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EXACT DISTRIBUTION AND CRITICAL VALUES OF A UNIT ROOT TEST IN THE PRESENCE OF CHANGE IN VARIANCE *

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Abstract

In this paper, using the Imhof (1961) method, we show the method of evaluating numerically the exact distribution of a unit root test statistic when the error variance changes. Based on the method, we examine the effect of the change in variance on the exact distribution, and we tabulate numerically exact critical values when the sample size is small and moderate.

Keywords: Change in error variance; Critical value; Exact distribution; Unit root test

JEL classification: C12; C22

Running title: Excat distribution and critical values

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1 Introduction

Consider the first order autoregressive process,

$$y_t = \rho y_{t-1} + u_t, \qquad t = 1,\ 2,\ \dots,n, \quad y_0 = 0. \tag{1}$$

Then, the ordinary least squares (OLS) estimator of ρ is

$$\widehat{\rho} = \frac{\sum_{t=1}^{n} y_t y_{t-1}}{\sum_{t=1}^{n} y_{t-1}^2}. \tag{2}$$

Since a limiting distribution of $\widehat{\rho}$ is not normal when $\rho = 1$, Fuller (1976) showed the critical values (or percentiles) of the distribution of $n(\widehat{\rho}-1)$, which can be used to test the null hypothesis $H_0 : \rho = 1$. Although Fuller's (1976) critical values were obtained based on Monte Carlo experiments, Evans and Savin (1981) showed how the numerically exact distribution of $n(\widehat{\rho}-1)$ can be evaluated by using Imhof's (1961) method. Also, using Imhof's (1961) method, Ohtani (2000, 2002) evaluated numerically exact critical values of unit root tests based on $n(\widehat{\rho}-1)$. Since the usual critical values of a unit root test are evaluated under the assumption that there is no structural change, these critical values are not valid if there is a structural change. Perron (1989, 1990) and Perron and Vogelsang (1992) derived limiting distributions for unit root test statistics, which take a shift in a constant term and a slope parameter into account, and they evaluated empirical distributions. Also, Hamori and Tokihisa (1997) considered the model where a constant term and a slope parameter do not change but the error variance changes. They derived limiting distributions of unit root test statistics when the error variance changes, and showed by Monte Carlo experiments that the size distortions are severe if the usual critical values are used.

In this paper, using the Imhof (1961) method, we show the method of evaluating numerically the exact distribution of $n(\widehat{\rho}-1)$ when the error variance changes. Based on the method, we examine the effect of the change in variance on the exact distribution of $n(\widehat{\rho}-1)$, and we tabulate numerically exact critical values when the sample size is small and moderate.

2 Distribution

Following Hamori and Tokihisa (1997), we consider the following model:

$$y_t = \rho y_{t-1} + u_t, \qquad t = 1,\ 2,\ \cdots,\ n, \qquad y_0 = 0, \tag{3}$$

where

$$u_t = \epsilon_t + d_t \eta_t, \tag{4}$$

$$d_t = \begin{cases} 0 & t = 1,\ 2,\ \dots,\ t^* \\ 1 & t = t^*+1,\ t^*+2,\ \dots,\ n. \end{cases} \tag{5}$$

$$\epsilon_t \sim NID(0,\ \sigma_1^2), \qquad \eta_t \sim NID(0,\ \sigma_2^2), \tag{6}$$

and ϵ_t and η_s are mutually independent for all t and s. In this model, the variance changes from σ_1^2 to $\sigma_1^2 + \sigma_2^2$ at the point in time $t^* + 1$, where t^* is assumed to be known.

Let $y = (y_1,\ y_2,\ \dots,\ y_n)$, $y_{-1} = (y_0,\ y_1,\ \dots,\ y_{n-1})$, and

$$L = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}. \tag{7}$$

Then, we have $y_{-1} = Ly$, and

$$\widehat{\rho} = \frac{y' y_{-1}}{y'_{-1} y_{-1}} = \frac{y' L y}{y' L' L y}. \tag{8}$$

Since $y_t = \sum_{i=1}^{t} u_t$ when $\rho = 1$, the mean and variance of y_t are as follows:

$$E[y_t] = \sum_{i=1}^{t} E[u_i] = 0, \tag{9}$$

$$\begin{aligned} V(y_t) &= E\Big[\Big(\sum_{i=1}^{t} u_i\Big)^2\Big] \\ &= E\Big[\sum_{i=1}^{t}\sum_{j=1}^{t} u_i u_j\Big] \\ &= E\Big(\sum_{i=1}^{t}\sum_{j=1}^{t} (\epsilon_i + d_i\eta_i)(\epsilon_j + d_j\eta_j)\Big] \\ &= E\Big[\sum_{i=1}^{n} (\epsilon_i^2 + 2d_i\epsilon_i\eta_i + d_i^2\eta_i^2 + \sum\sum_{i \neq j} (\epsilon_i\epsilon_j + d_i\eta_i\epsilon_j + d_j\epsilon_i\eta_j + d_i d_j \eta_i\eta_j)\Big] \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{t} (\sigma_1^2 + d_i \sigma_2^2) \\
&= t\sigma_1^2 + \Big(\sum_{i=1}^{t} d_i \sigma_2^2\Big). \qquad (10)
\end{aligned}
$$

Thus, the variance of y_t is finally written as

$$
V(y_t) = \begin{cases} t\sigma_1^2 & t = 1,\ 2,\ \ldots, t^* \\ t\sigma_1^2 + \sum_{i=t^*+1}^{t} \sigma_2^2 = t\sigma_1^2 + (t - t^*)\sigma_2^2 & t = t^*+1,\ t^*+2,\ \ldots, n. \end{cases} \qquad (11)
$$

The covariance between y_t and y_s $(s > t)$ is

$$
\begin{aligned}
Cov(y_t, y_s) = E[y_t y_s] &= E\Big[\sum_{i=1}^{t} u_i \sum_{j=1}^{s} u_j\Big] \\
&= E\Big[\sum_{i=1}^{t} u_i \Big(\sum_{j=1}^{t} u_j \sum_{j=t+1}^{s} u_j\Big)\Big] \\
&= E\Big[\sum_{i=1}^{t} u_i \sum_{j=1}^{t} u_j + \sum_{i=1}^{t} u_i \sum_{j=t+1}^{s} u_j\Big]. \qquad (12)
\end{aligned}
$$

Since $E[\sum_{i=1}^{t} u_i \sum_{j=t+1}^{s} u_j] = 0$, the formula of covariance is the same as that of variance given in (??).

Thus, the covariance matrix of y_t is written as

$$
\begin{array}{c} \\ \sigma_1^2 \Sigma = \sigma_1^2 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} 1 & 2 & 3 & \cdots & t^* & t^*+1 & \cdots & t \end{array} \\
\begin{pmatrix}
1 & 1 & 1 & \cdots & 1 & 2 & \cdots & 1 \\
1 & 2 & 2 & \cdots & 2 & 2 & \cdots & 2 \\
1 & 2 & 3 & \cdots & 3 & 3 & \cdots & 3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \\
1 & 2 & 3 & \cdots & t^* & t^* & \cdots & t^* \\
1 & 2 & 3 & \cdots & t^* & t^*+1+\theta & \cdots & t^*+1+\theta \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
1 & 2 & 3 & \cdots & t^* & t^*+1+\theta & \cdots & t+(t-t^*)\theta
\end{pmatrix}
\end{array} \qquad (13)
$$

where $\theta = \sigma_2^2/\sigma_1^2$. When $\theta = 0$, the covariance matrix reduces to the usual covariance matrix without the change in variance.

Since Σ is a symmetric positive definite matrix, there exists a symmetric positive definite matrix, $\Sigma^{-1/2}$, such that $\Sigma^{-1/2}\Sigma\Sigma^{-1/2} = I_n$, where I_n is an $n \times n$ identity matrix.

Defining $z = \Sigma^{-1/2}y/\sigma_1$, z is distributed as $N(0,\ I_n)$, and $\widehat{\rho}$ is rewritten as

$$\widehat{\rho} = \frac{z'\Sigma^{1/2\prime}L\Sigma^{1/2}z}{z'\Sigma^{1/2\prime}L'L\Sigma^{1/2}z}. \tag{14}$$

Thus, the distribution function of $n(\widehat{\rho}-1)$ is

$$\begin{aligned} F(c) &= Pr\Big(n(\widehat{\rho}-1) < c\Big) \\ &= Pr\Big(\widehat{\rho} < (1+\frac{c}{n})\Big) \\ &= Pr(z'Az < 0), \end{aligned} \tag{15}$$

where c is a constant, $Pr(E)$ is the probability of an event E, and

$$A = \Sigma^{1/2}\Big[\frac{1}{2}(L+L') - (1+\frac{c}{n})L'L\Big]\Sigma^{1/2}. \tag{16}$$

It is well known that the probability of the quadratic form given in (??) can be computed by the Imhof (1961) method.

3 Critical Values

To evaluate the distribution function numerically, the algorithm shown in Koerts and Abrahamse (1969) has been used. The numerical evaluations were executed on a personal computer, using FORTRAN code. Also, both the desired truncation error and the desired accuracy of numerical integration were set at 10^{-7}. The distribution function of $n(\widehat{\rho}-1)$ for $n = 60$ and some selected values of θ are shown in Figure 1. We see from Figure 1 that there are severe upward biases in size, and the actual level increases as the value of θ increases. This confirms the Monte Carlo results of Hamori and Tokihisa (1997) that the size distortion gets severe as the value of θ increases.

To conduct a unit root test based on $n(\widehat{\rho}-1)$ when the error variance changes, we have computed 1%, 5% and 10% critical values for small and moderate sample size and for several values of θ. The results are shown in Tables 1 to 3. In the tables, λ is defined as $\lambda = t^*/n$. Thus, for example, if it is suspected that the variance changes from $t^*+1 = 30$ when $n = 60$, the critical value located in $\lambda = 0.5$ is used.

Figure 1. Exact distribution functions of $n(\hat{\rho}-1)$ for $n=60$ and $\alpha=0.05$

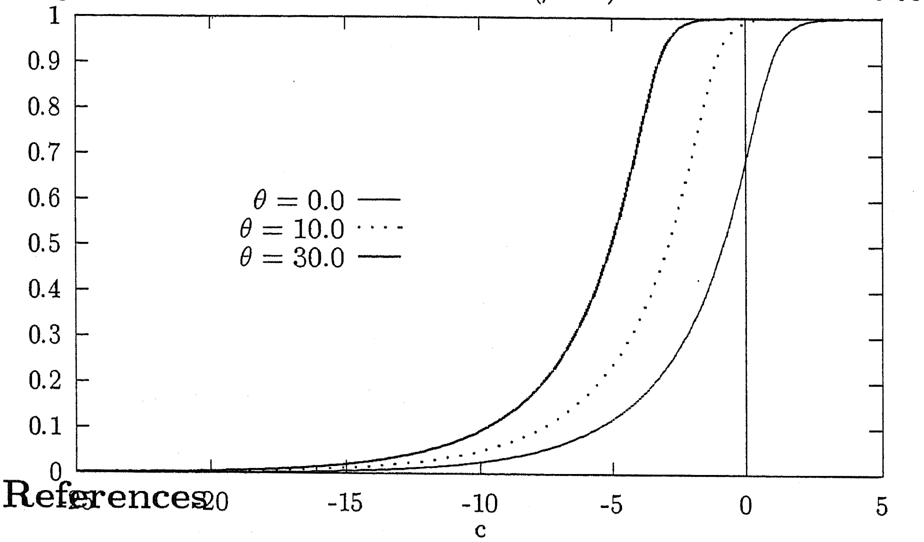

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Table 1. Exact critical values for $\alpha = 0.01$

		λ								
n	θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	1.0	-11.856	-11.933	-11.984	-12.020	-12.038	-12.035	-12.003	-11.937	-11.825
	3.0	-12.133	-12.427	-12.645	-12.832	-12.982	-13.073	-13.071	-12.919	-12.482
	5.0	-12.258	-12.711	-13.064	-13.385	-13.669	-13.887	-13.983	-13.843	-13.183
	7.0	-12.330	-12.898	-13.359	-13.788	-14.188	-14.528	-14.738	-14.665	-13.891
	10.0	-12.394	-13.084	-13.672	-14.230	-14.774	-15.275	-15.655	-15.727	-14.933
	15.0	-12.451	-13.272	-14.012	-14.732	-15.459	-16.175	-16.808	-17.158	-16.564
	20.0	-12.483	-13.386	-14.236	-15.076	-15.942	-16.828	-17.672	-18.295	-18.046
	30.0	-12.518	-13.519	-14.517	-15.530	-16.598	-17.738	-18.919	-20.019	-20.617
40	1.0	-12.874	-12.965	-13.024	-13.063	-13.081	-13.071	-13.027	-12.941	-12.802
	3.0	-13.204	-13.551	-13.810	-14.031	-14.204	-14.300	-14.279	-14.068	-13.546
	5.0	-13.361	-13.899	-14.324	-14.710	-15.048	-15.299	-15.388	-15.178	-14.381
	7.0	-13.452	-14.130	-14.689	-15.211	-15.697	-16.101	-16.328	-16.186	-15.222
	10.0	-13.535	-14.362	-15.080	-15.767	-16.439	-17.050	-17.490	-17.509	-16.435
	15.0	-13.610	-14.598	-15.508	-16.404	-17.316	-18.214	-18.982	-19.317	-18.277
	20.0	-13.652	-14.742	-15.790	-16.843	-17.940	-19.068	-20.120	-20.773	-19.903
	30.0	-13.697	-14.909	-16.145	-17.425	-18.795	-20.273	-21.784	-23.014	-22.640
60	1.0	-13.249	-13.346	-13.407	-13.449	-13.467	-13.455	-13.408	-13.316	-13.168
	3.0	-13.599	-13.967	-14.242	-14.477	-14.660	-14.760	-14.733	-14.505	-13.950
	5.0	-13.768	-14.339	-14.793	-15.206	-15.568	-15.834	-15.924	-15.690	-14.837
	7.0	-13.867	-14.588	-15.186	-15.747	-16.269	-16.701	-16.940	-16.775	-15.733
	10.0	-13.957	-14.839	-15.608	-16.349	-17.075	-17.735	-18.207	-18.212	-17.034
	15.0	-14.039	-15.094	-16.072	-17.041	-18.032	-19.012	-19.848	-20.198	-19.031
	20.0	-14.085	-15.250	-16.378	-17.519	-18.716	-19.953	-21.110	-21.814	-20.818
	30.0	-14.135	-15.431	-16.764	-18.154	-19.656	-21.290	-22.971	-24.331	-23.870
80	1.0	-13.444	-13.543	-13.606	-13.649	-13.667	-13.655	-13.606	-13.511	-13.359
	3.0	-13.805	-14.183	-14.467	-14.709	-14.897	-15.000	-14.970	-14.733	-14.161
	5.0	-13.980	-14.568	-15.038	-15.465	-15.839	-16.113	-16.204	-15.957	-15.074
	7.0	-14.083	-14.826	-15.445	-16.027	-16.568	-17.016	-17.261	-17.083	-16.000
	10.0	-14.177	-15.087	-15.884	-16.654	-17.408	-18.096	-18.585	-18.581	-17.347
	15.0	-14.263	-15.352	-16.367	-17.375	-18.409	-19.433	-20.308	-20.664	-19.428
	20.0	-14.310	-15.515	-16.686	-17.874	-19.126	-20.423	-21.638	-22.370	-21.299
	30.0	-14.362	-15.703	-17.088	-18.539	-20.112	-21.832	-23.610	-25.044	-24.521
100	1.0	-13.563	-13.664	-13.728	-13.771	-13.790	-13.777	-13.727	-13.630	-13.476
	3.0	-13.930	-14.315	-14.605	-14.852	-15.043	-15.147	-15.115	-14.872	-14.291
	5.0	-14.110	-14.709	-15.188	-15.623	-16.005	-16.285	-16.375	-16.121	-15.220
	7.0	-14.215	-14.972	-15.605	-16.199	-16.752	-17.210	-17.458	-17.272	-16.162
	10.0	-14.312	-15.239	-16.054	-16.841	-17.614	-18.318	-18.818	-18.808	-17.537
	15.0	-14.399	-15.511	-16.548	-17.581	-18.642	-19.694	-20.592	-20.952	-19.667
	20.0	-14.449	-15.677	-16.876	-18.093	-19.379	-20.714	-21.966	-22.714	-21.589
	30.0	-14.502	-15.870	-17.288	-18.776	-20.395	-22.169	-24.007	-25.489	-24.917

Table 2. Exact critical values for $\alpha = 0.05$

n	θ	λ								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	1.0	-7.469	-7.550	-7.580	-7.594	-7.595	-7.580	-7.546	-7.487	-7.406
	3.0	-7.659	-7.925	-8.086	-8.200	-8.277	-8.306	-8.262	-8.108	-7.778
	5.0	-7.738	-8.125	-8.400	-8.613	-8.779	-8.884	-8.882	-8.701	-8.142
	7.0	-7.782	-8.250	-8.616	-8.915	-9.164	-9.347	-9.407	-9.239	-8.497
	10.0	-7.820	-8.370	-8.840	-9.245	-9.603	-9.897	-10.061	-9.945	-9.011
	15.0	-7.853	-8.484	-9.075	-9.617	-10.120	-10.573	-10.906	-10.915	-9.820
	20.0	-7.871	-8.550	-9.223	-9.867	-10.486	-11.070	-11.554	-11.700	-10.573
	30.0	-7.890	-8.624	-9.399	10.188	-10.981	-11.772	-12.508	-12.922	-11.933
40	1.0	-7.865	-7.953	-7.987	-8.004	-8.005	-7.987	-7.948	-7.880	-7.782
	3.0	-8.080	-8.371	-8.553	-8.684	-8.774	-8.808	-8.760	-8.585	-8.211
	5.0	-8.172	-8.601	-8.913	-9.158	-9.353	-9.479	-9.484	-9.278	-8.680
	7.0	-8.224	-8.746	-9.162	-9.507	-9.800	-10.021	-10.105	-9.914	-9.155
	10.0	-8.269	-8.885	-9.422	-9.892	-10.315	-10.671	-10.883	-10.763	-9.852
	15.0	-8.309	-9.018	-9.695	-10.327	-10.925	-11.477	-11.899	-11.950	-10.942
	20.0	-8.330	-9.095	-9.867	-10.621	-11.359	-12.073	-12.685	-12.926	-11.933
	30.0	-8.353	-9.181	-10.072	-10.998	-11.949	-12.921	-13.854	-14.466	-13.660
60	1.0	-8.007	-8.097	-8.133	-8.150	-8.152	-8.134	-8.093	-8.023	-7.920
	3.0	-8.230	-8.532	-8.721	-8.859	-8.954	-8.992	-8.943	-8.761	-8.374
	5.0	-8.328	-8.772	-9.098	-9.356	-9.562	-9.697	-9.707	-9.496	-8.882
	7.0	-8.382	-8.925	-9.360	-9.723	-10.034	-10.271	-10.365	-10.174	-9.398
	10.0	-8.430	-9.071	-9.634	-10.129	-10.577	-10.960	-11.193	-11.081	-10.155
	15.0	-8.472	-9.211	-9.921	-10.589	-11.224	-11.816	-12.279	-12.357	-11.337
	20.0	-8.495	-9.292	-10.102	-10.899	-11.684	-12.453	-13.123	-13.412	-12.413
	30.0	-8.520	-9.383	-10.318	-11.298	-12.312	-13.360	-14.382	-15.085	-14.293
80	1.0	-8.080	-8.171	-8.208	-8.226	-8.227	-8.209	-8.167	-8.096	-7.991
	3.0	-8.307	-8.614	-8.808	-8.949	-9.047	-9.086	-9.037	-8.853	-8.459
	5.0	-8.407	-8.860	-9.193	-9.458	-9.671	-9.810	-9.822	-9.609	-8.986
	7.0	-8.463	-9.017	-9.462	-9.835	-10.155	-10.400	-10.500	-10.308	-9.522
	10.0	-8.513	-9.166	-9.743	-10.252	-10.713	-11.110	-11.355	-11.248	-10.310
	15.0	-8.556	-9.310	-10.038	-10.724	-11.379	-11.994	-12.478	-12.571	-11.542
	20.0	-8.580	-9.394	-10.224	-11.043	-11.854	-12.651	-13.353	-13.669	-12.666
	30.0	-8.605	-9.487	-10.445	-11.454	-12.501	-13.590	-14.662	-15.415	-14.640
100	1.0	-8.124	-8.216	-8.253	-8.272	-8.273	-8.255	-8.213	-8.141	-8.034
	3.0	-8.354	-8.664	-8.861	-9.004	-9.103	-9.144	-9.095	-8.909	-8.511
	5.0	-8.456	-8.914	-9.252	-9.520	-9.737	-9.879	-9.893	-9.678	-9.050
	7.0	-8.513	-9.073	-9.524	-9.903	-10.229	-10.479	-10.583	-10.391	-9.598
	10.0	-8.563	-9.225	-9.809	-10.326	-10.797	-11.201	-11.454	-11.350	-10.405
	15.0	-8.607	-9.371	-10.110	-10.807	-11.474	-12.102	-12.601	-12.703	-11.667
	20.0	-8.632	-9.456	-10.298	-11.132	-11.958	-12.773	-13.495	-13.828	-12.822
	30.0	-8.657	-9.550	-10.523	-11.549	-12.617	-13.731	-14.834	-15.620	-14.856

Table 3. Exact critical values for $\alpha = 0.1$

		λ								
n	θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	1.00	-5.468	-5.553	-5.580	-5.586	-5.580	-5.561	-5.527	-5.473	-5.408
	3.00	-5.610	-5.850	-5.996	-6.082	-6.130	-6.134	-6.080	-5.931	-5.676
	5.00	-5.667	-6.000	-6.244	-6.417	-6.535	-6.592	-6.560	-6.370	-5.925
	7.00	-5.698	-6.090	-6.411	-6.658	-6.844	-6.962	-6.969	-6.773	-6.161
	10.00	-5.724	-6.174	-6.579	-6.920	-7.198	-7.403	-7.481	-7.313	-6.495
	15.00	-5.747	-6.252	-6.750	-7.208	-7.612	-7.948	-8.150	-8.070	-7.015
	20.00	-5.759	-6.296	-6.853	-7.397	-7.903	-8.350	-8.670	-8.693	-7.497
	30.00	-5.772	-6.345	-6.973	-7.633	-8.291	-8.918	-9.445	-9.673	-8.377
40	1.00	-5.687	-5.775	-5.806	-5.813	-5.808	-5.788	-5.750	-5.692	-5.613
	3.00	-5.843	-6.100	-6.259	-6.358	-6.415	-6.425	-6.369	-6.213	-5.915
	5.00	-5.908	-6.268	-6.537	-6.732	-6.869	-6.944	-6.919	-6.727	-6.237
	7.00	-5.943	-6.369	-6.724	-7.004	-7.220	-7.366	-7.393	-7.201	-6.563
	10.00	-5.974	-6.464	-6.914	-7.299	-7.622	-7.872	-7.992	-7.837	-7.044
	15.00	-6.000	-6.552	-7.106	-7.626	-8.096	-8.502	-8.779	-8.734	-7.806
	20.00	-6.014	-6.602	-7.222	-7.841	-8.429	-8.968	-9.392	-9.480	-8.512
	30.00	-6.029	-6.656	-7.357	-8.107	-8.875	-9.629	-10.309	-10.669	-9.771
60	1.00	-5.764	-5.854	-5.885	-5.894	-5.888	-5.868	-5.830	-5.770	-5.688
	3.00	-5.926	-6.189	-6.353	-6.457	-6.517	-6.529	-6.474	-6.316	-6.009
	5.00	-5.993	-6.363	-6.642	-6.845	-6.990	-7.071	-7.052	-6.858	-6.361
	7.00	-6.030	-6.469	-6.836	-7.128	-7.356	-7.513	-7.551	-7.361	-6.721
	10.00	-6.062	-6.567	-7.033	-7.437	-7.777	-8.044	-8.182	-8.038	-7.253
	15.00	-6.090	-6.659	-7.234	-7.778	-8.273	-8.707	-9.014	-8.997	-8.094
	20.00	-6.105	-6.711	-7.355	-8.002	-8.623	-9.198	-9.664	-9.797	-8.871
	30.00	-6.120	-6.768	-7.495	-8.280	-9.090	-9.896	-10.638	-11.075	-10.250
80	1.00	-5.803	-5.894	-5.926	-5.935	-5.930	-5.909	-5.871	-5.810	-5.726
	3.00	-5.968	-6.234	-6.402	-6.507	-6.569	-6.583	-6.528	-6.369	-6.057
	5.00	-6.037	-6.411	-6.695	-6.903	-7.052	-7.137	-7.120	-6.927	-6.426
	7.00	-6.074	-6.520	-6.894	-7.192	-7.426	-7.589	-7.632	-7.445	-6.802
	10.00	-6.107	-6.620	-7.095	-7.507	-7.856	-8.134	-8.281	-8.144	-7.360
	15.00	-6.135	-6.714	-7.299	-7.856	-8.365	-8.813	-9.137	-9.135	-8.242
	20.00	-6.151	-6.767	-7.423	-8.085	-8.723	-9.318	-9.807	-9.963	-9.056
	30.00	-6.167	-6.825	-7.565	-8.369	-9.201	-10.035	-10.811	-11.290	-10.500
100	1.00	-5.827	-5.918	-5.951	-5.960	-5.955	-5.934	-5.895	-5.835	-5.749
	3.00	-5.994	-6.262	-6.431	-6.538	-6.601	-6.616	-6.561	-6.401	-6.087
	5.00	-6.063	-6.441	-6.728	-6.938	-7.090	-7.178	-7.162	-6.969	-6.465
	7.00	-6.101	-6.550	-6.929	-7.231	-7.469	-7.636	-7.682	-7.496	-6.852
	10.00	-6.134	-6.652	-7.133	-7.550	-7.905	-8.188	-8.341	-8.209	-7.425
	15.00	-6.163	-6.747	-7.339	-7.903	-8.421	-8.878	-9.212	-9.220	-8.332
	20.00	-6.179	-6.801	-7.465	-8.135	-8.784	-9.391	-9.894	-10.066	-9.168
	30.00	-6.195	-6.859	-7.609	-8.423	-9.270	-10.120	-10.918	-11.423	-10.655