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Analysis of micro-macro material properties and mechanical effects of damaged material containing periodically distributed elliptical microcracks

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Abstract

An existence of numerous microcracks causes changes of stiffness or fracture toughness of materials. In this paper, the manifestations of mechanical properties of the damaged materials caused by the microcracks are evaluated by present homogenization method based on the superposition method together with the VNA solution. On the other hand, it is known that the fracture toughness of macrocracked material increases due to the stress shielding effect caused by the microcracks. In this paper, in order to evaluate such mechanical effect of the microcracks on the macrocrack, the component separation method for the evaluation of mixed-mode stress intensity factors of the macrocrack in the damaged materials is newly developed by using the evaluation method of the path independent J integral. In this paper, various numerical analyses are achieved for two topics such as the mechanical properties of the damaged materials, and the mechanical behavior of the macrocrack in the damaged materials.

Key Words: Fracture Mechanics, Damaged Material, Anisotropy, Homogenization Method, Superposition Method, VNA Solution, Effective Elastic Moduli, Component Separation Method, Stress Intensity Factor, Microcrack Toughening.

1 Introduction

An existence of numerous microcracks in materials causes changes of material properties. The microcracks are often generated due to cyclic load or manufacturing processes. Therefore distribution patterns of the microcracks generally denote an anisotropy. The existence of numerous microcracks causes stiffness reduction or reduction in fracture toughness. On the contrary, another mechanical effects are induced by the distribution of the microcracks in the material. When the macrocrack front locates in the domain of stress shielding that is induced by the microcracks, the macrocrack experiences the reduction in stress concentration. Therefore, the stress shielding effect enhances the fracture toughness of the macrocracked material. Such mechanical effect is called microcrack-toughening effect.

Cai and Faber (1992) have studied the interaction problem of the macrocrack with multiple microcracks. They showed the effects of the strong stress shielding and the strong stress amplification induced by the microcrack on the macrocrack. Huang and Karihaloo (1993) have developed weight function method in order to analyze the three-dimensional problem of interaction between half-plane crack and penny-shaped microcracks. Laures and Kachanov (1991) have also studied the interaction problem of the half-plane crack with numerous microcracks. Ortiz (1987) evaluated the crack shielding in ceramics by using the asymptotic stress strain, and displacement field that is determined in closed form. Ortiz and Giannakopoulos (1989) considered the semi-infinite crack-microcrack problem. They have developed new technique for evaluation of the crack shielding. Their analyses rely on the damage model to provide the necessary connectivity between microcrack densities and the elastic moduli of the solid. They showed that the shielding effect is maximized when all microcracks are oriented at right angles to the maximum tensile direction.

Recently, homogenization method (Sanchez-Palencia, 1980; Suquet, 1987) is noticed as homogenized technique for microscopic and macroscopic coupled analysis (Guedes and Kikuchi, 1990; Hollister and Kikuchi, 1992). By using the homogenization method, we can evaluate manifestation of mechanical property of materials consisted of microstructures. The equivalent mechanical properties of the materials are evaluated by using the numerical analysis of the unit cell. Further, detailed stress fields at microscopic and macroscopic scales for macroscopic problems are precisely evaluated by using the homogenization method.

In this paper, the superposition method (Yagawa, Nishioka, Ando and Ogura, 1975) that uses analytical solutions and finite element solutions is applied for the homogenization analysis. In this technique, the VNA solution (Vijayakumar and Atluri, 1981; Nishioka and Atluri, 1983) is applied to the analytical solution, in order to evaluates the singularity of multiple elliptical cracks (Kato and Nishioka, 2002). Therefore the superposition method makes it possible to solve the elliptical cracks

problem by using a relatively coarse regular mesh pattern that does not describe the shapes of the elliptical cracks.

In this paper, manifestations of mechanical properties of the damaged materials are evaluated by the present homogenization method based on the superposition method. The manifestations of mechanical properties are expressed by the variations of effective elastic moduli in this study. Numerical analyses for various damaged materials containing periodically distributed elliptical microcracks are carried out in order to estimate the effective elastic moduli of the damaged materials for varying the aspect ratio of the elliptical microcrack and the crack density of the damaged material.

Furthermore, we evaluate the mechanical effect of the microcracks on the macrocrack by newly developed Component Separation Method. Using the effective elastic moduli obtained by present homogenization method, the mechanical effect of the microcracks on the macrocrack is evaluated by the variations of the stress intensity factor of the macrocrack in this study. The periodic distribution of elliptical microcracks causes the anisotropy of mechanical properties to macroscopic behavior of the damaged materials. Therefore, in order to evaluate the mixed-mode stress intensity factor of a crack in orthotropic materials, the component separation method based on the three-dimensional path independent J integral (Nishioka and Atluri, 1983) is newly developed in this study. The mechanical effects of the microcracks on the macrocrack are estimated by the evaluation of the J integral value and the stress intensity factors of the macrocrack. These numerical results are considered the connectivity between the macroscopic mechanical properties of the damaged material and the microstructure of the material such as the lattice distribution of the microcracks and microcrack aspect ratios.

2 Homogenization method for analysis of damaged material containing periodically distributed elliptical cracks

The homogenization method is a micro-macro coupled analytical method for a structure consisting of periodic microstructure (Guedes and Kikuchi, 1990; Hollister and Kikuchi, 1992). Basic period of the periodically distributed microscopic structure is called as unit cell.

Let y be the microscopic coordinate system, and x be the macroscopic coordinate system. The macroscopic coordinate system and microscopic coordinate system are related by

$$y = \frac{x}{\eta} \tag{1}$$

where η is the ratio that denotes the relation between microscopic scale of the unit cell and macroscopic structures.

For the homogenization theory, the unit cell is the periodically distributed basic element.

Therefore the displacement field variables can be expressed by an asymptotic expansion

$$u_i^\eta(x) = u_i^0(x) + \eta u_i^1(x, y) \quad (2)$$

where u_i^η is the exact value of the displacement field, u_i^0 is the macroscopic value of the displacement field, and u_i^1 is the perturbation value of the displacement field due to the microstructure.

Substituting asymptotic expansion of displacement field (2) into the equation of the principle of virtual work, we obtain the equilibrium equation

$$\int_{\Omega} (\delta \varepsilon_{ij}^0 + \delta \varepsilon_{ij}^1) D_{ijkl}^\eta (\varepsilon_{kl}^0 + \varepsilon_{kl}^1) d\Omega = \int_{\Gamma} t_i (\delta u_i^0 + \delta u_i^1) d\Gamma \quad (3)$$

where Ω represents the macroscopic domain, Γ represents the load-applied boundaries, ε_{ij}^0 is the macroscopic strain, and ε_{ij}^1 is the microscopic strain, respectively.

The microscopic displacements are assumed to be related to the macroscopic strain

$$u_i^1(x, y) = -\chi_i^{kl}(y) \varepsilon_{kl}^0 \quad (4)$$

where χ_i^{kl} is called the characteristic displacement. The characteristic displacement represents microscopic displacement per macroscopic unit strain due to the microstructure. Therefore the microscopic fields can be connected with macroscopic fields due to the microstructure by using the characteristic displacement.

Introducing the averaging principle into the equilibrium Equation (3) and assuming that the limit η tend to zero, we obtain microscopic and macroscopic equations as follows

$$\int_Y \delta \varepsilon_{ij}^1 \left(D_{ijkl} - D_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) dY = 0 \quad (5)$$

$$\int_{\Omega} \delta \varepsilon_{ij}^0 D_{ijkl}^H \varepsilon_{kl}^0 d\Omega = \int_{\Gamma} \bar{t}_i \delta u_i^0 d\Gamma \quad (6)$$

$$D_{ijkl}^H = \frac{1}{|Y|} \int_Y \left(D_{ijkl} - D_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) dY \quad (7)$$

where $|Y|$ is the volume of the unit cell and D_{ijkl}^H is homogenized elastic tensor, respectively.

In this study, the microscopic Equation (5) of the homogenization method is solved by the superposition method (Kato and Nishioka, 2002). Introducing the superposition method, the displacement u_i , strain ε_{ij} and stress σ_{ij} are expressed by the sum of two terms as follows

$$u_i = u_i^F + u_i^A = u_i^F + \sum_{n=1}^{N_C} u_i^{(n)} \quad (8)$$

$$\varepsilon_{ij} = \varepsilon_{ij}^F + \varepsilon_{ij}^A = \varepsilon_{ij}^F + \sum_{n=1}^{N_c} \varepsilon_{ij}^{(n)} \quad (9)$$

$$\sigma_{ij} = \sigma_{ij}^F + \sigma_{ij}^A = \sigma_{ij}^F + \sum_{n=1}^{N_c} \sigma_{ij}^{(n)} \quad (10)$$

where (n) and N_c denote the crack number and the number of cracks, respectively. The superscript F represents a function of the conventional finite element method, and the superscript A represents the analytical solution. For present superposition method, to analyze interacting multiple elliptical cracks in the unit cell, the VNA solution is applied to the individual elliptical crack. Therefore, to evaluate the interaction effects of elliptical cracks, the displacement, strain and stress fields are expressed by the sum of the displacement, strain, and stress fields caused by individual elliptical cracks (see Equations (8)-(10)).

The microscopic displacements (4) are expressed by the sum of two terms of the characteristic displacement (Kato and Nishioka, 2002)

$$u_i^l(x,y) = - \left[\{\chi_i^{kl}(y)\}^F + \{\chi_i^{kl}(y)\}^A \right] \varepsilon_{kl}^0 \quad (11)$$

Introducing the superposition of stress, strain and displacement into Equations (5) and (7), and using the microscopic displacements (11), we obtain the following microscopic equation

$$\int_Y \left\{ (\delta \varepsilon_{ij}^1)^F + (\delta \varepsilon_{ij}^1)^A \right\} \left[D_{ijkl} - D_{ijmn} \left\{ \frac{\partial (\chi_m^{kl})^F}{\partial y_n} + \frac{\partial (\chi_m^{kl})^A}{\partial y_n} \right\} \right] dY = 0 \quad (12)$$

The homogenized elastic tensor can be evaluated by following equation

$$D_{ijkl}^H = \frac{1}{|Y|} \int_Y \left[D_{ijkl} - D_{ijmn} \left\{ \frac{\partial (\chi_m^{kl})^F}{\partial y_n} + \frac{\partial (\chi_m^{kl})^A}{\partial y_n} \right\} \right] dY \quad (13)$$

Furthermore, the microscopic stress fields are evaluated by using the superposition of the characteristic displacements

$$\sigma_{ij} = \left[D_{ijkl} - D_{ijmn} \left\{ \frac{\partial (\chi_m^{kl})^F}{\partial y_n} + \frac{\partial (\chi_m^{kl})^A}{\partial y_n} \right\} \right] \varepsilon_{kl}^0 \quad (14)$$

For the VNA solution, the stresses σ , strains ε and displacements u can be summarized in matrix forms by using displacement matrix U , strain matrix Q , and stress matrix P of the VNA solution (Nishioka, Kato and Itoh, 1997)

$$u^A = \sum_{n=1}^{N_c} U^{(n)} C^{(n)} \quad (15)$$

$$\varepsilon^A = \sum_{n=1}^{N_c} Q^{(n)} C^{(n)} \quad (16)$$

$$\boldsymbol{\sigma}^A = \sum_{n=1}^{N_c} \mathbf{P}^{(n)} \mathbf{C}^{(n)} \quad (17)$$

where $\mathbf{C}$ is the undetermined coefficients. Introducing Equations (15), the microscopic displacement Equation (11) can be expressed in matrix form

$$\mathbf{u}^1 = - \left\{ (\boldsymbol{\chi}^{kl})^F + \mathbf{U}^* (\mathbf{C}^*)^{kl} \right\} \boldsymbol{\varepsilon}^0. \quad (18)$$

Furthermore, substituting the microscopic displacement expressed by Equation (18) into Equation (12), and introducing Equations (15)-(17), we obtain the following equations

$$\begin{bmatrix} \mathbf{K}^{FF} & \mathbf{K}^{FA} \\ \mathbf{K}^{AF} & \mathbf{K}^{AA} \end{bmatrix} \begin{Bmatrix} (\boldsymbol{\chi}^{kl})^F \\ (\mathbf{C}^*)^{kl} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}^F \\ \mathbf{F}^A \end{Bmatrix} \quad (19)$$

$$\mathbf{K}^{FF} = \int_Y \mathbf{B}^T \mathbf{D} \mathbf{B} dY \quad (20)$$

$$\mathbf{K}^{AA} = \int_{\partial Y} (\mathbf{U}^*)^T \mathbf{n} \mathbf{P}^* dS \quad (21)$$

$$\mathbf{K}^{FA} = (\mathbf{K}^{AF})^T = \int_{\partial Y} \mathbf{N}^T \mathbf{n} \mathbf{P}^* dS \quad (22)$$

$$\mathbf{F}^F = \int_Y \mathbf{N}^T \mathbf{D}^{kl} dY \quad (23)$$

$$\mathbf{F}^A = \int_Y (\mathbf{Q}^*)^T \mathbf{D}^{kl} dY \quad (24)$$

and

$$\mathbf{U}^* = [\mathbf{U}^{(1)}, \mathbf{U}^{(2)}, \dots, \mathbf{U}^{(N_c)}] \quad (25)$$

$$\mathbf{Q}^* = [\mathbf{Q}^{(1)}, \mathbf{Q}^{(2)}, \dots, \mathbf{Q}^{(N_c)}] \quad (26)$$

$$\mathbf{P}^* = [\mathbf{P}^{(1)}, \mathbf{P}^{(2)}, \dots, \mathbf{P}^{(N_c)}] \quad (27)$$

$$\mathbf{C}^* = \{\mathbf{C}^{(1)}, \mathbf{C}^{(2)}, \dots, \mathbf{C}^{(N_c)}\}^T. \quad (28)$$

where $\mathbf{N}$ is the shape function matrix, $\mathbf{B}$ is the nodal displacement-strain matrix, $\mathbf{n}$ is the direction cosine matrix of a unit vector for each elliptical crack, and the superscript * denotes the assembled matrix, respectively. $\mathbf{D}^{kl}$ is the unit stress vector caused by an applied unit strain $\bar{\boldsymbol{\varepsilon}}^{kl}$ for each kl . Therefore, $\mathbf{D}^{kl}$ is expressed by the product of the k -th line components of the stress-strain matrix $\mathbf{D}$ and the applied unit strain $\bar{\boldsymbol{\varepsilon}}^{kl}$.

$$\mathbf{D}^{kl} = \mathbf{D} \bar{\boldsymbol{\varepsilon}}^{kl} \quad (29)$$

For three-dimensional problem, the characteristic displacement $(\boldsymbol{\chi}^{kl})^F$ and the coefficients $(\mathbf{C})^{kl}$ for the VNA solution are determined by solving Equation (19) for six different kl cases. Therefore,

the unknown coefficients for the VNA solution for each elliptical microcrack in the damaged material can be obtained by the following equation

$$\mathbf{C}^* = -\mathbf{C}^\Delta \mathbf{B} \mathbf{d}^0. \quad (30)$$

$$\mathbf{C}^\Delta = [(\mathbf{C}^*)^{11}, (\mathbf{C}^*)^{22}, (\mathbf{C}^*)^{33}, (\mathbf{C}^*)^{12}, (\mathbf{C}^*)^{23}, (\mathbf{C}^*)^{31}]. \quad (31)$$

Using the determined coefficients $(\mathbf{C})^{kl}$, we can evaluate the homogenized elastic tensor and the microscopic stress fields

$$\mathbf{D}^H = \frac{1}{|\mathbf{Y}|} \int_{\mathbf{Y}} \left\{ \mathbf{D} - \mathbf{D} (\mathbf{B} \boldsymbol{\chi}^\Delta + \mathbf{Q}^* \mathbf{C}^\Delta) \right\} d\mathbf{Y} \quad (32)$$

$$\boldsymbol{\sigma} = \left\{ \mathbf{D} - \mathbf{D} (\mathbf{B} \boldsymbol{\chi}^\Delta + \mathbf{Q}^* \mathbf{C}^\Delta) \right\} \boldsymbol{\epsilon}^0 \quad (33)$$

and

$$\boldsymbol{\chi}^\Delta = [\boldsymbol{\chi}^{11}, \boldsymbol{\chi}^{22}, \boldsymbol{\chi}^{33}, \boldsymbol{\chi}^{12}, \boldsymbol{\chi}^{23}, \boldsymbol{\chi}^{31}] \quad (34)$$

Furthermore, we can also evaluate the mixed-mode stress intensity factors of all elliptical microcracks in the damaged material by using the determined coefficients $(\mathbf{C})^{kl}$ (Kato and Nishioka, 2002).

3 Evaluation method for three-dimensional path independent J integral and stress intensity factor of a crack in an orthotropic material

The manifestations of mechanical properties of the damaged materials are expressed by mixed-mode stress intensity factors of the macrocrack in damaged materials in this study. The component separation method for the evaluation of the mixed-mode stress intensity factors is newly developed using the evaluation method of the path independent J integral. The damaged material is regarded as the material containing periodically lattice distributed elliptical microcracks in present homogenization analysis. The three-dimensional path independent J integral and the stress intensity factor of the macrocrack in the damaged material are evaluated by using the value of the effective elastic moduli of the damaged material that is obtained by homogenization analysis.

Using the global Cartesian coordinate system as shown in Figure 1, the three-dimensional path independent dynamic J integral is defined in following equation by Nishioka and Atluri (1983).

$$J'_k = \int_{\Gamma + \Gamma_c} [(W + K) n_k - t_i u_{i,k}] dS + \int_{V_\Gamma} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \quad (35)$$

where W and K are the strain and kinetic energy densities, respectively. Furthermore, Nishioka and co-workers (1995) derived the new expression of the dynamic J integral using the Equivalent Domain Integral (EDI) method as follows

$$J'_k = \frac{1}{f} \left\{ \int_{V_\Gamma} [\sigma_{ij} u_{i,k} s_{,j} - (W+K) s_{,k} + \rho \ddot{u}_i u_{i,k} s - \rho \dot{u}_i \dot{u}_{i,k} s] dV + \int_{\Gamma_1+\Gamma_2} [(W+K) n_k - \sigma_{ij} n_j u_{i,k}] s dS \right\} \quad (36)$$

where s is a continuous function of a segment of the crack-front (see Figure 2). The value of s is not zero on the near-field path Γ_e , and s is zero on the far-field paths $\Gamma+\Gamma_c$. f is the integral value of the s function along a segment of the crack-front.

In this paper, the 20-noded isoparametric elements are used as the elements of finite element analyses. Using the shape function of the 20-noded isoparametric elements, the s function can be interpolated by following equation, which are also used for the displacement field

$$s = \sum_{q=1}^{20} N_q(\xi, \eta, \zeta) s_q \quad (37)$$

where N_q is the shape function and s_q is the nodal value of the s function at the node q . Substitution of Equation (37) into Equation (36), we obtain following equations

$$J'_k = \frac{1}{f} \sum_{n=1}^{N_v} \sum_{q=1}^{20} R_{knq} s_q + \frac{1}{f} \sum_{n=1}^{N_s} \sum_{q=1}^8 Q_{knq} s_q \quad (38)$$

and

$$R_{knq} = \int_{V_n} [\sigma_{ij} u_{i,k} N_{q,j} - (W+K) N_{q,k} + \{\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}\} N_q] dV \quad (39)$$

$$Q_{knq} = \int_{S_n} [(W+K) n_k - \sigma_{ij} n_j u_{i,k}] N_q dS \quad (40)$$

where N_v and N_s are the total number of elements in V_Γ and on $\Gamma_1+\Gamma_2$, respectively. V_n and S_n are the volume and the surface of the n -th element, respectively. The value of dynamic J-integral is obtained by taking the total on all elements in the path.

Generally, if the side surfaces of the segment $\Gamma_1+\Gamma_2$ is on the inside of the domain of interest, the surface integral becomes useless due to setting the s function to be zero on each side surface. If the s function can be set as zero on the side surfaces of the segment, Γ_1 and Γ_2 , the second term of the dynamic J integral in Equation (38) can be set as zero. Therefore, in most cases, the dynamic J integral can be evaluated by only the volume integration term. Furthermore, if the dynamic effects are set as zero, the dynamic J integral agrees with the static J integral derived by Rice (1968). Therefore, the EDI method is effective technique for the evaluation of three-dimensional static J integral such as numerical analyses in present study.

The crack-axis components of the dynamic J integral can be expressed by using the coordinate transformation (Nishioka, 1994)

$$J_k^0 = \alpha_{kl} J_l \quad (41)$$

where α_{kl} is the coordinate transformation tensor. Equation (41) is also very effective expression for the evaluation of the crack-axis components of the dynamic J integral along the curved crack-front. The use of regular elements without the singularity results in inaccurate values for J_2^0 and J_3^0 under a mixed-mode condition. On the other hand, J_1^0 can be calculated accurately by using the ordinary element, since it becomes to be $(n_1^0)^\pm \rightarrow 0$ by $r \rightarrow 0$ on Γ_c . Using the aforementioned properties, the component separation method for evaluation of the mixed-mode stress intensity factor was proposed by Nishioka and co-workers (1991). Using the component separation method, the evaluation method of the mixed-mode stress intensity factors of a crack in the orthotropic material is newly developed in this study.

The relations between energy release rates and stress intensity factors for the crack in the orthotropic material is given in the following equations (Fracture, Volume II, Liebowitz edited)

$$G_M = R_M K_M^2 \quad (M = I, II, III) \quad (42)$$

$$R_I = \left(\frac{b_{11} b_{22}}{2} \right)^{1/2} \left[\left(\frac{b_{22}}{b_{11}} \right)^{1/2} + \frac{2b_{12} + b_{66}}{2b_{11}} \right]^{1/2} \quad (43)$$

$$R_{II} = \frac{b_{11}}{\sqrt{2}} \left[\left(\frac{b_{22}}{b_{11}} \right)^{1/2} + \frac{2b_{12} + b_{66}}{2b_{11}} \right]^{1/2} \quad (44)$$

$$R_{III} = \frac{1}{2 [c_{55} c_{66} - c_{56}^2]^{1/2}} \quad (45)$$

and

$$b_{11} = \frac{a_{11}a_{33} - a_{13}^2}{a_{33}} \quad (46)$$

$$b_{22} = \frac{a_{22}a_{33} - a_{23}^2}{a_{33}} \quad (47)$$

$$b_{12} = \frac{a_{12}a_{33} - a_{13}a_{23}}{a_{33}} \quad (48)$$

$$b_{66} = \frac{a_{44}a_{33} - a_{34}^2}{a_{33}} \quad (49)$$

where a_{ij} is the components of the strain-stress matrix and c_{ij} is the components of the inverse matrix of the strain-stress matrix. The relations between the crack-opening displacements δ_I , δ_{II} and δ_{III} near the crack tip, and the mixed-mode stress intensity factors are expressed in the following equations (Fracture, Volume II, Liebowitz edited)

$$\frac{K_{II}}{K_I} = \frac{\delta_{II}}{\delta_I} \frac{b_{22} \operatorname{Re} \left[i \frac{(s_1 + s_2)}{s_1 s_2} \right]}{b_{11} \operatorname{Im} [(s_1 + s_2)]} \quad (50)$$

$$\frac{K_{III}}{K_I} = \frac{\delta_{III}}{\delta_I} \frac{b_{22} \operatorname{Re} \left[i \frac{(s_1 + s_2)}{s_1 s_2} \right]}{(c_{55}c_{66} - c_{56}^2)^{1/2}} \quad (51)$$

and

$$s_1 s_2 = - (b_{22}/b_{11})^{1/2} \quad (52)$$

$$s_1 + s_2 = i \sqrt{2} \left[\left(\frac{b_{22}}{b_{11}} \right)^{1/2} + \frac{b_{12} + b_{66}}{2b_{11}} \right]^{1/2} \quad (53)$$

As the dynamic J integral is equivalent to the energy release rate, the J integral on the crack tip coordinate system is expressed as following equation (Nishioka, 1994).

$$J_1^0 = G_I + G_{II} + G_{III} \quad (54)$$

Substituting Equation (42), (50) and (51) into Equation (54), the evaluation method of the mixed-mode stress intensity factors for the crack in the orthotropic material is obtained

$$K_M = \delta_M Q_M \bar{K} \quad (M = I, II, III) \quad (55)$$

and

$$\bar{K} = \left(\frac{J_1^0}{\delta_I^2 R_I Q_I^2 + \delta_{II}^2 R_{II} Q_{II}^2 + \delta_{III}^2 R_{III} Q_{III}^2} \right)^{1/2} \quad (56)$$

$$Q_I = 1 \quad (57)$$

$$Q_{II} = \frac{b_{22} \operatorname{Re} \left[i \frac{(s_1 + s_2)}{s_1 s_2} \right]}{b_{11} \operatorname{Im} [(s_1 + s_2)]} \quad (58)$$

$$Q_{III} = \frac{b_{22} \operatorname{Re} \left[i \frac{(s_1 + s_2)}{s_1 s_2} \right]}{(c_{55}c_{66} - c_{56}^2)^{1/2}} \quad (59)$$

For the component separation method, the signs of the crack-opening displacements near the crack tip determine the signs of the stress intensity factors.

4 Estimation of mechanical properties of damaged material

4.1. COMPARISON WITH OTHER STUDIES

The damaged material containing lattice distributed elliptical microcracks is expressed by collection of unit cells of identical dimensions as shown in Figure 3. The sizes of the unit cell in the X_1 , X_2 , and X_3 directions are denoted by L_1 , L_2 and L_3 , and elliptical crack axes in the X_3 and X_1 directions are expressed by a_1 and a_2 . The aspect ratio of elliptical microcrack in the unit cell is assumed to be $a_2/a_1=0.5$, and the sizes of the unit cell are set by $L_1/L_3=0.5$ and $L_2/L_3=1.0$. Poisson ratio of the material is assumed to be $\nu=0.3$. The finite element mesh pattern for present analyses is

shown in Figure 4. We can see that each element is coarse regular mesh pattern that does not describe the shape of the embedded elliptical crack.

Figures 5 and 6 show the variations of the effective Young's modulus and Poisson's ratio of the damaged material against the crack density ε defined by Budiansky and O'Connell (1976). The elastic moduli E^0 and ν^0 are the effective elastic moduli of the non-damaged material, and the effective elastic moduli are normalized by these values. The crack density ε for the unit cell containing the elliptical microcrack can be expressed in following equation (Nishioka and Kato, 1998).

$$\varepsilon = (1-k^2)a_1^3\pi N_c/4L_1L_2L_3E(k) \quad (60)$$

where $k^2=1-(a_2/a_1)^2$, $E(k)$ is the complete elliptic integral of the second kind, and N_c is the number of microcracks per unit volume. Furthermore, the effective elastic moduli evaluated by Budiansky and O'Connell (1976), and Dilute Distribution method (Nemat-Nasser and Hori, 1993) are shown in Figures 5 and 6. The effective Young's moduli of the present results are in agreement with the results obtained by other method. However, the present results are slightly different from those of other method. This can be attributed to the following reason. The results by Budiansky and O'Connell are the effective elastic moduli of the damaged material containing randomly distributed microcrack, and the results by Nemat-Nasser and Hori are the effective elastic moduli of the damaged material containing randomly distributed penny shaped microcrack. In other words, the present results are the orthotropic effective elastic moduli for periodic configurations while the results by other method are isotropic effective elastic moduli for random configurations.

Furthermore, the effective Poisson's ratio ν_{31}^* agrees with the non-axial effective Poisson's ratio ν^* obtained by other methods. The effective Poisson's ratio ν_{32}^* is different from ν^* at large crack density due to the influence of the elliptical shape of the microcracks in the damaged material on the effective Poisson's ratios.

4.2. EVALUATION OF EFFECTIVE ELASTIC MODULI OF DAMAGED MATERIAL

To investigate the effects of the aspect ratios of the microcracks on the macroscopic mechanical properties of the damaged material, the numerical analyses of the damaged material are carried out for $a_2/a_1=0.2, 0.4, 0.6$, and 0.8 . The size of the unit cell are set as $L_1/L_3=0.2, 0.4, 0.6, 0.8$ and $L_2/L_3=1.0$, respectively.

The variations of the effective Young's moduli against the crack density for various aspect ratios of the microcrack are shown in Figures 7a-d. For a same microcrack density, the effective

Young's modulus E_2^* decreases for decreasing aspect ratio. This is because the microcracks with smaller aspect ratios have larger cracked areas than the microcracks with larger aspect ratios for the same microcrack density. For all aspect ratios of the microcracks, the effective Young's moduli E_1^* and E_3^* are almost constant for increasing crack density.

The variations of the effective Poisson's ratios are shown in Figures 8a-d, and Figures 9a-d. For all aspect ratios of the microcracks, the effective Poisson's ratio ν_{31}^* is almost constant for increasing crack density. For a same microcrack density, the values of the effective Poisson's ratios ν_{12}^* and ν_{23}^* for the microcracks with smaller aspect ratios are larger than those of the microcracks with larger aspect ratios. On the contrary, the effective Poisson's ratio ν_{32}^* of the microcracks with larger aspect ratios is smaller than those of the microcracks with smaller aspect ratios.

Furthermore, the variations of effective shear moduli are shown in Figures 10a-d. For all aspect ratios of the microcracks, the effective shear modulus G_{31}^* is almost constant for increasing crack density. The effective shear moduli G_{12}^* and G_{23}^* decrease linearly for increasing crack density. The gradient of the variation of G_{12}^* increases for increasing aspect ratio. For the case of $a_2/a_1=0.2$, G_{12}^* is almost constant for increasing crack density.

5 Estimation of mechanical effect of microcracks on macrocrack

The analysis of the macrocrack in the damaged material (see Figure 11) is carried out in this section. The macrocrack is assumed to be sufficiently long at X_3 direction. The elliptical microcracks are assumed to be sufficiently small in comparison with the macrocrack geometry. From this analytical model, we can conclude that the positional relation between each microcrack and the macrocrack tip are not influenced on analytical results of the macrocrack. Therefore, Equation (6) at the macroscopic scale in the homogenization analysis is solved under the plane strain state in this study.

Solving Equation (6), both stress fields and strain fields at the macroscopic scale are evaluated by using the effective elastic moduli that is obtained in section 4.2. For such macroscopic fields, three-dimensional path independent J integral is evaluated by using EDI method. The crack-axis components of the static J integral are denoted by J^0 . Integral paths of the J integral are shown in Figure 12. The broken lines in the Figure 12 show the integral paths, and these of the integral paths from the inside are numbered from Path No.1 to Path No.3.

In Figure 13, the variations of the effective J integral J_1^{0*} of the macrocrack against the integral paths are shown for the microcrack density $\epsilon=0.03$. $\bar{J}_1^0$ is the J integral of the macrocrack in

the non-damaged material. For each aspect ratio of the microcrack, the variations of J_1^{0*} of each integral path show good path independency. The variations of J_1^{0*} for each aspect ratio of the microcracks against the crack density are shown in Figure 14. For all aspect ratios, J_1^{0*} increases for increasing crack density. For a same crack density, J_1^{0*} increases for decreasing aspect ratios.

The variations of effective stress intensity factor K_I^* of the macrocrack in the damaged materials against the crack density are shown in Figure 15. The stress intensity factor K_I^* is normalized by the stress intensity factor of the macrocrack in the non-damaged material $\bar{K}_I$. For all aspect ratios, K_I^* decreases in comparison with $\bar{K}_I$ for increasing crack density. This is because the macrocrack experiences dissipations of stress due to the stiffness reduction caused by the existence of cluster of microcracks. We can see that the variations of effective stress intensity factor K_I^* are similar to the variations of effective Young's modulus E_2^* for varying microcrack aspect ratio.

6 Concluding remarks

The applicability of the present homogenization method based on the superposition method together with the VNA solution was demonstrated for damaged material containing lattice distributed elliptical microcracks in this study. The mechanical effects appearing in the elastic moduli were evaluated for varying crack density, and for different aspect ratios of the elliptical microcracks. The differences in the variations of the mechanical properties of the damaged materials were clarified by the evaluation of the variations of the effective elastic moduli of the damaged material in this study.

For present homogenization method, the VNA solution evaluates the singularity of the elliptical cracks in detail. Therefore, it is possible to evaluate the effective elastic moduli of the damaged material by using rough mesh pattern. Furthermore, the present homogenization method can also evaluate detailed stress fields around the microcrack and the stress intensity factor of each microcrack in the damaged material for the macroscopic problem.

Furthermore, the macrocrack in the damaged material was analyzed in order to evaluate the mechanical effects of the microcracks on the macrocrack. The evaluation method of mixed-mode stress intensity factors of a crack in orthotropic materials by the component separation method based on the three-dimensional path independent J integral was newly developed in this study. The shielding effects appearing in the stress intensity factors were investigated for varying crack density, for different aspect ratios of the elliptical microcracks in the damaged material. The stress intensity factors of the macrocrack were reduced for increasing crack density, in comparison with the stress

intensity factor in the non-damaged material. These results denote the improvement of fracture toughness of the macrocracked material due to the mechanical effects of the distribution of microcrack on the macrocrack.

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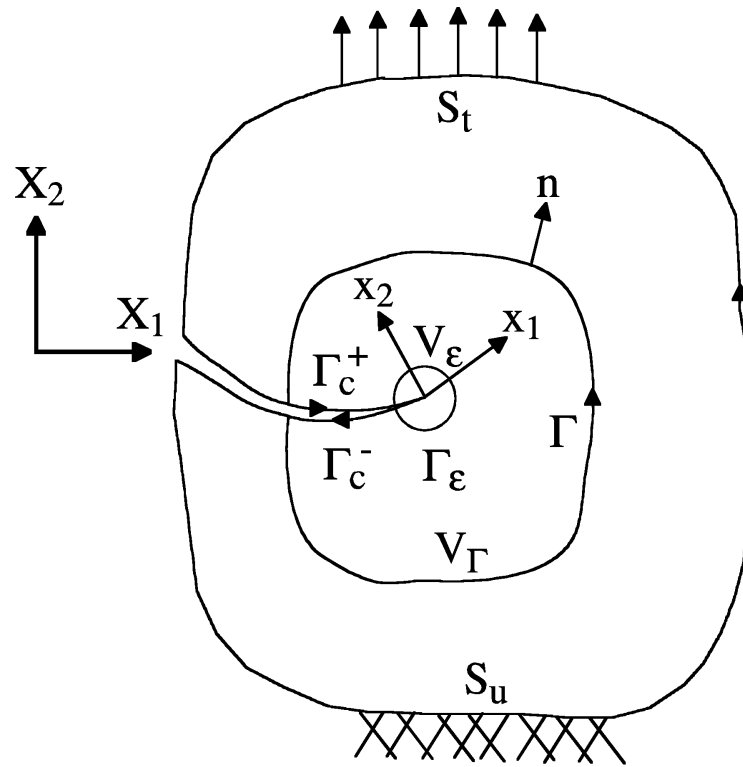


Figure 1 Contours for the path independent J integral

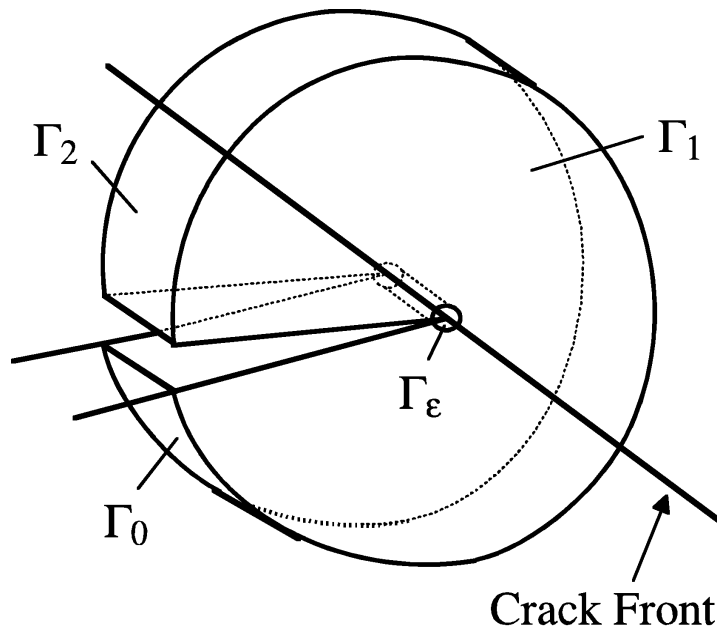


Figure 2 Crack front segment for the evaluation of the J integral

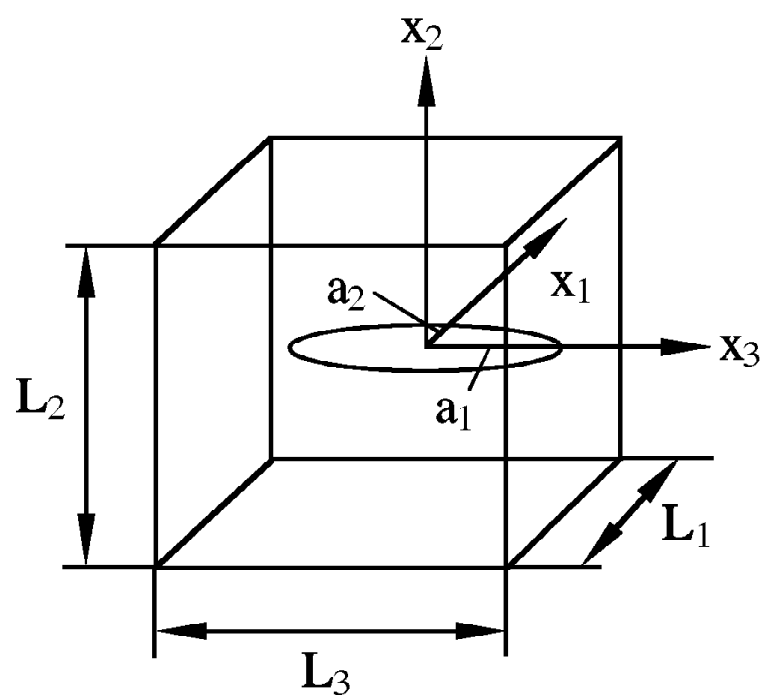


Figure 3 Unit cell

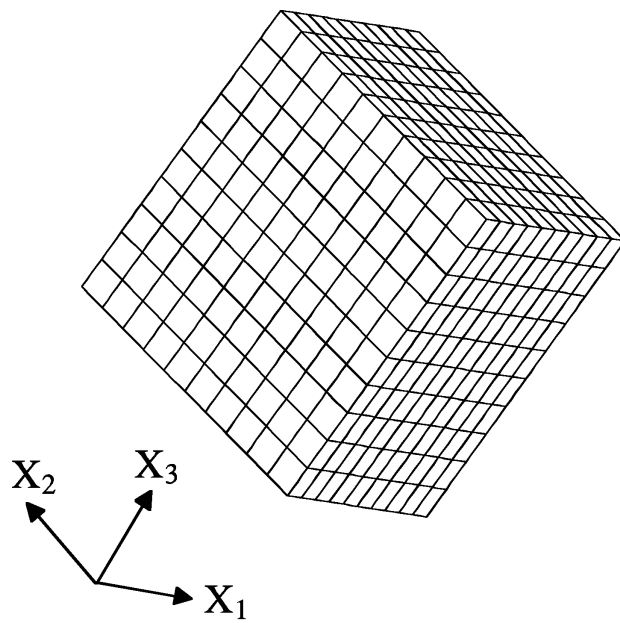


Figure 4 Finite element mesh pattern for the unit cell

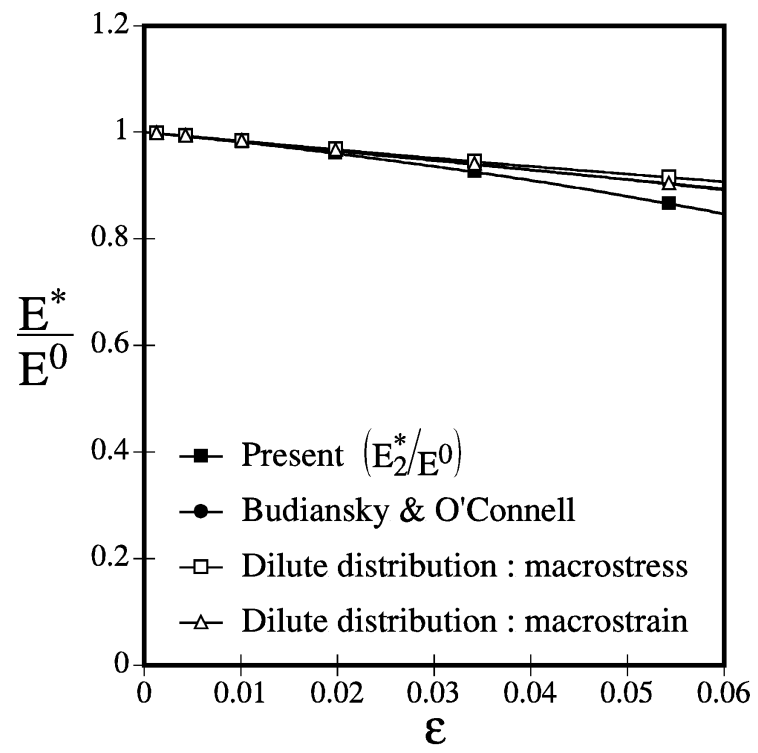


Figure 5 Comparison with other studies (Effective Young's moduli)

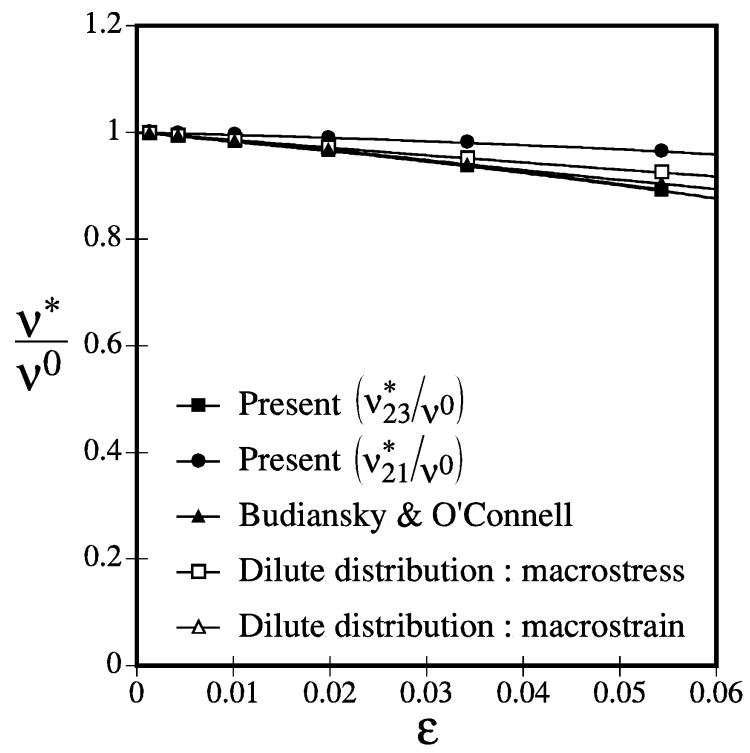
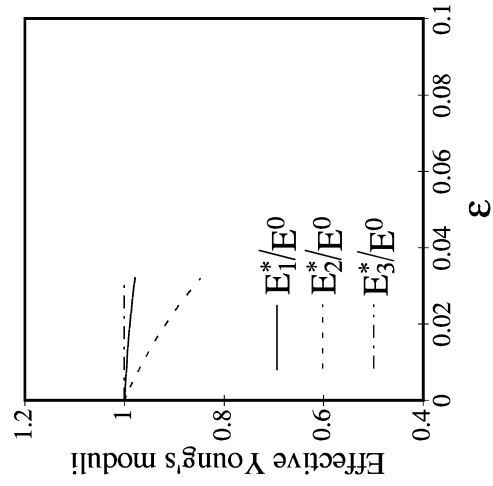
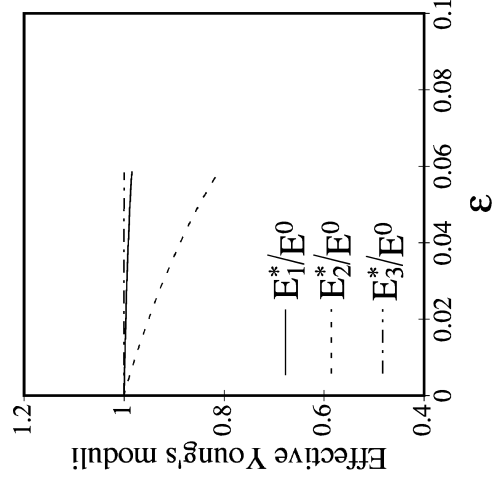


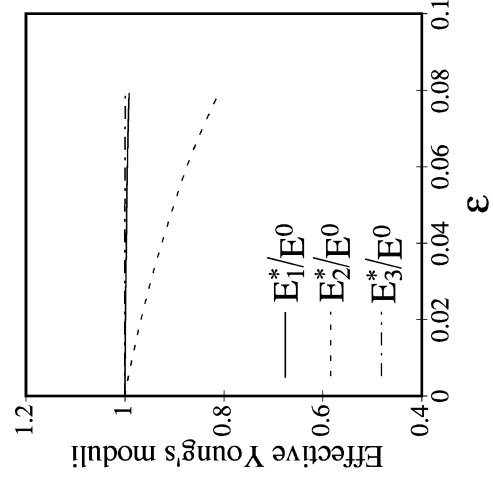
Figure 6 Comparison with other studies (Effective Poisson's ratios)



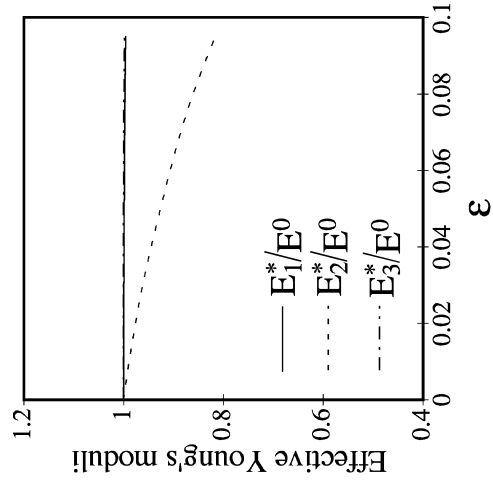
(a) $a_2/a_1=0.2$



(b) $a_2/a_1=0.4$

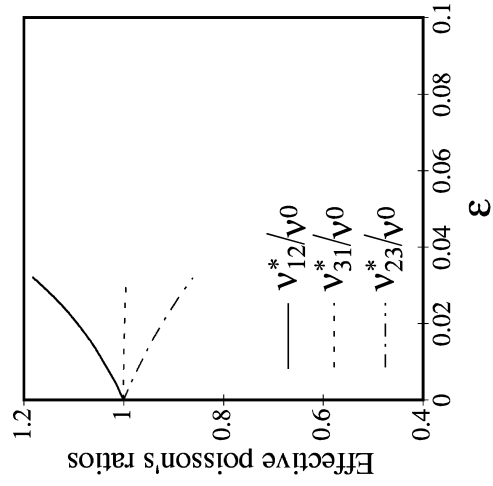


(c) $a_2/a_1=0.6$

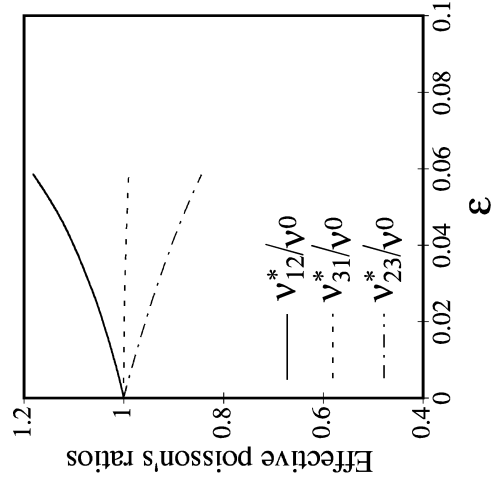


(d) $a_2/a_1=0.8$

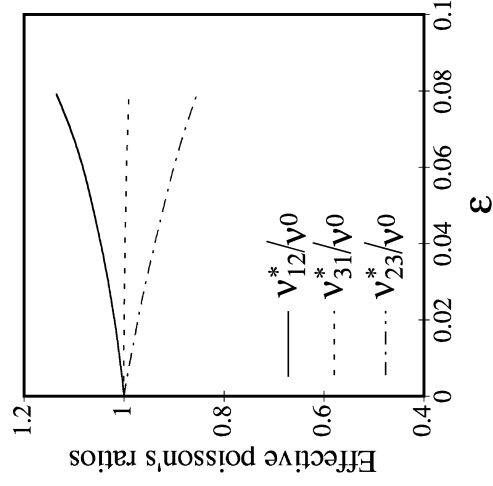
Figure 7 Effective Young's moduli of the damaged materials



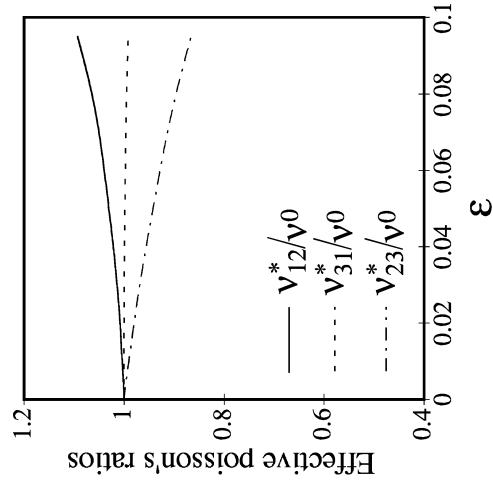
(a) $a_2/a_1=0.2$



(b) $a_2/a_1=0.4$



(c) $a_2/a_1=0.6$



(d) $a_2/a_1=0.8$

Figure 8 Effective Poisson's ratios of the damaged materials

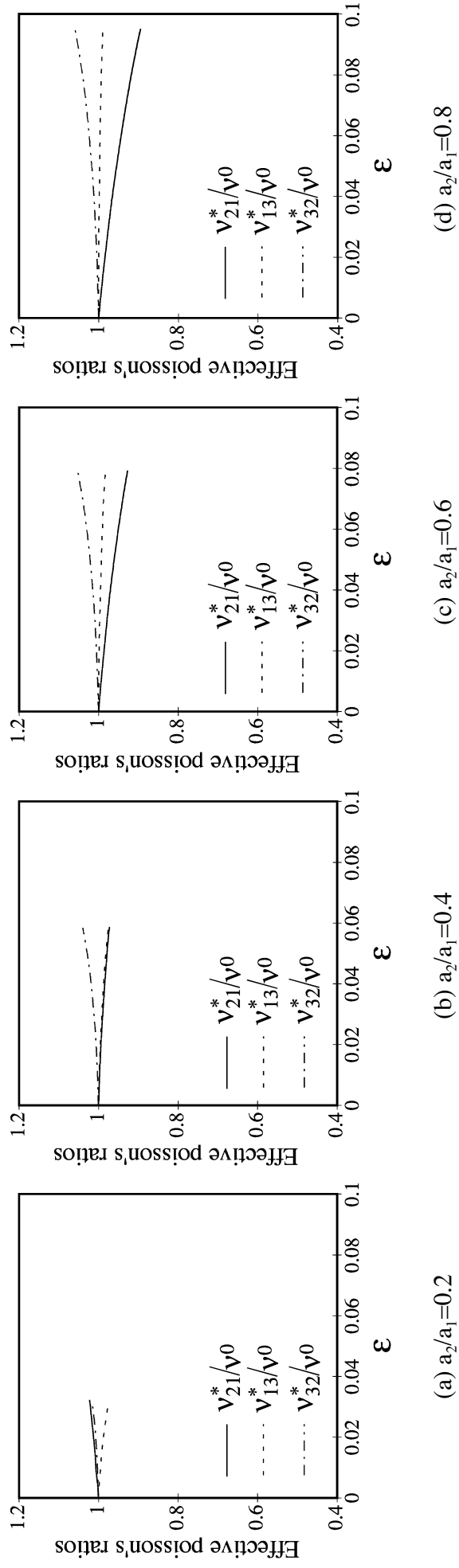
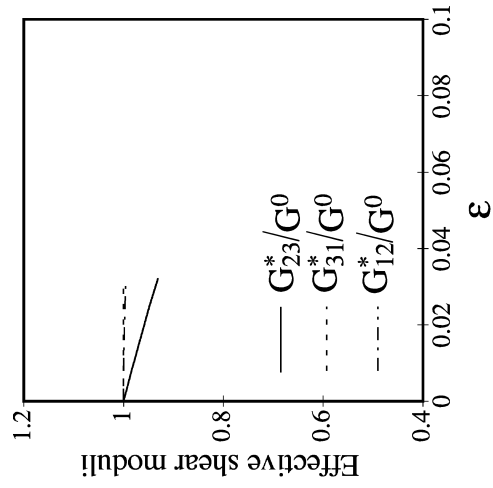
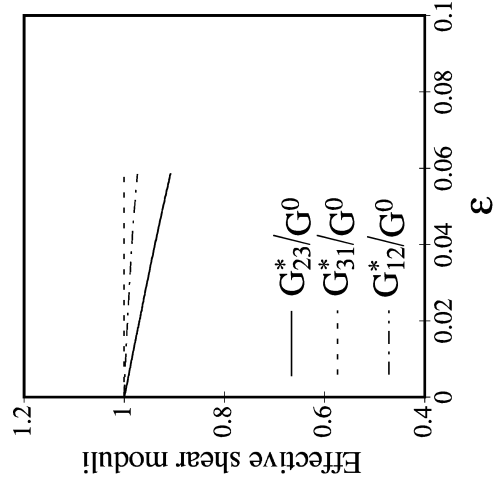


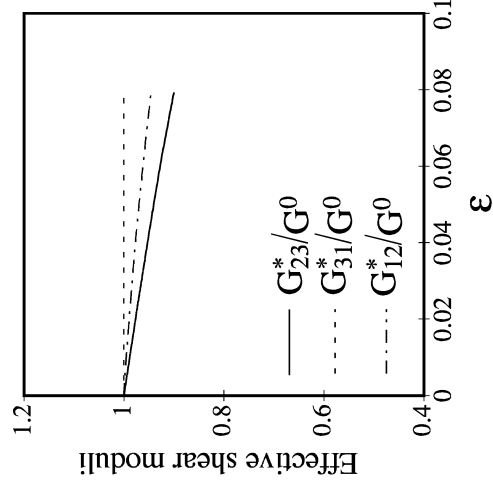
Figure 9 Effective Poisson's ratios of the damaged materials



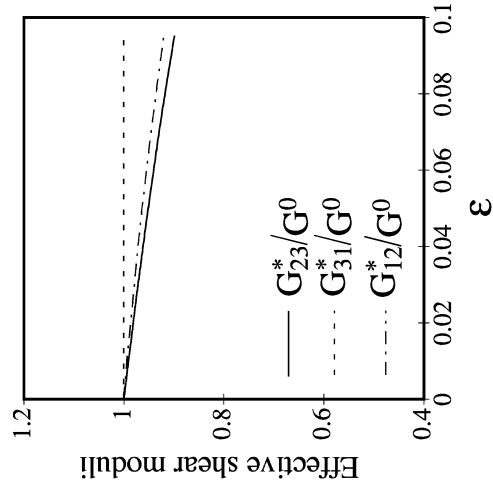
(a) $a_2/a_1=0.2$



(b) $a_2/a_1=0.4$



(c) $a_2/a_1=0.6$



(d) $a_2/a_1=0.8$

Figure 10 Effective Shear moduli of the damaged materials

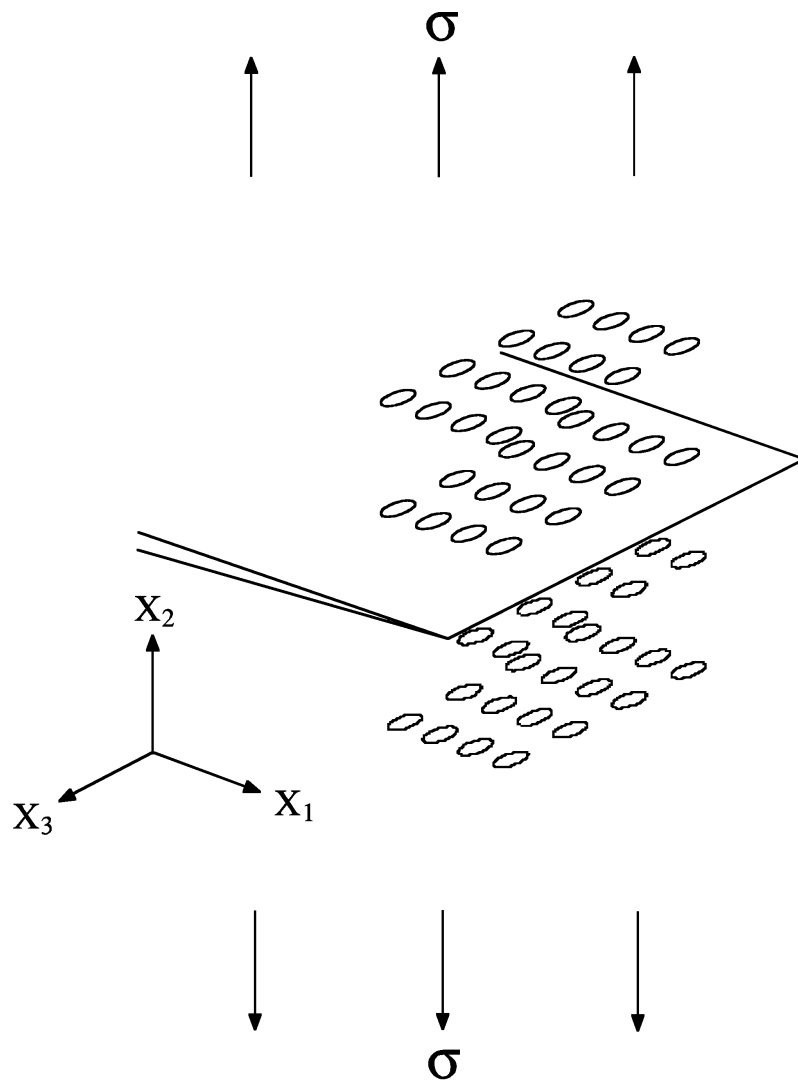


Figure 11 A macrocrack in a damaged material containing lattice distributed parallel micro elliptical cracks

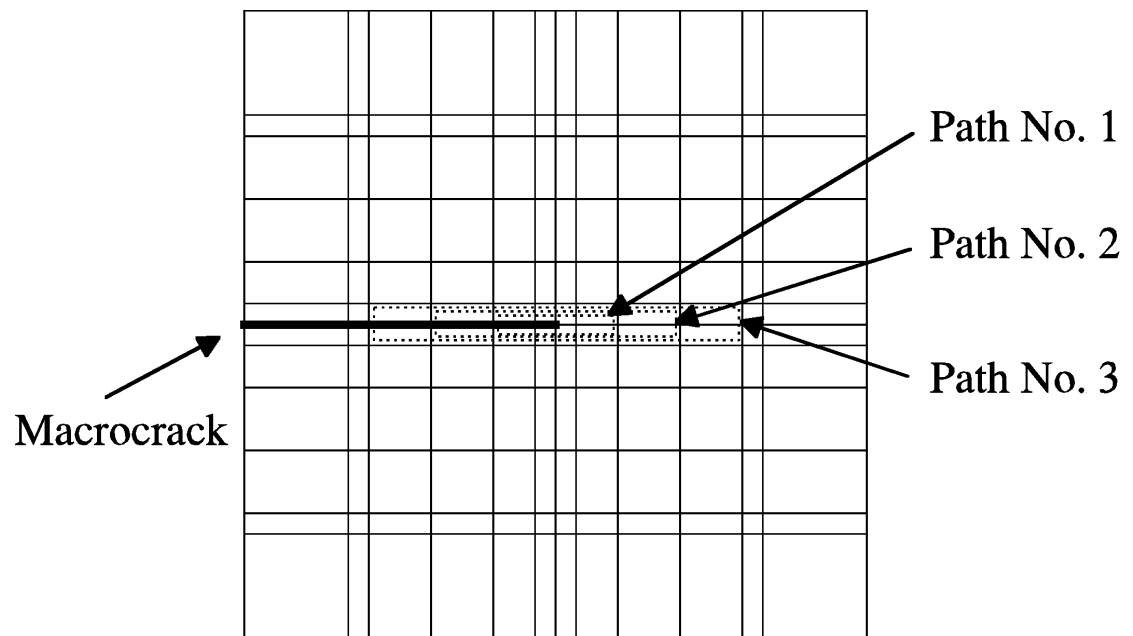


Figure 12 Integral paths of the J integral for the macrocrack in the damaged materials

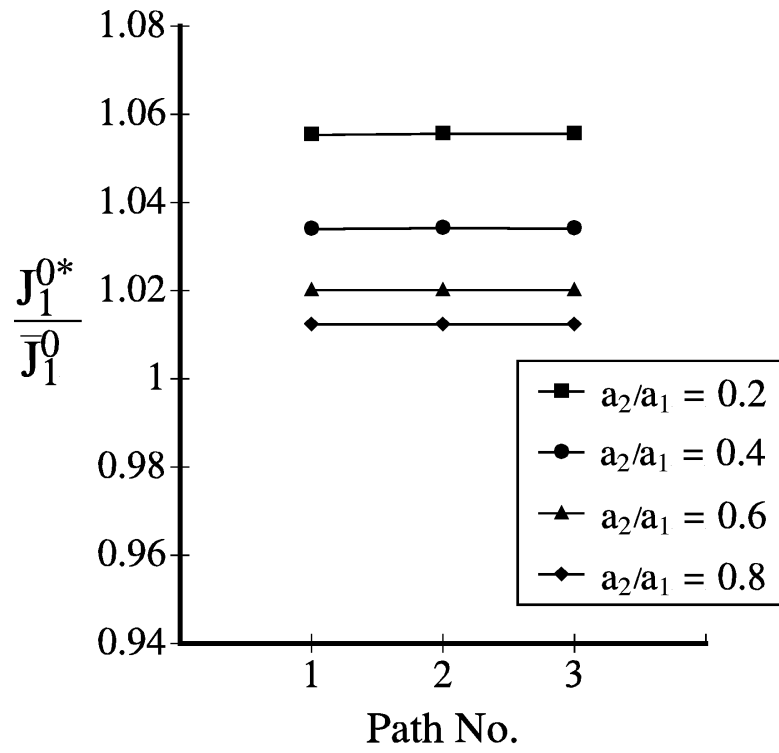


Figure 13 J integrals for the macrocrack in the damaged materials ($\varepsilon=0.03$)

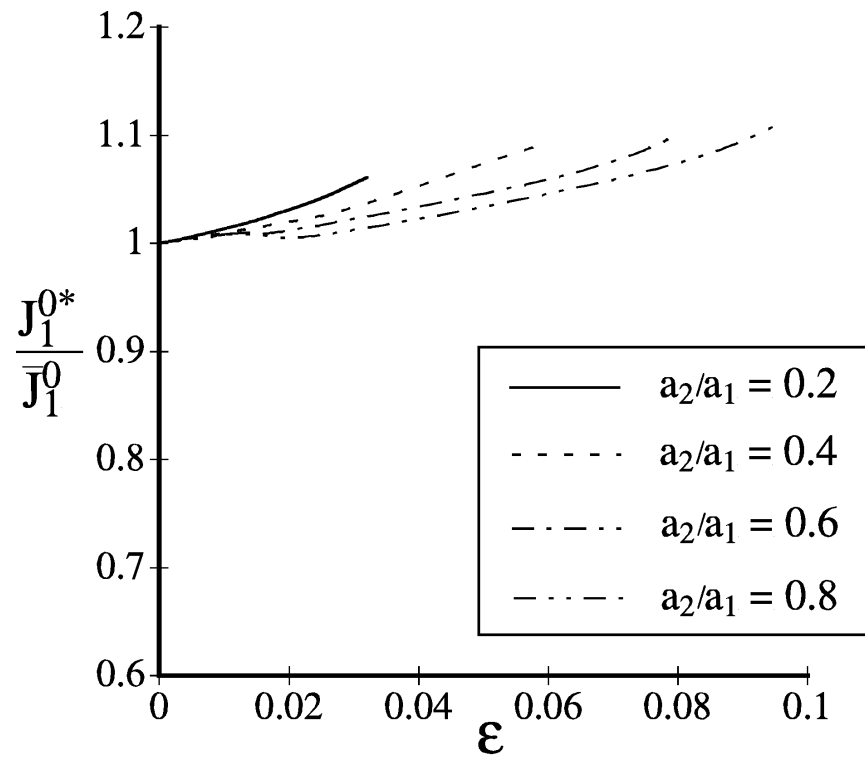


Figure 14 Variations of J integrals for the macrocrack in the damaged materials

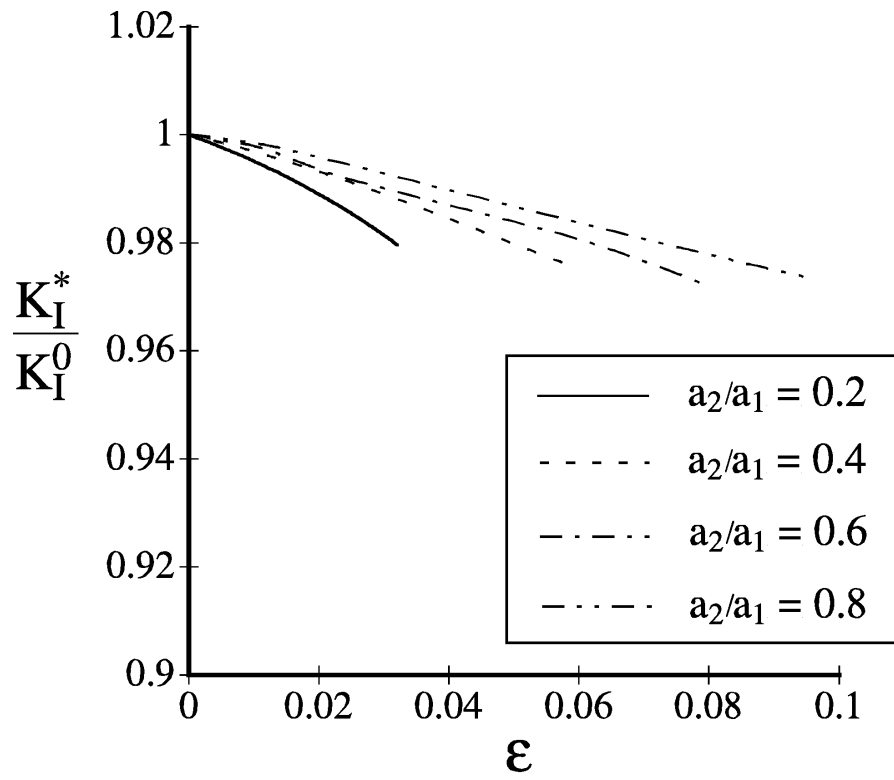


Figure 15 Variations of stress intensity factors for the macrocrack in the damaged materials