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Uncertainty of voters' preferences and differentiation in a runoff system*

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Abstract

Haan and Volkerink (2001, this journal) show that the principle of minimum differentiation holds in two-round elections for any number of candidates. We show that the principle of minimum differentiation may not hold, when uncertainty of the position of the median voter is introduced.

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Key Words: Runoff elections, minimum differentiation, uncertainty, location, Hotelling

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1 Introduction

Since the seminal work of Hotelling (1929), which shows the well-known result (the principle of Minimum differentiation (MD)), the model of spatial competition has been viewed by many subsequent researchers as an attractive framework for analyzing product differentiation. In the literature of political science, many researchers use spatial models similar to that of Hotelling (1929) (see, for instance, Downs (1957) and Davis, Hinich, and Ordeshook (1970)). Some researchers show conditions that MD no longer holds. For instance, if there are more than two candidates, MD does not hold (see Eaton and Lipsey (1975)).

In this journal (*EJPE*), Haan and Volkerink (2001) show that MD is restored under a runoff system, where only the two most successful candidates in the first round are allowed to run in the second. Their results remain valid even when changes in the number of candidates, the possibility of entering races, and the distribution of voters' preferences are introduced.

We introduce the uncertainty of the median voter's position into the model of Haan and Volkerink (2001). Such uncertainty is natural in a political election. Especially, when a political issue is vague, voters' preferences are unfocused, and candidates find it difficult to identify voters' preferences. We show that MD may not hold when voters' preferences are uncertain and there are more than two candidates.

The remainder of this paper is organized as follows. The basic setting is explained in Section 2. The results are presented in Section 3, and Section 4 is the conclusion.

2 The model

Consider the following model, which is based on Haan and Volkerink (2001). We have an election with two rounds. In the first round, n candidates participate ($n \geq 3$). The two candidates with the highest share of the vote proceed to the second round. The candidate with the largest share of the vote in the second round wins the election.[1]

[1] For this paper, it is assumed that the objective of the candidates is only to maximize their eventual share of the votes. This assumption is employed by Haan and Volkerink (2001). The authors of several papers, however, have assumed that parties have policy preferences (see Wittman (1977, 1983), Calvert (1985), Alesina and Rosenthal (2000), and Ossokina and Swank (2004)). The objective of this paper is to show an example in which the result of Haan and Volkerink (2001) does not hold. Therefore, a traditional approach developed by Downs (1957) is used here. Moreover, Ortuño-Ortín (2002) distinguishes parties, which have political preferences, from candidates, who only care about winning the election, and shows that parties may be better off when candidates manage the party. That is, they may be better off in a case in which political competition takes place between candidates who only care about winning the election. The result of Ortuño-Ortín (2002) may make the assumption presented in this paper plausible.

Preferences are represented by a horizontal line, normalized to $[0, 1]$. Before the first round, every candidate i chooses his position P_i. We assume that this position cannot be changed between rounds. Voters are distributed on $[0, 1]$. Voters always vote for the candidate with the position that is closest to theirs, that is, we assume sincere voting. In case of a tie, they decide randomly which candidate to vote for. In the deterministic case, Haan and Volkerink (2001) establish the following:

Result (Haan and Volkerink (2001)) *For any continuous distribution of voters' preferences, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter* (p_m)*:* $P_i = p_m \; \forall i$.

The intuition of the result is as follows. By choosing a position different from that of the median voter, a candidate may win the first round. In the second round, however, he will be beaten by one of the candidates who has chosen the median voter's position. Therefore, the deviated candidate wins with probability zero.

In contrast to Haan and Volkerink (2001), if there is uncertainty about the median voter's position, the result may not hold. We add the following setting. The *true* distribution of voters is unknown to the candidates, but the *probability* distribution of voters' preferences is known to the candidates. After the first-round election, the uncertainty is resolved. In the second round, the two candidates know the distribution of voters but cannot change their positions.[2]

3 Results

In this section, we show two results. One is an example in which the result of Haan and Volkerink (2001) does not hold. The other is an example that shows the equilibrium outcome in our setting.

3.1 MD does not hold

We first consider the following probability structure concerning the distribution of voters. $F(y)$ is the probability that the realized median voter's position is less than or equal to y. That is, $F(y)$ is a cumulative distribution function defined on $[0, 1]$ ($F\colon [0, 1] \to [0, 1]$). In this setting, Proposition 1 holds:

[2] The idea that parties are uncertain about voters' preferences is supported by the fact that parties often turn to polls to ascertain voters' preferences. In the literature different possible sources for parties's uncertainty have been mentioned, among which: uncertain voter turnout (see Ledyard (1984)), unanticipated changes in voters' preferences (see Ossokina and Swank (2004)). The latter source of uncertainty is relevant for the model of this paper.

Proposition 1 *Suppose that $F(y)$ is the probability that the realized median voter's position is less than or equal to y. When $F(y)$ is continuous, MD does not appear as an equilibrium outcome.*

Proof: To show that MD does not hold, we consider three cases: (1) All candidates agglomerate at $P \in [0, \bar{y})$, where $\bar{y}$ satisfies $F(\bar{y}) = 1/2$; (2) all candidates agglomerate at $P = \bar{y}$; and (3) all candidates agglomerate at $P \in (\bar{y}, 1]$. If all candidates agglomerate at the same position, each candidate wins with probability $1/n$. In the first case, if a candidate chooses a position at $P_i' \in (P, \bar{y})$, the candidate wins the first and the second rounds when the position of the median voter is $y' \in [P_i', 1]$. He wins with a probability greater than $1/2$ because $1 - F(P_i')$ is larger than $1/2$. In the second case, if a candidate chooses a position at $P_i' \in (\bar{y}', \bar{y})$, where $\bar{y}'$ satisfies $1/n < F(\bar{y}') < 1/2$ (note that, if $F(y)$ is continuous, we can find $\bar{y}'$), the candidate wins the first and the second rounds when the position of the median voter is $y' \in [0, P_i']$. He wins with a probability greater than $1/n$ because $F(P_i')$ is larger than $1/n$. The third case is similar to the first one. Therefore, for any $P \in [0, 1]$, MD does not appear in equilibrium. Q.E.D.

3.2 Equilibrium

We now describe an example that shows the equilibrium outcome in our setting. To find an equilibrium outcome, we have to specify *ex post* distribution functions of voters because the specification is needed to find the first round result unless all candidates agglomerate at the same point.[3] Since the specification of voters' distribution functions is complex, we think that it is not possible to find an equilibrium in the general model, although it is possible to do so in a more specific model. The example presented below does not fit the general model. All proofs are available upon request.[4]

We consider the following probability concerning the distribution of voters. Voters' preferences are uniformly distributed on $[m - 1/2, m + 1/2]$, where m is the median voter that is y ($y \in (0, 1/4)$) with probability $1/2$ and $1 - y$ with probability $1/2$ (see Figure 1). We assume that three candidates participate.

(Figure 1)

The setting may suit the presidential election in the Liberal Democratic Party of Japan (Jimin-To). There are often a few candidates. The main political issue may be whether the policy agenda

[3] In the case of agglomeration, as discussed in the proof of Proposition 1, we have to compare only two points.

[4] The proofs are also available on the following Web site: http://norick.sakura.ne.jp/research/paper.html

should be radical or conservative (that is, a binary choice). The president is elected by vote in two rounds, if necessary.[5]

In the setting, the following location pattern is an equilibrium outcome (see Figure 2).

Proposition 2 *Suppose that voters' preferences are uniformly distributed on* $[m - 1/2, m + 1/2]$*, where* m *is the median voter that is* y $(y \in (0, 1/4))$ *with probability* $1/2$ *and* $1 - y$ *with probability* $1/2$*. When three candidates participate, the following location pattern is an equilibrium outcome: two candidates locate at* y*, and the other candidate locates at* $1 - y$*.*

(Figure 2)

In Figure 2, the left-hand side candidates win with probability 1/4, and the right-hand-side candidate wins with probability 1/2. The result presents an interesting property. In this setting, when a runoff system is employed in an election, one of three candidates may be able to take an advantageous position in which the candidate's winning probability is 1/2.

A sketch of the proof and the intuition behind the result are as follows. To show that the location pattern is an equilibrium outcome, we consider two cases: (1) One of the left-hand-side candidates relocates and (2) the right-hand-side candidate relocates. In the first case, the relocating candidate is not able to win in the second round. Given that the relocating candidate does not locate at y or $1 - y$, when $m = y$ (*resp.* $m = 1 - y$), the candidate locating at y (*resp.* $1 - y$) wins in the second round. In the second case, when $m = y$, the right-hand-side candidate is not able to win in the second round unless it locates at m. For the right-hand-side candidate, locating at m does not improve the probability of winning (the probability is 1/3). From these two cases, we find that Proposition 2 holds.

In this setting, if a runoff system does not exist, the location pattern mentioned above is not an equilibrium outcome, and pure strategy equilibria do not exist. As discussed by Haan and Volkerink (2001), the runoff system may change the equilibrium location patterns.

4 Conclusion

In this paper, we reconsider the location model of Hotelling (1929) under a runoff system, which is considered by Haan and Volkerink (2001). We introduce uncertainty of the median voter's position into the model of Haan and Volkerink (2001). We show examples that MD does not hold.

[5] When any candidate does not acquire more than half of the votes, the first- and the second-place candidates run in the second round. The system is similar to that discussed in the paper.

The results are presented here in a political context. When a political issue is clearly determined, the opinions of candidates opinions tend to be similar in a runoff system. When a political issue is vague, the opinions of candidates will vary even in a runoff election.

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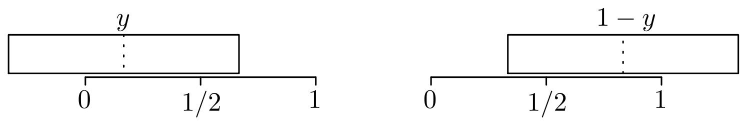

$m = y$
(Probability 1/2)

$m = 1 - y$
(Probability 1/2)

Figure 1: Uncertainty of voters' preferences

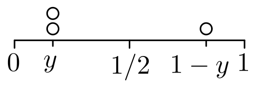

Figure 2: The locations in equilibrium