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Berth allocation at indented berths for mega-containerships

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Abstract

This paper addresses the berth allocation problem at a multi-user container terminal

with indented berths for fast handling of mega-containerships. In a previous research conducted

by the authors, the berth allocation problem at a conventional form of the multi-user terminal

was formulated as a nonlinear mathematical programming, where more than one ship are

allowed to be moored at a specific berth if the berth and ship lengths restriction is satisfied. In

this paper, we first construct a new integer linear programming formulation for easier

calculation and then the formulation is extended to model the berth allocation problem at a

terminal with indented berths, where both mega-containerships and feeder ships are to be

served for higher berth productivity. The berth allocation problem at the indented berths is

solved by genetic algorithms. A wide variety of numerical experiments were conducted and

interesting findings were explored.

Keywords: Berth allocation; Container transportation; Mega-containership;

Container terminal; Mathematical Programming

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1. Introduction

In major ports in Japan and the US such as Kobe, Yokohama, Los Angeles and Oakland, shipping lines have been leasing container terminals (referred to as *Dedicated Terminal or DT*) in order for them to be directly involved in the terminal operations as they aim to achieve higher productivity and economies of scale. Whereas this may be warranted in the case of a firm that handles a large amount of containers with a corresponding number of ship calls, it may not be justified for smaller quantities, as this will increase costs and consequently impact negatively key financial performance indicators. Over the past several years, port related charges in Japanese ports have been consistently higher than those in other major ports of the world. One of the reasons cited for the increased costs is the over-investment in ports with relatively small cargo volume.

A "Multi-User Container Terminal (MUT)" may be defined as a terminal with a long berth that is able to handle simultaneously a number of vessels, which are dynamically allocated to the berth and are not always assigned to specific berth locations. Some major container ports provide an MUT, while most of the other major ports of feature DTs. Examples of MUTs are Hong Kong International Terminal (HIT) in Hong Kong, Pusan East Container Terminal (PECT) in Pusan, and Delta Multi-User Terminal (DMU) in Rotterdam. In addition, most container terminals in China are used as MUTs, since the limited terminal space due to investment budget constraints has to be utilized efficiently in order to meet the very high volume of container traffic. As it was cited in Imai et al. (2001), Japan's central government decided to introduce the MUT concept in major container ports in Japan in order to strengthen their competitiveness.

The MUTs will be further raising their importance due to the planned introduction of mega-containerships, which are capable of carrying more than 10,000 TEUs. Logically, such huge vessels will have limited their ports of call as they aim to take advantage of their economies of scale in the sea voyage leg. Thus, a key issue is how to maximize the operating capacity of a terminal for mega-ships. The solution is to operate the terminal as an MUT. To our knowledge, the only example of an MUT that could be used for mega-ships is an indented

terminal at the port of Amsterdam. While an ordinary MUT is featured by the handling system that all calling ships are loaded and unloaded from one side as shown in Fig. 1 (a), an indented MUT is characterized by its capability of fast handling from both sides of the ship as shown in Fig. 1 (b). However, due to its configuration the indented MUT may also impose a constraint in serving efficiently small feeder ships as portrayed in Fig. 2, i.e., ships at the inner part of the indented berths cannot depart before those ships at the outer part depart.

Figs. 1 & 2

It is also noted that an MUT can be characterized as a hub since a lot of container traffic is transshipped between trunk and feeder lines; therefore ships of various sizes are served at the same period. In previous studies, the authors have examined the berth allocation problems (BAPs) in both discrete and continuous location space indices. The derived conclusion was that while the BAP in continuous location is preferable in terms of flexible and efficient utilization of the terminal, it turns out a very difficult problem to be solved. On the other hand, the BAP in discrete location (BAPD) is relatively easy to be solved although the solution yields a less efficient utilization. For more efficiency in discrete berth allocation, the authors also studied the BAP with multiple ships being permitted to be served at the same berth subject to berth length constraint.

In light of the above discussion, this study addresses the BAPD where multiple ships are permitted to be simultaneously served at a specific berth. The authors have already studied such a problem, but it was formulated as a nonlinear problem; therefore this study first introduces a new, linear formulation. Then, for the berth allocation at an indented terminal for mega-containerships, we extend the formulation to deal with efficient berth allocation of small feeder ships at the indented terminal.

The paper is organized as follows: The next section provides a literature review on berth allocation planning. The problem formulation is discussed in Section 3. A solution algorithm is introduced in Section 4, followed by numerical experiments to assess the productivity of the indented terminal for mega-ship services in Section 5. The final section

concludes the paper.

2. Literature review on the BAP

As there is an ever-growing demand of operating MUTs more efficiently due to the continuous increasing container traffic, the issues pertaining to the efficient berth allocation at an MUT have been receiving much attention these days.

Lai and Shih (1992) propose a heuristic algorithm for berth allocation, which is motivated by more efficient terminal (actually berth) usage in the HIT terminal of Hong Kong. Their problem considers a First-Come-First-Served (FCFS) allocation strategy, which is not the case in our problem. Brown et al. (1994, 1997) examine ship handling in naval ports. They identify the optimal set of ship-to-berth assignments that maximize the sum of benefits for ships while in port. Berth planning in naval ports has substantial differences from berth planning in commercial ports. In the former, a berth shift occurs when for proper service, a newly arriving ship must be assigned to a berth where another ship is already being served. This treatment is unlikely in commercial ports. Berth shifting as well as other factors less relevant to commercial ports are taken into account in their paper, thus making their study inappropriate for commercial ports.

Imai et al. (1997) address a BAPD for commercial ports. Most service queues are in general processed on the FCFS basis. They concluded that in order to achieve high port productivity, an optimal set of ship-to-berth assignments should be found without employing the FCFS rule. However, this service principle may result in certain ships being dissatisfied with their order of service. In order to deal with the two conflicting evaluation criteria, i.e., berth performance and dissatisfaction with the order of service, they developed a heuristic to find a set of non-inferior solutions while maximizing the former and minimizing the latter. Their study assumes a static situation, where ships to be served for a planning horizon have all arrived at the port before the berth allocation planning process. Thus, their study can be applied only to extremely busy ports. As far as container shipping is concerned, such busy ports are not

competitive and this type of situation is not "realistic" due to the long delays experienced in the interchange process at the ports of call. In this context, Imai et al. (2001, 2005a) extended the static version of the BAPD into a dynamic treatment that is similar to the static treatment, but with the difference that some ships arrive while work is in progress. As the first step in this dynamic treatment, only one objective, berth performance, is considered. Due to the difficulty in finding an exact solution, they developed a heuristic by using a subgradient method with Lagrangian relaxation. Their study assumes the same water depth for all berths, although in practice certain ports have berths with different water depths. Nishimura et al. (2001) extend further the dynamic version of the BAPD for the multi-water depth configuration. They employ genetic algorithms to solve that problem. In some real situations, the terminal operator assigns different priorities to calling vessels. For instance, at a terminal in China, small feeder ships are given priority, as handling work associated with them does not keep the other big vessels waiting for a long time. On the other hand, a terminal in Singapore treats large vessels with higher priority, because they are considered valuable customers for the terminal. Imai et al. (2003) extend the BAPD in Imai et al. (2001, 2005a) to treat ships with different priorities and see how the extended BAP differentiates ship handling in terms of their associated service time.

There is also another type of approach to the berth allocation problem, which is the one with a continuous location index (referred to as *BAPC*). While in the above mentioned studies the entire terminal space is partitioned into several parts (or berths) and the allocation is planned based on the divided berth space, under this approach ships are allowed to be served wherever the empty spaces are available to physically accommodate the ships via a continuous location system. This type of problem resembles more or less the cutting-stock problem where a set of commodities is packed into some boxes in an efficient manner. A ship in wait and in service at a berth can be shown by a rectangle in a time-space representation or Gantt chart, therefore efficient berth usage is sort of packing "ship rectangles" into a berth-time availability box with some limited packing scheme such that no rotation of ship rectangles is allowed. Lim (1998) addresses a problem with the objective of minimizing the maximum amount of quay space used at any time with the assumption that once a ship is berthed, it will not be moved to any other place along the quay before it departs. He also assumes that every ship is berthed as

soon as it arrives at the port. On the other hand, Li et al. (1998) solve the BAPC both for with and without the ship's movement restriction. Their objective is to minimize the makespan of the schedule. Guan et al. (2002) developed a heuristic for the BAPC with the objective that minimizes the total weighted completion time of ship services. Park and Kim (2002) study the BAPC with an objective that minimizes the costs of delayed departures of ships due to the undesirable service order and the costs associated with additional complexity in handling containers when ships are served at non-optimal mooring locations in port. Their work is more practical than the aforementioned BAPC studies since the factors assessed in the objective function depend on the quay locations of ships. Kim and Moon (2003) address the same BAPC as the one tackled by Park and Kim (2002), though the former employ the simulated annealing method while the latter apply the subgradient optimization method. It is noticed that all the above BAPC studies assume the ship handling time independent from the berthing location unlike Nishimura et al. (2001). Park and Kim (2003) study the BAPC with a similar objective to those of Park and Kim (2002), and Kim and Moon (2003). The difference in objective is that in addition to the costs considered in previous studies, Park and Kim (2003) take into account the costs resulting from early or late start of ship handling against the estimated times of ship arrival. Their study features an interesting characteristic in that they determine the optimal start times of ship service and associated mooring locations and at the same time they determine the optimal assignment of quay cranes to those ships. In their study, the handling time for a particular ship is a function of the number of quay cranes engaged in the ship; however, the handling time is independent from the mooring location of the ship. Imai et al. (2005b) address a BAPC, which differs noticeably from the other BAPCs in that the handling time depends on the berthing location of ship. They developed a heuristic for that problem in conjunction with a heuristic for the dynamic BAPD in Imai et al. (2001, 2005a). The conclusion of their study is that the best approximate solution is identified with the best solution in discrete location where the berth length is the maximum length of ships involved in the problem. This implies that the solution in discrete location is applicable in practice in berth allocation planning and that an improved solution can be obtained from it.

3. Problem formulation

In this section, we first formulate the BAPD with multiple ships served at the same berth in a terminal of conventional form (BAPM) and then the BAP with the same context but in a terminal with indented berths (BAPI).

3.1. BAPM formulation

In the following discussion, the term "berth" refers to a unit of quay space having a specific length e.g., 400 m. The assumptions we made are as follows:

- (a) The handling time of a ship depends on the berth assigned to the ship.
- (b) Up to two ships can be served at the same berth simultaneously if their total length is less than the overall berth length.
- (c) There is no precedence constraint in berthing two ships, which are to stay at the same berth simultaneously, as will be applied for the BAPI (see Section 3.3 for details).
- (d) All berths have the same water depths unlike Nishimura et al. (2001), who first introduced the BAPM.

The difficulty of formulating the BAPM is based on the problem of serving multiple ships at a specific berth simultaneously, which results in the nonlinearity of the formulation in Nishimura et al.

We introduce here a linear formulation of the BAPM, which centers on decision variables of berth-ship-order assignment, x_{ijk} s, used in the BAPD formulation in Imai et al. (2001, 2005a). The linear formulation of the BAPM is as follows:

[PM] Minimize
$$Z = \sum_{j \in V} \left\{ \sum_{i \in B} f_{ij} - A_j \right\}$$
 (1)

subject to
$$\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1$$
 $\forall j \in V$, (2)

$$\sum_{i \in V} x_{ijk} \le 1 \qquad \forall i \in B, k \in U, \qquad (3)$$

$$b_{ij} \ge \sum_{k \in U} \max \{S_i, A_j\} x_{ijk} \qquad \forall i \in B, j \in V,$$
(4)

$$f_{ij} = b_{ij} + \sum_{k \in U} C_{ij} x_{ijk} \qquad \forall i \in B, j \in V,$$
(5)

$$\sum_{k \in U} k x_{ijk} \ge \sum_{k' \in U} k' x_{ij'k'} + (\tau_{ijj'} - 1) TM \qquad \forall i \in B, j, j' \in V,$$
(6)

$$b_{ij} \le b_{ij'} + \left(1 - \tau_{ijj'}\right) TM \qquad \forall i \in B, j, j' \in V, \tag{7}$$

$$f_{ij} < b_{ij'} + (1 + \omega_{ijj'} - \tau_{ijj'})TM$$
 $\forall i \in B, j, j' \in V,$ (8)

$$\omega_{ij'}(L_j + L_{j'}) \le BL_i \qquad \forall i \in B, j, j' \in V, \qquad (9)$$

$$\omega_{iji'} \le \tau_{iji'} \qquad \forall i \in B, j, j' \in V, \tag{10}$$

$$\sum_{k \in U} \left(x_{ijk} + x_{ij'k} - 1 \right) \leq \tau_{ijj'} + \tau_{ij'j} \leq \frac{\sum_{k \in U} \left(x_{ijk} + x_{ij'k} \right)}{2}$$

$$\forall i \in B, j, j' \in V, \tag{11}$$

$$\sum_{i'} \omega_{ijj'} \le 1 \qquad \forall i \in B, j \in V, \qquad (12)$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U, \qquad (13)$$

$$\tau_{iii'} \in \{0,1\} \qquad \forall i \in B, j, j' \in V, \qquad (14)$$

$$\omega_{iii'} \in \{0,1\} \qquad \forall i \in B, j, j' \in V, \qquad (15)$$

$$b_{ii} \ge 0, \quad f_{ii} \ge 0 \qquad \forall i \in B, j \in V, \tag{16}$$

where

 $i (= 1, ..., I) \in B$ set of berths

 $j (= 1,...,T) \in V$ set of ships

 $k (=1,...,T) \in U$ set of service orders

TM a very large number

 A_i arrival time of ship j

 BL_i length of berth i

 L_j length of ship j

 S_i time when berth i becomes idle for the planning horizon (set to negative value)

 C_{ij} handling time spent by ship j at berth i

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x_{ijk} =1 if ship j is served as the k th ship at berth i, and =0 otherwise
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 $\tau_{ijj'}$ =1 if both ships j and j' are served in berth i where j is earlier than j', and =0 otherwise

 $\omega_{iij'}$ =1 if both ships j and j' are served at the same time in berth i, and =0 otherwise

 b_{ii} start time of handling ship j at berth i

 f_{ij} completion time of handling ship j at berth i

The decision variables are x_{ijk} s, τ_{ijj} s, ω_{ijj} s, b_{ij} s and f_{ij} s. Objective (1) minimizes the total service time. Constraint set (2) ensures that every ship must be served at any berth in any order of service. Constraints (3) enforce that every berth serves up to one ship in any order of service. Constraints (4) assure that a ship is served after the largest time between the time when that ship arrives at the port and the time when the assigned berth becomes idle in the planning horizon. Constraint set (5) defines the completion time of ship handling. Constraints (6) guarantee that if ship j is served earlier than ship j', the former takes an earlier service order than the latter. Constraints (7) ensure that if ship j is served earlier than ship j', the former starts it's handling earlier than the latter. Constraint set (8) enforces that if two ships, j and j', are served at the same berth while the former is served earlier than the latter and they do not at any time share the same berth, the start of serving the latter is later than the completion of serving the former. Constraint set (9) enables two ships to be moored at the same berth if the total of their lengths is less than the berth length. Inequality set (10) assures that if two ships are served at the same berth regardless of which ship is served first, their service time may be overlapping. In other words, if they are served at different berths, they are never served simultaneously at the same berth. Constraints (11) ensure the proper definition of the variables pertaining to the service precedence of two ships. Also they guarantee that unless both of the two ships are served at the same berth, neither $\tau_{ij'}$ nor $\tau_{ij'j}$ is set to one. Constraints (12) ensure that a ship can be served at most with one other ship at a berth simultaneously.

As stated above, the new formulation uses an auxiliary variable ω_{ij} , for defining simultaneous service to two ships. The introduction of auxiliary variable τ_{ij} , together with ω_{ij} , in constraints (7), (8) and (11) enables [PM] to have linear constraints for simultaneous

berthing. With respect to constraints (11), if none of the two ships j and j' or one of them is served at berth i, the left-hand side does not have a positive value and the right-hand side has a value no more than 0.5; therefore $\tau_{ijj'} + \tau_{ij'',j}$ is null. On the other hand, if they are both served at berth i, both of the left and right-hand sides have 1. This guarantees that either $\tau_{ijj'}$ or $\tau_{ij',j'}$ is 1. For constraints (7) and (8), if ship j' begins its handling earlier than ship j, i.e., $\omega_{ijj'} = 0$ and $\tau_{ijj'} = 0$; no relationship among b_{ij} , $b_{ij'}$ and f_{ij} is defined. If ship j is earlier than j', while they are both at berth i but do not share the berth at the same time, i.e., $\omega_{ijj'} = 0$ and $\tau_{ijj'} = 1$; $b_{ij} \leq b_{ij'}$ and $f_{ij} < b_{ij'}$. Furthermore, if ship j is earlier than j' and they share berth i at the same time, i.e., $\omega_{ijj'} = 1$ and $\tau_{ijj'} = 1$; then $b_{ij} \leq b_{ij'}$ and $f_{ij} < b_{ij'}$ or $f_{ij} \geq b_{ij'}$. In the last case, however, we optimally have $f_{ij} \geq b_{ij'}$ by beginning the service of ship j' when the berth and ship lengths allow, because all the ships are planned to start handling tasks as early as possible in order to minimize the objective function. Thus, auxiliary variables $\omega_{iji'}$ and $\tau_{ijj'}$ are properly defined.

3.2. BAPM formulation with mega-ships

As mentioned in Section 1, this study is concerned with employing a more efficient container liner service for mega-ships. In a solution to the BAPM, every ship could potentially be forced to be delayed in being handling service at an MUT. However, a mega-ship cannot be delayed as an enormous capital expenditure has been outlaid for this mega-ship service. For this apparent reason, we assume that priority is given to mega-ships and they are served without any delay when they arrive at the terminal.

The formulation with mega-ships is based on [PM], but with priority service for mega-ship, as follows:

[PM'] Minimize Z
subject to (2)-(14)

$$b_{ij} = \sum_{k=U} A_j x_{ijk} \qquad \forall i \in B, j \in VM, \qquad (17)$$

Where VM is the set of mega-ships.

Constraint set (17) ensures that mega-ships start their handling service as soon as they arrive at the terminal. For convenience, formulation [PM'] is hereafter referred to as [PM].

3.3. BAPI formulation

We next introduce the BAPI with the assumption that mega-ships are served only at the indented berths, which also serve feeder ships when idle. From this point of view, the BAPI is solved such that the indented berths are efficiently utilized for feeder ships if no mega-ship is served. As illustrated in Fig. 2, the BAPI imposes the restriction that when a ship is moored at section 1 of one berth of the indented berths (the blackened ship in Fig. 2, no ship can enter and exit section 2 regardless of whether or not there are ships at the opposite side (or berth) of the indented berths across the water basin. In other words, a ship served at section 1 with another ship at section 2 of the same berth, must be moored there no earlier and depart no later than the ship at section 2.

The above restriction may not reflect an actual limit in arriving at and/or leaving the indented berths with a narrow water basin; because the white ship at section 2 of Fig. 2 may in practice enter or leave the berth, even if in addition to the blackened ship, the gray ships are moored at the opposite side of the indented berth in Fig. 2. Thus, the previous assumption was made mainly due to the simplicity of the formulation. However, this restriction is reasonable because if the white ship decided to move, even in the case that there are no gray ships at the opposite side of the indented berth, the white ship would take a high risk of crashing against the quay wall of the opposite side due to the velocity towards the quay by her side-thrusters.

The BAPI may be formulated as follows:

$$[PI]$$
 Minimize Z (1)

subject to (2)-(14)

$$b_{ij} = \sum_{i \in V} A_j x_{ijk} \qquad \forall i \in B^*, j \in VM, \qquad (18)$$

$$b_{ij} = \sum_{k \in U} A_j x_{ijk} \qquad \forall i \in B^*, j \in VM, \qquad (18)$$

$$b_{ij} - (\delta_{ijj'} - 1)TM > b_{ij'} + \sum_{k \in U} C_{ij'} x_{ij'k} + d_{ij'} \qquad \forall i \in B^*, j, j' \in V, \qquad (19)$$

$$b_{ij} + \sum_{k \in U} C_{ij} x_{ijk} + d_{ij} - (\phi_{ijj'} - 1) TM \ge b_{ij'} + \sum_{k \in U} C_{ij'} x_{ij'k} + d_{ij'}$$

$$\forall i \in B^*, j, j' \in V, \tag{20}$$

$$b_{ii} \le b_{ii'} - (\phi_{iii'} - 1)TM \qquad \forall i \in B^*, j, j' \in V, \tag{21}$$

$$b_{ij'} + \sum_{k \in U} C_{ij'} x_{ij'k} + d_{ij'} - (\rho_{ijj'} - 1)TM \ge b_{ij} + \sum_{k \in U} C_{ij} x_{ijk} + d_{ij}$$

$$\forall i \in B^*, j, j \in V, \tag{22}$$

$$b_{ii'} \le b_{ii} - (\rho_{iii'} - 1)TM \qquad \forall i \in B^*, j, j' \in V, \tag{23}$$

$$b_{ij'} - (\sigma_{ijj'} - 1)TM > b_{ij} + \sum_{k \in U} C_{ij} x_{ijk} + d_{ij} \qquad \forall i \in B^*, j, j' \in V,$$
(24)

$$\frac{\sum_{k \in U} (x_{ijk} + x_{ij'k})}{2} - 0.5 \le \delta_{ijj'} + \phi_{ijj'} + \rho_{ijj'} + \sigma_{ijj'} \le \frac{\sum_{k \in U} (x_{ijk} + x_{ij'k})}{2}$$

$$\forall i \in B^*, j, j \in V, \tag{25}$$

$$\delta_{iii'} \in \{0,1\} \qquad \forall i \in B^*, j, j \in V, \tag{26}$$

$$\phi_{iii'} \in \{0,1\} \qquad \forall i \in B^*, j, j' \in V, \tag{27}$$

$$\rho_{iii'} \in \{0,1\} \qquad \forall i \in B^*, j, j' \in V, \tag{28}$$

$$\sigma_{ijj'} \in \{0,1\} \qquad \forall i \in B^*, j, j' \in V, \tag{29}$$

$$d_{ij} \ge 0 \qquad \forall i \in B, j \in V \,, \tag{30}$$

where

B* set of indented berths

 δ_{ijj} =1 if ship j' stays at either section 1 or 2 of the indented berth i before ship j regardless of ship j being at section 1 or 2 (see Fig. 3), =0 otherwise

 ϕ_{ijj} =1 if ship j' stays at section 1 of the indented berth i when ship j stays at section 2, =0 otherwise

 ρ_{ijj} =1 if ship j' stays at section 2 of the indented berth i when ship j stays at section 1, =0 otherwise

 σ_{ijj} =1 if ship j' stays at either section 1 or 2 of the indented berth i after ship j regardless of ship j being at section 1 or 2 (See Fig. 3), =0 otherwise

 d_{ij} the extended time for departure of ship j at section 2 of berth i because of precedence constraint.

The extended time d_{ij} is considered in the case where a departing ship from section 2 of an

indented berth is blocked by a ship at section 1.

The decision variables are x_{ijk} s, $\tau_{ijj'}$ s, $\omega_{ijj'}$ s, b_{ij} s, f_{ij} s, $\delta_{ijj'}$ s, $\phi_{ijj'}$ s, $\sigma_{ijj'}$ s, $\sigma_{ijj'}$ s and d_{ij} s. Constraints (18) offer priority to mega-ships at an indented berth. Constraints (19) -(29) ensure that a ship served at section 1 simultaneously with a ship at section 2 comes to the berth later and leave the berth earlier than the ship at section 2. For particular ship *j* at section 2, there are three cases of the relationship with another ship j' at section 1 or 2 as shown in Fig. 3(a). Constraint set (19) is for case (i), sets (20) and (21) for case (ii) and set (24) for case (iii), respectively. On the other hand, if ship j stays at section 1, there are also three cases as illustrated in Fig. 3(b). Case (i) is defined by constraint set (19), case (ii#) by sets (22) and (23), and case (iii) by set (24). Whether ship j is either at section 1 or 2, one of the three cases is applied. For instance, regarding constraint set (19), with $\delta_{ijj'} = 1$ this constraint set is reduced to $b_{ij} \ge b_{ij'} + \sum_{k \in U} C_{ij'} x_{ij'k} + d_{ij'}$. When $\delta_{ijj'} = 0$, the left-hand sides of the constraint set becomes an enormous number, therefore we do not need to care about the constraint set. Constraint set (25) ensures that constraints (19) – (24) are applied only if ships j and j' are served at the same berth. If neither ship j nor j' stays at berth i, we do not care about constraints (19) – (24). This is guaranteed because the left-hand side of constraint set (25) is -0.5 and the right hand side is zero, resulting in δ_{ij} , $+\phi_{ij}$, $+\phi_{ij}$, $+\sigma_{ij}$, =0. If either ships j or j' stays at berth i, we do not care about constraints (19) – (24) either. This is caused by the fact that the left-hand side of constraint set (25) is zero and the right is 0.5. If both ships stay at the berth, the left is 0.5 and the right is 1; therefore one of $\delta_{ijj'}$, $\phi_{ijj'}$, $\rho_{ijj'}$ and $\sigma_{ijj'}$ is set to one.

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Fig. 3

4. Solution procedure

4.1. Solution procedure using the genetic algorithm

(1) Outline of solution procedure

Although the formulation [PI] is linear, it is complicated and it is not likely that an efficient exact solution method exists; therefore, to facilitate the solution procedure we employ a GA-based heuristic, which is widely used in solving difficult problems and has a practical, short computational time.

GA is like a heuristic method in that the optimality of the answers cannot be determined. It works on the principle of evolving a population of trial solutions over many iterations, to adapt them to the fitness landscape expressed in the objective function. The objective function value and solution alternatives of [PI] correspond to the fitness value and individuals, respectively. For our heuristic, the number of individuals in a generation is set to 30.

(2) Representation

In the GA's application that was developed, we have chosen to work with scheduling order rather than berth schedules (and berthing times). Furthermore, instead of using the classical binary bit string representation, the chromosomes are represented as character strings. In the paper of Nishimura et al. (2001), two types of representation were examined. The long string representation yielded better solutions for many cases; however, for this study we apply the short string representation as shown in Fig. 4 because of its small computational memory space without any significant loss in solution quality. The length of the string of digits is the number of ships plus the number of berths minus one. For a two-berth problem, the string consists of two parts separated by a zero, each of which represents a service queue for one of the two berths. The example of Fig. 4 shows a schedule with ships 2, 8, 5, and 9 serviced in that order at berth 1, and ships 4, 7, 3, 1, and 6 at berth 2.

Fig. 4

(3) Fitness

The problem [PI] is a minimization problem; thus, the smaller the objective function value is, the higher the fitness value must be. In such a case, the fitness function could be defined as the reciprocal of the objective function (Kim and Kim, 1996). However, applying this definition will yield a best solution that is likely to have an extremely good fitness value among solutions obtained without any significant difference between them in the objective function value. As this chromosome is always selected as a parent, it is difficult to maintain the variety of chromosome by crossover. Other alternatives of the fitness function are the exponential and sigmoid functions. As a result of some tests we conducted with these functions, the sigmoid function as defined in (31) was found to be better where y(x) denotes the objective function value:

$$f(x) = 1/(1 + \exp(y(x)/10000)), \tag{31}$$

Note that f(x) has a value ranging from 0 to 0.5.

4.2. Obtaining a feasible solution with respect to two overlapping ship services

A chromosome representation simply defines the relationship among berth-ship-service order. The start time of handling each ship has to be determined in order to compute the objective function value of formulation [PI]. The determination of the start time must be made so that ships are served simultaneously at a berth subject to the berth length and ships' entries and departures at an indented berth being physically feasible.

The outline of the calculation process to determine the start time of handling ships at the indented berth is as follows. The process determines the section of the indented berth where ships are to be served and the start and completion times of their handling, one by one and in ascending order of their service order which was obtained by the GA computation. Basically, a ship is initially assigned to section 2 (this ship is referred to as ship 2); however, if its potential handling period overlaps with that of another ship at section 2 and their total length does not exceed the berth length, the ship under the process is assigned to section 1. In this case, as ships

with earlier service order do not necessarily arrive earlier than the others due to no FCFS rule applied by the GA process, the ship at section 1 (referred to as ship 1) may start its handling earlier than ship 2. If so, handling of ship 1 is postponed till ship 2 starts its handling. In the case of simultaneous services of ships 1 and 2, if ship 1 completes its handling later than ship 2, ship 2 is kept at the berth till ship 1 departs. Notice that if the ship under the process does not depart before the next mega-ship arrives, the ship is served after the mega-ship.

A more detailed description of the process is presented as follows. Note that the start time for ships not served at the indented berth can be determined by a simpler procedure because of no precedence constraint.

The following parameters are used:

```
j (= 1,...,T) \in V set of ships
```

 $k (= 1, ..., F) \in O$ set of service orders

- j(k) ship number who is served as the k th ship
- S_i time when berth i becomes idle for the berth allocation planning
- *BL* length of the indented berth
- L_i length of ship j
- A_i arrival time of ship j
- C_i handling time spent by ship j at the indented berth
- b_i start time of handling for ship j
- f_i departure time of ship j
- A_{MS} arrival time of a mega-ship
- f_{MS} departure time of a mega-ship
- ship number at section 1 of the indented berth
- ship number at section 2 of the indented berth
- d1 departure time of ship at section 1
- d2 departure time of ship at section 2
- y_j =1 if ship j is served at section 1, =2 if ship j is served at section 2

[Calculation of the start time of ship handling]

Step 0: Set
$$k = 1$$
. Berth ship $j(k)$ at section 2 and let $s2 = j(k)$ and $y_{j(k)} = 2$.

If
$$S_i > A_{j(k)}$$
,

Then, let $b_{i(k)} = S_i$,

Else, let $b_{j(k)} = A_{j(k)}$.

Let
$$y_{j(k)} = 2$$
, $f_{j(k)} = b_{j(k)} + C_{j(k)}$, $d2 = f_{j(k)}$.

Step 1: Let k := k + 1.

Step 2: If k > T, then STOP.

Step 3: If ship 2 and ship j(k) are not supposed to be served simultaneously, i.e., if $A_{j(k)} > d2$;

Then, berth ship j(k) at section 2 after ship s2,

i.e., let
$$b_{j(k)} = A_{j(k)}$$
, $s2 = j(k)$, $y_{j(k)} = 2$, $f_{j(k)} = b_{j(k)} + C_{j(k)}$, $d2 = f_{j(k)}$ and go to Step 6,

Else, go to the next step.

Step 4: If the total length of ship s2 and ship j(k) exceeds the length of the indented berth, i.e., if $L_{s2} + L_{j(k)} > BL$;

Then, berth ship j(k) at section 2,

i.e., let
$$b_{j(k)} = d2$$
, $s2 = j(k)$, $y_{j(k)} = 2$, $f_{j(k)} = b_{j(k)} + C_{j(k)}$, $d2 = f_{j(k)}$ and go to Step 6.

Else, berth ship j(k) at section 1,

i.e., if
$$A_{j(k)} > b_{s2}$$
,
Then, let $b_{j(k)} = A_{j(k)}$,
Else, let $b_{j(k)} = b_{s2}$.
Let $s1 = j(k)$, $y_{j(k)} = 1$, $f_{j(k)} = b_{j(k)} + C_{j(k)}$, $d1 = f_{j(k)}$ and

go to the next step. $J_{j(k)} - J_{j(k)} - J_{j(k)} + C_{j(k)}$, $u_1 - J_{j(k)}$ and

Step 5: Examine if ship s1 departs earlier than ship s2, i.e., if d1 < d2;

Then, go to Step 1.

Else, postpone the departure of ship s1 till the departure of ship s2,

i.e., let
$$f_{s1} = f_{s2}$$
 and $d1 = f_{s1}$ and go to Step 1.

Step 6: Examine if ship j(k) at section 2 departs after the next mega-ship arrives, i.e., if $d2 > A_{MS}$;

Then, berth ship j(k) at section 2 after the mega-ship,

i.e., let
$$b_{j(k)} = f_{MS}$$
, $s2 = j(k)$, $y_{j(k)} = 2$, $f_{j(k)} = b_{j(k)} + C_{j(k)}$, $d2 = f_{j(k)}$

Else, go to Step 1.

In the above procedure, a ship, which is too large to stay simultaneously with another ship at the indented berth, is forced to stay at section 2 of the indented berth. Note that in step 5, if the ship under process departs from section 1 earlier than ship s2, ship s1 is not checked in step 6 on whether or not its handling is completed before the arrival of the next mega-ship. This is simply because it was already confirmed that ship s2 (consequently ship s1 too) departs before the mega-ship.

5. Numerical experiments

5.1. Handling time estimation

In order to carry out the entire experiment, we carefully set up a major input parameter of the BAPI, C_{ij} , which defines the handling time of a ship at a specific berth location. In previous studies conducted by the authors dealing with berth allocation problems, no insight was provided for C_{ij} or equivalent parameters. In the numerical experiments of this study, we perform quantitative analyses on comparisons between the conventional berth and indented berth arrangements. For precise estimation of C_{ij} , we implemented a simulation model of container handling tasks by quay cranes, yard-trailers and transfer-cranes at a terminal and performed various cases of simulations by the model. In summary, we obtained a regression model to estimate the handling time using the number of containers to be handled (x_1) , the number of yard-trailers hauling containers between a quay crane and container stack on the yard (x_2) , and the distance between the quay carne and the container stack (x_3) as explanatory variables. By simulations with four berths, we constructed the following linear model with the

coefficient of the determination of 0.88:

$$y = 0.75x_1 - 0.77x_2 + 0.29x_3 + 1.71$$
 (R² = 0.88)

Note that the estimation model (32) was constructed with the data generated by the simulations, which assume a conventional terminal. Exactly speaking, it is not certain that (32) guarantees the C_{ii} estimation at an indented terminal; however, we assume that it does.

5.2. Experiments

The program codes for the BAPM and BAPI were implemented in "C" language on a Sun Blade 1500 workstation. By preliminary experiments, we identified parameters for GAs as population size=50, mutation rate=0.09 and the number of generations=1000. The objective function value of the problem converges approximately at the 650th generation out of 1000 with the above parameter settings.

In the experiments, we assess the handling capability of terminal with indented berths by comparing it with a conventional type of terminal as illustrated in Fig. 1. In order to assess these two types from a cost-effectiveness point of view, both terminals are set to have the same size in area and the same quay length. This setting results from the fact that terminal construction cost is comprised of two cost elements: yard area and quay. Both terminals are comprised of four terminal blocks, each having a 400X400 m² space. Each block (or berth) has five quay cranes. In the indented terminal, a mega-ship employs a total of 10 cranes, 5 cranes from either side of the indented berths. On the other hand, in the conventional terminal, a mega-ship is handled by 7 cranes, two of them being obtained from neighboring berths. From this point of view, the handling time at the neighboring berths may be affected by the existence of the mega-ship due to the number of available cranes at the berths. For simplicity purposes we assume the same handling time at the neighboring berths regardless of the mega-ship handling. To resolve such a complexity, we may assume to use 5 cranes instead of 7 for the mega-ship handling; however, such a mega-ship service is not sensible due to the long turnaround. Note that it is physically impossible to deploy 10 cranes for one-side handling of a mega-ship in the conventional terminal because of the relationship between berth length and

inter-crane margin. Although the practical situation depends on the physical characteristics of mega-ship, we assume that up to 7 cranes can be located on one side of the mega-ship. As mentioned before it is assumed that the mega-ship deploys 10 cranes in total for two-side handling, 5 on either side. We do not assume to have more than 5 cranes on each side, because of inter-crane interference caused by long crane beam stretching to the other side of the ship width.

It is assumed that ships served at the MUTs are categorized into three classes by associating ship length with the number of containers loaded and unloaded and the number of quay cranes employed as shown in Table 1. These ships arrive randomly at the MUTs for seven days, following two types of distributions: exponential and 2-Erlangian. For the handling time C_{ij} , ships of the three categories are generated randomly by a uniform distribution. As already mentioned, a mega-ship has a total of 16000 containers loaded and unloaded by 7 quay cranes in the conventional terminal and 10 cranes in the indented terminal, respectively.

Table 1

Computation settings we made reflect a relatively busy MUT. We generated four cases with different ship arrival patterns; Case 1: average arrival interval of 3 hours by exponential distribution, Case 2: 3-hour interval by 2-Erlangian distribution, Case 3: 4-hour interval by exponential distribution, and Case 4: 4-hour interval by 2-Erlangian distribution. For each computation case, we made two scenarios, one with 4 yard-trailers connecting a quay crane site and container stacking block on the yard, the other with 5 trailers. For a more precise analysis, given solutions to the BAPM and BAPI, we run a detailed yard operation simulation with complicated yard-trailer runs. This implies that the total service time (consequently the handling time and waiting time) is not necessarily the one obtained as the objective function value of solutions to the formulations [PM] and [PI].

As mentioned above, we have two scenarios for the number of trailers associated with each quay crane: 4 and 5. Each trailer takes the shortest path from a ship to the dedicated container block on the yard. The container block arrangement and corridor system for

yard-trailer run are illustrated in Fig. 5. Arrows in corridors show the direction of the trailer passage. A container block in the conventional terminal is comprised of 20 containers of 20 feet longitudinally. There are two types of blocks in the indented terminal: one is the same as the block in the conventional terminal and the other with 40 containers longitudinally. All blocks of two types have 6 containers wide and 4 containers high. There are always two gantry cranes employed at each block. To obtain the value of C_{ij} , the container block is assigned for a specific ship randomly with uniform distribution; this assignment determines the distance between the ship and its container block. Then, by Eq. (32), C_{ij} is computed with the distance as well as other relevant parameters: the numbers of containers and associated quay cranes, both defined by Table 1.

Fig. 5

Fig. 6 depicts the handling time of the mega-ship, which is not a consequence of the BAPI solution because the mega-ship has a priority without any delay. Fig. 7 illustrates the resulting total service time of all the ships involved which is comprised of the handling time and waiting time. As logically expected, the handling time for the mega-ship is shorter in the indented terminal than in the conventional one, simply because of more quay cranes employed at the indented terminal. As the delay in container exchange between a quay crane and a yard-trailer is shortened by more yard-trailers, the handling time with 5 trailers is shorter than 4 trailers both in the conventional and indented terminals. In Fig. 7, the total service time is surprisingly larger in the indented terminal than the conventional terminal for any computation settings. This results from the longer waiting time in the indented terminal, because when a mega-ship is handled at the indented berths, it actually occupies two berths, out of the four berths, across the water basin. Therefore, while the indented terminal accomplishes fast handling of a mega-ship, the berth productivity is not high, at least not as high as the conventional terminal. Note that although the general trend is as described above, the variance in the total service time between the conventional and indented terminals decreases as the arrival interval increases. This implies that the disadvantage of the indented terminal with

respect to the total service time is less significant in the cases of decreased traffic where enough handling capacity is provided and thus ships do not experience long waits for idle berths.

Figs. 6 & 7

Other findings from the computation results shown in Fig. 7 are the following: As expected, the total handling time of all ships does not significantly vary for ship arrival pattern and average interval time. On the other hand, the total waiting time (and consequently the total service time) is larger with exponential arrival than 2-Erlangian arrival for a specific interval time. For a specific arrival pattern, the waiting time is larger with shorter interval time. This result is quite apparent because a shorter interval results in a busier terminal state. The handling with more trailers, obviously shortens the total handling time; however, more interestingly it contributes to decrease significantly the waiting time. Typically we can observe this tendency for the indented terminal in Cases 1 and 3, and for the conventional terminal in Case 2.

From the above results and consequent impacts we consider that the indented terminal is not necessarily efficient for shipping services with mega-containerships, especially when taking into account the transit time of the entire shipping network involving mega-ships and small feeder ships.

The berth allocation scheduling we proposed here is based on given ship arrival data. If the terminal of concern is not so busy, then ship services are assigned to appropriate berths (or the berths with the minimum handling time among the others) and ship arrivals are scheduled so that the ships come exactly when the assigned berths become idle for the relevant ships. However, such a berth allocation scheme oriented by berth idle state is practical only when the terminal is not so busy. Also this is possible with the ideal case that ships always depart from the previous calling ports as voyage schedule and sail normally to the next ports. Usually, ships cannot depart from calling ports without delay and furthermore this delay sometimes cannot be overcome before arriving at the next port despite of increasing voyage speed.

6. Conclusions

In the context of the raising importance of the Multi-User Container Terminals (MUTs) especially due to the anticipated employment of mega-containerships, this paper addressed the berth allocation problem in the MUT with a consideration of serving simultaneously multiple small ships at a berth. In a previous paper, this problem was considered by the authors, but its formulation was nonlinear. This paper introduced a linear formulation for the problem and extended it to apply for an indented terminal, which is considered to be more efficient for serving mega-ships. To solve the problem, we developed a GA-based solution algorithm. In order to assess the capability of the indented terminal for mega-ships as well as small feeder services, we carried out numerical experiments for both indented and conventional terminals of the same size. From the derived results it was concluded that while the indented terminal served the mega-ship faster than the conventional terminal, the total service time for all ships was longer than the one in the conventional terminal. Therefore, the conclusion obtained is that the indented terminal is less efficient for an entire shipping service, which includes mega-containerships and small feeder ships, especially in terms of the total service time and consequent transit time of the whole shipping network.

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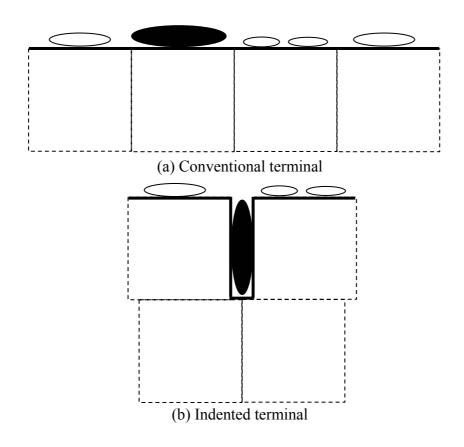


Fig. 1. MUT layout alternatives

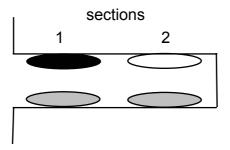


Fig. 2. Berthing small ships at indented berths

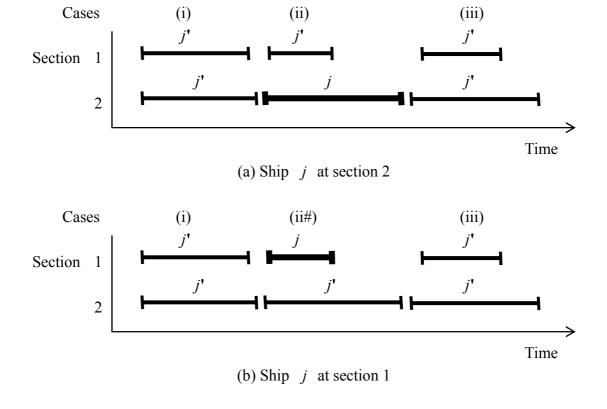


Fig. 3. Precedence constraints

Chromosome 2859047316 i 1111 22222 k 1234 12345

Fig. 4. Chromosome representation

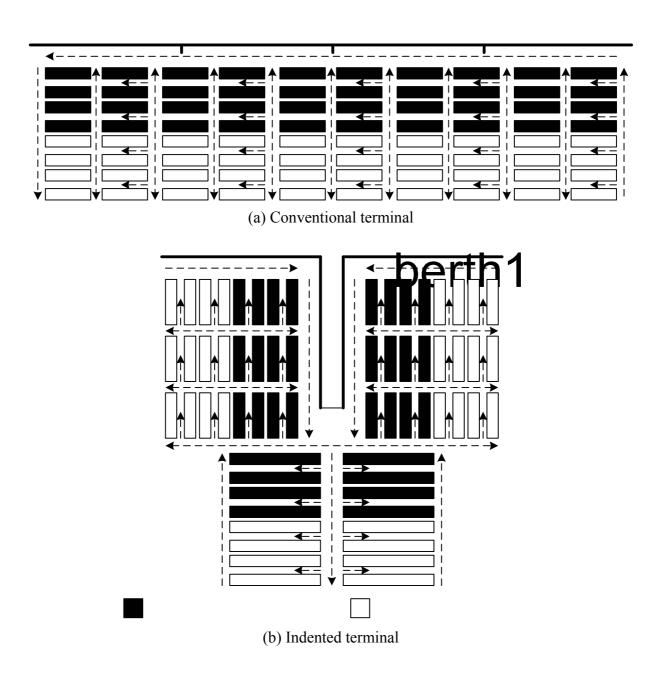
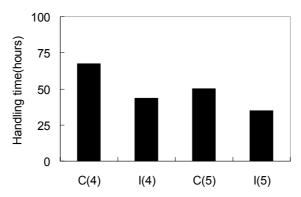


Fig. 5. Corridors for yard-trailer route



Legend

C(4): Conventional terminal with 4 yard-trailers for a quay crane

I(4): Indented terminal with 4 yard-trailers for a quay crane

C(5): Conventional terminal with 5 yard-trailers for a quay crane

I(5): Indented terminal with 5 yard trailers for a quay crane

Fig. 6. Handling time of mega-ship

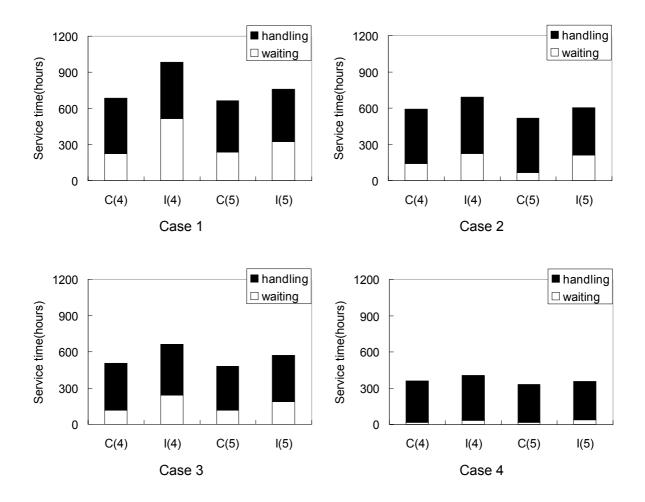


Fig. 7. Total service time

Table 1. Ship category

Ship length	# of containers handled	# of quay cranes
150 - 200	50 - 300	1
301 - 600	301 - 600	2
601 - 1200	251 - 300	3