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Calculable one-loop contributions to *S* and *T* parameters in the gauge-Higgs unification

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We investigate the one-loop contributions to S and T oblique parameters in gauge-Higgs unification. We show that these parameters are finite in five dimensional space-time, but are divergent in more than five dimensions. Remarkably, however, we find that a particular linear combination of S and T parameters, $S - 4\cos\theta_W T$, becomes finite for six dimensional space-time, though each of these parameters are divergent. This is because, in the gauge-Higgs unification scenario, the operators relevant for S and T parameters are not independent, but are included in a unique higher dimensional gauge invariant operator. Thus the predictable linear combination is model independent, irrespective of the detail of the matter content.

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I. INTRODUCTION

Solving the hierarchy problem motivates us to go beyond the standard model (SM). Gauge-Higgs unification is one of the attractive approaches to solving the hierarchy problem without supersymmetry. In this scenario, the Higgs field is identified with the zero mode of the extra component of the gauge field in higher dimensional gauge theories [1,2] and the gauge symmetry breaking occurs dynamically through the vacuum expectation value of Wilson line phase: the Hosotani mechanism [3]. One of the remarkable features is that the quantum correction to the Higgs mass-squared becomes finite thanks to the higher dimensional local gauge invariance, once all Kaluza-Klein (K-K) modes are summed up in the intermediate state of the loop diagram, thus solving the hierarchy problem at quantum level (the problem of "quadratic divergence") [4–7].

Recently, the scenario has been further developed and extended. In the case of gauge-Higgs unification on a simply-connected curved space S^2 as the extra space, the quantum correction to the Higgs mass turns out not only to be finite but also to vanish identically [8]. A similar mechanism is found to work in the "gravity-gauge-Higgs unification" scenario, where the Higgs field is identified with the extra-space component of a higher dimensional graviton field [9]. The argument of the finiteness can be extended to the two-loop level [10], and for Abelian gauge theories the finiteness is proved to hold at any order of the perturbative expansion [11]. Furthermore, attempts to construct a realistic model embodied with the idea of gauge-Higgs unification and investigations into the possible applications of the scenario in various aspects have been carried out [12–29].

In order to understand the gauge-Higgs unification scenario more deeply and to make a realistic model, it is important to ask whether there are other finite ("calculable") physical observables besides Higgs mass, which are genuine predictions of the scenario. Let us note that the reason why the Higgs mass is calculable in higher dimensional gauge theories, which are argued to be nonrenormalizable, is that the gauge-Higgs sector is controlled by the higher dimensional local gauge invariance, and no local gauge invariant operator responsible for the Higgs mass exists. It will be natural to ask whether there exist other calculable observables protected by higher dimensional gauge symmetry.

In this paper we consider the "oblique" parameters S and T [30] in the framework of Gauge-Higgs unification scenario, as one of the good candidates of such calculable physical observables. The parameters are defined as

$$S = -\frac{16\pi}{g^2\tan\theta_W}\Pi'_{3Y}, \tag{1.1}$$

$$T = -\frac{4\pi}{g^2\sin^2\theta_W}\frac{\Delta M^2}{M_W^2}, \tag{1.2}$$

where $\Pi'_{3Y} \equiv \frac{d^2}{dp^2}\Pi_{3Y}(p^2)|_{p^2=0}$, with $\Pi_{3Y}(p^2)$ being the $g_{\mu\nu}$ part of the gauge boson self-energy between W^3_μ and B_μ (W^3_μ, B_μ:$SU(2)_L$, $U(1)_Y$ gauge bosons) and θ_W denotes the Weinberg angle. $\Delta M^2 \equiv \delta M^2_{W^3} - \delta M^2_{W^\pm}$, with $\delta M^2_{W^3}$, $\delta M^2_{W^\pm}$ being quantum corrections to the neutral and charged gauge boson mass-squared.

The reason to take these parameters as the candidates is twofold. First, both S and T parameters are described in four dimensional (4D) space-time by (the coefficients of) "irrelevant" gauge invariant operators with higher ($d = 6$) mass dimension, such as $(\phi^\dagger D_\mu\phi)(\phi^\dagger D^\mu\phi)$ for T and $(\phi^\dagger W^a_{\mu\nu}\frac{\tau^a}{2}\phi)B^{\mu\nu}$ for S (ϕ: Higgs doublet, $W^a_{\mu\nu}$, $B_{\mu\nu}$: field strengths of $SU(2)_L$ and $U(1)_Y$ gauge fields). Since in the gauge-Higgs unification scenario the Higgs ϕ is unified with the gauge field, there is a possibility that these two operators are also unified in a single gauge invariant local

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operator with respect to higher dimensional gauge field, whose mass dimension is 6 from 4D point of view. This means that the structure of the divergence of the S and T parameters is not independent and some particular linear combination of these parameters is expected to be free from UV divergence, thus making it theoretically predictable. Second the S and T parameters have been severely constrained through the precision electroweak measurements. Thus studying these parameters is very useful in searching for a viable model based on the scenario.

The model we take in this paper is minimal SU(3) gauge-Higgs unification model compactified on an orbifold S^1/Z_2 with a triplet fermion as the matter field. We confirm by explicit one-loop calculations that our expectation stated above is the case.

The S and T parameters are calculated in two approaches. One approach is to perform the dimensional regularization for the 4D momentum integral before taking the K-K mode sum, which has the advantage that the 4D gauge invariance is manifest in each K-K mode, though the whole structure of divergence becomes clear only after the mode sum. The other one is to take the mode sum before the momentum integral, which is also useful to extract the whole structure of possible UV divergence and to make the higher dimensional gauge invariance manifest. In order to generalize our argument to more than 5D space-times, we first derive general formulas where the dimensionality D in the dimensional regularization is left arbitrary.

As the result, we show that one-loop contributions to S and T parameters are both finite in 5D space-time, but are divergent for higher space-time dimensions. The remarkable result is that in 6D space-time, although one-loop corrections to S and T parameters themselves are certainly divergent, a particular linear combination of these parameters, $S - 4\cos\theta_W T$, becomes finite (calculable) and predictable. We also show that the ratio of the coefficients in the linear combination just coincides with what we obtain from a single gauge invariant operator with respect to higher dimensional gauge fields $(D_L F_{MN})$ $(D^L F^{MN})$, which means that the predictable observable is fixed in a model independent way, irrespective of the detail of each model's matter content. This is a crucial difference from the situation in the universal extra dimension (UED) scenario [31].

This paper is organized as follows. In the next section, we introduce our model. In Sec. III, the one-loop contribution to the T-parameter in 5D is calculated in two different approaches stated above. A similar calculation on the one-loop contribution to the S-parameter in 5D is given in Sec. IV. In Sec. V, we generalize our results to higher space-time dimensions, and discuss the finiteness of a particular linear combination of S and T parameters for the 6D case. Section VI is devoted to the summary and some concluding remarks. In the appendix, some technical detail of the calculation for the T parameter is given.

II. THE MODEL

In this paper, we adopt a minimal model of SU(3) gauge-Higgs unification with an orbifold S^1/Z_2 as the extra space, in order to avoid unnecessary complications in investigating the divergence structure of the one-loop contributions to the S and T parameters, though the predicted Weinberg angle is unrealistic, $\sin^2\theta_W = \frac{3}{4}$.[1] As the matter field we introduce an SU(3) triplet fermion, which we identify with "top and bottom" quarks and their K-K "excited states," although the top quark mass vanishes and the bottom quark mass $m_b = M_W$ in this toy model. (For instance, the T parameter is sensitive to the mass splitting between m_t and m_b, not their absolute values, anyway.) In this work we neglect the presence of generations.

The SU(3) symmetry is broken into SU(2) × U(1) by the orbifolding S^1/Z_2 and adopting a nontrivial Z_2 parity assignment for the members of an irreducible repr. of SU(3), as stated below. The remaining gauge symmetry SU(2) × U(1) is supposed to be broken by the VEV of the K-K zero-mode of A_5, the extra-space component of the gauge field behaving as the Higgs doublet, through the Hosotani-mechanism [3], though we do not address the question how the VEV is obtained by minimizing the loop-induced effective potential for A_5 [12].

The Lagrangian is simply given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{MN}F^{MN}) + i\bar{\Psi}\not{D}\Psi \tag{2.1}$$

where $\Gamma^M = (\gamma^\mu, i\gamma^5)$,

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig_5[A_M, A_N] \quad (M, N = 0, 1, 2, 3, 5), \tag{2.2}$$

$$\not{D} = \Gamma^M(\partial_M - ig_5 A_M) \quad \left(A_M = A_M^a \frac{\lambda^a}{2} (\lambda^a\text{: Gell-Mann matrices})\right), \tag{2.3}$$

$$\Psi = (\psi_1, \psi_2, \psi_3)^T. \tag{2.4}$$

The periodic boundary conditions are imposed along S^1 for all fields and the nontrivial Z_2 parities are assigned for each field as follows,

[1] There is no tree level contribution to these parameters in our model. As pointed out in [23], if we add an extra U(1) to obtain the desirable Weinberg angle, a tree level contribution to the T-parameter appears, and a certain constraint on the compactification scale must be imposed.

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, \quad A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \tag{2.5}$$

$$\Psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}, \tag{2.6}$$

where $(+,+)$ means that Z_2 parities are even at the fixed points $y=0$ and $y=\pi R$, for instance. y is the fifth coordinate and R is the compactification radius. $\psi_{1L} \equiv \frac{1}{2} \times (1-\gamma_5)\psi_1$, etc. A remarkable feature of this manipulation of "orbifolding" is that in the gauge-Higgs sector, exactly what we need for the formation of the standard model is obtained at low energies; one can see that SU(3) is broken to $SU(2)_L \times U(1)_Y$ and the Higgs doublet $\phi = (\phi^+, \phi^0)^t$ emerges. Namely the K-K zero-mode of the gauge-Higgs sector takes the form,

$$A_\mu^{(0)} = \frac{1}{2}\begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{pmatrix}, \quad A_5^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \phi^- & \phi^{0*} & 0 \end{pmatrix}, \tag{2.7}$$

with W_μ^3, $W_\mu^\pm$, B_μ being the $SU(2)_L$, $U(1)_Y$ gauge fields, respectively, while in the zero-mode of the triplet fermion t_R is lacking,

$$\Psi^{(0)} = \begin{pmatrix} t_L \\ b_L \\ b_R \end{pmatrix}. \tag{2.8}$$

The VEV to break $SU(2)_L \times U(1)_Y$ is written as

$$\langle A_5 \rangle = \frac{v}{2}\lambda_6 \qquad \left(\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}\right). \tag{2.9}$$

Following these boundary conditions, K-K mode expansions for the gauge fields and the fermions are carried out:

$$A_{\mu,5}^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}}\left[A_{\mu,5}^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty} A_{\mu,5}^{(n)}(x) \times \cos(ny/R)\right], \tag{2.10}$$

$$A_{\mu,5}^{(-,-)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty} A_{\mu,5}^{(n)}(x)\sin(ny/R), \tag{2.11}$$

$$\psi_{1L,2L,3R}^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}}\left[\psi_{1L,2L,3R}^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty}\psi_{1L,2L,3R}^{(n)}(x)\cos(ny/R)\right], \tag{2.12}$$

$$\psi_{3L,1R,2R}^{(-,-)}(x,y) = i\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\psi_{3L,1R,2R}^{(n)}(x)\sin(ny/R). \tag{2.13}$$

In this paper we discuss one-loop contributions to the S and T parameters due to fermions, which potentially give sizable effects, though, e.g., the contribution due to gauge boson self interactions to the T parameter is handled by $U(1)_Y$ gauge coupling g' and is expected to be relatively not significant. For such purpose, only the term containing fermions, $\mathcal{L}_{\text{fermion}} = i\bar{\Psi}\not{D}\Psi$, in the Lagrangian (2.1) is enough to consider. Substituting the above K-K expansions for the triplet fermion and the zero-modes for the gauge-Higgs bosons in the term and integrating over the fifth coordinate y, we obtain a 4D effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{fermion}}^{(4D)} = \sum_{n=1}^{\infty}\Bigg\{ & i(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)})\gamma^\mu\partial_\mu\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix} + \frac{g}{2}(\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)})\begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix} \\ & - (\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)})\begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n & -m \\ 0 & -m & m_n\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}\Bigg\} + i\bar{t}_L\gamma^\mu\partial_\mu t_L + \bar{b}(i\gamma^\mu\partial_\mu - m)b \\ & + \frac{g}{\sqrt{2}}(\bar{t}\gamma_\mu L b W^{+\mu} + \bar{b}\gamma_\mu L t W^{-\mu}) + \frac{g}{2}(\bar{t}\gamma_\mu L t - \bar{b}\gamma_\mu L b)W_3^\mu + \frac{\sqrt{3}g}{6}(\bar{t}\gamma_\mu L t + \bar{b}\gamma_\mu L b - 2\bar{b}\gamma_\mu R b)B^\mu,\end{aligned} \tag{2.14}$$

where $L \equiv \frac{1}{2}(1-\gamma_5)$, $m_n = \frac{n}{R}$. $g = \frac{g_5}{\sqrt{2\pi R}}$ is the 4D gauge coupling and $m = \frac{gv}{2}(= M_W)$ is the bottom quark mass m_b. Let us note that nonzero K-K modes have both chiralities, as is seen in (2.6), and their gauge interactions are vectorlike, described by Dirac particles constructed as

$$\psi^{(n)}_{1,2,3} = \psi^{(n)}_{1,2,3R} + \psi^{(n)}_{1,2,3,L}(n > 0). \qquad (2.15)$$

Concerning fermion zero-mode, $b = b_R + b_L$ is a Dirac spinor, while t quark remains a Weyl spinor t_L. We realize that the fermion zero-modes have exactly the same gauge interaction as those in the SM, though $\sin^2\theta_W = \frac{3}{4}$ and

$$m_t = 0, \qquad m_b = m(= M_W). \qquad (2.16)$$

In deriving the 4D effective Lagrangian (2.14), a chiral rotation

$$\psi_{1,2,3} \to e^{-i(\pi/4)\gamma_5}\psi_{1,2,3} \qquad (2.17)$$

has been made in order to get rid of $i\gamma_5$.

We easily see that the mass matrix for the nonzero K-K modes can be diagonalized by use of the mass eigenstates $\tilde{\psi}^{(n)}_2$, $\tilde{\psi}^{(n)}_3$,

$$\begin{pmatrix} \psi^{(n)}_1 \\ \tilde{\psi}^{(n)}_2 \\ \tilde{\psi}^{(n)}_3 \end{pmatrix} = U \begin{pmatrix} \psi^{(n)}_1 \\ \psi^{(n)}_2 \\ \psi^{(n)}_3 \end{pmatrix}, \qquad U = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad (2.18)$$

as

$$U\begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n & -m \\ 0 & -m & m_n \end{pmatrix}U^\dagger = \begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n + m & 0 \\ 0 & 0 & m_n - m \end{pmatrix}. \qquad (2.19)$$

Note that a mixing occurs between the SU(2) doublet component and singlet component, accompanied by the mass splitting $m_n \pm m$. Each of the mass eigenvalues has a periodicity with respect to m: $m_n \pm (m + \frac{1}{R}) = m_{n\pm1} \pm m$, which is a remarkable feature of gauge-Higgs unification, not shared by the UED scenario, where the masses of nonzero K-K modes behave as $\sqrt{m_n^2 + m^2}$.

In terms of the mass eigenstates for nonzero K-K modes, the Lagrangian reads as

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\Bigg\{(\bar{\psi}^{(n)}_1, \bar{\tilde{\psi}}^{(n)}_2, \bar{\tilde{\psi}}^{(n)}_3) \times \begin{pmatrix} i\gamma^\mu\partial_\mu - m_n & 0 & 0 \\ 0 & i\gamma^\mu\partial_\mu - (m_n + m) & 0 \\ 0 & 0 & i\gamma^\mu\partial_\mu - (m_n - m) \end{pmatrix}\begin{pmatrix} \psi^{(n)}_1 \\ \tilde{\psi}^{(n)}_2 \\ \tilde{\psi}^{(n)}_3 \end{pmatrix}$$
$$+ \frac{g}{2}(\bar{\psi}^{(n)}_1, \bar{\tilde{\psi}}^{(n)}_2, \bar{\tilde{\psi}}^{(n)}_3)\begin{pmatrix} W^3_\mu + \frac{\sqrt{3}B_\mu}{3} & W^+_\mu & W^+_\mu \\ W^-_\mu & -\frac{W^3_\mu}{2} - \frac{\sqrt{3}B_\mu}{6} & -\frac{W^3_\mu}{2} + \frac{\sqrt{3}B_\mu}{2} \\ W^-_\mu & -\frac{W^3_\mu}{2} + \frac{\sqrt{3}B_\mu}{2} & -\frac{W^3_\mu}{2} - \frac{\sqrt{3}B_\mu}{6} \end{pmatrix}\gamma^\mu\begin{pmatrix} \psi^{(n)}_1 \\ \tilde{\psi}^{(n)}_2 \\ \tilde{\psi}^{(n)}_3 \end{pmatrix}\Bigg\} + \text{zero-mode part.} \qquad (2.20)$$

The relevant Feynman rules for our calculation can be readily read off from this Lagrangian.

III. CALCULATION OF *T*-PARAMETER IN 5D

In this section, we calculate the one-loop contribution to the T-parameter from the matter fermions in 5D space-time. For that purpose, we calculate the mass-squared difference between neutral and charged W-bosons $\Delta M^2 \equiv \delta M^2_{W^3} - \delta M^2_{W^\pm}$. We first derive general formulas in the space-time $M^D \times S^1/Z_2$ for later use, and finally set $D = 4$. The contributions from nonzero K-K modes are obtained from the diagrams in Figs. 1 and 2.

Let us first calculate the diagrams contributing to the neutral W-boson mass-squared shown in Fig. 1.

We first note that diagrams 1(a)–1(c) actually do not contribute. This is simply because the gauge couplings of nonzero K-K modes of the fermions are vectorlike and therefore these diagrams are just the same as the quantum correction to the photon mass in ordinary QED, which should vanish due to the gauge invariance. We have confirmed this is the case by performing D-dimensional momentum integral by use of dimensional regularization for each K-K mode n. The contribution of the remaining diagram 1(d) is calculated to be

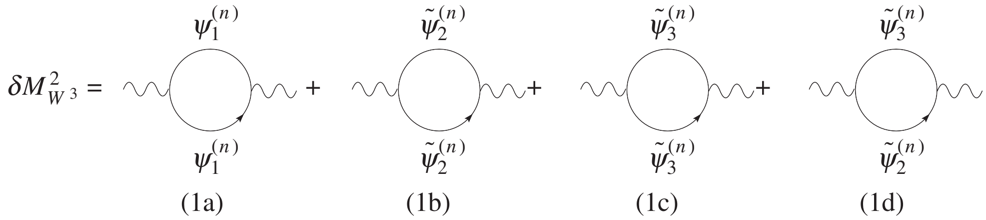

FIG. 1. One-loop diagrams contributing to the neutral W boson mass-squared due to the nonzero K-K modes of fermions. The external lines denote W_μ^3 having no external momenta.

$$\begin{aligned}
(1\mathrm{d}) &= i\frac{g^2N_c}{8}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2-m^2)}{[k^2-(m_n-m)^2][k^2-(m_n+m)^2]}\\
&= i\frac{g^2N_c}{8}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2-m^2)}{[k^2-(m_n^2+m^2)+2m_nm(2t-1)]^2}\\
&= -\frac{g^2N_c}{4(4\pi)^{D/2}}2^{D/2}\Gamma\left(2-\frac{D}{2}\right)\sum_{n=1}^{\infty}\int_0^1 dt\frac{m_nm(2t-1)-m^2}{[m_n^2+m^2-2m_nm(2t-1)]^{2-D/2}},
\end{aligned}\tag{3.1}$$

where $N_c=3$ is the color degree of freedom. In the last line, we adopted the dimensional regularization for the$D$-dimensional momentum integral.

Calculating the diagrams 2(a) and 2(b) contributing to the charged W boson mass-squared in a similar way, we obtain

$$\begin{aligned}
(2\mathrm{a})+(2\mathrm{b}) &= i\frac{g^2N_c}{4}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2+m_nm)}{[k^2-m_n^2][k^2-(m_n+m)^2]}+(m\to -m)\\
&= i\frac{g^2N_c}{4}\sum_{n=1}^{\infty}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2+m_nm)}{[k^2-m_n^2-t(2m_nm+m^2)]^2}\frac{2^{D/2}}{D}+(m\to -m)\\
&= \frac{g^2N_c}{4(4\pi)^{D/2}}2^{D/2}\Gamma\left(2-\frac{D}{2}\right)\sum_{n=1}^{\infty}\int_0^1 dt\frac{(2t-1)m_nm+tm^2}{[m_n^2+t(2m_nm+m^2)]^{2-D/2}}+(m\to -m),
\end{aligned}\tag{3.2}$$

where we note (2b) $= m\to -m$ in (2a). Thus, we get the contribution of the nonzero K-K modes to the T-parameter as [setting $N_c=3$ and using (1.2) with $\sin^2\theta_W=\frac{3}{4}$]

$$\begin{aligned}
T_{(n\neq 0)} &= -i\frac{2\pi}{M_W^2}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2-m^2)}{[k^2-(m_n-m)^2][k^2-(m_n+m)^2]}\\
&\quad+\left\{i\frac{4\pi}{M_W^2}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{(2-D)k^2+D(m_n^2+m_nm)}{[k^2-m_n^2][k^2-(m_n+m)^2]}+(m\to -m)\right\}\\
&= -i\frac{2\pi}{M_W^2}\frac{2^{D/2}}{D}\sum_{n=1}^{\infty}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\left[\frac{(2-D)k^2+D(m_n^2-m^2)}{[k^2-(m_n^2+m^2)+2m_nm(2t-1)]^2}\right.\\
&\quad\left.-2\left\{\frac{(2-D)k^2+D(m_n^2+m_nm)}{[k^2-m_n^2-t(2m_nm+m^2)]^2}+(m\to -m)\right\}\right]\\
&= -\frac{2^{D/2}}{(4\pi)^{D/2-1}M_W^2}\sum_{n=1}^{\infty}\int_0^1 dt\Gamma\left(2-\frac{D}{2}\right)\left[\frac{m^2+m_nm(1-2t)}{[m_n^2+m^2+2m_nm(1-2t)]^{2-D/2}}\right.\\
&\quad\left.-\left\{\frac{-m_nm+t(2m_nm+m^2)}{[m_n^2+t(2m_nm+m^2)]^{2-D/2}}+(m\to -m)\right\}\right].
\end{aligned}\tag{3.3}$$

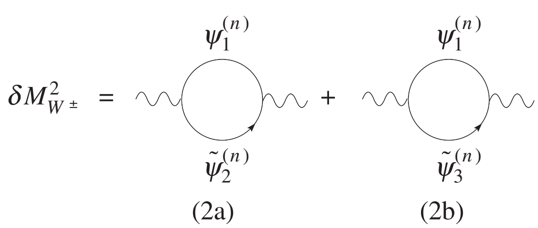

FIG. 2. One-loop diagrams contributing to the charged W boson mass-squared due to the nonzero K-K modes. The external lines denote $W_\mu^\pm$.

As we expect, $T_{(n\neq 0)}$ vanishes in the limit $m \to 0$, which corresponds to the limit of the custodial symmetry in our model.

Recalling $M_W = m$ in our toy model, we have

$$T_{(n\neq 0)} = -\frac{2^{D/2}}{(4\pi)^{D/2-1}m^2}\sum_{n=1}^{\infty}\int_0^1 dt\Gamma\left(2-\frac{D}{2}\right)\left[\frac{m^2+m_n m(1-2t)}{[m_n^2+m^2+2m_n m(1-2t)]^{2-D/2}} - \left\{\frac{-m_n m + t(2m_n m + m^2)}{[m_n^2 + t(2m_n m + m^2)]^{2-D/2}} + (m \to -m)\right\}\right]. \tag{3.4}$$

In (3.3), the$D$-dimensional momentum integral was performed before the K-K mode sum. We can equally perform the K-K mode sum first. In this approach, it is convenient to include the zero-mode ($n = 0$) contribution. The zero-mode contribution is calculated from Figs. 3 and 4:

$$\Delta M^2_{(n=0)} = ig^2N_c\frac{2^{D/2}}{8}\frac{(2-D)}{D}\int\frac{d^Dk}{(2\pi)^D}\frac{m^4}{k^2(k^2-m^2)^2} = \frac{g^2N_c}{(4\pi)^{D/2}}\frac{2^{D/2}}{8}\frac{D-4}{D}\Gamma\left(2-\frac{D}{2}\right)(m^2)^{D/2-1}. \tag{3.5}$$

Let us note that the zero-mode contribution just coincides with the half of what we obtain by setting $n = 0$, instead of the summation $\sum_{n>0}$, in (3.1) minus (3.2). Thus, by using (3.1) minus (3.2) the whole contribution to the T-parameter can be neatly written in terms of $\sum_{n=-\infty}^{\infty}$ as

$$\Delta M^2 = i\frac{g^2N_c}{16}\frac{2^{D/2}}{D}\sum_{n=-\infty}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\left[\frac{(2-D)k^2+D(m_n^2-m^2)}{[k^2-(m_n-m)^2][k^2-(m_n+m)^2]} - 4\frac{(2-D)k^2+D(m_n^2+m_n m)}{[k^2-m_n^2][k^2-(m_n+m)^2]}\right]. \tag{3.6}$$

We rewrite (3.6) as follows,

$$\Delta M^2 = i\frac{g^2N_c}{16}\frac{2^{D/2}}{D}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\sum_{n=-\infty}^{\infty}\left[\frac{2D}{k^2-m_n^2}+\frac{D}{k^2-(m_n+m)^2}+\frac{2(k^2-Dm^2)}{[k^2-(m_n+(2t-1)m)^2+4t(t-1)m^2]^2} - \frac{2(4k^2-Dm^2)}{[k^2-(m_n+tm)^2+t(t-1)m^2]^2}\right]. \tag{3.7}$$

Using the formulas,

$$\sum_{n=-\infty}^{\infty}\frac{1}{x^2+(a+2n\pi)^2} = \frac{\sinh x}{2x(\cosh x - \cos a)}, \tag{3.8}$$

$$\sum_{n=-\infty}^{\infty}\frac{1}{[x^2+(a+2n\pi)^2]^2} = -\frac{1}{2x}\frac{\partial}{\partial x}\sum_{n=-\infty}^{\infty}\frac{1}{x^2+(a+2n\pi)^2} = -\frac{1}{4x}\frac{\partial}{\partial x}\left[\frac{\sinh x}{x(\cosh x-\cos a)}\right], \tag{3.9}$$

we obtain the expression for ΔM^2 after taking the sum over n

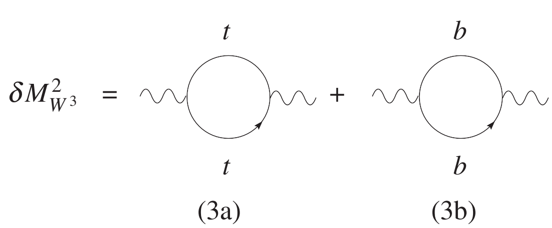

FIG. 3. One-loop diagrams contributing to the neutral W boson mass-squared due to the zero modes of fermions. The external lines denote W_μ^3.

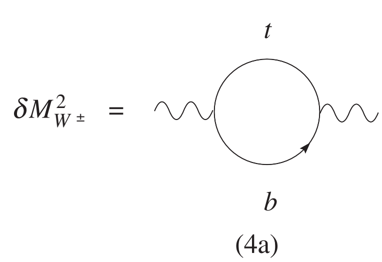

FIG. 4. A one-loop diagram contributing to the charged W boson mass-squared due to the zero modes of fermions. The external lines denote $W_\mu^\pm$.

$$\Delta M^2 = -\frac{g^2 N_c}{16}\frac{2^{D/2}}{D}L^{2-D}\int_0^1 dt\int\frac{d^D\rho}{(2\pi)^D}\bigg[-\frac{D\sinh\rho}{\rho(\cosh\rho-1)}-\frac{D\sinh\rho}{2\rho(\cosh\rho-\cos\alpha)} + 2(\rho^2+D\alpha^2)\Big(\frac{1}{4\rho}\Big)\frac{\partial}{\partial\rho}\bigg\{\frac{1}{\sqrt{\rho^2+4t(1-t)\alpha^2}}+\frac{1}{\sqrt{\rho^2+4t(1-t)\alpha^2}} \times\bigg(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos[(2t-1)\alpha]}-1\bigg)\bigg\} - 2(4\rho^2+D\alpha^2)\Big(\frac{1}{4\rho}\Big)\frac{\partial}{\partial\rho}\bigg\{\frac{1}{\sqrt{\rho^2+t(1-t)\alpha^2}}+\frac{1}{\sqrt{\rho^2+t(1-t)\alpha^2}}\bigg(\frac{\sinh\sqrt{\rho^2+t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+t(1-t)\alpha^2}-\cos(t\alpha)}-1\bigg)\bigg\}\bigg] \tag{3.10}$$

where $L \equiv 2\pi R$, $\rho^\mu \equiv Lk_E^\mu$, with k_E^μ being the Euclidean momentum, and $\alpha \equiv Lm$ is the "Aharanov-Bohm" phase.

Since the quantities in the integrand,

$$\frac{1}{\sqrt{\rho^2+4t(1-t)\alpha^2}}\bigg(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos[(2t-1)\alpha]}-1\bigg) \tag{3.11}$$

etc., do not have UV or IR divergence when it is multiplied by $\rho^{D-1}\frac{(\rho^2+D\alpha^2)}{\rho}$, etc., it is useful to perform the integration by parts to obtain

$$T = T_{(\mathrm{div})} + T_{(\mathrm{sc})}, \tag{3.12}$$

$$T_{(\mathrm{div})} = \frac{\pi}{\alpha^2}\frac{2^{D/2}}{D}L^{4-D}\int_0^1 dt\int\frac{d^D\rho}{(2\pi)^D}\bigg[-\frac{3D}{2\rho}-\frac{\rho^2+D\alpha^2}{2[\rho^2+4t(1-t)\alpha^2]^{3/2}}+\frac{4\rho^2+D\alpha^2}{2[\rho^2+t(1-t)\alpha^2]^{3/2}}\bigg] = -\pi\frac{2^{(3/2)D-3}}{(4\pi)^{D/2}}\frac{(1-2^{3-D})(D-1)}{D(3-D)}\frac{\Gamma(\frac{5-D}{2})\Gamma(\frac{D-1}{2})^2}{\Gamma(\frac{3}{2})\Gamma(D-1)}L^{4-D}\alpha^{D-3}, \tag{3.13}$$

$$T_{(\mathrm{sc})} = \frac{\pi}{\alpha^2}\frac{2^{D/2}}{D}L^{4-D}\int_0^1 dt\int\frac{d^D\rho}{(2\pi)^D}\bigg[-\frac{D}{\rho}\Big(\frac{\sinh\rho}{\cosh\rho-1}-1\Big)-\frac{D}{2\rho}\Big(\frac{\sinh\rho}{\cosh\rho-\cos\alpha}-1\Big) - \frac{D}{2}\frac{(1+(D-2)\frac{\alpha^2}{\rho^2})}{\sqrt{\rho^2+4t(1-t)\alpha^2}}\bigg(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos[(2t-1)\alpha]}-1\bigg) + \frac{D}{2}\frac{(4+(D-2)\frac{\alpha^2}{\rho^2})}{\sqrt{\rho^2+t(1-t)\alpha^2}}\bigg(\frac{\sinh\sqrt{\rho^2+t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+t(1-t)\alpha^2}-\cos(t\alpha)}-1\bigg)\bigg], \tag{3.14}$$

where $T_{(\mathrm{div})}$ denotes a possibly divergent part, a part which seems to be UV divergent relying on a naive power counting, though it is actually finite in 5D space-time ($D = 4$). $T_{(\mathrm{sc})}$ denotes an apparently superconvergent part. (3.14) is the exact formula, valid for arbitrary m (α), and can be evaluated by performing the convergent integrals, if necessary by numerical computation.

Now let us discuss the T-parameter in 5D space-time by taking the limit $D \to 4$. We first utilize the approach to carry out the momentum integral before taking the mode sum. In the limit $D \to 4$, the contribution of $n \neq 0$ modes (3.4) reduces to

$$T_{(n\neq 0)}(5\mathrm{D}) = -\frac{1}{\pi m^2}\sum_{n=1}^{\infty}\int_0^1 dt[(-(1-2t)m_n m + tm^2)\ln[m_n^2 + t(2m_n m + m^2)] + ((1-2t)m_n m + tm^2)\ln[m_n^2 + t(-2m_n m + m^2)] - (m^2 + (1-2t)m_n m)\ln[m_n^2 + m^2 + 2(1-2t)m_n m]], \tag{3.15}$$

where the pole term of $\Gamma(2-\frac{D}{2})$ is known to vanish, as $\int_0^1(1-2t)dt = 0$. Therefore, the T-parameter in 5D turns out to be finite.

The finite part can be explicitly evaluated if we adopt a reasonable approximation, $m \ll \frac{1}{R}$, i.e. $m/m_n \ll 1$. Thus, expanding the integrand in the powers of m/m_n up to $\mathcal{O}((m/m_n)^4)$ and integrating over t, we obtain

$$T_{(n\neq 0)}(5\mathrm{D}) \simeq \frac{2}{5\pi m^2}\sum_{n=1}^{\infty}\frac{m^4}{m_n^2} = \frac{\pi}{15}(mR)^2, \tag{3.16}$$

where $\sum_{n=1}^{\infty} n^{-2} = \zeta(2) = \pi^2/6$ is used. The fact that the leading order term of each K-K mode's contribution is proportional to $\frac{m^2}{m_n^2}$, corresponding to the leading contribution of $\mathcal{O}(\frac{m^4}{m_n^2})$ in ΔM^2, is the consequence of that the dominant contribution of the heavy $n \neq 0$ K-K modes to the T parameter (ΔM^2) is obtained by the insertion of VEV for the Higgs field ϕ in the 4D operator with mass dimension six, responsible for the parameter, $(\phi^\dagger D_\mu \phi)\times(\phi^\dagger D^\mu \phi)$, accompanied by the coefficient suppressed by $1/m_n^2$ (the "decoupling" of $n \neq 0$ K-K modes). The effects of the operators with higher mass dimensions are further suppressed.

This finite value of the T-parameter can be also derived from the second approach where the K-K mode sum is taken before the momentum integration, discussed above, which is useful to see the structure of UV divergence. Namely, for the 5D case ($D = 4$), we find that the possibly divergent part (3.13) becomes

$$T_{(\mathrm{div})}(5\mathrm{D}) = \frac{1}{4\pi m^2}\frac{3\pi^2}{8}(mR)m^2 \tag{3.17}$$

and is actually finite. It is found to be proportional to mR. In order to obtain the result consistent with (3.16), this term should be canceled by a term in the superconvergent part. We can see that this is indeed the case. After some lengthy calculations,[2] we get the superconvergent part for the 5D case ($D = 4$):

$$T_{(\mathrm{sc})}(5\mathrm{D}) \simeq \frac{1}{4\pi m^2}\left[m^2 - \frac{3\pi^2}{8}(mR)m^2 + \frac{4\pi^2}{15}(mR)^2 m^2\right]. \tag{3.18}$$

Combining (3.17) and (3.18), we obtain

$$T(5\mathrm{D}) \simeq \frac{1}{4\pi}\left(1 + \frac{4\pi^2}{15}(mR)^2\right). \tag{3.19}$$

One can see that the mR term (3.17) from the possibly divergent part is exactly canceled by the mR term from the superconvergent part (3.18). The constant term in the bracket in (3.19) is known to coincide with the zero-mode contribution (3.5) with $D = 4$. The remaining $(mR)^2$ term agrees with the finite result of nonzero K-K mode contribution (3.16), which was calculated by performing the momentum integral before taking the mode sum.

[2]The details of calculation is explained in the appendix.

IV. CALCULATION OF S-PARAMETER IN 5D

In this section, we calculate one-loop contribution to the S-parameter, which is calculated from the coefficient Π'_{3Y} of $p^2 g_{\mu\nu}$ term in the self-energy between two neutral gauge bosons W^3_μ and B_μ, $\Pi_{3Y}(p^2)_{\mu\nu} = \Pi'_{3Y}p^2 g_{\mu\nu} + \cdots$. The diagrams we have to calculate are listed in Fig. 5 and 6 where the former shows nonzero K-K mode contribution and the latter does zero-mode contribution, respectively. Let us first calculate one-loop contribution to the S-parameter due to the $n \neq 0$ K-K modes, by performing momentum integration first by use of dimensional regularization. The Taylor expansion of the first diagram (S1) in terms of external momentum p_μ yields a contribution at the order $\mathcal{O}(p^2)$,

$$\Pi^{(S1)}_{3Y}(p^2)_{\mu\nu} \simeq i\frac{\sqrt{3}g^2N_c}{72}2^{D/2}\int\frac{d^Dk}{(2\pi)^D}\sum_{n=1}^{\infty}\left[\left(\frac{3}{(k^2-m_n^2)^2} - \frac{4k^2}{D(k^2-m_n^2)^3}\right)p^2 g_{\mu\nu} - \frac{2p_\mu p_\nu}{(k^2-m_n^2)^2}\right] \tag{4.1}$$

$$= -\frac{\sqrt{3}g^2N_c}{36}\frac{2^{D/2}}{(4\pi)^{D/2}}\Gamma\left(2-\frac{D}{2}\right)\times\sum_{n=1}^{\infty}(m_n^2)^{(D/2)-2}(p^2 g_{\mu\nu} - p_\mu p_\nu) \tag{4.2}$$

$$= i\frac{\sqrt{3}g^2N_c}{36}2^{D/2}\int\frac{d^Dk}{(2\pi)^D}\times\sum_{n=1}^{\infty}\frac{1}{(k^2-m_n^2)^2}(p^2 g_{\mu\nu} - p_\mu p_\nu) \tag{4.3}$$

where $\simeq$ means that the only $\mathcal{O}(p^2)$ terms relevant for the S-parameter are picked up. Superficially, (4.1) and (4.3) look different, but they can be identified through dimensional regularization (4.2). The obtained result satisfies a conserved vector current (CVC) relation $p^\mu \Pi^{(S1)}_{3Y\mu\nu} = 0$, which is again the reflection of the fact that the gauge couplings of $n \neq 0$ fermions are vectorlike, just as in the ordinary QED. The CVC relation also holds for the diagrams (S2) and (S3), as we will see below.

The remaining diagrams due to $n \neq 0$ modes can be calculated in a similar manner:

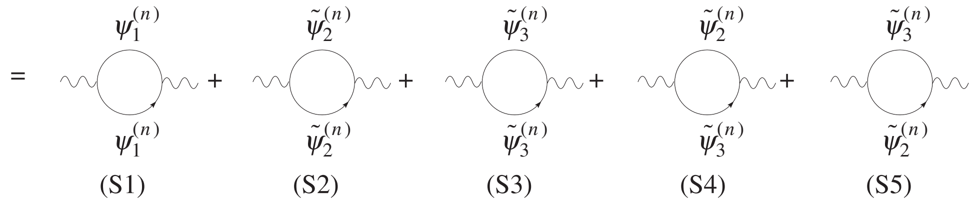

FIG. 5. One-loop diagrams contributing to the self-energy between W_μ^3 and B_μ from the nonzero K-K modes of fermions.

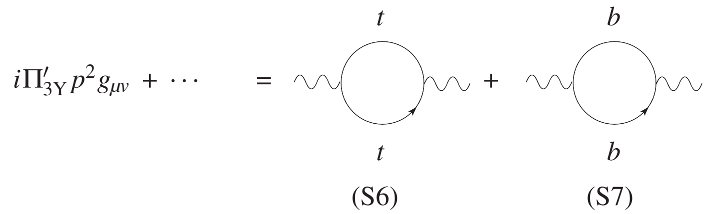

FIG. 6. One-loop diagrams contributing to the self-energy between W_μ^3 and B_μ from the zero mode of fermions.

$$\begin{aligned}\Pi_{3Y}^{(S2)+(S3)}(p^2)_{\mu\nu} &\simeq -\frac{\sqrt{3}g^2N_c}{144}\frac{2^{D/2}}{(4\pi)^{D/2}}\Gamma\left(2-\frac{D}{2}\right)\sum_{n=1}^{\infty}[(m_n+m)^2]^{(D/2)-2}(p^2g_{\mu\nu}-p_\mu p_\nu)+(m\to -m)\\ &= i\frac{\sqrt{3}g^2N_c}{144}2^{D/2}\int\frac{d^Dk}{(2\pi)^D}\sum_{n=1}^{\infty}\frac{1}{[k^2-(m_n+m)^2]^2}(p^2g_{\mu\nu}-p_\mu p_\nu)+(m\to -m),\end{aligned} \tag{4.4}$$

$$\begin{aligned}\Pi_{3Y}^{(S4)+(S5)}(p^2)_{\mu\nu} \simeq{}& \frac{\sqrt{3}g^2N_c}{8}\frac{2^{D/2}}{(4\pi)^{D/2}}\int_0^1 dtt(1-t)\sum_{n=1}^{\infty}\left[\Gamma\left(2-\frac{D}{2}\right)\frac{1}{[m_n^2+m^2+2(2t-1)m_nm]^{2-(D/2)}}(p^2g_{\mu\nu}-p_\mu p_\nu)\right.\\ &\left.-\Gamma\left(3-\frac{D}{2}\right)\frac{(2t-1)[m_n+(2t-1)m]m+4t(1-t)m^2}{[m_n^2+m^2+2(2t-1)m_nm]^{3-(D/2)}}p^2g_{\mu\nu}\right]+(m\to -m)\\ ={}& -i\frac{\sqrt{3}g^2N_c}{8}2^{D/2}\int_0^1 dtt(1-t)\int\frac{d^Dk}{(2\pi)^D}\\ &\times\sum_{n=1}^{\infty}\left[\frac{1}{[k^2-(m_n+(2t-1)m)^2-4t(1-t)m^2]^2}(p^2g_{\mu\nu}-p_\mu p_\nu)\right.\\ &\left.+2\frac{(2t-1)[m_n+(2t-1)m]m+4t(1-t)m^2}{[k^2-(m_n+(2t-1)m)^2-4t(1-t)m^2]^3}p^2g_{\mu\nu}\right]+(m\to -m)\end{aligned} \tag{4.5}$$

where we made use of the property that the diagrams (S3) and (S5) are the same diagrams as (S2) and (S4), respectively, if we replace m by $-m$.

Thus, nonzero K-K mode contributions to the S-parameter (1.1), after the momentum integral, are summarized as follows (with $\tan\theta_W = \sqrt{3}$ and $N_c = 3$),

$$\begin{aligned}S_{(n\neq 0)} ={}& \frac{\pi}{3}2^{D/2}\frac{\Gamma(2-\frac{D}{2})}{(4\pi)^{D/2}}\sum_{n=1}^{\infty}\left[\frac{4}{(m_n^2)^{2-D/2}}+\frac{1}{[(m_n+m)^2]^{2-D/2}}+\frac{1}{[(m_n-m)^2]^{2-D/2}}\right.\\ &\left.-18\int_0^1 dtt(1-t)\left\{\frac{1}{[m_n^2+m^2+2(2t-1)m_nm]^{2-D/2}}+\frac{1}{[m_n^2+m^2-2(2t-1)m_nm]^{2-D/2}}\right\}\right]\\ &+6\pi 2^{D/2}\frac{\Gamma(3-\frac{D}{2})}{(4\pi)^{D/2}}\sum_{n=1}^{\infty}\int_0^1 dt\left[\frac{t(1-t)[(2t-1)(m_n+(2t-1)m)m+4t(1-t)m^2]}{[m_n^2+m^2+2(2t-1)m_nm]^{3-D/2}}\right.\\ &\left.+\frac{t(1-t)[(2t-1)(-m_n+(2t-1)m)m+4t(1-t)m^2]}{[m_n^2+m^2-2(2t-1)m_nm]^{3-D/2}}\right].\end{aligned} \tag{4.6}$$

We can easily check that UV divergence (for the case of $D = 4$) is cancelled out for a fixed K-K mode as

$$\left[2 + 1 - 18\int_0^1 dtt(1-t)\right] \times (\text{log divergence}) = 0. \tag{4.7}$$

Next we take another approach, i.e. we perform the K-K mode sum before the momentum integral. First let us consider zero-mode contributions. They are given by

$$\Pi_{3Y}^{(S6)}(p^2)_{\mu\nu} = i\frac{\sqrt{3}g^2N_c}{24}2^{D/2}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\frac{(\frac{2-D}{D}k^2 + t(1-t)p^2)g_{\mu\nu} - 2t(1-t)p_\mu p_\nu}{[k^2 + t(1-t)p^2]^2}, \tag{4.8}$$

$$\Pi_{3Y}^{(S7)}(p^2)_{\mu\nu} = -i\frac{\sqrt{3}g^2N_c}{24}2^{D/2}\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\frac{(\frac{2-D}{D}k^2 + t(1-t)p^2 - 2m^2)g_{\mu\nu} - 2t(1-t)p_\mu p_\nu}{[k^2 + t(1-t)p^2 - m^2]^2}. \tag{4.9}$$

Noticing the fact

$$\Pi_{3Y}^{(S6)}(p^2)_{\mu\nu} = \frac{1}{2}\Pi_{3Y}^{(S1)}(p^2)_{\mu\nu} \quad \text{with} \quad m_n = 0, \tag{4.10}$$

$$\Pi_{3Y}^{(S7)}(p^2)_{\mu\nu} = \Pi_{3Y}^{(S2)}(p^2)_{\mu\nu} + \Pi_{3Y}^{(S4)}(p^2)_{\mu\nu} \quad \text{with} \quad m_n = 0, \tag{4.11}$$

the sum of all K-K mode contributions can be written as

$$\Pi'_{3Y}(0) = i\frac{\sqrt{3}g^2N_c}{144}2^{D/2}\int\frac{d^Dk}{(2\pi)^D}\sum_{n=-\infty}^{\infty}\left[\frac{2}{(k^2 - m_n^2)^2} + \frac{1}{[k^2 - (m_n + m)^2]^2} - 18\int_0^1 dtt(1-t)\right.$$
$$\left.\times\left\{\frac{1}{[k^2 - (m_n + (2t-1)m)^2 - 4t(1-t)m^2]^2} + 2\frac{(2t-1)[m_n + (2t-1)m]m + 4t(1-t)m^2}{[k^2 - (m_n + (2t-1)m)^2 - 4t(1-t)m^2]^3}\right\}\right]. \tag{4.12}$$

The first term in the right-hand side of (4.12) contains IR divergence for $n = 0$ (and for $D = 4$), which reflects the IR divergence we have when we take the limit $m_t \to 0$ in the $\ln(m_t/m_b)$ term of the ordinary (t, b) doublet contribution to the S-parameter. The IR divergence will be cured below.

In addition to (3.8) and (3.9), using the formulas

$$\sum_{n=-\infty}^{\infty}\frac{1}{[x^2 + (a + 2n\pi)^2]^3} = \frac{1}{8}\frac{1}{x}\frac{\partial}{\partial x}\frac{1}{x}\frac{\partial}{\partial x}\sum_{n=-\infty}^{\infty}\frac{1}{x^2 + (a + 2n\pi)^2} = \frac{1}{16}\frac{1}{x}\frac{\partial}{\partial x}\frac{1}{x}\frac{\partial}{\partial x}\left[\frac{\sinh x}{x(\cosh x - \cos a)}\right], \tag{4.13}$$

etc., we can make the K-K mode sum explicitly to obtain

$$S = S_{(\text{div})} + S_{(\text{sc})} \tag{4.14}$$

$$S_{(\text{div})} = \frac{\pi}{3}2^{D/2}L^{4-D}\int\frac{d^D\rho}{(2\pi)^D}\left\{\frac{3}{4}\frac{1}{\rho^3} - 18\int_0^1 dtt(1-t)\left(\frac{1}{4(\rho^2 + 4t(1-t)\alpha^2)^{3/2}} - \frac{3t(1-t)\alpha^2}{2}\frac{1}{(\rho^2 + 4t(1-t)\alpha^2)^{5/2}}\right)\right\}$$
$$= \frac{9\pi 2^{3D/2-5}}{(4\pi)^{D/2}\Gamma(5/2)}\frac{D-1}{D-3}\frac{\Gamma(\frac{5-D}{2})\Gamma(\frac{D+1}{2})^2}{\Gamma(D+1)}(2\pi R)m^{D-3}. \tag{4.15}$$

$$S_{(\text{sc})} = \frac{\pi}{3}2^{D/2}(D-2)L^{4-D}\int\frac{d^D\rho}{(2\pi)^D}\left[\frac{1}{2\rho^3}\left(\frac{\sinh\rho}{\cosh\rho - \cos\epsilon} - 1\right) + \frac{1}{4\rho^3}\left(\frac{\sinh\rho}{\cosh\rho - \cos\alpha} - 1\right)\right.$$
$$+ \frac{9}{8}\int_0^1 dt\left\{(2t(1-t) - 1)\frac{1}{\rho^2} + 2t(1-t)\alpha^2\frac{D-4}{\rho^4}\right\}\frac{1}{\sqrt{\rho^2 + 4t(1-t)\alpha^2}}$$
$$\left.\times\left(\frac{\sinh\sqrt{\rho^2 + 4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2 + 4t(1-t)\alpha^2} - \cos[(2t-1)\alpha]} - 1\right)\right], \tag{4.16}$$

where ϵ was introduced in the "superconvergent" part $S_{(sc)}$ to avoid the IR divergence due to $m_t = 0$ in our model. (The zero-mode contribution due to (t, b) doublet is well-known and is not of our main interest in this work, anyway).

Now, we discuss the one-loop contribution to the S-parameter in 5D space-time. Here we adopt the result of the approach to perform the momentum integral first. We have already seen that the coefficient of pole term in (4.6) disappears. Therefore, one-loop contribution to the S-parameter is also finite in 5D case. Then, the remaining finite part in (4.6) can be obtained by expanding the logarithmic terms up to $\mathcal{O}((m/m_n)^2)$, as was done in the calculation of the T-parameter, and also evaluating the finite term proportional to $\Gamma(3 - \frac{D}{2})$:

$$S(5D) \simeq \frac{\pi}{3}\frac{1}{(2\pi)^2}\sum_{n=1}^{\infty}\left(\frac{28}{5}+\frac{18}{5}\right)\left(\frac{m}{m_n}\right)^2 = \frac{23\pi}{180}(mR)^2 \tag{4.17}$$

where 28/5 part comes from the logarithmic terms, and 18/5 part is due to the $\Gamma(3 - \frac{D}{2})$ term. $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$ is used in the last equality. The behavior of $(m/m_n)^2$ of each $n \neq 0$ K-K mode's contribution is consistent with what we expect from the 4D gauge invariant operator with mass dimension 6 responsible for the S-parameter, $(\phi^\dagger W^a_{\mu\nu}\frac{\tau^a}{2}\phi)B^{\mu\nu}$, whose coefficient is suppressed by m_n^{-2} as the result of the decoupling of massive $n \neq 0$ K-K modes.

V. THE S AND T PARAMETERS IN HIGHER THAN 5 DIMENSIONAL SPACE-TIME

In the previous sections, we have shown that one-loop contributions to the S and T parameters are finite in the gauge-Higgs unification scenario in 5D space-time. Here, we would like to clarify whether these parameters are finite or not in the cases higher than 5 dimensions.

Before discussing this issue, let us recall why Higgs mass in the gauge-Higgs unification is finite. In the gauge-Higgs unification, the Higgs field is identified with the zero-mode of the extra component of the gauge field in higher dimensional gauge theories. This implies that the local mass term for Higgs $\frac{1}{2}m^2A_5^2$ (for 5D case) is strictly forbidden by the higher dimensional local gauge invariance. Although the Higgs mass is induced by the effect of Wilson-loop (A-B) phase, it is a nonlocal (global) operator. Therefore, the Higgs mass in the gauge-Higgs unification is free from UV divergence.

Then, a question we should ask is whether there are local gauge invariant operators with respect to the higher dimensional gauge field A_M, which are responsible for the S and T parameters. Let us recall that in 4D space-time these parameters are given by the coefficients of dimension six operators such as $(\phi^\dagger W_{\mu\nu}\phi)B^{\mu\nu}$ for S-parameter and $(\phi^\dagger D_\mu\phi)(\phi^\dagger D^\mu\phi)$ for T-parameter. Thus the operator should contain these dimension six operators when reduced to the 4D theory. At the first glance, such operators do not seem to exist, since the operators obtained by replacing the Higgs doublet ϕ by A_i (i: the index to denote extra space component) contradict with the shift symmetry under the higher dimensional gauge transformation $A_i \to A_i + \text{const}$ (for Abelian theories), just as in the case of Higgs mass-squared. Therefore, we may tend to conclude that S and T parameters in gauge-Higgs unification become finite. However, this argument is too naive and not correct: we find an operator to describe these parameters.

To see this, we first note that the contribution of heavy K-K states should be dominated by the gauge invariant operators with the lowest mass dimension. The contributions of the operators with higher mass dimension will be suppressed further by the inverse powers of the compactification scale $M_c \equiv 1/R$; the decoupling of the heavy K-K modes. (As for the "nondecoupling" contributions of the zero modes (t, b), such operators will equally contribute.) Thus we focus on the gauge invariant operators with respect to A_M with mass dimension 6 (when A_M is replaced by 4D field with mass dimension one).

Interestingly enough, such operator is unique:

$$\mathrm{Tr}[(D_L F_{MN})(D^L F^{MN})]. \tag{5.1}$$

Let us note that by use of the Bianchi identity, other possible operators all reduce to this one. In fact,

$$\begin{aligned}&\mathrm{Tr}[(D_L F_{MN})(aD^L F^{MN} + bD^M F^{NL} + cD^N F^{LM})]\\ &\quad = \left(a - \frac{b+c}{2}\right)\mathrm{Tr}[(D_L F_{MN})(D^L F^{MN})],\end{aligned} \tag{5.2}$$

for arbitrary constants a, b and c.

As far as there exist operators to describe the parameters, there is no reason for the S and T parameters to be UV finite in higher dimensional space-time. On the other hand, the fact that the S and T parameters are both described by a coefficient of a single operator means that the UV divergences appearing in the parameters are no longer independent of each other, but should be mutually related. In other words, if we take a specific linear combination of the S and T parameters, the 1-loop contribution to the combination should be finite, although these parameters themselves are divergent. It is important to note that the operator is uniquely determined just by the higher dimensional gauge symmetry. Thus the ratio of the coefficients in the linear combination should be independent of the detail of the matter content of the theory.

To find out the specific linear combination, let us explicitly write down the relevant operator in terms of 4D gauge fields and the VEV of Higgs doublet,

$$\mathrm{Tr}[(D_L F_{MN})(D^L F^{MN})] \supset 4(g\langle A_5\rangle)^4[(W_\mu^3)^2 + \tfrac{1}{4}\{(W_\mu^1)^2 + (W_\mu^2)^2\}] + \sqrt{3}(g\langle A_5\rangle)^2(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)(\partial^\mu B^\nu - \partial^\nu B^\mu) + 2\sqrt{3}(g\langle A_5\rangle)^2(\partial_\mu W_\nu^3)(\partial^\mu B^\nu) \tag{5.3}$$

$$= \tfrac{1}{2}(8m^4)(W_\mu^3)^2 + (2m^4)W_\mu^+ W^{-\mu} + 2\sqrt{3}m^2(p^2 g_{\mu\nu} - p_\mu p_\nu)W^{3\mu}B^\nu + 2\sqrt{3}m^2 p^2 g_{\mu\nu} W^{3\mu}B^\nu \tag{5.4}$$

where the partial integration is carried out in the last equality and the transformation into the momentum space and $m = g\langle A_5\rangle$ are understood. We can readily read off the contributions of an operator $C\,\mathrm{Tr}[(D_L F_{MN})(D^L F^{MN})]$ (C: constant) to ΔM^2 and Π'_{3Y} as

$$C\,\mathrm{Tr}[(D_L F_{MN})(D^L F^{MN})] \rightarrow \begin{cases} \Delta M^2 = 6Cm^4 \\ \Pi'_{3Y} = 4\sqrt{3}Cm^2, \end{cases} \tag{5.5}$$

Thus, we can expect that the linear combination $\Pi'_{3Y} - \frac{2}{\sqrt{3}m^2}\Delta M^2$ is free from UV divergence, since it does not get a contribution from the local operator. Equivalently, identifying $\sqrt{3}/2$ and m^2 with $\sin\theta_W$ and M_W^2 respectively and using (1.1) and (1.2), we expect that $S - 4\cos\theta_W T$ ($S - 2T$ in our model) is finite even in more than five dimensions.

Let us confirm that this expectation really holds for 6D space-time. For such purpose, we focus on the (possibly) divergent parts of the S and T parameters. For the T-parameter, it is given by (3.13),

$$T_{(\mathrm{div})} = -\pi \frac{2^{(3/2)D-3}}{(4\pi)^{D/2}} \frac{(1 - 2^{3-D})(D-1)}{D(3-D)} \frac{\Gamma(\frac{5-D}{2})\Gamma(\frac{D-1}{2})^2}{\Gamma(\frac{3}{2})\Gamma(D-1)} \times (2\pi R)m^{D-3}. \tag{5.6}$$

As for the S-parameter, it is given by (4.15),

$$S_{(\mathrm{div})} = -\frac{9\pi 2^{3D/2-5}}{(4\pi)^{D/2}\Gamma(5/2)} \frac{D-1}{3-D} \frac{\Gamma(\frac{5-D}{2})\Gamma(\frac{D+1}{2})^2}{\Gamma(D+1)} \times (2\pi R)m^{D-3}. \tag{5.7}$$

From these expressions, we can find that the ratio indicated by the operator analysis (5.5) indeed appears in 6D space-time, as we expected:

$$S_{(\mathrm{div})} = \frac{3(5-1)}{8(1-2^{3-5})} T_{(\mathrm{div})} = 2T_{(\mathrm{div})}. \tag{5.8}$$

Thus we have confirmed $S - 2T$ is finite as we expected.

We can also show that $S - 2T$ is finite in 6D case by using the results due to the momentum integration (by use of dimensional regularization) before the mode summation. Going back to the result (3.3)), for the case of $D = 5$ (6D), and Taylor expanding the integrand in the powers of m/m_n up to $\mathcal{O}((m/m_n)^6)$, $T(6D)$ can be calculated as

$$T_{(n\neq 0)}(6D) = -\frac{\sqrt{2}}{5\pi} \sum_{n=1}^{\infty} \left[-\frac{m^2}{m_n} + \frac{1}{12}\frac{m^4}{m_n^3} \right]. \tag{5.9}$$

The first term is actually "logarithmically" divergent, once the K-K mode sum is taken. Similarly, $S(6D)$ is calculated from (4.6) up to $\mathcal{O}((m/m_n)^4)$,

$$S_{(n\neq 0)}(6D) = -\frac{2\sqrt{2}}{5\pi} \sum_{n=1}^{\infty} \left[-\frac{m^2}{m_n} + \frac{3}{14}\frac{m^4}{m_n^3} \right]. \tag{5.10}$$

The first term is also logarithmically divergent. By taking the specific linear combination of these results (5.9) and (5.10), we obtain a finite result (at the leading order),

$$S_{(n\neq 0)}(6D) - 2T_{(n\neq 0)}(6D) = \frac{11\sqrt{2}}{210\pi} m^4 R^3 \zeta(3) \tag{5.11}$$

where $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 = 1.202\,056\,930\,3\cdots$.

Some comments are in order. In our model on $M^D \times (S^1/Z_2)$, only one extra spatial dimension is regarded to be compactified. One might think that our argument of finiteness for higher than five dimensional cases ($D > 4$) is meaningless, since the noncompact space-time is five dimensions not four dimensions for 6D case, for example. However, our argument with respect to the UV divergence will not be affected, irrespective of the compactness of extra dimensions. This is because the information of compactification is a global aspect, namely, the IR nature of the theory, so the structure of UV divergence has nothing to do with that. Therefore, the finiteness of the quantity $S - 4\cos\theta_W T$ holds true even in the 6D theory compactified on T^2/Z_2, for example, although the remaining finite value itself might be changed.

Another issue to be addressed is that the finiteness of $S - 4\cos\theta_W T$ does not seem to hold for higher than six dimensional cases ($D > 5$), as suggested from (5.6) and (5.7). Let us note that, for more than six dimensional cases, each of S and T parameters gets divergent contributions also from the gauge invariant operators, whose mass dimensions are higher than six (from 4D point of view). Thus the divergent contributions come from the multiple operators and it is no longer possible to find out a finite observable in a model independent way.

One-loop contributions to the S and T parameters also have been calculated in the UED scenario [31], where these parameters become finite in five dimensions, but divergent in more than five dimensions. Thus, the gauge-Higgs unification and the UED scenarios share the same divergence structure at this point. However, as was shown above, the divergences of the S and T parameters are not independent and a particular linear combination of these parameters is predictable in the gauge-Higgs unification, in a model independent way. On the other hand, in the UED scenario, even if some combination of the S and T parameters in 6D case is related, the combination will dependent on the detail (the choice of matter fields, etc.) of each model. This is essentially because in the UED scenario the operators responsible for the parameters are mutually independent as in the SM. This is the crucial difference between the gauge-Higgs unification scenario and the UED scenario.

VI. SUMMARY AND CONCLUDING REMARKS

In this paper, we have discussed the one-loop contributions to the S and T parameters in the gauge-Higgs unification scenario. Taking a minimal SU(3) gauge-Higgs unification model with a triplet fermion as the matter fields, we have calculated the S and T parameters in two different approaches. One is an approach to perform the momentum integral by use of dimensional regularization before taking the K-K mode sum. The other approach is to take the mode sum first before the momentum integration. The former has a natural approach from the point of view to make the 4D local gauge symmetry and the custodial symmetry manifest. On the other hand, the latter approach also has an advantage to make the higher dimensional gauge invariance and the structure of UV divergences manifest.

In five dimensional space-time, we have shown that the one-loop contributions to the S and T parameters are both finite, and evaluated their finite values explicitly, adopting two different approaches stated above. In more than five dimensions, we find that the S and T parameters themselves are divergent as in the universal extra dimension (UED) scenario. However, we have derived a genuine prediction of the gauge-Higgs unification scenario, i.e. that a particular linear combination of the S and T parameters, $S - 4\cos\theta_W T$, is calculable (UV finite) for the case of six dimensional space-time. The relative ratio of the coefficients appearing in the linear combination turns out to coincide with what is derived from an analysis of single higher dimensional gauge invariant operator, and therefore is determined in a model independent way. This is the crucial difference from the situation in the UED scenario.

The investigation done in this paper proves the predictability of the gauge-Higgs unification scenario concerning the S and T parameters, even though higher dimensional gauge theories are understood to be nonrenormalizable. Thus, in order to verify the feasibility of the scenario and/or to search for the genuine predictions of the scenario, it is very interesting to study these parameters in more realistic gauge-Higgs unification models, having reasonable Weinberg angle and quark masses, and to extract the phenomenological consequences utilizing existing very precise data on the oblique parameters.

It will be natural to expect that calculable observables controlled by the higher dimensional gauge invariance, other than the oblique parameters, still remain to be found in the scenario. We will continue to search for such observables.

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APPENDIX: DERIVATION OF THE SUPER-CONVERGENT PART OF T-PARAMETER (3.18)

In this appendix, we show the detailed calculations to arrive at the result (3.18). The starting point is

$$T_{(\mathrm{sc})} = \frac{\pi}{\alpha^2}\frac{2^{D/2}}{D}L^{4-D}\int_0^1 dt\int\frac{d^D\rho}{(2\pi)^D}\bigg[-\frac{D}{\rho}\Big(\frac{\sinh\rho}{\cosh\rho-1}-1\Big)-\frac{D}{2\rho}\Big(\frac{\sinh\rho}{\cosh\rho-\cos\alpha}-1\Big)$$
$$-\frac{D}{2}\frac{(1+(D-2)\frac{\alpha^2}{\rho^2})}{\sqrt{\rho^2+4t(1-t)\alpha^2}}\Big(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos[(2t-1)\alpha]}-1\Big)$$
$$+\frac{D}{2}\frac{(4+(D-2)\frac{\alpha^2}{\rho^2})}{\sqrt{\rho^2+t(1-t)\alpha^2}}\Big(\frac{\sinh\sqrt{\rho^2+t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+t(1-t)\alpha^2}-\cos[t\alpha]}-1\Big)\bigg]. \tag{A1}$$

Using a formula

$$\frac{1-x^2}{1-2x\cos\theta+x^2} = 1+\sum_{n=1}^{\infty}2(\cos n\theta)x^n, \tag{A2}$$

we obtain

$$\int\frac{d^D\rho}{(2\pi)^D}\frac{1}{\rho}\Big(\frac{\sinh\rho}{\cosh\rho-\cos\alpha}-1\Big)$$
$$=\frac{4\pi^{D/2}}{(2\pi)^D\Gamma(D/2)}\sum_{n=1}^{\infty}\frac{\cos(n\alpha)}{n^{D-1}}\int_0^\infty d\rho\rho^{D-2}e^{-\rho}$$
$$=\frac{4}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma(D/2)}\sum_{n=1}^{\infty}\frac{\cos(n\alpha)}{n^{D-1}}. \tag{A3}$$

Next we consider the following type of integral,

$$F(a, b, \theta) \equiv i \int \frac{d^D \rho}{(2\pi L)^D} \rho^{-a} \frac{1}{\sqrt{\rho^2 + b^2}} \times \left(\frac{\sinh\sqrt{\rho^2 + b^2}}{\cosh\sqrt{\rho^2 + b^2} - \cos\theta} - 1 \right), \quad \text{(A4)}$$

$$= \frac{4i\pi^{D/2}}{(2\pi L)^D \Gamma(D/2)} \sum_{n=1}^{\infty} \cos(n\theta) \times \int_0^\infty d\rho \rho^{D-1-a} \frac{1}{\sqrt{\rho^2 + b^2}} e^{-n\sqrt{\rho^2+b^2}} \quad \text{(A5)}$$

where (A2) is used in the second line. Rescaling $n\rho \to \rho$ and the change of the integration variable $\rho \to x = \sqrt{\rho^2 + (nb)^2}$ lead to

$$F(a, b, \theta) = \frac{4iL^{-D}}{(4\pi)^D \Gamma(D/2)} \sum_{n=1}^{\infty} \frac{\cos(n\theta)}{n^{D-a-1}} \times \int_{nb}^\infty dx (x^2 - (nb)^2)^{(D-2-a)/2} e^{-x}. \quad \text{(A6)}$$

For $D = 4$ and $a = 0$ or 2, the above integral can be performed to get

$$F(0, b, \theta) = \frac{8iL^{-4}}{(4\pi)^2} \sum_{n=1}^{\infty} \frac{\cos(n\theta)}{n^3} (nb + 1) e^{-nb}, \quad \text{(A7)}$$

$$= \frac{8iL^{-4}}{(4\pi)^2} \left[\zeta(3) + \frac{\theta^2 + b^2}{4} \ln(\theta^2 + b^2) - \frac{3}{4}\theta^2 - \frac{1}{4}b^2 - \frac{1}{6}b^3 - \frac{1}{288}\theta^4 - \frac{1}{48}\theta^2 b^2 + \frac{1}{96}b^4 + \cdots \right], \quad \text{(A8)}$$

$$F(2, b, \theta) = \frac{4iL^{-4}}{(4\pi)^2} \sum_{n=1}^{\infty} \frac{\cos(n\theta)}{n} e^{-nb}, \quad \text{(A9)}$$

$$= \frac{4iL^{-4}}{(4\pi)^2} \left[-\frac{1}{2} \ln(b^2 + \theta^2) + \frac{1}{2} b - \frac{1}{24} b^2 + \frac{1}{24} \theta^2 + \cdots \right], \quad \text{(A10)}$$

where the following expansion formula for small θ and b are used in the second line.

$$\sum_{n=1}^{\infty} \frac{\cos(n\theta)}{n^2} e^{-nb} = \frac{\pi^2}{6} + \frac{b}{2} \ln(\theta^2 + b^2) + \theta \tan^{-1}\left(\frac{b}{\theta}\right) - \frac{\pi}{2}\theta - b + \frac{\theta^2}{4} - \frac{b^2}{4} - \frac{\theta^2 b}{24} + \frac{b^3}{72} + \cdots, \quad \text{(A11)}$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\theta)}{n^3} e^{-nb} = \zeta(3) - \frac{\pi^2}{6} b + \frac{\theta^2 - b^2}{4} \ln(\theta^2 + b^2) - \theta b \tan^{-1}\left(\frac{b}{\theta}\right) - \frac{3}{4}(\theta^2 - b^2) + \frac{\pi}{2}\theta b - \frac{1}{4}\theta^2 b + \frac{b^3}{12} - \frac{\theta^4}{288} + \frac{\theta^2 b^2}{48} - \frac{b^4}{288} + \cdots. \quad \text{(A12)}$$

Thus, $T_{(sc)}$ for 5D case is given by

$$T_{(sc)}(5D) = \frac{1}{4\pi\alpha^2} \int_0^1 dt \Big[-8\zeta(3) - 4\left(\zeta(3) + \frac{\alpha^2}{4} \ln\alpha^2 - \frac{3}{4}\alpha^2 - \frac{\alpha^4}{288} \right) - 2\tilde{F}(0, \sqrt{4t(1-t)}\alpha, (2t-1)\alpha) - 4\alpha^2 \tilde{F}(2, \sqrt{4t(1-t)}\alpha, (2t-1)\alpha) + 8\tilde{F}(0, \sqrt{t(1-t)}\alpha, t\alpha) + 4\alpha^2 \tilde{F}(2, \sqrt{t(1-t)}\alpha, t\alpha) \Big] \quad \text{(A13)}$$

where

$$F(a, b, \theta) \equiv \frac{4iL^{-4}}{(4\pi)^2} \tilde{F}(a, b, \theta). \quad \text{(A14)}$$

From (A8) and (A10), we find

$$T_{(sc)}(5D) \simeq \frac{1}{4\pi\alpha^2} \int_0^1 dt \Big[(-12t + 6 + 4t \ln t - 2 \ln t)\alpha^2 + \left(\frac{8}{3}(t(1-t))^{3/2} - 2\sqrt{t(1-t)} \right) \alpha^3 + \frac{1}{36} \times (-48t^4 + 104t^3 - 102t^2 + 50t - 5)\alpha^4 \Big], \quad \text{(A15)}$$

$$= \frac{1}{4\pi m^2} \left(m^2 - \frac{3\pi^2}{8}(mR)m^2 + \frac{4\pi^2}{15}(mR)^2 m^2 \right). \quad \text{(A16)}$$

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