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Finite anomalous magnetic moment in the gauge-Higgs unification

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We show that the anomalous magnetic moment of fermion in the gauge-Higgs unification is finite in *any* spacetime dimensions, which is a new predictive physical observable similar to the Higgs mass.

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The gauge-Higgs unification [1,2] is one of the fascinating scenarios since it predicts some finite physical observables due to the higher dimensional gauge invariance, though it is regarded as a nonrenormalizable theory. As far as we know, the Higgs mass is the unique finite physical observable and its finiteness has been examined from the various points of view [3–9]. In this scenario, the Higgs scalar field is identified with the extra spatial components of the higher dimensional gauge field, which immediately forbids the local mass term relying on the gauge invariance. Then, the finite Higgs mass is generated by Wilson loop dynamics and is independent of the cutoff scale of the theory, which is therefore very predictive.

It is a natural question to ask whether there are any other finite physical observables in the gauge-Higgs unification. If we have such an observable, we can guess it is in the gauge-Higgs sector of the theory, just as the case of the Higgs mass. Along this line of thought, the divergence structure of S and T parameters has been investigated in [10], but the local gauge invariant operator for S and T parameters is allowed and they are found to be divergent in more than five dimensions (although a particular linear combination of them becomes finite even in six dimensions).

In this paper, we find a new finite physical observable in *any* spacetime dimensions in the context of the gauge-Higgs unification. It is the anomalous magnetic moment of fermion. Before calculating it in detail, it is instructive to give an argument of the operator analysis. In four dimensions, the gauge invariant dimension six operator relevant for the anomalous magnetic moment is given by

$$\bar{\Psi}_L \sigma_{\mu\nu} \Psi_R F^{\mu\nu} \langle H \rangle + \text{H.c.} \quad (1)$$

where $\Psi_{L(R)}$ denotes the standard model fermions and H is the standard model Higgs doublet. The field strength of photon is denoted by $F^{\mu\nu}$. In the gauge-Higgs unification on $M^D \times S^1$ where M^D is the D dimensional Minkowski spacetime, the Higgs is identified with the extra component of the gauge field A_y. From the lesson in the discussion for S and T parameters in the gauge-Higgs unification [10], we learn that A_y should be replaced by the extra space component of the covariant derivative D_y in order to preserve the higher dimensional gauge invariance. This observation leads us to the statement that the local gauge invariant operator relevant for the anomalous magnetic moment takes the form

$$i\bar{\Psi}\sigma_{MN} D_A \Gamma^A \Psi F^{MN}. \quad (2)$$

To be precise, the operator describing the effective 3-point vertex reads as $i\bar{\Psi}\sigma_{MN}\langle D_A\rangle\Gamma^A\Psi F^{MN}$, where $\langle D_A\rangle$ is obtained by replacing A_M with its vacuum expectation value (VEV), $\langle A_M\rangle = \delta^y_M\langle A_y\rangle$. On the other hand, the on-shell condition for the fermion implies $\Gamma^M\langle D_M\rangle\Psi = 0$ for the fermion without a bulk mass, which immediately tells us that the local operator describing the magnetic moment vanishes.

Our argument for the finiteness of the anomalous magnetic moment does not hold true provided the fermion has a gauge invariant bulk mass, since the chirality flip is realized by the insertion of the bulk mass, even if there is no insertion of the Higgs doublet. Though such gauge invariant bulk mass is responsible for QED, in a realistic gauge-Higgs unification model incorporating the standard model, the zero-mode fermions (quarks and leptons) get their masses only through the VEV of the Higgs field. Namely, our results obtained in this paper remain correct in a realistic gauge-Higgs unification.

As for the brane localized operators in the orbifold case, we notice that the operator

$$i\bar{\Psi}\sigma_{MN} A_y \Gamma^y \Psi F^{MN} \quad (3)$$

seems to be allowed on the branes. One might worry that the divergences localized on the branes will appear. However, it is known that the shift symmetry $A_y \to A_y + \text{const}$ [11], which is a remnant of the higher dimensional gauge symmetry, is operative even at branes. Therefore, the brane localized operator (3) is forbidden by this shift symmetry.

If this is true, this is a very remarkable result since there is a possibility that the anomalous magnetic moment is finite in *any* spacetime dimensions similar to the Higgs mass, in spite of the fact that the higher dimensional gauge theories are argued to be nonrenormalizable. As far as we know, the finite physical observable other than the Higgs mass in the gauge-Higgs unification is not known.

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Let us check the above expectation by calculating the 1-loop diagrams contributing to the anomalous magnetic moment in $(D+1)$ dimensional QED compactified on S^1 with the radius R. The action we consider is given by

$$S = \int d^D x dy \left[-\frac{1}{4} F_{MN} F^{MN} + \bar{\Psi} i \not{D}_{D+1} \Psi + \mathcal{L}_{\rm GF} \right], \quad (4)$$

where $\not{D}_{D+1} = \not{D} - i\Gamma_y D_y$, $\Gamma_y^2 = 1$, $D_M = \partial_M - igA_M (M = 0, 1, 2, 3, \cdots, D)$ is the covariant derivative. g is the $(D+1)$ dimensional gauge coupling constant. The coordinates of D dimensional spacetime and the circle are denoted as $x^\mu (\mu = 0, 1, 2, 3, \cdots, D-1)$ and y. We take the metric as $\eta_{MN} = \mathrm{diag}(+, -, \cdots, -)$. We choose the gauge fixing term as

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi} (\partial_\mu A^\mu + \xi \partial_y A^y)^2, \quad (5)$$

where ξ is a gauge parameter. Then, the gauge part of the action becomes

$$S_G = \int d^D x dy \frac{1}{2} [-(\partial_\mu A_\nu)^2 + (1 - \xi^{-1})(\partial_\nu A_\nu)^2 - (\partial_y A_\nu)^2 - (\partial_\mu A_y)^2 - \xi (\partial_y A_y)^2]. \quad (6)$$

Expanding the gauge field in terms of the Kaluza-Klein (KK) modes,

$$A_M(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} A_M^{(n)}(x^\mu) \exp\left(in \frac{y}{R} \right), \quad (7)$$

where $A_M^{(n)*} = A_M^{(-n)}$ and integrating out y coordinate, the D-dimensional action is written as

$$S_G = \int d^D x \sum_{n=-\infty}^{\infty} \frac{1}{2} [-|\partial_\mu A_\nu^{(n)}|^2 + (1 - \xi^{-1})|\partial_\nu A_\nu^{(n)}|^2 + M_n^2 |A_\nu^{(n)}|^2 + |\partial_\mu A_y^{(n)}|^2 - \xi M_n^2 |A_y^{(n)}|^2], \quad (8)$$

where $M_n = n/R$ is the KK mass. The order parameter α is defined as $\langle A_y \rangle \equiv -\alpha/(gR)$.[1] Hereafter, our calculation is done in the 't Hooft-Feynman gauge ($\xi = 1$).

Next, expanding the fermion in terms of the KK modes,

$$\Psi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \Psi^{(n)}(x^\mu) \exp\left(in \frac{y}{R} \right), \quad (9)$$

and integrating out y coordinate, the fermion part is written as[2]

$$S_m = \int d^D x \sum_{m,n} \bar{\Psi}^{(m)} \Big(i\delta_{nm}(\not{\partial} + iM_{n+\alpha}) + \sum_l \delta_{ml+n} (g_D \not{A}_\mu^{(l)} + g_D A_y^{(l)}) \Big) \Psi^{(n)} \quad (10)$$

where the D dimensional gauge coupling constant g_D is defined as $g_D = g/\sqrt{2\pi R}$. The zero-mode fermion is known to have a mass $M_\alpha = \alpha/R$. From (8) and (10), Feynman rules we need can be read off.

Now, we are ready to calculate the anomalous magnetic moment of the zero-mode fermion. The diagrams we should calculate are shown in Fig. 1 and are calculated as

$$(A) = \int \frac{d^D k}{(2\pi)^D i} \sum_{n=-\infty}^{\infty} g_D^3 \times \frac{\gamma_\nu (\not{p}' + \not{k} + M_{n+\alpha}) \gamma_\mu (\not{p} + \not{k} + M_{n+\alpha}) \gamma^\nu}{[(p'+k)^2 - M_{n+\alpha}^2][(p+k)^2 - M_{n+\alpha}^2](k^2 - M_n^2)}, \quad (11)$$

$$(B) = -\int \frac{d^D k}{(2\pi)^D i} \sum_{n=-\infty}^{\infty} g_D^3 \times \frac{(\not{p}' + \not{k} + M_{n+\alpha}) \gamma_\mu (\not{p} + \not{k} + M_{n+\alpha})}{[(p'+k)^2 - M_{n+\alpha}^2][(p+k)^2 - M_{n+\alpha}^2](k^2 - M_n^2)} \quad (12)$$

where $p_\mu (p'_\mu)$ is the external momentum of $\Psi (\bar{\Psi})$. Noting that the numerators can be cast into the forms consisting of γ_μ, $p_\mu + p'_\mu$ by use of the on-shell condition of Ψ and that the only $(p_\mu + p'_\mu)$ term contributes to the anomalous magnetic moment, we obtain the contribution of each diagram to the anomalous magnetic moment $a \equiv (g-2)/2$,

$$a(A) \equiv -\frac{2M_\alpha}{g_D}(A) = -4 g_D^2 M_\alpha \int \frac{d^D k}{(2\pi)^D i} \sum_{n=-\infty}^{\infty} \int_0^1 dx \int_0^{1-x} dy \times \frac{[2 - (D-2)(x+y)](x+y) M_\alpha + [4 - D(x+y)] M_n}{[k^2 - (M_n + (x+y) M_\alpha)^2]^3}, \quad (13)$$

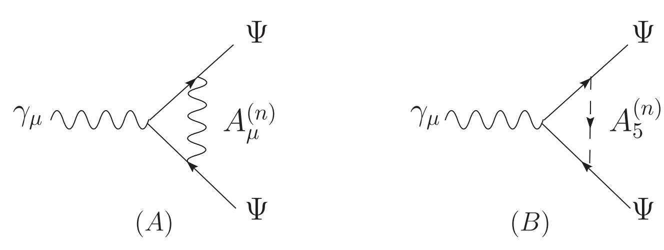

FIG. 1. The diagrams contributing to the anomalous magnetic moment in the $(D+1)$ dimensional QED on $M^D \times S^1$. The diagram (A)[(B)] is the photon [Higgs] KK mode exchange diagram, respectively.

[1] This sign definition is just a convention. This definition is useful because it is easy to check whether our results reproduce those of the standard model.

[2] In the case of odd D, $\Psi^{(n)}(x^\mu)$ in (10) represents two D dimensional spinors, simultaneously.

$$a(B) \equiv -\frac{2M_\alpha}{g_D}(B) = -4g_D^2 M_\alpha \int \frac{d^D k}{(2\pi)^D i} \sum_{n=-\infty}^{\infty} \int_0^1 dx \int_0^{1-x} dy \times \frac{(x+y)(M_n + (2-x-y)M_\alpha)}{[k^2 - (M_n + (x+y)M_\alpha)^2]^3} \tag{14}$$

where x, y are Feynman parameters and $g_D/(2M_\alpha)$ is a Bohr magneton. We note that the vertex under consideration are sandwiched between the wave functions of zero-mode fermion $\bar{\psi}^{(0)}(p')$ and $\psi^{(0)}(p)$. If $\not{p}(\not{p}')$ operates to $\psi^{(0)}(p)(\bar{\psi}^{(0)}(p'))$ from the left (right), it can be replaced by $-g\langle A_y \rangle = \alpha/R = M_\alpha$ because of the equations of motion. To obtain the final result, we make use of the symmetry under $x \leftrightarrow y$.

As a check, if we consider the case with $n = 0$, $D = 4$, we can confirm that $a(A)$ and $a(B)$ coincide with the results of the anomalous magnetic moment due to the photon and Higgs exchange diagrams in the standard model [12] provided the Higgs mass and Yukawa coupling are set to zero and g_D, respectively.

Let us first calculate $a(A)$ and $a(B)$ by taking the mode sum before the momentum integral. We use the following formula.

$$S_1(x,\alpha) \equiv \sum_{n=-\infty}^{\infty} \frac{1}{[x^2 + (\alpha + 2n\pi)^2]^3} = \frac{1}{16x}\left[\frac{x^2 \sinh x - 3(x\cosh x - \sinh x)}{x^4(\cosh x - \cos\alpha)} - \frac{3\sinh x(x\cosh x - \sinh x)}{x^3(\cosh x - \cos\alpha)^2} + \frac{2\sinh^3 x}{x^2(\cosh x - \cos\alpha)^3}\right] \tag{15}$$

$$\rightarrow \frac{3}{16x^5}(x \rightarrow \infty), \tag{16}$$

$$S_2(x,\alpha) \equiv \sum_{n=-\infty}^{\infty} \frac{2n\pi}{[x^2 + (\alpha + 2n\pi)^2]^3} = -\frac{(x\cosh x - \sinh x)\sin\alpha}{16x^3(\cosh x - \cos\alpha)^2} + \frac{\sinh^2 x \sin a}{8x^2(\cosh x - \cos\alpha)^3} - \alpha S_1(x,\alpha) \tag{17}$$

$$\rightarrow -\alpha S_1(\infty, \alpha)(x \rightarrow \infty). \tag{18}$$

From these observations, it is useful to separate these functions into the possibly divergent part $S^{(\mathrm{div})}$ and the superconvergent part $S^{(\mathrm{sc})}$ as

$$S_1^{(\mathrm{div})}(x,\alpha) = \frac{3}{16x^5}, \tag{19}$$

$$S_1^{(\mathrm{sc})}(x,\alpha) = \frac{1}{16x}\left[\frac{3}{x^4}\left(\frac{\sinh x}{(\cosh x - \cos\alpha)} - 1\right) - \frac{3\cosh x}{x^3(\cosh x - \cos\alpha)} - \frac{2\sinh x}{x^2(\cosh x - \cos\alpha)} - \frac{3\sinh x \cos\alpha}{x^2(\cosh x - \cos\alpha)^2} + \frac{3\sinh^2 x}{x^3(\cosh x - \cos\alpha)^2} + \frac{2\sinh^3 x}{x^2(\cosh x - \cos\alpha)^3}\right], \tag{20}$$

$$S_2^{(\mathrm{div})}(x,\alpha) = -\alpha S_1^{(\mathrm{div})}(x,\alpha), \tag{21}$$

$$S_2^{(\mathrm{sc})}(x,\alpha) = -\frac{(x\cosh x - \sinh x)\sin\alpha}{16x^3(\cosh x - \cos\alpha)^2} + \frac{\sinh^2 x \sin\alpha}{8x^2(\cosh x - \cos\alpha)^3} - \alpha S_1^{(\mathrm{cs})}(x,\alpha). \tag{22}$$

From these quantities, the possibly divergent part of (13) and (14) can be read as

$$a(A)_{\mathrm{div}} = 8g_D^2 M_\alpha \int \frac{d^D k_E}{(2\pi)^D} \int_0^1 dt (2\pi R)^5 (2\pi\alpha) \times t^2(-1+t) S_1^{(\mathrm{div})}(2\pi k_E R, 2\pi t\alpha), \tag{23}$$

$$a(B)_{\mathrm{div}} = 8g_D^2 M_\alpha \int \frac{d^D k_E}{(2\pi)^D} \int_0^1 dt (2\pi R)^5 (2\pi\alpha) \times t^2(1-t) S_1^{(\mathrm{div})}(2\pi k_E R, 2\pi t\alpha). \tag{24}$$

where $x + y \equiv t$ and k_E is an Euclidean momentum. Clearly, we immediately see $a(A)_{\mathrm{div}} + a(B)_{\mathrm{div}} = 0$. We found that the possibly divergent part is exactly canceled irrespective of the dimensionality D and a Feynman parameter t, as expected.

It is straightforward to obtain the explicit expression for the anomalous magnetic moment as the sum of remaining superconvergent parts as

$$a(A)_{\mathrm{sc}} + a(B)_{\mathrm{sc}} = 4g_D^2 M_\alpha \int \frac{d^D k_E}{(2\pi)^D} \int_0^1 dt (2\pi R)^5 t(4 - (D-1)t)\sin(2\pi t\alpha)\left[-\frac{1}{16(2\pi R k_E)^2[\cosh(2\pi R k_E) - \cos(2\pi t\alpha)]} - \frac{((2\pi R k_E)\cos(2\pi t\alpha) - \sinh(2\pi R k_E))}{16(2\pi R k_E)^3[\cosh(2\pi R k_E) - \cos(2\pi t\alpha)]^2} + \frac{\sinh^2(2\pi R k_E)}{8(2\pi R k_E)^2[\cosh(2\pi R k_E) - \cos(2\pi t\alpha)]^3}\right]. \tag{25}$$

Let us perform the momentum integral in (25). In order to do this, we need some formula in which the integrand is rewritten by the sum as

$$\frac{1}{x^2(\cosh x - \cos\alpha)} = \frac{2}{x^2}\sum_{n=1}^{\infty}\frac{\sin n\alpha}{\sin\alpha}e^{-nx}, \tag{26}$$

$$\frac{1}{x^2(\cosh x - \cos\alpha)^2} = -\frac{2}{x^2\sin\alpha}\sum_{n=1}^{\infty} \times \frac{n\cos n\alpha\sin\alpha - \sin n\alpha\cos\alpha}{\sin^2\alpha}e^{-nx}, \tag{27}$$

$$\frac{\sinh x}{x^3(\cosh x - \cos\alpha)^2} = \frac{2}{x^3}\sum_{n=1}^{\infty} n\frac{\sin n\alpha}{\sin\alpha}e^{-nx}, \tag{28}$$

$$\frac{\sinh^2 x}{x^2(\cosh x - \cos\alpha)^3} = \frac{1}{x^2}\sum_{n=1}^{\infty}\left[\frac{\sin n\alpha}{\sin^3\alpha} - \frac{n\cos n\alpha\cos\alpha}{\sin^2\alpha} + n^2\frac{\sin n\alpha}{\sin\alpha}\right]e^{-nx}. \tag{29}$$

Combining these results, we obtain the final expression for the anomalous magnetic moment,

$$a(A)_{\rm sc} + a(B)_{\rm sc} = \frac{g_D^2\Gamma(D-1)M_\alpha}{(2\pi R)^{D-5}(4\pi)^{D/2}\Gamma(\frac{D}{2})(D-3)} \times \int_0^1 dt t(4-(D-1)t) \times \sum_{n=1}^{\infty}\frac{\sin(2\pi n t\alpha)}{n^{D-4}} \tag{30}$$

where

$$\Gamma(z) = \int_0^\infty dt t^{z-1}e^{-t} \tag{31}$$

is used.

As a consistency check, let us evaluate the finite value of the anomalous magnetic moment by an alternative method, i.e. by doing the momentum integral before the mode sum invoking Poisson resummation,

$$\begin{aligned} a(A) &= 4g_D^2 M_\alpha\int_0^1 dt\int\frac{d^Dk}{(2\pi)^D}\sum_{n=-\infty}^{\infty}[t^2(2-(D-2)t)M_\alpha + t(4-Dt)M_n]\int_0^\infty ds\frac{s^2}{\Gamma(3)}e^{-[k^2+(M_n+tM_a)^2]s} \\ &= 2g_D^2M_\alpha\int_0^1 dt\int_0^\infty ds\frac{s^{2-(D/2)}}{(4\pi)^{D/2}} \times \sum_{n=-\infty}^{\infty}[t^2(-2+2t)M_aR\sqrt{\frac{\pi}{s}} + t(4-Dt)R^2\sqrt{\frac{\pi}{s^3}}(i\pi n)]e^{-((\pi Rn)^2/s)-2\pi int\alpha} \end{aligned} \tag{32}$$

where t is a Feynman parameter. In the first line, we used the integral expression for the Gamma function

$$\frac{1}{\Delta^s} = \int_0^\infty dt\frac{t^{s-1}}{\Gamma(s)}e^{-\Delta t}. \tag{33}$$

In the second line, the Gaussian integral for the momentum is performed and Poisson resummation formulas listed below are used, with the replacement $m \to n$ being done afterwards,

$$\sum_{n=-\infty}^{\infty}e^{-((n+t\alpha)/R)^2s} = \sum_{m=-\infty}^{\infty}R\sqrt{\frac{\pi}{s}}e^{-((\pi Rm)^2/s)-2\pi im\alpha t}, \tag{34}$$

$$\sum_{n=-\infty}^{\infty}\left(\frac{n+t\alpha}{R}\right)e^{-((n+t\alpha)/R)^2s} = \sum_{m=-\infty}^{\infty}R^2\sqrt{\frac{\pi}{s^3}}(i\pi m)e^{-((\pi Rm)^2/s)-2\pi im\alpha t}. \tag{35}$$

For $n = 0$ case in the second line of (32) ("zero-winding" sector [13]), the integral concerning s diverges at $s = 0$, which gives the divergent part

$$\tilde{a}(A)_{\rm div} = -\frac{g_D^2\alpha M_\alpha\sqrt{\pi}}{3(4\pi)^{D/2}}\int_0^\infty ds s^{(3-D)/2}. \tag{36}$$

The remaining finite part ($n \neq 0$) is

$$a(A)_{\rm finite} = \frac{2g_D^2\sqrt{\pi}M_\alpha}{(4\pi)^{D/2}}\int_0^1 dt\Gamma\left(\frac{D-5}{2}\right) \times \sum_{n=1}^{\infty}\left[4t^2(-1+t)\alpha\frac{\cos(2\pi nt\alpha)}{(\pi Rn)^{D-5}} + \left(\frac{D-5}{2}\right)2t(4-Dt)R\frac{\sin(2\pi nt\alpha)}{(\pi Rn)^{D-4}}\right]. \tag{37}$$

Similarly to the case (A), the corresponding divergent and finite parts of (B) are calculated to be

$$\tilde{a}(B)_{\rm div} = \frac{g_D^2\alpha M_\alpha\sqrt{\pi}}{3(4\pi)^{D/2}}\int_0^\infty ds s^{(3-D)/2} = -\tilde{a}(A)_{\rm div}, \tag{38}$$

$$a(B)_{\rm finite} = g_D^2\frac{2M_\alpha\sqrt{\pi}}{(4\pi)^{D/2}}\int_0^1 dt\Gamma\left(\frac{D-5}{2}\right)\sum_{n=1}^{\infty}\left[4t^2(1-t)\alpha \times \frac{\cos(2\pi nt\alpha)}{(\pi Rn)^{D-5}} + 2t^2R\frac{\frac{D-5}{2}\sin(2\pi nt\alpha)}{(\pi Rn)^{D-4}}\right] \tag{39}$$

where we can confirm that the divergences from (A) and (B) are exactly canceled, namely $\tilde{a}(A)_{\rm div} + \tilde{a}(B)_{\rm div} = 0$. Thus, we obtain the final result for the finite anomalous magnetic moment

$$\begin{aligned} a &= a(A)_{\rm finite} + a(B)_{\rm finite} \\ &= \frac{4g_D^2\Gamma(\frac{D-3}{2})M_\alpha}{(4\pi)^{D/2}\sqrt{\pi}(\pi R)^{D-5}}\int_0^1 dtt(4-(D-1)t) \\ &\quad \times \sum_{n=1}^{\infty}\frac{\sin(2\pi nt\alpha)}{n^{D-4}}. \end{aligned} \tag{40}$$

One can easily check $a(A)_{\rm finite} + a(B)_{\rm finite} = a(A)_{\rm sc} + a(B)_{\rm sc}$.

To see how the contribution of nonzero KK modes behaves, it will be useful to expand the final result (40) [or (30)] in terms of the ratio of the fermion mass to the compactification scale $M_\alpha/(R^{-1}) = \alpha$ under a plausible assumption $\alpha \ll 1$. We note that the sum over n in (40) is exactly given for $D = 4$ as

$$\sum_{n=1}^{\infty}\sin(2\pi nt\alpha) = \frac{1}{2}\cot(\pi t\alpha) \simeq \frac{1}{2\pi t\alpha} - \frac{\pi t\alpha}{6}. \tag{41}$$

Thus, for $D = 4$ (five-dimensional spacetime), the anomalous magnetic moment is given as

$$a(D=4) \simeq \frac{5g_4^2}{16\pi^2} - \frac{7g_4^2}{288}\alpha^2. \tag{42}$$

The first term corresponds to the zero-mode contribution, though it does not completely agree with the result in four dimensional QED, since in our case the contribution of the Higgs exchange diagram is comparable to that of the photon exchange diagram. The second term of $\mathcal{O}(\alpha^2) = \mathcal{O}(M_\alpha^2/R^{-2})$ is the contribution of nonzero KK modes, which is suppressed by the inverse power of $1/R$: the "decoupling" of massive KK modes.

In the case with spacetime dimension higher than five ($D > 4$), the separation of the contribution of nonzero KK modes is not straightforward. In fact, the leading order term in the power series expansion of α does not correspond to the four dimensional result. For instance, for $D = 5$ the leading term behaves as $\sim g_5^2 M_\alpha$, since

$$\sum_{n=1}^{\infty}\frac{\sin(2\pi nt\alpha)}{n} = \frac{\pi}{2}(1-2t\alpha). \tag{43}$$

The origin of this problem is that we are assuming only one space dimension is compactified on S^1, even for six dimensional spacetime ($D = 5$). To get a meaningful result, it will be necessary to work in a realistic situation where the extra space is, say, T^2 with KK modes (n, n'), and the four dimensional contribution can be extracted as the contribution of (0, 0) sector. However, we leave this issue for a future publication, since our main purpose in this article is to show the finiteness of the anomalous magnetic moment, not its precise value.

In summary, we have shown that the anomalous magnetic moment is UV finite and predictive in a $(D+1)$ dimensional QED gauge-Higgs unification model compactified on S^1. This result is naturally expected because the local gauge invariant operator relevant for the anomalous magnetic moment is forbidden by the higher dimensional gauge invariance, the Lorentz invariance and the on-shell condition. The finite value was derived by two independent methods, i.e., taking first the KK mode sum before the momentum integral and vice versa.

Some comments are in order. We have considered the case where only one of the spatial dimension is compactified. We note that the argument for UV finiteness discussed here is unchanged even if we consider a more realistic compactification though the finite value might be changed. This is because the compactification affects only the IR physics not the UV one.

The anomalous magnetic moment of the muon has been also calculated in the universal extra dimension (UED) scenario [14], where it is finite in five-dimensional spacetime, but divergent for more than five dimensions. This is the natural result from the argument of power counting. Thus, we can see that the predictions for the anomalous magnetic moment from the gauge-Higgs unification and UED scenarios are quite distinct.

The final comment is a concern about the electric dipole moment of a fermion in the gauge-Higgs unification. From the viewpoint of operator analysis, we notice that the only difference of the gauge invariant local operator relevant for the electric dipole moment from that for the magnetic moment is a Γ_y insertion. This immediately suggests that the gauge invariant operator for the electric dipole moment is also forbidden due to the higher dimensional gauge invariance and the on-shell condition, as discussed in this paper. Therefore, we can expect that the electric dipole moment in the gauge-Higgs unification also becomes finite in any spacetime dimensions if it ever exists.

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