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Information gain in an optical bistable system by stochastic resonance

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We have shown an experimental demonstration of information gain due to the stochastic resonance in an optical bistable system, that is, information hidden in the input wave form appears in the output of a nonlinear system when the input noise amplitude is adequate. The optical bistable system is a hybrid type comprising a $LiNbO_3$ crystal with an electric feedback loop, and the input of the system is the sum of a binary bit series and a Gaussian colored noise. The information gain is proved to be prominent when the noise cutoff frequency is larger than the bit rate.

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I. INTRODUCTION

The transmission of a coherent signal is enhanced by adding random noise in a nonlinear system. This cooperative phenomenon is called stochastic resonance (SR) and has attracted much interest in various fields [1]. Particularly, intensive studies have been made on the information gain due to SR [2–5]. If SR brings about information gain, information hidden in the input wave form appears in the output of a nonlinear system. In systems such as superconditing quantum interference device [2], neuronal systems [3], level crossing detector [4], and CdS crystal [5], the output signal-to-noise ratio (SNR) exceeds the input one for periodic signals. However, since periodic signals contain no finite information, the gain of SNR cannot mean the information gain.

SR with aperiodic signals, which have finite information, has been studied and definitions of measures of the transinformation in place of the conventional SNR were proposed [6–14]. Barbay *et al.* studied the dynamical behavior in a vertical cavity surface emitting laser with a binary input signal [6]. The mutual information was used as a measure [7,8] in studies on neuronal systems using models with threshold nonlinearity with a binary input signal. However, information gain has not been discussed in the above studies [6–14]. In a recent paper, Misono *et al.* showed that information shows gain in a bistable system for a binary input signal by numerical simulation [9].

In the present study, we show an experimental demonstration of information gain in an optical bistable system for a binary input signal. Transmission of information is proved to show the resonance peculiar to SR with the addition of noise, and information hidden in the input wave form appears in the output of the system when the noise amplitude is adequate. The influence of the bandwidth of noise in this system is also studied, and we show that a broadband noise is favorable for SR and information gain. In addition to the interest in fundamental characteristics, this subject is practically significant because instruments, such as a noise generator or an amplifier, have limited bandwidths.

II. EXPERIMENT

The experimental setup is illustrated in Fig. 1. The light source is a cw Ti:sapphire laser pumped by an argon-ion laser. The laser output is vertically polarized and its wavelength is 780 nm. The intensity of light is modulated by an electro-optic modulator (EOM). The driving voltage of the EOM is the sum of binary bit series $s(t)$, a colored noise $\xi(t)$, and bias voltage.

Signal $s(t)$ and noise $\xi(t)$ are generated in a computer, and are transferred to arbitrary wave form generators, AWG1 and AWG2, respectively. For $s(t)$, a pseudorandom bit series with a period of $2^{15}-1$ bits is used, and its bit rate R is 1 kbit/s. Data of 3 kbits are used in a single measurement. Noise $\xi(t)$ is the Ornstein-Uhlenbeck (OU) noise [15],

$$\frac{d\xi(t)}{dt}=-\frac{1}{\tau}\xi(t)+\frac{\sqrt{D}h}{\tau}\xi_w(t), \qquad (1)$$

where τ is the correlation time of noise, D is the noise intensity, and $\xi_w(t)$ is the white noise. The noise cutoff frequency is $f_c=1/(2\pi\tau)$. The OU noise in the present experiment has a Gaussian amplitude distribution with zero mean. The noise data consist of 8 bit/word$\times 10^6$ words.

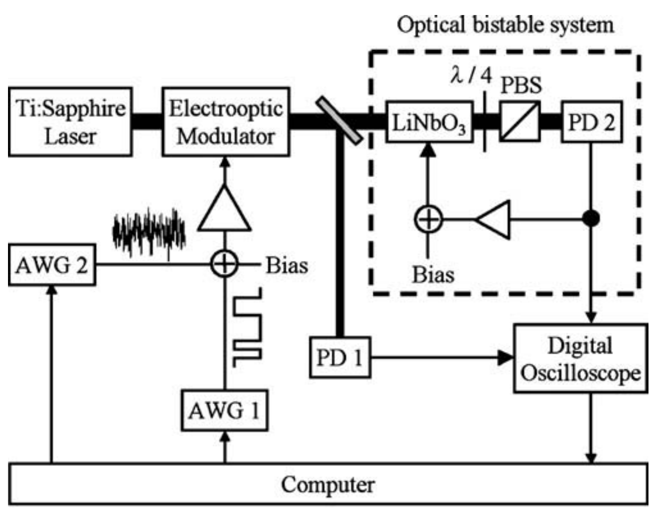

FIG. 1. Schematic diagram of the experimental setup. PD1 and PD2, photodiodes, AWG1 and AWG2, arbitrary wave form generators, PBS, polarization beam splitter.

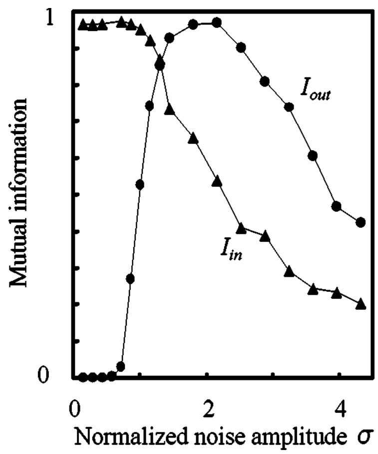

FIG. 2. Noise amplitude dependence of the output mutual information I_{out} (circles) and input one I_{in} (triangles), where the horizontal axis is the normalized noise amplitude σ. The signal amplitude A is 0.8, and f_c/R, the ratio of the noise cutoff frequency and the bit rate, is 10.

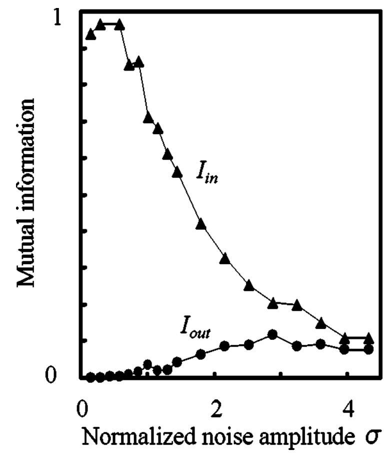

FIG. 3. Noise amplitude dependence of the output mutual information I_{out} (circles) and the input I_{in} (triangles), where the horizontal axis is the normalized noise amplitude σ. The signal amplitude A is 0.8, and f_c/R, the ratio of the noise cutoff frequency and the bit rate, is 0.1.

The output of the EOM is fed as the input to an optical bistable system. A part of the EOM output is used to monitor the input intensity $z_{in}(t)$ to the bistable system with a photodiode (PD1).

The optical bistable system is a hybrid one comprising an amplitude modulator with an electric feedback loop [16,17]. The amplitude modulator is composed of an electro-optic crystal ($LiNbO_3$, $1\times4\times30$ mm^3), a $\lambda/4$ plate, and a polarizer. The output intensity of the amplitude modulator is converted to an electric signal by a photodiode (PD2). The signal is amplified and is then fed back to the $LiNbO_3$ crystal with the bias voltage. The bandwidth of the amplifier is 40 kHz, and this limits the response of the optical bistable system. The output of the bistable system is $z_{out}(t)$.

Wave forms, $z_{in}(t)$ and $z_{out}(t)$ are introduced as the input to a digital oscilloscope (10^5 samples/s) and are transferred to a computer. Each bit in these wave forms is decided to be high or low by the computer. Thus the input bit series $s_{in}(t)$ and the output bit series $s_{out}(t)$ are obtained from $z_{in}(t)$ and $z_{out}(t)$, respectively. The decision timings in each bit for the input and for the output wave forms are at the center and at the end of each bit, respectively. Bit series $s_{in}(t)$ and $s_{out}(t)$ are compared with the original bit series $s(t)$, thus the input mutual information $I_{in}=I(s(t),s_{in}(t))$ and the output mutual information $I_{out}=I(s(t),s_{out}(t))$ are obtained.

The mutual information is defined as follows. Here a bit in the original bit series x is high with a probability p_x, and is low with a probability $p_{\bar{x}}=1-p_x$. Similarly a bit in the input or in the output bit series y is high with a probability p_y and is low with a probability $p_{\bar{y}}=1-p_y$. Conditional probability p_{yx} is that y is high under the condition that x is high, and the other conditional probabilities are defined in a similar way. Mutual information $I(x,y)$ between the bit series x and y is defined as [7]

$$I(x,y)=H(y)-H(y|x), \tag{2}$$

$$H(y)=-p_y\log_2 p_y-p_{\bar{y}}\log_2 p_{\bar{y}}, \tag{3}$$

$$H(y|x)=p_x(-p_{yx}\log_2 p_{yx}-p_{\bar{y}x}\log_2 p_{\bar{y}x}) + p_{\bar{x}}(-p_{y\bar{x}}\log_2 p_{y\bar{x}}-p_{\bar{y}\bar{x}}\log_2 p_{\bar{y}\bar{x}}), \tag{4}$$

where $H(y)$ is the entropy of y and $H(y|x)$ is the conditional entropy of y for a given value of x. When $p_x=p_{\bar{x}}=p_y=p_{\bar{y}}=1/2$, for example, $I(x,y)$ takes the following values. If bit series y is identical to the original bit series x, that is, all of the information is transmitted, then conditional probabilities p_{yx} and $p_{\bar{y}\bar{x}}$ become unity, while $p_{\bar{y}x}$ and $p_{y\bar{x}}$ are 0, and thus $I(x,y)$ is unity. On the other hand, if y has no correlation with x, that is, no information is transmitted, then all of the conditional probabilities are 1/2, and thus $I(x,y)$ is 0.

III. RESULTS AND DISCUSSION

In the optical bistable system a hysteresis loop appears, which is obtained by increasing and by decreasing the input intensity continuously. The input signal amplitude, the difference of light intensity between high and low bits, is normalized by range h of the input intensity which has two stable outputs. Normalized noise amplitude σ is obtained by dividing twice the standard deviation of noise by h. In our experiment, the normalized signal amplitude A is 0.8, and the bias voltage of the EOM is set at a value at which the input intensity of the optical bistable system is at the center of the bistable range when there is no signal or noise.

Figure 2 shows the noise amplitude dependence of the input and output mutual information, where the horizontal axis is the normalized noise amplitude σ, and the noise cutoff frequency f_c is 10 kHz ($f_c/R=10$). While the input mutual information I_{in} decreases monotonically as σ increases, the output mutual information I_{out} shows the resonancelike enhancement. When $\sigma>1.3$, I_{out} exceeds I_{in}. This means that information hidden in the input wave form appears in the output of the optical bistable system even if the bit series is buried in noise whose amplitude is sufficiently large.

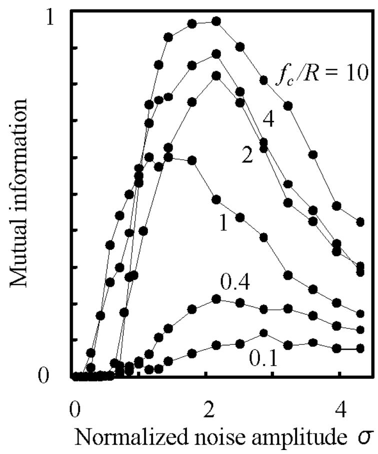

FIG. 4. Dependence of the resonance curve on the noise cutoff frequency f_c for $f_c/R=0.1$, 0.4, 1, 2, 4, and 10, where R is the bit rate. The signal amplitude A is 0.8.

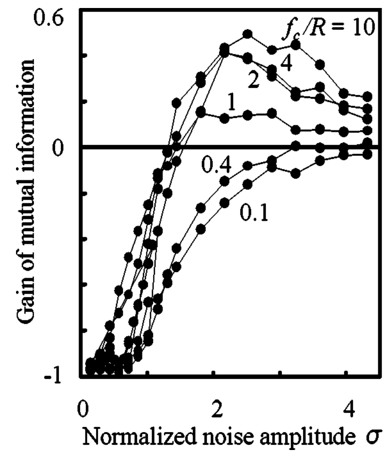

FIG. 5. Dependence of the GMI, $I_{out}-I_{in}$, on the noise cutoff frequency f_c for $f_c/R=0.1$, 0.4, 1, 2, 4, and 10, where R is the bit rate. The signal amplitude A is 0.8. The positive value of GMI means that more information is obtained from the output of the bistable system than from the input.

Figure 3 shows the noise amplitude dependence of the mutual information when f_c is 100 Hz $(f_c/R=0.1)$. The output mutual information I_{out} takes small values and hardly shows resonance. The noise with a small value of f_c/R causes the interwell motion less frequently even when the noise amplitude is optimized because noise makes small change in the duration of a single bit. In the range shown in Fig. 3, I_{out} does not exceed I_{in}.

Figure 4 shows the dependence of the resonance curve of I_{out} on noise cutoff frequency f_c. The values of f_c are 100 Hz, 400 Hz, 1 kHz, 2 kHz, 4 kHz, and 10 kHz ($f_c/R=0.1$, 0.4, 1, 2, 4, and 10) in increasing order. The curves for larger values of f_c show more remarkable resonance.

The gain of the mutual information (GMI), $I_{out}-I_{in}$, is shown in Fig. 5 for the same values of f_c shown in Fig. 4. When GMI is positive, more information is obtained from the output of the optical bistable system than from the input. GMI takes large positive values for large values of f_c. In the present experiment, no gain is obtained when f_c is 100 Hz $(f_c/R=0.1)$. The experimental results have the same characteristics as those of the simulation results [9] (note that the noise intensity $D=\sigma^2/(2\pi f_c)$ is used in Ref. [9]).

In Fig. 5, mutual information has remarkable gain when ratio f_c/R is more than unity, and has small or no gain in the opposite case. When $f_c/R=10$, the maximum value of GMI is 0.5, a large enhancement of transmitting of information is achieved by the optical bistable system. This enhancement can also be seen in Fig. 2. When $\sigma=2.2$, while I_{in} takes a small value, 0.5, I_{out} is nearly unity. This kind of increase in the mutual information by transmitting through the optical bistable system is realized when $\sigma>1.3$.

In Fig. 2, when $\sigma=1.2$, for example, transmitting the wave form through the bistable system in itself results in the degradation of the mutual information. Nevertheless, the enhancement of the transinformation is realized in the following way. Intentionally we added more noise to the input wave form to reach $\sigma=2.2$, and then transmitted the wave form through the optical bistable system. Thus the enhancement of the mutual information is achieved by adding noise in spite of its paradoxical appearance. When f_c/R is small, enhancement of transinformation cannot be achieved by the procedures described above.

IV. CONCLUSION

We have studied SR with a binary input signal in an optical bistable system employing mutual information as a measure of the transinformation. When f_c/R is large, the output mutual information of the optical bistable system shows remarkable resonance, and information hidden in the input wave form appears in the output of the optical bistable system, that is, information has gain. When f_c/R is small, the output mutual information hardly shows resonance, and the benefits of the information gain are not significant.

The information gain in the optical bistable system is significant for fundamental interest, and, moreover, it is beneficial to applications, such as optical communication and image processing [9]. The results of the present study can be applied to higher-speed signals because the shapes of resonance curves and GMI curves depend not on the value of f_c itself, but also on the ratio of f_c/R. Processing higher-speed signals is practicable by broadening the bandwidth of the feedback loop in the optical bistable system.

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