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Maru, Nobuhito
Sakai, Norisuke
Uekusa, Nobuhiro

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Radius stabilization by constant boundary superpotentials in warped spaceNobuhito Maru,^{1,*} Norisuke Sakai,^{2,†} and Nobuhiro Uekusa^{3,‡}¹*Department of Physics, Kobe University, Kobe 657-8501, Japan*²*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*³*High Energy Physics Division, Department of Physical Sciences, University of Helsinki, and Helsinki Institute of Physics, P.O. Box 64, FIN-00014 Helsinki, Finland*

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A warped space model with a constant boundary superpotential has been an efficient model both to break supersymmetry and to stabilize the radius, when a hypermultiplet, a compensator, and a radion multiplet are taken into account. In such a model of radius stabilization, the radion and moduli masses, the gravitino mass, and the induced soft masses are studied. We find that a lighter physical mode composed of the radion and the moduli can have mass of the order of a TeV and that the gravitino mass can be of the order of 10^7 GeV. It is also shown that soft mass induced by the anomaly mediation can be of the order of 100 GeV and can be dominant compared to that mediated by bulk fields. Localized F -terms and D -terms are discussed as candidates for canceling the cosmological constant. We find that there is no flavor changing neutral current problem in a wide range of parameters.

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I. INTRODUCTION

Supersymmetry is a well-motivated extension to the standard model, which plays a crucial role in solving the gauge hierarchy problem [1]. Extra dimensions with flat space [2] or with warped space [3] are also an alternative solution to the gauge hierarchy problem. Considering both ingredients is natural in the context of string theory and is often taken as the starting point in the phenomenological model of the brane world scenarios.

There is another motivation to consider the brane world scenario in the context of supersymmetry-breaking mediation in supergravity (SUGRA) [4]. In 4D SUGRA, once supersymmetry is broken in the hidden sector, its breaking effects can be mediated to the visible sector through the Planck suppressed interactions and soft masses can be of the order of the gravitino mass. Although experiments severely constrain the soft supersymmetry-breaking masses to be almost flavor diagonal, there is no symmetry reason for such a flavor structure in 4D SUGRA. Therefore 4D SUGRA models generically suffer from the flavor-changing–neutral-current (FCNC) problem. If the two sectors are separated from each other along the direction of extra dimensions [4,5], interactions between the visible and the hidden sectors are naturally suppressed. In this setup, soft supersymmetry-breaking terms in the visible sector are generated through the superconformal anomaly (anomaly mediation) and the resultant mass spectrum is found to be flavor blind, namely, there is no FCNC problem. If anomaly mediation dominates, tree level gravity contributions should be suppressed (for a review, see [6]). Four-dimensional realizations of this idea have also been

discussed [7–9]. More recently, attempts to probe anomaly mediation through TeV scale measurements have been discussed [10,11].

In the brane world scenario, there is an important issue of radius stabilization. In order for the scenario to be phenomenologically viable, the compactification radius should be stabilized. In the previous paper [12], we gave a model of radius stabilization. Here we investigate supersymmetry-breaking effects caused by constant (field independent) superpotentials localized at fixed points in the supersymmetric Randall-Sundrum model. By taking into account the hypermultiplet, the compensating multiplet, and the radion multiplet, we have shown that the radius is stabilized by the presence of constant boundary superpotentials.¹ However, a number of important phenomenological issues remain to be examined, such as FCNC and masses of superparticles which necessarily appear in the model.

In this paper, we calculate the radion and moduli masses, the gravitino mass, and the induced soft masses in the model given by Ref. [12]. We find that a lighter physical mode composed of the radion and the moduli can have masses of the order of a TeV and that the gravitino mass can be of the order of 10^7 GeV. It is also shown that induced mass mediated by anomaly can be of the order of 100 GeV and can be dominant compared to that mediated by bulk fields. We discuss canceling the cosmological constant with F -terms and D -terms localized at $y = 0$ so as not to

*Email address: maru@people.kobe-u.ac.jp

†Email address: nsakai@th.phys.titech.ac.jp

‡Email address: nobuhiro.uekusa@helsinki.fi

¹Supersymmetry breaking by a (bulk) constant superpotential related to Scherk-Schwarz supersymmetry breaking has been discussed in the literature [13,14] in which it has been shown that the radius is not stabilized in the Randall-Sundrum model if only the gravity multiplet is used. Such a destabilization was shown also in [15]. For other discussions on stabilization, see [16–21], for example.

affect radius stabilization. We find that there is no FCNC problem in a wide range of parameters.

The paper is organized as follows. The model is introduced in Sec. II. The radion and moduli masses are calculated in Sec. III. Soft masses induced by anomaly mediation are obtained in Sec. IV. In Sec. V we give Kaluza-Klein (KK) masses of the hyperscalar and gravitino and then estimate soft masses induced by mediation of all Kaluza-Klein modes. Section VI is devoted to discussion about localized F -terms. Soft masses induced by localized F -terms are calculated. In Sec. VII, cancellation of the cosmological constant by D -terms is analyzed. Conclusions are given in Sec. VIII. The details of calculations of mass spectra of the hyperscalar and gravitino are shown in the appendixes.

II. MODEL

We consider a five-dimensional supersymmetric model of a single hypermultiplet on the Randall-Sundrum background, whose metric is

$$ds^2 = e^{-2R\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad \sigma(y) \equiv k|y|, \quad (2.1)$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$, R is the radius of S^1 of the orbifold S^1/Z_2 , k is the AdS_5 curvature scale, and the angle of S^1 is denoted by y ($0 \leq y \leq \pi$). In terms of superfields for four manifest supersymmetry, our Lagrangian reads [12,22]

$$\begin{aligned} \mathcal{L}_5 = & \int d^4\theta \frac{1}{2} \varphi^\dagger \varphi (T + T^\dagger) \\ & \times e^{-(T+T^\dagger)\sigma} (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger} - 6M_5^3) \\ & + \int d^2\theta \left[\varphi^3 e^{-3T\sigma} \left\{ \Phi^c \left[\partial_y - \left(\frac{3}{2} - c \right) T \sigma' \right] \Phi + W_b \right\} \right. \\ & \left. + \text{H.c.} \right], \end{aligned} \quad (2.2)$$

where the compensator chiral supermultiplet φ (of supergravity) and the radion chiral supermultiplet T are denoted as²

$$\varphi = 1 + \theta^2 F_\varphi, \quad T = R + \theta^2 F_T, \quad (2.3)$$

respectively, and the chiral supermultiplets representing the hypermultiplet are denoted as Φ , Φ^c . The Z_2 parity is assigned to be even (odd) for Φ (Φ^c). The derivative with respect to y is denoted by $'$, such as $\sigma' \equiv d\sigma/dy$. The five-

²In the present paper, we are interested in the (real part of the) radion and the hyperscalars. If we also consider the imaginary part of the radion supermultiplet naively in the present model, we find it to be massless since the potential depends only on the real part of the radion. Therefore, we may require some other mechanisms which give the imaginary part a mass or suppress its coupling to the standard model fields in order to make our model fully compatible with experiments.

dimensional Planck mass is denoted as M_5 . Here we consider a model with a constant (field independent) superpotential localized at the fixed point $y = 0$,

$$W_b \equiv 2M_5^3 w_0 \delta(y), \quad (2.4)$$

where w_0 is a dimensionless constant. As far as the analysis in Sec. III is concerned, our Lagrangian turns out to be equivalent, with the replacement $w_0 \rightarrow (3/2)w_0$ in (2.4), to the Lagrangian³ proposed in Ref. [18] based on a more accurate treatment of the radion superfield using 5D SUGRA.

As shown in Ref. [12], the background solutions for the scalar components at the leading order of w_0 are given by

$$\phi(y) = N_2 \exp \left[\left(\frac{3}{2} - c \right) R \sigma \right], \quad (2.5)$$

$$\begin{aligned} \phi^c(y) = & \hat{\epsilon}(y) \left(\frac{\phi^\dagger \phi}{6M_5^3} - 1 \right)^{-1} \left(\frac{\phi^\dagger \phi}{6M_5^3} \right)^{(5/2-c)/(3-2c)} \\ & \times \left[c_1 + c_2 \left(\frac{\phi^\dagger \phi}{6M_5^3} \right)^{-[(1-2c)/(3-2c)]} \left(\frac{\phi^\dagger \phi}{6M_5^3} + \frac{2}{1-2c} \right) \right] \end{aligned} \quad (2.6)$$

where $c \neq 1/2, 3/2$, and

$$\hat{\epsilon}(y) \equiv \begin{cases} +1 & 0 < y < \pi, \\ -1 & -\pi < y < 0. \end{cases} \quad (2.7)$$

The solution contains three complex integration constants: c_1 , c_2 which are the coefficients of two independent solutions for ϕ^c , and the overall complex constant N_2 for the flat direction ϕ . Two out of these three complex integration constants are determined by the boundary conditions. The single remaining constant (which we choose as N_2) is determined through the minimization of the potential (stabilization). With these backgrounds, the potential is obtained as

$$\begin{aligned} V = & \frac{3M_5^3 k w_0^2}{2} \left\{ \frac{-2(1-2c)}{(1-2c)(e^{2Rk\pi} - 1)\hat{N} + 2(e^{(2c-1)Rk\pi} - 1)} \right. \\ & \times \hat{N}^{4-2c-[1/(3-2c)]} \\ & \left. + \frac{\hat{N}}{1-\hat{N}} \left(-4c^2 + 12c - 6 + \frac{3-2c}{3(1-\hat{N})} \right) \right\}, \end{aligned} \quad (2.8)$$

³The Lagrangian was explicitly given in Ref. [12] [Eq. (2.10) of that reference]. For other aspects of the Lagrangian, it is quite complicated to derive the Lagrangian from the 5D SUGRA and to identify the radion. Although there have been some attempts to understand the radion [18,19,21,23] and its imaginary part [20] in 5D SUGRA, we leave it to future work. Analyses on anomaly mediation and the gravitino are given by the following well-known formulations in Refs. [24,25], respectively. We study localized F - and D -terms based on 4D SUGRA as they are confined on the brane at $y = 0$ where the warp factor is trivial.

where $\hat{N} \equiv |N_2|^2/(6M_5^3)$. There is a unique nontrivial minimum with a finite value of both the radius R and the overall constant N_2 for the direction ϕ , provided $c < c_{\text{cr}}$ with

$$c_{\text{cr}} \equiv \frac{17 - \sqrt{109}}{12}. \quad (2.9)$$

The stationary conditions are solved with

$$\hat{N} = e^{-(3-2c)Rk\pi}. \quad (2.10)$$

For $c = c_{\text{cr}} - \Delta c$ with a small Δc , the stationary point at the leading order of Δc is obtained as

$$\begin{aligned} R &\approx \frac{-1}{[2(1 - c_{\text{cr}})(3 - 2c_{\text{cr}}) + 1]k\pi} \\ &\times \ln \left[\frac{(3 - 2c_{\text{cr}})(\frac{17}{3} - 4c_{\text{cr}})}{2(2c_{\text{cr}} - 1)(2 - c_{\text{cr}} - \frac{1 - c_{\text{cr}}}{3 - 2c_{\text{cr}}})} \Delta c \right] \\ &\approx \frac{1}{10k} \left(\ln \frac{1}{\Delta c} - 3.4 \right), \end{aligned} \quad (2.11)$$

which means that the radius is stabilized with the size of $R > 1/k$ for $\Delta c < 10^{-6}$. In our setup, the gauge hierarchy problem can be solved by supersymmetry, which is unbroken in the limit of the vanishing boundary superpotential W_b . Therefore we do not necessarily need an enormous hierarchy by the warp factor, such as $e^{-\pi Rk} \sim 10^{-16}$.

At the stationary point the potential becomes

$$V \approx -10^{37}(kw_0)^2(\Delta c)^{1.2}. \quad (2.12)$$

This negative vacuum energy may be shifted by contributions of other sources for supersymmetry breaking. As we will show in Sec. VI, the localized F -term is a candidate of such a source and can make the energy at the stationary point approximately zero (or a tiny positive cosmological constant within the observational bound). With this cancellation of the cosmological constant, we will work with 4D flat background rather than AdS_4 background.⁴

III. RADION AND MODULI MASSES

In this section, we calculate the masses for the quantum fluctuations of the radion and moduli in our model by computing the piece of the Lagrangian quadratic in fluctuations consisting of kinetic terms and mass terms,

$$\mathcal{L}_{\text{quadratic}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}}. \quad (3.1)$$

We start with the kinetic part of the quantum fluctuations in order to find the diagonalized and canonically normalized fluctuation fields. From Eq. (2.2), the kinetic Lagrangian is

written as

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \sigma(\partial_\mu R)(\partial^\mu R)e^{-2R\sigma}(\phi^\dagger \phi + \phi^{c\dagger} \phi^c - 6M_5^3) \\ &\quad - \frac{1}{2}(1 - 2R\sigma)e^{-2R\sigma}(\partial_\mu R)\partial^\mu(|\phi|^2 + |\phi^c|^2) \\ &\quad - \text{Re}^{-2R\sigma}[(\partial_\mu \phi^\dagger)(\partial^\mu \phi) + (\partial_\mu \phi^{c\dagger})(\partial^\mu \phi^c)], \end{aligned} \quad (3.2)$$

where we performed a partial integral and dropped four-dimensional total derivatives. Since the field ϕ^c is of higher order of w_0 , we will omit ϕ^c in Eq. (3.2). Without loss of generality, we can choose the phase of the background classical solution in Eq. (2.5) as

$$N_2 = N_2^\dagger. \quad (3.3)$$

We now introduce quantum fluctuation fields around the background classical solution to define the radion \tilde{R} and the moduli field \tilde{N}_2 :

$$R + \tilde{R}, \quad N_2 + \tilde{N}_2, \quad \tilde{N}_2 = \tilde{N}_{2R} + i\tilde{N}_{2I}. \quad (3.4)$$

Substituting Eq. (2.5) into \mathcal{L}_{kin} , we find that the 4D kinetic Lagrangian for the quantum fluctuations has a mixing between radion \tilde{R} and the real part of the moduli field \tilde{N}_{2R} ,

$$\begin{aligned} \int_0^\pi dy \mathcal{L}_{\text{kin}} &= -(\partial_\mu \tilde{R}, \partial_\mu \tilde{N}_{2R}) \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} \begin{pmatrix} \partial^\mu \tilde{R} \\ \partial^\mu \tilde{N}_{2R} \end{pmatrix} \\ &\quad - f_{22} \partial_\mu \tilde{N}_{2I} \partial^\mu \tilde{N}_{2I}, \end{aligned} \quad (3.5)$$

where

$$\begin{aligned} f_{11} &\equiv \frac{|N_2|^2}{(1 - 2c)^3 R^2 k} e^{(1-2c)Rk\pi} \left\{ -\left(\frac{3}{2} - c\right)\left(\frac{1}{2} + c\right) \right. \\ &\quad \times (1 - 2c)^2 (Rk\pi)^2 + 2(1 - 2c)(Rk\pi) - 2 \\ &\quad \left. + 2e^{-(1-2c)Rk\pi} \right\} + \frac{3M_5^3}{2R^2 k} \{1 - e^{-2Rk\pi}(1 + 2Rk\pi)\}, \end{aligned} \quad (3.6)$$

$$f_{12} \equiv \frac{\pi}{2} N_2^\dagger e^{(1-2c)Rk\pi} = \frac{\pi}{2} N_2 e^{(1-2c)Rk\pi} = f_{21}, \quad (3.7)$$

$$f_{22} \equiv \frac{e^{(1-2c)Rk\pi} - 1}{(1 - 2c)k}. \quad (3.8)$$

To transform this Lagrangian into canonical forms, we can take the following basis,

$$\tilde{N}_{2I} = \sqrt{2f_{22}}\tilde{N}_{2I}, \quad (3.9)$$

and

$$\begin{pmatrix} \tilde{R} \\ \tilde{N}_{2R} \end{pmatrix} = \begin{pmatrix} \sqrt{\lambda_+} & \\ & \sqrt{\lambda_-} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{R} \\ \tilde{N}_{2R} \end{pmatrix}, \quad (3.10)$$

where λ_\pm and the rotation angles are given by

⁴In AdS_4 background, supersymmetry breaking by a constant boundary superpotential is closely related to Scherk-Schwarz supersymmetry breaking. In the case of Scherk-Schwarz supersymmetry breaking in AdS_4 the radion potential and soft masses were given in Ref. [26].

$$\lambda_{\pm} = f_{11} + f_{22} \pm \sqrt{(f_{11} - f_{22})^2 + 4f_{12}^2}, \quad (3.11)$$

$$\tan\theta = \frac{1}{2f_{12}}\{f_{22} - f_{11} + \sqrt{(f_{11} - f_{22})^2 + 4f_{12}^2}\}. \quad (3.12)$$

Let us now evaluate the radion and moduli masses. The mass terms are given by

$$\int_0^\pi dy \mathcal{L}_{\text{mass}} = -\frac{1}{2}(\bar{R}, \bar{N}_{2R}) \mathcal{M}^2 \begin{pmatrix} \bar{R} \\ \bar{N}_{2R} \end{pmatrix} - \frac{1}{2} \mathcal{M}_{2I}^2 (\bar{N}_{2I})^2, \quad (3.13)$$

with the mass matrices

$$\mathcal{M}_{2I}^2 = \frac{1}{2f_{22}} \frac{\partial^2 V}{\partial N_2^\dagger \partial N_2}, \quad (3.14)$$

$$\mathcal{M}^2 = \begin{pmatrix} \frac{1}{\sqrt{\lambda_+}} & \\ & \frac{1}{\sqrt{\lambda_-}} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{\partial^2 V}{\partial R^2} & \frac{\partial^2 V}{\partial R \partial N_2} \\ \frac{\partial^2 V}{\partial N_2^\dagger \partial R} & \frac{\partial^2 V}{\partial N_2^\dagger \partial N_2} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\lambda_+}} & \\ & \frac{1}{\sqrt{\lambda_-}} \end{pmatrix}, \quad (3.15)$$

where the potential V is given in Eq. (2.8). For $c = c_{\text{cr}} - \Delta c$ with a small Δc , the mass-squared matrix at the leading order of $e^{-Rk\pi}$ is obtained as

$$\mathcal{M}^2 \approx k^2 w_0^2 \begin{pmatrix} \frac{2(1-2c)^2}{3-2c} (Rk\pi)^2 e^{(-4c^2+12c-10)Rk\pi} & \frac{(2c-1)^{5/2}}{(3-2c)^2} (Rk\pi) e^{(-4c^2+11c-17/2)Rk\pi} \\ \frac{(2c-1)^{5/2}}{(3-2c)^2} (Rk\pi) e^{(-4c^2+11c-17/2)Rk\pi} & \frac{2c-1}{4} (-4c^2 + 12c - 6 + \frac{4(3-2c)}{3}) e^{(2c-3)Rk\pi} \end{pmatrix}.$$

In this approximation, the mass squared of the imaginary part of the moduli field \bar{N}_{2I} is the same as the right-bottom component of this mass-squared matrix. Diagonalizing the mass-squared matrix and using Eqs. (2.10) and (2.11), we find that the lighter physical mode is almost exclusively made of the radion,

$$\begin{aligned} m_{\text{light}}^2 &\approx k^2 w_0^2 \frac{2(1-2c)^2}{3-2c} (Rk\pi)^2 e^{(-4c^2+12c-10)Rk\pi} \\ &\approx k^2 w_0^2 0.38(3.4 + \ln\Delta c)^2 (\Delta c)^{1.7}. \end{aligned} \quad (3.16)$$

The heavier eigenmode is found to be exclusively made of the real part of the moduli field,

$$\begin{aligned} m_{\text{heavy}}^2 &\approx k^2 w_0^2 \frac{(2c-1)}{4} \left[-4c^2 + 12c - 6 + \frac{4}{3}(3-2c) \right] \\ &\quad \times \frac{e^{(2c-3)Rk\pi}}{1 - e^{-(2c-1)Rk\pi}} \\ &\approx k^2 w_0^2 0.47(\Delta c)^{0.70}. \end{aligned} \quad (3.17)$$

The imaginary part of the moduli field has the same mass as the real part of the moduli field in this approximation.

For $\Delta c < 10^{-6}$ corresponding to $Rk > 1$, the physical mass of the light field (radion) is given by

$$m_{\text{light}} < k w_0 \times 10^{-4}. \quad (3.18)$$

The physical mass of the heavy field (complex moduli) is given by

$$m_{\text{heavy}} < k w_0 \times 10^{-2}. \quad (3.19)$$

We estimate the mass of the lighter physical mode (almost exclusively made of the radion) and that of the heavier mode (almost exclusively made of the complex moduli field) as

$$m_{\text{light}} \sim 1 \text{ TeV}, \quad m_{\text{heavy}} \sim 100 \text{ TeV} \quad (3.20)$$

for $w_0 \sim (10^7 \text{ GeV}/k)$ and $\Delta c \sim 10^{-6}$.

IV. SOFT MASS BY ANOMALY MEDIATION

In this section, we calculate soft masses induced by anomaly mediation in our hyperscalar background. In a supersymmetric Randall-Sundrum model, anomaly-mediated scalar mass is given by [24]

$$\tilde{m}_{\text{AMSB}} \sim \frac{g^2}{16\pi^2} \left\langle \frac{F_\omega}{\omega} \right\rangle \quad (4.1)$$

where the superfield ω is defined as a rescaled compensator multiplet $\omega = \varphi e^{-T\sigma}$ and we denoted its lowest component also as ω , and g is the gauge coupling constant for visible sector fields. In our model, the anomaly-mediated scalar mass becomes

$$\tilde{m}_{\text{AMSB}} \sim \frac{g^2}{16\pi^2} (F_\varphi - F_T \sigma)|_{y=\pi}. \quad (4.2)$$

The relevant F component is

$$F_\varphi - F_T \sigma = -\frac{e^{-R\sigma}}{R} \left[-\frac{1}{6M_5^3} \phi^\dagger \partial_y \phi^{c\dagger} - \frac{1}{3M_5^3} \phi^{c\dagger} \partial_y \phi^\dagger + \frac{1}{6M_5^3} \phi^\dagger \phi^{c\dagger} \left(\frac{9}{2} - c \right) R\sigma' - \frac{1}{2M_5^3} W_b - \frac{3}{r} \phi^{c\dagger} \partial_y \phi^\dagger - \frac{3}{r} W_b + \frac{1}{r} \phi^{c\dagger} \phi^\dagger \left(\frac{3}{2} - c \right) R\sigma' \right]. \quad (4.3)$$

With background solutions (2.5) and (2.6), Eq. (4.3) at the boundary $y = \pi$ becomes

$$(F_\varphi - F_T \sigma)|_{y=\pi} = \hat{\epsilon} \sigma' e^{-Rk\pi} \frac{\hat{N} e^{3Rk\pi} w_0}{\hat{N}(e^{2Rk\pi} - 1) + \frac{2}{1-2c}(e^{-(1-2c)Rk\pi} - 1)} \approx -\hat{\epsilon} \sigma' w_0 \frac{2c_{\text{cr}} - 1}{3 - 2c_{\text{cr}}} \sim -\hat{\epsilon} \sigma' w_0 \times 0.05, \quad (4.4)$$

where we used the stationary condition (2.10) in the second equality. Therefore we obtain the anomaly-mediated scalar mass as

$$\tilde{m}_{\text{AMSB}} \sim \mathcal{O}(10^{-4}) \times g^2 k w_0. \quad (4.5)$$

As shown in the previous section, a lighter physical mass among the radion and moduli can be of the order of a TeV for $w_0 \sim (10^7 \text{ GeV}/k)$. For $g^2 k w_0 \sim 10^6 \text{ GeV}$, we obtain

$$\tilde{m}_{\text{AMSB}} \sim 100 \text{ GeV}, \quad (4.6)$$

which is a typical soft mass.

For gaugino mass, anomaly mediation is also dominant as long as additional interactions with gauge singlets are not included. The gaugino mass is of the same order as the scalar mass.

V. SOFT MASS BY BULK FIELD MEDIATION

In this section, we examine soft mass induced by mediation of bulk fields. In order to perform this computation, we need the Kaluza-Klein mass spectrum of all bulk fields. Since the constant superpotential here only affects the hyperscalar and gravitino, what we have to do is calculate the mass spectrum of these two fields. The mass spectrum of the other fields is already known and can be read from that of the hyperscalar and gravitino as the $w_0 \rightarrow 0$ limit. By using this resulting mass spectrum, we calculate the soft mass induced by mediation of bulk fields.

A. Kaluza-Klein masses of the hyperscalar

Let us calculate the mass spectrum of the hyperscalar. The analysis is similar to Ref. [12]. However, we should carefully adopt approximations to use asymptotic forms of higher transcendental functions for $\Delta c \sim 10^{-6}$. In the case where $\Delta c \sim 10^{-6}$ corresponding to almost no exponential suppression $e^{-Rk\pi} \sim e^{-\pi} \sim 0.04$, there are two candidates of approximation:

$$(I) \quad m_n/k \ll 1 \quad \text{and} \quad m_n e^{kR\pi}/k \gg 1, \quad (5.1)$$

$$(II) \quad m_n/k \gg 1 \quad \text{and} \quad m_n e^{kR\pi}/k \gg 1. \quad (5.2)$$

Approximation (I) is used also in our previous work in Ref. [12], where we have studied only the case with $w_\pi \neq 0$, $w_0 = 0$ in detail. We will use approximation (I) in the

present model of $w_0 \neq 0$, $w_\pi = 0$ in order to obtain the excitation spectra for masses much smaller compared to the mass scale k of the AdS space. On the other hand, approximation (II) is more appropriate in the $\Delta c \sim 10^{-6}$ model since it is usable for most of n . Then, eigenfunctions are found to behave as cosine or sine similarly to the flat case, as easily derived from properties of Bessel functions.

We will perform an analysis similar to Ref. [12] with both approximations (I) and (II). The details of the calculations are shown in Appendix 8. For approximation (I), we find the hyperscalar mass

$$m_n = k e^{-Rk\pi} \times \begin{cases} (n + \frac{c+1}{2})\pi + \frac{c(1-c)}{2n\pi} (1 + 10^{-3}(\frac{n\pi}{w_0} e^{-Rk\pi})^2), \\ (n + \frac{c+1}{2})\pi - \frac{2n\pi}{c(c-1)} 10^{-3}(\frac{n\pi}{w_0} e^{-Rk\pi})^2, \end{cases} \quad (5.3)$$

where $c = c_{\text{cr}} - \Delta c$ and $\Delta c \sim 10^{-6}$. As expected, the w_0 -dependent terms are highly suppressed by exponential factors. For approximation (II), we find the hyperscalar mass

$$m_n \approx \frac{k}{e^{Rk\pi} - 1} \times \left(n\pi \pm \frac{w_0}{2\sqrt{3}} \right). \quad (5.4)$$

In the case where $w_0 \ll 1$, perturbative treatment in Eq. (5.3) seems to be broken. Then only Eq. (5.4) gives a valid representation for the hyperscalar mass.

B. Kaluza-Klein masses of the gravitino

Let us calculate the mass spectrum of the gravitino, which is the other superparticle affected by w_0 . The relevant gravitino Lagrangian in the bulk is given by⁵ [25]

$$\mathcal{L}_{\text{bulk}} = M_5 \sqrt{-g} \left[i \bar{\Psi}_M^i \gamma^{\text{MNP}} D_N \Psi_P^i - \frac{3}{2} \sigma' \bar{\Psi}_M^i \gamma^{\text{MN}} (\sigma_3)^{ij} \Psi_N^j \right], \quad (5.5)$$

$$\Psi_M^1 = (\psi_{M\alpha}^1, \bar{\psi}_M^{2\alpha})^T, \quad \Psi_M^2 = (\psi_{M\alpha}^2, -\bar{\psi}_M^{1\alpha})^T, \quad (5.6)$$

⁵Our convention is compatible with Refs. [27,28] and is summarized in Appendix B.

$$D_M = \partial_M + \omega_M, \quad \omega_M = (\omega_\mu, \omega_4) = (\sigma' \gamma_4 \gamma_\mu / 2, 0), \quad (5.7)$$

$$\begin{aligned} \gamma^{M_1 M_2 \dots M_N} &= \gamma^{[M_1} \gamma^{M_2} \dots \gamma^{M_N]} \\ &\equiv \frac{1}{N!} (\gamma^{M_1} \gamma^{M_2} \dots \gamma^{M_N} \\ &\quad + \text{antisymmetric permutations}), \end{aligned} \quad (5.8)$$

where the 5D curved indices are labeled by $M, N = 0, 1, 2, 3, 4$. The gamma matrix with curved indices is defined using the 5D vielbein e_A^M as $\gamma^M = e_A^M \gamma^A$, where A denote tangent space indices. In the second term in Eq. (5.5), $SU(2)_R$ indices are contracted by (σ_3) .

Boundary terms for the gravitino are also contained in the term with the boundary superpotential W_b in the superfield Lagrangian in Eq. (2.2). By restoring the fermionic part, we find [29]

$$\begin{aligned} \mathcal{L}_{\text{bound sup}} &= \int d^2 \theta \varphi^3 e^{-3T\sigma} W_b \\ &= 3 \left[F_\varphi - \frac{1}{M_5^2} \psi_\mu^\dagger \sigma^{[\mu} \bar{\sigma}^{\nu]} \psi_\nu^\dagger + \text{H.c.} \right] W_b + \dots, \end{aligned} \quad (5.9)$$

using $\varphi = 1 + \theta^2 F_\varphi$. Therefore we obtain a boundary mass term for the gravitino associated with the boundary superpotential,

$$\mathcal{L}_{\text{boundary}} = -\frac{3W_b}{M_5^2} [\psi_\mu^\dagger \sigma^{[\mu} \bar{\sigma}^{\nu]} \psi_\nu^\dagger + \bar{\psi}_\mu^\dagger \bar{\sigma}^{[\mu} \sigma^{\nu]} \bar{\psi}_\nu^\dagger], \quad (5.10)$$

where W_b is the constant superpotential localized at $y = 0$ given in Eq. (2.4) and we assumed the Z_2 parity of $\psi_\mu^{(2)}$ to be even (odd). From the Lagrangian given above, we calculate the mass spectrum. The details are shown in Appendix B.

For the lightest mode, we consider the limit

$$\frac{m_n}{k} \ll 1, \quad \frac{m_n}{k} e^{Rk\pi} \ll 1. \quad (5.11)$$

In this limit, we find

$$m_{\text{lightest}} \approx 6w_0 k, \quad (5.12)$$

which can be 10^7 GeV for $w_0 \sim (10^7 \text{ GeV}/k)$. This shows that the 4D gravitino (lightest mode) is much heavier than the radion as well as scalars of the visible sector. This is similar to the supersymmetry-breaking mediation model considered previously in Ref. [24].

For heavier KK modes of the gravitino, we consider two limits:

$$(I) \quad \frac{m_n}{k} \ll 1, \quad \frac{m_n}{k} e^{Rk\pi} \gg 1, \quad (5.13)$$

$$(II) \quad \frac{m_n}{k} \gg 1, \quad \frac{m_n}{k} e^{Rk\pi} \gg 1. \quad (5.14)$$

In limit (I), we find

$$m_n \approx 6w_0 k, \quad (n + \frac{1}{4})\pi k e^{-Rk\pi} \quad (5.15)$$

where n is an integer satisfying $(n + \frac{1}{4})\pi e^{-Rk\pi} \ll 1$. The former one is the lightest mode solved above. In limit (II), we find the mass

$$m_n \approx \left(n - \frac{6w_0}{2\pi} \right) \pi k e^{-Rk\pi} \quad (5.16)$$

where n is an integer satisfying $(n - \frac{6w_0}{2\pi})\pi e^{-Rk\pi} \gg 1$.

C. Induced masses by Kaluza-Klein modes

In general, scalars in the visible sector can receive masses due to supersymmetry-breaking effects by mediation of bulk fields or by mediation of all modes in Kaluza-Klein decomposition. For example, take masses of bosons and fermions to be n/R or $(n + 1/2)/R$, where the mass splitting is $1/(2R)$. This type of mass splitting has been considered also in the context of string theory [4,30]. In these models, it is known that induced soft mass is of the order of

$$\tilde{m}_{\text{KK-med}} \sim 10^{-1} \times \frac{m_{3/2}^2}{M_4}. \quad (5.17)$$

The constant superpotential w_0 at $y = 0$ induces soft mass by mediation of all Kaluza-Klein modes. Although we might be able to explicitly calculate the induced mass as in [4,30], here we adopt a different way. In our model the mass splitting between the bosons and fermions can be much smaller than $1/(2R)$ in the case above, since the mass splittings are proportional to positive powers of w_0 as seen from Eqs. (5.4), (5.12), (5.13), (5.14), (5.15), and (5.16). Thus the induced mass in our model should be small compared to Eq. (5.17). The soft mass by mediation of Kaluza-Klein modes is evaluated as

$$\tilde{m}_{\text{KK-med}} \lesssim 10^{-1} \times \frac{m_{\text{lightest}}^2}{M_4} \sim 10^{-5} \text{ GeV} \ll \tilde{m}_{\text{AMSB}}, \quad (5.18)$$

where we used $m_{\text{lightest}} \sim 10^7$ GeV given below Eq. (5.12). We see that soft masses by mediation of Kaluza-Klein modes in our model are smaller than that of anomaly mediation. Therefore our model passes the FCNC constraint also with respect to bulk field mediation while $\tilde{m}_{\text{AMSB}} \sim 100$ GeV.

VI. CANCELLATION OF THE COSMOLOGICAL CONSTANT BY F_X

As mentioned in Sec. II, the potential in our model is negative at the stationary point. On the other hand, we have assumed 4D flat background. In order for our analysis to be consistent, we should check whether additional sources can

cancel the cosmological constant. As candidates of such sources, we consider an F -term contribution and a D -term contribution. In this section, we examine cancellation of the cosmological constant by a localized F -term and check whether the induced soft mass is small compared to that of anomaly mediation. In the next section, we will examine the cancellation of the cosmological constant by the Fayet-Iliopoulos D -term.

We consider the setup where the hidden sector spurion⁶ chiral multiplet X is localized at $y = 0$ and the visible sector is localized at $y = \pi$. The Lagrangian is given by

$$\mathcal{L}_X = \left[\int d^4\theta \varphi^\dagger \varphi X^\dagger X + \int d^2\theta (\varphi^3 m^2 X + \text{H.c.}) \right] \delta(y), \quad (6.1)$$

with

$$X = F_X \theta^2, \quad (6.2)$$

where m is a mass parameter. The auxiliary field Lagrangian of (6.1) is

$$\mathcal{L}_{X\text{aug}} = [|F_X|^2 + (m^2 F_X + \text{H.c.})] \delta(y). \quad (6.3)$$

After solving equations of motion for F_X from this equation and performing the y integral, we obtain

$$\int dy \mathcal{L}_{X\text{aug}} = -m^4 = -|F_X|^2. \quad (6.4)$$

As seen from (6.3), the F component of the spurion field does not mix with the F components of Φ , Φ^c , φ , T . Therefore it does not change the spectrum of Φ , Φ^c , φ , T at tree level. However, at loop level it can contribute to visible sector soft masses. In this case, the gravity multiplet can transmit supersymmetry breaking to the visible sector at one loop (which is called brane-to-brane mediation by gravity). According to Ref. [31], the induced soft mass is given by

$$\begin{aligned} \Delta \tilde{m}_{\text{bbg}}^2 &= -\frac{c_w k^4}{18\pi^2 M_5^6} e^{-4k\pi R} |F_X|^2 \\ &= -\frac{c_w k^2}{18\pi^2 M_4^4} e^{-4k\pi R} |F_X|^2, \end{aligned} \quad (6.5)$$

where $M_5^3 \simeq k M_4^2$ is used in the second equality and the dimensionless coefficient is of order unity: $c_w \sim \mathcal{O}(1)$. This mass should be suppressed compared to the

⁶Here we assume that the vacuum expectation value of the scalar component of X vanishes: $\langle X \rangle = 0$. If X is dynamical, one might expect that the Coleman-Weinberg potential could generate nonzero $\langle X \rangle$. A possible radiatively induced Kähler potential $-(X^\dagger X / (4\pi M_X))^2$ could indeed give a negative contribution to the mass squared of X as $-(|F_X| / (4\pi M_X))^2 X^\dagger X$, where M_X is the mass of X . As we will see in (6.11), in the present model $F_X \sim (10^{10} \text{ GeV})^2$. Our assumption $\langle X \rangle = 0$ can be supported by a positive mass-squared term $M_X^2 X^\dagger X$ for X with $M_X \gtrsim 10^{10} \text{ GeV}$.

anomaly-mediated scalar mass because it is tachyonic,

$$\frac{\Delta \tilde{m}_{\text{bbg}}^2}{\tilde{m}_{\text{AMSB}}^2} \sim -\frac{c_w}{18\pi^2} \omega_0^4 \frac{|F_X|^2}{\tilde{m}_{\text{AMSB}}^2 M_4^2}, \quad \left| \frac{\Delta \tilde{m}_{\text{bbg}}^2}{\tilde{m}_{\text{AMSB}}^2} \right| < 10^{-2}. \quad (6.6)$$

This gives a constraint to F_X ,

$$F_X < 3\sqrt{2}\pi \tilde{m}_{\text{AMSB}} M_4 \omega_0^{-2} \times 10^{-1} \sim 10^{22} \text{ GeV}^2 \quad (6.7)$$

for $Rk \sim 1$,

where $\tilde{m}_{\text{AMSB}} \sim 100 \text{ GeV}$ and $\omega_0 = e^{-Rk\pi}$ are used.

On the other hand, the contribution of the brane-to-brane mediation by a hypermultiplet can be estimated following Ref. [16],

$$\Delta \tilde{m}_{\text{bbh}}^2 \sim \frac{c_{ij}}{16\pi^2} \left(\frac{F_X}{\sqrt{3}M_4} \right)^2 \left(\frac{k}{M_4} \right)^2 \left(\frac{1-2c}{e^{(1-2c)Rk\pi} - 1} \right)^2 e^{(3-2c)Rk\pi} \quad (6.8)$$

where $c_{ij} \sim \mathcal{O}(1)$. This contribution should also be suppressed because it is flavor violating,

$$\begin{aligned} \frac{\Delta \tilde{m}_{\text{bbh}}^2}{\tilde{m}_{\text{AMSB}}^2} &\sim \frac{c_{ij}}{16\pi^2} \left(\frac{F_X}{\sqrt{3}M_4} \right)^2 \left(\frac{k}{M_4} \right)^2 \left(\frac{2c-1}{e^{(1-2c)Rk\pi} - 1} \right)^2 \\ &\times \frac{e^{(3-2c)Rk\pi}}{\tilde{m}_{\text{AMSB}}^2} < 10^{-3}. \end{aligned} \quad (6.9)$$

This leads to the condition

$$\begin{aligned} F_X &< \frac{4\sqrt{3}\pi}{10^3} M_4 \left(\frac{M_4}{k} \right) \frac{1 - e^{(1-2c)Rk\pi}}{(2c-1)} e^{-(1/2)(3-2c)Rk\pi} \tilde{m}_{\text{AMSB}} \\ &\sim 10^{18} \left(\frac{10^{18}}{k} \right) \text{ GeV}^2, \end{aligned} \quad (6.10)$$

for $Rk \sim 1$. Taking into account the upper bound for the AdS curvature scale $k < M_4 = 10^{18} \text{ GeV}$, we find that the constraints (6.7) and (6.10) are satisfied by $\sqrt{F_X} \lesssim 10^{11} \text{ GeV}$.

We estimate the size of $\sqrt{F_X}$ to cancel the negative cosmological constant in Eq. (2.12). The size for $k\omega_0 \sim 10^7 \text{ GeV}$ is obtained as

$$\sqrt{F_X} \approx 10^{12} (\Delta c)^{0.3} \text{ GeV} \approx 10^{10} \text{ GeV} \quad \text{for } \Delta c \sim 10^{-6}. \quad (6.11)$$

This shows that the cancellation of the cosmological constant can occur below (although near) the critical point for the FCNC constraint.

The contribution of F_X to the gravitino mass (irrespective of the value of $\langle X \rangle$) is

$$m_{3/2} \sim \frac{F_X}{M_4} \approx 100 \text{ GeV} \quad (6.12)$$

which is estimated based on effective 4D supergravity below the compactification scale. This is small compared to the contribution from the constant superpotential (5.12).

VII. CANCELLATION OF THE COSMOLOGICAL CONSTANT BY THE FAYET-ILIOPOULOS SECTOR

In this section, we examine the possibility of cancellation of the cosmological constant by considering coupling of the Fayet-Iliopoulos sector localized at $y = 0$.

We begin with a simple extension of the four-dimensional Fayet-Iliopoulos model supplemented by the compensator given by the following Lagrangian,⁷

$$\begin{aligned} \mathcal{L}_V = \delta(y) & \left[\int d^4\theta \varphi^\dagger \varphi (A_1^\dagger e^{eV} A_1 + A_2^\dagger e^{-eV} A_2 + 2\kappa V) \right. \\ & + \left\{ \int d^2\theta \left(\frac{1}{4} (W^\alpha W_\alpha + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}) \right. \right. \\ & \left. \left. + \varphi^3 (W(A_1, A_2) + \text{H.c.}) \right) \right\} \Big], \end{aligned} \quad (7.1)$$

with the superpotential

$$W(A_1, A_2) = mA_1 A_2, \quad (7.2)$$

where A_1, A_2 are charged chiral superfields, V is a $U(1)$ vector superfield, and W^α is its field strength chiral superfield. The gauge coupling, the Fayet-Iliopoulos parameter, and the A_1, A_2 mass parameter are denoted as e, κ , and m , respectively. The part of the Lagrangian containing auxiliary fields of this model reads

$$\begin{aligned} \mathcal{L}_{V_{\text{aux}}} = \delta(y) & \left[\frac{1}{2} D^2 + \kappa D + \frac{e}{2} (A_1^* A_1 - A_2^* A_2) D \right. \\ & + |F_\varphi|^2 (|A_1|^2 + |A_2|^2) + |F_1|^2 + |F_2|^2 \\ & + \left\{ F_\varphi (A_1 F_1^* + A_2 F_2^*) + 3F_\varphi W + \frac{\partial W}{\partial A_1} F_1 \right. \\ & \left. \left. + \frac{\partial W}{\partial A_2} F_2 + \text{H.c.} \right\} \right]. \end{aligned} \quad (7.3)$$

The equations of motion for $D, F_{1,2}$ are solved as

$$0 = \frac{\partial \mathcal{L}_{V_{\text{aux}}}}{\partial D} = D + \kappa + \frac{e}{2} (A_1^* A_1 - A_2^* A_2), \quad (7.4)$$

$$0 = \frac{\partial \mathcal{L}_{V_{\text{aux}}}}{\partial F_1^*} = F_\varphi A_1 + F_1 + \left(\frac{\partial W}{\partial A_1} \right)^*, \quad (7.5)$$

$$0 = \frac{\partial \mathcal{L}_{V_{\text{aux}}}}{\partial F_2^*} = F_\varphi A_2 + F_2 + \left(\frac{\partial W}{\partial A_2} \right)^*. \quad (7.6)$$

Putting these equations into (7.3) gives

⁷Fayet-Iliopoulos terms have been related closely to R symmetry in the context of supergravity [21, 32–36]. As an aspect of supergravity it is straightforward to see from $\kappa \ll M_5$ that our model does not suffer from an Einstein term induced on the brane by the D -term. The authors thank Hiroyuki Abe for a valuable comment on this point.

$$\begin{aligned} \Delta \mathcal{L} = \delta(y) & \left[-\frac{1}{2} D^2 - \left| \frac{\partial W}{\partial A_1} \right|^2 - \left| \frac{\partial W}{\partial A_2} \right|^2 \right. \\ & \left. + \left\{ F_\varphi \left(3W - A_1 \frac{\partial W}{\partial A_1} - A_2 \frac{\partial W}{\partial A_2} \right) + \text{H.c.} \right\} \right] \end{aligned} \quad (7.7)$$

which implies that, if

$$0 = 3W - A_1 \frac{\partial W}{\partial A_1} - A_2 \frac{\partial W}{\partial A_2} = mA_1 A_2, \quad (7.8)$$

the hidden sector does not affect equations of motion for the compensator.

Next we have to examine whether condition (7.8) is compatible with the minimization conditions of the hidden sector potential. The hidden sector potential is given by

$$\begin{aligned} \Delta V = \delta(y) & \left[\frac{1}{2} (\kappa + \frac{e}{2} (A_1^* A_1 - A_2^* A_2))^2 + |mA_1|^2 \right. \\ & \left. + |mA_2|^2 - \{F_\varphi mA_1 A_2 + \text{H.c.}\} \right]. \end{aligned} \quad (7.9)$$

The minimization conditions of the potential are

$$\begin{aligned} 0 = \frac{\partial \Delta V}{\partial A_1^*} & = \left(\kappa + \frac{e}{2} (A_1^* A_1 - A_2^* A_2) \right) \left(\frac{e}{2} A_1 \right) \\ & + |m|^2 A_1 - (F_\varphi mA_2)^*, \end{aligned} \quad (7.10)$$

$$\begin{aligned} 0 = \frac{\partial \Delta V}{\partial A_2^*} & = \left(\kappa + \frac{e}{2} (A_1^* A_1 - A_2^* A_2) \right) \left(-\frac{e}{2} A_2 \right) \\ & + |m|^2 A_2 - (F_\varphi mA_1)^*. \end{aligned} \quad (7.11)$$

We find that a solution is given by

$$A_1 = A_2 = 0. \quad (7.12)$$

At this minimum, the value of the additional potential (after y integration) is

$$\Delta V = \frac{1}{2} \kappa^2. \quad (7.13)$$

This positive contribution can cancel the negative vacuum energy found in Eq. (2.12), if $\sqrt{\kappa} \approx 10^{10}$ GeV. If matter in the visible sector is neutral under $U(1)$, additional contributions to scalar masses are not generated. The contribution of this sector to the gravitino mass is small, similarly to (6.12).

VIII. CONCLUSION

We have studied radion and moduli masses and induced soft masses in the radius stabilization model proposed previously in Ref. [12]. We have simultaneously obtained supersymmetry breaking and radius stabilization, as well as soft mass without the FCNC problem, large gravitino mass, and radion mass of the order of TeV. These all give evidence that our model is phenomenologically viable.

In this model the potential has a negative value at the stationary point. In order to cancel the negative cosmological constant, we have introduced a localized F -term at

$y = 0$ as another source of supersymmetry breaking and analyzed it in 4D flat space. In our model there are four parameters: the 5D Planck mass M_5 , the 5D curvature k , the bulk mass parameter for the hypermultiplet c , and the constant boundary superpotential w_0 (up to the visible sector gauge coupling constant g). Among them, we can choose one of the dimensionful quantities to give the unit of mass dimension. In numerically evaluating various masses, we have chosen $M_5 \sim (M_4^2 k)^{1/3}$, $w_0 \sim (10^7 \text{ GeV}/k)$, and $c = c_{\text{cr}} - \Delta c$ where $c_{\text{cr}} \approx 0.546$ and $\Delta c \sim 10^{-6}$. This c corresponds to the stabilized radius $R \sim k^{-1}$.

The quantum fluctuation of radion mixes with complex moduli. We have found that the lightest physical mode of the admixture has the mass of the order of 1 TeV. Such a comparatively small radion mass appears as a common feature of the warped space model [37], and its value is in the experimentally allowed region [38].

We have also found that soft mass is of the order of 100 GeV and is generated by anomaly mediation. Therefore there is no FCNC problem. The gravitino mass is found to be 10^7 GeV . Such a large gravitino mass is similar to that of the supersymmetry-breaking mediation scenario given in Ref. [24]. We have found that the hyperscalar mass is of the order of k and is much heavier than other fields. Therefore the hyperscalar primarily acts as a part of the background configuration.

As for the cancellation of the cosmological constant, the FCNC constraint for the localized F -term leads to $\sqrt{F_X} \lesssim 10^{11} \text{ GeV}$. The cosmological constant is canceled for $\sqrt{F_X} \approx 10^{10} \text{ GeV}$. This justifies our analysis based on a 4D flat background. Similarly, in the scenario with the D -term contribution, the cosmological constant is canceled for the Fayet-Iliopoulos parameter $\sqrt{\kappa} \approx 10^{10} \text{ GeV}$.

Some issues still remain to be examined in our model. We have not considered the imaginary part for the lowest component of the radion supermultiplet. In the present model, the mass seems to vanish because the potential can be seen to include only the real part of the radion supermultiplet. However, the mass would be induced by a quantum loop effect similarly to the case of the gauge-Higgs unification model [39]. If the induced mass is very small, it would have to be examined whether the coupling with other particles can be very small as in the case of the axion.

The issue of radius stabilization by Casimir energy [40] also remains to be examined. Analysis about this point will be given in a separate paper [41].

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APPENDIX A: CALCULATIONS OF THE MASS SPECTRUM OF THE HYPERSCALAR

In this appendix, we give the details of the calculations of the mass spectrum of the hyperscalar. Let us consider the n th Kaluza-Klein effective field $\phi_n^I(x)$ with its mode functions $b_n^I(y)$ as the $\phi(x, y)$ component and $b_n^{cI}(y)$ as the $\phi^c(x, y)$ component,

$$\begin{pmatrix} \phi(x, y) \\ \phi^c(x, y) \end{pmatrix} = \sum_n \sum_{I=1,2} \phi_n^I(x) \begin{pmatrix} b_n^I(y) \\ \hat{e}(y) b_n^{cI}(y) \end{pmatrix}, \quad (\text{A1})$$

where I are the indices corresponding to the two independent mass eigenfunctions of effective fields.

Assuming that the effective four-dimensional field $\phi_n(x)$ has mass m_n , we easily find solutions in the bulk in terms of the Bessel functions $J_\alpha(J_\beta)$ and $Y_\alpha(Y_\beta)$ [25],

$$b_n(y) = \frac{e^{2R\sigma}}{N_n} [J_\alpha(m_n e^{R\sigma}/k) + b_\alpha(m_n) Y_\alpha(m_n e^{R\sigma}/k)],$$

$$\alpha = \left\lfloor c + \frac{1}{2} \right\rfloor, \quad (\text{A2})$$

$$b_n^c(y) = \frac{e^{2R\sigma}}{N_n^c} [J_\beta(m_n e^{R\sigma}/k) + b_\beta(m_n) Y_\beta(m_n e^{R\sigma}/k)],$$

$$\beta = \left\lfloor c - \frac{1}{2} \right\rfloor. \quad (\text{A3})$$

In the equations of motion, singular terms give the boundary conditions. The first boundary condition comes from δ^2 terms,

$$0 = -2b_n^c(0) + w_0 b_n(0), \quad (\text{A4})$$

$$0 = b_n^c(\pi). \quad (\text{A5})$$

The second boundary condition comes from the δ function,

$$0 = \frac{7}{3} w_0 b_n^c(0) - \frac{1}{3} w_0 \left[2b_n^c(0) + \frac{m_n}{k} \frac{1}{N_n^c} \{J'_\beta(m_n/k) + b_\beta(m_n) Y'_\beta(m_n/k)\} \right] - 4b_n(0) + 2\left(\frac{3}{2} - c\right) b_n(0) - \frac{2m_n}{k} \left[\frac{1}{N_n} \{J'_\alpha(m_n/k) + b_\alpha(m_n) Y'_\alpha(m_n/k)\} \right], \quad (\text{A6})$$

$$0 = (1 + 2c) b_n(\pi) + \frac{2m_n}{k} \left[\frac{e^{3Rk\pi}}{N_n} \{J'_\alpha(m_n e^{Rk\pi}/k) + b_\alpha(m_n) Y'_\alpha(m_n e^{Rk\pi}/k)\} \right], \quad (\text{A7})$$

with $J'(z) = dJ(z)/dz$.

Let us first consider approximation (I) in Eq. (5.1) to solve (A4)–(A7). From Eqs. (A4) and (A6), we find

$$b_\alpha(m_n) \sim -\frac{4\beta}{(\beta+5)w_0} \frac{N_n}{N_n^c} \left(\frac{m_n}{2k}\right)^{\alpha+\beta} \frac{\pi}{\Gamma(\beta+1)\Gamma(\alpha)}, \quad (\text{A8})$$

$$b_\beta(m_n) \sim -\frac{12\alpha}{(\beta+5)w_0} \left(\frac{m_n}{2k}\right)^{\alpha+\beta} \frac{\pi}{\Gamma(\alpha+1)\Gamma(\beta)} \frac{N_n^c}{N_n} \quad (\text{A9})$$

where we used⁸ $1+2c-2\alpha=0$ for $c=c_{\text{cr}}-\Delta c$, $\Delta c > 0$, and for simplicity we ignored w_0 -independent terms. To solve the remaining equations (A5) and (A7), we use the asymptotic behavior of the Bessel functions for $|z| \gg 1$,

$$J_\alpha(z) \sim \sqrt{\frac{2}{\pi z}} \left(\cos\left(z - \frac{2\alpha+1}{4}\pi\right) - \frac{4\alpha^2-1}{8z} \sin\left(z - \frac{2\alpha+1}{4}\pi\right) \right), \quad (\text{A10})$$

$$Y_\alpha(z) \sim \sqrt{\frac{2}{\pi z}} \left(\sin\left(z - \frac{2\alpha+1}{4}\pi\right) + \frac{4\alpha^2-1}{8z} \cos\left(z - \frac{2\alpha+1}{4}\pi\right) \right). \quad (\text{A11})$$

After changing variables,

$$\frac{m_n}{k} e^{Rk\pi} \equiv x, \quad \frac{2\alpha+1}{4}\pi \equiv a, \quad \frac{2\beta+1}{4}\pi \equiv b, \quad (\text{A12})$$

we can rewrite the boundary conditions at $y=\pi$ and obtain the mass eigenvalue equation similarly to Ref. [12],

$$\tan^2(x-a) - A(x)\tan(x-a) + B(x) = 0 \quad (\text{A13})$$

with

$$A(x) = \frac{c(1-c)}{2x} (1-B(x)), \quad (\text{A14})$$

$$B(x) = -\frac{48\alpha\beta}{(\beta+5)^2 w_0^2} \left(\frac{x e^{-Rk\pi}}{2}\right)^{2(\alpha+\beta)} \times \frac{\pi^2}{\Gamma(\alpha+1)\Gamma(\alpha)\Gamma(\beta+1)\Gamma(\beta)} \quad (\text{A15})$$

where $\cos(x-a) \neq 0$, $x \neq 0$. Solving this equation, we find the hyperscalar mass in Eq. (5.3).

Next we consider approximation (II) in Eq. (5.2) instead of Eq. (5.1) to solve (A4)–(A7). After substituting Eq. (A11) into the boundary conditions, we obtain the mass eigenvalue equation

⁸When we consider the supersymmetric limit $w_0 \rightarrow 0$, we should take $1+2c-2\alpha=0$ after $w_0 \rightarrow 0$.

$$(\tan[(1-e^{-Rk\pi})x])^2 \approx \frac{w_0^2}{12}. \quad (\text{A16})$$

By solving this equation, we find the hyperscalar mass in Eq. (5.4).

APPENDIX B: CALCULATIONS OF THE MASS SPECTRUM OF THE GRAVITINO

In this appendix, we give our convention and some details of the calculations of the mass spectrum of the gravitino.

The gamma matrices are given by [27,28,42]

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \quad (\text{B1})$$

where $m = 0, 1, 2, 3$, and

$$\gamma^4 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}. \quad (\text{B2})$$

We need to choose γ^4 to be anti-Hermitian, since

$$\gamma^A \gamma^B + \gamma^B \gamma^A = -2\eta^{AB}, \quad (\text{B3})$$

$$\eta^{AB} = (-1, +1, +1, +1, +1).$$

We define the Hermitian chiral gamma matrix as

$$\gamma_5 \equiv i\gamma^4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (\text{B4})$$

The charge conjugation matrix is given by

$$C = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 & & 0 & 1 \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \quad (\text{B5})$$

where $\varepsilon_{12} = -1$, $\varepsilon^{12} = +1$, and

$$C\gamma^A C^{-1} = \gamma^{AT}. \quad (\text{B6})$$

The four-component spinors Ψ^1 and Ψ^2 satisfy the symplectic Majorana condition

$$\Psi^i = \varepsilon^{ij} C \bar{\Psi}^j T \quad (\text{B7})$$

where the Dirac conjugate is defined as

$$\bar{\Psi} = \Psi^\dagger \gamma^0. \quad (\text{B8})$$

The Z_2 projection is defined as

$$\Psi_M^i(-y) = (\sigma^3)^{ij} \gamma_5 \Psi^j(y) \quad (\text{B9})$$

which can be rewritten in terms of two-component spinors as

$$\psi_{M\alpha}^1(-y) = \psi_{M\alpha}^1(y), \quad (\text{B10})$$

$$\psi_{M\alpha}^2(-y) = -\psi_{M\alpha}^2(y). \quad (\text{B11})$$

We now give some details of the calculations of the mass spectrum of the gravitino. Since we are interested in the 4D gravitino and its KK modes, we ignore the extra-dimensional component ψ_y . From the Lagrangian (5.5) and (5.10), the equations of motion for the gravitino are

$$0 = i\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\rho}\partial_{\mu}\psi_{\nu}^1 - \bar{\sigma}^{[\nu}\sigma^{\rho]}\left(\frac{1}{R}\partial_y - \frac{3}{2}\sigma'\right)\bar{\psi}_{\nu}^2 + \frac{6w_0}{R}\delta(y)\bar{\sigma}^{[\nu}\sigma^{\rho]}\bar{\psi}_{\nu}^1, \quad (\text{B12})$$

$$0 = i\sigma^{[\mu}\bar{\sigma}^{\nu}\sigma^{\rho]}\partial_{\mu}\bar{\psi}_{\nu}^2 + \sigma^{[\nu}\bar{\sigma}^{\rho]}\left(\frac{1}{R}\partial_y + \frac{3}{2}\sigma'\right)\psi_{\nu}^1 \quad (\text{B13})$$

which are further simplified to

$$0 = -i\bar{\sigma}^{\mu}\partial_{\mu}\psi_{\nu}^1 + \left(\frac{1}{R}\partial_y - \frac{3}{2}\sigma'\right)\bar{\psi}_{\nu}^2 - \frac{6w_0}{R}\delta(y)\bar{\psi}_{\nu}^1, \quad (\text{B14})$$

$$0 = -i\sigma^{\mu}\partial_{\mu}\bar{\psi}_{\nu}^2 - \left(\frac{1}{R}\partial_y + \frac{3}{2}\sigma'\right)\psi_{\nu}^1 \quad (\text{B15})$$

up to the gauge fixing $\bar{\sigma}^{\mu}\psi_{\mu}^{1,2} = 0$ and $\partial^{\mu}\psi_{\mu}^{1,2} = 0$.

Let us take mode expansions

$$\psi_{\rho}^{1,2}(x, y) = \sum_n \psi_{\rho}^{1,2(n)}(x) f^{1,2(n)}(y), \quad (\text{B16})$$

where the 4D effective fields $\psi_{\rho}^{1,2(n)}(x)$ have mass m_n ,

$$-i\bar{\sigma}^{\mu}\partial_{\mu}\psi_{\rho}^{1(n)} = m_n\bar{\psi}_{\rho}^{2(n)}, \quad (\text{B17})$$

$$-i\sigma^{\mu}\partial_{\mu}\bar{\psi}_{\rho}^{2(n)} = m_n\psi_{\rho}^{1(n)}. \quad (\text{B18})$$

The solutions in the bulk are obtained as [25]

$$f_1^{(n)}(y) = \frac{e^{R\sigma/2}}{N_n} \left[J_2\left(\frac{m_n}{k} e^{R\sigma}\right) + b_2(m_n) Y_2\left(\frac{m_n}{k} e^{R\sigma}\right) \right], \quad (\text{B19})$$

$$f_2^{(n)}(y) = \hat{\epsilon}(y) \frac{e^{R\sigma/2}}{N_n} \left[J_1\left(\frac{m_n}{k} e^{R\sigma}\right) + b_1(m_n) Y_1\left(\frac{m_n}{k} e^{R\sigma}\right) \right] \quad (\text{B20})$$

where $\hat{\epsilon}(y)$ is defined in Eq. (2.7).

The boundary conditions we should consider are

$$0 = \bar{f}_2^{(n)}(0) - \frac{6w_0}{2} \bar{f}_1^{(n)}(0), \quad (\text{B21})$$

$$0 = \bar{f}_2^{(n)}(\pi), \quad (\text{B22})$$

$$0 = \left(\partial_y + \frac{3}{2}R\sigma'\right) f_1^{(n)}(\pi), \quad (\text{B23})$$

where $f_2^{(n)}(y) \equiv \hat{\epsilon}(y) \bar{f}_2^{(n)}(y)$. Equation (B21) gives

$$0 = \left[J_1\left(\frac{m_n}{k}\right) + b_1(m_n) Y_1\left(\frac{m_n}{k}\right) \right] - \frac{6w_0}{2} \left[J_2\left(\frac{m_n}{k}\right) + b_2(m_n) Y_2\left(\frac{m_n}{k}\right) \right]. \quad (\text{B24})$$

Equations (B22) and (B23) give

$$b_1(m_n) = -\frac{J_1\left(\frac{m_n}{k} e^{Rk\pi}\right)}{Y_1\left(\frac{m_n}{k} e^{Rk\pi}\right)} = b_2(m_n). \quad (\text{B25})$$

For the lightest mode, we consider the limit

$$\frac{m_n}{k} \ll 1, \quad \frac{m_n}{k} e^{Rk\pi} \ll 1. \quad (\text{B26})$$

Using the asymptotic form of Bessel functions for $|z| \ll 1$ and (B25), the boundary condition (B24) can be rewritten as

$$0 \approx \left[\frac{m_n}{2k} - \frac{\frac{m_n}{2k} e^{Rk\pi}}{-\frac{1}{\pi} \frac{2k}{m_n} e^{-Rk\pi}} \left(-\frac{1}{\pi} \right) \frac{2k}{m_n} \right] - \frac{6w_0}{2} \left[\frac{1}{2} \left(\frac{m_n}{2k} \right)^2 - \frac{\frac{m_n}{2k} e^{Rk\pi}}{(-\frac{1}{\pi} \frac{2k}{m_n} e^{-Rk\pi})} \left(-\frac{1}{\pi} \right) \left(\frac{2k}{m_n} \right)^2 \right]. \quad (\text{B27})$$

From this equation, we find the mass spectrum in Eq. (5.12).

For heavier modes, we consider two limits:

$$(I) \quad \frac{m_n}{k} \ll 1, \quad \frac{m_n}{k} e^{Rk\pi} \gg 1, \quad (\text{B28})$$

$$(II) \quad \frac{m_n}{k} \gg 1, \quad \frac{m_n}{k} e^{Rk\pi} \gg 1. \quad (\text{B29})$$

Using the approximations of the Bessel function for $|z| \gg 1$ and Eq. (B25), we can reduce Eq. (B24) in limit (I) in Eq. (B28) to

$$0 \approx \left[\frac{m_n}{2k} - \cot\left(\frac{m_n}{k} e^{Rk\pi} - \frac{3}{4}\pi\right) \left(-\frac{1}{\pi} \right) \frac{2k}{m_n} \right] - \frac{6w_0}{2} \left[\frac{1}{2} \left(\frac{m_n}{2k} \right)^2 - \cot\left(\frac{m_n}{k} e^{Rk\pi} - \frac{3}{4}\pi\right) \left(-\frac{1}{\pi} \right) \left(\frac{2k}{m_n} \right)^2 \right]. \quad (\text{B30})$$

From this equation, we find the mass spectrum in Eq. (5.15).

On the other hand, in limit (II) in Eq. (B29) the boundary condition (B24) reduces to

$$0 \approx \left[\cos\left(\frac{m_n}{k} - \frac{3}{4}\pi\right) - \cot\left(\frac{m_n}{k} e^{Rk\pi} - \frac{3}{4}\pi\right) \sin\left(\frac{m_n}{k} - \frac{3}{4}\pi\right) \right] - \frac{6w_0}{2} \left[\cos\left(\frac{m_n}{k} - \frac{5}{4}\pi\right) - \cot\left(\frac{m_n}{k} e^{Rk\pi} - \frac{3}{4}\pi\right) \times \sin\left(\frac{m_n}{k} - \frac{5}{4}\pi\right) \right]. \quad (\text{B31})$$

Thus we find the mass spectrum in Eq. (5.16).

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