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Strong coupling quantum Einstein gravity at a $z = 2$ Lifshitz point

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We solve a renormalized Wheeler-DeWitt equation for Einstein gravity in $D + 1$ dimensions with $D =$ odd in the strong coupling limit, which is expected to be suited to probe quantum geometry at short distances, in order to test Hořava's idea that quantum gravity at short distances will be described by a nonrelativistic system with dynamical critical exponent $z > 1$. Our results support the idea and show that the Wheeler-DeWitt equation possesses a solution associated with a $z = 2$ Lifshitz point but no other $z > 2$ solutions to leading order of the strong coupling expansion.

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I. INTRODUCTION

Hořava [1,2] has recently proposed an interesting idea that quantum gravity at short distances will be described by a nonrelativistic system with an anisotropy between space and time. The anisotropy is characterized by the dynamical critical exponent z . A number of studies have been recently made on the Hořava-Lifshitz gravity to examine its cosmological applications [3], black hole physics [4], theoretical aspects [5], etc. However, little attention has been paid to a possibility that quantum theory of Einstein general relativity will have a $z > 1$ Lifshitz point at short distances, or that quantum Einstein gravity will be described effectively by a nonrelativistic theory at high energies, where Lorentz invariance should be broken spontaneously in the UV¹ rather than emerging accidentally in the infrared. This is the perspective we adopt in this paper.

In [7,8], we have presented a renormalization prescription of the $3 + 1$ -dimensional Wheeler-DeWitt equation [9], and found a solution in the strong coupling expansion, which is expected to probe short-distance behavior of quantum gravity [10]. Since the formalism is particularly suited to test Hořava's idea, it will be worthwhile reinvestigating the previous study of the Wheeler-DeWitt equation from a new aspect of Hořava's proposal, and extending the analysis to higher dimensions. The purpose of this paper is to answer the question whether the renormalized Wheeler-DeWitt equation for Einstein gravity in $D + 1$ dimensions can possess any solutions associated with $z > 1$ Lifshitz points to leading order of the strong coupling expansion. Our answer is positive, and we find a $z = 2$ solution but no other $z > 2$ solutions for $D =$ odd.

This paper is organized as follows. In Sec. II, we explain our renormalization prescription for the Wheeler-DeWitt equation in $D + 1$ dimensions. In Sec. III, we discuss consistency of the constraints with our renormalization procedure. In Sec. IV, we solve the renormalized

Wheeler-DeWitt equation to leading order of the strong coupling expansion. Section V is devoted to conclusions.

II. RENORMALIZATION SCHEME

In this section, we present a renormalization prescription for the Wheeler-DeWitt equation in $D + 1$ dimensions. The prescription has been developed by Mansfield [11] for the functional Schrödinger equation of the Yang-Mills theory and then generalized to the Wheeler-DeWitt equation [7,8]. In the following, we extend the technique given in [7,8] to arbitrary dimensions. Since actual computations are tedious and lengthy, we will omit the details in this paper. Instead, we will refer the reader to the appendices of [8] for $D = 3$. We will ignore boundary terms and freely drop integrals of all total derivative terms. The inclusion of those effects is beyond the scope of the paper.

The (unregulated) Wheeler-DeWitt equation [9] is given by

$$\left[-16\pi G G_{ijkl}(x) \frac{\delta}{\delta g_{kl}(x)} \frac{\delta}{\delta g_{ij}(x)} + \frac{\sqrt{g(x)}}{16\pi G} (R(x) + 2\Lambda) \right] \Psi[g] = 0, \quad (1)$$

where G is the “Newton” constant in $D + 1$ dimensions and $G_{ijkl}(x)$ is the metric on superspace

$$G_{ijkl}(x) = \frac{1}{2\sqrt{g(x)}} \left(g_{ik}(x)g_{jl}(x) + g_{il}(x)g_{jk}(x) - \frac{2}{D-1} g_{ij}(x)g_{kl}(x) \right). \quad (2)$$

The $R(x)$ denotes the scalar curvature constructed from the D -dimensional metric $g_{ij}(x)$ and Λ is a cosmological constant. Eq. (1) is ill defined because a product of two functional derivatives at the same point

$$\Delta(x) \equiv G_{ijkl}(x) \frac{\delta}{\delta g_{kl}(x)} \frac{\delta}{\delta g_{ij}(x)} \quad (3)$$

could produce the square of δ -functions, like $(\delta^D(x, x'))^2$,

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¹An attempt has been made in [6].

which is meaningless. To make (3) well defined, we need to replace $\Delta(x)$ by a renormalized operator $\Delta_R(x)$. To this end, we first define a regularized differential operator $\Delta(x; s)$ by point-splitting the functional derivatives by use of a heat kernel [7,8]:

$$\Delta(x; s) \equiv \int d^D x' K_{i'j'kl}(x', x; s) \frac{\delta}{\delta g_{kl}(x)} \frac{\delta}{\delta g_{i'j'}(x')}, \quad (4)$$

where $K_{i'j'kl}(x', x; s)$ is a bitensor at both x' and x and satisfies the heat equation

$$-\frac{\partial}{\partial s} K_{i'j'kl}(x', x; s) = -\nabla'_{i'} \nabla'^{i'} K_{i'j'kl}(x', x; s) \quad (5)$$

with the initial condition

$$\lim_{s \rightarrow 0} K_{i'j'kl}(x', x; s) = G_{i'j'kl}(x) \delta^D(x', x), \quad (6)$$

which assures that $\Delta(x; s)$ reduces to $\Delta(x)$ in the naive limit $s \rightarrow 0$. Here, ∇'_p denotes the covariant derivative with respect to x' . Taking s small but nonzero in (4) gives a regularized operator of $\Delta(x)$. We have chosen the factor ordering written in (4). Other choices of factor ordering will lead to different numerical values of our results but will not change the qualitative features.

The heat Eq. (5) can be solved by the standard technique [12,13]. Let \mathcal{O} be D -dimensional integrals of local functions of g_{ij} . The action of $\Delta(x; s)$ on \mathcal{O} will give an expansion in powers of s . These powers of s may be determined from general coordinate invariance and dimensional analysis. For example, we have

$$\Delta(x; s) \int d^D y \sqrt{g(y)} = \frac{\sqrt{g(x)}}{s^{D/2}} \left\{ \alpha_0 + \sum_a \sum_{n=1}^{\infty} s^n \alpha_n^a \mathcal{O}_{2n}^a(x) \right\}, \quad (7)$$

$$\begin{aligned} \Delta(x; s) \int d^D y \sqrt{g(y)} R(y) &= \frac{\sqrt{g(x)}}{s^{D/2+1}} \left\{ \beta_0 + s \beta_1 R(x) \right. \\ &\quad \left. + \sum_a \sum_{n=1}^{\infty} s^{n+1} \beta_{n+1}^a \mathcal{O}_{2(n+1)}^a(x) \right\}, \end{aligned} \quad (8)$$

where $\{\mathcal{O}_{2n}^a(x)\}$ symbolically denote a set of independent local scalar functions of mass dimension $2n$, and α_n^a, β_n^a are dimensionless numerical constants. The first few coefficients are found to be

$$\begin{aligned} \alpha_0 &= -\frac{D(D^2 - 2)}{4(D-1)(4\pi)^{D/2}}, & \beta_0 &= \frac{(D-2)D(D+1)}{8(4\pi)^{D/2}}, \\ \beta_1 &= -\frac{(D-2)(D^3 + 10D^2 - 13D - 34)}{48(D-1)(4\pi)^{D/2}}. \end{aligned} \quad (9)$$

These results agree with the values given in [7,8] for $D = 3$. Note that $\beta_0 = \beta_1 = 0$ for $D = 2$. This is consistent

with the fact that the term $\int d^D y \sqrt{g} R$ is a topological invariant for $D = 2$.

The second step of our renormalization prescription is to extract a finite part from $\Delta(x; s)\mathcal{O}$. We define $\Delta_R(x)\mathcal{O}$ from $\Delta(x; s)\mathcal{O}$ by analytic continuation [11]:

$$\Delta_R(x)\mathcal{O} \equiv \lim_{s \rightarrow +0} s \int_0^\infty d\varepsilon \varepsilon^{s-1} \phi(\varepsilon) \Delta(x; s = \varepsilon^2) \mathcal{O}. \quad (10)$$

Note that if $\Delta(x; s = 0)\mathcal{O}$ is finite, $\Delta_R(x)\mathcal{O}$ reduces to $\Delta(x; s = 0)\mathcal{O}$ provided that the differentiable function $\phi(\varepsilon)$ rapidly decreases to zero at infinity with

$$\phi(0) = 1. \quad (11)$$

According to the above renormalization prescription, we have, for example,

$$\begin{aligned} \Delta_R(x) \int d^D y \sqrt{g(y)} &= \sqrt{g(x)} \left\{ \frac{\phi^{(D)}(0)}{D!} \alpha_0 \right. \\ &\quad \left. + \sum_a \sum_{n=1}^{[D/2]} \frac{\phi^{(D-2n)}(0)}{(D-2n)!} \alpha_n^a \mathcal{O}_{2n}^a(x) \right\}, \end{aligned} \quad (12)$$

$$\begin{aligned} \Delta_R(x) \int d^D y \sqrt{g(y)} R(y) &= \sqrt{g(x)} \left\{ \frac{\phi^{(D+2)}(0)}{(D+2)!} \beta_0 + \frac{\phi^{(D)}(0)}{D!} \right. \\ &\quad \times \beta_1 R(x) + \sum_a \sum_{n=1}^{[D/2]} \frac{\phi^{(D-2n)}(0)}{(D-2n)!} \\ &\quad \left. \times \beta_{n+1}^a \mathcal{O}_{2(n+1)}^a(x) \right\}, \end{aligned} \quad (13)$$

where $\phi^{(n)}(0) \equiv d^n \phi(0)/d\varepsilon^n$, and $[D/2]$ denotes the Gauss symbol ($[D/2] = (D-1)/2$ for $D = \text{odd}$, $[D/2] = D/2$ for $D = \text{even}$). The results depend on the arbitrary function ϕ . This is an inevitable consequence of isolating finite quantities from divergent ones. Physical quantities must be independent of this arbitrariness, so that coupling “constants” should be regarded as functions of ϕ . This is the basic problem of renormalization [7,8,11]. We will return to this point later.

III. CONSISTENCY OF CONSTRAINTS

We have chosen the renormalization prescription to preserve D -dimensional general coordinate invariance. This does not, however, guarantee the whole symmetry of the theory at quantum level. We have to ensure that our renormalization procedure is consistent with the constraints which are generators of the symmetry.

The constraints consist of the momentum constraint $\mathcal{H}_i(x)$ and the Hamiltonian constraint $\mathcal{H}(x)$. Since our renormalization procedure preserves D -dimensional general coordinate invariance, no anomalous terms may appear in commutators with the momentum operators. In our renormalization prescription, $\mathcal{H}(x)$ should be replaced by the renormalized Hamiltonian constraint

$$\mathcal{H}_R(x) \equiv -16\pi G \Delta_R(x) + \frac{\sqrt{g(x)}}{16\pi G} (R(x) + 2\Lambda). \quad (14)$$

Anomalous terms would appear in the commutator of \mathcal{H}_R 's:

$$\begin{aligned} & \left[\int d^D x \eta_1(x) \mathcal{H}_R(x), \int d^D y \eta_2(y) \mathcal{H}_R(y) \right] \\ &= i \int d^D x (\eta_1(x) (\nabla_i \eta_2(x)) \\ & \quad - (\nabla_i \eta_1(x)) \eta_2(x)) \mathcal{H}^i(x) + \Delta\Gamma, \end{aligned} \quad (15)$$

where η_1 and η_2 are arbitrary scalar functions. The anomalous term $\Delta\Gamma$ is expected to be of the form

$$\Delta\Gamma = \int d^D x \sqrt{g(x)} \sum_a \sum_{n=1}^{[D/2]} \frac{\phi^{(D-2n)}(0)}{(D-2n)!} \tilde{\mathcal{O}}_{2(n+1)}(\eta_1, \eta_2; x). \quad (16)$$

We notice that dimension zero and two operators $\tilde{\mathcal{O}}_0$ and $\tilde{\mathcal{O}}_2$ will not appear on the right-hand-side of (16) because there are no such operators satisfying the antisymmetry under the exchange of η_1 and η_2 . There exists a dimension four operator $\tilde{\mathcal{O}}_4$, which is the lowest operator satisfying the antisymmetric property, i.e. $\tilde{\mathcal{O}}_4 = (\eta_1(\nabla_i \eta_2) - (\nabla_i \eta_1)\eta_2)\nabla^i R$, up to a constant. In [7,8], it has been shown that $\Delta\Gamma$ is proportional to $\phi^{(1)}(0)\tilde{\mathcal{O}}_4$ with a nonzero coefficient for $D = 3$, as expected in (16).

We could, in principle, compute the right-hand-side of (16), but it is practically impossible for any D because the size of computations will increase rapidly with D . It will not, however, be unreasonable to assume that every $\tilde{\mathcal{O}}_{2(n+1)}$ is nonvanishing for $n = 1, 2, \dots, [D/2]$ in (16) even for $D > 3$ since there is no symmetry to prevent every term in (16) from appearing. Thus, we conclude that the anomaly-free condition $\Delta\Gamma = 0$ requires that

$$\phi^{(D-2)}(0) = \phi^{(D-4)}(0) = \dots = \phi^{(1)}(0) = 0 \quad (17)$$

for $D = \text{odd}$. For $D = \text{even}$, there is an obstacle. If $\tilde{\mathcal{O}}_{D+2}$ is nonzero, $\Delta\Gamma$ cannot be zero for $D = \text{even}$ because of the condition (11). We will hereafter restrict ourselves to the case of $D = \text{odd}$, unless stated otherwise.

IV. STRONG COUPLING EXPANSION AND $z > 1$ LIFSHITZ POINTS

Let us now solve the renormalized Wheeler-DeWitt equation

$$\left[-16\pi G \Delta_R(x) + \frac{\sqrt{g(x)}}{16\pi G} (R(x) + 2\Lambda) \right] \Psi[g] = 0 \quad (18)$$

with the anomaly-free condition (17). Since we are interested in short-distance behavior, we do not probably need to solve it exactly. The strong coupling expansion will be well suited to probe quantum geometry at short distances

[7,8,10]. We then assume that the wave functional $\Psi[g]$ has the form

$$\Psi[g] \equiv \exp\{-S[g]\} = \exp\left\{-\sum_{n=1}^{\infty} \left(\frac{1}{16\pi G}\right)^{2n} S_n[g]\right\}. \quad (19)$$

Substituting (19) into (18), we have the leading-order equation

$$\Delta_R(x) S_1[g] = -\sqrt{g(x)} (R(x) + 2\Lambda). \quad (20)$$

Since we are looking for solutions that realize Hořava's idea [1,2], we restrict the form of $S_1[g]$ to a finite sum of integrals of local functions of g_{ij} :

$$S_1[g] = \int d^D x \sqrt{g(x)} \sum_a \sum_{n=0}^N \gamma_n^a \mathcal{O}_{2n}^a(x), \quad (21)$$

where γ_n^a is a dimensionful constant of mass dimension $-D - 2n + 2$. According to the discussions given in [1,2], the solution (21) turns out to realize a $z = 2N$ Lifshitz point if one of the coefficients γ_N^a for the highest operators given by the form $\mathcal{O}_{2N}^a \sim (\nabla)^{2N-2} R$ is nonvanishing.

It is easy to see that the relations (12) and (13) together with the constraints (17) lead to a solution with $N = 1^2$:

$$S_1[g] = \int d^D x \sqrt{g(x)} \{\gamma_0 + \gamma_1 R(x)\}, \quad (22)$$

where

$$\begin{aligned} \gamma_0 &= \frac{4(D-1)^2(D-2)!(4\pi)^{D/2}}{(D^2-2)\phi^{(D)}(0)} \\ &\quad \times \left(2\Lambda + \frac{6D(D-1)\phi^{(D+2)}(0)}{(D+2)(D^3+10D^2-13D-34)\phi^{(D)}(0)} \right), \\ \gamma_1 &= \frac{48(D-1)D!(4\pi)^{D/2}}{(D-2)(D^3+10D^2-13D-34)\phi^{(D)}(0)}. \end{aligned} \quad (23)$$

Thus, we conclude that the renormalized Wheeler-DeWitt equation has a $z = 2$ solution to leading order of the strong coupling expansion.

The form of the leading solution (22) is good news for the renormalizability of the theory and our strong coupling approximation. In our formulation, the renormalizability requires that all physical quantities must be independent of the arbitrary function $\phi(\varepsilon)$ or $\phi^{(n)}(0)$. Fortunately, this can be achieved, at least to leading order, by absorbing $\phi^{(D)}(0)$ and $\phi^{(D+2)}(0)$ into the redefinition of G and Λ because the leading-order wave functional becomes independent of ϕ . This implies that the combination $G^2(\mu)\mu^D$, where μ is a mass parameter defined by $\phi^{(D)}(0) \sim \mu^D$, should be inde-

²In [14], Kodama has pointed out that the exponential of the Chern-Simons action is an exact solution of the Hamiltonian constraint in the holomorphic representation of the Ashtekar formalism [15]. Connections with our solution are unclear.

pendent of μ . This fact makes a physical meaning of our approximation clear. Since the actual dimensionless expansion parameter is $1/[G^2(\mu)\mu^{2(D-1)}]$ and since it tends to zero as the mass scale μ increases (if $D > 2$), our strong coupling expansion is thus expected to give a good approximation scheme at high energies, as mentioned before.

Let us next try to construct solutions associated with higher Lifshitz points. Suppose that $S'_1[g]$ is another solution to (20). Then, $S'_1[g] - S_1[g]$ has to satisfy

$$\Delta_R(x)(S'_1[g] - S_1[g]) = 0. \quad (24)$$

If $S'_1[g]$ would correspond to a $z = 2N$ solution, $S'_1[g] - S_1[g]$ should be expanded as³

$$S'_1[g] - S_1[g] = \int d^D x \sqrt{g(x)} \sum_a \sum_{n=0}^N \delta_n^a \mathcal{O}_{2n}^a(x). \quad (25)$$

It turns out that the action of $\Delta_R(x)$ on (25) will lead to

$$\Delta_R(x)(S'_1[g] - S_1[g]) = \sqrt{g(x)} \sum_a \sum_{n=0}^N \delta_n^a \mathcal{O}_{2n}^a(x). \quad (26)$$

Since \mathcal{O}_{2n}^a 's are independent each other, all the coefficients δ_n^a have to vanish to be a solution to (24). Note that the number of the parameters $\{\delta_n^a\}$ is, in general, less than that of $\{\delta_n^a\}$ because if \mathcal{O}_{2n}^a is written as a total derivative like $\nabla_i \nabla^i R$, it does not contribute to (25) but can appear in (26). Therefore, all the coefficients δ_n^a in (25) should be trivial as long as there is no accidental degeneracy in the relations between δ_n^a and δ_n^a . For $D = 3$ with $N = 2$, we can explicitly verify that there is no nontrivial solution $S'_1[g]$. We thus conclude that the solution (22) is unique to leading order in the strong coupling expansion in a class of integrals of local functions given in (21).⁴ This result shows that the quantum Einstein gravity in our Wheeler-DeWitt formulation possesses a unique $z = 2$ Lifshitz point in the strong coupling limit.

Before closing this section, we would like to make a few comments in order. First, it should be stressed that the strong coupling expansion is not a derivative expansion because higher derivative terms, like $R^m (m > 1)$, could appear on the right-hand side of (22), but it happens that their coefficients are zero as a solution to Eq. (20). Since our results show that in the strong coupling limit the $D + 1$ -dimensional quantum Einstein gravity reduces to the

D -dimensional Einstein gravity, one might expect that by the inverse Wick rotation the $(D - 1) + 1$ -dimensional theory could further reduce to the $(D - 1)$ -dimensional one. This is not, however, the case because our formulation can apply only for the case of $D = \text{odd}$ due to the anomaly in (15). The final comment is that we need the ‘‘cosmological’’ term γ_0 in (22) in order for (22) to become a solution to (20) even if $\Lambda = 0$.

V. CONCLUSIONS

In this paper, we have solved the renormalized Wheeler-DeWitt equation in $D + 1$ dimensions in the strong coupling limit to test Hořava's idea that quantum gravity at short distances will be described by a nonrelativistic theory with $z > 1$. Our results indicate that the dimensional reduction [16] from $D + 1$ to D dimensions for $D = \text{odd}$ occurs in the strong coupling limit and the quantum Einstein gravity has a $z = 2$ Lifshitz point but no other higher Lifshitz points. Although we have not found a $z = 3$ solution [2,17], which will lead to a power-counting renormalizable quantum gravity theory in $3 + 1$ dimensions, we may still have a chance to get a finite theory because the 3-dimensional Einstein gravity has no local excitation and can be described by a topological field theory [18].

Our analysis can also be applied to the functional Schrödinger equation for the Yang-Mills theory. In [11,19,20], it has been suggested that the dimensional reduction from $3 + 1$ to 3 dimensions occurs in the infrared region, and a vacuum wave functional for the $3 + 1$ -dimensional Yang-Mills theory is given by the exponential of the 3-dimensional Yang-Mills action. This result now has a new interpretation that the Yang-Mills theory can be described by a $z = 2$ nonrelativistic theory at low energies.⁵ It would be of great interest to investigate the Yang-Mills theory at low energies from a new perspective.

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³We can add a gravitational Chern-Simons term discussed in [2] to (25). However, the conclusion given below will not change.

⁴This conclusion does not mean that there are no nonlocal solutions which include infinitely many higher derivatives.

⁵This happens in the infrared region, although Hořava [21] supposed that this situation will occur at high energies.

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