10,I}^1 < k_{10,I}^2 < 1/2$. As we assume $k > \max\{1/2, k_0\}$, we obtain $g_{10,I}^{TS} > 0$, which leads to $\Psi_{10}^{TS} - \Phi_I > 0$.

Next, we consider $\Psi_{21}^{TS} - \Phi_I$.

$$\Psi_{21}^{TS} - \Phi_I = \frac{(1-c)^2(1-\gamma)k^2\ g_{21,I}^{TS}}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\left(-4\gamma^2+8\gamma+8\right)k-3+\gamma\right]^2},$$

where $g_{21,I}^{TS} \equiv 32(10\gamma^6 + 4\gamma^5 - 23\gamma^4 - 14\gamma^3 + 98\gamma^2 + 128\gamma + 40)k^4 - 96(5\gamma^5 - 2\gamma^4 - 12\gamma^3 + 7\gamma^2 + 26\gamma + 12)k^3 + 4(65\gamma^4 - 78\gamma^3 - 87\gamma^2 + 118\gamma + 90)k^2 + 4(-15\gamma^3 + 28\gamma^2 + \gamma - 14)k + 5\gamma^2 - 12\gamma + 5$.

The $\text{sign}\{\Psi_{21}^{TS} - \Phi_I\}$ depends only on $g_{21,I}^{TS}$. To prove $\Psi_{21}^{TS} - \Phi_I > 0$, we show (i) $g_{21,I}^{TS} > 0$ at $k = 1/2$ and $\partial g_{21,I}^{TS}/\partial k > 0$, and (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$.

First, we show (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$.

$$\frac{\partial^2 g_{21,I}^{TS}}{\partial k^2} = 8\left[\begin{array}{l} 48\left(10\gamma^6+4\gamma^5-23\gamma^4-14\gamma^3+98\gamma^2+128\gamma+40\right)k^2 \\ -72\left(5\gamma^5-2\gamma^4-12\gamma^3+7\gamma^2+26\gamma+12\right)k+65\gamma^4-78\gamma^3-87\gamma^2+118\gamma+90 \end{array}\right].$$

Solving $\partial^2 g_{21,I}^{TS}/\partial k^2 = 0$ for k, we obtain two roots. Using numerical calculation, we can show that all roots are less than 1/2. Because the coefficient of k^2 in the equation $\partial^2 g_{21,I}^{TS}/\partial k^2$ is positive, and we assume $k > \max\{1/2, k_0\}$, $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$. Further, substituting $k = 1/2$ into $\partial g_{21,I}^{TS}/\partial k$, we have $\left(\partial g_{21,I}^{TS}/\partial k\right)\big|_{k=1/2} = 4(40\gamma^6 - 74\gamma^5 + 9\gamma^4 + 67\gamma^3 + 207\gamma^2 + 163\gamma + 20) > 0$. Thus, we obtain (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$, which leads to $\partial g_{21,I}^{TS}/\partial k > 0$ $\forall k > 1/2$.

Since we already had $\partial g_{21,I}^{TS}/\partial k > 0$, to prove (i) $g_{21,I}^{TS} > 0$ at $k = 1/2$ and $\partial g_{21,I}^{TS}/\partial k > 0$, we show only that $g_{21,I}^{TS} > 0$ at $k = 1/2$: $g_{21,I}^{TS}\big|_{k=1/2} = 20\gamma^6 - 52\gamma^5 + 43\gamma^4 + 8\gamma^3 + 86\gamma^2 + 52\gamma + 3 > 0$.

Therefore, $\forall k > 1/2$, we have $g^{TS}_{21,I} > 0$, which implies that $\Psi^{TS}_{21} - \Phi_I > 0$. From this and $\Psi^{TS}_{10} - \Phi_I > 0$, we obtain Lemma 2.

Lemma 2. $\Phi_I < \min\{\Psi^{TS}_{10}, \Psi^{TS}_{21}\}$.

Here, we show $\Psi^{TS}_{21} > \Phi_N$ and $\Psi^{TS}_{20} > \Phi_N$. We consider $\Psi^{TS}_{21} - \Phi_N$.

$$\Psi^{TS}_{21} - \Phi_N = \frac{(1-c)^2(1-\gamma)k^2\ g^{TS}_{21,N}}{[(4\gamma+2)k-1]^2\left[\left(-4\gamma^2+8\gamma+8\right)k+\gamma-3\right]^2},$$

where $g^{TS}_{21,N} \equiv 4(4\gamma^4 - 16\gamma^3 + 3\gamma^2 + 26\gamma + 10)k^2 - 4(2\gamma^3 - 7\gamma^2 + 3\gamma + 5)k + \gamma^2 - 3\gamma + 1$.

The sign$\{\Psi^{TS}_{21} - \Phi_N\}$ depends only on $g^{TS}_{21,N}$. Solving $g^{TS}_{21,N} = 0$ for k, we have two roots, $k^1_{21,N}$ and $k^2_{21,N}$, where $k^1_{21,N} < k^2_{21,N}$. The coefficient of k^2 in $g^{TS}_{21,N}$ is positive, so we have $g^{TS}_{21,N} > 0$ if $k < k^1_{21,N}$ or $k > k^2_{21,N}$. In addition, using numerical calculation, we can show that $k^1_{21,N} < k^2_{21,N} < 1/2$. Because we assume $k > \max\{1/2, k_0\}$, we obtain $g^{TS}_{21,N} > 0$, which leads to $\Psi^{TS}_{21} - \Phi_N > 0$.

Next, we consider $\Psi^{TS}_{20} - \Phi_N$.

$$\Psi^{TS}_{20} - \Phi_N = -\frac{(c-1)^2(\gamma-1)k^2\ g^{TS}_{20,N}}{4(1-2(\gamma+2)k)^2((4\gamma+2)k-1)^2\left(\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right)^2},$$

where $g^{TS}_{20,N} \equiv 64(13\gamma^6 - 8\gamma^5 - 134\gamma^4 - 44\gamma^3 + 200\gamma^2 + 176\gamma + 40)k^4 - 32(27\gamma^5 - 49\gamma^4 - 214\gamma^3 + 72\gamma^2 + 280\gamma + 100)k^3 + 12(27\gamma^4 - 78\gamma^3 - 131\gamma^2 + 140\gamma + 114)k^2 - 4(13\gamma^3 - 50\gamma^2 - 17\gamma + 54)k + 3\gamma^2 - 14\gamma + 7$.

The sign$\{\Psi^{TS}_{20} - \Phi_N\}$ depends only on $g^{TS}_{20,IN}$. To prove $\Psi^{TS}_{20} - \Phi_N > 0$, we show (i) $g^{TS}_{20,N} > 0$ at $k = 1/2$ and $\partial g^{TS}_{20,N}/\partial k > 0$; and (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$.

First, we show (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$.

$$\frac{\partial^2 g^{TS}_{20,N}}{\partial k^2} = 24\left[\begin{array}{l} 32(13\gamma^6 - 8\gamma^5 - 134\gamma^4 - 44\gamma^3 + 200\gamma^2 + 176\gamma + 40)k^2 \\ -8(27\gamma^5 - 49\gamma^4 - 214\gamma^3 + 72\gamma^2 + 280\gamma + 100)k + 27\gamma^4 - 78\gamma^3 - 131\gamma^2 + 140\gamma + 114 \end{array}\right].$$

Solving $\partial^2 g^{TS}_{20,N}/\partial k^2 = 0$ for k, we obtain two roots. Using numerical calculation, we can show that both roots are less than $1/2$. Because the coefficient of k^2 in $\partial^2 g^{TS}_{20,N}/\partial k^2$ is positive and we assume $k > \max\{1/2, k_0\}$, $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$. In addition, substituting $k = 1/2$ into $\partial g^{TS}_{20,N}/\partial k$, we have $\left(\partial g^{TS}_{20,N}/\partial k\right)\big|_{k=1/2} = 4(104\gamma^6 - 226\gamma^5 - 697\gamma^4 + 685\gamma^3 + 825\gamma^2 + 165\gamma + 8) > 0$. Therefore, we obtain (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$, which leads to $\partial g^{TS}_{20,N}/\partial k > 0$ for any $k > 1/2$.

Since we already had $\partial g^{TS}_{20,N}/\partial k > 0$, to prove (i) $g^{TS}_{20,N} > 0$ at $k = 1/2$ and $\partial g^{TS}_{20,N}/\partial k > 0$, we show only that $g^{TS}_{20,N} > 0$ at $k = 1/2$: $g^{TS}_{20,N}\big|_{k=1/2} = 52\gamma^6 - 140\gamma^5 - 259\gamma^4 + 420\gamma^3 + 222\gamma^2 + 24\gamma + 1 > 0$. Therefore, $\forall k > 1/2$, $g^{TS}_{20,N} > 0$, which implies that $\Psi^{TS}_{20} - \Phi_N > 0$. Hence, we obtain Lemma 3.

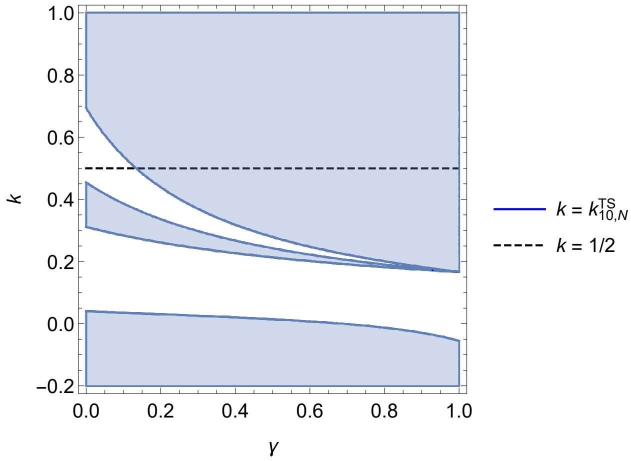

Figure III.2: Region of $g^{TS}_{10,N}(k,\gamma) > 0$

Lemma 3. $\Psi^{TS}_{21} > \Phi_N$ *and* $\Psi^{TS}_{20} > \Phi_N$.

Finally, we compare Ψ^{TS}_{10} to Φ_N.

$$\Psi^{TS}_{10} - \Phi_N = \frac{(1-c)^2(1-\gamma)k^2\ g^{TS}_{10,N}}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\left(-4\gamma^2+8\gamma+8\right)k+\gamma-3\right]^2},$$

where $g^{TS}_{10,N} \equiv 32(22\gamma^6-16\gamma^5-223\gamma^4-62\gamma^3+274\gamma^2+208\gamma+40)k^4-96(7\gamma^5-14\gamma^4-56\gamma^3+17\gamma^2+62\gamma+20)k^3+4(55\gamma^4-180\gamma^3-297\gamma^2+296\gamma+234)k^2-4(7\gamma^3-34\gamma^2-13\gamma+40)k+\gamma^2-8\gamma+5$.

Note that we can depict all roots for the above equation as in Figure III.2. The blue curves are the set of pairs (γ,k) that satisfies $g^{TS}_{10,N}(\gamma,k)=0$. The dashed line is $k=1/2$. Hence, given γ, we can implicitly define $k=k^{TS}_{10,N}$ as the largest root if it exists for $k>1/2$. In the shaded area, $g^{TS}_{10,N}(\gamma,k)>0$.

This result yields Lemma 4.

Lemma 4. $\Psi^{TS}_{10} > \Phi_N$ *if* $k > k^{T}_{10,N}$; $\Psi^{TS}_{10} \le \Phi_N$ *if* $\max\{1/2,k_0\} < k \le k^{T}_{10,N}$.

From Proposition 2 and Lemmas 1 and 4, we have three thresholds for k: $1/(4\gamma)$, k^{TS}, and $k^{TS}_{10,N}$. Depicting them, we have Figure III.3. Then, we obtain $k^{TS}_{10,N} < \min\{k^{TS},\, 1/(4\gamma)\}$. However, we cannot conclude that $k^{TS}_{10,N} < k^{TS}$ because $k^{TS}_{10,N}$ is implicitly defined and diverges to infinity as $\gamma \to 0$. Hence, we potentially have five regions: (i) $\max\{1/2,k_0\} < k \le k^{TS}_{10,N}$, (ii) $\max\{1/2,k_0,k^{TS}_{10,N}\} < k \le \min\{1/(4\gamma),k^{TS}\}$, (iii) $\max\{1/2,k_0,k^{TS}\} < k \le 1/(4\gamma)$, (iv) $\max\{1/2,k_0,1/(4\gamma)\} < k \le k^{TS}$, and (v) $\max\{1/2,k_0,k^{TS},1/(4\gamma)\} < k$. We depict each case in Figure III.4.

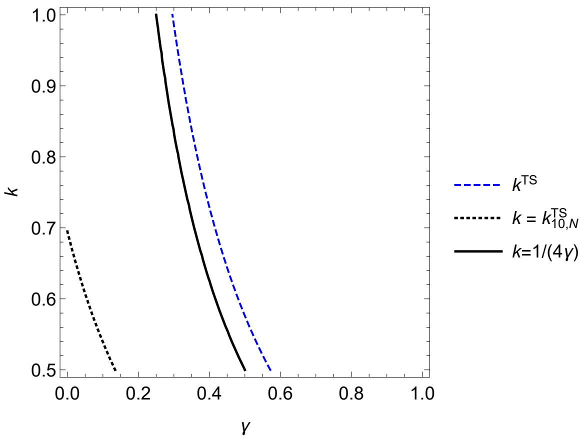

Figure III.3: Thresholds

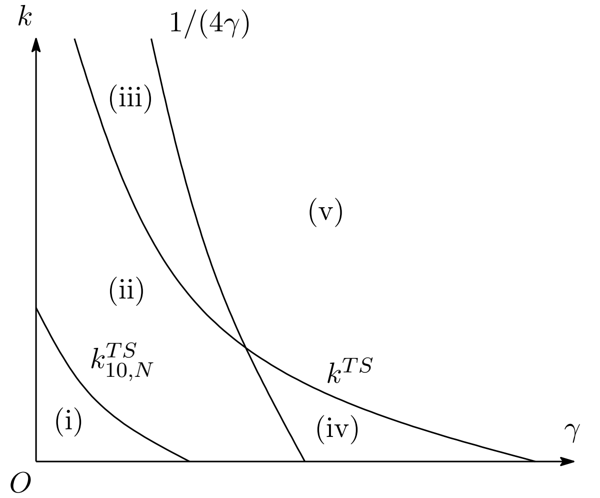

Figure III.4: Threshold ranking cases

From Proposition 2, Lemmas 2–4, and (1), we obtain the threshold ranking:

$$\begin{aligned}
&\Phi_I < \Psi_{10}^{TS} \le \Phi_N < \Psi_{20}^{TS} < \Psi_{21}^{TS} \text{ if (i) } \max\{1/2, k_0\} < k \le k_{10,N}^{TS},\\
&\Phi_I \le \Phi_N < \Psi_{10}^{TS} \le \Psi_{20}^{TS} \le \Psi_{21}^{TS} \text{ if (ii) } \max\{1/2, k_0, k_{10,N}^{TS}\} < k \le \min\{1/(4\gamma), k^{TS}\},\\
&\Phi_I \le \Phi_N < \Psi_{21}^{TS} < \Psi_{20}^{TS} < \Psi_{10}^{TS} \text{ if (iii) } \max\{1/2, k_0, k^{TS}\} < k \le 1/(4\gamma),\\
&\Phi_N < \Phi_I < \Psi_{10}^{TS} \le \Psi_{20}^{TS} \le \Psi_{21}^{TS} \text{ if (iv) } \max\{1/2, k_0, 1/(4\gamma)\} < k \le k^{TS},\\
&\Phi_N < \Phi_I < \Psi_{21}^{TS} < \Psi_{20}^{TS} < \Psi_{10}^{TS} \text{ if (v) } \max\{1/2, k_0, k^{TS}, 1/(4\gamma)\} < k.
\end{aligned} \tag{2}$$

Combining this ranking with Lemma 1 and Proposition 2, we can identify the condition for underinvestment downstream. Therefore, the proof is complete. □