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Industry profits and free entry in input markets*

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Abstract

When upstream firms compete in quantity and freely enter the input market, competition among downstream firms reduces the input price (the marginal cost of downstream firms). The industry profits of downstream firms competing in quantity may increase with the number of downstream firms.

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Key Words: Cournot competition, input markets, free entry, profits

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1 Introduction

In the standard Cournot model of oligopoly, as the number of firms competing in the market increases, industry profits (the sum of the firms' profits) decrease through increased product market competition. The nature of the relationship between the number of firms and industry profits influences the incentives for firms. For instance, when firms collude in a product market, they control their own quantities supplied like a monopolist, because their monopolistic behavior maximizes the industry profits.

In this paper, we show that, under free entry into input markets, the relationship between industry profits and the number of downstream firms depends on fixed costs (the ease of entry) in the input markets. When the fixed costs are large, industry profits can increase with the number of downstream firms. We also show that, as the number of downstream firms increases, the input price decreases.

Naylor (2002) also shows a situation in which industry profits increase as the number of downstream firms increases. However, the structure of his model is quite different from ours. Naylor (2002) considers a quantity-setting model in which input prices are not exogenous but are determined by bargaining in a bilateral oligopoly. A pair made up of a labor union (an upstream firm) and a downstream firm bargain for the wages of the laborers (the wholesale price). The bargaining structure is similar to that in Horn and Wolinsky (1988), who discuss Nash bargaining.

On the contrary, in our model, upstream firms compete in quantity and freely enter into input markets. Each upstream firm faces the derived demand of downstream firms for input. The wholesale price is determined by the quantities supplied by upstream firms. Negotiations between upstream and downstream firms do not exist.

The driving force of our result is as follows. The number of downstream firms affects the derived demand for input. The increase in the number of downstream firms enhances the derived demand and attracts potential entrants to the input market. The additional entries reduce the input price and are beneficial to the downstream firms. Under some conditions, the reduction of the input price has a significant effect on industry profits.

As mentioned earlier, in our model, as the number of downstream firms increases, the input price decreases. This property is related to that in Lahiri and Ono (1995). They introduce the

Cournot oligopoly to the Heckscher-Ohlin model and show that free trade reduces the oligopoly price and increases welfare under free entry. Free trade enlarges the market size for the oligopolists and induces additional entries. The entries reduce the oligopoly price. The mechanism of their result is similar to that of ours, but they do not consider industry profits and a vertical relationship between upstream and downstream firms.

The rest of the paper is organized as follows. Section 2 contains the basic model. Section 3 has the main results. Section 4 concludes the paper.

2 The model

The setting of the model is somewhat similar to those in Salinger (1988) and Lin (2004). There are m upstream firms and n downstream firms. As discussed later, m is endogenously determined, and n is exogenously given. All downstream firms buy an input from the upstream firms and then transform it into the final product. One unit of the final product requires exactly one unit of input. The unit cost of producing the input is c. For simplicity, c and the cost of transforming the input into the final product are normalized to zero. The demand for the final product is given by $p = a - bQ$, where Q is the quantity supplied by the downstream firms. Upstream firms freely enter the input market. When an upstream firm enters the market, it incurs a fixed cost, F. Free entry into input markets is not considered by Salinger (1988) and Lin (2004).

The input price is determined by Cournot competition at both levels of the industry: The downstream firms choose their output levels given the input price, leading to the derived demand for input; the upstream firms then compete in a Cournot fashion with respect to the derived demand.

3 Results

Given the input price w, downstream firm i $(i = 1, 2, \ldots, n)$ maximizes the following function:

$$\pi_i = (a - b(Q_{-i} + q_i) - w)q_i,$$

where Q_{-i} is the sum of the quantity supplied by the other firms. The first-order conditions lead to

$$q_i = \frac{a-w}{b(n+1)}, \quad p = \frac{a+nw}{n+1}, \quad \pi_i = \frac{1}{b}\left(\frac{a-w}{n+1}\right)^2, \quad Q = \frac{n(a-w)}{b(n+1)}.$$

Using the last equation, we have the derived demand for input:

$$w = a - \frac{b(n+1)}{n}Q_d. \tag{1}$$

Upstream firm j ($j = 1, 2, \ldots, m$) maximizes the following function:

$$\pi_j = \left(a - \frac{b(n+1)}{n}(Q_{-j} + q_j)\right) q_j.$$

The first-order conditions lead to

$$q_j = \frac{na}{(m+1)(n+1)b}, \quad w = \frac{a}{m+1}, \quad \pi_j = \frac{na^2}{(m+1)^2(n+1)b}, \quad Q_d = \frac{mna}{(m+1)(n+1)b}. \tag{2}$$

Upstream firms freely enter the input market. Their actions lead the profit of each upstream firm to zero. Therefore, $\pi_j - F = 0$, that is,

$$\pi_j = \frac{na^2}{(m+1)^2(n+1)b} = F, \quad \rightarrow \quad m+1 = a\sqrt{\frac{n}{b(n+1)F}}, \quad w = \sqrt{\frac{b(n+1)F}{n}}. \tag{3}$$

To assure that the number of upstream firms is larger than 1, we assume that $F < a^2/(8b)$. From (3), we have the following lemma:

Proposition 1 *As the number of downstream firms increases, the input price w decreases.*

Under free entry into the input market, the gross profit of each upstream firm must be equal to F. As the number of downstream firms increases, the slope of the demand function for the input becomes gentler (see w in (1)). The average total cost curve and the slope of the demand function for the input must come in contact with each other (see Figure 1). We find that the gentler the slope of the demand function for the input is, the lower the input price is. Therefore, Proposition 1 holds.

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Figure 1

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In this paper, we assume a linear demand function. If we assume a general inverse demand function $P(Q)$ which satisfies $P'''(Q) \geq 0$, we can derive Proposition 1 (see Appendix).

Note that, in the analysis, we ignore the integer problem in the number of upstream firms. When we take the problem into account, the number of entrants may not change with the number of downstream firms. Even though we take the integer problem into account, the qualitative property of our result holds, if the rise in the number of downstream firms increases the number of upstream firms. (Obviously, the rise in the number of downstream firms never decreases the number of upstream firms.)

Substituting the last equation in (3) into the profit of each downstream firm, we have:

$$\pi_i = \frac{1}{b}\left(\frac{a-w}{n+1}\right)^2 = \frac{1}{b}\left(\frac{a}{n+1} - \sqrt{\frac{bF}{n(n+1)}}\right)^2.$$

We now compare the industry profits in the case of a downstream monopoly and downstream duopoly. The profits are:

$$\begin{aligned}
\Pi_M &\equiv \pi_1 = \frac{1}{b}\left(\frac{a}{2} - \sqrt{\frac{bF}{2}}\right)^2, \\
\Pi_D &\equiv \sum_{i=1}^{2} \pi_i = \frac{2}{b}\left(\frac{a}{3} - \sqrt{\frac{bF}{6}}\right)^2.
\end{aligned}$$

From the equations, we have

Proposition 2 *If the following inequality holds, the industry profits in the case of a downstream duopoly are larger than those in the case of a downstream monopoly:*

$$0.0486a^2/b \simeq \frac{(85 - 60\sqrt{2} - 48\sqrt{3} + 34\sqrt{6})a^2}{6b} < F < \frac{a^2}{8b}.$$

The result provides a market structure in which industry profits increase with the number of downstream firms. In this paper, as mentioned in Proposition 1, the input price decreases with the number of downstream firms. This feature crucially depends on the free entry condition in the input market. When the number of upstream firms is exogenously given, the increase in the number of downstream firms does not affect the input price but affects the quantity supplied by each upstream firm (see w and q_j in (2)). Under free entry, the increase in the derived demand for input leads to a decrease in input price. When the fixed cost is large, the reduction of the input price is effective because the number of upstream firms is not large, and the input price is then set at a high level (see w in (3)). Therefore, Proposition 2 holds true.

Finally, we briefly discuss the industry profits in the case of n downstream firms. The industry profit is larger than in the case of the downstream monopoly if and only if

$$K(n)a^2/b < F < a^2/8b, \quad \text{where} \quad K(n) \equiv \frac{(n+1-2\sqrt{n})^2}{2(n+1)(\sqrt{n+1}-\sqrt{2})^2}.$$

$K(n)$ is increasing in n (see Figure 2). When $n = 2$ or $n = 3$, the industry profits may be larger than those in the case of a downstream monopoly.

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Figure 2

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4 Conclusion

In the paper, we consider an oligopoly model in which upstream firms compete in quantity and freely enter the input market. Each upstream firm faces the derived demand of downstream firms for input. The wholesale price is determined by the quantities supplied by upstream firms. Taking into account the wholesale price, each downstream firm sets its quantity supplied. We show that competition among downstream firms reduces the input price. Due to the reduction in the input price, the industry profits of downstream firms competing in quantity may increase with the number of downstream firms.

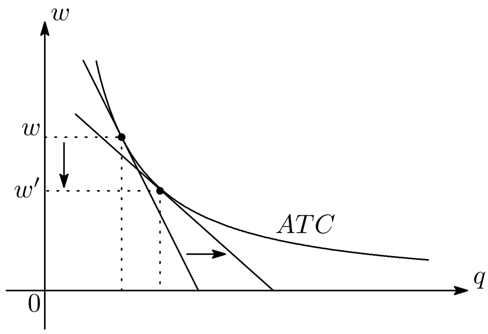

Figure 1: The input price(s) under free entry into the input market.

(The arrow indicates the increase in the number of downstream firms)

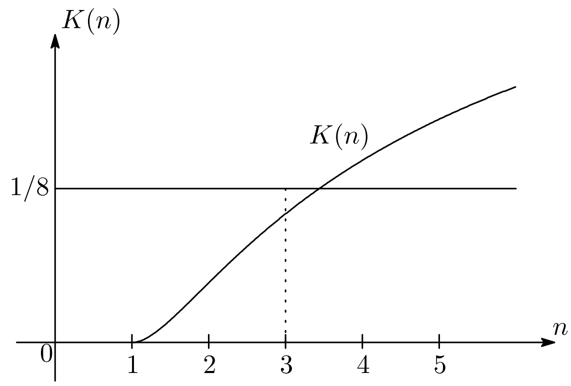

Figure 2

Appendix

We now suppose that the demand for the final product is given by $P(Q)$, where Q is the quantity supplied by the downstream firms. Given the input price w, downstream firm i ($i = 1, 2, \ldots, n$) maximizes the following function:

$$\pi_i = (P(Q_{-i} + q_i) - w)q_i,$$

where Q_{-i} is the sum of the quantity supplied by the other firms. The first-order conditions lead to

$$P(Q) - w + P'(Q)q_i = 0, \quad \Rightarrow \quad nP(Q) - nw + P'(Q)Q = 0.$$

Using the last equation, we have the derived demand for input:

$$w(Q, n) = P(Q) + \frac{P'(Q)Q}{n}. \tag{4}$$

Upstream firm j ($j = 1, 2, \ldots, m$) maximizes the following function:

$$\pi_j = w(Q_{-j} + q_j, n)q_j.$$

The first-order conditions lead to

$$\frac{\partial w(Q, n)}{\partial Q} q_j + w(Q, n) = 0, \quad \Rightarrow \quad \frac{\partial w(Q, n)}{\partial Q} Q + mw(Q, n) = 0. \tag{5}$$

Upstream firms freely enter the input market. Their actions lead to the profit of each upstream firm becoming zero. Therefore, $\pi_j - F = 0$, that is,

$$w(Q, n)q_i = F, \quad \rightarrow \quad w(Q, n)Q = mF. \tag{6}$$

Substituting m in (6) into (5), we have:

$$F \frac{\partial w(Q^*(n), n)}{\partial Q} + [w(Q^*(n), n)]^2 = 0, \tag{7}$$

where $Q^*(n)$ is the total quantities supplied by the downstream firms in equilibrium. In equilibrium, the input price, $w(Q, n)$, is $w(Q^*(n), n)$.

We now consider the relation between $w(Q^*(n), n)$ and n. Differentiating $w(Q^*(n), n)$ with respect to n, we have:

$$\frac{dw(Q^*(n), n)}{dn} = \frac{\partial w(Q^*(n), n)}{\partial Q} \frac{dQ}{dn} + \frac{\partial w(Q^*(n), n)}{\partial n}. \tag{8}$$

The total quantity supplied, $Q^*(n)$, depends on n. To check the property of $Q^*(n)$, we derive the total differential of (7):

$$\left(F\frac{\partial^2 w(Q^*(n),n)}{\partial Q^2}+2w(Q^*(n),n)\frac{\partial w(Q^*(n),n)}{\partial Q}\right)dQ + \left(F\frac{\partial^2 w(Q^*(n),n)}{\partial Q\partial n}+2w(Q^*(n),n)\frac{\partial w(Q^*(n),n)}{\partial n}\right)dn=0. \tag{9}$$

Substituting (9) into (8), we have:

$$\frac{dw(Q^*(n),n)}{dn}=F\cdot\frac{\dfrac{\partial w}{\partial n}\dfrac{\partial^2 w}{\partial Q^2}-\dfrac{\partial w}{\partial Q}\dfrac{\partial^2 w}{\partial Q\partial n}}{F\dfrac{\partial^2 w}{\partial Q^2}+2w\dfrac{\partial w}{\partial Q}}. \tag{10}$$

We now check the sign of the fraction in (10). To check the sign, we consider two components of the fraction: the denominator and the numerator.

First, we show that the denominator of the fraction is negative. Substituting (7) into the denominator, we have:

$$\begin{aligned} F\frac{\partial^2 w}{\partial Q^2}+2w\frac{\partial w}{\partial Q} &= -\frac{[w(Q^*(n),n)]^2}{\frac{\partial w(Q^*(n),n)}{\partial Q}}\frac{\partial^2 w(Q^*(n),n)}{\partial Q^2}+2w(Q^*(n),n)\frac{\partial w(Q^*(n),n)}{\partial Q} \\ &= -\frac{w(Q^*(n),n)}{\frac{\partial w(Q^*(n),n)}{\partial Q}}\left[w(Q^*(n),n)\frac{\partial^2 w(Q^*(n),n)}{\partial Q^2}-2\left(\frac{\partial w(Q^*(n),n)}{\partial Q}\right)^2\right]. \end{aligned} \tag{11}$$

The first-order condition of each upstream firm is (5) and the second-order condition is

$$2\frac{\partial w(Q,n)}{\partial Q}+\frac{\partial^2 w(Q,n)}{\partial Q^2}q_i<0. \tag{12}$$

We now show that the bracket in (11) is negative. Substituting the first-order condition into the bracket, we have:

$$\begin{aligned} &\left[w(Q^*(n),n)\frac{\partial^2 w(Q^*(n),n)}{\partial Q^2}-2\left(\frac{\partial w(Q^*(n),n)}{\partial Q}\right)^2\right] \\ =\ & -\frac{\partial w(Q^*(n))}{\partial Q}\left(q_i^*(n)\frac{\partial^2 w(Q^*(n),n)}{\partial Q^2}+2\frac{\partial w(Q^*(n),n)}{\partial Q}\right)<0. \end{aligned}$$

Second, we consider the sign of the numerator. If the numerator of the right-hand fraction in (10) is positive, the sign of the fraction in (10) is negative. We now show the condition where the denominator of the fraction is positive. Using (4), we calculate the terms of the denominator and summarize as follows:

$$\begin{aligned}&\frac{\partial w}{\partial n}\frac{\partial^2 w}{\partial Q^2}-\frac{\partial w}{\partial Q}\frac{\partial^2 w}{\partial Q \partial n}\\&= \frac{[P''(Q^*(n))^2 - P'(Q^*(n))P'''(Q^*(n))]Q^*(n)^2 + (n+1)P'(Q^*(n))^2}{n^3}. \qquad (13)\end{aligned}$$

If $P'''(Q^*(n)) \geq 0$, (13) is positive, that is, the denominator in (10) is positive. We find that $w(Q^*(n), n)$ is decreasing in n if $P'''(Q^*(n)) \geq 0$. Q.E.D.

References

Horn, H. and Wolinsky, A. "Bilateral monopolies and incentives for merger." *Rand Journal of Economics*, Vol. 19 (1988), pp. 408–419.

Lin, P. "Strategic spin-offs of input divisions." mimeo, 2004, forthcoming *European Economic Review*.

Lahiri, S. and Ono, Y. "The role of free entry in an oligopolistic Heckscher-Ohlin model." *International Economic Review*, Vol. 36 (1995), pp. 609–626.

Naylor, R. A. "Industry profits and competition under bilateral oligopoly." *Economics Letters*, Vol. 77 (2002), pp. 169–175.

Salinger, M. A. "Vertical mergers and market foreclosure." *Quarterly Journal of Economics*, Vol. 103 (1988), pp. 345–356.