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**Title:**

# A note on the acoustic properties of a double-leaf permeable membrane

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## 1. Introduction

The authors have so far studied the acoustic properties of membrane materials for building purposes and double-leaf membrane structures of various configurations using them [1-7]. Membrane materials show different acoustical properties according to their acoustic permeability. Therefore, different combinations of permeable and impermeable membranes produce various double-leaf membrane structure of different acoustic properties. The double-leaf membranes with impermeable leaves only [5], with impermeable and permeable leaves [6,7] have been investigated. However, those with two permeable leaves were only studied under the normal incidence condition [8,9], and its acoustical properties for diffuse field incidence should be studied.

In this study, a double-leaf membrane structure with two permeable leaves (DLPM) is analysed theoretically by using Helmholtz-Kirchhoff integral formulation to obtain a general solution, and discussed its acoustic properties for diffuse field incidence through numerical examples.

## 2. Theoretical considerations

Figure 1 shows the geometry and model for analysis of a DLPM. The two permeable membranes (PM1 and PM2) are supposed to be of infinite extent, with the surface density $m_1$, $m_2$ (kg/m$^2$), flow resistance $r_1$, $r_2$ (Pa s/m), and the tension $T_1$, $T_2$ (N/m), respectively. A plane sound wave of unit pressure amplitude $p_i$ is supposed to be incident upon the PM1 at the angle of incidence $\theta$. Fig.1

Using a Helmholtz-Kirchhoff integral, the surface pressure of the illuminated side of PM1 is expressed by the following equation:

$$p_1(x,0) = 2p_i(x,0) + \frac{i}{2}\int_{-\infty}^{\infty}\left[\rho_0\omega^2 w_1(x_0) + ik_0 A_{m1}\Delta p_1(x_0)\right]H_0^{(1)}\left(k_0|x - x_0|\right)dx_0 \qquad (1)$$

Here, $A_{mj}=\rho_0 c_0/r_j$ ($j$=1,2), with $\rho_0 c_0$ the air impedance, $k_0=\omega/c_0$ the wavenumber in the air, $\omega$ the angular frequency, $w_j$ the vibration displacement of the $j$-th membrane, $\Delta p_j$ the pressure difference of the both side surfaces of the $j$-th membrane.

A similar expression is obtained for the surface pressure on the transmitted side (back side) of PM2:

$$p_3(x,D) = -\frac{i}{2}\int_{-\infty}^{\infty}\left[\rho_0\omega^2 w_2(x_0) - ik_0 A_{m2}\Delta p_2(x_0)\right]H_0^{(1)}\left(k_0|x - x_0|\right)dx_0 \qquad (2)$$

The sound pressure and particle velocity in the air cavity are written in the following forms:

$$p_2(x,z) = \left(Xe^{ik_0 Z\cos\theta} + Ye^{-ik_0 Z\cos\theta}\right)e^{ik_0 x\sin\theta} \qquad (3)$$

$$v_2(x,z) = \frac{\cos\theta}{\rho_0 c_0}\left(Xe^{ik_0 Z\cos\theta} - Ye^{-ik_0 Z\cos\theta}\right)e^{ik_0 x\sin\theta} \qquad (4)$$

Here, $X$ and $Y$ are the pressure amplitude of the wave propagating into $+z$ and $-z$ direction, respectively, which are to be determined. From eqs.(1)-(4) and the equations of motion of PM1 and PM2, using Fourier transform technique, the reflected pressure $p_r$ and transmitted pressure $p_t$ are solved as follows:

$$p_r(x,z) = \left[1 + \frac{i\rho_0\omega^2 L_1(k_0 \sin\theta) - k_0 A_{m1} J_1 \{H_1 L_1(k_0 \sin\theta) + H_2 L_2(k_0 \sin\theta) + H_3\}}{k_0 \cos\theta}\right] \times \exp[i(k_0 \sin\theta x - k_0 \cos\theta z)] \quad (5)$$

$$p_t(x,z) = \left[\frac{-i\rho_0\omega^2 L_2(k_0 \sin\theta) + k_0 A_{m2} J_2 \{I_1 L_1(k_0 \sin\theta) + I_2 L_2(k_0 \sin\theta) + I_3\}}{k_0 \cos\theta}\right] \times \exp[i(k_0 \sin\theta x + k_0 \cos\theta z)] \quad (6)$$

Where, $H_j$, $I_j$, $J_j$, $L_j$ are considerably complicated functions including various parameters relating to the membranes and the air cavity, which cannot be presented here for lack of space. From the above equations the absorption and transmission coefficients, $\alpha$, and $\tau$, are obtained as follows:

$$\alpha = 1 - |p_r(x,0)|^2, \quad \tau = |p_t(x,D)|^2 \quad (7)$$

For evaluating the sound absorption performance, it is necessary to evaluate the energy dissipated in the structure only, therefore the difference of absorption and transmission coefficients, $\alpha - \tau$, is used throughout. For diffuse field incidence the field-incidence average (average over 0-78 degrees of half sphere) is taken.

## 3. Numerical examples and discussion

A typical example of calculated results of $\alpha - \tau$ of a DLPM is shown in Fig. 2. At high frequencies fluctuation with alternating peaks and dips appear as observed in conventional porous absorbers, which demonstrates the effect of the standing wave in the air cavity. This shows that the PM2 is behaving as the back wall in a conventional porous sound absorbing system. The decrease in $\alpha - \tau$ at low frequencies is caused by the decrease in acoustic resistance

Fig.2

of the membrane due to the sound induced vibration, which is commonly observed in various permeable materials [3].

Figure 3 shows the effect of the surface density of the membranes on $\alpha-\tau$ of a DLPM: (a) shows that of PM1 ($m_1$), and (b) shows that of PM2 ($m_2$). In general the acoustic resistance of PM1 increases with increasing $m_1$, thus monotonic increase in $\alpha-\tau$ can be expected. However, the value of $\alpha-\tau$ decreases at mid frequencies, but increases at low frequencies with increasing $m_1$. This is due to somewhat complex behaviours of the absorption and the transmission coefficients of the entire system: the both coefficients decrease with increasing $m_1$, but the amounts of their change are different and their frequency dependences are different. Therefore, the behaviour of $\alpha-\tau$ of a DLPM to $m_1$ is rather complex. On the other hand, the effect of the surface density of PM2 is rather simple: it appears low to mid frequencies only, and $\alpha-\tau$ increases with increasing $m_2$. This is considered to be caused by the decrease in transmitted energy and the increase in the acoustic resistance of PM2. Fig.3

Figure 4 shows the effect of the flow resistance of the membranes on $\alpha-\tau$ of a DLPM: (a) shows that of PM1 ($r_1$), and (b) shows that of PM2 ($r_2$). When $r_1$ is extremely small, only PM2 works as sound absorbing element showing the same absorption characteristics as a single permeable membrane (Fig. 4 (a)(1)). On the other hand if it is extremely large, the system becomes a double-leaf membrane with an impermeable leaf on the illuminated side and a permeable leaf on the back side. Therefore, the characteristics shows a resonance peak due to the membrane-type absorption which is similar to an impermeable double-leaf membrane [4] (Fig. 4(a)(5)). Regarding the effect of $r_2$, when it is extremely small the system behaves like a single permeable membrane with PM1 only (Fig. 4(b)(1)), but when it is extremely large PM2 behaves as an impermeable membrane, and the system behaves as a double-leaf membrane with a permeable membrane on the illuminated side and an impermeable membrane on the back side, and the characteristics becomes those of a double-leaf with the combination of permeable and impermeable leaves [5] (Fig. 4 (b)(5)). Fig.4

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**Captions of figures**

Fig. 1: Geometry of the model of a DLPM for theoretical analysis.

Fig. 2: A typical example of $\alpha-\tau$ of DLPM. $m_1$=$m_2$=1.0kg/m$^2$, $r_1$=$r_2$=816Pa s/m, $D$=0.05m.

Fig. 3: Effect of (a) $m_1$ and (b) $m_2$. (1)0.25, (2)0.5, (3)1.0, (4)2.0 and (5) 4.0 kg/m$^2$. Other parameters are the same as in Fig. 2.

Fig. 4: Effect of (a) $r_1$ and (b) $r_2$. (1)0.001, (2)100, (3)816, (4)1000 and (5) 10000 Pa s/m. Other parameters are the same as in Fig. 2.

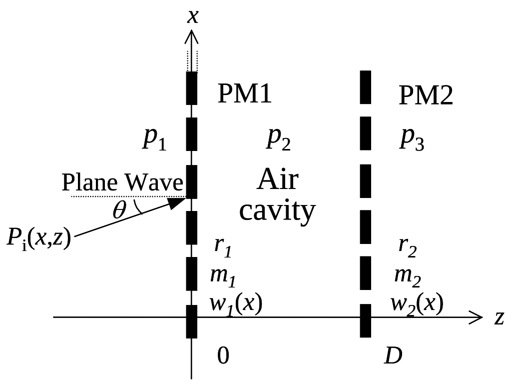
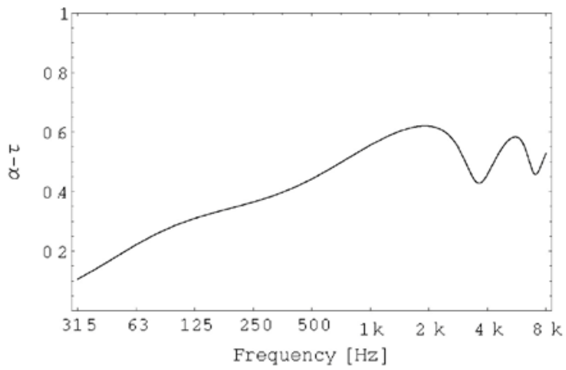



Fig. 1: Geometry of the model of a DLPM for theoretical analysis.

[65mm]

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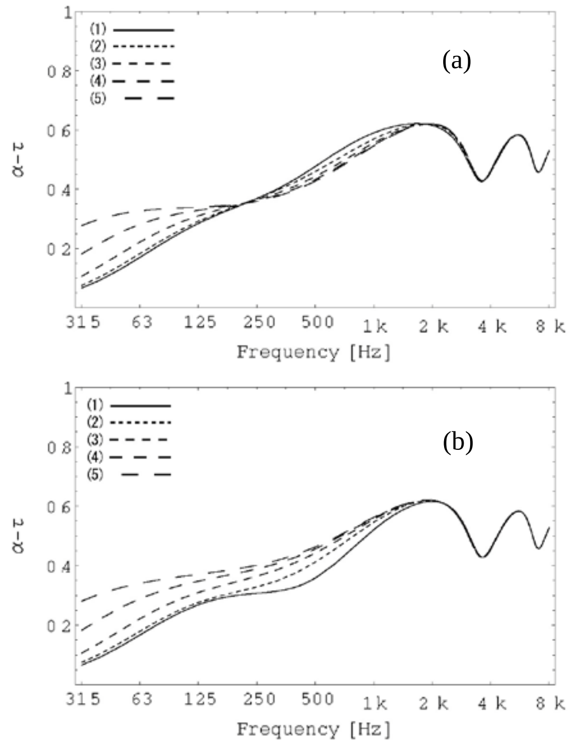
[65mm]

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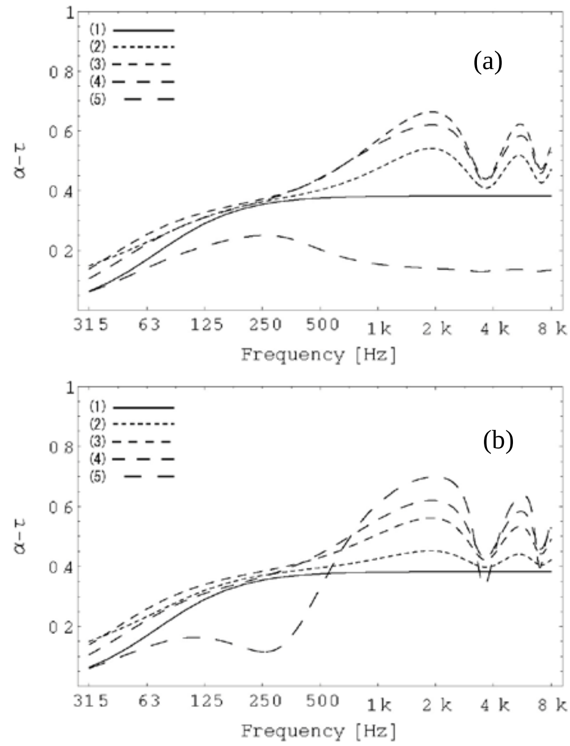
[65mm]

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