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E C O N O M I C S  B U L L E T I N

# On Chamberlinian−Ricardian trade patterns with many industries

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## *Abstract*

This note explores the determinants of trade patterns by extending a Chamberlinian−Ricardian monopolistic competition trade model to have a larger number of industries as did Dornbush, Fischer and Samuelson (1977). It will be shown that the degree of cross−country technical differences among industries plays an important role as a determinant of trade within each industry.

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# 1 Introduction

Chamberlinian monopolistic competition models of trade have been extensively studied since the seminal work of Krugman (1979). Those models are very successful to explain the emergence of intra-industry trade (i.e., two-way trade of differentiated products) and the industrial agglomeration. To focus on the role of increasing returns and imperfect competition, a standard model assumes the cross-country technical homogeneity: each firm in the monopolistically competitive sector incurs an identical fixed cost ($\alpha$) and a constant marginal cost ($\beta$). As a result, there has been little investigation of the role of technical heterogeneity among countries. However, the Ricardian comparative advantage, which plays a basic role in traditional international-trade context, is worthy of more attention. To address this point, Kikuchi (2004) explored cross-country technical heterogeneity in both fixed costs and marginal costs as a determinant of trade patterns. Within a two-country, one-industry framework, he showed that manufacturing sector is agglomerated in a country and the intra-industry trade is very unlikely in a trading equilibrium.

The present note takes Kikuchi (2004) as its point of departure, and extends his analysis to include a large number of industries as did Dornbush, Fischer and Samuelson (1977). In each industry, both fixed costs and marginal costs can differ between the countries. It will be shown that the equilibrium specialization pattern is determined by the technology index. It will also be shown that trade patterns, particularly the emergence of intra-industry trade, are crucially dependent on the shape of the technology index schedule, which is taken from Dornbush, Fischer and Samuelson (1977). That is, if technical standardization occurs and the share of similar industries becomes larger between countries, the possibility of intra-industry trade rises.

This study is closely related to Venables (1999), which explores the division of industries between countries in a multi-industry framework. However, he uses a framework in which there are both transport costs and linkages through intermediate inputs. In contrast, in this study we assume away such aspects and focus on the interaction between cross-country technological differences and trade patterns.

The next section develops a Chamberlinian-Ricardian model with many industries. Section 3 deals with the determinants of trade patterns. Section 4 discusses some implications of the analysis.

# 2 The Model

Suppose that there are two countries in the world, Home and Foreign. Home (Foreign) is endowed with $L$ ($\tilde{L}$) units of labor and the only source of income is the wage, $w$ ($\tilde{w}$). We assume that there are $M$ manufacturing industries.[1] Industry specific variables will be indexed by industry label $i$. Consumers have Cobb-Douglas preferences and purchase equal values of the output of all industries.

Each industry is modelled as a Dixit-Stiglitz (1977) monopolistically competitive industry. The quantity index of industry $i$ takes the form

$$X^i = \left( \sum_{k=1}^{n^i} (d_k^i)^\theta + \sum_{\tilde{k}=1}^{\tilde{n}^i} (d_{\tilde{k}}^i)^\theta \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \tag{1}$$

where $n^i$ ($\tilde{n}^i$) is the number of products produced in industry $i$ in Home (Foreign), $d_k^i$ ($d_{\tilde{k}}^i$) is the quantity of product $k$ ($\tilde{k}$) in Home market, and $1/(1-\theta) > 1$ is the elasticity of substitution between every pair of products. The price index of industry $i$ can be obtained as:

$$P^i = \left( \sum_{k=1}^{n^i} (p_k^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{k}=1}^{\tilde{n}^i} (p_{\tilde{k}}^i)^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}, \tag{2}$$

where $p_k^i$ ($p_{\tilde{k}}^i$) is the price of the $k$ ($\tilde{k}$)-th differentiated product produced in industry $i$ in Home (Foreign). Note that the total revenue in Home is $wL$, which will be equally expended in each industry due to the assumption of the Cobb-Douglas preferences. Solving consumers' maximization problem yields the following demand functions for Home consumers:

$$d_k^i = \frac{(p_k^i)^{\frac{1}{\theta-1}}}{\sum_{j=1}^{n^i} (p_j^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{j}=1}^{\tilde{n}^i} (p_{\tilde{j}}^i)^{\frac{\theta}{\theta-1}}} \frac{wL}{M}, \tag{3}$$

$$d_{\tilde{k}}^i = \frac{(p_{\tilde{k}}^i)^{\frac{1}{\theta-1}}}{\sum_{j=1}^{n} (p_j^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{j}=1}^{\tilde{n}} (p_{\tilde{j}}^i)^{\frac{\theta}{\theta-1}}} \frac{wL}{M}. \tag{4}$$

[1] Although it is easier to set the model up with a finite number of manufacturing industries, we will later think of there being a continuum of them.

Assuming that the products are transported free between countries, then the prices of each product in two countries are equal. Therefore, the demand functions for Foreign consumers are

$$\tilde{d}_k^i = \frac{(p_k^i)^{\frac{1}{\theta-1}}}{\sum_{j=1}^{n^i}(p_j^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{j}=1}^{\tilde{n}^i}(p_{\tilde{j}}^i)^{\frac{\theta}{\theta-1}}} \frac{\tilde{w}\tilde{L}}{M},$$

and

$$\tilde{d}_{\tilde{k}}^i = \frac{(p_{\tilde{k}}^i)^{\frac{1}{\theta-1}}}{\sum_{j=1}^{n}(p_j^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{j}=1}^{\tilde{n}}(p_{\tilde{j}}^i)^{\frac{\theta}{\theta-1}}} \frac{\tilde{w}\tilde{L}}{M},$$

respectively.

Differentiated products are supplied by monopolistically competitive firms. There is cross-country technical heterogeneity: each Home (Foreign) firm in industry $i$ has both $\alpha^i$ ($\tilde{\alpha}^i$) units of labor as a fixed input and $\beta^i$ ($\tilde{\beta}^i$) units of labor as a variable input. With the number of firms being very large, the elasticity of demand for each product becomes $1/(1-\theta)$. Thus, each product is priced at a markup over marginal cost:

$$p_k^i = \frac{\beta^i w}{\theta}, \quad p_{\tilde{k}}^i = \frac{\tilde{\beta}^i \tilde{w}}{\theta}.$$

Using these pricing equations, the summation in equation (3) takes the form

$$\sum_{k=1}^{n^i}(p_k^i)^{\frac{\theta}{\theta-1}} + \sum_{\tilde{k}=1}^{\tilde{n}^i}(p_{\tilde{k}}^i)^{\frac{\theta}{\theta-1}} = n^i\left(\frac{\beta^i w}{\theta}\right)^{\frac{\theta}{\theta-1}} + \tilde{n}^i\left(\frac{\tilde{\beta}^i \tilde{w}}{\theta}\right)^{\frac{\theta}{\theta-1}}.$$

Substituting this into the demand function yields the profit function of each Home firm[2]

$$\begin{aligned}\pi^i =& (p^i - \beta^i w)x - \alpha^i w \\ =& \frac{1-\theta}{\theta}\beta^i w(d_k^i + \tilde{d}_k^i) - \alpha^i w\end{aligned}$$

[2]Hereafter, the subscript $k$ is often dropped for simplicity.

$$= \frac{(1-\theta)\left(\frac{\beta^i w}{\theta}\right)^{\frac{\theta}{\theta-1}}}{n^i\left(\frac{\beta^i w}{\theta}\right)^{\frac{\theta}{\theta-1}} + \tilde{n}^i\left(\frac{\tilde{\beta}^i \tilde{w}}{\theta}\right)^{\frac{\theta}{\theta-1}}} \frac{wL + \tilde{w}\tilde{L}}{M} - \alpha^i w. \tag{5}$$

Similarly, the profit function of each Foreign firm is

$$\tilde{\pi}^i = \frac{(1-\theta)\left(\frac{\tilde{\beta}^i \tilde{w}}{\theta}\right)^{\frac{\theta}{\theta-1}}}{n^i\left(\frac{\beta^i w}{\theta}\right)^{\frac{\theta}{\theta-1}} + \tilde{n}^i\left(\frac{\tilde{\beta}^i \tilde{w}}{\theta}\right)^{\frac{\theta}{\theta-1}}} \frac{wL + \tilde{w}\tilde{L}}{M} - \tilde{\alpha}^i \tilde{w}. \tag{6}$$

Now turn to the specialization pattern of industry $i$. In the trading equilibrium with zero transport costs, we need non-positive profits in industry $i$ in each country, with profits being equal to zero if production takes place. Thus, by setting profits equal to zero for both countries ($\pi^i = \tilde{\pi}^i = 0$), we would like to test whether the co-existence of both countries' firms is consistent with equilibrium.

First, let us draw attention to the condition that, if both countries' firms in industry $i$ co-exist, profits must be identical for each country's firms, i.e.,

$$\pi^i = \tilde{\pi}^i. \tag{7}$$

This is the condition that must be satisfied if $\pi^i = \tilde{\pi}^i = 0$ is to hold. Substituting (5) and (6) into (7), we obtain

$$\frac{\frac{wL + \tilde{w}\tilde{L}}{M}(1-\theta)\ \theta^{\frac{\theta}{1-\theta}}}{n^i\left(\frac{\beta^i w}{\theta}\right)^{\frac{\theta}{\theta-1}} + \tilde{n}^i\left(\frac{\tilde{\beta}^i \tilde{w}}{\theta}\right)^{\frac{\theta}{\theta-1}}} = \frac{\alpha^i w - \tilde{\alpha}^i \tilde{w}}{(\beta^i w)^{\frac{\theta}{\theta-1}} - (\tilde{\beta}^i \tilde{w})^{\frac{\theta}{\theta-1}}}. \tag{8}$$

Inserting the RHS of (8) into the profit function yields

$$\pi^i = \frac{(\beta^i w)^{\frac{\theta}{\theta-1}}(\alpha^i w - \tilde{\alpha}^i \tilde{w})}{(\beta^i w)^{\frac{\theta}{\theta-1}} - (\tilde{\beta}^i \tilde{w})^{\frac{\theta}{\theta-1}}} - \alpha^i w,$$

$$\tilde{\pi}^i = \frac{(\tilde{\beta}^i \tilde{w})^{\frac{\theta}{\theta-1}}(\alpha^i w - \tilde{\alpha}^i \tilde{w})}{(\beta^i w)^{\frac{\theta}{\theta-1}} - (\tilde{\beta}^i \tilde{w})^{\frac{\theta}{\theta-1}}} - \tilde{\alpha}^i \tilde{w}.$$

It is important to note that profits are independent of both the total number of firms and market size.

Before turning to the case of co-existence, note that the equilibrium number of firms for the case in which only one country's firms exist is

$$n^{iT}_{\{\tilde{n}^i=0\}} = \frac{(1-\theta)(wL+\tilde{w}\tilde{L})}{M\alpha^i w},$$
$$\tilde{n}^{iT}_{\{n^i=0\}} = \frac{(1-\theta)(wL+\tilde{w}\tilde{L})}{M\tilde{\alpha}^i \tilde{w}},$$

where $T$ denotes a trading equilibrium value.

Using these results, one can obtain the necessary condition for the co-existence of firms. Let us define a technology index for industry $i$:

$$A^i \equiv \left(\frac{\tilde{\alpha}^i}{\alpha^i}\right)^{1-\theta}\left(\frac{\tilde{\beta}^i}{\beta^i}\right)^{\theta}. \tag{9}$$

In the free-trade equilibrium the profit must be zero: $\pi^i = \tilde{\pi}^i = 0$. Simple calculations show that the equations are satisfied only if the technology index, $A^i$, is equal to the relative wage rate $\omega \equiv w/\tilde{w}$.

**Proposition 1** *If $A^i > (<)\ \omega$, only Home (Foreign) firms produce the differentiated products in industry $i$. Intra-industry trade in industry $i$ (i.e., the co-existence of both countries' firms) occurs only if $A^i = \omega$.*

[Proof] Suppose that $A^i > \omega$. In this case, both countries' firms cannot co-exist. To see that the case where only Home firms are active is an equilibrium, note that

$$\tilde{\pi}^i_{\{n^i=n^{iT},\ \tilde{n}^i=0\}} = \left(\frac{\beta^i w}{\tilde{\beta}^i \tilde{w}}\right)^{\frac{\theta}{1-\theta}} \alpha^i w - \tilde{\alpha}^i \tilde{w} = \tilde{\alpha}^i \tilde{w}\left[\left(\frac{\omega}{A^i}\right)^{\frac{1}{1-\theta}} - 1\right].$$

This becomes negative if $A^i > \omega$ since $\theta \in (0,1)$. Therefore, Foreign firms have no incentive to enter given that $n^{iT}$ Home firms are active. On the other hand, the case in which only Foreign firms are active cannot support a free trading equilibrium. This is because that

$$\pi^i_{\{n^i=0,\ \tilde{n}^i=\tilde{n}^{iT}\}} = \left(\frac{\tilde{\beta}^i \tilde{w}}{\beta^i w}\right)^{\frac{\theta}{1-\theta}} \tilde{\alpha}^i \tilde{w} - \alpha^i w = \alpha^i w\left[\left(\frac{A^i}{\omega}\right)^{\frac{1}{1-\theta}} - 1\right]$$

is positive, and hence, Home firms have an incentive to enter the world market. Therefore, only Home firms produce the differentiated products in industry $i$ in the free trade equilibrium. The case of $A^i < \omega$ can be proven analogously. [Q.E.D.]

# 3 Trade Patterns

To obtain the world trading equilibrium, we index industries in order of diminishing Home comparative advantage.[3]

$$\frac{dA^i}{di} \leq 0,$$

where $A^i$ is defined in (9). This schedule is drawn in Figure 1 as the downward sloping locus $AA$. Now assume that there is flat segment in the $AA$ schedule: a partition of industries (from $\underline{m}$ to $\bar{m}$) is assumed to have equal level of technology index. We can interpret this as, even firms in each industry produce differentiated products, production technologies of these industries have become standardized due to increased information flow between industries.

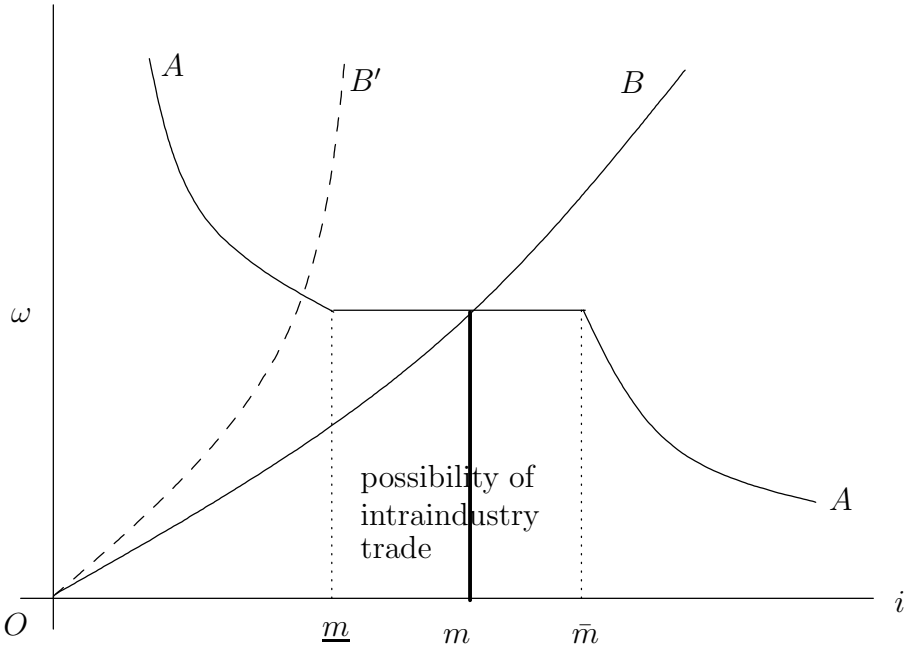


Figure 1: Intraindustry trade

Let $m$ denote hypothetical dividing line between Home- and Foreign-produced commodities, equilibrium in the market for Home products requires

[3]See footnote 1.

that Home labor income $wL$ equals world spending on Home-produced products:

$$wL = \frac{m}{M}(wL + \tilde{w}\tilde{L}).$$

This schedule is drawn in Figure 1 as the upward sloping locus $OB$ and is obtained by rewriting the equation in the form:

$$\omega = \frac{m}{M - m}\ \frac{\tilde{L}}{L}.$$

The equilibrium relative wage is obtained as the intersection of schedules $AA$ and $OB$. Now assume that the intersection is obtained at the flat segment in the $AA$ schedule. Thus, the following condition holds.

$$\omega = A^i,\ \ \underline{m} \leq i \leq \bar{m}.$$

In this case, from Proposition 1, firms within these industries can be located in both countries. Therefore, intra-industry trade within these industries will occur.

**Proposition 2** *Given that there is a flat segment in the AA schedule and the OB schedule cuts that segment, intra-industry trade occurs between countries.*

Using Figure 1, let us examine the effect of an increase in the relative size of Foreign. An increase in $\tilde{L}/L$ shifts schedule $OB$ upward. If the new intersection occurs in the flat segment of $AA$, this shift only changes the portion of intra-industry trade and the relative wage remains unchanged. If the upward shift is sufficiently large and the new intersection occurs in the downward-sloping segment of $AA$, no intra-industry trade occurs in the trading equilibrium and the Home relative wage rises. Our model suggests that the share of intra-industry trade is smaller between countries that are dissimilar in size. This finding is consistent with empirical work by Helpman (1987).

# 4 Discussion

Several remarks are in order. First, we should note that these results are crucially dependent on the assumption of monopolistically competitive industries. If firms in each industry produce *homogeneous* products as in

Dornbush, Fischer, and Samuelson (1977), there are few incentives of intra-industry trade between countries. In our model, intra-industry trade occurs since each firm produces *differentiated* products and those firms are distributed between countries.

Second, intra-industry trade emerges as a result of the equalization of the relative wage rate and the technology index, which is also supported by the existence of the flat segment of the $AA$ schedule. If there is only one monopolistically competitive industry in the economy, intra-industry trade is obtained as a result of identical technologies between countries and wage rate equalization.[4] In our multi-industry model, however, wage rates need not be equalized to obtain intra-industry trade.

Krugman (1979, p. 479) argues that trade need not be a result of cross-country differences in technology. We would like to emphasize that the degree of cross-country technical differences *among industries* plays a more important role as a determinant of trade *within each industry*.

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