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POWER SYSTEM VOLTAGE STABILITY

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ABSTRACT

Power system voltage stability is characterized as being capable of maintaining load voltage magnitudes within specified operating limits under steady state conditions. In this paper, the first order delay model of a load admittance change is introduced. Then, using this model, a set of linearized dynamic equations is derived and stability conditions are obtained. An earlier result in the literature is shown to agree with that in this paper. The stability conditions are tested and verified in a 2-load, 2-power source system and a 13-node, 4-power source system.

INTRODUCTION

Transient or steady state stability is defined as the capability of a power system to operate stably without loss of synchronization among generators after a large or small disturbance, respectively. Voltage stability on the other hand, is the ability to maintain load voltage magnitudes within specified operating limits under steady state conditions. Because it has become increasingly difficult to obtain power plant sites in the vicinity of power consumers, electrical power is now often transported over long distances, by using large capacity lines. Under these circumstances, voltage stability may be a major problem, as well as transient and steady state stabilities.

There have been several reports of voltage instability phenomena in the USSR, Japan and France. Much work has been done to clarify voltage instability phenomena[1-9]. Various past studies have indicated that the voltage instability occurred gradually over a long period of time and that its process could be regarded as steady state rather than transient state.

In [3] voltage stability conditions for a multi-power source system were derived from a steady state analysis in which a time lag of the load admittance change was assumed for a small step change in source voltage or shunt capacitance at the load terminal. In [5] and [6], the stability conditions were further refined by using M-matrix properties. (See Appendices I and II.)

In [8], by assuming that the loads consist of constant impedance loads and induction motors, a first order delay load model was obtained. Then the stability condition, very similar to that given in [5], was derived.

In this paper, by examining the behavior of transformer tap changers, a dynamic load model is introduced. Then by inspecting eigenvalues of a set of linearized dynamic system equations, stability conditions are derived. One of the conditions is proved to be equivalent to that given in [5]. The results are exemplified by digital simulations on a 2-load, 2-power-source system and a 13-node, 4-power-source system.

REPRESENTATION OF LOAD DYNAMICS

Voltage stability depends heavily on the load-voltage characteristics. Induction motors, which tend to consume constant power irrespective of a terminal voltage change, and on-load tap changers, which operate as though the loads have constant power load characteristics, are known to be the critical components to worsen voltage stability. Therefore, in this section, load dynamics are derived from the operation of on-load tap changers.

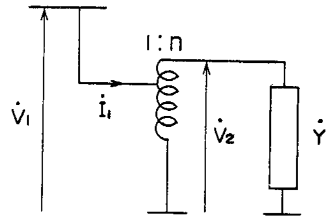

FIG. 1 MODEL OF AN ON-LOAD TAP CHANGER

Figure 1 shows an equivalent circuit for an on-load tap changer and a load connected to it. In the figure, $\dot{V}_1$ and $\dot{I}_1$ are primary complex voltage and current, respectively; 1 : n, a transformer winding ratio; $\dot{V}_2$, secondary complex voltage; and $\dot{Y}$, resultant admittance of the transformer leakage admittance and the load admittance. The following equations hold:

$$\dot{I}_1 = n\,\dot{Y}\,\dot{V}_2 \qquad (1)$$

$$n\,\dot{V}_1 = \dot{V}_2 \qquad (2)$$

Hence, $\dot{I}_1 = n^2\dot{Y}\,\dot{V}_1$.

Thus the load admittance $\dot{Y}_L$ seen from the primary side is given by

$$\dot{Y}_L = n^2\,\dot{Y} \qquad (3)$$

Let the secondary voltage magnitude V_2 $(=|\dot{V}_2|)$ be regulated as the constant value V_S. By assuming that the winding ratio n is continuously changeable, the dynamics of n according to the secondary voltage change are modeled by

$$\frac{dn}{dt} = \frac{1}{T_S}(V_S - V_2) \qquad (4)$$

where T_S (>0) is a time constant. Differentiating $\dot{Y}_L$ in (3) with respect to time t, by assuming $\dot{Y}$ is constant, gives

$$\frac{d\dot{Y}_L}{dt} = 2n\dot{Y}\frac{dn}{dt} \qquad (5)$$

In view of (2), (3) and (4), (5) reduces to

$$\frac{d\dot{Y}_L}{dt} = \frac{2V_1}{T_S}\left(\frac{V_S V_2}{V_1^2}\dot{Y} - \dot{Y}_L\right) \qquad (6)$$

From (6), the tap changer acts so that the load admittance $\dot{Y}_L$ is regulated to $V_S^2\dot{Y}/V_1^2$. This means that the power consumed at the load is regulated as constant with a first order delay, irrespective of primary voltage magnitude V_1. Therefore, as a load model, the following first order delay model is used in the following analysis.

$$\frac{dG_L}{dt} = -\frac{1}{T}(G_L - f_G(V)) \qquad (7)$$

$$\frac{dB_L}{dt} = -\frac{1}{T}(B_L - f_B(V)) \qquad (8)$$

where $\dot{Y}_L = G_L + jB_L$, T : time constant; f_G, f_B : steady state load voltage characteristics for G_L and B_L, respectively. In (5), the coefficient of $\dot{Y}_L$ includes V_1. Thus T may be a function of V_1, however, for simplicity

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T is assumed to be constant. The induction motor loads can also be represented by a first order delay model as seen from [8] and [10] .

VOLTAGE STABILITY CONDITIONS

Since voltage instability occurs under steady state conditions, generators can be modeled by power sources with constant voltage magnitudes. Now consider an (N + M) node power system with N loads and M power sources. Let nodes 1 to N be load nodes and nodes N + 1 to N + M be source nodes. For nodes i, the following nodal power equations hold:

$$B_{Li}V_i^2 = -Q_{bi} - \sum_{k \varepsilon s_i} Q_{ik}$$
$$G_{Li}V_i^2 = -\sum_{k \varepsilon s_i} P_{ik} \qquad i=1,\cdots,N \quad \cdots\cdots (9)$$

where $G_{Li}+jB_{Li}$: complex load admittance;
S_i : set of node numbers with lines connected to node i;
Q_{bi} : reactive power fed from susceptance b_i at node i;
Q_{ik} : reactive power flow from node i to node k at node i; and
P_{ik} : real power flow from node i to node k at node i.

The latter three variables are given, respectively, by

$$Q_{bi} = -b_iV_i^2$$
$$Q_{ik} = -B_{ik}V_i^2 + B_{ik}V_iV_k\cos(\theta_i-\theta_k) - G_{ik}V_iV_k\sin(\theta_i-\theta_k)$$
$$P_{ik} = -B_{ik}V_iV_k\sin(\theta_i-\theta_k) + G_{ik}(V_i^2 - V_iV_k\cos(\theta_i-\theta_k)) \quad \cdots\cdots (10)$$

where $G_{ik}+jB_{ik}$: complex admittance of line from node i to node k;
θ_i : voltage phase angle at node i in reference to node N + M.

The first order delay model as shown in (7) and (8) is used to represent load dynamics at node i. Namely,

$$\frac{dB_{Li}}{dt} = -\frac{1}{T_i}(B_{Li} - f_{Bi}(V_i))$$
$$\frac{dG_{Li}}{dt} = -\frac{1}{T_i}(G_{Li} - f_{Gi}(V_i)) \quad \cdots\cdots (11)$$

Because voltage instability occurs over a long period of time (from several minutes to hours), the power system under study can be considered to be in steady state conditions. Thus in the voltage stability analysis, the generators can be modeled as power sources, with constant source voltages, which follow the changes in power consumed at loads without any time delays. Then at generator nodes N + i, let the incremental generated outputs for the increase of the total transmission loss and of the total power consumed by loads be given by,

$$\Delta R_i = \Delta\Big(\sum_{j \varepsilon S_{i+N}} P_{i+N,j}\Big) - k_i\,\Delta\Big(P_{LOSS} + \sum_{j=1}^{N} G_{Lj}V_j^2\Big) = 0 \quad \cdots\cdots (12)$$
$$i = 1,\cdots, M-1$$

where k_i are incremental factors of the generated outputs at node N + i which satisfy

$$\sum_{i=1}^{M} k_i = 1 \quad \text{and} \quad k_i \geq 0.$$

The P_{LOSS} is the total transmission loss given by

$$P_{LOSS} = \sum_{\substack{i,j=1 \\ i \neq j}}^{N+M} P_{ij} = \sum_{(i,j)} (P_{ij}+P_{ji}) = \sum_{(i,j)} G_{ij}\{V_i^2+V_j^2-2V_iV_j\cos(\theta_i-\theta_j)\}$$

where (i, j) are all combinations of node numbers corresponding to both ends of a line.

Let k_i be constant and integrate (12).

$$R_i = \sum_{j \varepsilon S_{i+N}} P_{i+N,j} - k_i\Big(P_{LOSS} + \sum_{j=1}^{N} G_{Lj}V_j^2\Big) + C_i = 0 \quad \cdots\cdots (13)$$
$$i = 1,\cdots, M-1$$

where C_i are integral constants and

$$\sum_{i=1}^{M} C_i = 0.$$

In order to derive a stability condition at the equilibrium point $(B_L, G_L)_{t=\infty}$ using (11) and (12), let linearize (11) around the equilibrium point. Since V_i values in (11) are functions of B_{Lj}, G_{Lj} and R_k (j=1, ..., N, k=1, ..., M-1),

$$\Delta V = \frac{\partial V}{\partial B_L}\Delta B_L + \frac{\partial V}{\partial G_L}\Delta G_L + \frac{\partial V}{\partial R}\Delta R \quad \cdots\cdots (14)$$

holds, where

$$\Delta V = (\Delta V_1, \cdots, \Delta V_N)^t,\quad \Delta B_L = (\Delta B_{L1}, \cdots, \Delta B_{LN})^t,\quad \Delta G_L = (\Delta G_{L1}, \cdots, \Delta G_{LN})^t,\quad \Delta R = (\Delta R_1, \cdots, \Delta R_{M-1})^t,$$

$$\frac{\partial V}{\partial B_L} = \begin{bmatrix} \frac{\partial V_1}{\partial B_{L1}} & \cdots & \frac{\partial V_1}{\partial B_{LN}} \\ \vdots & & \vdots \\ \frac{\partial V_N}{\partial B_{L1}} & \cdots & \frac{\partial V_N}{\partial B_{LN}} \end{bmatrix},$$

$$\frac{\partial V}{\partial G_L} = \begin{bmatrix} \frac{\partial V_1}{\partial G_{L1}} & \cdots & \frac{\partial V_1}{\partial G_{LN}} \\ \vdots & & \vdots \\ \frac{\partial V_N}{\partial G_{L1}} & \cdots & \frac{\partial V_N}{\partial G_{LN}} \end{bmatrix}, \quad \frac{\partial V}{\partial R} = \begin{bmatrix} \frac{\partial V_1}{\partial R_1} & \cdots & \frac{\partial V_1}{\partial R_{M-1}} \\ \vdots & & \vdots \\ \frac{\partial V_N}{\partial R_1} & \cdots & \frac{\partial V_N}{\partial R_{M-1}} \end{bmatrix}.$$

Let

$$X_{Bi} = B_{Li}(t) - B_{Li}(\infty),$$
$$X_{Gi} = G_{Li}(t) - G_{Li}(\infty). \quad \cdots\cdots (15)$$

Then in view of (14), a set of linearized equations around the equilibrium point is given by

$$\frac{d}{dt}\begin{bmatrix} X_B \\ X_G \end{bmatrix} = \begin{bmatrix} -T^{-1}(E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}) & T^{-1}\frac{\partial f_B}{\partial V}\frac{\partial V}{\partial G_L} \\ T^{-1}\frac{\partial f_G}{\partial V}\frac{\partial V}{\partial B_L} & -T^{-1}(E - \frac{\partial f_G}{\partial V}\frac{\partial V}{\partial G_L}) \end{bmatrix} \begin{bmatrix} X_B \\ X_G \end{bmatrix} + \begin{bmatrix} T^{-1}\frac{\partial f_B}{\partial V}\frac{\partial V}{\partial R} \\ T^{-1}\frac{\partial f_G}{\partial V}\frac{\partial V}{\partial R} \end{bmatrix}\Delta R \quad \cdots\cdots (16)$$

where E is an N x N unit matrix and

$$X_B = (X_{B1}, \cdots, X_{BN})^t,\quad \frac{\partial f_B}{\partial V} = \begin{bmatrix} \frac{\partial f_{B1}(V_1)}{\partial V_1} & & 0 \\ & \ddots & \\ 0 & & \frac{\partial f_{BN}(V_N)}{\partial V_N} \end{bmatrix},$$

$$X_G = (X_{G1}, \cdots, X_{GN})^t,$$

$$T = \begin{bmatrix} T_1 & & 0 \\ & \ddots & \\ 0 & & T_N \end{bmatrix}, \quad \frac{\partial f_G}{\partial V} = \begin{bmatrix} \frac{\partial f_{G1}(V_1)}{\partial V_1} & & 0 \\ & \ddots & \\ 0 & & \frac{\partial f_{GN}(V_N)}{\partial V_N} \end{bmatrix}.$$

In order to evaluate $\partial V/\partial B_L$, $\partial V/\partial G_L$ and $\partial V/\partial R$, let (9) and (13) be expressed in vector forms as follows:

$$B_L = g_B(V, \theta)$$
$$G_L = g_G(V, \theta) \qquad \cdots\cdots \qquad (17)$$
$$R = g_R(V, \theta, G_L)$$

Taking small increments in (17) yields

$$\begin{bmatrix} \Delta B_L \\ \Delta G_L \\ \Delta R \end{bmatrix} = \begin{bmatrix} \frac{\partial g_B}{\partial V} & \frac{\partial g_B}{\partial \theta} \\ \frac{\partial g_G}{\partial V} & \frac{\partial g_G}{\partial \theta} \\ \frac{\partial g_R}{\partial V} & \frac{\partial g_R}{\partial \theta} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\partial g_R}{\partial G_L} \end{bmatrix} \Delta G_L. \cdots \qquad (18)$$

Because $\Delta R = 0$ is assumed in (12), (18) reduces to

$$\begin{bmatrix} \Delta B_L \\ \Delta G_L \\ -\frac{\partial g_R}{\partial G_L} \Delta G_L \end{bmatrix} = \begin{bmatrix} \frac{\partial g_B}{\partial V} & \frac{\partial g_B}{\partial \theta} \\ \frac{\partial g_G}{\partial V} & \frac{\partial g_G}{\partial \theta} \\ \frac{\partial g_R}{\partial V} & \frac{\partial g_R}{\partial \theta} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \theta \end{bmatrix}. \qquad \cdots\cdots \qquad (19)$$

Therefore, ΔV in (14) can be solved for from (19). Namely,

$$\begin{bmatrix} \Delta V \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} \frac{\partial g_B}{\partial V} & \frac{\partial g_B}{\partial \theta} \\ \frac{\partial g_G}{\partial V} & \frac{\partial g_G}{\partial \theta} \\ \frac{\partial g_R}{\partial V} & \frac{\partial g_R}{\partial \theta} \end{bmatrix}^{-1} \begin{bmatrix} \Delta B_L \\ \Delta G_L \\ -\frac{\partial g_R}{\partial G_L} \Delta G_L \end{bmatrix}. \qquad \cdots\cdots \qquad (20)$$

From the inverse mapping theorem,

$$\begin{bmatrix} \frac{\partial g_B}{\partial V} & \frac{\partial g_B}{\partial \theta} \\ \frac{\partial g_G}{\partial V} & \frac{\partial g_G}{\partial \theta} \\ \frac{\partial g_R}{\partial V} & \frac{\partial g_R}{\partial \theta} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{\partial V}{\partial B_L} & \frac{\partial V}{\partial G_L} & \frac{\partial V}{\partial R} \\ \frac{\partial \theta}{\partial B_L} & \frac{\partial \theta}{\partial G_L} & \frac{\partial \theta}{\partial R} \end{bmatrix}. \qquad \cdots\cdots \qquad (21)$$

Hence, $\partial V/\partial B_L$, $\partial V/\partial G_L$, and $\partial V/\partial R$ in (14) and (16) are given by (21), by considering g_R is a function of V and θ, and $\Delta R = -\partial g_R/\partial G_L \cdot \Delta G_L$. Subtitution of $\Delta R = -\partial g_R/\partial G_L \cdot \Delta G_L$ into (16) yields

$$\frac{d}{dt}\begin{bmatrix} X_B \\ X_G \end{bmatrix} = \begin{bmatrix} -T^{-1}\left(E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}\right) & T^{-1}\frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right) \\ T^{-1}\frac{\partial f_G}{\partial V}\frac{\partial V}{\partial B_L} & -T^{-1}\left[E - \frac{\partial f_G}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right)\right] \end{bmatrix} \begin{bmatrix} X_B \\ X_G \end{bmatrix}. \qquad \cdots\cdots \qquad (22)$$

The eigenvalues of the coefficient matrix in (22) are given by solving the following equation:

$$\det \begin{bmatrix} -T^{-1}\left(E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}\right) - \lambda E & T^{-1}\frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right) \\ T^{-1}\frac{\partial f_G}{\partial V}\frac{\partial V}{\partial B_L} & -T^{-1}\left[E - \frac{\partial f_G}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G}\right)\right] - \lambda E \end{bmatrix} = 0 \qquad \cdots\cdots \qquad (23)$$

In (23), subtracting from the i + Nth ($1 \leq i \leq N$) row elements the corresponding ith row elements multiplied by $(\partial f_{Bi}/\partial V_i)^{-1}(\partial f_{Gi}/\partial V_i)$ yields

$$\det \begin{bmatrix} -T^{-1}\left(E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}\right) - \lambda E & T^{-1}\frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right) \\ \left(\frac{\partial f_B}{\partial V}\right)^{-1}\left(\frac{\partial f_G}{\partial V}\right)(T^{-1} + \lambda E) & -T^{-1} - \lambda E \end{bmatrix} = 0. \qquad \cdots\cdots \qquad (24)$$

Then in (24), adding to the ith ($1 \leq i \leq N$) column elements the corresponding N + ith column elements multiplied by $(\partial f_{Bi}/\partial V_i)^{-1}(\partial f_{Gi}/\partial V_i)$ gives

$$\det \begin{bmatrix} -T^{-1}\left[E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L} - \frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right)\left(\frac{\partial f_B}{\partial V}\right)^{-1}\frac{\partial f_G}{\partial V}\right] - \lambda E & T^{-1}\frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right) \\ 0 & -T^{-1} - \lambda E \end{bmatrix} = 0. \qquad \cdots\cdots \qquad (25)$$

From (25), among the 2N eigenvalues in (23), the N eigenvalues are given by $-1/T_1$, $-1/T_2$, $\cdots\cdots$, $-1/T_N$. The remaining N eigenvalues are those of the following matrix:

$$-T^{-1}\left[E - \frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L} - \frac{\partial f_B}{\partial V}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right) \times \left(\frac{\partial f_B}{\partial V}\right)^{-1}\frac{\partial f_G}{\partial V}\right]$$

$$= -T^{-1}\frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}\left[\left(\frac{\partial V}{\partial B_L}\right)^{-1}\left(\frac{\partial f_B}{\partial V}\right)^{-1} - E - \left(\frac{\partial V}{\partial B_L}\right)^{-1} \times \left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right)\left(\frac{\partial f_B}{\partial V}\right)^{-1}\frac{\partial f_G}{\partial V}\right]$$

$$= -T^{-1}\frac{\partial f_B}{\partial V}\frac{\partial V}{\partial B_L}\left[\left(\frac{\partial V}{\partial B_L}\right)^{-1} - \frac{\partial f_B}{\partial V} - \left(\frac{\partial V}{\partial B_L}\right)^{-1}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right)\frac{\partial f_G}{\partial V}\right]\left(\frac{\partial f_B}{\partial V}\right)^{-1} \qquad \cdots\cdots \qquad (26)$$

Equation (26) is derived by using the fact that the matrices T, $\partial f_B/\partial V$, and $\partial f_G/\partial V$ are diagonal and that diagnal matrix multiplication is commutative. In (26), matrices $\partial f_B/\partial V$, and $(\partial f_B/\partial V)^{-1}$ are multiplied from both sides. Thus the N eigenvalues in (26) coincide with those of the following matrix:

$$-T^{-1}\frac{\partial V}{\partial B_L}\left[\left(\frac{\partial V}{\partial B_L}\right)^{-1} - \frac{\partial f_B}{\partial V} - \left(\frac{\partial V}{\partial B_L}\right)^{-1}\left(\frac{\partial V}{\partial G_L} - \frac{\partial V}{\partial R}\frac{\partial g_R}{\partial G_L}\right)\frac{\partial f_G}{\partial V}\right] \cdots \qquad (27)$$

In order to evaluate $\partial V/\partial B_L$, $\partial V/\partial G_L$, and $\partial V/\partial R$ in (21), let

$$\frac{\partial g_B}{\partial V} = F_1 , \quad \frac{\partial g_B}{\partial \theta} = A_1 \quad \begin{bmatrix} \frac{\partial g_G}{\partial V} \\ \frac{\partial g_R}{\partial V} \end{bmatrix} = F_2 , \quad \begin{bmatrix} \frac{\partial g_G}{\partial \theta} \\ \frac{\partial g_R}{\partial \theta} \end{bmatrix} = A_2.$$

Then from (21),

$$\frac{\partial V}{\partial B_L} F_1 + \left[\frac{\partial V}{\partial G_L} \;\; \frac{\partial V}{\partial R} \right] F_2 = 0, \quad \cdots\cdots (28)$$

$$\frac{\partial V}{\partial B_L} A_1 + \left[\frac{\partial V}{\partial G_L} \;\; \frac{\partial V}{\partial R} \right] A_2 = 0. \quad \cdots\cdots (29)$$

From (29),

$$\left[\frac{\partial V}{\partial G_L} \;\; \frac{\partial V}{\partial R} \right] = - \frac{\partial V}{\partial B_L} A_1 A_2^{-1}.$$

Substitution of the above equation into (28) gives

$$\frac{\partial V}{\partial B_L} = (F_1 - A_1 A_2^{-1} F_2)^{-1} \triangleq G^{-1} \quad \cdots\cdots (30)$$

where G is a reduced Jacobian matrix.
Then

$$\left[\frac{\partial V}{\partial G_L} \;\; \frac{\partial V}{\partial R} \right] = - G^{-1} A_1 A_2^{-1} \quad \cdots\cdots (31)$$

Substituting (30) and (31) into (27),

$$- T^{-1} G^{-1} \left\{ G - \frac{\partial f_B}{\partial V} + A_1 A_2^{-1} \begin{bmatrix} \frac{\partial f_G}{\partial V} \\ -\frac{\partial g_R}{\partial G_L} \cdot \frac{\partial f_G}{\partial V} \end{bmatrix} \right\}$$

$$= - T^{-1} G^{-1} \left\{ F_1 - \frac{\partial f_B}{\partial V} - A_1 A_2^{-1} \begin{bmatrix} \frac{\partial g_G}{\partial V} - \frac{\partial f_G}{\partial V} \\ \frac{\partial g_R}{\partial V} + \frac{\partial g_R}{\partial G_L} \cdot \frac{\partial f_G}{\partial V} \end{bmatrix} \right\}. \quad \cdots (32)$$

Matrix G in (32) is evaluated by assuming G_L and B_L are constant in (17), namely the loads are linear. The remaining matrix in the right hand side is evaluated using the following equations:

$$B_L = f_B(V) = g_B(V, \theta),$$
$$G_L = f_G(V) = g_G(V, \theta), \quad \cdots\cdots (33)$$
$$R = g_R(V, \theta, f_G(V)).$$

This implies that the loads are assumed to be nonlinear. Then let (32) be expressed as

$$- T^{-1} G_{LL}^{-1} G_{NL}$$

where LL and NL denote that loads are linear and nonlinear, respectively. Matrices G_{LL}, and G_{NL} correspond to those in [3] , [5] and [6] (see Appendix I). Let these be denoted as G_{LL}', and G_{NL}' (or generally as G'). The relationship between matrices G and G' is given by (see Appendix III)

$$G' = - \begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} G. \quad \cdots\cdots (34)$$

Therefore,

$$- T^{-1} G_{LL}^{-1} G_{NL} = - T^{-1} G_{LL}'^{-1} G_{NL}'. \quad \cdots\cdots (35)$$

From (35) the following voltage stability condition is derived:

Stability Condition I Load voltages are stable if and only if all of the real parts of the eigenvalues of (35) are negative.

If $G_{LL}^{-1} G_{NL}$ is an M-matrix (see Appendix II), $T \cdot G_{NL}^{-1} G_{NL}$ is also an M-matrix from theorem 2 in Appendix II. Therefore, from theorem 1, load voltages are stable. Thus the following stability condition is obtained.

Stability Condition II Load voltages are stable if $G_{LL}^{-1} \cdot G_{NL}$ (or $G_{LL}'^{-1} G_{NL}'$) is an M-matrix.

In [5] , [6] and [7] , G_{LL}' (hence $-G_{LL}$ from theorem 2) is shown to be nearly equal to an M-matrix. When G_{LL}' is an M-matrix, stability condition II is futher simplified and the following two stability conditions are derived:

Stability Condition III Load voltages are stable if G_{LL}' (or $-G_{LL}$) is an M-matrix, off-diagonal elements of G_{NL}' ($-G_{NL}$) are all non-positive and $G_{NL}' - G_{LL}' \geq 0$ ($G_{LL} - G_{NL} \geq 0$). (See Definition 1 in Appendix II.)

Proof From theorem 11 in Appendix II, $G_{NL}'^{-1} G_{LL}'$ ($G_{NL}^{-1} G_{LL}$) is an M-matrix. Then from theorem 2, $G_{NL}'^{-1} G_{LL}' T$ ($G_{NL}^{-1} G_{LL} T$) is also an M-matrix. From the collorary to theorem 11, all the real parts of eigenvalues of $T^{-1} G_{LL}'^{-1} G_{NL}'$ ($T^{-1} G_{LL}^{-1} G_{NL}$) are positive. Thus load voltages are stable.

The magnitudes of time constants Ti, whether they be large or small, do not affect the voltage stability at all.

Stability Condition IV Load voltages are stable if G_{LL}' ($-G_{LL}$) and G_{NL}' ($-G_{NL}$) are M-matrices, and $G_{NL}' - G_{LL}' \leq 0$ ($G_{LL} - G_{NL} \leq 0$).

Proof From theorem 11, $T^{-1} G_{LL}'^{-1} G_{NL}'$ ($T^{-1} G_{LL} \cdot G_{NL}$) is an M-matrix. Thus load voltages are stable.

In this case, from theorem 6, the voltage stability limit occurs when the determinant of G_{NL} (or G_{NL}') vanishes.[5,7]

When $G_{NL}' - G_{LL}'$ ($G_{LL} - G_{NL}$) has both positive and negative elements, $G_{LL}^{-1} G_{NL}$ may deviate from being an M-matrix, even when the load voltages are stable. In this case it is not clear whether it is a sufficient condition for voltage stability that G_{NL}' and G_{LL}' ($-G_{NL}$ and $-G_{LL}$) be M-matrices. What can be clarified so far is as follows: Let

$$G_{NL}' = G_{LL}' - P_1 + P_2$$

where $P_1, P_2 \geq 0$. Then load voltages are stable if G_{LL}' and $G_{LL}' - P_1$ are M-matrices, off diagonal elements of G_{NL}' are all non positive, the magnitudes of load voltages are the same, and the time constants of load dynamics are the same. (The proof can be made by using theorem 13, but is not discussed here.)

Stability conditions III and IV state that G_{LL}' and G_{NL}' ($-G_{LL}$ and $-G_{NL}$) being M-matrices is a sufficient condition for voltage stability. Also from continuity of the eigenvalues, it is assumed that G_{LL}' and G_{NL}' being M-matrices is a sufficient condition for voltage stability.

NUMERICAL EXAMPLES

Two-Load, Two-Power-Source System

Case 1 Consider the 2-load, 2-power-source system[3,5] shown in Fig. 2. Let

$Y_1 = Y_2 = Y_3 = Y_4 = -j\,1.0$ (p.u.),
$b_1 = 0.15$ (p.u.), $b_2 = 0.1$ (p.u.),
$P_{L1} = 0.6021$ (p.u.), $P_{L2} = 0.1374$ (p.u.),
$Q_{L1} = 0.0799$ (p.u.), $Q_{L2} = 0.0648$ (p.u.),
$P_{31} = 0.1736$ (p.u.).

All the loads have constant power characteristics.

Figure 3 shows loci of V_1 and V_2 by varying the value of E_1 and fixing that of E_2 to 1 p.u. For branch 1 of the figure, G_{LL}', G_{NL}' ($-G_{LL}$, $-G_{NL}$) are M-matrices and $G_{NL}' - G_{LL}' < 0$ when the incremental factor k_1=0.1, 0.5 and 0.9. Thus from stability condition IV, branch 1 is stable. Table I shows G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$, and $(G_{LL}'^{-1} G_{NL}')^{-1}$ for branch 1 with k_1=0.1 and E_1= 0.4 p.u..

In branches 2, 3 and 4, G_{NL}' is not an M-matrix. For k_1=0.9, there is a region where two eigenvalues of $G_{LL}^{-1} G_{NL}$ are positive. However, this stable region vanishes when k_1 changes to 0.5 (or 0.1). Thus the stability margin is very small and this sort of the stable region should not be judged as suitable from the stand-point of practical stable operation.[3,5]

Case 2 A leading load is assumed for node 1 and parameters are changed to

$$Q_{L1} = -0.0701\,(\text{p.u.})\ ,\quad b_1 = 0$$

and k_1 = 0.9.

The real power loads are assumed to have constant power characteristics as before, while reactive power loads are assumed to have the following voltage characteristics:

$$Q_{Li} = Q_{0i} + 2.0\,P_{0i}\,(V_i - V_{0i}) \qquad i = 1\ ,\ 2$$

where Q_{01} = -0.0701, P_{01} = 0.6021
Q_{02} = 0.0799, P_{02} = 0.1374
and $V_{01} = V_{02}$ = 1.0 (p.u.).

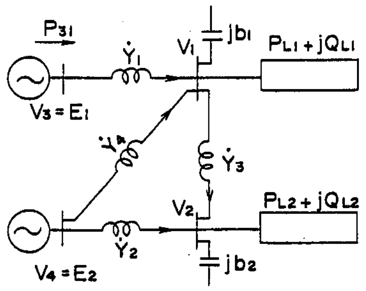

FIG. 2 TWO-LOAD, TWO-POWER-SOURCE SYSTEM

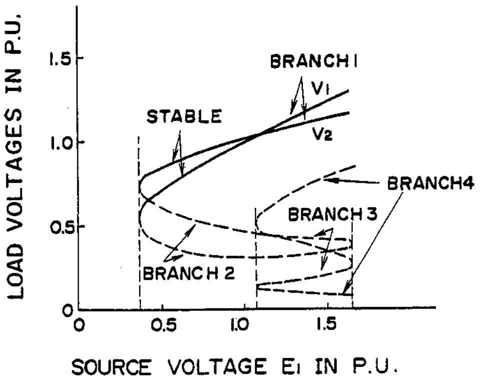

FIG. 3 LOAD VOLTAGE LOCI

Table I Matrices G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$, and $(G_{LL}'^{-1} G_{NL}')^{-1}$ for Branch 1 of Fig.3 with E_1=0.4 P.U.

G_{LL}'

	1	2
1	3.1204	-0.4888
2	-0.8181	1.4954

G_{NL}'

	1	2
1	1.0443	-0.6858
2	-0.8564	1.3227

$G_{LL}'^{-1} G_{NL}'$

	1	2
1	0.2679	-0.0888
2	-0.4261	0.8359

$(G_{LL}'^{-1} G_{NL}')^{-1}$

	1	2
1	4.4917	0.4773
2	2.2899	1.4395

Figure 4 shows loci of V_1 and V_2 by varying E_1 with E_2=1.0 p.u. For branch 1, G_{LL}', G_{NL}' are M-matrices but diagonal elements of $G_{NL}'-G_{LL}'$ are positive while the off-diagonal elements are negative. Table II shows G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$, and $(G_{LL}'^{-1} \cdot G_{NL}')^{-1}$ for branch 1 with E_1=0.3 p.u. Table III shows eigenvalues of $T^{-1} G_{LL}^{-1} G_{NL}$, by changing the ratio of time constants T_1 and T_2. This table suggests that even when $G_{LL}^{-1} \cdot G_{NL}$ is not an M-matrix, the condition that G_{NL} be an M-matrix is sufficient to determine voltage stability.

Table IV shows G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$, and $(G_{LL}'^{-1} G_{NL}')^{-1}$ for branch 2 with E_1=0.3 p.u. In this case G_{NL}' is an M-matrix, while G_{LL}' is not. The eigenvalues of $G_{LL}'^{-1} G_{NL}'$ are 6.12 and -3.51. For branch 2 one of the eigenvalues is negative. Thus branch 2 is unstable.

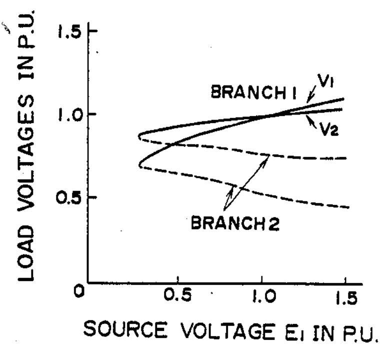

FIG. 4 LOAD VOLTAGE LOCI

Table II Matrices G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$ and $(G_{LL}'^{-1} G_{NL}')^{-1}$ for Branch 1 of Fig.4 with E_1=0.3 P.U.

G_{LL}'

	1	2
1	3.0867	-0.6231
2	-0.9290	1.7202

G_{NL}'

	1	2
1	3.5915	-0.8440
2	-0.9513	1.9052

$G_{LL}'^{-1} G_{NL}'$

	1	2
1	1.1806	-0.0560
2	0.0847	1.0772

$(G_{LL}'^{-1} G_{NL}')^{-1}$

	1	2
1	0.8438	0.0438
2	-0.0663	0.9249

Table III Eigenvalues of $T^{-1} G_{LL}^{-1} G_{NL}$ for Branch 1 of Fig.4 with E_1=0.3 P.U.

T_1	T_2	Eigenvalue 1	Eigenvalue 2
1000	1	1180	1.08
100	1	118	1.08
10	1	11.8	1.08
1	1	1.13+j0.091	1.13-j0.091
1	10	17.8	1.19
1	100	108	1.18
1	1000	1077	1.18

Table IV G_{LL}', G_{NL}', $G_{LL}'^{-1} G_{NL}'$ and $(G_{LL}'^{-1} G_{NL}')^{-1}$ for Branch 2 of Fig.4 with E_1=0.3 P.U.

G_{LL}'

	1	2
1	-0.3595	-1.2484
2	-0.8923	1.6181

G_{NL}'

	1	2
1	4.0403	-0.7655
2	-0.9207	1.8275

$G_{LL}'^{-1} G_{NL}'$

	1	2
1	-3.1778	-0.6149
2	-2.3211	0.7902

$(G_{LL}'^{-1} G_{NL}')^{-1}$

	1	2
1	-0.2006	-0.1561
2	-0.5893	0.8068

Thirteen-Node, Four-Power-Source System

Consider the 13-node, 4-power-source system shown in Fig. 5. Table V shows line impedances, and Table VI, specified generation, load and shunt capacitance. Table VII gives a load flow solution obtained by specifying P and V values except for nodes 3 and 5. The susceptance of shunt capacitance at each load node (except nodes 3 and 5) is determined so that the load voltage becomes 1.0 p.u. when P and Q values are specified.

The incremental factors are selected as $k_1=0.1$, $k_2=0.6$, $k_3=0.2$ and $k_4=0.1$. Load voltage characteristics are assumed to be

$$P_{Li} = P_{oi} + \alpha_i (V_i - V_{oi}), \qquad i = 1, \cdots, 9$$

$$Q_{Li} = Q_{oi} + \beta_i (V_i - V_{oi}).$$

For case 1,
$\alpha_i = 0.25$, $\beta_i = 0.05$ $i = 1, \ldots, 9$
and for case 2
$\alpha_i = 0.05$, $\beta_i = 0.05$
are assumed.

Table VIII shows $G_{LL}^{-1} G_{NL}$, $(G_{LL}^{-1}G_{NL})^{-1}$, and eigenvalues of $G_{LL}^{-1}G_{NL}$ for case 1. Off-diagonal elements of $G_{LL}^{-1}G_{NL}$ are non-positive except those of column three which are very small. Elements of $(G_{LL}^{-1} \cdot G_{NL})^{-1}$ are positive except the off-diagonal elements of column three. Thus $G_{LL}^{-1} G_{NL}$ is nearly equal to an M-matrix. Also the eigenvalues of $G_{LL}^{-1} G_{NL}$ are all positive. Thus the load voltages are stable. Matrices G_{LL}', and G_{NL}', although not shown in the table, are also nearly equal to M-matrices and elements of $G_{NL}' - G_{LL}'$ are negative except three off-diagonal elements. In this case stability condition IV shown in the previous section is satisfied.

Table IX shows $G_{LL}^{-1} G_{NL}$, $(G_{LL}^{-1}G_{NL})^{-1}$ and eigenvalues of $G_{LL}^{-1}G_{NL}$ for case 2. The elements of $(G_{LL}^{-1} G_{NL})^{-1}$ are negative except for column three, and one of the eigenvalues of $G_{LL}^{-1}G_{NL}$ is negative. Thus the load voltages are unstable. This is because G_{NL}' is not an M-matrix.

DISCUSSION OF RESULTS

The assumption of the continuous tap change used to derive (4) may be inappropriate for a single on-load tap changer. But when a number of on-load tap changers are connected to the same network node, aggregation of them can be considered to behave like the continuously variable tap changer. Thus in this case the assumption is justified. Also the first order delay load model given by (7) and (8) is known to represent other load dynamics such as induction motors and thermostatically controlled heating load.

By modeling the load admittance change as a first order delay, the necessary and sufficient condition for voltage stability is shown to be that the real parts of eigenvalues of $T^{-1} G_{LL}^{-1} G_{NL}$ are positive. This condition coincides with the condition that $-G_{LL}$ and $-G_{NL}$ are M-matrices when $G_{LL}-G_{NL} \leq 0$. This is the stability condition shown in [5], [6] and [7]. Due to the operation of on-load tap changers, and induction motors included in a load, actual load voltage characteristics tend to be between those of linear and constant power loads. Under this circumstance, the condition $G_{LL} - G_{NL} \leq 0$ holds as seen from the simulations. Therefore, in order to investigate voltage stability of an actual power system it is sufficient to examine $-G_{NL}$ and $-G_{LL}$ as being M-matrices.

There is a stable region where $-G_{LL}$ and $-G_{NL}$ are not M-matrices. However, the stability margin is very small and this region should be judged as unstable from the viewpoint of actual network operation. Thus, practically, the stability condition that $-G_{NL}$ and $-G_{LL}$ be M-matrices is considered to be a necessary and sufficient condition for voltage stability.

CONCLUSIONS

A first order delay model was derived for a load admittance change from the operation of an on-load tap changer . This model was then used to analyze voltage stability and stability conditions were derived from a set of linearized dynamic equations. A previous stability condition[5,6] which stated that the load voltages were stable if the reduced Jacobian matrix for nonlinear loads was (nearly equal to) an M-matrix was shown to be in agreement with that derived in this paper.

Digital simulations on sample systems showed that the above mentioned stability condition was sufficient to determine voltage stability for an actual power system.

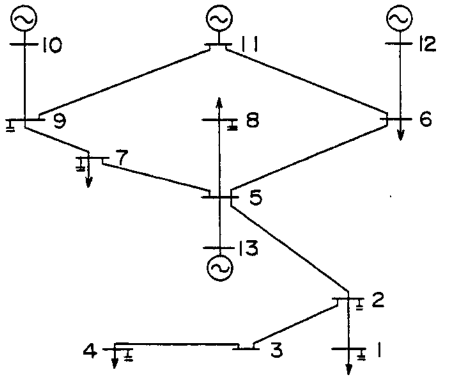

FIG. 5 13-NODE, 4-POWER-SOURCE SYSTEM

Table V Line Impedances

Node i-j	Impedance	Node i-j	Impedance
1-2	j0.029	5-13	j0.056
2-3	j0.019	6-11	j0.1
2-5	j0.019	6-12	j0.035
3-4	j0.029	7-9	j0.11
5-6	j0.03	9-10	j0.035
5-7	j0.034	9-11	j0.027
5-8	j0.11		

Table VI Specified Generation, Load and Shunt Capacitance (in P.U.)

Node	Susceptance of Shunt Capacitance	Generation P	Load P_L	Q_L
1	1.3500		3.8	1.14
2	2.5124		0.9	0.27
4	1.4895		3.8	1.14
6	1.1950		3.9	1.17
7	1.3437		2.0	0.60
8	0.8103		1.0	0.30
9	0.8704		2.9	0.87
10		1.9		
11		11.0		
12		3.9		
13		1.5		

Table VII Load Flow Solution

Node	Voltage Magnitude (in P.U.)	Phase Angle (in degree)
1	1.0000	-30.00
2	1.0000	-23.67
3	0.9960	-27.83
4	1.0000	-34.18
5	0.9902	-4.63
6	1.0000	2.37
7	1.0000	-3.04
8	1.0000	9.31
9	1.0000	14.95
10	1.0200	18.69
11	1.0200	25.59
12	1.0500	9.83
13	1.0500	0.0

ACKNOWLEDGEMENT

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APPENDIX I [3,5,6,7]

Voltage Stability Conditions from Steady State Analysis

For an (N + M)-node power system with M power sources and N nonlinear loads, voltage stability conditions are derived by the following three steps:

(I) Apply a small step change to a source voltage or shunt capacitance at any load terminal.

(II) The load admittances behave as linear loads at that instance because of their time lags.

(III) The load voltages change stepwise owing to (I) and (II). Due to these load voltage changes, the load admittances also begin to change. The equilibrium point is stable if the load admittances change towards the values of the load admittances at the new equilibrium point and is unstable if not.

By applying the above procedures, the following two stability conditions are derived:

$$\left.\frac{dV_j}{dE_i}\right|_{N\cdot L} \cdot \left.\frac{dV_j}{dE_i}\right|_{L\cdot L} > 0 \quad \text{for } i=1,\cdots,M,\ j=1,\cdots,N \qquad (I1)$$

and

$$\left.\frac{dV_j}{db_i}\right|_{N\cdot L} \cdot \left.\frac{dV_j}{db_i}\right|_{L\cdot L} > 0 \quad \text{for } i,j=1,\cdots,N \quad \cdots\cdots\cdots\cdots \quad (I2)$$

where $E_i = V_{N+i}$ is a source voltage magnitude at node N + i and is assumed to be constant. (Notations which are not defined here are the same as defined in the text section "Voltage Stability Conditions".)

By digital simulations, it is revealed that stability conditions (I1) and (I2) give not only the actually operable solutions, but also stable solutions which are not actually operable because of the small stability margin. In order to eliminate these solutions the stability conditions are refined to the following:

$$\left.\frac{dV_j}{dE_i}\right|_{N\cdot L} > 0 \quad \text{for almost all the combinations of } (i,j) \text{ where } i=1,\ldots,M,\ j=1,\ldots,N \quad \cdots \quad (I3)$$

$$\left.\frac{dV_j}{db_i}\right|_{N\cdot L} > 0 \quad \text{for almost all the combinations of } (i,j) \text{ where } i,j=1,\ \ldots,\ N \quad \cdots \quad (I4)$$

Equations (I3) and (I4) are evaluated by

$$G'_{NL}\ dV/dE = C_1 - A_1' A_2'^{-1} C_2 \quad \cdots\cdots\cdots\cdots \quad (I5)$$

$$G'_{NL}\ dV/db = D_1 \quad \cdots\cdots\cdots\cdots \quad (I6)$$

Table VIII Matrices $G_{LL}^{-1} G_{NL}$ and $(G_{LL}^{-1} G_{NL})^{-1}$, and Eigenvalues of $G_{LL}^{-1} G_{NL}$ for Case 1

Matrix $G_{LL}^{-1} G_{NL}$

	1	2	3	4	5	6	7	8	9
1	0.53265	-0.06956	0.00000	-0.40537	-0.00000	-0.04370	-0.03959	-0.00477	-0.00271
2	-0.37958	0.93043	0.00000	-0.40537	-0.00000	-0.04370	-0.03959	-0.00477	-0.00271
3	-0.37805	-0.06929	1.00001	-0.48480	-0.00000	-0.04352	-0.03944	-0.00475	-0.00270
4	-0.37958	-0.06957	0.00001	0.42503	-0.00000	-0.04370	-0.03959	-0.00477	-0.00271
5	-0.15703	-0.03208	0.00000	-0.15680	1.00000	-0.04327	-0.03921	-0.00472	-0.00268
6	-0.07459	-0.01612	0.00000	-0.07156	-0.00000	0.93444	-0.02485	-0.00364	-0.00322
7	-0.13128	-0.02772	0.00000	-0.12809	-0.00000	-0.04889	0.92449	-0.00540	-0.00175
8	-0.05892	-0.01345	0.00000	-0.05416	-0.00000	-0.03603	-0.02961	0.97709	-0.01415
9	-0.02859	-0.00683	0.00000	-0.02526	-0.00000	-0.02165	-0.02155	-0.01011	0.97561

Matrix $(G_{LL}^{-1} G_{NL})^{-1}$

	1	2	3	4	5	6	7	8	9
1	285.60083	47.76622	-0.00398	332.97632	0.00045	32.89406	29.58434	3.56082	2.06357
2	259.53491	44.57402	-0.00363	303.75293	0.00041	30.00711	26.98785	3.24830	1.88246
3	283.82617	47.65231	0.99602	332.28003	0.00045	32.81561	29.51378	3.55233	2.05865
4	312.54102	52.47333	-0.00438	366.99390	0.00050	36.13559	32.49971	3.91172	2.26693
5	108.49289	18.21860	-0.00152	126.96338	1.00017	12.57155	11.30663	1.36088	0.78866
6	53.97218	9.06421	-0.00076	63.15691	0.00009	7.30146	5.63176	0.67855	0.39445
7	94.78726	15.91809	-0.00133	110.92050	0.00015	10.99754	10.92321	1.19055	0.68856
8	43.29945	7.27257	-0.00061	50.66493	0.00007	5.03918	4.52899	1.56494	0.32857
9	22.01839	3.69850	-0.00031	25.76259	0.00004	2.56676	2.31052	0.28594	1.18471

Eigenvalues of $G_{LL}^{-1} G_{NL}$

1	2	3	4	5	6	7	8	9
0.9999	0.9999	1.0000	0.9888	0.9683	0.9659	0.9115	0.8713	0.1405×10^{-2}

Table IX Matrices $G_{LL}^{-1} G_{NL}$ and $(G_{LL}^{-1} G_{NL})^{-1}$, and Eigenvalues of $G_{LL}^{-1} G_{NL}$ for Case 2

Matrix $G_{LL}^{-1} G_{NL}$

	1	2	3	4	5	6	7	8	9
1	0.52411	-0.07554	0.00000	-0.41435	-0.00000	-0.04430	-0.04073	-0.00484	-0.00257
2	-0.38748	0.92446	0.00000	-0.41435	-0.00000	-0.04430	-0.04073	-0.00484	-0.00257
3	-0.38593	-0.07524	1.00001	-0.49484	-0.00000	-0.04412	-0.04057	-0.00482	-0.00256
4	-0.38748	-0.07554	0.00001	0.41430	-0.00000	-0.04430	-0.04073	-0.00484	-0.00257
5	-0.16087	-0.03551	0.00000	-0.16080	1.00000	-0.04387	-0.04033	-0.00479	-0.00255
6	-0.07656	-0.01800	0.00000	-0.07354	-0.00000	0.93378	-0.02577	-0.00384	-0.00323
7	-0.13464	-0.03085	0.00000	-0.13152	-0.00000	-0.04976	0.92288	-0.00547	-0.00143
8	-0.06061	-0.01515	0.00000	-0.05578	-0.00000	-0.03682	-0.03077	0.97668	-0.01396
9	-0.02946	-0.00775	0.00000	-0.02607	-0.00000	-0.02215	-0.02241	-0.01057	0.97554

Matrix $(G_{LL}^{-1} G_{NL})^{-1}$

	1	2	3	4	5	6	7	8	9
1	-16.86464	-3.20808	0.00025	-21.07719	-0.00003	-2.06441	-1.88424	-0.22377	-0.12130
2	-16.37373	-1.92447	0.00023	-19.21390	-0.00003	-1.88191	-1.71766	-0.20399	-0.11057
3	-17.93135	-3.20266	1.00024	-20.94250	-0.00003	-2.06093	-1.88106	-0.22339	-0.12109
4	-19.75941	-3.52917	0.00027	-21.98007	-0.00003	-2.27104	-2.07283	-0.24617	-0.13344
5	-6.88244	-1.22488	0.00009	-8.09051	0.99999	-0.76294	-0.69635	-0.08270	-0.04483
6	-3.43716	-0.61052	0.00005	-4.04441	-0.00000	0.66682	-0.34035	-0.03958	-0.02020
7	-6.02747	-1.07150	0.00008	-7.08947	-0.00001	-0.65367	0.43576	-0.07083	-0.03994
8	-2.76870	-0.49084	0.00004	-3.26095	-0.00000	-0.28447	-0.26287	0.98895	-0.00437
9	-1.41393	-0.25028	0.00002	-1.66656	-0.00000	-0.14094	-0.12651	-0.00677	1.01554

Eigenvalues of $G_{LL}^{-1} G_{NL}$

1	2	3	4	5	6	7	8	9
0.9999	0.9999	1.0000	0.9888	0.9682	0.9660	0.9100	0.8699	-0.2273×10^{-1}

where $G' = F_1' - A_1' A_2'^{-1} F_2'$ and is called the reduced Jacobian matrix,

$$F' = \begin{bmatrix} F_1' & A_1' \\ F_2' & A_2' \end{bmatrix} = \begin{bmatrix} \frac{\partial f_Q}{\partial V} & \frac{\partial f_Q}{\partial \theta} \\ \frac{\partial f_p}{\partial V} & \frac{\partial f_p}{\partial \theta} \end{bmatrix},$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = -\begin{bmatrix} \frac{\partial f_Q}{\partial E} \\ \frac{\partial f_p}{\partial E} \end{bmatrix}, \qquad D_1 = \frac{\partial f_Q}{\partial b} = \begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix},$$

$$f_{Q,i} = Q_{Li} + Q_{bi} + \sum_{k \in S_i} Q_{ik} = 0,$$

$$f_{p,i} = P_{Li} + \sum_{k \in S_i} P_{ik} = 0, \qquad i = 1, \cdots, N$$

$$f_{p,i+N} = \sum_{k \in S_{N+i}} P_{N+i,k} - k_i (P_{LOSS} + \sum_{j=1}^{N} P_{Li}) + C_i = 0, \qquad i = 1, \cdots, M-1$$

$$Q_{Li} = f_{Bi}(V_i) \cdot V_i^2,$$

$$P_{Li} = f_{Gi}(V_i) \cdot V_i^2.$$

From (I5) and (I6), the stability conditions (I3) and (I4) are equivalent to the conditions that almost all the elements of $G_{NL}'^{-1}$ and $C_1 - A_1' A_2'^{-1} C_2$ are positive. Under light load conditions, $G' \simeq F_1'$ and $C_1 - A_1' A_2'^{-1} \cdot C_2 \simeq C_1$ hold. Because the elements of C_1 are non-negative, the load voltages are stable, if the elements of $G_{NL}'^{-1}$ are all non-negative. This is equivalent to the condition that G_{NL}' is an M-matrix. Under heavy load conditions, there can be some positive off-diagonal elements in G_{NL}' due to the term $A_1' A_2'^{-1} F_2'$. But so long as the absolute values of positive off-diagonal elements are small compared with those of negative off-diagonal elements, elements of G^{-1} can be non-negative from the continuity of matrix inversion. Therefore, in this case also, the load voltages are stable if G_{NL}' is nearly equal to an M-matrix.

APPENDIX II

M-matrix[11,12,13]

Definition 1 For vector x, $x \geq 0$ ($x > 0$) means that all of the elements of x are non-negative (positive). For nxn matrices $A = (a_{ij})$ and $B = (b_{ij})$, $A \geq B$ ($A > B$) means that $a_{ij} \geq b_{ij}$ ($a_{ij} > b_{ij}$) for all i,j $1 \leq i, j \leq n$.

Definition 2 Set Z is a set of n x n matrices with non-positive off-diagonal elements.

Theorem 1 For $A \in Z$, the following five conditions are equivalent:

(1) All the leading principal minors are positive. Namely,

$$\det \begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \end{vmatrix} > 0 \qquad k = 1, \cdots, n \quad \cdots \text{(II1)}$$

(2) There exists $x > 0$, such that $Ax > 0$.

(3) There exists $y > 0$, such that $A^t y > 0$.

(4) Matrix A is a regular matrix and $A^{-1} \geq 0$.

(5) The real parts of eigenvalues of A are all positive.

Definition 3 A matrix A which satisfies either of the above conditions is said to be an M-matrix.

Theorem 2 If arbitrary columns or rows of an M-matrix are multiplied by positive numbers, the obtained matrix is also an M-matrix.

Theorem 3 Let $A, B \in Z$, $A \leq B$ and A be an M-matrix. Then the following relations hold :

(1) Matrices A and B are regular and $A^{-1} \geq B^{-1} \geq 0$

(2) $\det B \geq \det A > 0$.

Definition 4 An n x n matrix A is said to be reducible if there exists an n x n permutation matrix P such that

$$PAP^T = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \quad \cdots\cdots \text{(II2)}$$

where A_{11} : r x r matrix and $1 \leq r \leq n$

A_{22} : (n-r) x (n-r) matrix

A matrix A is irreducible if no such permutation matrix exists.

Definition 5 A directed graph is said to be strongly connected, if there exists a path from an arbitrary node to another arbitrary node.

Theorem 4 The necessary and sufficient condition of a matrix being irreducible is that the corresponding directed graph is strongly connected.

Theorem 5 Let A be an M-matrix. Then $A^{-1} > 0$ if and only if A is irreducible.

Theorem 6 Let A be an irreducible matrix. If for $\varepsilon > 0$, $A + \varepsilon E$ is an M-matrix where E is a unit matrix, then proper principal minors of A are all positive.

Definition 6 An n x n matrix $A = (a_{ij})$ is said to be diagonally dominant if

$$|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \qquad i = 1, \cdots, n \quad \cdots\cdots \text{(II3)}$$

holds for all i. An irreducible matrix A is said to be irreducibly diagonally dominant if for at least one i, a strict inequality holds.

Theorem 7 Let the diagonal elements of $A \in Z$ be positive. If A is irreducibly diagonally dominant, $A^{-1} > 0$ holds. If equalities in (II3) hold for all i, A is singular. And if (II3) do not hold for any i, A is not an M-matrix.

Theorem 8 Let $A \in Z$ be an irreducibly diagonally dominant M-matrix, and $A^{-1} = B = (b_{ij})$. Then the following relations hold:

$$b_{ii} \geq b_{ji} > 0 \qquad i, j = 1, \cdots, n$$

Theorem 9 For n x n matrix $A \geq 0$, the following two conditions are equivalent:

(1) $\alpha > \rho(A)$

(2) E-A is regular and $(\alpha E - A)^{-1} \geq 0$.

where $\rho(A)$ is the maximum absolute value of eigenvalues of A.

Definition 7 Let A, M, and N be n x n matrices and A = M-N. The separation of A into M and N is said to be a regular separation if M is regular and $M^{-1} \geq 0$, $N \geq 0$ hold.

Theorem 10 Let A = M-N be a regular separation of A. Then if $A^{-1} \geq 0$ holds,

$$\rho(M^{-1}N) = \frac{\rho(A^{-1}N)}{1 + \rho(A^{-1}N)} < 1 \quad \cdots\cdots \text{(II4)}$$

Likewise the same relation holds for $\rho(NM^{-1})$.

Theorem 11 Let A = M-N be a regular separation of A and A and M be M-matrices. Then AM^{-1}, and $M^{-1}A$ are also M-matrices.

Proof Because

$$AM^{-1} = (M-N)M^{-1} = E - NM^{-1},$$

$AM^{-1} \in Z$ holds. From Theorem 10, $\rho(NM^{-1}) < 1$, then in theorem 9, for $\alpha = 1$, $E - NM^{-1}$ is regular and $(E - NM^{-1})^{-1} \geq 0$ holds. Thus AM^{-1} is an M-matrix. Likewise $M^{-1}A$ is proved to be an M-matrix.

Collorary In theorem 11, the real parts of eigenvalues of MA^{-1} are all positive.

Proof Let the eigenvalue of $A M^{-1}$ be $\alpha + j\beta$. Then the eigenvalue of $(A M^{-1})^{-1} = M A^{-1}$ is given by

$$\frac{1}{\alpha + j\beta} = \frac{\alpha - j\beta}{\alpha^2 + \beta^2}$$

Since $A M^{-1}$ is an M-matrix, $\alpha > 0$. Thus the real parts of eigenvalue of MA^{-1} are all positive.

Theorem 12 Let A be an n x n positive definite matrix and, B be n x n real symmetric matrix, then all the eigen values of A + jB have positive real parts.

Proof Let

$$(A + jB)x = \lambda x \quad \cdots\cdots (II5)$$

where λ : eigenvalue

and x : corresponding eigen-vector.

Premultiplying x^* to (II5) where * denotes conjugate transpose, gives

$$x^*(A + jB)x = \lambda x^* x$$

Because B is real symmetric,

$$(x^* B x)^* = x^* B x$$

holds. Therefore, the real part of $x^* Ax$ coincides with that of $\lambda x^* x$. Since A is positive definite, the real part of λ is positive.

Theorem 13 Let A, B, and $B - D_1$ be M-matrices, where $A = B - D_1 + D_2$ $(D_1, D_2 \geq 0)$. Then $B^{-1}A$ does not have negative eigenvalues.

Proof Let $B^{-1}A$ have a negative eigenvalue $-\lambda$ with corresponding eigen-vector x :

$$B^{-1} A x = -\lambda x \quad (\lambda > 0)$$

Then

$$Ax = (B - D_1 + D_2)x = -\lambda B x$$

yields

$$[(1+\lambda)B - D_1 + D_2]x = (1+\lambda)\left[B - \frac{1}{1+\lambda}(D_1 - D_2)\right]x = 0 \quad \cdots\cdots (II6)$$

Equation (II6) implies that $B - (D_1 - D_2)/(1+\lambda)$ has a zero eigenvalue. Because $B - D_1$ is an M-matrix and

$$B - \frac{1}{1+\lambda}(D_1 - D_2) \geq B - D_1 + \frac{1}{1+\lambda}D_2 \geq B - D_1,$$

$B - (D_1 - D_2)/(1+\lambda)$ is an M-matrix from theorem 3. Thus the matrix cannot have a zero eigenvalue. Therefore, $B^{-1}A$ does not have a negative eigenvalue.

APPENDIX III

Relation between G and G'

Instead of using 2N equations concerning B_L and G_L in (17) or (33), the following equations are used to evaluate matrix G' :

$$B_{Li} V_i^2 (\text{or } f_{Bi}(V_i)V_i^2) - g_{Bi}(V,\theta)V_i^2 = 0$$
$$G_{Li} V_i^2 (\text{or } f_{Gi}(V_i)V_i^2) - g_{Gi}(V,\theta)V_i^2 = 0 \quad \cdots\cdots (III1)$$
$$i = 1, \cdots, N$$

Then F_1', F_2', A_1', and A_2' defined in Appendix I are expressed by using F_1, F_2, A_1 and A_2 in (30) as follows:

$$F_1' = -\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} F_1, \quad A_1' = -\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} A_1$$

$$F_2' = -\begin{bmatrix} V_1^2 & & & & 0 \\ & \ddots & & & \\ & & V_N^2 & & \\ & & & -1 & \\ 0 & & & & \ddots \\ & & & & & -1 \end{bmatrix} F_2 \quad A_2' = -\begin{bmatrix} V_1^2 & & & & 0 \\ & \ddots & & & \\ & & V_N^2 & & \\ & & & -1 & \\ 0 & & & & \ddots \\ & & & & & -1 \end{bmatrix} A_2 \quad (III2)$$

Therefore,

$$G' = F_1' - A_1' A_2'^{-1} F_2' = -\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} F_1$$

$$-\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} A_1 A_2^{-1} \begin{bmatrix} 1/V_1^2 & & & & 0 \\ & \ddots & & & \\ & & 1/V_N^2 & & \\ 0 & & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{bmatrix} \begin{bmatrix} V_1^2 & & & & 0 \\ & \ddots & & & \\ & & V_N^2 & & \\ 0 & & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{bmatrix} F_2$$

$$= -\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} (F_1 - A_1 A_2^{-1} F_2) = -\begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \end{bmatrix} G \cdots (III3)$$

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Discussion

G. K. Rao (Institute of Technology, Banaras Hindu University, Varanasi, India): The authors deserve congratulations for an excellent paper incorporating the necessary and sufficient conditions for load voltage stability through a sound mathematical basis and at the same-time going through physically justifiable assumptions. The characteristics of the M-matrix are well exploited. The same was used earlier by the authors to examine the convergence of load-flow solution at the neighborhood of voltage stability limit.

Numerical values of G_{LL} and G_{NL} are given in the paper for different sample systems. The usefulness of the paper would be greatly enhanced if step-by-step derivation of these values is given in the paper.

A logical extension of this analysis can be a control problem, that is, what control strategy to adopt to increase the voltage stability margin. Would the authors please comment on the possible effects of static reactive power compensaters on the voltage stability. The discusser thanks the authors for making available a copy of their paper.

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V. A. Venikov, V. A. Stroev (Moscow Power Engineering Institute, Moscos, USSR): The problems of voltage stability in power systems, as determined by the loads behaviour under small disturbances, play important part in operation and design of the systems.

The authors of the paper presented an elegant approach to the analysis of these problems based on dynamic description of the loads and the use of M-matrics for the formulation of sufficient stability conditions. An undoubted merit of their algorithm is its universal character in the sense of the applicability to the systems of any pattern and complexity.

The problems of power systems voltage stability have been investigated for many years in the USSR. Methods and algorithms we use now differ from those outlined in the paper in some general points. We would like to know the comments of the authors on these points which are outlined below.

1. In order to determine the stability liimit an initial stable operating condition is changed in finite steps along a specified trajectory, and for each of the calculated points the stability is checked using small disturbances method. Is the stability of an operating condition depends on the parameters of this condition and does not depend on the trajectory along which this condition is approached, the finite and small changes of the system variables in general case follow different rules. For instance, in equation (12) of the paper there may be $K_i + 0$ for finite changes and $K_i = 0$ for the stability analysis.

2. Neglecting dynamic properties of the system in the voltage stability analysis leads to the use of an algebraic sign of the last term of characteristic equation (An) as the stability criterion [1]. This criterion allows to estimate practically important conditions of aperiodic instability in the form of the collapse of voltage, and it is widely used in practice.

The above stability criterion is mathematically equivalent to the so-called practical criteria in the form of $dE_i/dV_j>0$, $doQ_j/dV_j<0$, $dP_i/d\theta_i>0$ [2,3] provided the system is stable for fixed independent variables in the derivatives (V_j = const, θ_i=const). In this case the stability analysis requires consideration of only one of the derivatives. Such an approach proved to be computationally effective for relatively simple systems, for instance, these with a single load. For complex power systems it is expedient to use the above criterion of a constant sign of An during the transition from a stable condition.

For the power system considered in the paper this criterion assumes the following form:

$$\text{sign}\begin{vmatrix} \left(\frac{\partial g_B}{\partial V} - \frac{\partial f_B}{\partial V}\right) & \frac{\partial g_B}{\partial \theta} \\ \left(\frac{\partial g_G}{\partial V} - \frac{\partial f_G}{\partial V}\right) & \frac{\partial g_G}{\partial \theta} \\ \left(\frac{\partial g_R}{\partial V} + \frac{\partial g_R}{\partial G_L}\frac{\partial f_G}{\partial V}\right) & \frac{\partial g_R}{\partial \theta} \end{vmatrix} = idem,$$

or, adopting the author's notation

$$\text{sign}\left\{ |A_2| \times \left| G_1 - \frac{\partial f_B}{\partial V} + A_1 A_2^{-1} \begin{bmatrix} \frac{\partial f_G}{\partial V} \\ -\frac{\partial g_R}{\partial G_L}\frac{\partial f_G}{\partial V} \end{bmatrix} \right| \right\} = idem.$$

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S. Abe, Y. Fukunaga, A. Isono and **B. Kondo:** We would like to thank Dr. Rao, Dr. Venikov and Dr. Stroev for their valuable comments.

As for Dr. Rao's comments, the voltage characteristics of static reactive power compensators directly influence the voltage stability margin. Intuitively it is clear that, than by conventional linear static condensers, the voltage stability is improved more effectively by nonlinear compensators which feed, for instance, constant reactive power, irrespective of terminal voltage changes. This can be explained by using M-matrix properties. Let the reactive power fed by the reactive power compensator be given by

$$Q_b = -b\,V^m \quad (b>0). \tag{D1}$$

Thus

$$\frac{\delta}{\delta V}(Q_b / V^2) = -b\,(m-2)\,V^{m-3} \quad \begin{cases} >0 \text{ for } m<2 \\ =0 \text{ for } m=2 \\ <0 \text{ for } m>2. \end{cases} \tag{D2}$$

Then from (9), (17) and (32), as the parameter m decreases, the corresponding diagonal element of $-G_{NL}$ increases. Thus from theorem 3 in Appendix II, the voltage stability margin is improved.[5],[6]

Now to effectively improve the system voltage stability, to which node should we install the nonlinear reactive compensator? This problem is also solved by using M-matrix properties. From theorem 7 in Appendix II, if the power system is unstable or is operating in the neighborhood of a voltage stability limit, some of the column (or row) sums of $-G_{NL}$ are negative. This means that if the reactive power compensator is installed at the node whose column (or row) sum is the smallest, system voltage stability is effectively improved.[5],[6]

Table IX shows column sums of $-G_{NL}$ for the 13-node, 4-power -source system used in the text section "Numerical Examples". In the table the column sum of node 2 is the smallest. Node 2 is the intermediate node between source nodes and load nodes. This verifies the validity of the well known intermediate reactive power compensation concept.

Table IX Column Sums of $-G_{NL}$

Node	Case 1	Case2
1	−3.05	−3.07
2	−7.93	−8.17
3	−1.01	−1.04
4	−3.19	−3.21
5	11.86	11.52
6	33.02	32.83
7	−3.06	−3.12
8	−2.12	−2.18
9	61.26	61.04

Dr. Venikov and Dr. Stroev pointed out that the system parameters in the stability analysis and in getting a trajectory should be different. They are quite right. In "Case 2" of "Two-Load, Two-Power Source System" in "Numerical Examples", we only showed the stability analysis for $k_1 = 0.9$. This was simply because in the stable region, changes in k_1 do not affect the voltage stability very much.

Basically our algorithms are equivalent to those proposed by Dr. Venikov.[5]-[7] Among the conditions that $-G_{NL}$ is an M-matrix, the determinat of $-G_{NL}$ is positive is the severest from theorem 6 in Appendix II. Thus to determine the system voltage stability under finite steps of changes in operating conditions, it will be enough to check the sign of determinant of $-G_{NL}$ as Dr. Venikov and Dr. Stroev pointed out. The sign of the determinant only tells us whether the voltages are stable. The stability margin is effectively estimated by[5],[6]

$$\gamma = \det G_{NL} / \det D_{LL}.$$

The easiest way to determine voltage stability is by load flow calculations. When the set of initial values is selected so that the reduced Jacobian matrix is (nearly) equal to an M-matrix (This condition is almost always satisfied by flat voltage selection), the load flow calculation converges to the solution whose voltages are the highest. This solution is actually operable (stable), when generator nodes and load nodes are, respectively, specified P-V and P-Q, since actual load voltage characteristics are considered to be between linear and constant power characteristics.[7]

The stability criteria

$$dE_i/dV_j>0 \qquad d\Delta Q_i/dV_j<0 \tag{D3}$$

are equivalent to the condition that $-G_{NL}$ is an M-matrix. But the condition

$$\det(-G_{NL})>0 \tag{D4}$$

is not a sufficient condition that $-G_{NL}$ is an M-matrix. Thus mathematically (D3) and (D4) are not equivalent. However, as we discussed earlier, practically they can be considered to be equivalent.

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