



MSE performance of the 2SHI estimator in a regression model with multivariate t error terms

Namba, Akio

(Citation)

Statistical Papers, 42(1):81-96

(Issue Date)

2001-01

(Resource Type)

journal article

(Version)

Accepted Manuscript

(URL)

<https://hdl.handle.net/20.500.14094/90000344>



MSE PERFORMANCE OF THE 2SHI ESTIMATOR IN A REGRESSION MODEL WITH MULTIVARIATE t ERROR TERMS

Akio Namba

Graduate School of Economics, Kobe-University
Rokko, Nada-ku, Kobe 657-8501, Japan

The date of receipt and acceptance will be inserted by the editor

In this paper, we consider a linear regression model with multivariate t error terms and derive the explicit formula of the mean squared error (MSE) of the two-stage hierarchical information (2SHI) estimator. It is shown by numerical evaluations that the 2SHI estimator has smaller MSE than the positive-part Stein-rule (PSR) estimator over a wide region of the parameter space.

1 INTRODUCTION

Consider a linear regression model,

$$y = X\beta + u, \quad (1)$$

where y is an $n \times 1$ vector of observations on a dependent variable, X is an $n \times k$ matrix of full column rank of observations on nonstochastic independent variables, β is a $k \times 1$ vector of regression coefficients.

In most of the theoretical and applied researches, it is assumed that error terms, u , has a normal distribution. However, there exists many economics data (specifically, financial data) which may be generated by distributions with fatter tails than normal distributions (see, for example, Fama (1965) and Blattberg and Gonedes (1974)). One example of such distribution is a multivariate t distribution. To examine the effects of departure from normality of error terms on the sampling performance of estimators and test statistics in a linear regression model, multivariate t distributions have often been used. Some examples are Zellner (1976), Prucha and Kelejian (1984), Ullah and Zinde-Walsh (1984), Judge *et al.* (1985), Sutradhar and Ali (1986), Sutradhar (1988), Singh (1988, 1991), Giles (1991, 1992), Ohtani (1991, 1993), Ohtani and Giles (1993) and Ohtani and Hasegawa (1993).

Thus, in this paper, we assume that the error terms, u , has a multivariate t distribution with the probability density function (p.d.f.) given by

$$p(u|\nu, \sigma) = \frac{\nu^{\nu/2} \Gamma((\nu + n)/2)}{\pi^{n/2} \Gamma(\nu/2) \sigma^n} \frac{1}{\{\nu + u'u/\sigma^2\}^{(n+\nu)/2}}, \quad (2)$$

where σ is a scale parameter and ν is a degrees of freedom parameter. For $\nu > 2$, it is easy to show that $E[u] = 0$ and $E[uu'] = [\nu\sigma^2/(\nu - 2)]I_n$. When $\nu \rightarrow \infty$, the p.d.f. of u approaches a multivariate normal distribution with mean 0 and covariance matrix $\sigma^2 I_n$.

As shown in Zellner (1976), the multivariate t distribution can be viewed as a mixture of multivariate normal and inverted gamma distributions:

$$p(u|\nu, \sigma) = \int_0^\infty p_N(u|\tau) p_{IG}(\tau|\nu, \sigma) d\tau, \quad (3)$$

where

$$p_N(u|\tau) = (2\pi\tau^2)^{-n/2} \exp[-u'u/2\tau^2], \quad (4)$$

$$p_{IG}(\tau|\nu, \sigma) = \frac{2(\nu\sigma^2/2)^{\nu/2}}{\Gamma(\nu/2)} \tau^{-(\nu+1)} \exp[-\nu\sigma^2/2\tau^2]. \quad (5)$$

The ordinary least squares (OLS) estimator of β is

$$b = S^{-1}X'y, \quad (6)$$

where $S = X'X$. In the context of linear regression, the Stein-rule (SR) estimator proposed by Stein (1956) is defined as

$$b_{SR} = \left(1 - \frac{ae'e}{b'Sb}\right) b, \quad (7)$$

where $e = y - Xb$, and a is a constant such that $0 \leq a \leq 2(k-2)/(n-k+2)$. Although the Gauss-Markov theorem ensures that the OLS estimator is the best linear unbiased estimator (BLUE) for $\nu > 2$, if we use the loss function

$$L(\bar{\beta}; \beta) = (\bar{\beta} - \beta)' S (\bar{\beta} - \beta), \quad (8)$$

where $\bar{\beta}$ is any estimator of β , and if normal error terms are assumed, the SR estimator dominates the OLS estimator in terms of mean squared error (MSE) for $k \geq 3$. As is shown in James and Stein (1961), the MSE of the SR estimator is minimized when $a = (k-2)/(n-k+2)$. Thus, we use this value of a hereafter.

Although the SR estimator dominates the OLS estimator, Baranchik (1970) showed under normality assumption that the SR estimator is further dominated by the positive-part Stein-rule (PSR) estimator defined as

$$b_{PSR} = \max \left[0, 1 - \frac{ae'e}{b'Sb} \right] b. \quad (9)$$

As one of improved estimators, Theil (1971) proposed the following minimum mean squared error (MMSE) estimator. However, Theil's (1971)

MMSE estimator includes unknown parameters, Farebrother (1975) suggested the following operational variant of the MMSE estimator:

$$b_M = \left(\frac{b'Sb}{b'Sb + e'e/(n-k)} \right) b. \quad (10)$$

Hereafter, we call the operational variant of the MMSE estimator the MMSE estimator simply. The MMSE estimator dominates the OLS estimator if $k \geq 3$ and normal error terms are assumed, since it satisfies Baranchik's (1970) condition.

As an extension of the MMSE estimator, Ohtani (1996) considered the following estimator which is obtained by adjusting the degrees of freedom of $b'Sb$ (i.e., k):

$$b_{AM} = \left(\frac{b'Sb/k}{b'Sb/k + e'e/(n-k)} \right) b. \quad (11)$$

To improve the SR estimator, Tran Van Hoa and Chaturvedi (1990) suggested the following family of two-stage hierarchical information (2SHI) estimators (see Tran van Hoa, 1985, for details):

$$b_H = \left(1 - c\omega \frac{e'e}{b'Sb} - c(1-\omega) \frac{e'e}{b'Sb + c^*e'e} \right) b, \quad (12)$$

where c and c^* are two non-negative characterizing scalars, and ω is a constant such that $0 \leq \omega \leq 1$. If $c \leq 2(k-2)/(n-k+2)$ and normal error terms are assumed, the 2SHI estimator dominates the OLS estimator since it satisfies Baranchik's (1970) condition. When $c^* = 0$ or when $\omega = 1$, the 2SHI estimator reduces to the SR estimator. When $\omega = 0$ and $c = c^* = 1/(n-k)$, the 2SHI estimator reduces to the MMSE estimator. Also, when $\omega = 0$ and $c = c^* = k/(n-k)$, the 2SHI estimator reduces to the AMMSE estimator. There are several studies on the properties of the 2SHI estimator. Some examples are Tran Van Hoa and Chaturvedi (1990, 1993). However, except for Tran Van Hoa's (1992, 1993) and Tran Van Hoa and Chaturvedi's (1997) Monte Carlo experiments, most of the results rely on asymptotic expansions. Furthermore, the studies on the small sample properties of the MMSE, AMMSE and 2SHI estimators have been restricted to the case of normal error terms, so far.

In this paper, we examine the small sample properties of the 2SHI estimator under multivariate t errors. We derive the explicit formula for the MSE of the 2SHI estimator in the next section. In section 3, we compare the MSE performance of the 2SHI estimator with that of the other estimators numerically. It is shown that the 2SHI estimator has smaller MSE than the PSR estimator over a wide region of the parameter space. Finally, some concluding remarks are given in section 4.

2 MSE OF THE 2SHI ESTIMATOR

The MSE of the 2SHI estimator is

$$MSE(b_H) = E[(b_H - \beta)'S(b_H - \beta)]$$

$$= E[b'_H S b_H] - 2E[\beta' S b_H] + \beta' S \beta. \quad (13)$$

If we define the functions, $H(p, q, r, \alpha)$ and $J(p, q, r, \alpha)$, as

$$H(p, q, r, \alpha) = E \left[\frac{(b' S b)^p (e' e)^q}{(b' S b + \alpha e' e)^r} \right], \quad (14)$$

$$J(p, q, r, \alpha) = E \left[\frac{(b' S b)^p (e' e)^q \beta' S b}{(b' S b + \alpha e' e)^r} \right], \quad (15)$$

and let $a_1 = c\omega$, $a_2 = c(1 - \omega)$ and $a_3 = c^*$, then the MSE of the 2SHI estimator is written as

$$\begin{aligned} & MSE(b_H) \\ &= H(1, 0, 0, 0) + a_1^2 H(0, 2, 1, 0) + a_2^2 H(1, 2, 2, a_3) \\ &\quad - 2a_1 H(0, 1, 0, 0) + 2a_1 a_2 H(0, 2, 1, a_3) - 2a_2 H(1, 1, 1, a_3) \\ &\quad - 2J(0, 0, 0, 0) + 2a_1 J(0, 1, 1, 0) + 2a_2 J(0, 1, 1, a_3) + \beta' S \beta. \end{aligned} \quad (16)$$

As shown in appendix, the explicit formulae of $H(p, q, r, \alpha)$ and $J(p, q, r, \alpha)$ are

$$\begin{aligned} H(p, q, r, \alpha) &= (\nu \sigma^2)^{\nu/2} \sum_{i=0}^{\infty} \left(\frac{\theta^i}{i!} \right) (\theta + \nu \sigma^2)^{p+q-r-i-\nu/2} \\ &\quad \times \frac{\Gamma(\nu/2 - p - q + r + i)}{\Gamma((n-k)/2) \Gamma(\nu/2)} G_i(p, q, r, \alpha), \end{aligned} \quad (17)$$

$$\begin{aligned} J(p, q, r, \alpha) &= \theta (\nu \sigma^2)^{\nu/2} \sum_{i=0}^{\infty} \left(\frac{\theta^i}{i!} \right) (\theta + \nu \sigma^2)^{p+q-r-i-\nu/2} \\ &\quad \times \frac{\Gamma(\nu/2 - p - q + r + i)}{\Gamma((n-k)/2) \Gamma(\nu/2)} G_{i+1}(p, q, r, \alpha), \end{aligned} \quad (18)$$

where $\theta = \beta' S \beta$ and

$$\begin{aligned} G_i(p, q, r, \alpha) &= \frac{\Gamma(n/2 + p + q - r + i)}{\Gamma(k/2 + i)} \\ &\quad \times \int_0^1 \frac{t^{k/2+p+i-1} (1-t)^{(n-k)/2+q-1}}{[\alpha + t(1-\alpha)]^r} dt. \end{aligned} \quad (19)$$

Substituting (17) and (18) in (16), we obtain the explicit formula of the MSE of the 2SHI estimator.

Since further theoretical analysis seems to be difficult, we compare the MSE of the 2SHI estimator with those of the other estimators in the next section.

3 NUMERICAL ANALYSIS

To compare the MSE performances of the estimators, we evaluated the values of relative MSE defined as $MSE(\bar{\beta})/MSE(b)$, where $\bar{\beta}$ is any estimator

of β . Thus, the estimator $\bar{\beta}$ has smaller MSE than the OLS estimator when the value of relative MSE is smaller than unity. It is easy to show that the relative MSE depends only on the value of θ/σ^2 .

Although their results rely on the asymptotic expansions, Tran Van Hoa and Chaturvedi (1990) showed that there exists a value of the characterizing scalar $c^*(> 0)$ such that the 2SHI estimator uniformly dominates the SR estimator if and only if $(k-4)/(n-k+4) \leq c \leq 2(k-2)/(n-k+2)$ and $k > 4$. When $k < 4$, however, $(k-4)/(n-k+4)$ is less than 0. Thus, in the numerical evaluations, we used the values of $c = c^* = c_H(k-2)/(n-k+2)$, where $c_H = 0.1, 0.2, 0.3, \dots, 2.0$. Also, we used the parameter values; $\omega = 0.0, 0.1, 0.2, \dots, 1.0$; $\theta/\sigma^2 =$ various values. The numerical evaluations were executed on a personal computer, using the FORTRAN code. In evaluating the integral in $G_i(p, q, r, \alpha)$ given in (19), we used Simpson's 3/8 rule with 500 equal subdivisions. The infinite series in $H(p, q, r, \alpha)$ and $J(p, q, r, \alpha)$ were judged to converge when the increment of the series got smaller than 10^{-12} .

First, we executed the numerical evaluations for $k = 5$ and $n = 10, 20, 30, 40$. Then we found that the 2SHI estimator with $c_H = 2.0$ and $\omega = 0.2$ has smaller MSE than the 2SHI estimators with the other values of c_H and ω in the neighborhood of $\theta/\sigma^2 = 0$. Thus, we use these values in the following numerical evaluations.

The numerical evaluations are executed for $k = 3, 4, 5, 6, 8, n = 10, 20, 30, 40$ and $\nu = 3, 5, 10, 15, 20, 30, \infty$ (normal). Since the results for $n = 20$ $k = 3, 5, 8$ and $\nu = 3, 20$ are qualitatively typical, we do not show the results for the other cases.

The relative MSE's for $k = 3, n = 20$ and $\nu = 3$ are shown in Table 1. The AMMSE estimator has the smallest MSE for all the values of θ/σ^2 , and the 2SHI estimator has the second smallest MSE.

The relative MSE's for $k = 3, n = 20$ and $\nu = 20$ are shown in Table 2. The AMMSE estimator has the smallest MSE for small values of θ/σ^2 (e.g., $\theta/\sigma^2 \leq 14.0$). However, the MMSE estimator has the smallest MSE when θ/σ^2 is large (e.g., $\theta/\sigma^2 \geq 18.0$).

When $k = 5$ and $\nu = 3$, we see from Table 3 that the 2SHI estimator has the smallest MSE over a wide region of θ/σ^2 (e.g., $\theta/\sigma^2 \leq 50.0$). Also, when $k = 5$ and $\nu = 20$, we see from Table 4 that the 2SHI estimator has the smallest MSE for small values of θ/σ^2 . However, the region where the MSE of the 2SHI estimator is smallest is narrower than that of $\nu = 3$ (e.g., $\theta/\sigma^2 \leq 3.0$). The AMMSE estimator has the smallest MSE for moderate values of θ/σ^2 (e.g., $4.0 \leq \theta/\sigma^2 \leq 40.0$) and the PSR estimator has the smallest MSE for large values of θ/σ^2 (e.g., $\theta/\sigma^2 \geq 50.0$).

When $k = 8$ and $\nu = 3$, we see from Table 5 that the MSE of the 2SHI estimator is smallest for all the values of θ/σ^2 . When $k = 8$ and $\nu = 20$, we see from Table 6 that the 2SHI estimator has the smallest MSE for small values of θ/σ^2 . Similar to the case of $k = 5$ and $\nu = 20$, the MSE of the AMMSE estimator is smallest for moderate values of θ/σ^2 , and the MSE of the PSR estimator is smallest for large values of θ/σ^2 . However, the region

where the 2SHI has the smallest MSE for $k = 8$ and $\nu = 20$ is wider than that of $k = 5$ and $\nu = 20$.

4 CONCLUDING REMARKS

In section 3, we showed that comparing the maximum gain in MSE with the maximum loss, the MSE performance of the 2SHI estimator is more desirable than that of the PSR estimator under the multivariate t error terms. Especially, the 2SHI estimator has smaller MSE than the PSR estimator over a wide region of θ/σ^2 when the degrees of freedom of the error terms, ν , is small, and the MSE of the 2SHI estimator is uniformly smaller than that of the PSR estimator when $\nu = 3$.

Since the values of c , c^* and ω we used in the numerical evaluations are limited and somewhat tentative, there may exist more appropriate values. However, to seek such values is beyond the scope of this paper and the remaining problem for future research.

APPENDIX

First, we derive the formula for $E\left[\frac{(b'Sb)^p(e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau\right]$. Let $u_1 = b'Sb/\tau^2$ and $u_2 = e'e/\tau^2$. Then, for given τ , u_1 is distributed as the noncentral chi-square distribution with k degrees of freedom and noncentrality parameter $\lambda = \beta'S\beta/\tau^2$, and u_2 is distributed as the chi-square distribution with $n - k$ degrees of freedom.

Using u_1 and u_2 , $E\left[\frac{(b'Sb)^p(e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau\right]$ is expressed as

$$\begin{aligned} & E\left[\frac{(b'Sb)^p(e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau\right] \\ &= E\left[\frac{(\tau^2 u_1)^p (\tau^2 u_2)^q}{(\tau^2 u_1 + \alpha \tau^2 u_2)^r} \middle| \tau\right] \\ &= (\tau^2)^{p+q-r} \sum_{i=0}^{\infty} K_i \int_0^{\infty} \int_0^{\infty} u_1^{k/2+i-1} u_2^{(n-k)/2-1} \\ & \quad \times \exp[-(u_1 + u_2)/2] \frac{u_1^p u_2^q}{(u_1 + \alpha u_2)^r} du_1 du_2, \quad (20) \end{aligned}$$

where

$$K_i = \frac{w_i(\lambda)}{2^{n/2+i} \Gamma((n-k)/2) \Gamma(k/2+i)}, \quad (21)$$

and $w_i(\lambda) = \exp(-\lambda/2)(\lambda/2)^i / i!$.

Making use of the change of variables, $v_1 = u_1/u_2$ and $v_2 = u_2$, (20) reduces to:

$$(\tau^2)^{p+q-r} \sum_{i=0}^{\infty} K_i \int_0^{\infty} \int_0^{\infty} \frac{v_1^{k/2+p+i-1} v_2^{n/2+p+q-r+i-1}}{(v_1 + \alpha)^r} \times \exp[-(1+v_1)v_2/2] dv_1 dv_2. \quad (22)$$

Again, making use of the change of variable, $z = (1+v_1)v_2/2$, (22) reduces to:

$$(\tau^2)^{p+q-r} \sum_{i=0}^{\infty} K_i 2^{n/2+p+q-r+i} \Gamma(n/2+p+q-r+i) \times \int_0^{\infty} \frac{v_1^{k/2+p+i-1}}{(1+v_1)^{n/2+p+q-r+i} (v_1 + \alpha)^r} dv_1. \quad (23)$$

Further, making use of the change of variable, $t = v_1/(1+v_1)$, and substituting (21) in (23), (23) reduces to:

$$E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] = (2\tau^2)^{p+q-r} \sum_{i=0}^{\infty} w_i(\lambda) G_i^*(p, q, r, \alpha), \quad (24)$$

where

$$G_i^*(p, q, r, \alpha) = \frac{\Gamma(n/2+p+q-r+i)}{\Gamma(k/2+i)\Gamma((n-k)/2)} \times \int_0^1 \frac{t^{k/2+p+i-1} (1-t)^{(n-k)/2+q-1}}{[\alpha + t(1-\alpha)]^r} dt. \quad (25)$$

Next, we derive the formula for $E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]$. Differentiating $E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]$ given in (24) with respect to β , we obtain:

$$\begin{aligned} & \frac{\partial E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]}{\partial \beta} \\ &= (2\tau^2)^{p+q-r} \sum_{i=0}^{\infty} \left[\frac{\partial w_i(\lambda)}{\partial \beta} \right] G_i^*(p, q, r, \alpha) \\ &= (2\tau^2)^{p+q-r} \sum_{i=0}^{\infty} \left[-\frac{S\beta}{\tau^2} w_i(\lambda) + \frac{S\beta}{\tau^2} w_{i-1}(\lambda) \right] G_i^*(p, q, r, \alpha) \\ &= -(S\beta/\tau^2) E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] \\ & \quad + (S\beta/\tau^2) (2\tau^2)^{p+q-r} \sum_{i=0}^{\infty} w_i(\lambda) G_{i+1}^*(p, q, r, \alpha), \end{aligned} \quad (26)$$

where we define $w_{-1}(\lambda) = 0$.

Expressing $E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]$ by $b'Sb$ and $e'e$, we have:

$$\begin{aligned} & E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] \\ &= \int_0^\infty \int_{-\infty}^\infty \frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} p_N(b|\tau) p_e(e'e|\tau) db d(e'e), \end{aligned} \quad (27)$$

where $p_e(e'e|\tau)$ is the conditional density function of $e'e$ given τ , and

$$p_N(b|\tau) = \frac{1}{(2\pi)^{k/2} |\tau^2 S^{-1}|^{1/2}} \exp \left[-\frac{(b - \beta)' S (b - \beta)}{2\tau^2} \right] \quad (28)$$

is the conditional density function of b given τ .

Differentiating $E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]$ given in (27) with respect to β , we obtain:

$$\begin{aligned} & \frac{\partial E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]}{\partial \beta} \\ &= \frac{1}{\tau^2} E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} Sb \middle| \tau \right] - \frac{S\beta}{\tau^2} E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right]. \end{aligned} \quad (29)$$

Equating (26) and (29), and multiplying β' from left, we obtain:

$$\begin{aligned} & E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] \\ &= \beta' S \beta (2\tau^2)^{p+q-r} \sum_{i=0}^\infty w_i(\lambda) G_{i+1}^*(p, q, r, \alpha). \end{aligned} \quad (30)$$

Finally, we derive the formulae for $H(p, q, r, \alpha)$ and $J(p, q, r, \alpha)$. From (3), $H(p, q, r, \alpha)$ and $J(p, q, r, \alpha)$ can be written as

$$H(p, q, r, \alpha) = \int_0^\infty E \left[\frac{(b'Sb)^p (e'e)^q}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] p_{IG}(\tau|\nu, \sigma) d\tau, \quad (31)$$

$$J(p, q, r, \alpha) = \int_0^\infty E \left[\frac{(b'Sb)^p (e'e)^q \beta' Sb}{(b'Sb + \alpha e'e)^r} \middle| \tau \right] p_{IG}(\tau|\nu, \sigma) d\tau. \quad (32)$$

Substituting (24) and (30) in (31) and (32), and making use of the change of variable, $u = (\theta + \nu\sigma^2)/2\tau^2$, we obtain (17) and (18) in the text.

ACKNOWLEDGMENT

The author is grateful to Kazuhiro Ohtani for his very helpful guidance, and to two anonymous referees for very useful comments.

REFERENCES

- Baranchik, A.J. (1970), A family of minimax estimators of the mean of a multivariate normal distribution, *Annals of Mathematical Statistics*, 41, 642-645.
- Blattberg, R.C. and N.J. Gonedes. (1974), A comparison of the stable and Student distributions as statistical models for stock prices, *Journal of Business*, 47, 244-280.
- Fama, E.F. (1965), The behavior of stock market prices, *Journal of Business*, 38, 34-105.
- Farebrother, R.W. (1975), The minimum mean square error linear estimator and ridge regression. *Technometrics*, 17, 127-128.
- Giles, J.A. (1991), Pre-testing for linear restrictions in a regression model with spherically symmetric disturbances, *Journal of Econometrics*, 50, 377-398.
- Giles, J.A. (1992), Estimation of the error variance after a preliminary test of homogeneity in a regression model with spherically symmetric disturbances, *Journal of Econometrics*, 53, 345-361.
- James, W. and C. Stein. (1961), Estimation with quadratic loss, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability 1* (Berkeley, University of California Press), 361-379.
- Judge, G., S. Miyazaki. and T. Yancey. (1985), Minimax estimators for the location vectors of spherically symmetric densities, *Econometric Theory*, 1, 409-417.
- Ohtani, K. (1991), Small sample properties of the interval constrained least squares estimator when error terms have a multivariate t distribution, *Journal of the Japan Statistical Society*, 21, 197-204.
- Ohtani, K. (1993), Testing for equality of error variances between two linear regressions with independent multivariate t errors, *Journal of Quantitative Economics*, 9, 85-97.
- Ohtani, K. (1996), On an adjustment of degrees of freedom in the minimum mean squared error estimator, *Communications in Statistics-Theory and Methods*, 25, 3049-3058.
- Ohtani, K. and H. Hasegawa. (1993), On small sample properties of R^2 in a linear regression model with multivariate t errors and proxy variables, *Econometric Theory*, 9, 504-515.
- Ohtani, K. and J. Giles. (1993), Testing linear restrictions on coefficients in a linear regression model with proxy variables and spherically symmetric disturbances, *Journal of Econometrics*, 57, 393-406.
- Prucha, I.R. and H. Kelejian. (1984), The structure of simultaneous equation estimators: a generalization towards nonnormal distributions, *Econometrica*, 52, 721-736.
- Singh, R.S. (1988), Estimation of error variance in linear regression models with error having multivariate Student- t distribution with unknown degrees of freedom, *Economics Letters*, 27, 47-53.

- Singh, R.S. (1991), James-Stein rule estimators in linear regression models with multivariate- t distributed error, *Australian Journal of Statistics*, 33, 145-158.
- Stein, C. (1956), Inadmissibility of the usual estimator for the mean of a multivariate normal distribution, *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley, University of California Press), 197-206.
- Sutradhar, B.C. (1988), Testing linear hypothesis with t error variable, *Sankhyā-B*, 50, 175-180.
- Sutradhar, B.C. and M.M. Ali. (1986), Estimation of the parameters of a regression model with a multivariate t error variable, *Communications in Statistics-Theory and Methods*, 15, 429-450.
- Tran Van Hoa (1985), The inadmissibility of the Stein estimator in normal multiple regression equations, *Economics Letters*, 19, 39-42.
- Tran Van Hoa (1992), Modelling Output Growth, *Economics Letters*, 38, 279-284.
- Tran Van Hoa (1993), The mixture properties of the 2SHI estimators in linear regression models, *Statistics and Probability Letters*, 16, 111-115.
- Tran Van Hoa and A. Chaturvedi. (1990), Further results on the two-stage hierarchical information (2SHI) estimators in the linear regression models, *Communications in Statistics-Theory and Methods*, 19, 4697-4704.
- Tran Van Hoa and A. Chaturvedi. (1993), Asymptotic approximations to the gain of the 2SHI over Stein estimators in linear regression models when the disturbances are small, *Communications in Statistics-Theory and Methods*, 22, 2777-2782.
- Tran Van Hoa and A. Chaturvedi. (1997), Performance of the 2SHI estimator under the generalised Pitman nearness criterion, *Communications in Statistics-Theory and Methods*, 26, 1227-1238.
- Ullah, A. and V. Zinde-Walsh. (1984), On the robustness of LM, LR, and Wald tests in regression models, *Econometrica*, 52, 1055-1066.
- Zellner, A. (1976), Bayesian and non-Bayesian analysis of the regression model with multivariate Student- t error terms, *Journal of the American Statistical Association*, 71, 400-405.

Table 1: Relative MSE's for $k = 3$, $n = 20$, $\nu = 3$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.70175	.57977	.63498	.36624	.47622
.1	.70500	.58436	.63812	.37117	.48116
.2	.70813	.58878	.64117	.37598	.48593
.3	.71114	.59305	.64413	.38068	.49056
.4	.71404	.59718	.64700	.38526	.49505
.6	.71955	.60503	.65252	.39411	.50362
.8	.72469	.61240	.65774	.40256	.51172
1.0	.72952	.61934	.66269	.41065	.51938
1.5	.74040	.63503	.67407	.42946	.53688
2.0	.74989	.64878	.68422	.44651	.55241
3.0	.76576	.67185	.70167	.47636	.57892
4.0	.77863	.69059	.71622	.50177	.60088
5.0	.78937	.70624	.72864	.52378	.61951
6.0	.79853	.71958	.73941	.54310	.63560
7.0	.80648	.73114	.74888	.56025	.64970
8.0	.81347	.74129	.75730	.57563	.66220
9.0	.81968	.75030	.76487	.58952	.67338
10.0	.82526	.75837	.77171	.60215	.68348
12.0	.83490	.77228	.78366	.62432	.70105
14.0	.84299	.78393	.79380	.64325	.71589
16.0	.84991	.79386	.80255	.65964	.72866
18.0	.85594	.80247	.81021	.67403	.73980
20.0	.86124	.81004	.81699	.68680	.74964
22.0	.86596	.81676	.82306	.69822	.75841
24.0	.87020	.82278	.82853	.70852	.76631
26.0	.87404	.82822	.83350	.71788	.77346
28.0	.87753	.83317	.83804	.72643	.77998
30.0	.88074	.83769	.84221	.73427	.78595
40.0	.89353	.85568	.85895	.76567	.80984
50.0	.90279	.86862	.87115	.78838	.82710
100.0	.92772	.90308	.90419	.84872	.87327
150.0	.93967	.91942	.92010	.87685	.89514
200.0	.94707	.92948	.92996	.89386	.90855
300.0	.95612	.94171	.94201	.91411	.92478

Table 2: Relative MSE's for $k = 3$, $n = 20$, $\nu = 20$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.70175	.57977	.63498	.36624	.47622
.1	.71053	.59215	.64346	.37956	.48955
.2	.71896	.60408	.65169	.39254	.50243
.3	.72706	.61558	.65966	.40519	.51490
.4	.73485	.62668	.66740	.41754	.52696
.6	.74956	.64770	.68218	.44131	.54994
.8	.76318	.66727	.69610	.46394	.57148
1.0	.77581	.68550	.70921	.48548	.59169
1.5	.80358	.72585	.73884	.53500	.63704
2.0	.82678	.75984	.76453	.57896	.67600
3.0	.86278	.81298	.80644	.65306	.73882
4.0	.88881	.85158	.83868	.71240	.78648
5.0	.90804	.88006	.86384	.76035	.82323
6.0	.92256	.90139	.88374	.79944	.85199
7.0	.93372	.91761	.89967	.83155	.87480
8.0	.94245	.93011	.91257	.85811	.89310
9.0	.94940	.93989	.92312	.88024	.90795
10.0	.95501	.94763	.93184	.89877	.92013
12.0	.96341	.95889	.94524	.92763	.93860
14.0	.96931	.96648	.95487	.94852	.95166
16.0	.97364	.97183	.96200	.96391	.96115
18.0	.97693	.97574	.96741	.97543	.96822
20.0	.97949	.97871	.97162	.98414	.97360
22.0	.98155	.98102	.97496	.99082	.97778
24.0	.98323	.98287	.97766	.99598	.98107
26.0	.98463	.98438	.97987	.99999	.98370
28.0	.98582	.98564	.98171	1.00314	.98584
30.0	.98683	.98670	.98326	1.00561	.98759
40.0	.99029	.99026	.98833	1.01205	.99293
50.0	.99230	.99230	.99109	1.01388	.99547
100.0	.99622	.99622	.99595	1.01168	.99888
150.0	.99749	.99749	.99739	1.00892	.99951
200.0	.99812	.99812	.99807	1.00712	.99972
300.0	.99875	.99875	.99873	1.00504	.99988

Table 3: Relative MSE's for $k = 5$, $n = 20$, $\nu = 3$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.47059	.35330	.72542	.33566	.27519
.1	.47407	.35756	.72692	.33872	.27940
.2	.47745	.36172	.72839	.34174	.28352
.3	.48075	.36578	.72982	.34470	.28754
.4	.48395	.36975	.73123	.34761	.29148
.6	.49013	.37743	.73395	.35329	.29909
.8	.49601	.38478	.73655	.35879	.30640
1.0	.50161	.39182	.73906	.36411	.31342
1.5	.51459	.40821	.74491	.37673	.32985
2.0	.52630	.42310	.75026	.38846	.34488
3.0	.54672	.44922	.75975	.40970	.37156
4.0	.56408	.47151	.76796	.42850	.39468
5.0	.57914	.49086	.77518	.44533	.41502
6.0	.59241	.50789	.78160	.46053	.43316
7.0	.60423	.52305	.78739	.47438	.44949
8.0	.61488	.53668	.79264	.48708	.46431
9.0	.62454	.54903	.79744	.49878	.47786
10.0	.63338	.56030	.80185	.50962	.49032
12.0	.64903	.58018	.80973	.52914	.51253
14.0	.66252	.59726	.81658	.54628	.53184
16.0	.67434	.61217	.82263	.56151	.54884
18.0	.68483	.62534	.82802	.57518	.56399
20.0	.69423	.63710	.83288	.58754	.57761
22.0	.70272	.64770	.83729	.59881	.58994
24.0	.71046	.65733	.84132	.60913	.60120
26.0	.71754	.66612	.84502	.61865	.61152
28.0	.72407	.67420	.84845	.62745	.62105
30.0	.73011	.68166	.85163	.63564	.62987
40.0	.75482	.71201	.86471	.66943	.66597
50.0	.77330	.73454	.87457	.69495	.69294
100.0	.82558	.79747	.90284	.76776	.76866
150.0	.85204	.82888	.91731	.80460	.80633
200.0	.86888	.84873	.92658	.82792	.82998
300.0	.88995	.87342	.93824	.85687	.85914

Table 4: Relative MSE's for $k = 5$, $n = 20$, $\nu = 20$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.47059	.35330	.72542	.33566	.27519
.1	.47998	.36480	.72948	.34394	.28656
.2	.48912	.37603	.73344	.35208	.29767
.3	.49799	.38699	.73731	.36007	.30852
.4	.50663	.39769	.74109	.36792	.31912
.6	.52320	.41833	.74840	.38322	.33960
.8	.53889	.43802	.75538	.39799	.35917
1.0	.55377	.45679	.76205	.41225	.37788
1.5	.58772	.50005	.77750	.44584	.42122
2.0	.61762	.53857	.79137	.47673	.46019
3.0	.66761	.60368	.81518	.53146	.52719
4.0	.70744	.65596	.83475	.57826	.58247
5.0	.73970	.69830	.85104	.61856	.62861
6.0	.76620	.73290	.86475	.65347	.66753
7.0	.78824	.76143	.87639	.68389	.70067
8.0	.80680	.78518	.88638	.71054	.72912
9.0	.82259	.80510	.89500	.73401	.75372
10.0	.83615	.82197	.90252	.75478	.77514
12.0	.85817	.84876	.91492	.78970	.81044
14.0	.87520	.86887	.92469	.81770	.83810
16.0	.88870	.88439	.93255	.84047	.86017
18.0	.89965	.89667	.93899	.85923	.87806
20.0	.90868	.90660	.94435	.87485	.89275
22.0	.91625	.91478	.94888	.88799	.90496
24.0	.92268	.92162	.95274	.89914	.91522
26.0	.92820	.92744	.95607	.90868	.92392
28.0	.93300	.93244	.95898	.91691	.93136
30.0	.93720	.93679	.96153	.92405	.93777
40.0	.95223	.95213	.97069	.94875	.95954
50.0	.96147	.96144	.97635	.96286	.97165
100.0	.98044	.98044	.98799	.98704	.99145
150.0	.98689	.98689	.99196	.99304	.99595
200.0	.99015	.99015	.99396	.99549	.99765
300.0	.99341	.99341	.99596	.99749	.99892

Table 5: Relative MSE's for $k = 8$, $n = 20$, $\nu = 3$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.35714	.26059	.80298	.32010	.20035
.1	.35980	.26365	.80370	.32205	.20332
.2	.36240	.26666	.80440	.32398	.20624
.3	.36495	.26962	.80509	.32588	.20912
.4	.36745	.27254	.80576	.32777	.21195
.6	.37232	.27824	.80708	.33146	.21748
.8	.37703	.28377	.80836	.33507	.22285
1.0	.38157	.28914	.80961	.33860	.22806
1.5	.39230	.30191	.81256	.34709	.24048
2.0	.40225	.31383	.81531	.35515	.25211
3.0	.42019	.33547	.82032	.37011	.27337
4.0	.43601	.35468	.82479	.38376	.29243
5.0	.45017	.37189	.82883	.39631	.30969
6.0	.46297	.38746	.83250	.40792	.32546
7.0	.47463	.40165	.83586	.41871	.33997
8.0	.48534	.41466	.83897	.42878	.35339
9.0	.49524	.42666	.84186	.43823	.36587
10.0	.50442	.43778	.84455	.44712	.37753
12.0	.52102	.45781	.84943	.46343	.39873
14.0	.53566	.47543	.85377	.47811	.41761
16.0	.54875	.49111	.85766	.49143	.43458
18.0	.56055	.50520	.86119	.50361	.44996
20.0	.57129	.51798	.86441	.51481	.46401
22.0	.58112	.52964	.86737	.52516	.47692
24.0	.59018	.54036	.87011	.53478	.48885
26.0	.59856	.55025	.87265	.54375	.49992
28.0	.60636	.55943	.87502	.55214	.51023
30.0	.61364	.56798	.87724	.56003	.51987
40.0	.64406	.60349	.88654	.59341	.56027
50.0	.66747	.63060	.89376	.61952	.59141
100.0	.73676	.70981	.91536	.69847	.68326
150.0	.77353	.75127	.92695	.74101	.73135
200.0	.79752	.77813	.93456	.76889	.76234
300.0	.82819	.81227	.94433	.80457	.80134

Table 6: Relative MSE's for $k = 8$, $n = 20$, $\nu = 20$

θ/σ^2	SR	PSR	MMSE	AMMSE	2SHI
.0	.35714	.26059	.80298	.32010	.20035
.1	.36430	.26884	.80491	.32537	.20837
.2	.37132	.27697	.80680	.33058	.21626
.3	.37821	.28496	.80865	.33572	.22402
.4	.38496	.29283	.81048	.34079	.23165
.6	.39807	.30819	.81404	.35076	.24655
.8	.41069	.32308	.81748	.36048	.26097
1.0	.42284	.33750	.82081	.36997	.27495
1.5	.45133	.37162	.82866	.39273	.30806
2.0	.47738	.40316	.83592	.41418	.33876
3.0	.52322	.45937	.84887	.45357	.39388
4.0	.56219	.50764	.86006	.48882	.44191
5.0	.59566	.54926	.86981	.52051	.48407
6.0	.62465	.58530	.87837	.54911	.52135
7.0	.64997	.61665	.88593	.57503	.55449
8.0	.67226	.64405	.89266	.59858	.58412
9.0	.69200	.66813	.89867	.62007	.61074
10.0	.70959	.68938	.90408	.63972	.63477
12.0	.73956	.72502	.91339	.67434	.67634
14.0	.76410	.75358	.92111	.70377	.71095
16.0	.78452	.77686	.92761	.72903	.74012
18.0	.80176	.79614	.93314	.75089	.76497
20.0	.81651	.81235	.93791	.76994	.78632
22.0	.82924	.82615	.94206	.78667	.80483
24.0	.84035	.83803	.94570	.80144	.82098
26.0	.85013	.84837	.94891	.81456	.83516
28.0	.85879	.85745	.95177	.82627	.84770
30.0	.86651	.86548	.95433	.83678	.85884
40.0	.89525	.89495	.96395	.87612	.89953
50.0	.91386	.91376	.97024	.90151	.92474
100.0	.95445	.95445	.98414	.95432	.97295
150.0	.96905	.96905	.98919	.97135	.98617
200.0	.97657	.97657	.99181	.97941	.99162
300.0	.98423	.98423	.99448	.98699	.99597