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Abstract

It is well known that when a rival introduces a new product, a firm's response is affected by conflicting factors. For example, a certain factor stimulates firms to introduce their new products in a quick and retaliatory manner if their rivals introduce new products. Based on this fact, we build a simple vertical relation model: two downstream firms decide whether to introduce a horizontally differentiated new product, whereas a single upstream supplier invests in cost-reducing research and development (R&D). We show that the equilibrium of downstream innovation depends on upstream efficiency. If upstream R&D efficiency is high, downstream innovation is a strategic complement; this corresponds to the scenario in which downstream firms act in a retaliatory manner against their rivals introducing new products. Conversely, if upstream efficiency is low, downstream innovation is a strategic substitute: this implies that downstream firms behave passively when their rivals introduce new products. We also find that upstream R&D efficiency works similarly to the R&D spillover parameter in the d'Aspremont and Jacquemin's (1988) model. When R&D spillover is high (low), the firm's innovation behavior is a strategic complement (substitute). Hence, we offer a new insight into the innovation literature.

Keywords: Upstream cost-reducing R&D; New product introduction; Strategic complement

JEL classification: D43; L13; O31

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1 Introduction

When rivals introduce or develop new products, how do firms react?[1] *Reaction time* is an important indicator that is knowing their responses. In their investigation, Bowman and Gatignon reported that in response to rival firms introducing new products, 31.4% of firms introduced their new products in less than one year, whereas 28.7% of firms took a year or more to introduce their new products (Bowman and Gatignon, 1995, Table 1 on p.46.)

It is well known that a firm's decision regarding whether to introduce its new product within a shorter time and aggressively depends on multiple factors. For example, according to empirical analysis by Bowman and Gatignon (1995) and Kuester et al. (1999), the factor "market growth" increases a firm's market size and profitability, and hence makes its market more attractive; that is, these studies empirically showed that the factor "market growth" motivates firms to aggressively and quickly introduce their new products in response to their rivals introducing new products.[2] It has also been empirically shown that the factor "firm market share (market share of the introducing firm)" makes firms adopt a wait-and-see attitude regarding their rivals introducing new products. This is because, if firms aggressively and quickly introduce new products, the market share of their existing products may be partly diminished by such retaliatory action. Hence, in this case, firms do not aggressively countervail the introduction of their rivals' new products.

If firms aggressively and quickly introduce their new products in response to their rivals introducing new products, the behavior of firms implies *strategic complements*.[3] Conversely, if

[1]Studies that investigated a firm's response to a rival introducing a new product were conducted in the 1980s. For example, MacMillan et al. (1985) conducted a representative study.

[2]Debruyne et al. (2002) also empirically showed that the larger the degree of market growth, the higher the possibility of a firm's retaliatory act against its rivals introducing new products.

[3]Several researchers have used strategic complements and substitutes to assess the reaction time of decision-making by firms. For example, in the analysis of "merger waves," the strategic complement is often used to capture the quickness of mergers among firms; see Fauli-Oller (2000), Qiu and Zhou (2007), and Yao and Zhou

firms have wait-and-see attitude and take much longer to introduce new products in response to their rivals introducing new products by, the behavior of firms is passive and it equals to *strategic substitutes*. Whether the outcome of firm behavior that results from strategic interaction among firms becomes a strategic complement is important from the viewpoint of industrial development (or to create innovation waves); that is, if strategic complementarity works well, a firm's new product introduction causes another firm's new product introduction, and, as a result, innovation in the whole society rapidly improves.[4] Furthermore, it is also important from the viewpoint of consumer welfare because product varieties increase and consumer benefit can be enhanced because of strategic complementarity. However, whether the outcome of strategic interaction among firms is a strategic complement or strategic substitute tended to be overlooked in previous studies.

In this paper, we show that a firm's strategic behavior against its rival introducing a new product depends on upstream production efficiency. We offer a vertical oligopoly model in which a firm's new product introduction behavior can become both a strategic complement and substitute because of the degree of upstream efficiency. Whereas an upstream monopoly supplier invests in cost-reducing research and development (R&D),[5] downstream duopoly firms introduce or develop new horizontally differentiated products. The production efficiency of the upstream monopolist that makes an effort to reduce costs is equivalent to the efficiency of its R&D.

In the above scenario, suppose a downstream firm introduces a new product. This behavior

(2015).

[4] Strategic complementarity represents player's *self-fulfilling expectations* (e.g., the reason why I invest is that I expected others to invest). Similarly, in studies on industrialization and agglomeration, great importance is attached to the role of self-fulfilling expectations; see, for example, Krugman (1991) and Matsuyama (1991).

[5] Fontana and Guerzoni (2008) empirically found that firms tend to attach the most importance to cost-reducing R&D when their market size is large. In our setting, as downstream new product introduction increases product varieties and its market size, the demand for inputs increases and the market size of inputs also increases. Hence, we can consider that assuming cost-reducing R&D upstream is consistent with the empirical findings.

increases demand for the inputs, so the upstream supplier actively invests to reduce its production cost, and hence, the input price decreases. Because the price-cost margin of downstream firms widens as the input price lowers and results in a scenario similar to market expansion, the incentive of the rival firm to introduce a new product becomes strong. Introducing a new product steals a part of the rival's market share, so the rival's incentive to introduce a new product weakens. At this time, if upstream R&D efficiency is high, the input price drops rapidly because of the introduction of a new product. The fall of the input price facilitates the rival to introduce a new product, so the race to introduce a new product starts to have the nature of a strategic complement. Conversely, if upstream R&D efficiency is low, because the effect of stealing the rival's market share is dominant, the race to introduce a new product starts to have the nature of a strategic substitute. Bowman and Gatignon (1995) and Kuester et al. (1999) empirically found that when the (i) market size is large, or (ii) the firm has less of a possibility to lose its market share because of the introduction of a new product, retaliatory action (new product introduction) often occurs. Hence, our model is consistent with the results of the empirical study.

Our study also contributes to innovation literature. In most cases, d'Aspremont and Jacquemin (1988)-type process R&D models have an exogenous knowledge spillover among firms. If spillover from the rival's R&D is small, the investment behavior is a strategic substitute. If R&D spillover is large, the investment behavior is a strategic complement. Conversely, there is no R&D spillover in our model. We show that through the input price, upstream efficiency works similarly to the spillover parameter in a representative process R&D model; that is, if upstream efficiency is low (high), downstream investment behavior is a strategic substitute (complement). In our model, downstream R&D is the introduction of a new product and it differs from cost-reducing R&D. However, we newly demonstrate the theoretical fact that an efficiency parameter upstream of a vertical structure model works similarly to the spillover rate in a horizontal process R&D

framework.

This paper is related to two strands in the literature. One is studies on new product introduction in an oligopoly (Basak and Mukherjee, 2018)[6] and the other is studies that focus on upstream innovation in vertically related markets (e.g., Chen and Sapppington, 2010; Hu et al., 2020; Pinopoulos, 2020; Stefanadis, 1997). Basak and Mukherjee (2018) considered new product introduction in a unionized duopoly. They showed that in their many different settings, the strategic substitute equilibrium such that "one firm only introduces a new product" appears,[7] whereas the strategic complimentary equilibrium appears if and only if labor unions are firm specific (i.e., decentralized labor unions) and product differentiation is *asymmetric*. Our model shows that, when an upstream supplier engages in cost-reducing investment, both the strategic complementary equilibrium and strategic substitute equilibrium appear. This is in sharp contrast to the results of Basak and Mukherjee (2018).

Although some researchers have also focused on upstream process innovation, their models and purposes differ substantially from ours. Chen and Sappington (2010) considered the effects of vertical integration and separation on upstream innovation. Hu et al. (2020) considered upstream R&D, but their purpose was to examine the relationship between upstream cost-reducing investment and cross-holdings among downstream firms. Pinopoulos (2020) considered some types of input pricing behaviors, for example, two-part tariffs, by an upstream firm that engages in cost-reducing R&D. Stefanadis (1997) considered R&D competition between two

[6] Additionally, Dawid et al. (2010) considered new product development in a duopoly setting. However, their model has no upstream sector, and the R&D types differ between the two firms: one firm engages in a project of new product development and the other firm engages in cost-reducing R&D. Although Dobson and Waterson (1996) and Grossman (2007) also considered a similar scenario in which firms choose their number of differentiated goods, there is no upstream market in these models.

[7] This case is the same as the scenario in which "one is a multi-product firm and the other is a single product firm." Hence, the strategic substitute type of equilibrium in our new product introduction model includes that scenario. Inomata (2018) and Kawasaki et al. (2014) also considered the coexistence of multi-product and single-product firms.

upstream suppliers, and also examined the possibility that the conclusion of exclusive supply contracts with a downstream firm discourages upstream innovation.

This paper is structured as follows: In Section 2, we provide the basic model, and in Section 3, we present the analysis of the model. In Section 4, we perform welfare analysis, and in Section 5, we present downstream price competition. Finally, in Section 6, we draw conclusions.

2 Model

We consider a vertically related market with an upstream firm (U) and two symmetric downstream firms (Di, $i = 1, 2$). Di uses one unit of input to produce one unit of the final product, and it competes in Cournot fashion.[8] For simplicity, we omit other production costs for Di. U decides the input price w and makes a take-it-or-leave-it offer. For example, in the United States, the Robinson–Patman Act is enforced, so U charges a uniform input price for Di.[9]

U engages in R&D to reduce the constant marginal cost $c \in (0, 1)$. To create demand by introducing a new product, Di chooses whether to conduct R&D paying a fixed cost f (> 0). Let Di's existing product be $q_{e,i}$ and its new product be $q_{n,i}$. When $D1$ and $D2$ introduce new products, inverse demand is[10]

$$\begin{aligned} p_{e,i} &= 1 - q_{e,i} - \gamma(q_{n,i} + q_{e,j} + q_{n,j}), \\ p_{n,i} &= 1 - q_{n,i} - \gamma(q_{e,i} + q_{e,j} + q_{n,j}), \end{aligned} \tag{1}$$

where $p_{e,i}$ ($p_{e,j}$) is the price of the existing product of Di (Dj) and $p_{n,i}$ ($p_{n,j}$) is the price of the new product of Di (Dj), $i \neq j$ and $i, j = 1, 2$. The parameter γ ($0 \leq \gamma < 1$) measures the degree of product substitutability among final products. Final products are independent if

[8] Our main results do not alter in Bertrand competition. For more details, see Section 4.

[9] Even if U conducts price discrimination, our results do not alter.

[10] The other possible setting is that the existing and new products are differentiated. The formula in such a case is $p_{e,i} = 1 - (q_{e,i} + q_{e,j}) - \gamma(q_{n,i} + q_{n,j})$, $i, j = 1, 2$; $i \neq j$. However, our main results do not alter, so we use a simpler form (1).

$\gamma = 0$, whereas they are homogeneous if $\gamma = 1$.

The gross profit of Di is

$$\pi_{Di}(q_{e,i}, q_{n,i}) \equiv (p_{e,i} - w)q_{e,i} + (p_{n,i} - w)q_{n,i}. \tag{2}$$

If Di innovates, its profit is $\pi_{Di}(q_{e,i}, q_{n,i}) - f$; otherwise, its profit is $\pi_{Di}(q_{e,i}, 0)$.

The profit of U is

$$\pi_U \equiv (w - (c - x))Q - kx^2, \tag{3}$$

where x is the investment level and kx^2 is the R&D cost. k (> 0) denotes R&D efficiency. Q is the demand for input. $Q = \sum_i q_{e,i}$ if no one innovates. $Q = \sum_i q_{e,i} + q_{n,j}$ if only Dj innovates. $Q = \sum_i q_{e,i} + \sum_i q_{n,i}$ if everyone innovates.

We consider the following four-stage game. In the first stage, $D1$ and $D2$ independently and simultaneously choose whether to conduct R&D by paying the fixed cost (I) or not (N). In the second stage, U decides the investment level. At the third stage, U charges the input price. Finally, downstream competes *à la* Cournot.

This timing structure corresponds to the difficulty in R&D. Generally, product development requires a sunk cost, such as a long-term contract with researchers, and it takes a much longer time. Hence, downstream R&D is at the first stage. Effort that produces a prototype and repeatedly tests its safety is not required, so upstream R&D is at the second stage. Downstream can flexibly adjust its production, so the quantity of final products is decided in the final stage. The solution concept is the subgame perfect Nash equilibrium.

3 Results

Depending on the downstream investment decisions, four regimes can arise: II, IN, NI, and NN. Because downstream firms are symmetric, IN and NI are the same. We call II the *all product-developers regime*, IN and NI the *mixed regime*, and NN the *no one invests regime*.

Using (1)–(3), we obtain the equilibrium solutions for each regime.

No one invests regime: *NN*. Each Di only produces the existing product; hence, $q_{n,i} = 0$. Thus, we obtain the following equilibrium outcomes:

$$\begin{aligned} w^{NN} &= \frac{(1+c)(\gamma+2)k-1}{2(\gamma+2)k-1}; \quad x^{NN} = \frac{1-c}{2(\gamma+2)k-1}; \quad \pi_U^{NN} = \frac{(1-c)^2 k}{2(\gamma+2)k-1}, \\ q_{e,i}^{NN} &= \frac{(1-c)k}{2(\gamma+2)k-1}; \quad \pi_{Di}^{NN} = \left(q_{e,i}^{NN}\right)^2 \quad \text{for } i = 1,2. \end{aligned} \tag{4}$$

Mixed regime: *IN* or *NI*. When Di invests and Dj does not, because Di produces both existing and new products but Dj only produces its existing product, $q_{e,i} > 0$, $q_{n,i} > 0$, $q_{e,j} > 0$, and $q_{n,j} = 0$ ($i, j = 1, 2$ and $i \neq j$). From this we obtain the following:

$$\begin{aligned} x^{IN} &= x^{NI} = \frac{(1-c)(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ w^{IN} &= w^{NI} = \frac{2(1+c)(2+2\gamma-\gamma^2)k-(3-\gamma)}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad \pi_U^{IN} = \frac{(1-c)^2(3-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ q_{e,1}^{IN} &= q_{n,1}^{IN} = \frac{(1-c)(2-\gamma)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}; \quad q_{e,2}^{IN} = \frac{2(1-c)k}{4(2+2\gamma-\gamma^2)k-(3-\gamma)}, \\ \pi_{D1}^{IN} &= \frac{2(1-c)^2(2-\gamma)^2(1+\gamma)k^2}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2}; \quad \pi_{D2}^{IN} = \left(q_{e,2}^{IN}\right)^2. \end{aligned} \tag{5}$$

Note that $q_{e,2}^{NI} = q_{n,2}^{NI} = q_{e,1}^{IN} = q_{n,1}^{IN}$, $q_{e,2}^{IN} = q_{e,1}^{NI}$, $\pi_{D2}^{NI} = \pi_{D1}^{IN}$, and $\pi_{D1}^{NI} = \pi_{D2}^{IN}$.

All product-developers regime: *II*. Four differentiated products are produced in this regime, so we obtain the following equilibrium outcomes:

$$\begin{aligned} w^{II} &= \frac{(1+c)(2\gamma+1)k-1}{(4\gamma+2)k-1}; \quad x^{II} = \frac{1-c}{(4\gamma+2)k-1}; \quad \pi_U^{II} = \frac{(1-c)^2 k}{(4\gamma+2)k-1}, \\ q_{e,i}^{II} &= q_{n,i}^{II} = \frac{(1-c)k}{2\left[(4\gamma+2)k-1\right]}; \quad \pi_{Di}^{II} = \frac{(1-c)^2(\gamma+1)k^2}{2\left[(4\gamma+2)k-1\right]^2} \quad \text{for } i = 1,2. \end{aligned} \tag{6}$$

To ensure the positive marginal cost of U after investment, we need Assumption 1.

Assumption 1. $k > k_0 \equiv \dfrac{1}{2c(1+2\gamma)}$.

We establish Proposition 1 from Equations (4)–(6).

Proposition 1. *(i) The equilibrium level of the R&D investment of U is largest in the all product-developers regime, intermediate size in the mixed regime, and smallest in the no one invests regime. More precisely,* $x^{II} > x^{IN} = x^{NI} > x^{NN}$. *(ii) In all regimes, higher R&D efficiency of U increases its investment level. Higher product substitutability decreases the level of R&D investment of U. More precisely,* $\partial x^r/\partial k < 0$ *and* $\partial x^r/\partial \gamma < 0$, *where* $r = II, IN, NI, NN$.

Proof. (i) $x^{II} - x^{IN} = \frac{2k(1-c)(1-\gamma)}{L_N} > 0$ and $x^{IN} - x^{NN} = \frac{2k(1-c)(1-\gamma)(2-\gamma)}{L_I} > 0$, where $L_N \equiv [(4\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]$ and $L_I \equiv [2(\gamma+2)k-1][4(2+2\gamma-\gamma^2)k-(3-\gamma)]$. (ii) The partial derivative of x^r with respect to k yields $\partial x^{II}/\partial k = -\frac{2(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial k = -\frac{4(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial k = -\frac{2(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} < 0$. The partial derivative of x^r with respect to γ yields $\partial x^{II}/\partial \gamma = -\frac{4(1-c)k}{[(4\gamma+2)k-1]^2} < 0$, $\partial x^{IN}/\partial \gamma = -\frac{4(1-c)(\gamma^2-6\gamma+8)k}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} < 0$, and $\partial x^{NN}/\partial \gamma = -\frac{2(1-c)k}{[2(\gamma+2)k-1]^2} < 0$. □

Proposition 1 immediately yields Corollary 1.

Corollary 1. *(i)* $w^{NN} > w^{IN} = w^{NI} > w^{II}$. *(ii)* $\partial w^r/\partial k > 0$ *and* $\partial w^r/\partial \gamma > 0$, *where* $r = II, IN, NI, NN$.

Proof. (i) $w^{NN} - w^{IN} = \frac{k(1-c)(2-\gamma)(1-\gamma)}{L_I} > 0$ and $w^{IN} - w^{II} = \frac{k(1-c)(1-\gamma)}{L_N} > 0$. (ii) The partial derivative of w^r with respect to k yields $\partial w^{II}/\partial k = \frac{(1-c)(2\gamma+1)}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial k = \frac{2(1-c)(3-\gamma)(2+2\gamma-\gamma^2)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial k = \frac{(1-c)(\gamma+2)}{[2(\gamma+2)k-1]^2} > 0$. The partial derivative of w^r with respect to γ yields $\partial w^{II}/\partial \gamma = \frac{2(1-c)k}{[(4\gamma+2)k-1]^2} > 0$, $\partial w^{IN}/\partial \gamma = \frac{2(1-c)k(4-\gamma)(2-\gamma)}{[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0$, and $\partial w^{NN}/\partial \gamma = \frac{(1-c)k}{[2(\gamma+2)k-1]^2} > 0$. □

The logic behind part (i) of Proposition 1 is as follows: As U engages in cost-reducing investment, it invests a large amount if it can sell a large amount of input. Innovation by Di increases the number of product varieties, so the demand for the input also expands. If $D1$ and $D2$ innovate, because the input demand is the largest among all regimes and the sales opportunity for inputs is similarly the largest, the investment level becomes the largest. Hence, when no one innovates, the investment size becomes the least among all regimes. If only Di innovates, the investment level becomes intermediate.

Part (ii) is intuitive. The first result is natural. Although larger γ makes competition tougher, in our model, it reduces downstream market size. The latter effect is dominant, so the input demand shrinks. This impedes upstream investment.

Proposition 1 immediately yields Corollary 1. Since a larger investment corresponds to a lower input price, we obtain the ranking of the input price. Part (ii) is a natural one. The effects of γ are similar to those in part (ii) of Proposition 1.

Our model has a similar timing structure to those of Banerjee and Lin (2003), Gilbert and Cvsa (2003), and Haucap and Wey (2004); that is, in their models, the downstream firm (in our model, Di) first executes cost-reducing R&D investment, and after observing it, the upstream agent (in our model, U) charges its price. Banerjee and Lin (2003), Gilbert and Cvsa (2003), and Haucap and Wey (2004) emphasized that the increasing upstream price extracts the benefits of downstream innovation.[11] Conversely, in our study, because market expansion that results from downstream R&D promotes upstream investment, the input price falls. Corollary 1 implies that upstream R&D is very influential in vertical structures.

To derive the equilibrium of the game, we introduce two threshold functions Φ_I and Φ_N: $\Phi_I \equiv \pi_{D1}^{IN} - \pi_{D1}^{NN} = \pi_{D2}^{NI} - \pi_{D2}^{NN}$ and $\Phi_N \equiv \pi_{D1}^{II} - \pi_{D1}^{NI} = \pi_{D2}^{II} - \pi_{D2}^{IN}$.[12] These thresholds are related to the gain (or loss) that results from the deviation from equilibrium regimes NN and II, respectively.

Φ_I and Φ_N are given by

[11] Banerjee and Lin (2003) showed that a fixed-price contract that uses the input price resolves this hold-up problem. Conversely, Takauchi and Mizuno (2019) demonstrated that a fixed-price contract can harm upstream and downstream.

[12] Chowdhury (2005) defined Φ_I as a *non-strategic benefit* of R&D and Φ_N as a *strategic benefit* of R&D.

$$\Phi_N \equiv \frac{(1-c)^2(1-\gamma)k^2\left[\begin{array}{l} 1+4\gamma-\gamma^2+16(2+6\gamma+6\gamma^2+2\gamma^3-\gamma^4)k^2 \\ -8(2+4\gamma+3\gamma^2-\gamma^3)k \end{array}\right]}{2[(4\gamma+2)k-1]^2[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0 \quad \text{and}$$

$$\Phi_I \equiv \frac{(1-c)^2(1-\gamma)k^2[8(8+8\gamma-4\gamma^4)k^2-8(2+2\gamma+\gamma^2-\gamma^3)k-2\gamma^2+5\gamma-1]}{[2(\gamma+2)k-1]^2[4(2+2\gamma-\gamma^2)k-(3-\gamma)]^2} > 0.$$

Di innovates if $f < \Phi_I$ and does not innovate if $\Phi_N < f$. Hence, the mixed regime, $IN\&NI$, can appear if $\Phi_N < \Phi_I$; and the complementary equilibrium, $NN\&II$, can appear if $\Phi_I < \Phi_N$. These arguments yield Proposition 2.

Proposition 2.

1. Suppose that $k \in (k_0, 1/(4\gamma))$. Then, $\Phi_I < \Phi_N$. (i) If $f < \Phi_I$, II appears; (ii) if $f > \Phi_N$, NN appears; and (iii) if $\Phi_I \leq f \leq \Phi_N$, $NN\&II$ can appear.

2. Suppose $k > 1/(4\gamma)$ or $1/(4\gamma) \leq k_0$. Then, $\Phi_N < \Phi_I$. (i) If $f < \Phi_N$, II appears; (ii) if $f > \Phi_I$, NN appears; and (iii) if $\Phi_N \leq f \leq \Phi_I$, $IN\&NI$ can appear.

Proof. See Appendix A.

When the fixed cost f is small (large) because I (N) is the dominant strategy, II (NN) appears. If f is an intermediate size, Di's strategy depends on upstream R&D efficiency k: (i) if k is small, $NN\&II$ can appear; and (ii) if k is large, $IN\&NI$ can appear (see Figure 1).

The intuition behind this is as follows: (i) When k is small, upstream R&D is efficient. In this case, if Di deviates from II, its market halves. Furthermore, the input price jumps, so Di does not deviate from II. If Di deviates from NN, the number of product markets increases. As upstream R&D efficiency is high and the range of the drop in input price is larger, downstream production costs largely fall. However, this promotes the rival's production and makes competition in the existing product market tougher, so the benefit of R&D can be canceled. Di does not deviate.

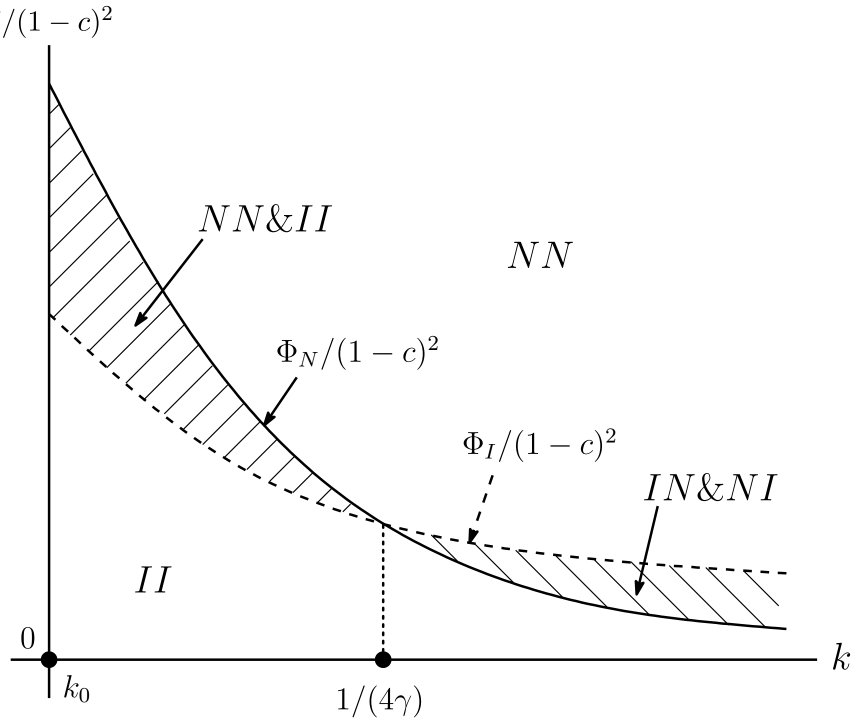

Figure 1: Equilibrium of the new product introduction game in $(k, \frac{f}{(1-c)^2})$-space ($k_0 < 1/(4\gamma)$)

Basak and Mukherjee (2018) found that, in a unionized duopoly, the emergence of the complementary equilibrium needs *asymmetric product differentiation* and *decentralized unions*. Conversely, we show that the complementary equilibrium appears even if there is no asymmetry in product differentiation. This implies that upstream R&D plays an important role in downstream innovation, and therefore, gives a new insight into the literature.

(ii) When k is large (or $1/(4\gamma) \leq k_0$), because upstream R&D is inefficient, the effects of upstream investment on the input price weakens. If Di deviates from II, its product market halves and input price rises. However, in this case, the input price is relatively high because of the large k (see Corollary 1). Thus, the production cost is also high, and the profit from the new product market becomes relatively small; that is, the R&D benefit is relatively small. As a deviation from II raises the input price, it also increases the rival's production cost. This results in a lessening of competition in the existing product market. Because the R&D benefit is small and competition in the existing product market loosens, Di has an incentive to choose N

if the rival chooses I. The deviation from NN increases the sales market of products and lowers the input price. Then, although the R&D benefit is small, the input price is high because k is large. Hence, a fall in production cost through the decrease in input price becomes attractive. Di chooses I when the rival chooses N.

We believe that our model makes two types of contributions to the literature. The first is that it offers results that are partially consistent with the results of an empirical study in the marketing area. As Bowman and Gatignon (1995) and Kuester et al. (1999) empirically showed, the factor "market growth" strengthens a firm's incentive to introduce a new product in a retaliatory manner; hence, it produces the equilibrium of strategic complementarity. Simultaneously, the factor "firm market share" strengthens the firm's incentive to wait and see; hence, it generates the equilibrium of strategic substitutability. In our model, when the parameter k is small, and, thus, upstream is efficient, the market expanding effect caused by introducing a new product is strong. This corresponds to the scenario in which the effects of the factor "market growth" are stronger. Conversely, when k is large, and, thus, the upstream is less efficient, the snatching effect of the existing-product market caused by introducing a new product becomes stronger. This is related to the scenario in which the effect of the factor "firm market share" is strong. Hence, our model undoubtedly has empirical relevance to marketing studies.

The second contribution is that our model offers a new finding with respect to the works of variables between the traditional R&D model and simple vertical structure model. To the best of our knowledge, one of the most popular models for cost-reducing R&D investment is the d'Aspremont and Jacquemin (1988) model, in which the R&D spillover rate β is an exogenous variable. More specifically, in that model, if A (> 0) is defined as a constant marginal cost of R&D firms, the cost function of firm i is defined as $C_i(q_i, x_i, x_j) = [A - x_i - \beta x_j]q_i$, $i, j = 1, 2$, $i \neq j$ with $0 < \beta < 1$. Then, the best response function is a strategic substitute (complement) if the spillover rate β is small (large).[13]

[13]Henriques (1990) showed that in the d'Aspremont and Jacquemin (1988) model, each firm's best response to

Conversely, in our model, there is no term related to R&D spillover "$-\beta x_j$." However, by way of the work in upstream input price w, if k is small (or upstream is efficient), strategic complementarity appears in downstream innovation. This corresponds to the case in which the spillover rate β is large in the d'Aspremont and Jacquemin (1988) model. Conversely, if k is large (or upstream is less efficient), strategic substitutability appears in downstream innovation. This corresponds to the case in which the spillover rate β is small in the d'Aspremont and Jacquemin model.

While d'Aspremont and Jacquemin (1988) considered process R&D, the downstream firm in our model conducts product R&D (new product introduction), so these two models consider different types of R&D. However, from the viewpoint of promotion or the restriction factor of innovation, the variable k in our model works similarly to the spillover rate β in the d'Aspremont and Jacquemin model. Because we newly found such an essential fact, our main result offers a new insight into the study of innovation activities.

4 Welfare analysis

Even if a downstream firm innovates, it will not capture all the surplus it generates. Therefore, if the innovation cost f is large, downstream firms give up on introducing new products, and underinvestment (i.e., welfare loss) occurs. In this section, we provide the conditions that can cause underinvestment.

To avoid unnecessary algebraic complexity and facilitate our welfare analysis, we must set a lower limiting value for k. Therefore, we make Assumption 2.

Assumption 2. $k \geq \dfrac{1}{2}$.

First, we discuss underinvestment in terms of consumer surplus. We define consumer surplus

the R&D level is a strategic complement (substitute) if $\beta > 1/2$ ($\beta < 1/2$).

and gross total surplus (excluding downstream R&D cost f) as follows:

$$CS = \frac{q_{e,1}^2 + q_{e,2}^2 + q_{n,1}^2 + q_{n,2}^2}{2} + \gamma \left[q_{e,1}(q_{e,2} + q_{n,1} + q_{n,2}) + q_{e,2}(q_{n,1} + q_{n,2}) + q_{n,1}q_{n,2} \right]$$

and $TS = CS + \pi_U + \sum_i \pi_{Di}$.

We have the following equilibrium surpluses: in the all product-developers regime,

$$CS^{II} = \frac{(1-c)^2(3\gamma+1)k^2}{2[(4\gamma+2)k-1]^2}; \quad TS^{II} = \frac{(1-c)^2 k[(13\gamma+7)k-2]}{2[(4\gamma+2)k-1]^2}.$$

In the mixed regime,

$$CS^{IN} = \frac{(1-c)^2(\gamma^3 - 7\gamma^2 + 8\gamma + 6)k^2}{[\gamma + (-4\gamma^2 + 8\gamma + 8)k - 3]^2},$$
$$TS^{IN} = \frac{(1-c)^2 k[(7\gamma^3 - 33\gamma^2 + 24\gamma + 42)k - (\gamma-3)^2]}{[\gamma + (-4\gamma^2 + 8\gamma + 8)k - 3]^2}.$$

Note that $CS^{IN} = CS^{NI}$ and $TS^{IN} = TS^{NI}$.

In the no one invests regime,

$$CS^{NN} = \frac{(1-c)^2(\gamma+1)k^2}{[1-2(\gamma+2)k]^2}; \quad TS^{NN} = \frac{(1-c)^2 k[(3\gamma+7)k-1]}{[1-2(\gamma+2)k]^2}.$$

By comparing the consumer surpluses, we obtain Result 1.

Result 1. *(i) Assume that "II" appears if the equilibrium regime is II and NN. Then, from the viewpoint of consumer surplus, underinvestment in downstream occurs if $f > \Phi_N$. (ii) Assume that "NN" appears if the equilibrium regime is II and NN. Then, from the viewpoint of consumer surplus, underinvestment in downstream occurs if $f > \min\{\Phi_N, \Phi_I\}$.*

Proof. See Appendix B.

Consumers always welcome an increase in the variety of goods they consume. Moreover, from Proposition 2, equilibrium regime II is not realized when the R&D cost f is large. Therefore, the consumer surplus is not maximized if f is large.

Next, we discuss underinvestment in terms of total surplus. We assume $k > \max\{1/2, k_0\}$. To consider the best regime that maximizes the total surplus, we define the gross benefits of an increase in the number of downstream firms conducting R&D: $\Psi_{21}^{TS} \equiv TS^{II} - TS^{IN} = TS^{II} - TS^{NI}$, $\Psi_{10}^{TS} \equiv TS^{IN} - TS^{NN} = TS^{NI} - TS^{NN}$, and $\Psi_{20}^{TS} \equiv (TS^{II} - TS^{NN})/2$. More precisely, a rise in the number of downstream firms conducting R&D increases the total surplus if the following conditions are satisfied: $\Psi_{21}^{TS} > f$, $\Psi_{10}^{TS} > f$, or $\Psi_{20}^{TS} > f$.

To provide the result for the total surplus, we need to compare the gross benefits of an increase in the number of downstream innovating firms. We define $g^{TS}(\gamma, k)$, which has the same sign as $\Psi_{21}^{TS} - \Psi_{10}^{TS}$. Additionally, we implicitly define $k^{TS}(> \max\{1/2, k_0\})$, which is the largest root of the following equation:

$$g^{TS}(\gamma, k) \equiv 64\gamma(9\gamma^4+\gamma^3-88\gamma^2-106\gamma-32)k^4 + 32(-15\gamma^4+20\gamma^3+142\gamma^2+112\gamma+20)k^3 \\ +4(29\gamma^3-97\gamma^2-298\gamma-126)k^2 - 4(\gamma^2-17\gamma-26)k - 3 - \gamma = 0.$$

Note that all roots of $g^{TS}(\gamma, k) = 0$ are depicted in Figure 2. The blue curves represent the set of pair (γ, k), which satisfies $g^{TS}(\gamma, k) = 0$. The dashed line represents $k = 1/2$. In the shadow area, $g^{TS}(\gamma, k) > 0$.

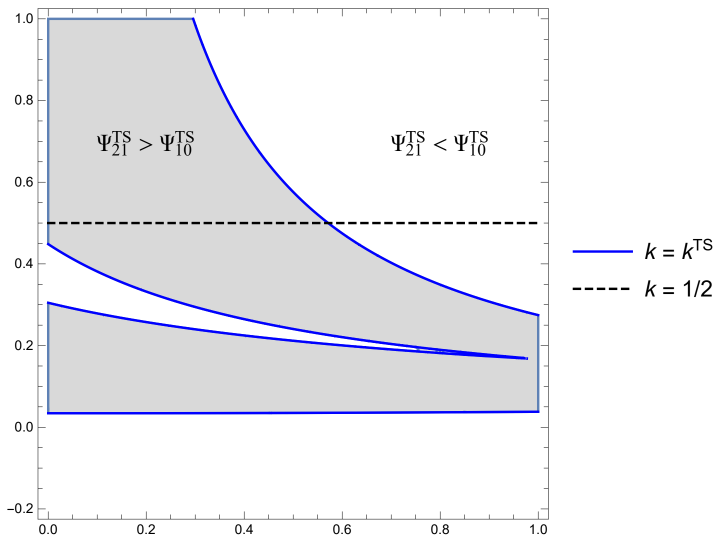

Figure 2: Region for $g^{TS}(k, \gamma) > 0$

Then, by comparing the total surpluses, we can show the condition under which underinvestment occurs.

Result 2. *Suppose $k > \max\{1/2, k_0\}$. (i) Assume that "NN" is realized if the equilibrium regimes are "II" and "NN." Then, from the viewpoint of the total surplus, the condition under which underinvestment for downstream development occurs is given as follows:*

$$\Phi_I < f < \Psi_{20}^{TS} \quad \text{if} \quad \max\{1/2, k_0\} < k \le 1/(4\gamma),$$
$$\Phi_N < f < \Psi_{20}^{TS} \quad \text{if} \quad \max\{1/2, k_0, 1/(4\gamma)\} < k \le k^{TS},$$
$$\Phi_N < f < \Psi_{10}^{TS} \quad \text{if} \quad \max\{1/2, k_0, k^{TS}, 1/(4\gamma)\} < k.$$

(ii) Assume that "II" is realized if the equilibrium regimes are "II" and "NN." Then, from the viewpoint of the total surplus, underinvestment for downstream development occurs if the following condition is satisfied:

$$\Phi_N < f < \Psi_{20}^{TS} \quad \text{if} \quad \max\{1/2, k_0\} < k \le \max\{1/(4\gamma), k^{TS}\},$$
$$\Phi_N < f < \Psi_{10}^{TS} \quad \text{if} \quad \max\{1/(4\gamma), k^{TS}\} < k.$$

Proof. See Appendix C.

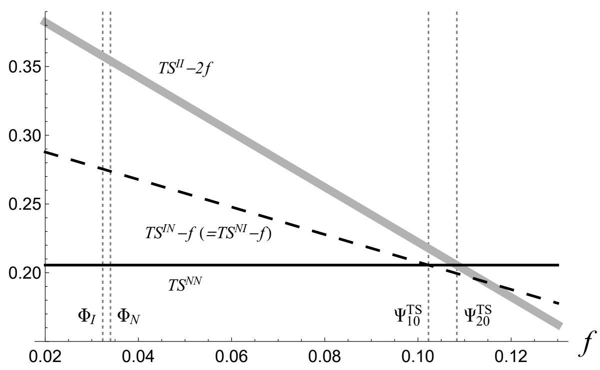

Figure 3: Total welfare comparison: $\boldsymbol{k = 1}$

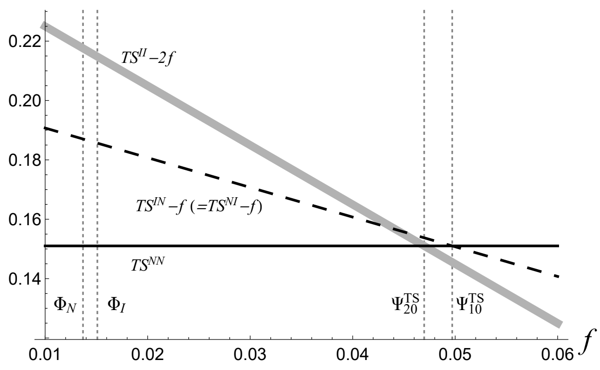

Figure 4: Total welfare comparison: $\boldsymbol{k = 5}$

Note: In both Figures 3 and 4, $c = 2/5$ and $\gamma = 1/5$.

We depict the results in Figures 3 and 4, in which the horizontal axis represents the fixed cost f of introducing a new product. For Figure 3 (or Figure 4), we assume $c = 2/5$, $k = 1$, and $\gamma = 1/5$ (or $2/5$, $k = 5$, and $\gamma = 1/5$). Each figure has three lines. The bold gray, dashed black, and solid black lines represent $TS^{II} - 2f$, $TS^{IN} - f(= TS^{NI} - f)$, and TS^{NN}, respectively. Each figure has four vertical dotted lines: Φ_I, Φ_N, Ψ_{10}^{TS}, and Ψ_{20}^{TS}. From Proposition 2, Φ_I and Φ_N determine the investment decision for downstream firms. Additionally, at $f = \Psi_{10}^{TS}$ and $f = \Psi_{20}^{TS}$, $TS^{IN} - f = TS^{NI} - f = TS^{NN}$ and $TS^{II} - 2f = TS^{NN}$, respectively.

We explain the intuition behind Result 2. First, we consider a case with small f. At $f = 0$, downstream firms always invest. Additionally, the total surplus increases with the number of investing downstream firms because downstream investment increases final demand. From the continuity of the total surplus function, both $D1$ and $D2$ invest, and this investment decision is socially optimal if f is small.

Second, we consider a case with large f. When no downstream firm invests, the total surplus is independent of f; when at least one downstream firm invests, the total surplus decreases with f. Hence, we obtain two threshold values: Ψ_{10}^{TS} and Ψ_{20}^{TS}. Because from Proposition 2, no downstream firm invests if f is large, the equilibrium number of investing downstream firms is socially optimal.

Finally, we consider a case in which f takes an intermediate value. In our model, $\max\{\Phi_I, \Phi_N\} < \max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\}$. Hence, for any $\max\{\Phi_I, \Phi_N\} < f < \max\{\Psi_{10}^{TS}, \Psi_{20}^{TS}\}$, no downstream firm invests while it is socially desirable for both downstream firms to invest. In the case with $\Phi_I < f < \Phi_N$ in Figure 3, equilibrium investment decisions are $II\&NN$. Hence, whether the underinvestment of downstream firms occurs depends on the manner in which the equilibrium is refined. If NN (or II) is realized under $\Phi_I < f < \Phi_N$, downstream investment is insufficient (or socially optimal).

5 Downstream price competition

In this section, we discuss the case in which downstream firms compete on price to show the robustness of the main result in the previous sections. In differentiated Bertrand competition, the demand function depends on whether downstream firms introduce a new product.

No one invests regime: NN. In this regime, the demand function is

$$q_{e,i} = \frac{(1-\gamma) - p_{e,i} + \gamma p_{e,j}}{1-\gamma^2}; \quad q_{e,j} = \frac{(1-\gamma) - p_{e,j} + \gamma p_{e,i}}{1-\gamma^2}, \quad i \neq j. \tag{7}$$

Mixed regime: IN or NI. As only Di introduces a new product, the demand function is

$$\begin{aligned} q_{e,i} &= \frac{(1-\gamma) - (\gamma+1)p_{e,i} + \gamma(p_{n,i} + p_{e,j})}{(1-\gamma)(2\gamma+1)}, \\ q_{e,j} &= \frac{(1-\gamma) - (\gamma+1)p_{e,j} + \gamma(p_{e,i} + p_{n,i})}{(1-\gamma)(2\gamma+1)}, \\ q_{n,i} &= \frac{(1-\gamma) - (\gamma+1)p_{n,i} + \gamma(p_{e,i} + p_{e,j})}{(1-\gamma)(2\gamma+1)}. \end{aligned} \tag{8}$$

All product-developers regime: II. In this regime, the demand function is

$$\begin{aligned} q_{e,i} &= \frac{(1-\gamma) - (2\gamma+1)p_{e,i} + \gamma(p_{n,i} + p_{n,j} + p_{e,j})}{(1-\gamma)(3\gamma+1)}, \\ q_{n,i} &= \frac{(1-\gamma) - (2\gamma+1)p_{n,i} + \gamma(p_{n,j} + p_{e,i} + p_{e,j})}{(1-\gamma)(3\gamma+1)}. \end{aligned} \tag{9}$$

The timing of the game is similar to that of the Cournot competition case. By applying the same procedure as that in the previous setting, we obtain each regime's equilibrium outcomes, which are shown in Appendix D. Using their equilibrium outcomes, we can derive the best response of downstream firms.

Suppose that Dj introduces a new product. If $f < \hat{\Phi}_I$, Di introduces a new product; otherwise, it does not, and

$$\hat{\Phi}_I \equiv (1-c)^2(\gamma-1)k^2 \left[\frac{\gamma+1}{(2(\gamma-2)(\gamma+1)k+1)^2} - \frac{2(2\gamma+1)(3\gamma+2)^2}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2} \right].$$

Suppose that Dj does not introduce a new product. If $f < \hat{\Phi}_N$, Di introduces a new product; otherwise, it does not, and

$$\hat{\Phi}_N \equiv \frac{1}{2}(1-c)^2(\gamma-1)(\gamma+1)k^2 \left[\frac{8(\gamma+1)^2(2\gamma+1)}{(\gamma(\gamma+5)+4(2\gamma+1)((\gamma-2)\gamma-2)k+3)^2} - \frac{3\gamma+1}{(\gamma-2(3\gamma+1)k+1)^2} \right].$$

These two thresholds, $\hat{\Phi}_I$ and $\hat{\Phi}_N$, have similar properties to those in the Cournot competition case; that is, even if downstream has a different competition form, we obtain similar results.[14] The logic behind the results in the Bertrand case is very similar to that of Proposition 2. Hence, we conclude that the complementary equilibrium appears without asymmetry in product differentiation and verify that Proposition 2 has a certain robustness.

6 Conclusion

In a standard vertically related market comprising an upstream monopoly supplier and downstream differentiated duopoly firms, we showed that whether the behavior of a downstream firm introducing a new product becomes a strategic complement or substitute depends on upstream efficiency; that is, the efficiency level of cost-reducing R&D by the upstream supplier. When upstream is efficient, the result of a new product introduction game is a strategic complement. A strategic complement in the behavior of introducing a new product implies that each firm quickly introduces or develops a new product if its rival introduces a new product, so the varieties of products expand and product innovation rapidly progresses in such a case. Thus, if the rival coincidentally introduces a new product in the equilibrium of strategic complementarities, consumer welfare can increase and further industrial development can be realized, and therefore, it may be desirable for the whole society.

Although we considered a simple vertical structure in this paper, its analysis is limited to a domestic or single region's market. When downstream firms have two options, that is, "produce the product for domestic consumers" and "introduce a new product for foreign consumers," it would be interesting to consider what equilibrium patterns arise. Additionally, we did not consider the effect of vertical integration/separation on upstream and downstream innovation.

[14] As the figure for equilibrium in the differentiated Bertrand competition is almost the same as that in the Cournot case, we omit it.

When upstream and a downstream firm are integrated, because their organizational forms alter, we expect that innovation activities would be affected. However, this consideration is beyond the scope of this study. Hence, we leave it to future research.

Acknowledgements

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Appendix A. Proof of Proposition 2.

By comparing Φ_N with Φ_I, we obtain

$$\Phi_N - \Phi_I = \frac{(1-c)^2(1-\gamma)^2k^2(1-4\gamma k)\ g(k,\gamma)}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2}$$

$$\text{and}\ \ g(k,\gamma) \equiv 16(3\gamma^4+5\gamma^3+16\gamma^2+22\gamma+8)k^3-12(5\gamma^3+5\gamma^2+8\gamma+6)k^2+24\gamma^2k-3\gamma+3.$$

We show that $g(k,\gamma) > 0$ and $\text{sign}\{\Phi_N - \Phi_I\}$ depend only on $1-4\gamma k$. To prove $g(k,\gamma) > 0$, it is sufficient to show that $g(k,\gamma)$ has its minimum value at $k = k_0$ and $c = 1$, and that value is positive.

First, we show that $g(k,\gamma)$ is an increasing function of k; that is, $g(k,\gamma)$ is smallest at $k = k_0$. The first derivative $g(k,\gamma)$ with respect to k is $\partial g(k,\gamma)/\partial k = 24[2(3\gamma^4+5\gamma^3+16\gamma^2+22\gamma+8)k^2-(5\gamma^3+5\gamma^2+8\gamma+6)k+\gamma^2]$. $\partial g(k,\gamma)/\partial k$ is a quadratic function of k and the coefficient of k^2 is positive. Hence, by solving $\partial g(k,\gamma)/\partial k > 0$ for k, we obtain $k < k_1$ and $k > k_2$, where k_1 and k_2 are roots of $g(k,\gamma) = 0$ for k and $k_1 < k_2$.

As $k_0 = 1/[2c(2\gamma+1)]$ decreases with c, k_0 has its minimum value at $c = 1$. We illustrate

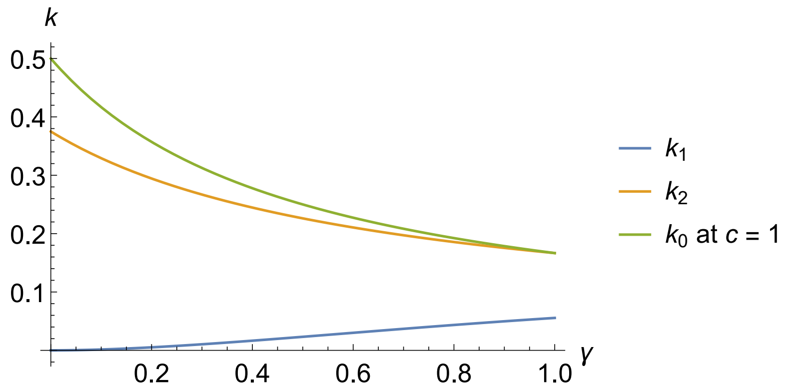

Figure 5: k_0 at $c = 1$ and the two roots of $g(k, \gamma) = 0$

k_1, k_2, and k_0 at $c = 1$ in Figure 5. Using numerical calculation, we find that for $\gamma \in [0, 1]$, the unique root of $k_2 - k_0|_{c=1} = 0$ is $\gamma = 1$. Hence, $\partial g(k, \gamma)/\partial k > 0$ for any $k > k_0$. Therefore, $g(k, \gamma)$ has its minimum value at $k = k_0$.

Second, we show $\partial g(k_0, \gamma)/\partial c < 0$. The derivation yields

$$\frac{\partial g(k_0, \gamma)}{\partial c} = \frac{\partial g(k_0, \gamma)}{\partial k} \times \frac{\partial k_0}{\partial c} = \frac{\partial g(k_0, \gamma)}{\partial k} \left[\frac{-1}{2c^2(2\gamma + 1)} \right] < 0.$$

The last inequality is satisfied because $\partial g(k, \gamma)/\partial k > 0$. Hence, $g(k_0, \gamma)$ is a decreasing function for c and it has its minimum value when $c = 1$.

From the above discussion, $g(k_0, \gamma)$ has the following minimum value at $k = k_0$ and $c = 1$: $g(k_0, \gamma)\big|_{c=1} = \frac{(1-\gamma)^2(\gamma+1)}{(2\gamma+1)^3} > 0$. Because $g(k_0, \gamma)\big|_{c=1}$ is positive, $\forall k > k_0$, $g(k, \gamma) > 0$. This result implies that $\text{sign}\{\Phi_N - \Phi_I\}$ depends only on "$1 - 4\gamma k$." Hence, $\Phi_N > \Phi_I$ iff $k < 1/(4\gamma)$. □

Appendix B. Proof of Result 1.

Case (i). From Proposition 2, the equilibrium regime is either "$IN\&NI$" or "NN" if $f > \Phi_N$; and the equilibrium regime is "II" if $f < \phi_N$. Hence, to prove the first part, we should show that $CS^{II} > CS^{IN} (= CS^{NI}) > CS^{NN}$. This is because underinvestment occurs only if $f > \Phi_N$.

Case (ii). Applying a similar discussion to that in case (i), we find that underinvestment occurs only if $f > \min\{\Phi_N, \Phi_I\}$, where the equilibrium regime is also either "$IN\&NI$" or

"NN." Hence, in both cases, if we show $CS^{II} > CS^{IN} > CS^{NN}$, the proof is complete.

First, we consider $\text{sign}\{CS^{II} - CS^{IN}\}$:

$$CS^{II} - CS^{IN} = \frac{(1-c)^2(1-\gamma)k^2\ \psi_1^{CS}}{2[(4\gamma+2)k-1)]^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2},$$

where $\psi_1^{CS} \equiv 8(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)k^2 - 8\gamma(2-\gamma^2)k - \gamma^2 + 2\gamma - 3$.

$\text{sign}\{CS^{II} - CS^{IN}\}$ depends only on ψ_1^{CS}. Because ψ_1^{CS} is a quadratic function of k and the coefficient of k^2 is positive, $\psi_1^{CS} = 0$ has two roots: k_1^{CS} and k_2^{CS}. By solving $\psi_1^{CS} > 0$ for k, we obtain $k < k_1^{CS}$ or $k > k_2^{CS}$, where

$$k_1^{CS} \equiv \frac{2\gamma(2-\gamma^2)-\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)};\quad k_2^{CS} \equiv \frac{2\gamma(2-\gamma^2)+\sqrt{6\gamma^4-40\gamma^3+34\gamma^2+52\gamma+12}}{4(2+10\gamma+9\gamma^2-4\gamma^3-2\gamma^4)}.$$

We compare k_2^{CS} with k_0. We consider the case $c = 1$. Using numerical calculation, we find $\forall\gamma \in [0,1)$, $k_0|_{c=1} > k_2^{CS}$. Because k_0 takes the minimum value at $c = 1$, $k_0 > k_2^{CS} > k_1^{CS}$ for any $c > 0$. Hence, $CS^{II} - CS^{IN} > 0$.

Next, we consider $CS^{IN} - CS^{NN}$ and apply a proof similar to that above:

$$CS^{IN} - CS^{NN} = -\frac{(c-1)^2(\gamma-1)k^2\ \psi_2^{CS}}{(1-2(\gamma+2)k)^2\left[\gamma+\left(-4\gamma^2+8\gamma+8\right)k-3\right]^2},$$

where $\psi_2^{CS} \equiv 4(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)k^2 - 4\gamma(\gamma^2-2\gamma+2)k - 3 + 2\gamma$.

$\text{sign}\{CS^{IN} - CS^{NN}\}$ depends only on ψ_2^{CS}. Because the coefficient of k^2 in ψ_2^{CS} is positive, by solving $\psi_2^{CS} > 0$ for k, we obtain $k < k_3^{CS}$ or $k > k_4^{CS}$, where

$$k_3^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)-(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)};\quad k_4^{CS} \equiv \frac{\gamma(\gamma^2-2\gamma+2)+(2-\gamma)\sqrt{\gamma^4-6\gamma^3+\gamma^2+14\gamma+6}}{2(3\gamma^4-6\gamma^3-6\gamma^2+16\gamma+8)}.$$

We show $k_0 > k_4^{CS} (> k_3^{CS})$. At $c = 1$, using numerical calculation, we find that, $\forall\gamma \in [0,1)$, $k_0|_{c=1} > k_4^{CS}$. Because k_0 has its minimum value at $c = 1$, for any $c > 0$, $k_0 > k_4^{CS} > k_3^{CS}$ holds. Therefore, $CS^{IN} - CS^{NN} > 0$. $\square$

Appendix C. Proof of Result 2.

In both regimes, we need to determine, given the downstream R&D cost f, which regime maximizes the total surplus.

We consider case (i) and compare the gross benefits. First, we consider $\Psi_{21}^{TS} - \Psi_{10}^{TS}$:

$$\Psi_{21}^{TS} - \Psi_{10}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2\ g^{TS}(k,\gamma)}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[(-4\gamma^2+8\gamma+8)\,k-3\gamma\right]^2}.$$

sign$\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$ depends only on $g^{TS}(\gamma,k)$. From Figure 2, we obtain $g^{TS}(\gamma,k) > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

Next, we consider $\Psi_{21}^{TS} - \Psi_{20}^{TS}$:

$$\Psi_{21}^{TS} - \Psi_{20}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2\ g^{TS}(\gamma,k)}{4[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[(-4\gamma^2+8\gamma+8)\,k-3+\gamma\right]^2}.$$

Hence, sign$\{\Psi_{21}^{TS} - \Psi_{20}^{TS}\}$ is the same as sign$\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$. Then, we obtain $\Psi_{21}^{TS} - \Psi_{10}^{TS} > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

Finally, we consider $\Psi_{20}^{TS} - \Psi_{10}^{TS}$:

$$\Psi_{20}^{TS} - \Psi_{10}^{TS} = \frac{(1-c)^2(1-\gamma)^2k^2\ g^{TS}(\gamma,k)}{4[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[(-4\gamma^2+8\gamma+8)\,k-3+\gamma\right]^2}.$$

sign$\{\Psi_{20}^{TS} - \Psi_{10}^{TS}\}$ is also the same as sign$\{\Psi_{21}^{TS} - \Psi_{10}^{TS}\}$. Thus, $\Psi_{20}^{TS} - \Psi_{10}^{TS} > 0$ if $\max\{1/2, k_0\} < k < k^{TS}$.

From these, we have the following ranking of the thresholds:

$$\left.\begin{array}{ll} \Psi_{10}^{TS} < \Psi_{20}^{TS} < \Psi_{21}^{TS} & \text{if} \quad \max\{1/2, k_0\} < k < k^{TS}, \\ \Psi_{21}^{TS} \leq \Psi_{20}^{TS} \leq \Psi_{10}^{TS} & \text{if} \quad k \geq k^{TS}. \end{array}\right\} \tag{10}$$

Then, we establish Lemma 1.

Lemma 1. *(i) For $\max\{1/2, k_0\} < k < k^{TS}$, the best regime for total welfare is "II" if $f \leq \Psi_{20}^{TS}$; and "NN" if $f > \Psi_{20}^{TS}$. (ii) For $k \geq k^{TS}$, the best regime for total welfare is "II" if $f \leq \Psi_{21}^{TS}$; "IN" or "NI" if $\Psi_{21}^{TS} < f \leq \Psi_{10}^{TS}$; and "NN" if $f > \Psi_{10}^{TS}$.*

By comparing Ψ_{10}^{TS}, Ψ_{20}^{PS}, Ψ_{21}^{TS}, Φ_I, and Φ_N, we show the ranking of thresholds. First, we show $\Phi_I < \min\{\Psi_{10}^{TS}, \Psi_{21}^{TS}\}$. The difference $\Psi_{10}^{TS} - \Phi_I$ yields

$$\Psi_{10}^{TS} - \Phi_I = \frac{(1-c)^2(1-\gamma)k^2\ g_{10,I}^{TS}}{[1-2(\gamma+2)k]^2\left[(-4\gamma^2+8\gamma+8)\,k-3+\gamma\right]^2},$$

where $g_{10,I}^{TS} \equiv 4(7\gamma^4 - 10\gamma^3 - 42\gamma^2 + 32\gamma + 40)k^2 - 4(4\gamma^3 - 11\gamma^2 - 6\gamma + 16)k + 2\gamma^2 - 7\gamma + 4$.

sign$\{\Psi_{10}^{TS} - \Phi_I\}$ depends only on $g_{10,I}^{TS}$. By solving $g_{10,I}^{TS} = 0$ for k, we obtain two roots $k_{10,I}^1$ and $k_{10,I}^2$, where $k_{10,I}^1 < k_{10,I}^2$. Because the coefficient of k^2 in $g_{10,I}^{TS}$ is positive, $g_{10,I}^{TS} > 0$ if $k < k_{10,I}^1$ or $k > k_{10,I}^2$. Additionally, using numerical calculation, we can show $k_{10,I}^1 < k_{10,I}^2 < 1/2$. As we assume $k > \max\{1/2, k_0\}$, we obtain $g_{10,I}^{TS} > 0$, which leads to $\Psi_{10}^{TS} - \Phi_I > 0$.

Next, we consider $\Psi_{21}^{TS} - \Phi_I$:

$$\Psi_{21}^{TS} - \Phi_I = \frac{(1-c)^2(1-\gamma)k^2\ g_{21,I}^{TS}}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[(-4\gamma^2+8\gamma+8)\,k-3+\gamma\right]^2},$$

where $g_{21,I}^{TS} \equiv 32(10\gamma^6 + 4\gamma^5 - 23\gamma^4 - 14\gamma^3 + 98\gamma^2 + 128\gamma + 40)k^4 - 96(5\gamma^5 - 2\gamma^4 - 12\gamma^3 + 7\gamma^2 + 26\gamma + 12)k^3 + 4(65\gamma^4 - 78\gamma^3 - 87\gamma^2 + 118\gamma + 90)k^2 + 4(-15\gamma^3 + 28\gamma^2 + \gamma - 14)k + 5\gamma^2 - 12\gamma + 5$.

sign$\{\Psi_{21}^{TS} - \Phi_I\}$ depends only on $g_{21,I}^{TS}$. To prove $\Psi_{21}^{TS} - \Phi_I > 0$, we show (i) $g_{21,I}^{TS} > 0$ at $k = 1/2$ and $\partial g_{21,I}^{TS}/\partial k > 0$; and (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$.

First, we show (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$:

$$\frac{\partial^2 g_{21,I}^{TS}}{\partial k^2} = 8\left[\begin{array}{l} 48\left(10\gamma^6+4\gamma^5-23\gamma^4-14\gamma^3+98\gamma^2+128\gamma+40\right)k^2 \\ -72\left(5\gamma^5-2\gamma^4-12\gamma^3+7\gamma^2+26\gamma+12\right)k+65\gamma^4-78\gamma^3-87\gamma^2+118\gamma+90 \end{array}\right].$$

By solving $\partial^2 g_{21,I}^{TS}/\partial k^2 = 0$ for k, we obtain two roots. Using numerical calculation, we can show that all roots are smaller than 1/2. Because the coefficient of k^2 in the equation $\partial^2 g_{21,I}^{TS}/\partial k^2$ is positive and we assume $k > \max\{1/2, k_0\}$, $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$. Furthermore, by substituting $k = 1/2$ into $\partial g_{21,I}^{TS}/\partial k$, we obtain $\left(\partial g_{21,I}^{TS}/\partial k\right)\big|_{k=1/2} = 4(40\gamma^6 - 74\gamma^5 + 9\gamma^4 + 67\gamma^3 + 207\gamma^2 + 163\gamma + 20) > 0$. Thus, we obtain (ii) $\partial g_{21,I}^{TS}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g_{21,I}^{TS}/\partial k^2 > 0$, which leads to $\partial g_{21,I}^{TS}/\partial k > 0\ \forall k > 1/2$.

Because we already have $\partial g_{21,I}^{TS}/\partial k > 0$, to prove (i) $g_{21,I}^{TS} > 0$ at $k = 1/2$ and $\partial g_{21,I}^{TS}/\partial k > 0$, we only show $g_{21,I}^{TS} > 0$ at $k = 1/2$: $g_{21,I}^{TS}\big|_{k=1/2} = 20\gamma^6 - 52\gamma^5 + 43\gamma^4 + 8\gamma^3 + 86\gamma^2 + 52\gamma + 3 > 0$.

Therefore, $\forall k > 1/2$, $g^{TS}_{21,I} > 0$, which implies $\Psi^{TS}_{21} - \Phi_I > 0$. From this and $\Psi^{TS}_{10} - \Phi_I > 0$, we obtain Lemma 2.

Lemma 2. $\Phi_I < \min\{\Psi^{TS}_{10},\ \Psi^{TS}_{21}\}$.

We show $\Psi^{TS}_{21} > \Phi_N$ and $\Psi^{TS}_{20} > \Phi_N$. We consider $\Psi^{TS}_{21} - \Phi_N$:

$$\Psi^{TS}_{21} - \Phi_N = \frac{(1-c)^2(1-\gamma)k^2\ g^{TS}_{21,N}}{[(4\gamma+2)k-1]^2\left[\left(-4\gamma^2+8\gamma+8\right)k+\gamma-3\right]^2},$$

where $g^{TS}_{21,N} \equiv 4(4\gamma^4-16\gamma^3+3\gamma^2+26\gamma+10)k^2-4(2\gamma^3-7\gamma^2+3\gamma+5)k+\gamma^2-3\gamma+1$.

$\text{sign}\{\Psi^{TS}_{21} - \Phi_N\}$ depends only on $g^{TS}_{21,N}$. By solving $g^{TS}_{21,N} = 0$ for k, we obtain two roots $k^1_{21,N}$ and $k^2_{21,N}$, where $k^1_{21,N} < k^2_{21,N}$. The coefficient of k^2 in $g^{TS}_{21,N}$ is positive; hence, $g^{TS}_{21,N} > 0$ if $k < k^1_{21,N}$ or $k > k^2_{21,N}$. Additionally, using numerical calculation, we can show $k^1_{21,N} < k^2_{21,N} < 1/2$. Because we assume $k > \max\{1/2, k_0\}$, we obtain $g^{TS}_{21,N} > 0$, which leads to $\Psi^{TS}_{21} - \Phi_N > 0$.

Next, we consider $\Psi^{TS}_{20} - \Phi_N$:

$$\Psi^{TS}_{20} - \Phi_N = -\frac{(c-1)^2(\gamma-1)k^2\ g^{TS}_{20,N}}{4[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\gamma+(-4\gamma^2+8\gamma+8)k-3\right]^2},$$

where $g^{TS}_{20,N} \equiv 64(13\gamma^6-8\gamma^5-134\gamma^4-44\gamma^3+200\gamma^2+176\gamma+40)k^4-32(27\gamma^5-49\gamma^4-214\gamma^3+72\gamma^2+280\gamma+100)k^3+12(27\gamma^4-78\gamma^3-131\gamma^2+140\gamma+114)k^2-4(13\gamma^3-50\gamma^2-17\gamma+54)k+3\gamma^2-14\gamma+7$.

$\text{sign}\{\Psi^{TS}_{20} - \Phi_N\}$ depends only on $g^{TS}_{20,IN}$. To prove $\Psi^{TS}_{20} - \Phi_N > 0$, we show (i) $g^{TS}_{20,N} > 0$ at $k = 1/2$ and $\partial g^{TS}_{20,N}/\partial k > 0$; and (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$.

First, we show (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$:

$$\frac{\partial^2 g^{TS}_{20,N}}{\partial k^2} = 24\left[\begin{array}{l} 32(13\gamma^6-8\gamma^5-134\gamma^4-44\gamma^3+200\gamma^2+176\gamma+40)k^2 \\ -8(27\gamma^5-49\gamma^4-214\gamma^3+72\gamma^2+280\gamma+100)k+27\gamma^4-78\gamma^3-131\gamma^2+140\gamma+114 \end{array}\right].$$

Solving $\partial^2 g^{TS}_{20,N}/\partial k^2 = 0$ for k, we obtain two roots. Using numerical calculation, we can show that both roots are smaller than 1/2. Because the coefficient of k^2 in $\partial^2 g^{TS}_{20,N}/\partial k^2$ is positive and we assume $k > \max\{1/2, k_0\}$, $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$. Additionally, by substituting $k = 1/2$ into $\partial g^{TS}_{20,N}/\partial k$, we obtain $\left(\partial g^{TS}_{20,N}/\partial k\right)\big|_{k=1/2} = 4\left(104\gamma^6-226\gamma^5-697\gamma^4+685\gamma^3+825\gamma^2+165\gamma+8\right) > 0$. Therefore, we obtain (ii) $\partial g^{TS}_{20,N}/\partial k > 0$ at $k = 1/2$ and $\partial^2 g^{TS}_{20,N}/\partial k^2 > 0$, which leads to

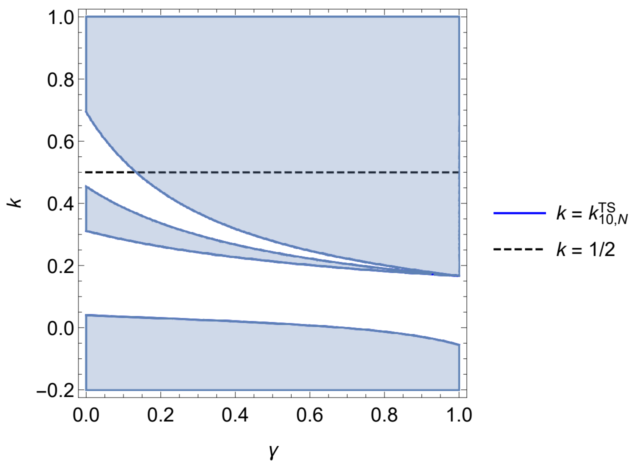

Figure 6: Region for $g^{TS}_{10,N}(k,\gamma) > 0$

$\partial g^{TS}_{20,N}/\partial k > 0$ for any $k > 1/2$.

Because we already have $\partial g^{TS}_{20,N}/\partial k > 0$, to prove (i) $g^{TS}_{20,N} > 0$ at $k = 1/2$ and $\partial g^{TS}_{20,N}/\partial k > 0$, we only show $g^{TS}_{20,N} > 0$ at $k = 1/2$: $g^{TS}_{20,N}|_{k=1/2} = 52\gamma^6 - 140\gamma^5 - 259\gamma^4 + 420\gamma^3 + 222\gamma^2 + 24\gamma + 1 > 0$. Therefore, $\forall k > 1/2$, $g^{TS}_{20,N} > 0$, which implies $\Psi^{TS}_{20} - \Phi_N > 0$. Hence, we obtain Lemma 3.

Lemma 3. $\Psi^{TS}_{21} > \Phi_N$ *and* $\Psi^{TS}_{20} > \Phi_N$.

Finally, we compare Ψ^{TS}_{10} with Φ_N:

$$\Psi^{TS}_{10} - \Phi_N = \frac{(1-c)^2(1-\gamma)k^2\ g^{TS}_{10,N}}{2[1-2(\gamma+2)k]^2[(4\gamma+2)k-1]^2\left[\left(-4\gamma^2+8\gamma+8\right)k+\gamma-3\right]^2},$$

where $g^{TS}_{10,N} \equiv 32(22\gamma^6 - 16\gamma^5 - 223\gamma^4 - 62\gamma^3 + 274\gamma^2 + 208\gamma + 40)k^4 - 96(7\gamma^5 - 14\gamma^4 - 56\gamma^3 + 17\gamma^2 + 62\gamma + 20)k^3 + 4(55\gamma^4 - 180\gamma^3 - 297\gamma^2 + 296\gamma + 234)k^2 - 4(7\gamma^3 - 34\gamma^2 - 13\gamma + 40)k + \gamma^2 - 8\gamma + 5$.

Note that all roots for the above equation are depicted in Figure 6. The blue curves represent the set of pair (γ, k), which satisfies $g^{TS}_{10,N}(\gamma, k) = 0$. The dashed line represents $k = 1/2$. Hence, given γ, we can implicitly define $k = k^{TS}_{10,N}$ as the largest root if for $k > 1/2$, it exists. In the shadow area, $g^{TS}_{10,N}(\gamma, k) > 0$.

This result yields Lemma 4.

Lemma 4. $\Psi^{TS}_{10} > \Phi_N$ *if* $k > k^{T}_{10,N}$; $\Psi^{TS}_{10} \leq \Phi_N$ *if* $\max\{1/2, k_0\} < k \leq k^{T}_{10,N}$.

From Proposition 2 and Lemmas 1 and 4, we obtain three thresholds for k: $1/(4\gamma)$, k^{TS}, and $k^{TS}_{10,N}$. They are depicted in Figure 7. Then, we obtain $k^{TS}_{10,N} < \min\{k^{TS}, 1/(4,\gamma)\}$. However, we cannot conclude that $k^{TS}_{10,N} < k^{TS}$ because $k^{TS}_{10,N}$ is implicitly defined and it diverges to infinity as $\gamma \to 0$. Hence, we potentially have five regions: (i) $\max\{1/2, k_0\} < k \leq k^{TS}_{10,N}$, (ii) $\max\{1/2, k_0, k^{TS}_{10,N}\} < k \leq \min\{1/(4\gamma), k^{TS}\}$, (iii) $\max\{1/2, k_0, k^{TS}\} < k \leq 1/(4\gamma)$, (iv) $\max\{1/2, k_0, 1/(4\gamma)\} < k \leq k^{TS}$, and (v) $\max\{1/2, k_0, k^{TS}, 1/(4\gamma)\} < k$. Each case is depicted in Figure 8.

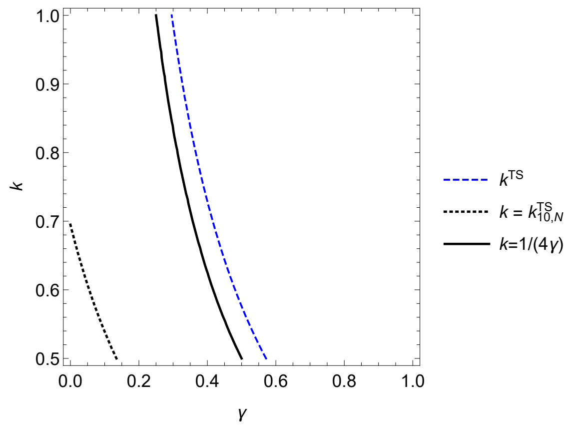

Figure 7: Thresholds

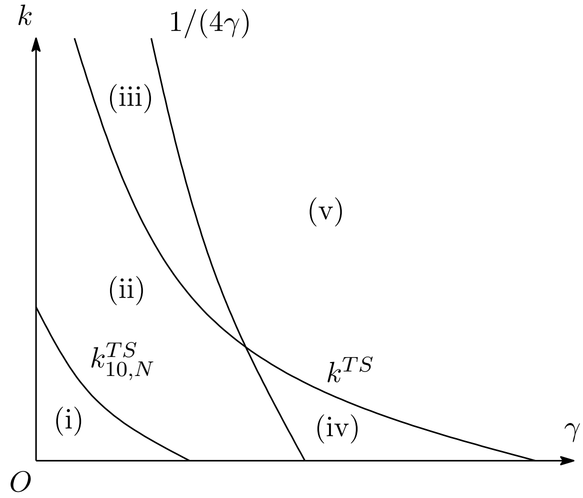

Figure 8: Cases for threshold ranking

From Proposition 2, Lemmas 2–4, and (10), we obtain the threshold ranking:

$$
\begin{aligned}
&\Phi_I < \Psi^{TS}_{10} \leq \Phi_N < \Psi^{TS}_{20} < \Psi^{TS}_{21} && \text{if (i)}\ \max\{1/2, k_0\} < k \leq k^{TS}_{10,N},\\
&\Phi_I \leq \Phi_N < \Psi^{TS}_{10} \leq \Psi^{TS}_{20} \leq \Psi^{TS}_{21} && \text{if (ii)}\ \max\{1/2, k_0, k^{TS}_{10,N}\} < k \leq \min\{1/(4\gamma), k^{TS}\},\\
&\Phi_I \leq \Phi_N < \Psi^{TS}_{21} < \Psi^{TS}_{20} < \Psi^{TS}_{10} && \text{if (iii)}\ \max\{1/2, k_0, k^{TS}\} < k \leq 1/(4\gamma),\\
&\Phi_N < \Phi_I < \Psi^{TS}_{10} \leq \Psi^{TS}_{20} \leq \Psi^{TS}_{21} && \text{if (iv)}\ \max\{1/2, k_0, 1/(4\gamma)\} < k \leq k^{TS},\\
&\Phi_N < \Phi_I < \Psi^{TS}_{21} < \Psi^{TS}_{20} < \Psi^{TS}_{10} && \text{if (v)}\ \max\{1/2, k_0, k^{TS}, 1/(4\gamma)\} < k.
\end{aligned}
$$

By combining this ranking with Lemma 1 and Proposition 2, we can identify the condition for downstream underinvestment. Therefore, the proof is complete. □

Appendix D. SPNE outcomes in downstream Bertrand.

To identify Bertrand rivalry, we attach "ˆ" to the variables of the equilibrium solutions.

No one invests regime: NN

$\hat{w}^{NN} = \frac{1-(c+1)(2-\gamma)(\gamma+1)k}{1-2(2-\gamma)(\gamma+1)k}$, $\hat{x}^{NN} = \frac{1-c}{2(2-\gamma)(\gamma+1)k-1}$, $\hat{\pi}_U^{NN} = \frac{(1-c)^2k}{2(2-\gamma)(\gamma+1)k-1}$,

$\hat{p}_{e,i}^{NN} = \frac{1-(\gamma+1)k(c-2\gamma+3)}{2(\gamma^2-\gamma-2)k+1}$, and $\hat{\pi}_{Di}^{NN} = \frac{(1-c)^2(1-\gamma)(\gamma+1)k^2}{(2(\gamma^2-\gamma-2)k+1)^2}$.

Mixed regime: IN or NI

$\hat{w}^{IN} = \frac{2(c+1)(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$,

$\hat{x}^{IN} = \frac{(1-c)(\gamma(\gamma+5)+3)}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$, $\hat{\pi}_U^{IN} = \frac{(1-c)^2(\gamma(\gamma+5)+3)k}{4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3)}$,

$\hat{p}_{e,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{p}_{n,i}^{IN} = \frac{-(2\gamma+1)k(c(\gamma+1)(\gamma+2)+5(1-\gamma)\gamma+6)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$,

$\hat{p}_{e,j}^{IN} = \frac{-2(2\gamma+1)k(2\gamma c+c+2(1-\gamma)\gamma+3)+\gamma(\gamma+5)+3}{\gamma(\gamma+5)-4(2\gamma+1)((2-\gamma)\gamma+2)k+3}$, $\hat{\pi}_{Di}^{IN} = \frac{2(1-c)^2(1-\gamma)(2\gamma+1)(3\gamma+2)^2k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$,

and $\hat{\pi}_{Dj}^{IN} = \frac{4(1-c)^2(1-\gamma)(\gamma+1)^3(2\gamma+1)k^2}{(4(2\gamma+1)((2-\gamma)\gamma+2)k-(\gamma(\gamma+5)+3))^2}$.

All product-developers regime: II

$\hat{w}^{II} = \frac{(c+1)(3\gamma+1)k-(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{x}^{II} = \frac{(1-c)(\gamma+1)}{(6\gamma+2)k-(\gamma+1)}$, $\hat{\pi}_U^{II} = \frac{(1-c)^2(\gamma+1)k}{(6\gamma+2)k-(\gamma+1)}$, $\hat{q}_{e,i}^{II} = \frac{(3\gamma+1)k(\gamma c+c-\gamma+3)-2(\gamma+1)}{4(3\gamma+1)k-2(\gamma+1)}$,

and $\hat{\pi}_{Di}^{II} = \frac{(1-c)^2(1-\gamma)(\gamma+1)(3\gamma+1)k^2}{2[2(3\gamma+1)k-(\gamma+1)]^2}$.

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