Kobe University Repository : Kernel

PDF issue: 2025-12-04

Power comparison of non-parametric tests: Small-sample properties from Monte Carlo experiments

Tanizaki, Hisashi

(Citation)
Journal of Applied Statistics,24(5):603-632

(Issue Date)
1997-10

(Resource Type)
journal article

(Version)
Accepted Manuscript

(URL)
https://hdl.handle.net/20.500.14094/90000570

POWER COMPARISON OF NONPARAMETRIC TESTS

— Small Sample Properties from Monte-Carlo Experiments —

Hisashi Tanizaki
Kobe University
Faculty of Economics
Nadaku, Kobe 657, JAPAN

Abstract: Nonparametric tests dealing with two samples includes scores tests (e.g., Wilcoxon rank sum test, normal scores test, logistic scores test, Cauchy scores test and so on) and Fisher's randomization test. Since in general the nonparametric tests require a large amount of computational burden, there are few studies on small sample properties although asymptotic properties from various aspects were studied in the past. In this paper, the nonparametric tests are compared with t-test through Monte-Carlo experiments. Aslo, we consider testing structural changes as an application to economics.

Key Words: t-Test, Nonparametric Test, Scores Test, Normal Scores Test, Logistic Scores Test, Cauchy Scores Test, Wilcoxon Rank Sum Test, Fisher Two Sample Test

1 INTRODUCTION

There are many kinds of nonparametric tests (distribution-free tests), i.e., scores tests, Fisher's test and so on. However, almost all the studies in the past are related to asymptotic properties. In this paper, we examine small sample properties of nonparametric two sample tests by Monte-Carlo experiments.

One of the features of nonparametric tests is that we do not have to impose any assumption on the underlying distribution. From no restriction on the distribution, it can be expected that nonparametric tests are less powerful than the conventional parametric tests such as t-test. However, Hodges and Lehman (1956) and Chernoff and Savage (1958) showed that Wilcoxon rank sum test is as powerful as t-test under the location-shift alternatives and moreover that Wilcoxon test is sometimes much more powerful than t-test. Especially, the remarkable fact about the Wilcoxon test is that it is about 95 per cent as powerful as the usual t-test for normal data. Chernoff and Savage (1958) proved that Pitman's asymptotic relative efficiency of the normal scores test relative to t-test is greater than one under the location-shift alternatives.[1] This implies that the power of normal scores test is always larger than that of t-test. According to Mehta and Patel (1992), normal scores test is less powerful than Wilcoxon test if the tails of the underlying distributions are diffuse.

Fisher test statistic is difference between two sample means, and therefore it is asymptotically equivalent to t-test (Bradley (1968)).[2] The scores tests are similar to the Fisher test except that the test statistic of the scores test statistics are sum of scores while Fisher test statistic is difference between two sample means. Both test statistics are discretely distributed and we have to obtain all the possible combinations for test.

It is quite difficult to obtain all the possible combinations and computaional time is also quite large. Mehta and Patel (1983, 1986a, 1986b), Mehta, Patel and Tsiatis (1984), Mehta, Patel and Gray (1985), Mehta, Patel and Wei (1988) made a program on Fisher permutation test (a generalization of the Fisher two-sample test treated in this paper, i.e., independence test by $r \times c$ contingency table) using a network algorithm.[3]

In this paper, we consider small sample properties of two-sample nonparametric tests (i.e., the scores tests and the Fisher test) by comparing with t-test which is the usual parametric test. Finally, testing structural change is examined as an application to economics.

2 OVERVIEW OF NONPARAMETRIC TESTS

It is well known for testing two-sample means that t-test gives us a uniform powerful test under normality assumption but not under non-normality. We consider distribution-free test in this paper, which is also called nonparametric test. Normal scores test, Wilcoxon (1945) rank sum test, and Fisher (1935) test are famous nonparametric tests, which are similar tests. We have two sample groupes. We test if two samples are generated from the same distribution. Let $x_1, x_2, \cdots, x_{n1}$ be mutually independently distributed as $F(x)$, and $y_1, \cdots, y_{n2}$ be mutually independently distributed as $G(x)$. $F(x)$ and $G(x)$ are continuous distribution functions. Under the assumptions, we consider the null hypothesis of no difference between two sample means. The null hypothesis H_0 is represented by

$$H_0 \ : \ F(x) = G(x).$$

Both the scores tests and the Fisher test are usually applied under the alternative of location shift.[4] One possible alternative hypothesis H_1 is given by

$$H_1 \ : \ F(x) = G(x - \mu), \qquad \mu > 0,$$

where a shift of the location parameter μ is tested.

Let $n1$ be the sample size of Group 1 and $n2$ be that of Group 2. We consider randomly taking $n1$ samples out of $n1 + n2$ samples, mixing two groups. Then, we have ${}_{n1+n2}C_{n1}$ combinations. Each event of ${}_{n1+n2}C_{n1}$ combinations occurs with equal probability $1/{}_{n1+n2}C_{n1}$. For both the scores tests and the Fisher test, all the possible combinations are compared with the original two samples.

[1] Note that Pitman's asymptotic relative efficiency relative to t-test is defined as follows. Let $N_0 = n1_0 + n2_0$ be the sample size required to obtain the same power as t-test for the sample size $N = n1 + n2$. Then, the limit of N/N_0 is called Pitman's asymptotic relative efficiency (see, for example, Kendall and Stuart (1979)), where $n1/N = n1_0/N_0$ and $n2/N = n2_0/N_0$. If we take $\mu = \mu'/\sqrt{N}$, Pitman's asymptotic relative efficiency does not depend on μ'.

[2] Bradley (1968) showed that t-test does not depend on the functional form of the underlying distribution for large $n1$ if there exists the fourth moment, which implies that t test is asymptotically a nonparametric test.

[3] *StatXact* is a computer software on nonparametric inferrence, which computes the exact probability using a nonparametric test. There, the network algorithm that Mehta and Patel (1983, 1986a, 1986b), Mehta, Patel and Tsiatis (1984), Mehta, Patel and Gray (1985), Mehta, Patel and Wei (1988) developed is used for a permutation program.

[4] For the nonparametric test in the case where we test if the functional form of the two distributions is different, we have the runs test (Kendall and Stuart (1979)).

2.1 Scores Test

For the scores tests, the two samples $\{x_i\}_{i=1}^{n1}$ and $\{y_j\}_{j=1}^{n2}$ are converted into the data ranked by size. Let $\{Rx_i\}_{i=1}^{n1}$ and $\{Ry_j\}_{j=1}^{n2}$ be the ranked samples corresponding to $\{x_i\}_{i=1}^{n1}$ and $\{y_j\}_{j=1}^{n2}$. The scores test statistic s_0 is represented by

$$s_0 = \sum_{i=1}^{n1} a(Rx_i), \tag{1}$$

where $a(\cdot)$ is a function to be specified.

For all the possible combinations of taking $n1$ samples out of $n1 + n2$ samples (i.e., ${}_{n1+n2}C_{n1}$ combinations), we compute sum of the scores.

Let the scores sum be s_m, $m = 1, 2, \cdots, {}_{n1+n2}C_{n1}$.[5] s_m occurs with equal probability (i.e., $1/{}_{n1+n2}C_{n1}$) for all the combinations. Comparing s_0 and s_m, the following probabilities can be computed.

$$\begin{aligned}
\text{Prob}(s < s_0) &= \frac{\text{the number of combinations less than } s_0 \text{ out of } s_m,\ m = 1, 2, \cdots, {}_{n1+n2}C_{n1}}{{}_{n1+n2}C_{n1} \text{ (the number of all the possible combinations)}}, \\
\text{Prob}(s = s_0) &= \frac{\text{the number of combinations equal to } s_0 \text{ out of } s_m,\ m = 1, 2, \cdots, {}_{n1+n2}C_{n1}}{{}_{n1+n2}C_{n1} \text{ (the number of all the possible combinations)}}, \\
\text{Prob}(s > s_0) &= \frac{\text{the number of combinations greater than } s_0 \text{ out of } s_m,\ m = 1, 2, \cdots, {}_{n1+n2}C_{n1}}{{}_{n1+n2}C_{n1} \text{ (the number of all the possible combinations)}},
\end{aligned}$$

where s is taken as a random variable generated from the scores test statistic.

If $\text{Prob}(s < s_0)$ is small enough, s_0 is located at the right tail of the distribution, which implies $F(x) < G(x)$ for all x. Similarly, if $\text{Prob}(s > s_0)$ is small enough, s_0 is located at the left tail of the distribution, which implies $F(x) > G(x)$ for all x. Therefore, in the case of the null hypothesis $H_0 : \ F(x) = G(x)$ and the alternative $H_1 : \ F(x) \neq G(x)$, the null hypothesis is rejected at the 10% significance level when $\text{Prob}(s < s_0) \leq 0.05$ or $\text{Prob}(s > s_0) \leq 0.05$.

We can consider various scores tests by specifying the function $a(\cdot)$. The scores tests examined in this paper are Wilcoxon rank sum test, normal scores test, logistic scores test, and Cauchy scores test.

Wilcoxon Rank Sum Test: One of the most famous nonparametric tests is the Wilcoxon rank sum test. Wilcoxon test statistic w_0 is the scores test defined as $a(Rx_i) = Rx_i$, which is as follows:

$$w_0 = \sum_{i=1}^{n1} Rx_i. \tag{2}$$

In the past, it was too difficult to obtain the exact distribution of w, from computational point of view. Therefore, under the null hypothesis, we have tested utilizing the fact that w has approximately normal distribution with mean $E(w)$ and variance $\text{Var}(w)$:

$$\begin{aligned}
\text{E}(w) &= \frac{n1(n1 + n2 + 1)}{2}, \\
\text{Var}(w) &= \frac{n1\ n2(n1 + n2 + 1)}{12}.
\end{aligned}$$

Accordingly, in the past, the following statistic was used for the Wilcoxson test statistic.

$$aw_0 = \frac{w_0 - \text{E}(w)}{\sqrt{\text{Var}(w)}}. \tag{3}$$

which is called the asymptotic Wilcoxon test statistic in this paper. aw is asymptotically distributed as a standard normal random variable. Mann and Whitney (1947) demonstrated that the normal approximation is quite axccurate when $n1$ and $n2$ are larger than 7 (see Mood, Graybill and Boes (1974)).

Hodges and Lehman (1956) showed that Pitman's asymptotic relative efficiency of the Wilcoxon test relative to t-test is quite good. They obtained the result that the asymptotic relative efficiency is greater than 0.864 under the null hypothesis of location shift. This result implies that the Wilcoxon test is not too poor, compared with t-test,

[5] Note that at least one of s_m, $m = 1, 2, \cdots, {}_{n1+n2}C_{n1}$, is equal to s_0.

and moreover that the Wilcoxon test may be much better than t-test. Especially, they showed that the relative efficiency of the Wilcoxon test is 1.33 when the density function $f(x)$ takes the following [6]:

$$f(x) = \frac{x^2 \exp(-x)}{\Gamma(3)}, \tag{4}$$

where $\Gamma(3)$ is a gamma function with parameter 3. In general, for the distributions with large tails, the Wilcoxon test is more powerful than t-test.

All the past studies are concerned with asymptotic properties. In Section 3, we examine the small sample cases, i.e., $n1 = n2 = 6, 8, 10$.

Normal Scores Test: The normal scores test statistic ns_0 is

$$ns_0 = \sum_{i=1}^{n1} \Phi^{-1}\left(\frac{Rx_i}{n1+n2+1}\right), \tag{5}$$

where $\Phi(\cdot)$ is a standard normal distribution.

The scores test that $a(\cdot)$ in equation (1) is assumed to be $a(x) = \Phi^{-1}(x/n1 + n2 + 1)$ is called the normal scores test.[7]

Chernoff and Savage (1958) proved that the asymptotic relative efficiency of the normal scores test relative to t-test is greater than or equal to one, i.e., that the normal scores test is equivalent to t-test under normality assumption and the power of the normal scores test is greater than that of t-test otherwise.

Logistic Scores Test: The logistic scores test statistic ls_0 is given by

$$ls_0 = \sum_{i=1}^{n1} F^{-1}\left(\frac{Rx_i}{n1+n2+1}\right), \tag{6}$$

where $F(x) = \dfrac{1}{1+e^{-x}}$, which is a logistic distribution.

Cauchy Scores Test: The Cauchy scores test statistic cs_0 is represented as

$$cs_0 = \sum_{i=1}^{n1} F^{-1}\left(\frac{Rx_i}{n1+n2+1}\right), \tag{7}$$

where $F(x) = 1/2 + (1/\pi)\tan^{-1} x$, which is a Cauchy distribution.

By specifyng a functional form for $a(\cdot)$, various scores tests can be constructed. In this paper, the four scores tests discussed above and the Fisher test in the following section are compared.

2.2 Fisher's Two Sample Test

While the Wilcoxon test statistic is the rank sum of the two samples, the Fisher test statistic uses difference bwteen two sample means, i.e., $\overline{x} - \overline{y}$ for the two samples $\{x_i\}_{i=1}^{n1}$ and $\{y_j\}_{j=1}^{n2}$. Thus, the test statistic is given by

$$f_0 = \overline{x} - \overline{y}. \tag{8}$$

For all the possible combinations (i.e., ${}_{n1+n2}C_{n1}$ combinations taking $n1$ out of $n1+n2$), we compute difference between the sample means. Let f_m, $m = 1, 2, \cdots, {}_{n1+n2}C_{n1}$, be the difference between two samples means for all the possible combinations.[8] For all m, f_m occurs with equal probability (i.e., $1/{}_{n1+n2}C_{n1}$). Comparing f_0 and f_m, we can compute Prob($f < f_0$), Prob($f = f_0$) and Prob($f > f_0$), where f is a random variable generated from the Fisher test statistic.

Fisher's two sample test is the same type as the scores tests in the sense of use of all the possible combinations, but the Fisher test uses more information than the scores tests bacause the scores tests utilize the ranked data as the test statistics while the Fisher test uses the original data. It might be expected that the Fisher test is more

[6] Let z be a chi-square random variable with 6 degrees of freedom. Define $x = z/2$. Then, x is distributed as $f(x)$.

[7] We can interpret the Wilcoxon test as the scores test assumed to be a uniform distribution for the inverse function of $a(\cdot)$. That is, the scores test defined as $a(x) = x/n1 + n2 + 1$ is equivalent to the Wilcoxon test.

[8] Note that at least one out of f_m, $m = 1, 2, \cdots, {}_{n1+n2}C_{n1}$, is equal to f_0.

powerful than the scores tests. Moreover, Bradley (1968) stated that Fisher test and t-test are asymptotically equivalent because both of them use difference bwteen two sample means as the test statistic.[9] Therefore, it can be shown that the asymptotic relative efficiency of the Fisher test is sometimes better or worse, compared with the Wilxoson test.

3 POWER COMPARISON (SMALL SAMPLE PROPERTIES)

In Section 2, we have introduced the four scores tests and the Fisher test. In this section, we examine small sample properties by Monte-Carlo experiments. Assuming a specific distribution for Group 1 sample $\{x_i\}_{i=1}^{n1}$ and Group 2 sample $\{y_j\}_{j=1}^{n2}$ and generating random draws, we compare the nonparametric tests and t-test with respect to the sample power. The underlying distributions examined in this section are summarized in Table 1.

Table 1: Monte-Carlo Experiments I – V

	Underlying Distribution of $\{x_i\}$ and $\{y_j\}$	Table	Section
Monte-Carlo Experiment I	Normal Distribution	Table 2	Section 3.1
Monte-Carlo Experiment II	Uniform Distribution	Table 3	Section 3.2
Monte-Carlo Experiment III	Logistic Distribution	Table 4	Section 3.3
Monte-Carlo Experiment IV	Cauchy Distribution	Table 5	Section 3.4
Monte-Carlo Experiment V	$\chi^2(6)/2$ distribution	Table 6	Section 3.5

3.1 Monte-Carlo Experiment I: Normal Distribution

First, generate normal random draws as $x_i \sim N(0,1)$ for $i = 1, \cdots, n1$ and $y_j \sim N(\mu, \sigma^2)$ for $j = 1, \cdots, n2$. Then, t-test[10] is compared with the nonparametric tests introduced in this paper. The null hypothesis is $H_0 : F(x) = G(y)$. We compare the sample powers for a shift in not only location parameter (i.e., μ) but also scale parameter (i.e., σ). Therefore, Here the alternative hypothesis is given by $H_1 : F(x) = G(\frac{x-\mu}{\sigma})$. We perform Monte-Carlo experiments in the following cases: $n1 = n2 = 6,\ 8,\ 10$, $\mu = 0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$, and $\sigma = 1.0,\ 1.5,\ 2.0,\ 2.5$ for each of significance levels, i.e., $\alpha = 0.10, 0.05, 0.01$.

The results are in Table 2.[11] t, w, aw, ns, ls, cs and f represent t-test, Wilcoxon test, asymptotic Wilcoxon test, normal scores test, logistic scores test, and Cauchy scores test, reppectively.

[9] Hoeffding (1952) showed that the Fisher test is asymptotically as powerful as t-test even under normality assumption.

[10] t-test statistic is represented as

$$t_0 = \frac{\overline{x} - \overline{y}}{s\sqrt{1/n1 + 1/n2}},$$

where the degree of freedom is $n1 + n2 - 2$. $\overline{x}$, $\overline{y}$ and s are given by

$$\overline{x} = \sum_i x_i, \quad \overline{y} = \sum_j y_j, \quad s^2 = \frac{(n1-1)s_x^2 + (n2-1)s_y^2}{n1+n2-2},$$

$$s_x^2 = \frac{1}{n1-1}\sum_i (x_i - \overline{x})^2, \quad s_y^2 = \frac{1}{n2-1}\sum_j (y_j - \overline{y})^2$$

In the case of $\mathrm{Var}(x) = \mathrm{Var}(y)$, t-test provides a uniformly most powerful test. Otherwise, the test statistic t_0 does not follow a t distribution and therefore t-test would be meaningless.

[11] In Tables 2, note as follows.

(i) Perform 1000 simulation runs, i.e., $m = 1000$. Given the significance level $\alpha = 0.10, 0.05, 0.01$, each value in Table 2 is a ratio of rejection numbers to 1000 simulation runs, which represents the following probabilities: $\mathrm{Prob}(t < -t_0) \le \alpha$, $\mathrm{Prob}(w < w_0) \le \alpha$, $\mathrm{Prob}(aw < -aw_0) \le \alpha$, $\mathrm{Prob}(ns < ns_0) \le \alpha$, $\mathrm{Prob}(ls < ls_0) \le \alpha$, $\mathrm{Prob}(cs < cs_0) \le \alpha$ and $\mathrm{Prob}(f < f_0) \le \alpha$, where t_0, w_0, aw_0, ns_0, ls_0, cs_0 and f_0 are the statistics from the original data for each simulation run. For the t-distribution and the normal distribution, given the significance level, we have the following critical points (i.e., t_o and aw_0):

Theoretically, in the case of $\sigma = 1$, t-test is more powerful than any other test because of normality and equal variance, and in the case of $\sigma \neq 1$, generally the nonparametric tests are more powerful than t-test.[12]

In the case of $\sigma = 1$, in spite of a shift in the location parameter, there is no significant difference among t-test, Wilcoxon test and Fisher test. When the location parameter is small (e.g., $\mu = 0.5$), Wilcoxon test shows a larger power than Fisher test and t-test. For a large scale parameter, Wilcoxon test gives us the most powerful test when $\mu = 0$. Moreover, there is larger difference between t-test and Wilcoxon test when the sample size is small than when it is large.

Logistic scores test is as powerful as the other tests when $\sigma = 1$, but it is less powerful than t-test as μ is large and/or as σ increases. Logistic scores test approaches t-test in power as the sample size increases. Normal scores test has the almost same resuls as Logistic scores test, but the former is more powerful than the latter over all.

The Cauchy scores test is as powerful as t-test when $\sigma = 1$ and mu is small, but it is the wrorst test of six as σ is large and/or as μ is large.

Judging from Table 2, it might be concluded that t-test, Wilcoxon test and Fisher test are superior to normal scores test, logistic scores test and Cauchy scores test.

Fisher two sample test uses more information than Wilcoxon test in that the Fisher test utilizes original data while Wilcoxon test utilizes the ranked data. Accordingly, it is easily expected that in small sample Fisher test is more powerful than Wilcoxon test. However, the Monte-Carlo experiment shows that there is no significant difference between the two tests. Moreover, even when t-test is available, Wilcoson test has the same power as t-test as a whole. From the above results, the large sample properties studied by Hoeffding (1952), Hodges and Lehman (1956) and Bradley (1968) can be applied directly to the samll sample cases.

3.2 Monte-Carlo Experiment II: Uniform Distribution

Let x_i and y_j be uniform random numbers. For a large shift in location parameter (i.e., large μ), Cauchy scores test shows the most powerful test of the six especially when $\sigma = 1$.

3.3 Monte-Carlo Experiment III: Logistic Distribution

In Sections 3.3 – 3.5, we take two examples of the underlying distributions such that Wilcoxon test has a larger asymptotic relative efficiency than t-test, which are logistic distribution, Cauchy distribution and $\chi^2(6)/2$ distribution (i.e., equation (4)).

We examine the alternative hypothesis of $H_1 : F(x) = G(\frac{x-\mu}{\sigma})$, where we consider a shift in location parameter as well as that in scale parameter.

In the case of logistic distribution, it is known that the asymptotic relative efficiency of wilcoxon test to normal scores test is 1.047 (see Kendall and Stuart (1979)).

α		0.10	0.05	0.01
t_0	$n1 = n2 = 6$	1.3722	1.8125	2.7638
	$n1 = n2 = 8$	1.3450	1.7613	2.6245
	$n1 = n2 = 10$	1.3304	1.7341	2.5524
aw_0		1.2816	1.6449	2.3263

Let $\widehat{p}_t$, $\widehat{p}_w$, $\widehat{p}_{aw}$, $\widehat{p}_{ns}$, $\widehat{p}_{ls}$, $\widehat{p}_{cs}$ and $\widehat{p}_f$ be the ratios for each test, which indicate the probabilities which reject the null hypothesis under the alternative, i.e., the sample powers.

(ii) The estimated variance of each value in Table 2 is given by $\mathrm{Var}(\widehat{p}_k) = \dfrac{\widehat{p}_k(1-\widehat{p}_k)}{m}$ for $k = t,\ w,\ aw,\ ns,\ ls,\ cs,\ f$ and $m = 1000$.

(iii) $\circ$, $\circ\circ$, $\circ\circ\circ$, $\times$, $\times\times$ and $\times\times\times$ in Table 2 represent comparison with t-test. We put the superscript $\circ$ in the corresponding values when $\dfrac{\widehat{p}_k - \widehat{p}_t}{\sqrt{\mathrm{Var}(\widehat{p}_t)}}$, $k = w,\ aw,\ ns,\ ls,\ cs,\ f$, is greater than 1, $\circ\circ$ when it is greater than 2, and $\circ\circ\circ$ when it is greater than 3. The superscript $\times$ is put in the corresponding values if $\dfrac{\widehat{p}_k - \widehat{p}_t}{\sqrt{\mathrm{Var}(\widehat{p}_t)}}$, $k = w,\ aw,\ ns,\ ls,\ cs,\ f$, is less than -1, $\times\times$ if it is less than -2, and $\times\times\times$ if it is less than -3.

Therefore, the values with $\circ$ indicate more powerful test than t-test. The values with $\times$ represent less powerful test than t-test. Moreover, the number of $\circ$ or $\times$ shows degree of the sample power.

[12] However, since the studies by Hodges and Lehman (1956) and Chernoff and Savage (1958) can be applied to a shift in the location parameter, it is not appropriate to conclude that the nonparametric tests are more powerful than t-test in the case of $\sigma \neq 1$.

3.4 Monte-Carlo Experiment IV: Cauchy Distribution

When the underlying distribution is Cauchy, it is known that the asymptotic relative efficiency of Wilcoxon test to normal scores test is 1.413 (see Kendall and Stuart (1979)).

Both Wilcoxon test and Fisher test have much more power than t-test. As a whole, the order of large power is given by Wilcoxon test, Fisher test and t-test.

3.5 Monte-Carlo Experiment V: $\chi^2(6)/2$ Distribution

When the underlying distribution is given by equation (4), i.e., $\chi^2(6)/2$, the asymptotic relative efficiency of Wilcoxon test to t-test is 1.33. (see Hodges and Lehman (1956)).

For small μ there is no significant difference among all the tests, but for large μ Wilcoxon test gives us the largest power.

From the fact that Fisher test is as powerful as t-test, it is known that t-test is a nonparametric test in a sense. If the original distribution has the fourth moment, the significance point of t-test does not depend on a functional form when the sample size is large.

As the tails of distribution are large, the asymptotic property that Wilcoxon test has more asymptotic relative efficiency than t-test holds in the case of the small sample from the Monte-Carlo experiments in Tables 2 – 6.

Intuitively, it is expected from an amount of information set included in the test statistics that Fisher test performs better than Wilcoxon test in the sample power. However, judging from the results obtained in Tables 5 – 6, Wilcoxon test is more powerful. A reason of the results comes from the following three facts:

(i) Both t-test and Fisher test take difference between the two sample means as the test statistics.

(ii) Both Fisher test and Wilcoxon test are the nonparametric tests based on combination.

(iii) For a broad range of distribution, Wilcoxon test has more asymptotic relative efficiency than t-test.

From these three facts, we can consider that Fisher test is between t-test and Wilcoxon test in a sense of the sample power.

Accordingly, the order of the three sample powers is given by

$$\text{Wilcoxon} \geq \text{Fisher} \geq t, \qquad \text{(for a broad range of distribution)},$$

or

$$\text{Wilcoxon} \leq \text{Fisher} \leq t, \qquad \text{(otherwise)}.$$

The theorem proved by Chernoff and Savage (1958) that the asymptotic relative efficiency of normal scores test to t-test is more than one under the alternative hypothesis of shifting location parameter holds in the case of small sample. In the case of $\sigma = 1$ in Table 2, there is no significant difference between t-test and normal scores test. Thus, t-test and normal scores test are similar but the latter is slightly better than the former.

In Table 5, however, clearly normal scores test performs better than t-test in the sample power.

Generally, in the case of small sample, it might be concluded from Tables 2 – 6 that the order of size of the sample power is given by Wilcoxon test, normal scores test and Fisher test.

4 EXAMPLE: TESTING STRUCTURAL CHANGES

In a regression analysis, usually, the disturbance term is assumed to be normal and we perform testing a hypothesis. However, sometimes, the normality assumption is too strong. In this paper, loosening the normality assumption, we test a structural change without assuming any distribution for the distrurbance term. The testing procedure is as follows.

$$y_t = x_t\beta + u_t, \qquad t = 1, \cdots, T,$$

where y_t, x_t, β and u_t are a dependent variable of time t, a $1 \times k$ vector of independent variable of time t, a $k \times 1$ unknown parameter vector to be estimated, and the distrurbance term of time t with mean zero and variance σ^2, respectively. The sample size is T.

Let us define $X_{t-1} = (x_1' \ x_2' \ \cdots \ x_{t-1}')'$ and $Y_{t-1} = (y_1 \ y_2 \ \cdots \ y_{t-1})'$. β_{t-1} denotes the OLS estimate of β using the data up to time $t-1$, i.e., $\beta_{t-1} = (X_{t-1}'X_{t-1})^{-1}X_{t-1}'Y_{t-1}$. The predicted error (i.e., $\omega_t =$

$\dfrac{(y_t - x_t\beta_{t-1})}{\sqrt{1 + x_t(X'_{t-1}X_{t-1})^{-1}x'_t}}$ for $t = k+1, \cdots, T$) can be estimated by recursive ordinary least squares estimation, which is distributed with mean zero and variance σ^2. This predicted error is called the recursive residual. ω_t, $t = k+1, \cdots, T$, are mutually independently distributed and normalized to mean zero and variance σ^2.

Based on the recursive residual ω_t, we perform testing the structural change.[13] We can judge the structural change if the structure of the recursive residuals change in a period. Dividing the sample into two groups, We test if both $\{\omega_t\}_{t=k+1}^{n1}$ and $\{\omega_t\}_{t=n1+1}^{T}$ are generated from the same distribution, where $T = n1 + n2$. The null hypothesis is represented by $H_0 : F(\omega) = G(\omega)$ while the alternative is $H_1 : F(\omega) \neq G(\omega)$. Let $F(\cdot)$ be the distribution of the first $n1$ recursive residuals and $G(\cdot)$ be that of the last $n2$ recursive residuals.

We take an example of Japanese import function. Annual data from *Annual Report on National Accounts* (Economic Planning Agency, Government of Japan) is used. Let GDP_t be Gross Domestic Product (1985 price, billions of Japanese yen), M_t be Imports of Goods and Services (1985 price, billions of Japanese yen), and P_t be Terms of Trade Index, which is given by Imports of Goods and Services Implict Price deflator (1985=1.00) divided by Gross Domestic Product Implicit Price deflator (1985=1.00).

The following two import functions are estimated and the recursive residuals are computed.[14]

$$\log M_t = \beta_0 + \beta_1 \log GDP_t + \beta_2 \log P_t, \tag{7}$$

$$\log M_t = \beta_0 + \beta_1 \log GDP_t + \beta_2 \log P_t + \beta_3 \log M_{t-1}, \tag{8}$$

where β_0, β_1, β_2 and β_3 are the unkown parameters to be estimated.

For each regression equation, we compute the recursive residuals from 1961 $\sim$ 1991, divide the period into two groups, and test if the recursive residuals in the first period are the same as those in the last period. The results are in Tables 7 and 8. In the tables, t, Fisher, Wilcoson, Asy Wil, Normal, Logistic, Cauchy and Chow denote t-test, Fisher test, Wilcoxon test, asymptotic Wilcoxon test, normal scores test, logistic scores test, Cauchy scores test and stepwise Chow test. Moreover, each test statistic is given by t_0, f_0, w_0, aw_0, ns_0, ls_0, cs_0 and F_0. The probability less than the corresponding test statistics are p-val.

The results of Stepwise Chow test are also in Tables 7 and 8. The Stepwise Chow test statistic is denoted by F_0 and the p-value corresponding to F_0 in given by p-val.

From the import function (7) in Table 7, the structural change occurs during the period 1973 – 1984 for t-test, the period 1973 – 1983 for Fisher test, normal scores test and logistic scores test, the period 1973 – 1982 for Wilcoxon test and asymptotic Wilcoxon test, the period 1974 – 1984 for Cauchy scores test, and the period 1968 – 1985 for stepwise Chow test, where the significance level is 1%. It is concluded that the recursive residuals in the first period are larger than those in the last period.

From the import function (8) in Table 8, the structural change occurs during the period 1974 for normal scores test, logistic scores test and Cauchy scores test, and the period 1961 – 1976, 1979 for stepwise Chow test, where the significance level is 1%. For t-test, Fisher test, Wilcoxon tesr and asymptotic Wilcoxon test, we are unable to accept the fact that the structural change occurred during the period 1961 – 1992.

[13] Why do we use the recursive residual, not the conventional OLS residual? Note that the OLS residuals have the following problem. Let e_t be the residuals obtained from the regression equation $y_t = x_t\beta + u_t$, $t = 1, \cdots, T$, i.e., $e_t = y_t - x_t\widehat{\beta}$. The residuals e_t, $t = 1, \cdots, T$, are not mutually independently distributed. Clearly, we have $\mathrm{E}(e_s e_t) \neq 0$ for $s \neq t$. Therefore, we utilize the recursive residuals which are mutually independently distributed.

[14] The estimation results by OLS are as follows.

$$\log M_t = \underset{(0.48285)}{-\ 5.32481} + \underset{(0.04037)}{1.25813}\ \log GDP_t - \underset{(0.09436)}{0.11073}\ \log P_t, \tag{7}$$

$$R^2 = 0.98393, \quad \overline{R}^2 = 0.98296, \quad D.W. = 0.4555, \quad s.e. = 0.10501,$$

Estimation Period: 1958 $\sim$ 1993,

$$\log M_t = \underset{(0.83159)}{-\ 1.07250} + \underset{(0.15653)}{0.39726}\ \log GDP_t - \underset{(0.06756)}{0.15735}\ \log P_t + \underset{(0.11231)}{0.62666}\ \log M_{t-1}, \tag{8}$$

$$R^2 = 0.99253, \quad \overline{R}^2 = 0.99185, \quad D.W. = 2.0564, \quad s.e. = 0.07589,$$

Estimation Period: 1957 $\sim$ 1993,

where the values in the parentheses are the standard errors. R^2, $\overline{R}^2$, $D.W.$ and $s.e.$ are the coefficient of multiple determination, the adjusted R^2, Durbin=Watson statistic, and the standard error of the distrubance, respectively. For both equations (7) and (8), the recursive residuals are obtained from 1961 to 1993 (33 periods).

5 SUMMARY

Nonparametric test statistics have been approximated by a normal random variable from both computational and programming points of view. Recently, however, we can perform the exact test by progress of a computer. In this paper, we have compared the sample powers in small sample, taking nonparametric two sample tests, i.e, scores tests (Wilcoxon rank sum test, normal scores test, logistic scores test and Cauchy scores test) and Fisher test.

In the case where we compare t-test, Wilcoxon test and Fisher test, it might be intuitively expected that

(i) When the underlying distribution is normal, t-test gives us the most powerful test.

(ii) In the situations that we cannot use t-test, two nonparametric tests are more powerful than t-test, and moreover Fisher test is better than Wilcoxon test because the former utilizes more information than the latter. Accordingly, in the case where the underlying distribution is nonnormal or two samples are heteroskedastic, we might have the following relationship among the sample powers: $\widehat{p}_t \leq \widehat{p}_w \leq \widehat{p}_f$

However, the results of the Monte-Carlo simulations are: Wilcoxon test is as powerful as t-test even in the case where the two samples are identically and normally distributed. Moreover, when the underlying distribution is Cauchy, Wilcoxon test is much better than t-test. In general, we have $\widehat{p}_t \leq \widehat{p}_f \leq \widehat{p}_w$ when we cannot use t-test. The fact proved by Chernoff and Savage (1958), which is the theorem that under the alternative hypothesis of a shifting location parameter the asymptotic relative efficiency of normal scores relative to t-test is more than one, holds even in the small sample case. Generally, in the small sample case, it might be concluded from Tables 2 – 6 that we have $\widehat{p}_f \leq \widehat{p}_{ns} \leq \widehat{p}_w$.

Finally, we take an example of testing structural change as an application to the nonparametric tests.

Using the nonparametric tests, we can test the hypothesis in spite of the functional form of distribution of the disturbances.

REFERENCES

Bradley, J.V. (1968) *Distribution-Free Statistical Tests*, Englewood Cliffs, New Jersey: Prentice-Hall.

Chernoff, H. and I.R. Savage (1958) "Asymptotic Normality and Efficiency of Certain Nonparametric test Statistics," *Annals of Mathematical Statistics*, Vol.29, pp.972–994.

Fisher, R.A. (1935) *The Design of Experiments* (eighth edition, 1966), New York: Hafner.

Harbison, S.P. and G.L. Steele Jr. (1987) *C: A Reference Manual* (second edition), Prentice-Hall.

Hodges, J.L. and E.L. Lehman (1956) "The Efficiency of Some Nonparametric Competitors of the t Test," *Annals of Mathematical Statistics*, Vol.27, pp.324–335.

Hoeffding, W. (1952) "The Large Sample Power of Tests Based on Permutations of Observations," *Annals of Mathematical Statistics*, Vol.23, pp.169–192.

Kendall, M. and A. Stuart (1979) *The Advanced Theory of Statistics, Vol.2, Inference and Relationship* (fourth edition), Charles Griffin & Company Limited.

Mann, H.B. and D.R. Whitney (1947) "On a Test of Whether One of Two Random Variables Is Stochastically Larger Than the Other," *Annals of Mathematical Statistics*, Vol.18, pp.50 – 60.

Mehta, C.R. and N.R. Patel (1983) "A Network Algorithm for Performing Fisher's Exact Test in $r \times c$ Contingency Tables," *Journal of the American Statistical Association*, Vol.78, No.382, pp.427–434.

Mehta, C.R., N.R. Patel and A.A. Tsiatis (1984) "Exact Significance Testing to Establish Treatment Equivalence for Ordered Categorical Data," *Biometrica*, Vol.40, pp.819–825.

Mehta, C.R., N.R. Patel and R. Gray (1985) "On Computing an Exact Confidence Interval for the Common Odds Ratio in Several 2×2 Contingency Tables," *Journal of the American Statistical Association*, Vol.80, No.392, pp.969–973.

Mehta, C.R. and N.R. Patel (1986a) "A Hybrid Algorithm for Fisher's Exact Test in Unordered $r \times c$ Contingency Tables," *Communications in Statistics*, Vol.15, No.2, pp.387–403.

Mehta, C.R. and N.R. Patel (1986b) "FEXACT: A Fortran Subroutine for Fisher's Exact Test on Unordered $r \times c$ Contingency Tables," *ACM Transactions on Mathematical Software*, Vol.12, No.2, pp.154–161.

Mehta, C.R., N.R. Patel and L.J. Wei (1988) "Computing Exact Permutational Distributions with Restricted Randomization Designs," *Biometrica*, Vol.75, No.2, pp.295–302.

Mood, A.M., F.A. Graybill and D.C. Boes (1974) *Introduction to the Theory of Statistics* (third edition), McGraw-Hill.

Mehta, C.R. and N.R. Patel (1992) *StatXact: User Manual*, CYTEL Software Corporation.

Wilcoxon, F. (1945) "Individual Comparisons by Ranking Methods," *Biometrics*, Vol.1, pp.80-83.

Table 2: Normal Distribution

$\mu \backslash \sigma$		$n1 = n2 = 5$			$n1 = n2 = 7$			$n1 = n2 = 9$		
		1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.5	2.0
(a) $\alpha = 0.10$										
	t	0.1061	0.1065	0.1084	0.1022	0.1029	0.1067	0.1055	0.1035	0.1054
	f	0.1086	0.1086	0.1100	0.1026	0.1029	0.1056	0.1061	0.1031	0.1041
	w	$0.1150^{\circ\circ}$	$0.1137^{\circ\circ}$	$0.1220^{\circ\circ\circ}$	0.1049	$0.1111^{\circ\circ}$	$0.1138^{\circ\circ}$	$0.1140^{\circ\circ}$	$0.1196^{\circ\circ\circ}$	$0.1204^{\circ\circ\circ}$
0.0	aw	$0.1150^{\circ\circ}$	$0.1137^{\circ\circ}$	$0.1220^{\circ\circ\circ}$	0.1049	$0.1111^{\circ\circ}$	$0.1138^{\circ\circ}$	$0.0986^{\times\times}$	$0.1003^{\times}$	0.1051
	ns	0.1071	0.1053	0.1074	0.1011	0.1012	$0.0972^{\times\times\times}$	0.1041	0.1010	$0.0970^{\times\times}$
	ls	0.1086	$0.1018^{\times}$	$0.0942^{\times\times\times}$	0.1018	$0.0998^{\times}$	$0.0933^{\times\times\times}$	0.1050	$0.1000^{\times}$	$0.0906^{\times\times\times}$
	cs	0.1086	$0.1018^{\times}$	$0.0942^{\times\times\times}$	0.1020	$0.0918^{\times\times\times}$	$0.0724^{\times\times\times}$	0.1054	$0.0868^{\times\times\times}$	$0.0605^{\times\times\times}$
	t	0.3018	0.2551	0.2251	0.3629	0.2918	0.2493	0.4095	0.3307	0.2808
	f	0.3085°	0.2598°	0.2286	0.3627	0.2916	0.2474	0.4100	0.3305	0.2789
	w	$0.3187^{\circ\circ\circ}$	$0.2703^{\circ\circ\circ}$	$0.2401^{\circ\circ\circ}$	0.3588	0.2952	0.2508	$0.4208^{\circ\circ}$	$0.3436^{\circ\circ}$	$0.2937^{\circ\circ}$
0.5	aw	$0.3187^{\circ\circ\circ}$	$0.2703^{\circ\circ\circ}$	$0.2401^{\circ\circ\circ}$	0.3588	0.2952	0.2508	$0.3867^{\times\times\times}$	$0.3116^{\times\times\times}$	$0.2654^{\times\times\times}$
	ns	0.3050	0.2526	$0.2127^{\times\times}$	$0.3500^{\times\times}$	$0.2777^{\times\times\times}$	$0.2249^{\times\times\times}$	$0.3986^{\times\times}$	$0.3124^{\times\times\times}$	$0.2520^{\times\times\times}$
	ls	0.3069°	$0.2429^{\times\times}$	$0.1930^{\times\times\times}$	$0.3479^{\times\times\times}$	$0.2727^{\times\times\times}$	$0.2141^{\times\times\times}$	$0.3987^{\times\times}$	$0.3073^{\times\times\times}$	$0.2409^{\times\times\times}$
	cs	0.3069°	$0.2429^{\times\times}$	$0.1930^{\times\times\times}$	$0.3381^{\times\times\times}$	$0.2506^{\times\times\times}$	$0.1727^{\times\times\times}$	$0.3689^{\times\times\times}$	$0.2582^{\times\times\times}$	$0.1581^{\times\times\times}$
	t	0.5898	0.4572	0.3773	0.7080	0.5618	0.4577	0.7865	0.6403	0.5224
	f	0.5986°	0.4650°	0.3839°	0.7083	0.5601	0.4556	0.7867	0.6400	0.5193
	w	$0.6021^{\circ\circ}$	$0.4718^{\circ\circ}$	$0.3958^{\circ\circ\circ}$	$0.6932^{\times\times\times}$	$0.5501^{\times\times}$	$0.4495^{\times}$	0.7861	0.6397	0.5244
1.0	aw	$0.6021^{\circ\circ}$	$0.4718^{\circ\circ}$	$0.3958^{\circ\circ\circ}$	$0.6932^{\times\times\times}$	$0.5501^{\times\times}$	$0.4495^{\times}$	$0.7611^{\times\times\times}$	$0.6056^{\times\times\times}$	$0.4894^{\times\times\times}$
	ns	$0.5818^{\times}$	$0.4482^{\times}$	$0.3647^{\times\times}$	$0.6879^{\times\times\times}$	$0.5349^{\times\times\times}$	$0.4152^{\times\times\times}$	$0.7722^{\times\times\times}$	$0.6097^{\times\times\times}$	$0.4746^{\times\times\times}$
	ls	$0.5734^{\times\times\times}$	$0.4298^{\times\times\times}$	$0.3247^{\times\times\times}$	$0.6869^{\times\times\times}$	$0.5286^{\times\times\times}$	$0.4034^{\times\times\times}$	$0.7702^{\times\times\times}$	$0.6045^{\times\times\times}$	$0.4579^{\times\times\times}$
	cs	$0.5734^{\times\times\times}$	$0.4298^{\times\times\times}$	$0.3247^{\times\times\times}$	$0.6556^{\times\times\times}$	$0.4789^{\times\times\times}$	$0.3236^{\times\times\times}$	$0.7126^{\times\times\times}$	$0.5060^{\times\times\times}$	$0.3121^{\times\times\times}$
(b) $\alpha = 0.05$										
	t	0.0518	0.0551	0.0587	0.0515	0.0526	0.0552	0.0551	0.0540	0.0550
	f	0.0538	0.0577°	0.0598	0.0513	0.0524	0.0554	0.0554	0.0538	0.0550
	w	$0.0798^{\circ\circ\circ}$	$0.0815^{\circ\circ\circ}$	$0.0866^{\circ\circ\circ}$	$0.0646^{\circ\circ\circ}$	$0.0683^{\circ\circ\circ}$	$0.0721^{\circ\circ\circ}$	$0.0599^{\circ\circ}$	$0.0635^{\circ\circ\circ}$	$0.0667^{\circ\circ\circ}$
0.0	aw	$0.0495^{\times}$	$0.0521^{\times}$	$0.0523^{\times\times}$	0.0503	0.0523	0.0542	$0.0506^{\times}$	0.0540	0.0570
	ns	$0.0583^{\circ\circ}$	0.0586°	0.0578	0.0508	0.0506	$0.0503^{\times\times}$	$0.0519^{\times}$	0.0529	$0.0524^{\times}$
	ls	$0.0583^{\circ\circ}$	0.0586°	0.0578	0.0513	0.0504	$0.0491^{\times\times}$	$0.0519^{\times}$	$0.0515^{\times}$	$0.0494^{\times\times}$
	cs	0.0544°	0.0537	$0.0524^{\times\times}$	0.0511	$0.0482^{\times}$	$0.0422^{\times\times\times}$	0.0529	$0.0469^{\times\times\times}$	$0.0374^{\times\times\times}$
	t	0.1861	0.1495	0.1281	0.2279	0.1747	0.1479	0.2725	0.2094	0.1678
	f	0.1897	0.1533°	0.1311	0.2278	0.1744	0.1482	0.2731	0.2101	0.1686
	w	$0.2427^{\circ\circ\circ}$	$0.2022^{\circ\circ\circ}$	$0.1767^{\circ\circ\circ}$	$0.2623^{\circ\circ\circ}$	$0.2051^{\circ\circ\circ}$	$0.1723^{\circ\circ\circ}$	$0.2856^{\circ\circ}$	$0.2234^{\circ\circ\circ}$	$0.1865^{\circ\circ\circ}$
0.5	aw	$0.1770^{\times\times}$	$0.1384^{\times\times\times}$	$0.1168^{\times\times\times}$	$0.2180^{\times\times}$	$0.1708^{\times}$	$0.1425^{\times}$	$0.2563^{\times\times\times}$	$0.1943^{\times\times\times}$	$0.1629^{\times}$
	ns	$0.1963^{\circ\circ}$	0.1564°	0.1278	$0.2230^{\times}$	$0.1675^{\times}$	$0.1300^{\times\times\times}$	$0.2665^{\times}$	$0.1986^{\times\times}$	$0.1497^{\times\times\times}$
	ls	$0.1963^{\circ\circ}$	0.1564°	0.1278	$0.2233^{\times}$	$0.1658^{\times\times}$	$0.1282^{\times\times\times}$	$0.2655^{\times}$	$0.1932^{\times\times\times}$	$0.1392^{\times\times\times}$
	cs	$0.1819^{\times}$	$0.1430^{\times}$	$0.1161^{\times\times\times}$	$0.2161^{\times\times}$	$0.1551^{\times\times\times}$	$0.1143^{\times\times\times}$	$0.2532^{\times\times\times}$	$0.1697^{\times\times\times}$	$0.1055^{\times\times\times}$
	t	0.4219	0.3171	0.2542	0.5523	0.4081	0.3094	0.6559	0.4895	0.3707
	f	0.4269°	0.3218°	0.2585	0.5524	0.4104	0.3106	0.6563	0.4893	0.3712
	w	$0.4982^{\circ\circ\circ}$	$0.3827^{\circ\circ\circ}$	$0.3190^{\circ\circ\circ}$	$0.5848^{\circ\circ\circ}$	$0.4406^{\circ\circ\circ}$	$0.3445^{\circ\circ\circ}$	0.6583	0.4946°	$0.3832^{\circ\circ}$
1.0	aw	$0.3969^{\times\times\times}$	$0.2908^{\times\times\times}$	$0.2247^{\times\times\times}$	$0.5252^{\times\times\times}$	$0.3860^{\times\times\times}$	$0.2956^{\times\times}$	$0.6206^{\times\times\times}$	$0.4563^{\times\times\times}$	$0.3510^{\times\times\times}$
	ns	0.4288°	0.3143	$0.2432^{\times\times}$	$0.5340^{\times\times\times}$	$0.3809^{\times\times\times}$	$0.2740^{\times\times\times}$	$0.6393^{\times\times\times}$	$0.4619^{\times\times\times}$	$0.3312^{\times\times\times}$
	ls	0.4288°	0.3143	$0.2432^{\times\times}$	$0.5345^{\times\times\times}$	$0.3795^{\times\times\times}$	$0.2696^{\times\times\times}$	$0.6368^{\times\times\times}$	$0.4538^{\times\times\times}$	$0.3154^{\times\times\times}$
	cs	$0.4055^{\times\times\times}$	$0.2912^{\times\times\times}$	$0.2188^{\times\times\times}$	$0.5206^{\times\times\times}$	$0.3569^{\times\times\times}$	$0.2382^{\times\times\times}$	$0.6047^{\times\times\times}$	$0.3985^{\times\times\times}$	$0.2430^{\times\times\times}$
(c) $\alpha = 0.01$										
	t	0.0092	0.0098	0.0114	0.0103	0.0103	0.0116	0.0117	0.0122	0.0133
	f	$0.0117^{\circ\circ}$	$0.0121^{\circ\circ}$	$0.0142^{\circ\circ}$	0.0097	0.0108	0.0127°	0.0113	0.0123	0.0136
	w	$0.0165^{\circ\circ\circ}$	$0.0171^{\circ\circ\circ}$	$0.0188^{\circ\circ\circ}$	$0.0133^{\circ\circ}$	$0.0158^{\circ\circ\circ}$	$0.0176^{\circ\circ\circ}$	$0.0141^{\circ\circ}$	0.0139°	$0.0170^{\circ\circ\circ}$
0.0	aw	$0.0076^{\times}$	$0.0083^{\times}$	$0.0099^{\times}$	0.0097	0.0096	0.0121	0.0111	0.0113	0.0130
	ns	$0.0165^{\circ\circ\circ}$	$0.0171^{\circ\circ\circ}$	$0.0188^{\circ\circ\circ}$	0.0111	$0.0124^{\circ\circ}$	0.0136°	0.0126	$0.0109^{\times}$	$0.0116^{\times}$
	ls	$0.0165^{\circ\circ\circ}$	$0.0171^{\circ\circ\circ}$	$0.0188^{\circ\circ\circ}$	0.0105	0.0121°	0.0130°	0.0124	$0.0107^{\times}$	$0.0110^{\times\times}$
	cs	$0.0165^{\circ\circ\circ}$	$0.0171^{\circ\circ\circ}$	$0.0188^{\circ\circ\circ}$	0.0110	0.0120°	0.0132°	0.0120	$0.0108^{\times}$	$0.0098^{\times\times\times}$
	t	0.0495	0.0401	0.0359	0.0706	0.0505	0.0416	0.0953	0.0671	0.0509
	f	$0.0552^{\circ\circ}$	$0.0458^{\circ\circ}$	$0.0430^{\circ\circ\circ}$	0.0697	0.0512	0.0427	0.0946	0.0668	0.0522
	w	$0.0711^{\circ\circ\circ}$	$0.0593^{\circ\circ\circ}$	$0.0514^{\circ\circ\circ}$	$0.0842^{\circ\circ\circ}$	$0.0610^{\circ\circ\circ}$	$0.0536^{\circ\circ\circ}$	$0.1082^{\circ\circ\circ}$	$0.0737^{\circ\circ}$	$0.0603^{\circ\circ\circ}$
0.5	aw	$0.0396^{\times\times\times}$	$0.0322^{\times\times\times}$	$0.0300^{\times\times\times}$	$0.0620^{\times\times\times}$	$0.0450^{\times\times}$	$0.0360^{\times\times}$	$0.0881^{\times\times}$	$0.0606^{\times\times}$	$0.0487^{\times}$
	ns	$0.0711^{\circ\circ\circ}$	$0.0593^{\circ\circ\circ}$	$0.0514^{\circ\circ\circ}$	0.0701	0.0520	0.0418	0.0947	$0.0601^{\times\times}$	$0.0452^{\times\times}$
	ls	$0.0711^{\circ\circ\circ}$	$0.0593^{\circ\circ\circ}$	$0.0514^{\circ\circ\circ}$	0.0684	0.0489	$0.0396^{\times}$	0.0949	$0.0586^{\times\times\times}$	$0.0429^{\times\times\times}$
	cs	$0.0711^{\circ\circ\circ}$	$0.0593^{\circ\circ\circ}$	$0.0514^{\circ\circ\circ}$	0.0685	0.0490	$0.0393^{\times}$	$0.0905^{\times}$	$0.0550^{\times\times\times}$	$0.0372^{\times\times\times}$
	t	0.1661	0.1070	0.0829	0.2604	0.1659	0.1163	0.3573	0.2257	0.1509
	f	$0.1798^{\circ\circ\circ}$	$0.1210^{\circ\circ\circ}$	$0.0940^{\circ\circ\circ}$	0.2629	0.1671	0.1226°	0.3569	0.2267	0.1561°
	w	$0.2104^{\circ\circ\circ}$	$0.1470^{\circ\circ\circ}$	$0.1120^{\circ\circ\circ}$	$0.2820^{\circ\circ\circ}$	$0.1874^{\circ\circ\circ}$	$0.1329^{\circ\circ\circ}$	$0.3731^{\circ\circ\circ}$	$0.2400^{\circ\circ\circ}$	$0.1680^{\circ\circ\circ}$
1.0	aw	$0.1320^{\times\times\times}$	$0.0890^{\times\times\times}$	$0.0717^{\times\times\times}$	$0.2315^{\times\times\times}$	$0.1427^{\times\times\times}$	$0.0977^{\times\times\times}$	$0.3359^{\times\times\times}$	$0.2083^{\times\times\times}$	$0.1433^{\times\times}$
	ns	$0.2104^{\circ\circ\circ}$	$0.1470^{\circ\circ\circ}$	$0.1120^{\circ\circ\circ}$	$0.2529^{\times}$	$0.1605^{\times}$	$0.1074^{\times\times}$	$0.3449^{\times\times}$	$0.2069^{\times\times\times}$	$0.1313^{\times\times\times}$
	ls	$0.2104^{\circ\circ\circ}$	$0.1470^{\circ\circ\circ}$	$0.1120^{\circ\circ\circ}$	$0.2483^{\times\times}$	$0.1540^{\times\times\times}$	$0.1017^{\times\times\times}$	$0.3423^{\times\times\times}$	$0.2014^{\times\times\times}$	$0.1233^{\times\times\times}$
	cs	$0.2104^{\circ\circ\circ}$	$0.1470^{\circ\circ\circ}$	$0.1120^{\circ\circ\circ}$	$0.2470^{\times\times\times}$	$0.1541^{\times\times\times}$	$0.1020^{\times\times\times}$	$0.3281^{\times\times\times}$	$0.1888^{\times\times\times}$	$0.1091^{\times\times\times}$

Table 3: Uniform Distribution

$\mu \backslash \sigma$		$n1 = n2 = 5$			$n1 = n2 = 7$			$n1 = n2 = 9$		
		1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.5	2.0
(a) $\alpha = 0.10$										
	t	0.0985	0.1004	0.1037	0.1018	0.1002	0.1015	0.0966	0.1023	0.1046
	f	0.1038°	0.1051°	0.1089°	0.1031	0.1010	0.1029	0.0979	0.1030	0.1053
	w	0.1127$^{\circ\circ\circ}$	0.1171$^{\circ\circ\circ}$	0.1234$^{\circ\circ\circ}$	0.1076°	0.1127$^{\circ\circ\circ}$	0.1187$^{\circ\circ\circ}$	0.1097$^{\circ\circ\circ}$	0.1187$^{\circ\circ\circ}$	0.1262$^{\circ\circ\circ}$
0.0	aw	0.1127$^{\circ\circ\circ}$	0.1171$^{\circ\circ\circ}$	0.1234$^{\circ\circ\circ}$	0.1076°	0.1127$^{\circ\circ\circ}$	0.1187$^{\circ\circ\circ}$	0.0940	0.1028	0.1106°
	ns	0.1042°	0.1043°	0.1044	0.1030	0.0975	0.0947$^{\times\times}$	0.1014°	0.0949$^{\times\times}$	0.0940$^{\times\times\times}$
	ls	0.1035°	0.0954$^{\times}$	0.0823$^{\times\times\times}$	0.1027	0.0931$^{\times\times}$	0.0886$^{\times\times\times}$	0.1009°	0.0897$^{\times\times\times}$	0.0827$^{\times\times\times}$
	cs	0.1035°	0.0954$^{\times}$	0.0823$^{\times\times\times}$	0.1030	0.0805$^{\times\times\times}$	0.0586$^{\times\times\times}$	0.1019°	0.0614$^{\times\times\times}$	0.0345$^{\times\times\times}$
	t	0.1463	0.1376	0.1340	0.1624	0.1456	0.1366	0.1764	0.1590	0.1485
	f	0.1535$^{\circ\circ}$	0.1435°	0.1385°	0.1646	0.1479	0.1373	0.1786	0.1605	0.1497
	w	0.1660$^{\circ\circ\circ}$	0.1551$^{\circ\circ\circ}$	0.1527$^{\circ\circ\circ}$	0.1712$^{\circ\circ}$	0.1548$^{\circ\circ}$	0.1502$^{\circ\circ\circ}$	0.1934$^{\circ\circ\circ}$	0.1733$^{\circ\circ\circ}$	0.1653$^{\circ\circ\circ}$
0.5	aw	0.1660$^{\circ\circ\circ}$	0.1551$^{\circ\circ\circ}$	0.1527$^{\circ\circ\circ}$	0.1712$^{\circ\circ}$	0.1548$^{\circ\circ}$	0.1502$^{\circ\circ\circ}$	0.1696$^{\times}$	0.1509$^{\times\times}$	0.1444$^{\times}$
	ns	0.1594$^{\circ\circ\circ}$	0.1393	0.1307	0.1722$^{\circ\circ}$	0.1378$^{\times\times}$	0.1241$^{\times\times\times}$	0.1919$^{\circ\circ\circ}$	0.1414$^{\times\times\times}$	0.1240$^{\times\times\times}$
	ls	0.1618$^{\circ\circ\circ}$	0.1285$^{\times\times}$	0.1039$^{\times\times\times}$	0.1747$^{\circ\circ\circ}$	0.1301$^{\times\times\times}$	0.1130$^{\times\times\times}$	0.1962$^{\circ\circ\circ}$	0.1359$^{\times\times\times}$	0.1128$^{\times\times\times}$
	cs	0.1618$^{\circ\circ\circ}$	0.1285$^{\times\times}$	0.1039$^{\times\times\times}$	0.1826$^{\circ\circ\circ}$	0.1146$^{\times\times\times}$	0.0762$^{\times\times\times}$	0.2130$^{\circ\circ\circ}$	0.0937$^{\times\times\times}$	0.0487$^{\times\times\times}$
	t	0.2098	0.1840	0.1676	0.2425	0.2063	0.1830	0.2792	0.2302	0.1996
	f	0.2172°	0.1916°	0.1730°	0.2471°	0.2088	0.1847	0.2821	0.2319	0.2008
	w	0.2367$^{\circ\circ\circ}$	0.1971$^{\circ\circ\circ}$	0.1825$^{\circ\circ\circ}$	0.2555$^{\circ\circ\circ}$	0.2097	0.1894°	0.3045$^{\circ\circ\circ}$	0.2378°	0.2110$^{\circ\circ}$
1.0	aw	0.2367$^{\circ\circ\circ}$	0.1971$^{\circ\circ\circ}$	0.1825$^{\circ\circ\circ}$	0.2555$^{\circ\circ\circ}$	0.2097	0.1894°	0.2683$^{\times\times}$	0.2122$^{\times\times\times}$	0.1881$^{\times\times}$
	ns	0.2299$^{\circ\circ\circ}$	0.1786$^{\times}$	0.1574$^{\times\times}$	0.2686$^{\circ\circ\circ}$	0.1943$^{\times\times}$	0.1580$^{\times\times\times}$	0.3135$^{\circ\circ\circ}$	0.2033$^{\times\times\times}$	0.1664$^{\times\times\times}$
	ls	0.2402$^{\circ\circ\circ}$	0.1686$^{\times\times\times}$	0.1271$^{\times\times\times}$	0.2740$^{\circ\circ\circ}$	0.1850$^{\times\times\times}$	0.1452$^{\times\times\times}$	0.3224$^{\circ\circ\circ}$	0.1952$^{\times\times\times}$	0.1486$^{\times\times\times}$
	cs	0.2402$^{\circ\circ\circ}$	0.1686$^{\times\times\times}$	0.1271$^{\times\times\times}$	0.2945$^{\circ\circ\circ}$	0.1635$^{\times\times\times}$	0.0983$^{\times\times\times}$	0.3616$^{\circ\circ\circ}$	0.1453$^{\times\times\times}$	0.0685$^{\times\times\times}$
(b) $\alpha = 0.05$										
	t	0.0513	0.0549	0.0603	0.0470	0.0506	0.0535	0.0485	0.0537	0.0565
	f	0.0540°	0.0550	0.0595	0.0470	0.0503	0.0533	0.0489	0.0537	0.0568
	w	0.0759$^{\circ\circ\circ}$	0.0818$^{\circ\circ\circ}$	0.0882$^{\circ\circ\circ}$	0.0623$^{\circ\circ\circ}$	0.0686$^{\circ\circ\circ}$	0.0729$^{\circ\circ\circ}$	0.0576$^{\circ\circ\circ}$	0.0628$^{\circ\circ\circ}$	0.0706$^{\circ\circ\circ}$
0.0	aw	0.0486$^{\times}$	0.0495$^{\times\times}$	0.0491$^{\times\times\times}$	0.0452	0.0525	0.0572°	0.0469	0.0525	0.0590°
	ns	0.0565$^{\circ\circ}$	0.0572°	0.0543$^{\times\times}$	0.0490	0.0501	0.0466$^{\times\times\times}$	0.0505	0.0498$^{\times}$	0.0487$^{\times\times\times}$
	ls	0.0565$^{\circ\circ}$	0.0572°	0.0543$^{\times\times}$	0.0497°	0.0493	0.0455$^{\times\times\times}$	0.0495	0.0467$^{\times\times\times}$	0.0421$^{\times\times\times}$
	cs	0.0504	0.0538	0.0510$^{\times\times\times}$	0.0495°	0.0478$^{\times}$	0.0388$^{\times\times\times}$	0.0500	0.0394$^{\times\times\times}$	0.0250$^{\times\times\times}$
	t	0.0799	0.0749	0.0743	0.0881	0.0807	0.0757	0.0939	0.0880	0.0824
	f	0.0821	0.0761	0.0739	0.0883	0.0806	0.0754	0.0940	0.0882	0.0822
	w	0.1151$^{\circ\circ\circ}$	0.1094$^{\circ\circ\circ}$	0.1110$^{\circ\circ\circ}$	0.1128$^{\circ\circ\circ}$	0.0987$^{\circ\circ\circ}$	0.0940$^{\circ\circ\circ}$	0.1070$^{\circ\circ\circ}$	0.0949$^{\circ\circ}$	0.0952$^{\circ\circ\circ}$
0.5	aw	0.0752$^{\times}$	0.0691$^{\times\times}$	0.0649$^{\times\times\times}$	0.0848$^{\times}$	0.0773$^{\times}$	0.0753	0.0893$^{\times}$	0.0808$^{\times\times}$	0.0796$^{\times}$
	ns	0.0882$^{\circ\circ\circ}$	0.0788°	0.0720	0.0929°	0.0733$^{\times\times}$	0.0638$^{\times\times\times}$	0.1036$^{\circ\circ\circ}$	0.0791$^{\times\times\times}$	0.0673$^{\times\times\times}$
	ls	0.0882$^{\circ\circ\circ}$	0.0788°	0.0720	0.0938$^{\circ\circ}$	0.0726$^{\times\times}$	0.0625$^{\times\times\times}$	0.1057$^{\circ\circ\circ}$	0.0736$^{\times\times\times}$	0.0600$^{\times\times\times}$
	cs	0.0826	0.0738	0.0673$^{\times\times}$	0.0954$^{\circ\circ}$	0.0711$^{\times\times\times}$	0.0532$^{\times\times\times}$	0.1144$^{\circ\circ\circ}$	0.0609$^{\times\times\times}$	0.0356$^{\times\times\times}$
	t	0.1194	0.1030	0.0961	0.1458	0.1158	0.1053	0.1689	0.1367	0.1182
	f	0.1215	0.1033	0.0956	0.1454	0.1159	0.1044	0.1687	0.1365	0.1174
	w	0.1667$^{\circ\circ\circ}$	0.1428$^{\circ\circ\circ}$	0.1364$^{\circ\circ\circ}$	0.1762$^{\circ\circ\circ}$	0.1363$^{\circ\circ\circ}$	0.1200$^{\circ\circ\circ}$	0.1824$^{\circ\circ\circ}$	0.1400	0.1257$^{\circ\circ}$
1.0	aw	0.1156$^{\times}$	0.0923$^{\times\times\times}$	0.0807$^{\times\times\times}$	0.1409$^{\times}$	0.1083$^{\times\times}$	0.0983$^{\times\times}$	0.1596$^{\times\times}$	0.1195$^{\times\times\times}$	0.1096$^{\times\times}$
	ns	0.1343$^{\circ\circ\circ}$	0.1063°	0.0911$^{\times}$	0.1548$^{\circ\circ}$	0.1037$^{\times\times\times}$	0.0836$^{\times\times\times}$	0.1839$^{\circ\circ\circ}$	0.1181$^{\times\times\times}$	0.0930$^{\times\times\times}$
	ls	0.1343$^{\circ\circ\circ}$	0.1063°	0.0911$^{\times}$	0.1569$^{\circ\circ\circ}$	0.1025$^{\times\times\times}$	0.0813$^{\times\times\times}$	0.1903$^{\circ\circ\circ}$	0.1143$^{\times\times\times}$	0.0830$^{\times\times\times}$
	cs	0.1273$^{\circ\circ}$	0.0998$^{\times}$	0.0846$^{\times\times\times}$	0.1643$^{\circ\circ\circ}$	0.1029$^{\times\times\times}$	0.0689$^{\times\times\times}$	0.2126$^{\circ\circ\circ}$	0.0967$^{\times\times\times}$	0.0510$^{\times\times\times}$
(c) $\alpha = 0.01$										
	t	0.0138	0.0148	0.0174	0.0099	0.0128	0.0153	0.0121	0.0127	0.0159
	f	0.0132	0.0163°	0.0179	0.0090	0.0114$^{\times}$	0.0138$^{\times}$	0.0105$^{\times}$	0.0120	0.0145$^{\times}$
	w	0.0170$^{\circ\circ}$	0.0203$^{\circ\circ\circ}$	0.0226$^{\circ\circ\circ}$	0.0121$^{\circ\circ}$	0.0149°	0.0187$^{\circ\circ}$	0.0130	0.0152$^{\circ\circ}$	0.0193$^{\circ\circ}$
0.0	aw	0.0090$^{\times\times\times}$	0.0105$^{\times\times\times}$	0.0133$^{\times\times\times}$	0.0084$^{\times}$	0.0094$^{\times\times\times}$	0.0114$^{\times\times\times}$	0.0100$^{\times}$	0.0121	0.0145$^{\times}$
	ns	0.0170$^{\circ\circ}$	0.0203$^{\circ\circ\circ}$	0.0226$^{\circ\circ\circ}$	0.0094	0.0119	0.0137$^{\times}$	0.0105$^{\times}$	0.0122	0.0119$^{\times\times\times}$
	ls	0.0170$^{\circ\circ}$	0.0203$^{\circ\circ\circ}$	0.0226$^{\circ\circ\circ}$	0.0089$^{\times}$	0.0114$^{\times}$	0.0130$^{\times}$	0.0105$^{\times}$	0.0112$^{\times}$	0.0105$^{\times\times\times}$
	cs	0.0170$^{\circ\circ}$	0.0203$^{\circ\circ\circ}$	0.0226$^{\circ\circ\circ}$	0.0088$^{\times}$	0.0116$^{\times}$	0.0131$^{\times}$	0.0115	0.0112$^{\times}$	0.0104$^{\times\times\times}$
	t	0.0202	0.0222	0.0254	0.0197	0.0201	0.0218	0.0251	0.0239	0.0233
	f	0.0205	0.0222	0.0245	0.0175$^{\times}$	0.0182$^{\times}$	0.0207	0.0229$^{\times}$	0.0221$^{\times}$	0.0220
	w	0.0264$^{\circ\circ\circ}$	0.0279$^{\circ\circ\circ}$	0.0300$^{\circ\circ}$	0.0229$^{\circ\circ}$	0.0240$^{\circ\circ}$	0.0248$^{\circ\circ}$	0.0257	0.0251	0.0261°
0.5	aw	0.0139$^{\times\times\times}$	0.0147$^{\times\times\times}$	0.0179$^{\times\times\times}$	0.0155$^{\times\times\times}$	0.0158$^{\times\times\times}$	0.0166$^{\times\times\times}$	0.0208$^{\times\times}$	0.0198$^{\times\times}$	0.0211$^{\times}$
	ns	0.0264$^{\circ\circ\circ}$	0.0279$^{\circ\circ\circ}$	0.0300$^{\circ\circ}$	0.0184	0.0195	0.0185$^{\times\times}$	0.0227$^{\times}$	0.0194$^{\times\times}$	0.0170$^{\times\times\times}$
	ls	0.0264$^{\circ\circ\circ}$	0.0279$^{\circ\circ\circ}$	0.0300$^{\circ\circ}$	0.0182$^{\times}$	0.0190	0.0179$^{\times\times}$	0.0229$^{\times}$	0.0191$^{\times\times\times}$	0.0154$^{\times\times\times}$
	cs	0.0264$^{\circ\circ\circ}$	0.0279$^{\circ\circ\circ}$	0.0300$^{\circ\circ}$	0.0178$^{\times}$	0.0193	0.0180$^{\times\times}$	0.0242	0.0187$^{\times\times\times}$	0.0143$^{\times\times\times}$
	t	0.0322	0.0311	0.0320	0.0357	0.0321	0.0303	0.0465	0.0387	0.0361
	f	0.0314	0.0292$^{\times}$	0.0318	0.0319$^{\times\times}$	0.0290$^{\times}$	0.0291	0.0431$^{\times}$	0.0362$^{\times}$	0.0345
	w	0.0396$^{\circ\circ\circ}$	0.0374$^{\circ\circ\circ}$	0.0387$^{\circ\circ\circ}$	0.0415$^{\circ\circ\circ}$	0.0358$^{\circ\circ}$	0.0338$^{\circ\circ}$	0.0518$^{\circ\circ}$	0.0416°	0.0380°
1.0	aw	0.0221$^{\times\times\times}$	0.0221$^{\times\times\times}$	0.0224$^{\times\times\times}$	0.0284$^{\times\times\times}$	0.0249$^{\times\times\times}$	0.0223$^{\times\times\times}$	0.0416$^{\times\times}$	0.0340$^{\times\times}$	0.0317$^{\times\times}$
	ns	0.0396$^{\circ\circ\circ}$	0.0374$^{\circ\circ\circ}$	0.0387$^{\circ\circ\circ}$	0.0364	0.0292$^{\times}$	0.0259$^{\times\times}$	0.0463	0.0338$^{\times\times}$	0.0260$^{\times\times\times}$
	ls	0.0396$^{\circ\circ\circ}$	0.0374$^{\circ\circ\circ}$	0.0387$^{\circ\circ\circ}$	0.0363	0.0289$^{\times}$	0.0247$^{\times\times\times}$	0.0469	0.0337$^{\times\times}$	0.0245$^{\times\times\times}$
	cs	0.0396$^{\circ\circ\circ}$	0.0374$^{\circ\circ\circ}$	0.0387$^{\circ\circ\circ}$	0.0359	0.0289$^{\times}$	0.0250$^{\times\times\times}$	0.0530$^{\circ\circ\circ}$	0.0326$^{\times\times\times}$	0.0225$^{\times\times\times}$

Table 4: Logistic Distribution

$\mu \backslash \sigma$		$n1 = n2 = 5$			$n1 = n2 = 7$			$n1 = n2 = 9$		
		1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.5	2.0
(a) $\alpha = 0.10$										
	t	0.1007	0.1015	0.1056	0.1056	0.1021	0.1031	0.1003	0.1019	0.1048
	f	0.1036	0.1057°	0.1092°	0.1046	0.1022	0.1010	0.0992	0.1007	0.1029
	w	0.1127$^{\circ\circ\circ}$	0.1153$^{\circ\circ\circ}$	0.1176$^{\circ\circ\circ}$	0.1076	0.1071°	0.1114$^{\circ\circ}$	0.1097$^{\circ\circ\circ}$	0.1166$^{\circ\circ\circ}$	0.1212$^{\circ\circ\circ}$
0.0	aw	0.1127$^{\circ\circ\circ}$	0.1153$^{\circ\circ\circ}$	0.1176$^{\circ\circ\circ}$	0.1076	0.1071°	0.1114$^{\circ\circ}$	0.0940$^{\times\times}$	0.1001	0.1045
	ns	0.1042°	0.1059°	0.1046	0.1030	0.1009	0.0960$^{\times\times}$	0.1014	0.0991	0.0965$^{\times\times}$
	ls	0.1035	0.1011	0.0928$^{\times\times\times}$	0.1027	0.0988$^{\times}$	0.0940$^{\times\times}$	0.1009	0.0985$^{\times}$	0.0918$^{\times\times\times}$
	cs	0.1035	0.1011	0.0928$^{\times\times\times}$	0.1030	0.0925$^{\times\times\times}$	0.0794$^{\times\times\times}$	0.1019	0.0865$^{\times\times\times}$	0.0662$^{\times\times\times}$
	t	0.2014	0.1806	0.1659	0.2242	0.1940	0.1766	0.2459	0.2099	0.1895
	f	0.2054	0.1830	0.1675	0.2225	0.1917	0.1745	0.2447	0.2081	0.1874
	w	0.2132$^{\circ\circ}$	0.1913$^{\circ\circ}$	0.1810$^{\circ\circ\circ}$	0.2296°	0.1982°	0.1872$^{\circ\circ}$	0.2686$^{\circ\circ\circ}$	0.2293$^{\circ\circ\circ}$	0.2113$^{\circ\circ\circ}$
0.5	aw	0.2132$^{\circ\circ}$	0.1913$^{\circ\circ}$	0.1810$^{\circ\circ\circ}$	0.2296°	0.1982°	0.1872$^{\circ\circ}$	0.2394$^{\times}$	0.2049$^{\times}$	0.1868
	ns	0.2003	0.1771	0.1625	0.2242	0.1871$^{\times}$	0.1648$^{\times\times\times}$	0.2447	0.2027$^{\times}$	0.1747$^{\times\times\times}$
	ls	0.1971$^{\times}$	0.1701$^{\times\times}$	0.1465$^{\times\times\times}$	0.2224	0.1820$^{\times\times\times}$	0.1575$^{\times\times\times}$	0.2433	0.1996$^{\times\times}$	0.1648$^{\times\times\times}$
	cs	0.1971$^{\times}$	0.1701$^{\times\times}$	0.1465$^{\times\times\times}$	0.2117$^{\times\times}$	0.1664$^{\times\times\times}$	0.1321$^{\times\times\times}$	0.2224$^{\times\times\times}$	0.1677$^{\times\times\times}$	0.1149$^{\times\times\times}$
	t	0.3432	0.2790	0.2412	0.4056	0.3265	0.2781	0.4591	0.3660	0.3070
	f	0.3496°	0.2860°	0.2445	0.4054	0.3242	0.2747	0.4574	0.3643	0.3032
	w	0.3589$^{\circ\circ\circ}$	0.3004$^{\circ\circ\circ}$	0.2623$^{\circ\circ\circ}$	0.4168$^{\circ\circ}$	0.3369$^{\circ\circ}$	0.2882$^{\circ\circ}$	0.4907$^{\circ\circ\circ}$	0.3929$^{\circ\circ\circ}$	0.3301$^{\circ\circ\circ}$
1.0	aw	0.3589$^{\circ\circ\circ}$	0.3004$^{\circ\circ\circ}$	0.2623$^{\circ\circ\circ}$	0.4168$^{\circ\circ}$	0.3369$^{\circ\circ}$	0.2882$^{\circ\circ}$	0.4522$^{\times}$	0.3561$^{\times\times}$	0.2977$^{\times\times}$
	ns	0.3436	0.2791	0.2367$^{\times}$	0.4014	0.3187$^{\times}$	0.2604$^{\times\times\times}$	0.4556	0.3524$^{\times\times}$	0.2846$^{\times\times\times}$
	ls	0.3370$^{\times}$	0.2678$^{\times\times}$	0.2122$^{\times\times\times}$	0.3979$^{\times}$	0.3102$^{\times\times\times}$	0.2499$^{\times\times\times}$	0.4530$^{\times}$	0.3437$^{\times\times\times}$	0.2699$^{\times\times\times}$
	cs	0.3370$^{\times}$	0.2678$^{\times\times}$	0.2122$^{\times\times\times}$	0.3692$^{\times\times\times}$	0.2761$^{\times\times\times}$	0.2012$^{\times\times\times}$	0.3966$^{\times\times\times}$	0.2788$^{\times\times\times}$	0.1849$^{\times\times\times}$
(b) $\alpha = 0.05$										
	t	0.0493	0.0514	0.0540	0.0473	0.0504	0.0531	0.0508	0.0513	0.0546
	f	0.0525°	0.0538°	0.0577°	0.0476	0.0516	0.0544	0.0513	0.0514	0.0556
	w	0.0759$^{\circ\circ\circ}$	0.0796$^{\circ\circ\circ}$	0.0833$^{\circ\circ\circ}$	0.0623$^{\circ\circ\circ}$	0.0664$^{\circ\circ\circ}$	0.0695$^{\circ\circ\circ}$	0.0576$^{\circ\circ\circ}$	0.0604$^{\circ\circ\circ}$	0.0644$^{\circ\circ\circ}$
0.0	aw	0.0486	0.0493	0.0512$^{\times}$	0.0452	0.0497	0.0527	0.0469$^{\times}$	0.0504	0.0543
	ns	0.0565$^{\circ\circ\circ}$	0.0563$^{\circ\circ}$	0.0568°	0.0490	0.0511	0.0505$^{\times}$	0.0505	0.0520	0.0502$^{\times}$
	ls	0.0565$^{\circ\circ\circ}$	0.0563$^{\circ\circ}$	0.0568°	0.0497°	0.0510	0.0501$^{\times}$	0.0495	0.0508	0.0467$^{\times\times\times}$
	cs	0.0504	0.0506	0.0501$^{\times}$	0.0495°	0.0487	0.0452$^{\times\times\times}$	0.0500	0.0454$^{\times\times}$	0.0390$^{\times\times\times}$
	t	0.1091	0.0965	0.0878	0.1295	0.1074	0.0965	0.1453	0.1200	0.1083
	f	0.1131°	0.0998°	0.0929°	0.1305	0.1096	0.0976	0.1461	0.1206	0.1087
	w	0.1547$^{\circ\circ\circ}$	0.1377$^{\circ\circ\circ}$	0.1318$^{\circ\circ\circ}$	0.1606$^{\circ\circ\circ}$	0.1351$^{\circ\circ\circ}$	0.1219$^{\circ\circ\circ}$	0.1607$^{\circ\circ\circ}$	0.1385$^{\circ\circ\circ}$	0.1246$^{\circ\circ\circ}$
0.5	aw	0.1045$^{\times}$	0.0895$^{\times\times}$	0.0825$^{\times}$	0.1256$^{\times}$	0.1048	0.0966	0.1379$^{\times\times}$	0.1173	0.1063
	ns	0.1202$^{\circ\circ\circ}$	0.1013°	0.0923°	0.1291	0.1049	0.0915$^{\times}$	0.1429	0.1188	0.0980$^{\times\times\times}$
	ls	0.1202$^{\circ\circ\circ}$	0.1013°	0.0923°	0.1289	0.1041$^{\times}$	0.0899$^{\times\times}$	0.1419	0.1152$^{\times}$	0.0925$^{\times\times\times}$
	cs	0.1114	0.0939	0.0830$^{\times}$	0.1225$^{\times\times}$	0.0982$^{\times\times}$	0.0806$^{\times\times\times}$	0.1341$^{\times\times\times}$	0.0998$^{\times\times\times}$	0.0708$^{\times\times\times}$
	t	0.2103	0.1658	0.1413	0.2650	0.2002	0.1640	0.3135	0.2326	0.1877
	f	0.2190$^{\circ\circ}$	0.1722°	0.1466°	0.2663	0.2032	0.1675	0.3154	0.2347	0.1889
	w	0.2811$^{\circ\circ\circ}$	0.2224$^{\circ\circ\circ}$	0.1965$^{\circ\circ\circ}$	0.3076$^{\circ\circ\circ}$	0.2377$^{\circ\circ\circ}$	0.2012$^{\circ\circ\circ}$	0.3407$^{\circ\circ\circ}$	0.2536$^{\circ\circ\circ}$	0.2104$^{\circ\circ\circ}$
1.0	aw	0.2013$^{\times\times}$	0.1550$^{\times\times}$	0.1310$^{\times\times}$	0.2625	0.1966	0.1639	0.3088$^{\times}$	0.2225$^{\times\times}$	0.1848
	ns	0.2236$^{\circ\circ\circ}$	0.1716°	0.1429	0.2621	0.1930$^{\times}$	0.1520$^{\times\times\times}$	0.3129	0.2235$^{\times\times}$	0.1716$^{\times\times\times}$
	ls	0.2236$^{\circ\circ\circ}$	0.1716°	0.1429	0.2619	0.1926$^{\times}$	0.1493$^{\times\times\times}$	0.3071$^{\times}$	0.2179$^{\times\times\times}$	0.1608$^{\times\times\times}$
	cs	0.2078	0.1580$^{\times\times}$	0.1281$^{\times\times\times}$	0.2493$^{\times\times\times}$	0.1787$^{\times\times\times}$	0.1302$^{\times\times\times}$	0.2801$^{\times\times\times}$	0.1864$^{\times\times\times}$	0.1238$^{\times\times\times}$
(c) $\alpha = 0.01$										
	t	0.0090	0.0093	0.0098	0.0079	0.0082	0.0103	0.0104	0.0104	0.0115
	f	0.0130$^{\circ\circ\circ}$	0.0149$^{\circ\circ\circ}$	0.0157$^{\circ\circ\circ}$	0.0095°	0.0102$^{\circ\circ}$	0.0129$^{\circ\circ}$	0.0104	0.0113	0.0129°
	w	0.0170$^{\circ\circ\circ}$	0.0185$^{\circ\circ\circ}$	0.0197$^{\circ\circ\circ}$	0.0121$^{\circ\circ\circ}$	0.0130$^{\circ\circ\circ}$	0.0162$^{\circ\circ\circ}$	0.0130$^{\circ\circ}$	0.0139$^{\circ\circ\circ}$	0.0149$^{\circ\circ\circ}$
0.0	aw	0.0090	0.0092	0.0113°	0.0084	0.0080	0.0096	0.0100	0.0112	0.0129°
	ns	0.0170$^{\circ\circ\circ}$	0.0185$^{\circ\circ\circ}$	0.0197$^{\circ\circ\circ}$	0.0094°	0.0102$^{\circ\circ}$	0.0118°	0.0105	0.0113	0.0122
	ls	0.0170$^{\circ\circ\circ}$	0.0185$^{\circ\circ\circ}$	0.0197$^{\circ\circ\circ}$	0.0089°	0.0097°	0.0111	0.0105	0.0110	0.0113
	cs	0.0170$^{\circ\circ\circ}$	0.0185$^{\circ\circ\circ}$	0.0197$^{\circ\circ\circ}$	0.0088°	0.0097°	0.0111	0.0115°	0.0101	0.0094$^{\times}$
	t	0.0263	0.0217	0.0207	0.0277	0.0249	0.0223	0.0392	0.0324	0.0291
	f	0.0317$^{\circ\circ\circ}$	0.0284$^{\circ\circ\circ}$	0.0289$^{\circ\circ\circ}$	0.0299°	0.0285$^{\circ\circ}$	0.0269$^{\circ\circ\circ}$	0.0411	0.0337	0.0318°
	w	0.0405$^{\circ\circ\circ}$	0.0354$^{\circ\circ\circ}$	0.0345$^{\circ\circ\circ}$	0.0379$^{\circ\circ\circ}$	0.0341$^{\circ\circ\circ}$	0.0323$^{\circ\circ\circ}$	0.0461$^{\circ\circ\circ}$	0.0392$^{\circ\circ\circ}$	0.0375$^{\circ\circ\circ}$
0.5	aw	0.0222$^{\times\times}$	0.0209	0.0209	0.0270	0.0229$^{\times}$	0.0216	0.0373	0.0309	0.0292
	ns	0.0405$^{\circ\circ\circ}$	0.0354$^{\circ\circ\circ}$	0.0345$^{\circ\circ\circ}$	0.0323$^{\circ\circ}$	0.0271°	0.0251°	0.0387	0.0324	0.0280
	ls	0.0405$^{\circ\circ\circ}$	0.0354$^{\circ\circ\circ}$	0.0345$^{\circ\circ\circ}$	0.0309°	0.0265°	0.0242°	0.0378	0.0317	0.0270$^{\times}$
	cs	0.0405$^{\circ\circ\circ}$	0.0354$^{\circ\circ\circ}$	0.0345$^{\circ\circ\circ}$	0.0311$^{\circ\circ}$	0.0264	0.0238°	0.0381	0.0303$^{\times}$	0.0231$^{\times\times\times}$
	t	0.0586	0.0450	0.0384	0.0857	0.0596	0.0487	0.1102	0.0769	0.0606
	f	0.0722$^{\circ\circ\circ}$	0.0549$^{\circ\circ\circ}$	0.0481$^{\circ\circ\circ}$	0.0927$^{\circ\circ}$	0.0651$^{\circ\circ}$	0.0537$^{\circ\circ}$	0.1158°	0.0820°	0.0646°
	w	0.0879$^{\circ\circ\circ}$	0.0687$^{\circ\circ\circ}$	0.0581$^{\circ\circ\circ}$	0.1068$^{\circ\circ\circ}$	0.0742$^{\circ\circ\circ}$	0.0622$^{\circ\circ\circ}$	0.1307$^{\circ\circ\circ}$	0.0948$^{\circ\circ\circ}$	0.0737$^{\circ\circ\circ}$
1.0	aw	0.0516$^{\times\times}$	0.0402$^{\times\times}$	0.0364$^{\times}$	0.0771$^{\times\times\times}$	0.0540$^{\times\times}$	0.0441$^{\times\times}$	0.1084	0.0773	0.0609
	ns	0.0879$^{\circ\circ\circ}$	0.0687$^{\circ\circ\circ}$	0.0581$^{\circ\circ\circ}$	0.0896°	0.0611	0.0486	0.1133	0.0759	0.0566$^{\times}$
	ls	0.0879$^{\circ\circ\circ}$	0.0687$^{\circ\circ\circ}$	0.0581$^{\circ\circ\circ}$	0.0863	0.0593	0.0466	0.1128	0.0750	0.0548$^{\times\times}$
	cs	0.0879$^{\circ\circ\circ}$	0.0687$^{\circ\circ\circ}$	0.0581$^{\circ\circ\circ}$	0.0867	0.0593	0.0463$^{\times}$	0.1057$^{\times}$	0.0695$^{\times\times}$	0.0492$^{\times\times\times}$

Table 5: Cauchy Distribution

		$n1 = n2 = 5$			$n1 = n2 = 7$			$n1 = n2 = 9$		
$\mu \backslash \sigma$		1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.5	2.0
(a) $\alpha = 0.10$										
	t	0.0853	0.0863	0.0857	0.0846	0.0848	0.0859	0.0886	0.0887	0.0890
	f	0.1027$^{\circ\circ\circ}$	0.1040$^{\circ\circ\circ}$	0.1047$^{\circ\circ\circ}$	0.0999$^{\circ\circ\circ}$	0.0978$^{\circ\circ\circ}$	0.1014$^{\circ\circ\circ}$	0.1001$^{\circ\circ\circ}$	0.1002$^{\circ\circ\circ}$	0.1012$^{\circ\circ\circ}$
	w	0.1127$^{\circ\circ\circ}$	0.1126$^{\circ\circ\circ}$	0.1158$^{\circ\circ\circ}$	0.1076$^{\circ\circ\circ}$	0.1059$^{\circ\circ\circ}$	0.1089$^{\circ\circ\circ}$	0.1097$^{\circ\circ\circ}$	0.1138$^{\circ\circ\circ}$	0.1188$^{\circ\circ\circ}$
0.0	aw	0.1127$^{\circ\circ\circ}$	0.1126$^{\circ\circ\circ}$	0.1158$^{\circ\circ\circ}$	0.1076$^{\circ\circ\circ}$	0.1059$^{\circ\circ\circ}$	0.1089$^{\circ\circ\circ}$	0.0940°	0.0968$^{\circ\circ}$	0.1036$^{\circ\circ\circ}$
	ns	0.1042$^{\circ\circ\circ}$	0.1044$^{\circ\circ\circ}$	0.1061$^{\circ\circ\circ}$	0.1030$^{\circ\circ\circ}$	0.1000$^{\circ\circ\circ}$	0.1016$^{\circ\circ\circ}$	0.1014$^{\circ\circ\circ}$	0.1019$^{\circ\circ\circ}$	0.1013$^{\circ\circ\circ}$
	ls	0.1035$^{\circ\circ\circ}$	0.1018$^{\circ\circ\circ}$	0.1001$^{\circ\circ\circ}$	0.1027$^{\circ\circ\circ}$	0.0991$^{\circ\circ\circ}$	0.0994$^{\circ\circ\circ}$	0.1009$^{\circ\circ\circ}$	0.1028$^{\circ\circ\circ}$	0.1009$^{\circ\circ\circ}$
	cs	0.1035$^{\circ\circ\circ}$	0.1018$^{\circ\circ\circ}$	0.1001$^{\circ\circ\circ}$	0.1030$^{\circ\circ\circ}$	0.0968$^{\circ\circ\circ}$	0.0924$^{\circ\circ}$	0.1019$^{\circ\circ\circ}$	0.0988$^{\circ\circ\circ}$	0.0903
	t	0.1467	0.1323	0.1242	0.1483	0.1335	0.1265	0.1502	0.1381	0.1305
	f	0.1777$^{\circ\circ\circ}$	0.1634$^{\circ\circ\circ}$	0.1524$^{\circ\circ\circ}$	0.1761$^{\circ\circ\circ}$	0.1583$^{\circ\circ\circ}$	0.1482$^{\circ\circ\circ}$	0.1737$^{\circ\circ\circ}$	0.1601$^{\circ\circ\circ}$	0.1496$^{\circ\circ\circ}$
	w	0.2073$^{\circ\circ\circ}$	0.1864$^{\circ\circ\circ}$	0.1762$^{\circ\circ\circ}$	0.2196$^{\circ\circ\circ}$	0.1955$^{\circ\circ\circ}$	0.1795$^{\circ\circ\circ}$	0.2588$^{\circ\circ\circ}$	0.2232$^{\circ\circ\circ}$	0.2053$^{\circ\circ\circ}$
0.5	aw	0.2073$^{\circ\circ\circ}$	0.1864$^{\circ\circ\circ}$	0.1762$^{\circ\circ\circ}$	0.2196$^{\circ\circ\circ}$	0.1955$^{\circ\circ\circ}$	0.1795$^{\circ\circ\circ}$	0.2278$^{\circ\circ\circ}$	0.1975$^{\circ\circ\circ}$	0.1826$^{\circ\circ\circ}$
	ns	0.1923$^{\circ\circ\circ}$	0.1709$^{\circ\circ\circ}$	0.1617$^{\circ\circ\circ}$	0.2037$^{\circ\circ\circ}$	0.1783$^{\circ\circ\circ}$	0.1636$^{\circ\circ\circ}$	0.2204$^{\circ\circ\circ}$	0.1904$^{\circ\circ\circ}$	0.1738$^{\circ\circ\circ}$
	ls	0.1811$^{\circ\circ\circ}$	0.1607$^{\circ\circ\circ}$	0.1481$^{\circ\circ\circ}$	0.1994$^{\circ\circ\circ}$	0.1736$^{\circ\circ\circ}$	0.1599$^{\circ\circ\circ}$	0.2145$^{\circ\circ\circ}$	0.1858$^{\circ\circ\circ}$	0.1667$^{\circ\circ\circ}$
	cs	0.1811$^{\circ\circ\circ}$	0.1607$^{\circ\circ\circ}$	0.1481$^{\circ\circ\circ}$	0.1777$^{\circ\circ\circ}$	0.1550$^{\circ\circ\circ}$	0.1382$^{\circ\circ\circ}$	0.1759$^{\circ\circ\circ}$	0.1521$^{\circ\circ\circ}$	0.1328
	t	0.2248	0.1919	0.1700	0.2326	0.1976	0.1753	0.2313	0.2002	0.1809
	f	0.2733$^{\circ\circ\circ}$	0.2338$^{\circ\circ\circ}$	0.2094$^{\circ\circ\circ}$	0.2676$^{\circ\circ\circ}$	0.2299$^{\circ\circ\circ}$	0.2049$^{\circ\circ\circ}$	0.2677$^{\circ\circ\circ}$	0.2285$^{\circ\circ\circ}$	0.2061$^{\circ\circ\circ}$
	w	0.3323$^{\circ\circ\circ}$	0.2848$^{\circ\circ\circ}$	0.2508$^{\circ\circ\circ}$	0.3795$^{\circ\circ\circ}$	0.3149$^{\circ\circ\circ}$	0.2752$^{\circ\circ\circ}$	0.4449$^{\circ\circ\circ}$	0.3701$^{\circ\circ\circ}$	0.3201$^{\circ\circ\circ}$
1.0	aw	0.3323$^{\circ\circ\circ}$	0.2848$^{\circ\circ\circ}$	0.2508$^{\circ\circ\circ}$	0.3795$^{\circ\circ\circ}$	0.3149$^{\circ\circ\circ}$	0.2752$^{\circ\circ\circ}$	0.4094$^{\circ\circ\circ}$	0.3339$^{\circ\circ\circ}$	0.2866$^{\circ\circ\circ}$
	ns	0.3071$^{\circ\circ\circ}$	0.2602$^{\circ\circ\circ}$	0.2270$^{\circ\circ\circ}$	0.3372$^{\circ\circ\circ}$	0.2808$^{\circ\circ\circ}$	0.2447$^{\circ\circ\circ}$	0.3758$^{\circ\circ\circ}$	0.3080$^{\circ\circ\circ}$	0.2661$^{\circ\circ\circ}$
	ls	0.2755$^{\circ\circ\circ}$	0.2332$^{\circ\circ\circ}$	0.2019$^{\circ\circ\circ}$	0.3305$^{\circ\circ\circ}$	0.2741$^{\circ\circ\circ}$	0.2369$^{\circ\circ\circ}$	0.3577$^{\circ\circ\circ}$	0.2935$^{\circ\circ\circ}$	0.2569$^{\circ\circ\circ}$
	cs	0.2755$^{\circ\circ\circ}$	0.2332$^{\circ\circ\circ}$	0.2019$^{\circ\circ\circ}$	0.2655$^{\circ\circ\circ}$	0.2230$^{\circ\circ\circ}$	0.1898$^{\circ\circ\circ}$	0.2540$^{\circ\circ\circ}$	0.2146$^{\circ\circ\circ}$	0.1838
(b) $\alpha = 0.05$										
	t	0.0274	0.0264	0.0283	0.0279	0.0285	0.0301	0.0305	0.0321	0.0326
	f	0.0536$^{\circ\circ\circ}$	0.0527$^{\circ\circ\circ}$	0.0529$^{\circ\circ\circ}$	0.0488$^{\circ\circ\circ}$	0.0515$^{\circ\circ\circ}$	0.0535$^{\circ\circ\circ}$	0.0502$^{\circ\circ\circ}$	0.0513$^{\circ\circ\circ}$	0.0526$^{\circ\circ\circ}$
	w	0.0759$^{\circ\circ\circ}$	0.0777$^{\circ\circ\circ}$	0.0794$^{\circ\circ\circ}$	0.0623$^{\circ\circ\circ}$	0.0631$^{\circ\circ\circ}$	0.0676$^{\circ\circ\circ}$	0.0576$^{\circ\circ\circ}$	0.0586$^{\circ\circ\circ}$	0.0617$^{\circ\circ\circ}$
0.0	aw	0.0486$^{\circ\circ\circ}$	0.0497$^{\circ\circ\circ}$	0.0498$^{\circ\circ\circ}$	0.0452$^{\circ\circ\circ}$	0.0477$^{\circ\circ\circ}$	0.0503$^{\circ\circ\circ}$	0.0469$^{\circ\circ\circ}$	0.0489$^{\circ\circ\circ}$	0.0511$^{\circ\circ\circ}$
	ns	0.0565$^{\circ\circ\circ}$	0.0564$^{\circ\circ\circ}$	0.0562$^{\circ\circ\circ}$	0.0490$^{\circ\circ\circ}$	0.0492$^{\circ\circ\circ}$	0.0516$^{\circ\circ\circ}$	0.0505$^{\circ\circ\circ}$	0.0510$^{\circ\circ\circ}$	0.0527$^{\circ\circ\circ}$
	ls	0.0565$^{\circ\circ\circ}$	0.0564$^{\circ\circ\circ}$	0.0562$^{\circ\circ\circ}$	0.0497$^{\circ\circ\circ}$	0.0498$^{\circ\circ\circ}$	0.0515$^{\circ\circ\circ}$	0.0495$^{\circ\circ\circ}$	0.0502$^{\circ\circ\circ}$	0.0506$^{\circ\circ\circ}$
	cs	0.0504$^{\circ\circ\circ}$	0.0499$^{\circ\circ\circ}$	0.0481$^{\circ\circ\circ}$	0.0495$^{\circ\circ\circ}$	0.0491$^{\circ\circ\circ}$	0.0479$^{\circ\circ\circ}$	0.0500$^{\circ\circ\circ}$	0.0500$^{\circ\circ\circ}$	0.0464$^{\circ\circ\circ}$
	t	0.0605	0.0531	0.0503	0.0632	0.0544	0.0503	0.0670	0.0583	0.0548
	f	0.1038$^{\circ\circ\circ}$	0.0917$^{\circ\circ\circ}$	0.0870$^{\circ\circ\circ}$	0.1061$^{\circ\circ\circ}$	0.0919$^{\circ\circ\circ}$	0.0850$^{\circ\circ\circ}$	0.1057$^{\circ\circ\circ}$	0.0927$^{\circ\circ\circ}$	0.0865$^{\circ\circ\circ}$
	w	0.1505$^{\circ\circ\circ}$	0.1362$^{\circ\circ\circ}$	0.1283$^{\circ\circ\circ}$	0.1529$^{\circ\circ\circ}$	0.1306$^{\circ\circ\circ}$	0.1187$^{\circ\circ\circ}$	0.1520$^{\circ\circ\circ}$	0.1336$^{\circ\circ\circ}$	0.1206$^{\circ\circ\circ}$
0.5	aw	0.0993$^{\circ\circ\circ}$	0.0872$^{\circ\circ\circ}$	0.0799$^{\circ\circ\circ}$	0.1215$^{\circ\circ\circ}$	0.1044$^{\circ\circ\circ}$	0.0956$^{\circ\circ\circ}$	0.1292$^{\circ\circ\circ}$	0.1143$^{\circ\circ\circ}$	0.1022$^{\circ\circ\circ}$
	ns	0.1112$^{\circ\circ\circ}$	0.0982$^{\circ\circ\circ}$	0.0889$^{\circ\circ\circ}$	0.1184$^{\circ\circ\circ}$	0.1012$^{\circ\circ\circ}$	0.0884$^{\circ\circ\circ}$	0.1292$^{\circ\circ\circ}$	0.1118$^{\circ\circ\circ}$	0.0989$^{\circ\circ\circ}$
	ls	0.1112$^{\circ\circ\circ}$	0.0982$^{\circ\circ\circ}$	0.0889$^{\circ\circ\circ}$	0.1180$^{\circ\circ\circ}$	0.0999$^{\circ\circ\circ}$	0.0870$^{\circ\circ\circ}$	0.1260$^{\circ\circ\circ}$	0.1087$^{\circ\circ\circ}$	0.0949$^{\circ\circ\circ}$
	cs	0.1008$^{\circ\circ\circ}$	0.0888$^{\circ\circ\circ}$	0.0792$^{\circ\circ\circ}$	0.1074$^{\circ\circ\circ}$	0.0899$^{\circ\circ\circ}$	0.0764$^{\circ\circ\circ}$	0.1097$^{\circ\circ\circ}$	0.0916$^{\circ\circ\circ}$	0.0775$^{\circ\circ\circ}$
	t	0.1159	0.0944	0.0802	0.1202	0.0938	0.0808	0.1220	0.0992	0.0852
	f	0.1799$^{\circ\circ\circ}$	0.1499$^{\circ\circ\circ}$	0.1310$^{\circ\circ\circ}$	0.1813$^{\circ\circ\circ}$	0.1477$^{\circ\circ\circ}$	0.1281$^{\circ\circ\circ}$	0.1818$^{\circ\circ\circ}$	0.1499$^{\circ\circ\circ}$	0.1304$^{\circ\circ\circ}$
	w	0.2607$^{\circ\circ\circ}$	0.2165$^{\circ\circ\circ}$	0.1900$^{\circ\circ\circ}$	0.2792$^{\circ\circ\circ}$	0.2247$^{\circ\circ\circ}$	0.1908$^{\circ\circ\circ}$	0.3027$^{\circ\circ\circ}$	0.2401$^{\circ\circ\circ}$	0.2018$^{\circ\circ\circ}$
1.0	aw	0.1791$^{\circ\circ\circ}$	0.1457$^{\circ\circ\circ}$	0.1281$^{\circ\circ\circ}$	0.2333$^{\circ\circ\circ}$	0.1877$^{\circ\circ\circ}$	0.1565$^{\circ\circ\circ}$	0.2708$^{\circ\circ\circ}$	0.2124$^{\circ\circ\circ}$	0.1773$^{\circ\circ\circ}$
	ns	0.1917$^{\circ\circ\circ}$	0.1585$^{\circ\circ\circ}$	0.1367$^{\circ\circ\circ}$	0.2137$^{\circ\circ\circ}$	0.1712$^{\circ\circ\circ}$	0.1421$^{\circ\circ\circ}$	0.2496$^{\circ\circ\circ}$	0.1976$^{\circ\circ\circ}$	0.1621$^{\circ\circ\circ}$
	ls	0.1917$^{\circ\circ\circ}$	0.1585$^{\circ\circ\circ}$	0.1367$^{\circ\circ\circ}$	0.2084$^{\circ\circ\circ}$	0.1699$^{\circ\circ\circ}$	0.1415$^{\circ\circ\circ}$	0.2385$^{\circ\circ\circ}$	0.1877$^{\circ\circ\circ}$	0.1524$^{\circ\circ\circ}$
	cs	0.1695$^{\circ\circ\circ}$	0.1409$^{\circ\circ\circ}$	0.1176$^{\circ\circ\circ}$	0.1817$^{\circ\circ\circ}$	0.1459$^{\circ\circ\circ}$	0.1178$^{\circ\circ\circ}$	0.1792$^{\circ\circ\circ}$	0.1450$^{\circ\circ\circ}$	0.1168$^{\circ\circ\circ}$
(c) $\alpha = 0.01$										
	t	0.0019	0.0019	0.0022	0.0020	0.0018	0.0024	0.0031	0.0030	0.0033
	f	0.0129$^{\circ\circ\circ}$	0.0141$^{\circ\circ\circ}$	0.0148$^{\circ\circ\circ}$	0.0094$^{\circ\circ\circ}$	0.0097$^{\circ\circ\circ}$	0.0111$^{\circ\circ\circ}$	0.0106$^{\circ\circ\circ}$	0.0105$^{\circ\circ\circ}$	0.0116$^{\circ\circ\circ}$
	w	0.0170$^{\circ\circ\circ}$	0.0180$^{\circ\circ\circ}$	0.0183$^{\circ\circ\circ}$	0.0121$^{\circ\circ\circ}$	0.0125$^{\circ\circ\circ}$	0.0135$^{\circ\circ\circ}$	0.0130$^{\circ\circ\circ}$	0.0134$^{\circ\circ\circ}$	0.0143$^{\circ\circ\circ}$
0.0	aw	0.0090$^{\circ\circ\circ}$	0.0089$^{\circ\circ\circ}$	0.0104$^{\circ\circ\circ}$	0.0084$^{\circ\circ\circ}$	0.0077$^{\circ\circ\circ}$	0.0086$^{\circ\circ\circ}$	0.0100$^{\circ\circ\circ}$	0.0101$^{\circ\circ\circ}$	0.0117$^{\circ\circ\circ}$
	ns	0.0170$^{\circ\circ\circ}$	0.0180$^{\circ\circ\circ}$	0.0183$^{\circ\circ\circ}$	0.0094$^{\circ\circ\circ}$	0.0096$^{\circ\circ\circ}$	0.0101$^{\circ\circ\circ}$	0.0105$^{\circ\circ\circ}$	0.0105$^{\circ\circ\circ}$	0.0118$^{\circ\circ\circ}$
	ls	0.0170$^{\circ\circ\circ}$	0.0180$^{\circ\circ\circ}$	0.0183$^{\circ\circ\circ}$	0.0089$^{\circ\circ\circ}$	0.0089$^{\circ\circ\circ}$	0.0094$^{\circ\circ\circ}$	0.0105$^{\circ\circ\circ}$	0.0110$^{\circ\circ\circ}$	0.0112$^{\circ\circ\circ}$
	cs	0.0170$^{\circ\circ\circ}$	0.0180$^{\circ\circ\circ}$	0.0183$^{\circ\circ\circ}$	0.0088$^{\circ\circ\circ}$	0.0088$^{\circ\circ\circ}$	0.0095$^{\circ\circ\circ}$	0.0115$^{\circ\circ\circ}$	0.0106$^{\circ\circ\circ}$	0.0098$^{\circ\circ\circ}$
	t	0.0077	0.0060	0.0054	0.0079	0.0069	0.0061	0.0100	0.0089	0.0083
	f	0.0336$^{\circ\circ\circ}$	0.0289$^{\circ\circ\circ}$	0.0269$^{\circ\circ\circ}$	0.0281$^{\circ\circ\circ}$	0.0258$^{\circ\circ\circ}$	0.0235$^{\circ\circ\circ}$	0.0341$^{\circ\circ\circ}$	0.0278$^{\circ\circ\circ}$	0.0244$^{\circ\circ\circ}$
	w	0.0418$^{\circ\circ\circ}$	0.0363$^{\circ\circ\circ}$	0.0327$^{\circ\circ\circ}$	0.0387$^{\circ\circ\circ}$	0.0337$^{\circ\circ\circ}$	0.0298$^{\circ\circ\circ}$	0.0455$^{\circ\circ\circ}$	0.0385$^{\circ\circ\circ}$	0.0338$^{\circ\circ\circ}$
0.5	aw	0.0240$^{\circ\circ\circ}$	0.0212$^{\circ\circ\circ}$	0.0201$^{\circ\circ\circ}$	0.0262$^{\circ\circ\circ}$	0.0239$^{\circ\circ\circ}$	0.0206$^{\circ\circ\circ}$	0.0375$^{\circ\circ\circ}$	0.0313$^{\circ\circ\circ}$	0.0266$^{\circ\circ\circ}$
	ns	0.0418$^{\circ\circ\circ}$	0.0363$^{\circ\circ\circ}$	0.0327$^{\circ\circ\circ}$	0.0309$^{\circ\circ\circ}$	0.0272$^{\circ\circ\circ}$	0.0233$^{\circ\circ\circ}$	0.0360$^{\circ\circ\circ}$	0.0311$^{\circ\circ\circ}$	0.0261$^{\circ\circ\circ}$
	ls	0.0418$^{\circ\circ\circ}$	0.0363$^{\circ\circ\circ}$	0.0327$^{\circ\circ\circ}$	0.0295$^{\circ\circ\circ}$	0.0257$^{\circ\circ\circ}$	0.0220$^{\circ\circ\circ}$	0.0349$^{\circ\circ\circ}$	0.0305$^{\circ\circ\circ}$	0.0251$^{\circ\circ\circ}$
	cs	0.0418$^{\circ\circ\circ}$	0.0363$^{\circ\circ\circ}$	0.0327$^{\circ\circ\circ}$	0.0292$^{\circ\circ\circ}$	0.0256$^{\circ\circ\circ}$	0.0218$^{\circ\circ\circ}$	0.0332$^{\circ\circ\circ}$	0.0277$^{\circ\circ\circ}$	0.0228$^{\circ\circ\circ}$
	t	0.0227	0.0161	0.0120	0.0281	0.0178	0.0132	0.0302	0.0210	0.0172
	f	0.0738$^{\circ\circ\circ}$	0.0574$^{\circ\circ\circ}$	0.0476$^{\circ\circ\circ}$	0.0743$^{\circ\circ\circ}$	0.0563$^{\circ\circ\circ}$	0.0475$^{\circ\circ\circ}$	0.0758$^{\circ\circ\circ}$	0.0581$^{\circ\circ\circ}$	0.0487$^{\circ\circ\circ}$
	w	0.0863$^{\circ\circ\circ}$	0.0670$^{\circ\circ\circ}$	0.0561$^{\circ\circ\circ}$	0.1001$^{\circ\circ\circ}$	0.0730$^{\circ\circ\circ}$	0.0607$^{\circ\circ\circ}$	0.1168$^{\circ\circ\circ}$	0.0857$^{\circ\circ\circ}$	0.0715$^{\circ\circ\circ}$
1.0	aw	0.0548$^{\circ\circ\circ}$	0.0430$^{\circ\circ\circ}$	0.0367$^{\circ\circ\circ}$	0.0719$^{\circ\circ\circ}$	0.0524$^{\circ\circ\circ}$	0.0427$^{\circ\circ\circ}$	0.0967$^{\circ\circ\circ}$	0.0709$^{\circ\circ\circ}$	0.0579$^{\circ\circ\circ}$
	ns	0.0863$^{\circ\circ\circ}$	0.0670$^{\circ\circ\circ}$	0.0561$^{\circ\circ\circ}$	0.0790$^{\circ\circ\circ}$	0.0571$^{\circ\circ\circ}$	0.0464$^{\circ\circ\circ}$	0.0911$^{\circ\circ\circ}$	0.0663$^{\circ\circ\circ}$	0.0542$^{\circ\circ\circ}$
	ls	0.0863$^{\circ\circ\circ}$	0.0670$^{\circ\circ\circ}$	0.0561$^{\circ\circ\circ}$	0.0727$^{\circ\circ\circ}$	0.0540$^{\circ\circ\circ}$	0.0449$^{\circ\circ\circ}$	0.0875$^{\circ\circ\circ}$	0.0631$^{\circ\circ\circ}$	0.0521$^{\circ\circ\circ}$
	cs	0.0863$^{\circ\circ\circ}$	0.0670$^{\circ\circ\circ}$	0.0561$^{\circ\circ\circ}$	0.0722$^{\circ\circ\circ}$	0.0530$^{\circ\circ\circ}$	0.0444$^{\circ\circ\circ}$	0.0734$^{\circ\circ\circ}$	0.0539$^{\circ\circ\circ}$	0.0438$^{\circ\circ\circ}$

Table 6: $\chi^2(6)/2$ Distribution

$\mu \backslash \sigma$		$n1 = n2 = 5$			$n1 = n2 = 7$			$n1 = n2 = 9$		
		1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.5	2.0
(a) $\alpha = 0.10$										
	t	0.0941	0.0827	0.0771	0.1020	0.0842	0.0759	0.1006	0.0871	0.0819
	f	0.0974°	0.0858°	0.0799°	0.1014	0.0820	0.0739	0.0987	0.0864	0.0798
	w	$0.1044^{\circ\circ\circ}$	0.0824	0.0790	0.1043	$0.0744^{\times\times\times}$	$0.0658^{\times\times\times}$	$0.1085^{\circ\circ}$	$0.0732^{\times\times\times}$	$0.0649^{\times\times\times}$
0.0	aw	$0.1044^{\circ\circ\circ}$	0.0824	0.0790	0.1043	$0.0744^{\times\times\times}$	$0.0658^{\times\times\times}$	$0.0928^{\times\times}$	$0.0622^{\times\times\times}$	$0.0553^{\times\times\times}$
	ns	0.0979°	$0.0738^{\times\times\times}$	$0.0656^{\times\times\times}$	0.0994	$0.0629^{\times\times\times}$	$0.0498^{\times\times\times}$	$0.0973^{\times}$	$0.0588^{\times\times\times}$	$0.0464^{\times\times\times}$
	ls	0.0969	$0.0692^{\times\times\times}$	$0.0545^{\times\times\times}$	0.0994	$0.0600^{\times\times\times}$	$0.0444^{\times\times\times}$	$0.0974^{\times}$	$0.0548^{\times\times\times}$	$0.0412^{\times\times\times}$
	cs	0.0969	$0.0692^{\times\times\times}$	$0.0545^{\times\times\times}$	0.1008	$0.0502^{\times\times\times}$	$0.0292^{\times\times\times}$	0.0989	$0.0408^{\times\times\times}$	$0.0199^{\times\times\times}$
	t	0.2089	0.1508	0.1264	0.2362	0.1775	0.1454	0.2519	0.1923	0.1557
	f	$0.2178^{\circ\circ}$	0.1532	0.1281	0.2351	0.1750	$0.1404^{\times}$	0.2506	0.1888	$0.1514^{\times}$
	w	$0.2296^{\circ\circ\circ}$	0.1532	$0.1221^{\times}$	$0.2506^{\circ\circ\circ}$	$0.1540^{\times\times\times}$	$0.1177^{\times\times\times}$	$0.2869^{\circ\circ\circ}$	$0.1682^{\times\times\times}$	$0.1256^{\times\times\times}$
0.5	aw	$0.2296^{\circ\circ\circ}$	0.1532	$0.1221^{\times}$	$0.2506^{\circ\circ\circ}$	$0.1540^{\times\times\times}$	$0.1177^{\times\times\times}$	0.2563°	$0.1484^{\times\times\times}$	$0.1081^{\times\times\times}$
	ns	0.2167°	$0.1415^{\times\times}$	$0.1081^{\times\times\times}$	$0.2482^{\circ\circ}$	$0.1404^{\times\times\times}$	$0.0959^{\times\times\times}$	$0.2725^{\circ\circ\circ}$	$0.1435^{\times\times\times}$	$0.0961^{\times\times\times}$
	ls	$0.2214^{\circ\circ\circ}$	$0.1349^{\times\times\times}$	$0.0922^{\times\times\times}$	$0.2497^{\circ\circ\circ}$	$0.1382^{\times\times\times}$	$0.0901^{\times\times\times}$	$0.2762^{\circ\circ\circ}$	$0.1394^{\times\times\times}$	$0.0896^{\times\times\times}$
	cs	$0.2214^{\circ\circ\circ}$	$0.1349^{\times\times\times}$	$0.0922^{\times\times\times}$	$0.2480^{\circ\circ}$	$0.1237^{\times\times\times}$	$0.0640^{\times\times\times}$	$0.2722^{\circ\circ\circ}$	$0.1118^{\times\times\times}$	$0.0496^{\times\times\times}$
	t	0.3630	0.2578	0.1947	0.4304	0.3164	0.2458	0.4710	0.3520	0.2709
	f	$0.3732^{\circ\circ}$	0.2634°	0.1970	0.4319	0.3144	0.2429	0.4700	0.3484	0.2673
	w	$0.3965^{\circ\circ\circ}$	0.2592	$0.1889^{\times}$	$0.4652^{\circ\circ\circ}$	$0.2938^{\times\times\times}$	$0.2040^{\times\times\times}$	$0.5249^{\circ\circ\circ}$	$0.3344^{\times\times\times}$	$0.2213^{\times\times\times}$
1.0	aw	$0.3965^{\circ\circ\circ}$	0.2592	$0.1889^{\times}$	$0.4652^{\circ\circ\circ}$	$0.2938^{\times\times\times}$	$0.2040^{\times\times\times}$	$0.4898^{\circ\circ\circ}$	$0.3003^{\times\times\times}$	$0.1957^{\times\times\times}$
	ns	$0.3789^{\circ\circ\circ}$	$0.2437^{\times\times\times}$	$0.1685^{\times\times\times}$	$0.4609^{\circ\circ\circ}$	$0.2796^{\times\times\times}$	$0.1754^{\times\times\times}$	$0.5132^{\circ\circ\circ}$	$0.3057^{\times\times\times}$	$0.1786^{\times\times\times}$
	ls	$0.3773^{\circ\circ}$	$0.2386^{\times\times\times}$	$0.1513^{\times\times\times}$	$0.4598^{\circ\circ\circ}$	$0.2751^{\times\times\times}$	$0.1654^{\times\times\times}$	$0.5155^{\circ\circ\circ}$	$0.3007^{\times\times\times}$	$0.1672^{\times\times\times}$
	cs	$0.3773^{\circ\circ}$	$0.2386^{\times\times\times}$	$0.1513^{\times\times\times}$	$0.4439^{\circ\circ}$	$0.2568^{\times\times\times}$	$0.1287^{\times\times\times}$	$0.4832^{\circ\circ}$	$0.2607^{\times\times\times}$	$0.1044^{\times\times\times}$
(b) $\alpha = 0.05$										
	t	0.0453	0.0347	0.0318	0.0490	0.0375	0.0325	0.0500	0.0401	0.0359
	f	0.0483°	0.0368°	0.0347°	0.0503	0.0382	0.0343°	0.0505	0.0410	0.0362
	w	$0.0724^{\circ\circ\circ}$	$0.0556^{\circ\circ\circ}$	$0.0523^{\circ\circ\circ}$	$0.0639^{\circ\circ\circ}$	$0.0433^{\circ\circ\circ}$	$0.0366^{\circ\circ}$	$0.0574^{\circ\circ\circ}$	0.0384	$0.0337^{\times}$
0.0	aw	0.0460	$0.0318^{\times}$	$0.0275^{\times\times}$	0.0491	$0.0321^{\times\times}$	$0.0284^{\times\times}$	0.0482	$0.0322^{\times\times\times}$	$0.0278^{\times\times\times}$
	ns	$0.0533^{\circ\circ\circ}$	0.0371°	0.0322	0.0508	$0.0302^{\times\times\times}$	$0.0232^{\times\times\times}$	0.0514	$0.0287^{\times\times\times}$	$0.0217^{\times\times\times}$
	ls	$0.0533^{\circ\circ\circ}$	0.0371°	0.0322	0.0504	$0.0297^{\times\times\times}$	$0.0223^{\times\times\times}$	0.0507	$0.0265^{\times\times\times}$	$0.0183^{\times\times\times}$
	cs	$0.0496^{\circ\circ}$	0.0341	$0.0298^{\times}$	0.0510	$0.0268^{\times\times\times}$	$0.0168^{\times\times\times}$	0.0498	$0.0228^{\times\times\times}$	$0.0120^{\times\times\times}$
	t	0.1078	0.0750	0.0597	0.1349	0.0858	0.0665	0.1511	0.0990	0.0789
	f	0.1138°	$0.0818^{\circ\circ}$	$0.0660^{\circ\circ}$	0.1392°	0.0898°	0.0709°	0.1532	0.1014	0.0806
	w	$0.1622^{\circ\circ\circ}$	$0.1083^{\circ\circ\circ}$	$0.0895^{\circ\circ\circ}$	$0.1734^{\circ\circ\circ}$	$0.0993^{\circ\circ\circ}$	$0.0719^{\circ\circ}$	$0.1729^{\circ\circ\circ}$	$0.0959^{\times}$	$0.0690^{\times\times\times}$
0.5	aw	0.1130°	$0.0685^{\times\times}$	$0.0526^{\times\times}$	0.1406°	$0.0787^{\times\times}$	$0.0574^{\times\times\times}$	0.1482	$0.0827^{\times\times\times}$	$0.0593^{\times\times\times}$
	ns	$0.1288^{\circ\circ\circ}$	0.0782°	0.0593	$0.1444^{\circ\circ}$	$0.0739^{\times\times\times}$	$0.0482^{\times\times\times}$	$0.1632^{\circ\circ\circ}$	$0.0795^{\times\times\times}$	$0.0499^{\times\times\times}$
	ls	$0.1288^{\circ\circ\circ}$	0.0782°	0.0593	$0.1449^{\circ\circ}$	$0.0739^{\times\times\times}$	$0.0476^{\times\times\times}$	$0.1633^{\circ\circ\circ}$	$0.0771^{\times\times\times}$	$0.0455^{\times\times\times}$
	cs	$0.1191^{\circ\circ\circ}$	0.0752	$0.0535^{\times\times}$	$0.1461^{\circ\circ\circ}$	$0.0696^{\times\times\times}$	$0.0394^{\times\times\times}$	$0.1622^{\circ\circ\circ}$	$0.0683^{\times\times\times}$	$0.0296^{\times\times\times}$
	t	0.2250	0.1360	0.1016	0.2882	0.1850	0.1264	0.3348	0.2185	0.1500
	f	$0.2369^{\circ\circ}$	$0.1483^{\circ\circ\circ}$	$0.1115^{\circ\circ\circ}$	0.2937°	0.1916°	0.1324°	0.3375	0.2220	0.1535
	w	$0.3062^{\circ\circ\circ}$	$0.1885^{\circ\circ\circ}$	$0.1418^{\circ\circ\circ}$	$0.3543^{\circ\circ\circ}$	$0.2062^{\circ\circ\circ}$	$0.1331^{\circ\circ}$	$0.3848^{\circ\circ\circ}$	0.2149	$0.1345^{\times\times\times}$
1.0	aw	0.2258	$0.1322^{\times}$	$0.0914^{\times\times\times}$	$0.3011^{\circ\circ}$	$0.1660^{\times\times\times}$	$0.1057^{\times\times\times}$	$0.3483^{\circ\circ}$	$0.1858^{\times\times\times}$	$0.1172^{\times\times\times}$
	ns	$0.2530^{\circ\circ\circ}$	$0.1475^{\circ\circ\circ}$	0.1013	$0.3133^{\circ\circ\circ}$	$0.1664^{\times\times\times}$	$0.0944^{\times\times\times}$	$0.3665^{\circ\circ\circ}$	$0.1888^{\times\times\times}$	$0.1039^{\times\times\times}$
	ls	$0.2530^{\circ\circ\circ}$	$0.1475^{\circ\circ\circ}$	0.1013	$0.3151^{\circ\circ\circ}$	$0.1676^{\times\times\times}$	$0.0932^{\times\times\times}$	$0.3643^{\circ\circ\circ}$	$0.1853^{\times\times\times}$	$0.0962^{\times\times\times}$
	cs	$0.2385^{\circ\circ\circ}$	0.1379	$0.0934^{\times\times}$	$0.3091^{\circ\circ\circ}$	$0.1599^{\times\times\times}$	$0.0803^{\times\times\times}$	$0.3542^{\circ\circ\circ}$	$0.1696^{\times\times\times}$	$0.0728^{\times\times\times}$
(c) $\alpha = 0.01$										
	t	0.0084	0.0066	0.0067	0.0082	0.0054	0.0051	0.0089	0.0064	0.0057
	f	$0.0115^{\circ\circ\circ}$	$0.0102^{\circ\circ\circ}$	$0.0095^{\circ\circ\circ}$	0.0094°	0.0065°	0.0065°	0.0095	$0.0084^{\circ\circ}$	0.0070°
	w	$0.0146^{\circ\circ\circ}$	$0.0125^{\circ\circ\circ}$	$0.0117^{\circ\circ\circ}$	$0.0125^{\circ\circ\circ}$	$0.0081^{\circ\circ\circ}$	$0.0076^{\circ\circ\circ}$	$0.0123^{\circ\circ\circ}$	0.0074°	0.0065°
0.0	aw	0.0089	0.0066	0.0061	0.0082	0.0050	0.0045	0.0096	0.0062	0.0053
	ns	$0.0146^{\circ\circ\circ}$	$0.0125^{\circ\circ\circ}$	$0.0117^{\circ\circ\circ}$	$0.0103^{\circ\circ}$	0.0060	0.0051	0.0101°	0.0057	$0.0044^{\times}$
	ls	$0.0146^{\circ\circ\circ}$	$0.0125^{\circ\circ\circ}$	$0.0117^{\circ\circ\circ}$	0.0099°	0.0055	0.0048	0.0104°	0.0057	$0.0043^{\times}$
	cs	$0.0146^{\circ\circ\circ}$	$0.0125^{\circ\circ\circ}$	$0.0117^{\circ\circ\circ}$	0.0099°	0.0060	0.0048	0.0105°	$0.0056^{\times}$	$0.0038^{\times\times}$
	t	0.0232	0.0143	0.0116	0.0328	0.0175	0.0111	0.0411	0.0203	0.0151
	f	$0.0304^{\circ\circ\circ}$	$0.0186^{\circ\circ\circ}$	$0.0170^{\circ\circ\circ}$	$0.0374^{\circ\circ}$	$0.0214^{\circ\circ}$	$0.0147^{\circ\circ\circ}$	0.0440°	$0.0254^{\circ\circ\circ}$	$0.0185^{\circ\circ}$
	w	$0.0401^{\circ\circ\circ}$	$0.0237^{\circ\circ\circ}$	$0.0203^{\circ\circ\circ}$	$0.0465^{\circ\circ\circ}$	$0.0243^{\circ\circ\circ}$	$0.0170^{\circ\circ\circ}$	$0.0516^{\circ\circ\circ}$	$0.0272^{\circ\circ\circ}$	0.0168°
0.5	aw	$0.0212^{\times}$	0.0132	0.0112	0.0328	$0.0156^{\times}$	0.0104	0.0405	0.0213	$0.0133^{\times}$
	ns	$0.0401^{\circ\circ\circ}$	$0.0237^{\circ\circ\circ}$	$0.0203^{\circ\circ\circ}$	$0.0387^{\circ\circ\circ}$	0.0189°	0.0122°	$0.0460^{\circ\circ}$	0.0214	$0.0114^{\times\times\times}$
	ls	$0.0401^{\circ\circ\circ}$	$0.0237^{\circ\circ\circ}$	$0.0203^{\circ\circ\circ}$	$0.0381^{\circ\circ}$	0.0180	0.0118	$0.0467^{\circ\circ}$	0.0211	$0.0107^{\times\times\times}$
	cs	$0.0401^{\circ\circ\circ}$	$0.0237^{\circ\circ\circ}$	$0.0203^{\circ\circ\circ}$	$0.0384^{\circ\circ\circ}$	0.0180	0.0116	$0.0476^{\circ\circ\circ}$	0.0206	$0.0096^{\times\times\times}$
	t	0.0606	0.0302	0.0200	0.0939	0.0424	0.0262	0.1225	0.0589	0.0345
	f	$0.0784^{\circ\circ\circ}$	$0.0430^{\circ\circ\circ}$	$0.0300^{\circ\circ\circ}$	$0.1022^{\circ\circ}$	$0.0515^{\circ\circ\circ}$	$0.0336^{\circ\circ\circ}$	$0.1295^{\circ\circ}$	$0.0674^{\circ\circ\circ}$	$0.0420^{\circ\circ\circ}$
	w	$0.0969^{\circ\circ\circ}$	$0.0545^{\circ\circ\circ}$	$0.0373^{\circ\circ\circ}$	$0.1249^{\circ\circ\circ}$	$0.0587^{\circ\circ\circ}$	$0.0350^{\circ\circ\circ}$	$0.1555^{\circ\circ\circ}$	$0.0687^{\circ\circ\circ}$	$0.0409^{\circ\circ\circ}$
1.0	aw	$0.0553^{\times\times}$	0.0289	0.0197	0.0931	0.0417	$0.0239^{\times}$	$0.1302^{\circ\circ}$	$0.0563^{\times}$	0.0333
	ns	$0.0969^{\circ\circ\circ}$	$0.0545^{\circ\circ\circ}$	$0.0373^{\circ\circ\circ}$	$0.1062^{\circ\circ\circ}$	$0.0480^{\circ\circ}$	0.0266	$0.1403^{\circ\circ\circ}$	0.0586	$0.0312^{\times}$
	ls	$0.0969^{\circ\circ\circ}$	$0.0545^{\circ\circ\circ}$	$0.0373^{\circ\circ\circ}$	$0.1053^{\circ\circ\circ}$	$0.0468^{\circ\circ}$	0.0255	$0.1392^{\circ\circ\circ}$	0.0588	$0.0297^{\times\times}$
	cs	$0.0969^{\circ\circ\circ}$	$0.0545^{\circ\circ\circ}$	$0.0373^{\circ\circ\circ}$	$0.1062^{\circ\circ\circ}$	$0.0467^{\circ\circ}$	0.0259	$0.1356^{\circ\circ\circ}$	$0.0553^{\times}$	$0.0262^{\times\times\times}$

Table 7: Testing Structural Change by Nonparametric Tests

Import Function (7)

Year	t		Fisher		Wilcoson		Asy Wil		Normal		Logistic		Cauchy		Chow	
	t_0	p-val	f_0	p-val	w_0	p-val	aw_0	p-val	ns_0	p-val	ls_0	p-val	cs_0	p-val	F_0	p-val
1961	.9503	.8254	.0848	.8182	28	.8182	1.1552	.8760	.9661	.8182	1.6094	.8182	1.7321	.8182	1.5534	.7788
1962	.4816	.6833	.0312	.6742	40	.6553	.4527	.6746	.5754	.6591	.9837	.6591	1.2165	.6742	1.8903	.8475
1963	.2695	.6053	.0145	.5882	53	.5352	.1252	.5498	.2673	.5634	.4890	.5645	.8162	.5867	1.8224	.8356
1964	.3144	.6223	.0149	.6083	72	.5738	.2206	.5873	.4208	.5874	.7327	.5861	1.0089	.5806	2.0108	.8664
1965	.3106	.6209	.0134	.6086	88	.5484	.1506	.5599	.3455	.5654	.6113	.5652	.9134	.5594	2.5013	.9216
1966	.4048	.6558	.0162	.6462	108	.5988	.2801	.6103	.5756	.6007	.9790	.5966	1.2070	.5659	3.5358	.9736
1967	.8039	.7862	.0302	.7837	137	.7752	.7927	.7860	1.6741	.7585	2.8248	.7473	3.3967	.6506	4.0381	.9841
1968	1.0030	.8382	.0357	.8382	160	.8348	1.0082	.8433	2.1474	.8051	3.5870	.7902	4.0394	.6573	5.2710	.9951
1969	1.2675	.8928	.0431	.8949	186	.9037	1.3339	.9089	2.8961	.8688	4.8108	.8518	5.1935	.6773	7.5633	.9993
1970	1.7732	.9570	.0571	.9602	217	.9660	1.8411	.9672	4.3286	.9496	7.3122	.9399	9.3155	.7485	8.2567	.9996
1971	2.0532	.9757	.0634	.9782	244	.9853	2.1768	.9853	5.1810	.9732	8.7175	.9656	10.7198	.7574	9.5572	.9999
1972	2.1446	.9800	.0646	.9820	265	.9890	2.2829	.9888	5.4891	.9777	9.2122	.9706	11.1202	.7541	11.9225	1.0000
1973	3.0382	.9976	.0847	.9982	298	.9981	2.8370	.9977	7.6592	.9978	13.3866	.9975	32.1127	.9889	10.3183	.9999
1974	3.8916	.9998	.1002	.9999	330	.9998	3.3512	.9996	9.3546	.9998	16.4311	.9998	39.0679	.9997	11.4613	1.0000
1975	3.6195	.9995	.0946	.9996	341	.9993	3.1093	.9991	8.8813	.9995	15.6690	.9995	38.4252	.9995	10.4418	.9999
1976	3.6761	.9996	.0953	.9997	359	.9994	3.1339	.9991	8.9566	.9996	15.7904	.9996	38.5207	.9995	10.4135	.9999
1977	3.7292	.9996	.0962	.9997	376	.9994	3.1339	.9991	8.9566	.9996	15.7904	.9996	38.5207	.9995	10.2567	.9999
1978	3.9506	.9998	.1004	.9998	398	.9997	3.3263	.9996	9.3473	.9998	16.4161	.9998	39.0363	.9997	10.3053	.9999
1979	4.4222	.9999	.1087	1.0000	422	.9999	3.6062	.9998	9.9066	.9999	17.3218	.9999	39.8227	.9999	17.8416	1.0000
1980	4.0266	.9998	.1036	.9998	429	.9997	3.2791	.9995	9.0542	.9997	15.9164	.9997	38.4184	.9996	19.6243	1.0000
1981	3.7299	.9996	.1000	.9996	438	.9991	3.0313	.9988	8.4047	.9994	14.8578	.9994	37.4649	.9994	20.8868	1.0000
1982	3.3324	.9989	.0941	.9986	443	.9963	2.6351	.9958	7.3062	.9975	13.0120	.9977	35.2752	.9980	17.7729	1.0000
1983	2.7212	.9947	.0826	.9936	447	.9860	2.1937	.9859	6.0581	.9905	10.8804	.9913	32.3859	.9934	12.5561	1.0000
1984	2.4844	.9907	.0791	.9887	455	.9704	1.8999	.9713	5.3094	.9823	9.6566	.9845	31.2318	.9914	11.1331	1.0000
1985	1.8828	.9654	.0646	.9614	458	.9122	1.3863	.9172	3.8769	.9421	7.1552	.9488	27.1097	.9731	8.1768	.9996
1986	.7506	.7707	.0282	.7736	459	.7620	.7486	.7730	1.7068	.7628	2.9808	.7590	6.1172	.7361	2.7787	.9418
1987	−.1333	.4474	−.0054	.4598	461	.5272	.0934	.5372	.0114	.5021	−.0637	.4936	−.8380	.4540	.8033	.4981
1988	−.4836	.3160	−.0209	.3251	467	.3217	−.4519	.3257	−.9547	.3246	−1.6732	.3257	−2.5700	.3448	.5065	.3192
1989	−.1691	.4334	−.0080	.4502	492	.4682	−.0552	.4780	−.3052	.4371	−.6146	.4277	−1.6165	.3752	.5910	.3743
1990	.4115	.6582	.0222	.6783	522	.7544	.7515	.7738	.9429	.7122	1.5171	.6974	1.2728	.6296		
1991	.3710	.6434	.0241	.6723	536	.7273	.6790	.7514	.7128	.6951	1.1493	.6856	.9791	.6458		
1992	.1119	.5442	.0101	.5455	546	.5455	.2100	.5832	.1535	.5455	.2436	.5455	.1927	.5455		

Table 8: Testing Structural Change by Nonparametric Tests

Import Function (8)

Year	t		Fisher		Wilcoson		Asy Wil		Normal		Logistic		Cauchy		Chow	
	t_0	p-val	f_0	p-val	w_0	p-val	aw_0	p-val	ns_0	p-val	ls_0	p-val	cs_0	p-val	F_0	p-val
1961	.9736	.8311	.0631	.8182	28	.8182	1.1552	.8760	.9661	.8182	1.6094	.8182	1.7321	.8182	4.5203	.9942
1962	.3779	.6460	.0178	.6572	41	.6799	.5281	.7013	.6580	.6799	1.1147	.6780	1.3317	.6875	6.0978	.9989
1963	.2691	.6052	.0106	.6212	60	.6932	.5636	.7135	.8115	.6851	1.3584	.6782	1.5244	.6510	5.8350	.9986
1964	.0156	.5062	.0005	.5216	74	.6150	.3310	.6297	.5814	.6199	.9906	.6159	1.2308	.5969	5.3262	.9976
1965	.1731	.5682	.0054	.5805	94	.6606	.4519	.6743	.8115	.6505	1.3584	.6430	1.5244	.5969	6.4682	.9992
1966	−.1613	.4365	−.0047	.4478	103	.5091	.0467	.5186	.1620	.5286	.2998	.5300	.5709	.5315	6.0486	.9989
1967	−.0083	.4967	−.0002	.5064	124	.5768	.2202	.5871	.4701	.5778	.7945	.5742	.9713	.5464	6.0117	.9988
1968	−.0569	.4775	−.0015	.4857	142	.5896	.2521	.5995	.5454	.5860	.9158	.5813	1.0668	.5452	6.0758	.9989
1969	−.3322	.3710	−.0084	.3769	152	.4763	−.0404	.4839	−.0139	.4979	.0101	.5009	.2804	.5108	5.4878	.9980
1970	−.0496	.4804	−.0012	.4862	177	.5988	.2742	.6080	.6356	.5934	1.0687	.5882	1.2339	.5433	5.5784	.9981
1971	.5878	.7195	.0140	.7223	209	.7912	.8402	.7996	2.3310	.8021	4.1132	.7996	8.1890	.7186	5.5545	.9981
1972	1.1452	.8695	.0263	.8691	239	.9001	1.3098	.9049	3.5791	.9008	6.2449	.8960	11.0783	.7536	5.8334	.9986
1973	2.1089	.9784	.0455	.9771	272	.9689	1.8790	.9699	5.7492	.9810	10.4193	.9831	32.0709	.9888	4.9017	.9962
1974	2.4037	.9888	.0504	.9880	298	.9856	2.1856	.9856	6.4980	.9903	11.6430	.9910	33.2250	.9921	5.2552	.9974
1975	1.8937	.9662	.0406	.9656	301	.9498	1.6631	.9519	5.0655	.9623	9.1416	.9646	29.1029	.9695	4.6489	.9949
1976	1.7215	.9524	.0371	.9522	316	.9410	1.5850	.9435	4.9120	.9569	8.8980	.9599	28.9102	.9675	4.2245	.9919
1977	1.6458	.9450	.0356	.9453	333	.9410	1.5850	.9435	4.9120	.9569	8.8980	.9599	28.9102	.9675	3.9484	.9888
1978	1.8218	.9609	.0392	.9615	356	.9634	1.8078	.9647	5.3853	.9711	9.6601	.9723	29.5528	.9729	4.0106	.9896
1979	2.0590	.9760	.0441	.9768	380	.9808	2.0763	.9811	5.9445	.9832	10.5658	.9833	30.3393	.9790	5.1674	.9971
1980	1.4929	.9272	.0333	.9286	382	.9362	1.5474	.9391	4.2491	.9345	7.5213	.9334	23.3841	.9106	3.4344	.9795
1981	1.1956	.8795	.0274	.8806	390	.8862	1.2350	.8916	3.5003	.8957	6.2975	.8979	22.2300	.8982	3.1825	.9723
1982	.8871	.8091	.0210	.8086	397	.8020	.8784	.8101	2.6479	.8329	4.8922	.8418	20.8257	.8831	2.7466	.9527
1983	.4951	.6880	.0121	.6841	401	.6428	.3917	.6524	1.3999	.6989	2.7605	.7180	17.9364	.8499	1.9946	.8783
1984	.6670	.7452	.0168	.7420	423	.7178	.6063	.7279	1.7906	.7546	3.3863	.7679	18.4520	.8637	2.1574	.9010
1985	.3198	.6244	.0084	.6171	428	.5410	.1260	.5501	.6921	.6087	1.5404	.6351	16.2623	.8481	2.0168	.8817
1986	−.3731	.3558	−.0103	.3483	429	.2802	−.5725	.2835	−1.4780	.2680	−2.6340	.2673	−4.7303	.3032	.8122	.4723
1987	−.7050	.2430	−.0205	.2393	441	.1988	−.8402	.2004	−1.8687	.2030	−3.2597	.2070	−5.2459	.2649	.8779	.5108
1988	−.3059	.3808	−.0096	.3701	468	.3394	−.4017	.3440	−1.0163	.3141	−1.8543	.3084	−3.8416	.2866	1.0166	.5849
1989	.4818	.6833	.0166	.6698	499	.6150	.3310	.6297	.4162	.5863	.6471	.5768	.2805	.5228		
1990	1.1739	.8753	.0451	.8739	528	.8561	1.1272	.8702	1.5147	.8167	2.4929	.8000	2.4702	.7214		
1991	.7508	.7708	.0352	.7405	534	.6799	.5281	.7013	.5486	.6534	.8835	.6420	.7381	.6117		
1992	.3469	.6345	.0228	.5152	545	.5152	.1050	.5418	.0753	.5152	.1214	.5152	.0955	.5152		

Figure 1: p-Values of Nonparametric Tests

— Import Function (7): Table 7 —

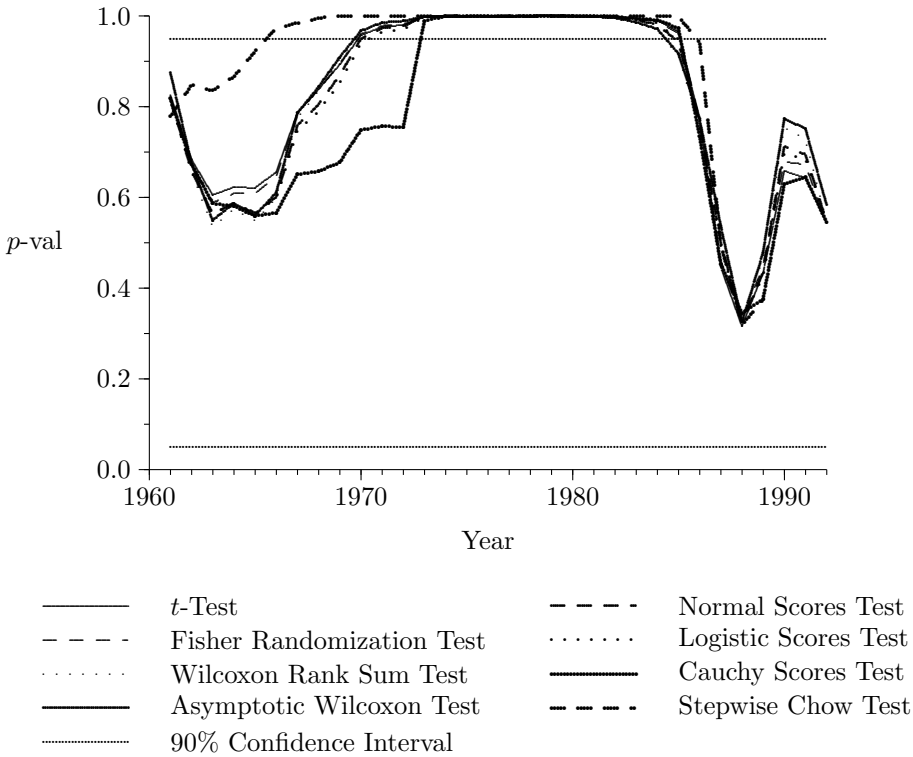
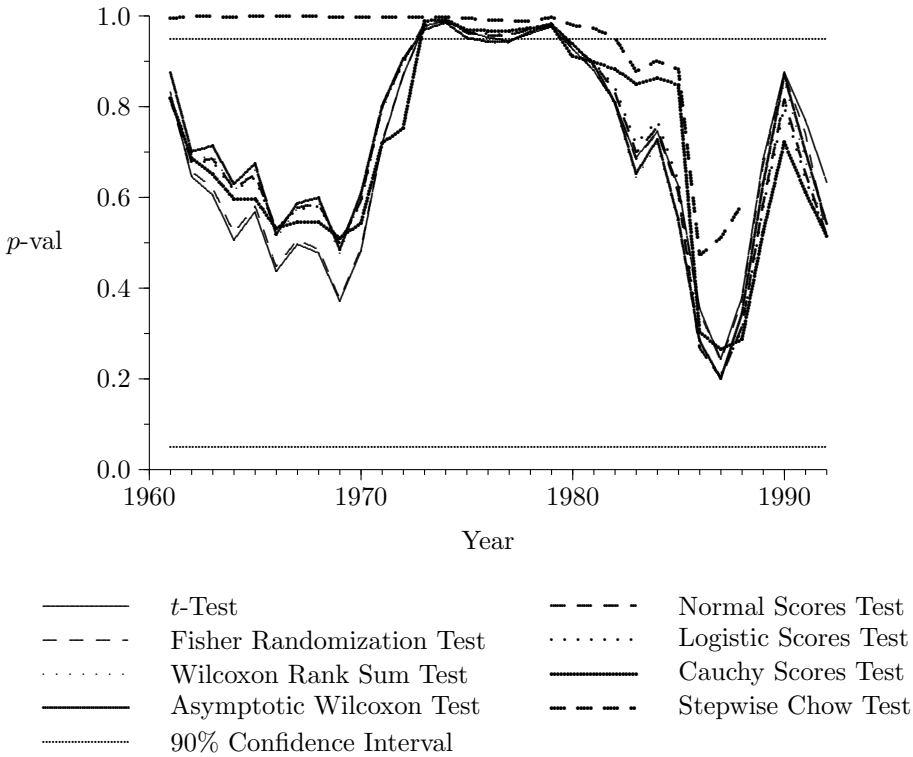


Figure 2: p-Values of Nonparametric Tests

— Import Function (8): Table 8 —