Towards a realistic grand gauge-Higgs unification

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Abstract

We investigate a 5D $SU(6)$ grand gauge-Higgs unification model compactified on an orbifold S^1/Z_2. Ordinary quarks and leptons, together with right-handed neutrinos, are just accommodated into a minimal set of representations of the gauge group, without introducing any exotic states in the same representations. The proton decay turns out to be forbidden at least at the tree level. We also find a correct electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ is easily realized by introducing suitable number of adjoint fermions.

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The hierarchy problem, especially the problem of how to stabilize the Higgs mass under the quantum correction, has played a key role to motivate the physics beyond the standard model. Almost all possible scenarios to solve the problem invoke to some sort of symmetry in order to protect the Higgs mass at the quantum level. Supersymmetry is the most popular scenario and has been extensively discussed.

Recently the gauge-Higgs unification scenario [1, 2, 3] has obtained a revived interest as a possible new avenue to solve the problem [4]. In this scenario the Higgs is regarded as the extra space component of higher dimensional gauge fields and the Higgs mass is protected by higher dimensional gauge symmetry without relying on the supersymmetry. Rich structure of the theory and its phenomenology have been investigated [1]-[27].

Another possible interesting scenario is to regard the Higgs as a pseudo Nambu-Goldstone (PNG) boson due to the breakdown of some global symmetry. As far as the global symmetry is larger than local gauge symmetry, even though some N-G bosons are absorbed to gauge bosons via Higgs mechanism on the spontaneous symmetry breaking, there should remain some physical PNG bosons, which can be identified as the Higgs bosons [28]-[34]. The scenario faces a difficulty at quantum level, once gauge interactions are switched on. Namely, the Higgs mass suffers from a quadratic divergence, essentially because the original global symmetry is partly gauged and therefore the global symmetry is hardly broken by the gauge couplings. However, such difficulty may be avoided, once direct products of identical global symmetries are taken. It is interesting to note that such "dimensional deconstruction" or related "little Higgs" scenarios [35] can be regarded as a kind of gauge-Higgs unification, where the extra space has finite number of lattice points. In fact, it is a recent remarkable progress to have established the relation between a four dimensional theory with global symmetry G and a five dimensional gauge theory with gauge symmetry G, through the AdS/CFT correspondence (holographic approach) [36].

The gauge hierarchy problem was originally discussed in the framework of Grand Unified Theory (GUT) as the problem to keep the discrepancy between the GUT scale and the weak scale. So it will be meaningful to test the possible scenarios in the framework of GUT.

The PNG boson scenario in the framework of GUT was discussed long time ago [28]. Since the global symmetry needs to be larger than the gauge symmetry $SU(5)$, a minimal model with an $SU(6)$ global symmetry was proposed. As explained above, unfortunately the Higgs boson suffers from a quadratic divergence at quantum level, and SUSY was introduced to eliminate the unwanted divergence.

In this paper, in view of the recent progress mentioned above, we attempt to construct a GUT model based on the scenario of gauge-Higgs unification, say "grand gauge-Higgs

unification". A nice thing in our scenario is that the Higgs mass is automatically stabilized at the quantum level without relying on the SUSY.

What we adopt is a minimal grand gauge-Higgs unification model, i.e. 5-dimensional (5D) GUT with an $SU(6)$ gauge symmetry. It is interesting to note that an $SU(6)$ symmetry emerges again, as suggested by the AdS/CFT correspondence. This is because in the gauge-Higgs unification, the gauge symmetry needs to be enlarged from the minimal one $SU(5)$, since Higgs inevitably belongs to the adjoint representation of the gauge group while the Higgs should behave as the fundamental representation of $SU(5)$.

In addition to the finite Higgs mass, as a bonus, we find that the sector of fermionic zero-mode of the theory just accommodates three generations of quarks and leptons. Namely, we do not encounter the problem of introducing exotic particles in the representations quarks and leptons belong to, which often happens in the gauge-Higgs unification.

Another remarkable feature, we will see below, is that the dangerous proton decay due to the exchange of GUT particles turns out to be prohibited at least at the tree level without any symmetry. This is due to the splitting multiplet mechanism.

We will also discuss how desirable gauge symmetry breaking is realized via Hosotani mechanism [3]. We will see the desirable pattern of gauge symmetry breaking is realized without introducing additional scalar matter fields.

In ref. [37], an elegant 5D $SU(6)$ grand gauge-Higgs unification model was discussed, but as a SUSY theory. In this context, a non-SUSY $SU(6)$ grand gauge-Higgs unification model has been already studied [10], where the main focus was in the pattern of electroweak symmetry breaking and a viable Higgs mass satisfying the experimental lower limit was obtained.

The set-up of our model concerning gauge-Higgs sector just follows the model of [10] and [37]. The 5D space-time we consider has an extra space compactified on an orbifold S^1/Z_2 with a radius R, whose coordinate is y. On the fixed points $y = 0, \pi R$, the different Z_2 parities are assigned as

$$P = \text{diag}(+, +, +, +, +, -) \text{ at } y = 0, \quad P' = \text{diag}(+, +, -, -, -, -) \text{ at } y = \pi R, \qquad (1)$$

which implies the gauge symmetry breaking pattern

$$SU(6) \to SU(5) \times U(1) \text{ at } y = 0, \qquad (2)$$

$$SU(6) \to SU(2) \times SU(4) \times U(1) \text{ at } y = \pi R. \qquad (3)$$

in each fixed point. These symmetry breaking patterns are inspired by [10, 28, 37]. It is instructive to see the concrete parity assignments of each component of the 4D gauge

field A_μ and 4D scalar field A_5,

$$
A_\mu \;=\;
\left(
\begin{array}{cc|ccc|c}
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) \\
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) \\
\hline
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\
\hline
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+)
\end{array}
\right), \tag{4}
$$

$$
A_5 \;=\;
\left(
\begin{array}{cc|ccc|c}
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) \\
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) \\
\hline
(-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\
(-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\
(-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\
\hline
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-)
\end{array}
\right), \tag{5}
$$

where $(+,-)$ means that the Z_2 parity is even (odd) at $y = 0 (y = \pi R)$, for instance. Kaluza-Klein (KK) mode expansion in each type of the parity assignment is given by

$$
\Phi_{(+,+)}(x,y) \;=\; \frac{1}{\sqrt{2\pi R}}\left[\phi_{(+,+)}^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty}\phi_{(+,+)}^{(n)}(x)\cos\left(\frac{n}{R}y\right)\right], \tag{6}
$$

$$
\Phi_{(+,-)}(x,y) \;=\; \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi_{(+,-)}^{(n)}(x)\cos\left(\frac{n+\frac{1}{2}}{R}y\right), \tag{7}
$$

$$
\Phi_{(-,+)}(x,y) \;=\; \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi_{(-,+)}^{(n)}(x)\sin\left(\frac{n+\frac{1}{2}}{R}y\right), \tag{8}
$$

$$
\Phi_{(-,-)}(x,y) \;=\; \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi_{(-,-)}^{(n)}(x)\sin\left(\frac{n}{R}y\right). \tag{9}
$$

Noting that the 4D massless KK zero mode appears only in the $(+,+)$ component, the gauge symmetry breaking by orbifolding is found (see A_μ parity assignment) to be

$$
SU(6) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X. \tag{10}
$$

Here the hypercharge $U(1)_Y$ is contained in the upper-left 5×5 block of Georgi-Glashow $SU(5)$. Therefore, we obtain

$$
g_3 = g_2 = \sqrt{\frac{5}{3}}g_Y, \tag{11}
$$

at the unification scale, which will be not far from $1/R$. This means that the Weinberg angle is just the same as the Georgi-Glashow $SU(5)$ GUT, namely $\sin^2\theta_W = 3/8$ (θ_W: Weinberg angle) at the classical level. In fact, we can explicitly confirm it for a $\mathbf{6}^*$ representation given in (14) below,

$$
\sin^2\theta_W = \frac{\mathrm{Tr}I_3^2}{\mathrm{Tr}Q^2} = \frac{(\frac{1}{2})^2 + (-\frac{1}{2})^2}{(\frac{1}{3})^2 \times 3 + (-1)^2} = \frac{3}{8} \tag{12}
$$

where I_3 is the third component of $SU(2)_L$ and Q is an electric charge. In order to compare with the experimental data, the gauge coupling running effects have to be taken into account. Such a study is beyond the scope of this paper. Since Higgs belongs to the doublet of $SU(2)_L$ and the electroweak gauge symmetry is embedded into ordinary $SU(5)$, the Z boson mass is given as

$$M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W} = \sqrt{\frac{8}{5}} M_W \simeq 102 \text{GeV} \tag{13}$$

at the classical level.

On the other hand, concerning A_5, the zero mode appears only in the doublet component, as we wish. The colored Higgs has a mass at least of the order of $1/R$, and the doublet-triplet splitting is realized [38].

Let us now turn to the non-trivial question of how quarks and leptons can be accommodated into the representations of $SU(6)$, without introducing exotic states in the zero mode sector. Key observation is that the fundamental representation **6** of $SU(6)$ contains a doublet of $SU(2)_L$ and symmetric products of **6** easily introduces a triplet of $SU(2)_L$, which is exotic. We therefore focus on the possibility of totally antisymmetric tensor representations of $SU(6)$. We find that the minimal set to accommodate one generation of quarks and leptons contains two $\mathbf{6^*}$ and one **20** representations. Their parity assignments are fixed according to (1),

$$\mathbf{6^*} = \begin{cases} \mathbf{6^*}_L = \underbrace{(\mathbf{3^*}, \mathbf{1})^{(+,-)}_{(1/3,-1)} \oplus l_L(\mathbf{1}, \mathbf{2})^{(+,+)}_{(-1/2,-1)}}_{\mathbf{5^*}} \oplus \underbrace{(\mathbf{1},\mathbf{1})^{(-,-)}_{(0,5)}}_{\mathbf{1}} \\ \mathbf{6^*}_R = \underbrace{(\mathbf{3^*}, \mathbf{1})^{(-,+)}_{(1/3,-1)} \oplus (\mathbf{1}, \mathbf{2})^{(-,-)}_{(-1/2,-1)}}_{\mathbf{5^*}} \oplus \underbrace{\nu_R(\mathbf{1},\mathbf{1})^{(+,+)}_{(0,5)}}_{\mathbf{1}} \end{cases} \tag{14}$$

$$\mathbf{6^*} = \begin{cases} \mathbf{6^*}_L = \underbrace{(\mathbf{3^*}, \mathbf{1})^{(-,-)}_{(1/3,-1)} \oplus (\mathbf{1}, \mathbf{2})^{(-,+)}_{(-1/2,-1)}}_{\mathbf{5^*}} \oplus \underbrace{(\mathbf{1},\mathbf{1})^{(+,-)}_{(0,5)}}_{\mathbf{1}} \\ \mathbf{6^*}_R = \underbrace{d_R^*(\mathbf{3^*}, \mathbf{1})^{(+,+)}_{(1/3,-1)} \oplus (\mathbf{1}, \mathbf{2})^{(+,-)}_{(-1/2,-1)}}_{\mathbf{5^*}} \oplus \underbrace{(\mathbf{1},\mathbf{1})^{(-,+)}_{(0,5)}}_{\mathbf{1}} \end{cases} \tag{15}$$

$$\mathbf{20} = \begin{cases} \mathbf{20}_L = \underbrace{q_L(\mathbf{3}, \mathbf{2})^{(+,+)}_{(1/6,-3)} \oplus (\mathbf{3^*}, \mathbf{1})^{(+,-)}_{(-2/3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(+,-)}_{(1,-3)}}_{\mathbf{10}} \\ \quad \oplus \underbrace{(\mathbf{3^*}, \mathbf{2})^{(-,+)}_{-1/6,3} \oplus (\mathbf{3}, \mathbf{1})^{(-,-)}_{(2/3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,-)}_{(-1,3)}}_{\mathbf{10^*}} \\ \mathbf{20}_R = \underbrace{(\mathbf{3}, \mathbf{2})^{(-,-)}_{(1/6,-3)} \oplus (\mathbf{3^*}, \mathbf{1})^{(-,+)}_{(-2/3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,+)}_{(1,-3)}}_{\mathbf{10}} \\ \quad \oplus \underbrace{(\mathbf{3^*}, \mathbf{2})^{(+,-)}_{-1/6,3} \oplus u_R(\mathbf{3}, \mathbf{1})^{(+,+)}_{(2/3,-3)} \oplus e_R(\mathbf{1},\mathbf{1})^{(+,+)}_{(-1,3)}}_{\mathbf{10^*}} \end{cases} \tag{16}$$

where the numbers written by the bold face in the parenthesis are the representations under $SU(3)_C \times SU(2)_L$. The numbers written in the subscript denote the charges under

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$U(1)_Y \times U(1)_X$. $L(R)$ means the left(right)-handed chirality. Note that the difference between the first and the second $\mathbf{6}^*$ representations lies in the relative sign of the parity at $y = 0$. The corresponding representations of $SU(5)$ are also displayed. It is interesting that the charged lepton doublet l_L and the right-handed down quark singlet d_R^* are separately embedded in different $\mathbf{5}^*$ representations. Similarly, the quark doublet q_L and the right-handed up quark u_R, electron e_R are separately embedded in different $\mathbf{10}$ representations.

A remarkable fact is that one generation of quarks and leptons (including ν_{eR}) is elegantly embedded as the zero modes of the minimal representations, without introducing any exotic particles. Since the zero mode sector is nothing but the matter content of the standard model (including ν_{eR}), we have no 4D gauge anomalies with respect to the standard model gauge group. As the wave functions of zero modes are y-independent and the non-zero KK modes are vector-like, there is no anomalies even in the 5D sense. As for the remaining $U(1)_X$, we can easily see that the symmetry is anomalous and is broken at the quantum level. Thus its gauge boson should become heavy and is expected to be decoupled from the low energy sector of the theory [8].

Next, let us study whether we can obtain the correct pattern of electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$. We will see below that for such purpose the minimal set of matter fields is not sufficient and we need to introduce several massless fermions belonging to the adjoint representation of $SU(6)$.

One-loop induced Higgs (A_5) potential due to the matter fields $N_{\mathrm{ad}} \times \mathbf{35} \oplus 3 \times (2 \times \mathbf{6}^* \oplus \mathbf{20})$ (N_{ad} and 3 denote the number of adjoint fermions and 3 generations, respectively) is calculated as[3]

$$
\begin{aligned}
V(\alpha) \;=\; C &\left[(4N_{\mathrm{ad}} - 3) \sum_{n=1}^{\infty} \frac{1}{n^5} \left\{ \cos(2\pi n\alpha) + 2\cos(\pi n\alpha) + 6(-1)^n \cos(\pi n\alpha) \right\} \right. \\
&\left. + 48 \sum_{n=1}^{\infty} \frac{1 + (-1)^n}{n^5} \cos(\pi n\alpha) \right]
\end{aligned}
\tag{17}
$$

where $C \equiv \frac{3}{128\pi^7 R^5}$. The dimensionless parameter α is defined by $\langle A_5 \rangle \equiv \frac{\alpha}{gR} \frac{\lambda_{27}}{2}$ where λ_{27} is the twenty seventh generator of $SU(6)$ possessing the values in the (2, 6) component of 6×6 matrix. As can be seen in Fig. 1, if we have no adjoint fermion $N_{\mathrm{ad}} = 0$, the potential is minimized at $\alpha = 1$ where the desired electroweak symmetry breaking is not realized, namely $SU(2) \times U(1) \to U(1) \times U(1)$. On adding the adjoint fermions, we can

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[3]The difference between the one-loop effective Higgs potential of [10] and ours lies in the matter content. In [10], complex scalars and fermions in the adjoint and fundamental representation are considered. In our case, the fermions in the adjoint, fundamental and third-rank antisymmetric representations are considered, but scalars are not. More concretely, as can be seen from the second term of the potential in (17), the contributions of fermions of both $\mathbf{6}^*$ and $\mathbf{20}$ with the periodic and the antiperiodic boundary conditions are equally included in our matter content, which is not necessary the feature of [10].
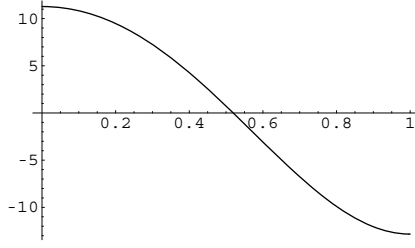
Figure 1: One-loop Higgs potential with no adjoint fermion, massless fermions of $3 \times (\mathbf{6}^* + \mathbf{6}^*)$ and $3 \times \mathbf{20}$. The potential minimum is located at $\alpha = 1$.
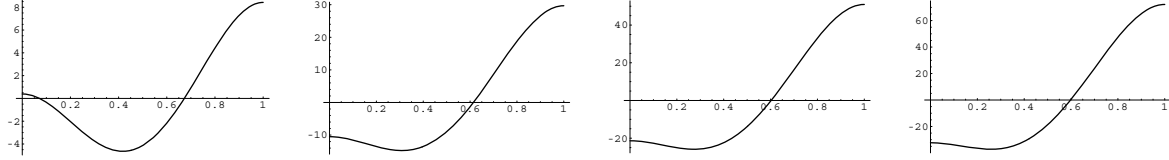

Figure 2: One-loop Higgs potential with massless fermions of several adjoints, $3 \times (\mathbf{6}^* + \mathbf{6}^* + \mathbf{20})$ representations. The plots from the left to the right correspond to the cases with one to four adjoint fermions. Their minimum is located at $\alpha = 0.417571, 0.307592, 0.27334, 0.256505$, respectively.

see from Fig. 2 that the nontrivial minimum appears in the range $0 < \alpha < 1$ where the desired electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ is realized. Note that the value of α at the minimum tends to become smaller, as the number of the adjoint fermions is larger. This feature is useful in order to make a Higgs mass heavy. Higgs mass can be obtained from the second derivative of the potential as

$$
\begin{aligned}
m_H^2 &= g^2 R^2 \frac{d^2 V(\alpha)}{d\alpha^2}\bigg|_{\alpha=\alpha_0} \\
&= -\frac{3 g_4^2 M_W^2}{32 \pi^4 \alpha^2} \Bigg[(4 N_{\mathrm{ad}} - 3) \sum_{n=1}^{\infty} \frac{1}{n^3} \left\{ 2\cos(2\pi n \alpha) + \cos(\pi n \alpha) + 3(-1)^n \cos(\pi n \alpha) \right\} \\
&\quad + 24 \sum_{n=1}^{\infty} \frac{1 + (-1)^n}{n^3} \cos(\pi n \alpha) \Bigg]\Bigg|_{\alpha=\alpha_0}
\end{aligned}
\tag{18}
$$

where α_0 denotes the value of α at the minimum of the potential. The relation derived from the gauge-Higgs unification $M_W = \alpha_0/R$ is used in the last expression. The gauge coupling in four dimensions g_4 is related to the gauge coupling in five dimensions g through $g_4^2 = \frac{g^2}{2\pi R}$. The Higgs masses for several choices of N_{ad} are numerically calculated and tabulated in the table below.

Adj No.	α_0	Higgs mass
20	0.216557	113.9 g_4 GeV
21	0.216083	116.9 g_4 GeV

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We can obtain a viable Higgs mass satisfying the experimental lower bound if more than 20 adjoint fermions are introduced (g_4 is assumed to be $\mathcal{O}(1)$). Here we note that this result is just an existence proof not a realistic example for getting a relatively heavy Higgs mass. In our results, the compactification scale is a little bit low, $1/R = M_W/\alpha_0 \simeq 370$ GeV, which contradics with the current experimental data. Therefore, we need further investigations for obtaining a viable Higgs mass with more realsitic situation. As one of the possibilities, it would be interesting to analyze the Higgs potential with only three pairs of fermions in the representations ($\mathbf{6^*} + \mathbf{6^*} + \mathbf{20}$) on the warped space since it has been suggested that the Higgs mass on the warped space is enhanced comparing to the case of flat space [17].

The extension to the case of massive fermion is straightforward. We can incorporate a Z_2-odd bulk mass of the type $M\epsilon(y)$ ($\epsilon(y)$: sign function) for fermions. The Higgs potential is then given by [39]

$$
\begin{aligned}
V(\alpha) \;=\; & C\Bigg[(4N_{\mathrm{ad}} - 3)\sum_{n=1}^{\infty}\frac{1}{n^5}\left(1 + nz + \frac{1}{3}n^2 z^2\right)e^{-nz} \\
& \times \left\{\cos(2\pi n\alpha) + 2\cos(\pi n\alpha) + 6(-1)^n\cos(\pi n\alpha)\right\} \\
& + 48\sum_{n=1}^{\infty}\left(1 + nz + \frac{1}{3}n^2 z^2\right)e^{-nz}\frac{1 + (-1)^n}{n^5}\cos(\pi n\alpha)\Bigg]
\end{aligned}
\tag{19}
$$

where $z \equiv 2\pi RM$ and the bulk masses of fermions are taken to be a common value M for simplicity. We will skip all the detail of the analysis by use of this potential, except reporting that there do not appear any drastic qualitative and quantitative change from the case of $M = 0$.

The relation $M_W = \alpha_0/R$ tells us that the compactification scale $1/R$ is not so far from the weak scale M_W, and therefore the GUT scale also cannot be extremely greater than the weak scale M_W (Above the compactification scale, a power-law running of gauge couplings is expected [40]). Thus we have to worry about possible too rapid proton decay. Interestingly, such baryon number violating amplitude concerning KK zero modes turns out to be forbidden at least at the tree level. This is essentially because the quarks and leptons are separated into different representations in our model although they are accommodated in the same representation in ordinary $SU(5)$ GUT. From (16) we learn the type of baryon number (and lepton number) changing vertices of A_μ and A_5 is limited. Namely concerning fermionic zero-modes, only possibility is $u_R \leftrightarrow e_R$ due to "colored" 4D gauge boson A_μ with $(SU(3),\ SU(2))$ quantum number $(\mathbf{3}, \mathbf{1})$ and $q_L \leftrightarrow e_R$ due to "lepto-quark" 4D scalar A_5 with $(\mathbf{3}, \mathbf{2})$. Let us note these relevant A_μ and A_5 are "bosonic partners" of colored Higgs and X, Y gauge bosons in ordinary $SU(5)$ GUT. Though these bosons couple to baryon number changing currents, these interactions do not lead to net baryon number violation, since each of gauge or Higgs boson couples to unique baryon

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number violating current. In the diagrams where these bosons are exchanged, one vertex with $\Delta B = 1/3$ and another vertex with $\Delta B = -1/3$ which is the Hermitian conjugate of the other necessarily appear, thus leading to no net baryon number violation. In other words, we can assign definite baryon (and lepton) number to each boson, and in such a sense baryon number is preserved at each vertex. It will be definitely necessary to consider whether such mechanism to preserve net baryon number is also operative at loop diagrams, though it is not discussed here.

In summary, we have investigated a 5D $SU(6)$ grand gauge-Higgs unification model compactified on an orbifold S^1/Z_2, with a realistic matter content. Three generation of quarks and leptons with additional right-handed neutrinos are just embedded as the zero modes of the minimal set of representations, $3 \times (2 \times \mathbf{6}^* + \mathbf{20})$, without introducing any exotic particles in the same representations. As a remarkable feature of the model, we have found the dangerous proton decay is forbidden at the tree level. We have also found that the desired pattern of electroweak symmetry breaking is dynamically realized by introducing suitable number of adjoint fermions. Higgs mass was also calculated and shown to become heavy, if certain number of the adjoint fermions are introduced. Searching for a simpler matter content yielding a reasonable Higgs mass is desirable and a very nontrivial task. This is left for a future work.

There are still many issues to be studied. The construction of the realistic hierarchy of Yukawa couplings is a fundamental problem in the gauge-Higgs unification since Yukawa coupling is naively the gauge coupling which is flavor independent. One of the promising proposals to avoid the problem [7] is that the mixing between the bulk massive fermions and the brane localized quarks and leptons generates non-local Yukawa couplings after integrating out the bulk massive fermions. The huge hierarchy is then realized by an order one tuning of the bulk mass. It is very important to study whether this proposal can be incorporated into the present model. To study the energy evolution of gauge couplings and their unification is another important issue. These issues will be discussed elsewhere.

Acknowledgments

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