



Observational heterarchy enhancing active coupling

Gunji, Yukio-Pegio

Kamiura, Moto

(Citation)

Physica D: Nonlinear Phenomena, 198(1-2):74-105

(Issue Date)

2004-11-01

(Resource Type)

journal article

(Version)

Accepted Manuscript

(URL)

<https://hdl.handle.net/20.500.14094/90001519>



Gunji, Y.-P. & Kamiura, M. (2004)
Observational heterarchy enhancing active coupling
Physica D 198, 74-105.

Observational Heterarchy Enhancing Active Coupling

Yukio-Pegio Gunji and Moto Kamiura

*Department of Earth & Planetary Sciences, Faculty of Science, Kobe University, Nada Kobe 657-8501,
JAPAN*

yukio@kobe-u.ac.jp

Abstract

Heterarchy and the agent of self-organization are ones of most challengeable topics in complex systems. When the notion of heterarchy is generalized and dilated to general phenomena and is replaced by observational heterarchy, the agent of self-organization (i.e., internal observer carrying measurement) is found even in non-hierarchical system. A usual dynamics in the sense of complex system is articulated into two kinds of dynamics, Intent and Extent dynamics, and the interaction between them, where Intent corresponds to an attribute to a given phenomenon and Extent corresponds to a collection of objects satisfying the phenomenon. We formalize observational heterarchy by introducing pre-equivalence (pre-adjunction) between Intent and Extent, and apply it to a coupled map system. Since the model inherits evolvability of dynamics, we call it active coupling to distinguish usual (passive) coupling from it. We show that perturbation can modify the dynamics itself and enhance robust behavior opened to emergence in the system of active coupling, and discuss the significance of observational heterarchy in complex systems.

Keywords; heterarchy, agent, internal measurement, endo-physics, coupled map system

1. Introduction

Recently, new concepts are addressed in the field of complex systems; the one is heterarchy [1], and the other is the agent carrying the adjustment of measurement [2]. The concept of heterarchy is originally proposed by McCulloch to reveal indefinite hierarchical system [3], and then it is addressed in abstract system theory including Autopoiesis and cognitive science [4-6]. In the late behalf of 1990's, such a notion is taken in the field of social sciences including economics, theory of management and sociology [7, 8]. In 2003, E. Jen who is a Santa-Fe mathematician dealing with

cellular automata points out the importance of heterarchy with respect to the difference between stability and robustness [1]. In about same days, Kauffmann who is also a Santa-Fe researcher declares that the most important problem in complex systems is to describe the agent who can adjust the way of measurement by consequence of measurement [2]. The idea results from Maxwell's demon, and in 1980's it is described as the notion of internal measurement [9] and endo-physics [10]. Although one of the authors also commits formalization of internal measurement and endo-physics [11-16], the notion is not developed to a large field of researches. We should note that these two ideas, heterarchy and the agent carrying measurement, are addressed in the Santa-Fe Institute, one of centers of complex systems, and that these two notions are strongly relevant with each other.

Originally, heterarchy reveals that it is impossible to determine to which subsystems an element belongs. Since a man belongs to his family and also belongs to the company to which he is employed, his action sometimes depends on his family and sometimes depends on the company. As a result, the attribute of his action cannot be determined to which it essentially belongs. Recently, the feature is focused such that the layer of an element (e.g., man) perpetually interacts with other layers (e.g., family or company), and dynamical structural change results in robust behavior [17, 18]. Jen points out that retained function in complex system results from dynamical interaction of layers in heterarchical system (e.g., biological hierarchical system [19-21]), and that it is not stable but robust. It is necessary to see the form of heterarchy in mathematical sense.

Independent of the arguments of heterarchy, Kauffman refers to the agent of self-organizing system such as a windmill that can change the way of receipt of winds, and says that the windmill "measures" wind (i.e., receives wind) and the consequence of the measurement can change and adjust the direction and rotation of the axis of the windmill (i.e., means of measurement). Sometimes much more effective way of measurement can be created, and sometimes it fails. He says that such a windmill is a metaphor of adaptable agent. It is easy to see that such a windmill is expressed as a structure of heterarchy. While measurement is operating a state, the consequence of measurement results in work. These two concepts are different from each other with respect to logical category. In spite of the difference it is necessary to see the mixture or interaction between them. It is an essential and minimal form of heterarchy, and it is nothing but an internal observer proposed by Matsuno [9, 22] and Roessler [10, 23].

How can one describe the mixture of different logical categories? In a usual sense, it entails to a self-reference resulting in a contradiction [24]. In the context of Autopoiesis there are some attempts to propose the mathematical framework in which a self-reference is regarded as a non-trivial fixed point with positive significance [25,26]. Actually domain theory developed in computer science is consistent with this kind of view [27, 28] and it leads to the development of the reflection theory in computer science [29, 30]. The idea of self-organizing agent has, however, little something to do with a fixed point, since it inherits evolvability and/or changeability. By contrast, fixed points

always retain repetition of the same operation. The agent is relevant rather for connection with the outside of the logical framework. If one retains in the logical framework, a self-reference implies a contradiction, and otherwise, does not [31, 32]. It is consistent with the feature of Autopoiesis, autonomous circulation convoluting the outside (i.e., its own environment) [33]. Some attempts in the context of internal measurement are proposed in the form of invalidation of self-reference [34, 35]. We here show such a form by more natural way in the sense of heterarchy.

Previously, the arguments concerning about the self-organizing agent, internal measurement, endophysics and Autopoiesis are too speculative, philosophical and abstract to develop in the science of complex systems. It was not clear to manifest the significance of such agents and to see the difference between external and internal perspective (i.e., the perspective with internal agent). If the idea of heterarchy is generalized and applied to dynamical systems, we can change the perspective of stability to that of robustness. Imagine a population of insects. In usual sense, the number of individuals is defined and a dynamics with respect to the number or concentration is described. However, the concept such as a population of the insect is described not only by a generalized attribute but by a collection of individuals. As far as one describes it as a particular dynamics, he ignores the feature of collection of individuals or he unconsciously assumes that description in the form of dynamics is equivalent to a collection of individuals. If a population is large enough, the assumption is reasonable. Otherwise, inconsistency between two items of description enhances the dynamical change, and that implies the change of dynamics itself. Although the generalized attribute and a collection of individuals, these two items are different with each other in a term of logical category, the interaction or mixture between them cannot be ignored, and that enhances dynamical change or robust behavior opened to evolvability. That is why it is easy to see heterarchy in such aspects.

Independent of the theory of dynamics, some attempts are recently proposed to describe a phenomenon as a concept as a pair of two items. One is called a formal concept, and each concept is expressed as a pair of sets connecting by polar and bipolar operations [36]. It results from a pair of Intent and Extent in a set theory. Another attempt is a description with infomorphism [37]. Each concept or perspective of observation as a triplet called a classification, sets of types (attribute), tokens (objects) and the binary relation between them. The binary relation represents a set of pairs consisting of types and tokens. The operation between two classifications is defined as a pair of maps, called infomorphism. Although they introduce the way to describe observed phenomenon as a pair of two items that are different with respect to logical category, they are too static to describe evolution and dynamical behavior. We previously claim that it is little relevant for dynamical behavior since there is no inconsistency between two items, and modify them to dynamical concept [38, 39] and to dynamical infomorphism [40, 41] by introducing inconsistency between two items. Since these models are based on a discrete set, it is difficult to develop them in the field of

dynamical system and/or complex system. The formal model for observational heterarchy proposed is developed from the idea of dynamical infomorphism however it is defined with continuous state.

In this paper, we firstly organize some terms for heterarchy, and manifest the essence of heterarchy. Secondly, we dilate the idea of heterarchy to general phenomena, and propose the idea of observational heterarchy. In the sense of observational heterarchy, we deal with the interaction between attribute of insect and a collection of individuals for a phenomenon of the population of insect. The process of measurement and description cannot be separated from what is observed and measured, and epistemology cannot be separated from ontology [42]. It results in dynamical description and dynamical ontology. Finally, we apply the form of observational heterarchy to a coupled map system [43-45] and shows how perturbation enhances robust behavior opened to evolvability in the framework of observational heterarchy, and discuss the role in complex system.

2. Heterarchy and Observational Heterarchy

2-1. Heterarchy and concept

A heterarchy is defined as a dynamical hierarchical system with latent evolvability. For example, imagine economic interaction between countries. In Japan, goods are valid in a term of Japanese Yen, and in the United States they are valid in a term of Dollar. There is an inter-country operation that is the exchange of currencies. Note that exchange itself is valid (i.e., the exchange needs cost), and the exchange is also valid in a term of Yen or Dollar, and it reveals the change of the correspondence between goods and Yen (or dollar). It leads to newly constructing exchange because of addition of the exchange as a particular good, and that is fallen into an infinite regression (Fig.1). Although it looks like a logical contradiction, a real economic system as a whole behaves as a robust system with evolvability. That is a heterarchy existing at the logically critical state close to a contradiction.

The essential property of heterarchy is summarized by the follows. It consists of at least two layers containing an intra-layer operation (e.g., a map from goods to value in a term of currency), and inter-layer operation between them (e.g., exchange). It also inherits the mixture between intra- and inter-layer operations (e.g., exchange is also valid), and that is expressed such that the inter-layer operation itself can have the status of the intra-layer operation. The next problem arises, how one can formally describe the inter-layer operation inhering the mixture of intra- and inter-layer operation, against a contradiction. It is the central issue of heterarchy.

To formalize the heterarchy, we firstly refer to the notion of a concept in a set theory before Russell (not in a ZF-set theory). We investigate a particular relationship between complete concept

and a paradox. That is why a set theory before Russell is referred. In the set theory, a concept is defined so as to satisfy; for all x there exists y such that

$$A(x) \Leftrightarrow x \in y. \quad (1)$$

The first expression, $A(x)$ represents the attribute of a concept, and is called Intent, and second expression represents a collection of objects, and is called Extent. For example, a concept as even numbers is expressed as, $2n \Leftrightarrow \{0, 2, 4, \dots\}$.

Intent and Extent are expressed as maps. If a set of truth value, $2 = \{\text{true}, \text{false}\}$, Intent is expressed as a designation of a particular proposition, where a proposition is expressed as a map $p: X \rightarrow 2$ with X , an arbitrary set. Therefore, Intent A is a map, $A: X \rightarrow \text{Hom}(X, 2)$, where $\text{Hom}(X, 2) = \{p: X \rightarrow 2\}$. Extent is expressed as a particular binary relation that is also expressed as a map $\in: X \times X \rightarrow 2$ where for all (x, y) in $X \times X$, $\in(x, y) = \text{true}$, if $x \in y$; $\in(x, y) = \text{false}$, otherwise. The equivalence between Intent and Extent is obtained by the following. Define a map, $A(a) = \in(a, y)$ by fixing a on x . Note that $A(a)$ is an element of $\text{Hom}(X, 2)$, and $A: X \rightarrow \text{Hom}(X, 2)$ is expressed as a designation of a particular proposition. Because $A(a)$ is also a map, we can apply $A(a)$ to y , and obtains that $A(a)(y) = \in(a, y)$. In varying x again, we obtain

$$A(x)(y) = \in(x, y). \quad (2)$$

The equation (2) represents a particular relationship between Intent and Extent [46]. Consider a map $\iota: \text{Ext}(X \times X, 2) \rightarrow \text{Int}(X, \text{Hom}(X, 2))$ such that for \in in $\text{Ext}(X \times X, 2)$, $\iota(\in) = A$ with $A(x) = \in(x, y)$, where $\text{Int}(A, B)$ and $\text{Ext}(C, D)$ also represent a set of maps from A to B , and from C to D , respectively. It is easy to see that ι is onto mapping. Because A is a one-variable map, and \in is a two-variable map, for all A , there exists \in in $\text{Ext}(X \times X, 2)$ such that $\iota(\in) = A$. Secondly, assume that $\iota(\in) = \iota(\in')$ with $\in \neq \in'$. We obtain $A = A'$, $A(x)(y) = A'(x)(y)$, and then $\in(x, y) = \in'(x, y)$, $\in = \in'$, and that reveals a contradiction. It leads that if $\in \neq \in'$, then $\iota(\in) \neq \iota(\in')$. Therefore, ι is a one-to-one map. As a result, the equivalence between Intent and Extent makes sense in the form of isomorphism such that

$$\text{Int}(X, \text{Hom}(X, 2)) \cong \text{Ext}(X \times X, 2). \quad (3)$$

It reveals a one-to-one onto mapping between a set of Intent and one of Extent, and the equivalence between Intent and Extent.

In the isomorphism (3), there are two operations between a set of Intent and Extent, which are called functors in category theory, $\text{Hom}(X, -)$ and $X \times (-)$ [46, 47]. In this case, $\text{Hom}(X, -)$

transforms each set, A , to a set as $\text{Hom}(X, A)$, and each map, $f:A \rightarrow B$ to a map, $\text{Hom}(X, f): \text{Hom}(X, A) \rightarrow \text{Hom}(X, B)$ such that for a map, m in $\text{Hom}(X, A)$, $\text{Hom}(X, f)(m) = fm$ in $\text{Hom}(X, B)$. The functor $X \times (-)$ transforms a set, A to a product set, $X \times A$, and a map, $f:A \rightarrow B$ to a product map, such as $\text{id}_X \times f$ where $(\text{id}_X \times f)(x, y) = (x, f(y))$. A functor is defined so as to satisfy the law of preservation of composition, namely, composition of applying a functor to a map is equivalent to applying a functor to the composition. Actually, two functors, $\text{Hom}(X, -)$ and $X \times (-)$ satisfy the condition. Given $f:A \rightarrow B$ and $g:B \rightarrow C$, for all m in $\text{Hom}(X, A)$, $\text{Hom}(X, gf)(m) = gfm = \text{Hom}(X, g)(fm) = \text{Hom}(X, g)\text{Hom}(X, f)(m)$. As for $X \times (-)$, one obtains that $X \times gf = (X \times g)(X \times f)$. By using those notations, equation (2) is expressed as

$$\text{ev}(\text{id}_X \times A)(x, y) = \in(x, y). \quad (4)$$

In this formulation, a universal map, $\text{ev}: X \times \text{Hom}(X, 2) \rightarrow 2$ such that $\text{ev}(x, f) = f(x)$ is introduced. Diagrammatically it is shown as

$$\begin{array}{ccc} \text{Hom}(X, 2) & & X \times \text{Hom}(X, 2) \\ \uparrow A & \xrightarrow{X \times (-)} & \uparrow \text{id}_X \times A \\ X & & X \times X \end{array} \quad \begin{array}{c} \searrow \text{ev} \\ \nearrow \in \\ \rightarrow 2 \end{array} \quad (5)$$

Dually, there is another universal map, $\eta: X \rightarrow \text{Hom}(X, X \times X)$ such that $\eta(x)(y) = (x, y)$. By the universal map, η , we obtain that

$$\text{Hom}(X, \in)\eta(x) = A(x). \quad (6)$$

It also reveals that $\text{Hom}(X, \in)\eta(x)(y) = \in\eta(x)(y) = \in(x, y) = A(x)(y)$ as mentioned in equation (2). Diagrammatically it is shown as

$$\begin{array}{ccc} X \times X & & \text{Hom}(X, X \times X) \\ \downarrow \in & \xrightarrow{X \times (-)} & \downarrow \text{Hom}(X, \in) \\ 2 & & \text{Hom}(X, 2) \end{array} \quad \begin{array}{c} \nwarrow \eta \\ \nearrow A \\ X \end{array} \quad (7)$$

The isomorphism (3) is sometimes called adjunction, and functors constituting adjunction are called adjoint functors. Note that adjunction is based on two functors satisfying the law of preservation of composition. Diagrammatically adjunction is shown in Fig. 2A. The property carried by diagrams (5) and (6) is called universal mapping property that holds for any adjoint functors.

In the scheme of adjunction (3), the mixture of a functor and a map reveals a paradox. In denoting a functor as F , applying a functor to a map f such as $F(f)$ reveals that difference between a functor, F , and a map, f with respect to the logical status of operation. Namely, a functor is an operator and a map is an operand in this case. The mixture of a functor and a map is expressed as

$$F(f) = hf. \quad (8)$$

In this form, h is an arbitrary map, and applying a functor to a map is replaced by composition of maps. If $\text{Hom}(X, -)$ is replaced by $[\text{Hom}(X, -)]$ featuring equation (8), $[\text{Hom}(X, f)](m) = hfm$. In applying such a functor-like operation to \in , we obtain that $[\text{Hom}(X, \in)](\eta(x))(y) = h\in\eta(x)(y) = h\in(x, y)$. On the other hand, commutative diagram (7) reveals that $[\text{Hom}(X, \in)](\eta(x))(y) = A(x)(y) = \in(x, y)$, and it entails that $h(\in(x, y)) = \in(x, y)$. Any points are fixed points with respect to an arbitrary map h , and that is a contradiction. The mixture of a functor and a map in the form of equation (8) can entail to a contradiction (also see [40]).

With respect to a concept of a set theory, it is summarized that a concept consists of two intra-layer operations (elements in $\text{Int}(X, \text{Hom}(X, 2))$ and ones in $\text{Ext}(X \times X, 2)$), and a pair of inter-layer operations (a pair of functors, $\text{Hom}(X, -)$ and $X \times (-)$), and it constitutes equivalence between two intra-layers (adjunction). If the mixture between intra- and inter-layer operations occurs, it entails to a contradiction. Such a property is generalized by an adjunction derived from a pair of functors, F and G . Conversely, our goal to describe heterarchy is expressed as a kind of “weak and dynamic adjunction” based on an operation that is not a functor, inheriting the mixture of functor and a map, as shown in Fig. 2B. The property is as same as heterarchy’s. A question arises how one can formally describe adjunction-like structure at the logically critical state close to a contradiction. It is replaced by the question, how one can formalize the inter-layer operation inheriting the mixture of intra- and inter-layer operations against a contradiction.

The next question also arises, whether heterarchy is ubiquitously found or not. Although hierarchical structures are ubiquitously found, they are often regarded as nested structure in a term of agent size, and dynamical behavior of each level is characterized by damping time constant. As a result, a level of most of hierarchical structures can be regarded as independent level separated from each other with respect to dynamical properties. No property of heterarchy such as the mixture of inter- and intra-layer dynamics can be found in most hierarchies. We, however, claim that heterarchy is ubiquitously found. The key notion is an observational heterarchy.

2-2. Observational heterarchy as an incomplete concept

We here introduce the idea of the observable as an observational heterarchy, derived from a concept in a set theory, or a pair of Intent- and Extent set [36] and of type and token [37]. Observational heterarchy is defined as an incomplete concept consisting of a pair of perspectives, Intent and Extent perspectives. Completeness in mathematic, is often defined such that each subset of a given structured set with a particular operation is closed with respect to the operation. Actually, a formal concept [36] is defined in that manner. Given a triplet, two sets, Object and Attribute and a binary relation between them, a formal concept is defined by a pair of subsets, Intent in Attribute and Extent in Object such that each element of Intent is related to all elements of Extent with respect to the binary relation, and vice versa. It implies that Intent can be explained by Extent, and vice versa. By contrast, incomplete concept inherits inconsistency between Intent and Extent. The term, incomplete concept is used by referring to inconsistency and/or inequality.

Firstly, we illustrate observational heterarchy for dynamical behavior of a species of protein, for example, splitting enzyme for lactose. In Intent perspective, one focuses on a general attribute of the protein, and it results in his ignoring differences among individual enzymes. It is as same as Intent in a set theory. A population of the proteins can be grasped as a whole, and then one defines a concentration of proteins as a particular variable. It leads that the dynamical behavior of the protein is expressed as a differential equation with respect to the variable. By contrast, in Extent perspective, we focus on individual differences of the protein, and express the feature as a collection of individual objects. In this sense, Extent perspective is same as Extent in a set theory. Actually, there are some differences among individual proteins with respect to three-dimensional structures. In a microscopic point of view, each protein is folded dependent on weak interactions like hydrogen bonds, and such interactions depend on microscopic environments where hydrogen clusters are distributed. Dependent on three-dimensional structures, there are some variations in a term of dynamical property or functions. A collection of individual objects can be replaced by a collection of attributes carried by individual proteins.

In general, Intent perspective is assumed to be equivalent to Extent one, as well as a concept in a set theory or formal concept in a concept lattice. Due to the equivalence, one does not pay attention to one of perspectives. As a result, all what is needed to describe dynamical behavior of proteins is to determine a particular differential equation or state transition of the concentration of proteins. However it is a usual and convenient way, the equivalence between Intent and Extent perspectives results just from an approximation and/or an assumption. Even if a collection of individual objects is finite under a concrete observation, it is opened to a virtually infinite set. Virtually microscopic distribution of hydrogen clusters and some other substrates are not numerable.

If the observed distribution of hydrogen clusters is slightly changed, it can bring about possibly evolvable changes of protein functions. That is why there can be some variants in Extent perspective that cannot be covered by Intent perspective. It reveals inconsistency between Intent and Extent. It gives rise to the idea of incomplete concept or dynamical negotiation between Intent and Extent perspectives. At every instance of observation, inconsistency between Intent and Extent remains. Such an inconsistency perpetually drives negotiation between Intent and Extent with respect to observation. It can give rise to drastic change of a pair of Intent and Extent. An observational heterarchy, therefore, inherits evolvability and robustness (Fig. 3).

A heterarchy like economic interaction mentioned first results from the naïve realism in which reality is uniquely and objectively represented by the observable. Once it is observed, there is no ambiguity between reality and the observable, and that constitutes the naïve realism. Dynamical property (i.e., the mixture of intra- and inter-layer operations) results from a specific situation that the exchange itself is valid. By contrast, an observational heterarchy consists of a pair of intra-layer perspectives and inter-layer operations. The inter-layer operations provide the way to verify the equivalence between two intra-layer operations (i.e., Intent and Extent perspectives), while the process of verification and intra-layer operation has to proceed simultaneously because a pair of Intent and Extent perspectives is the observable standing at an instance. Such a simultaneous operation of intra-layer operations and verifications results in the mixture of intra- and inter-layer operations. That is why different from a real hierarchical system, an observational heterarchy is always accompanied with the mixture leading a dynamical structural change. An observational heterarchy is not a naively real but a virtual-real complex.

An observational heterarchy is, therefore, always accompanied with the mixture of intra- and inter-layer operations, and it inherits indefiniteness between reality and the observable in principle. Indefiniteness perpetually remains in the observable as a pair of Intent and Extent perspective, because it is replaced by inconsistency between Intent and Extent. In naïve realism, once an observation and/or measurement are finished, reality is separated from the observable. In observational heterarchy, the process of observation is inherited and it constitutes the observable. That is why it inherits evolvability, while it exists as a robust concept.

Adaptive mutation in Lactose operon illustrates the property of observational heterarchy [48-50]. Splitting enzyme for Lactose is controlled by the operon on DNA. If it switches on, the enzyme is elaborated, and if off, not. In the experiment of adaptive mutation, *E coli* bacteria are cultured in culture media only with Lactose. The DNA of bacteria is converted not to elaborate the splitting enzyme corresponding to Lactose (i.e., broken Lactose operon). The bacteria are wasted because of absence of the enzyme, and that gives rise to malfunction of DNA-enzyme production system. Due to the malfunction, mutation rate becomes high, and mutation hits the broken gene

corresponding the splitting enzyme for Lactose. As a result, the bacteria can acquire the ability to use Lactose as energy resource.

After the success of the experiment of adaptive gene, the adaptive behavior is explained by using some cryptic genes [51]. The cryptic genes that are not used in normal conditions plays an essential role in adaptive mutation under a stressed condition. If the dynamical behavior of broken Lactose operon is expressed as a complete concept consisting of a pair of Intent and Extent, the Intent reveals that no Lactose splitting enzyme is produced because of the conversion of DNA. The Extent, therefore, does not involve the varieties of function carried by cryptic genes. In other words, all what are related to virtual and latent function are not mentioned. All what it is described by complete concept is only functions of broken Lactose operon under a normal condition. It never refers to adaptive mutation. By contrast, the observable as observational heterarchy of broken Lactose operon inherits adaptive mutation. Latent function related to cryptic gene appears from negotiation between Intent and Extent perspectives, and it leads to the emergent Intent dynamics of which the enzyme for Lactose can be produced.

The situation of adaptive mutation is as same as the situation of a particular machine, broken machine. Imagine that one is estimating which part is essential to execute computation. For this purpose, he removes a part by part from a machine. Call the part that is essential for execution of computation in the sense of architect, part-A. When he removes part-A, the machine begins to vibrates due to malfunction. It leads that the tuning peg inside is put off and it gets stuck between two some circuits, and that it gives rise to some connection to execute computation. Accidentally, he cannot find that part-A is essential. Although no one can describe such a situation in advance, the situation inherits in the working machine in environment in which the machine works. Such an environment is destined to be opened to the outside.

Observational heterarchy finally consists of two sets of intra-layer maps, called Intent and Extent perspectives, and inter-layer operations. The inter-layer operations have to satisfy the following condition. (1) The inter-layer operations inherit the mixture of intra- and inter-layer operations. (2) There is a procedure by which the inter-layer operation can be regarded as an adjoint functor. If the inter-layer operation satisfies the condition (1)-(2), it is called a pre-functor.

Formally, observational heterarchy consists of two sets of maps, $\text{Ext}(\langle F \rangle X, Y)$ and $\text{Int}(X, \langle G \rangle Y)$, or $\text{Ext}(FX, Y)$ and $\text{Int}(X, \langle G \rangle Y)$ (i.e., either of adjoint functors is a pre-functor) where $\langle F \rangle$ or $\langle G \rangle$ are pre-functors (Brackets are used to distinguish pre-functor from functor). Formal model of an observational heterarchy is derived from adjunction. In a pair of $\text{Ext}(FX, Y)$ and $\text{Int}(X, \langle G \rangle Y)$, both Extent and Intent perspectives are expressed as a set of maps. It is easy to see that even a collection of objects (Extent) can be regarded as a particular map as mentioned in a concept of the set theory. A pair of functors yields logical difference between Intent and Extent perspectives. Especially, in an observational heterarchy, the inter-layer operation has to carry the mixture of intra-

(maps) and usual inter-layer operations (functors). That is why we newly define the notion of pre-functor. Actually, a pre-functor is defined by a pair of map, $\langle f, f^* \rangle$, and is expressed as

$$\begin{array}{ccc}
 X & \xleftarrow{f} & \langle F \rangle X \\
 \phi \downarrow & & \downarrow \langle F \rangle \phi = f^* \phi f \\
 X & \xrightarrow{f^*} & \langle F \rangle X.
 \end{array} \quad (9)$$

A pre-functor, $\langle F \rangle: \text{Int} \rightarrow \text{Ext}$ is mapping a set, X , to a set, $\langle F \rangle X$, and a map ϕ to a map, $f^* \phi f$, where $f f^*(x) = x$ for all x in $f(X)$ with $f: \langle F \rangle X \rightarrow X$. In this sense we call f^* pseudo-inverse of f . Because applying a pre-functor to a map is expressed as composition of maps, it satisfies the condition (1). It also satisfies the condition (2). The approximation is defined by the assumption that f is a one-to-one onto map. If one accepts the approximation, $f^* = f^{-1}$ holds. It leads that a pre-functor can become a functor. Given two maps, $\phi, \psi: X \rightarrow X$,

$$\begin{aligned}
 \langle F \rangle(\phi) \langle F \rangle(\psi) &= (f^* \phi f)(f^* \psi f) = f^* \phi (f f^*) \psi f = f^* \phi (f f^{-1}) \psi f \\
 &= f^* \phi \psi f = \langle F \rangle(\phi \psi).
 \end{aligned} \quad (10)$$

It implies that F preserves the composition of maps, ϕ and ψ .

By introducing a pre-functor instead of a functor, pre-adjunction is also newly defined. Instead of the equivalence via two adjoint functors, equivalence via a map is introduced such that,

$$\begin{array}{ccc}
 X & \xrightarrow{[f]} & \langle F \rangle X \\
 \phi \downarrow & & \downarrow \psi \\
 \langle G \rangle Y & \xleftarrow{[g]} & Y.
 \end{array} \quad (11)$$

In this scheme, there are two pre-functors, $\langle F \rangle: \text{Int} \rightarrow \text{Ext}$ and $\langle G \rangle: \text{Ext} \rightarrow \text{Int}$ ($\langle G \rangle(\gamma) = g^* \gamma g$, and g^* is also a pseudo-inverse of g). As mentioned before, it is possible to construct pre-adjunction by using a pair of a functor and a pre-functor, where $[f]$ and $[g]$ represent particular maps derived from a pair of $\langle f, f^* \rangle$ and $\langle g, g^* \rangle$. The diagram is commutative such that

$$[g]\psi[f] = \phi \quad (12)$$

Indeed, for all x in X and all y in Y ,

$$\phi(x) = [g](y) \Leftrightarrow \psi[f](x) = y. \quad (13)$$

It is clear to see that equivalence (12) is always obtained if g is a one-to-one map. It can be verified by the following. In assuming that $\phi(x) = [g](y)$, one obtains

$$[g]\psi[f](x) = [g](y), \quad (14)$$

from commutative diagram (11). Because $[g]$ is a one-to-one map, (14) leads $\psi[f](x) = y$. Conversely, in assuming that $\psi[f](x) = y$, $[g]\psi[f](x) = [g](y)$ is obtained, and then $\phi(x) = [g](y)$ from (12). In this sense, if one approximates that $[g]$ is a one-to-one map, the pre-adjunction can be reduced just to a commutative diagram (12).

In order to formalize observational heterarchy, we introduce the device carrying the negotiation between Intent and Extent perspectives. If a pre-functor is approximated to be a functor, universal mapping property is also obtained, such that

$$\begin{array}{ccc} \begin{array}{c} \langle G \rangle X \\ \uparrow \phi \\ X \end{array} & \xrightarrow{\langle F \rangle} & \begin{array}{c} \langle F \rangle \langle G \rangle X \\ \uparrow f^* \phi^f \\ \langle F \rangle X \end{array} \\ & & \begin{array}{c} \mu \searrow \\ \psi' \nearrow \\ Y \end{array} \end{array} \quad (15)$$

where μ is a universal map. The following universal mapping property is dually obtained,

$$\begin{array}{ccc} \begin{array}{c} \langle F \rangle X \\ \downarrow \psi \\ X \end{array} & \xrightarrow{\langle G \rangle} & \begin{array}{c} \langle G \rangle \langle F \rangle X \\ \downarrow g^* \psi g \\ \langle G \rangle X \end{array} \\ & & \begin{array}{c} \eta \nwarrow \\ \phi' \swarrow \\ Y \end{array} \end{array} \quad (16)$$

where η is also a universal map. The universal mapping property is no longer consistent with pre-adjunction, (11). That is why the new map, ψ' and ϕ' , are distinguished from ψ and ϕ . Finally, by introducing time transition, the negotiation between Intent and Extent perspectives are defined by

$$[g]\psi^t[f] = \phi. \quad (17)$$

$$\psi^{t+1} = \mu f^* \phi^t f \quad (18)$$

$$\phi^{t+1} = g^* \psi^t g \eta. \quad (19)$$

As mentioned before, an observational heterarchy consists of $\text{Ext}(\langle F \rangle X, Y)$ and $\text{Int}(X, \langle G \rangle Y)$, or $\text{Ext}(FX, Y)$ and $\text{Int}(X, \langle G \rangle Y)$, and then if only one of adjoint functors is replaced by a pre-functor, time development is defined by eq.(17) and either eq.(18) or (19). Actually, in the next chapter, we introduce such forms.

The most important characteristic of the formal model of an observational heterarchy is the operation featuring a pseudo-inverse map, f^* . Because of definition, a pseudo-inverse of a continuous non-monotonous map has to be a discontinuous map for many cases. Suppose that $f: R \rightarrow R$ is a continuous non-monotonous map, where R is an interval of real numbers, and remember that f^* satisfies the condition, $ff^*(x)=x$ for all x in $f(R)$. It can be verified that a pseudo-inverse, f^* is discontinuous map. In supposing that f^* is also a continuous map, for all y in R , $\forall \varepsilon \exists \delta$, with $\varepsilon, \delta > 0$,

$$f^*(U_\delta(y)) \subseteq U_\varepsilon(f^*(y)). \quad (20)$$

Because f is also continuous,

$$f(f^*(U_\delta(y))) \subseteq f(U_\varepsilon(f^*(y))). \quad (21)$$

$$U_\delta(y) \subseteq f(U_\varepsilon(f^*(y))). \quad (22)$$

The second statement is obtained from that $ff^*(x)=x$ for all x in $f(R)$. Therefore,

$$y + \delta < f(f^*(y) + \varepsilon). \quad (23)$$

Because f is a non-monotonous map, there exists z in R such that

$$f^*(z) < f^*(z) + \varepsilon \quad (24a)$$

$$f(f^*(z)) > f(f^*(z) + \varepsilon) \quad (24b)$$

From inequalities, (23) and (24b), the following inequality is obtained such as,

$$z + \delta < f(f^*(z) + \varepsilon) < f(f^*(z)) < z. \quad (25)$$

That is a contradiction, and then f^* is a discontinuous map.

Fig. 4 shows pseudo-inverse maps derived from various non-monotonous continuous maps. A pseudo-inverse map for a map, $f: [0.0, 1.0] \rightarrow [0.0, 1.0]$ is defined by;

$$f^*(x) = y \ (y \leq z, \ \forall z \in \{z \mid f(x) = z\}) \quad (26)$$

All pseudo-inverse maps are discontinuous maps for non-monotonous continuous maps.

As a result, a pre-functor maps a continuous map to a discontinuous map by the pinch of continuous non-monotonous map and discontinuous map. As mentioned later, such a characteristic plays an essential role in emergence and robustness in an observational heterarchy.

The property of the formal model of an observational heterarchy is summarized as follows:

- (i) It consists of a pair of Intent and Extent perspectives, defined by $\text{Ext}(\langle F \rangle X, Y)$ and $\text{Int}(X, \langle G \rangle Y)$ (or $\text{Ext}(FX, Y)$ and $\text{Int}(X, \langle G \rangle Y)$).
- (ii) Each perspective is transformed by a pre-functor that often maps a continuous map to a discontinuous map, and they satisfy pre-adjunction.
- (iii) Because each perspective is transformed by a pre-functor, the difference between Intent and Extent perspective corresponds to the difference in a term of logical status, such as level and meta-level.
- (iv) If pre-functor is approximated to a functor, then an observational heterarchy can reveal the equivalence between Intent and Extent perspectives, such that $\text{Ext}(FX, Y) \cong \text{Int}(X, GY)$, where F and G represent functors approximated from pre-functors, $\langle F \rangle$ and $\langle G \rangle$.
- (v) Inconsistency between the commutative diagram derived from universal mapping property and pre-adjunction reveals perpetual negotiation between Intent and Extent perspectives, or the time development of a pair of dynamics.

In the next section, we apply the notion of an observational heterarchy to a coupled map system, and shows that an observational heterarchy exists as a robust and emergent system. Especially we discuss the property of an observational heterarchy such that perturbation enhances robust behavior.

3. Active Coupling

3-1. Intent and Extent perspective in a cell

A dynamical system that is a usual expression for dynamical behavior of some biochemical substrate-species is regarded just as an Intent perspective, from the point of view based on an observational heterarchy. It results from the approximation revealing the equivalence between Intent and Extent perspectives. Imagine some biochemical substrates in a cell of two-cells system. Usually, one grasps a population of the substrate-species as a whole, and denotes the concentration of the population as a variable, x . The dynamical behavior of the population is defined by a particular dynamics, and the transportation of substrates is defined by a diffusion-like manner. It results only from Intent perspective. For the sake of convenience, the dynamics is here expressed as a difference equation, and the time development is represented by t . The time development of the concentration in a cell is expressed as

$$x_i^{t+1} = (1 - c)\phi(x_i^t) + f(\phi(x_j^t)) \quad (27)$$

with $i, j = 0, 1$, and $x_i^t \in [0.0, 1.0]$. The map, f represents the flow of materials from cell to cell, and $f(x)$ is defined by a map that can be approximated by a linear map,

$$f(x) \sim cx. \quad (28)$$

If $f(x) = cx$, the interaction is expressed as a linear coupling between two cells. The dynamics of intra-cellular behavior is expressed as a logistic map (e.g., [52]),

$$\phi(x_i^t) = \alpha x_i^t(1 - x_i^t) \quad (29)$$

where α is a logistic parameter, $0.0 < \alpha \leq 4.0$. This system does not lose generality of a usual coupled map lattice [43-45].

We apply the framework of an observational heterarchy to this two-cells system. A population of the substrate in a cell is divided into Intent and Extent perspectives. As mentioned before, Intent perspective reveals a general attribute of substrates and Extent one reveals individual variations of substrates. If the amount of substrate is small, one focuses on individual variations. Therefore, Intent corresponds to major endogenetic parts of population, and Extent corresponds to minor exotic parts derived from the flow. If Intent and Extent are denoted by ϕ and ψ , respectively, the time development of the concentration is expressed as

$$x_i^{t+1} = (1 - c)\phi(x_i^t) + \psi(x_j^t) \quad (30)$$

In this expression, if

$$\psi(x) = f(\phi(x)), \quad (31)$$

then eq.(30) can be reduced to eq.(27).

In the framework of an observational heterarchy, if the approximation is possible, Intent and Extent are equivalent to each other via a pair of adjoint functors derived from pre-functors. To make such a condition of an observational heterarchy, we define a pair of functor and pre-functor. Such a formulation needs some terms in a category theory. In the before section, Intent and Extent perspectives are expressed as sets of maps. In other words, each perspective consists of a collection of sets and one of maps. A category is regarded as a general form of that perspective, and consists of a collection of objects and arrows that are operations between objects. A category has to satisfy the following condition; (i) each object, A , has an identity arrow denoted by $\text{id}_A: A \rightarrow A$ such that $f \text{id}_A = f$ and $\text{id}_A g = g$ for any arrow, $f: A \rightarrow X$ and $g: Y \rightarrow A$, (ii) Given $f: A \rightarrow B$, $g: B \rightarrow C$, and $h: C \rightarrow D$, there is a composition, hgf , and $hgf = (hg)f = h(gf)$. A functor, $F: \mathbf{C} \rightarrow \mathbf{D}$, is, in strict speaking, defined as an operation from a category, \mathbf{C} , to a category, \mathbf{D} , satisfying $F(\text{id}_C) = \text{id}_{FC}$ and $F(gf) = F(g)F(f)$, where id_C, f and g are arrows of a category, \mathbf{C} . An observational heterarchy for a coupled map lattice needs a comma category, denoted by \mathbf{C}/A . An object of \mathbf{C}/A is a map whose range is a set, A (i.e., $g: X \rightarrow A$), and an arrow of \mathbf{C}/A is a map between domains of objects, satisfying the commutative diagram (i.e., $p: X \rightarrow Y$ with $g: X \rightarrow A$ and $h: Y \rightarrow A$ such that $hp = g$). It is ready to introduce an observational heterarchy for two-cells system.

Given, a map, $f: A \rightarrow B$, a composition functor, $\Sigma_f: \mathbf{C}/A \rightarrow \mathbf{C}/B$, and a pseudo-pull-back pre-functor, $\langle \Pi_f \rangle: \mathbf{C}/B \rightarrow \mathbf{C}/A$, are defined. In the case of two-cells system mentioned before, the map, f represents the flow and $A = B = [0.0, 1.0]$. A composition functor is defined by;

$$\begin{array}{ccc}
 \begin{array}{ccc} X & \xrightarrow{p} & X' \\ g \searrow & & \nearrow h \\ & A & \end{array} & \xrightarrow{\Sigma_f} & \begin{array}{ccc} X & \xrightarrow{p} & X' \\ fg \searrow & & \nearrow fh \\ & B & \end{array}
 \end{array} \quad (32)$$

where $hp = g$ and $fhp = fg$. For an object, g in \mathbf{C}/A , $\Sigma_f(g) = fg$ is obtained in \mathbf{C}/B , and for an arrow, p in \mathbf{C}/A , $\Sigma_f(p) = p$ in \mathbf{C}/B is obtained. It is easy to see that composition of arrows is always preserved (i.e., verification of preserving composition of arrows with respect to a functor, Σ_f). Given

$$\begin{array}{ccccc}
X & \xrightarrow{p} & X' & \xrightarrow{q} & X'' \\
& \searrow g & \downarrow h & \swarrow i & \\
& & A & &
\end{array}
\tag{33}$$

such that $hp = g$ and $iq = h$, a composition qi ,

$$\Sigma_f(qp) = qp = \Sigma_f(q) \Sigma_f(p). \tag{34}$$

Because of commutative diagram in C/A , the commutative diagram in C/B also holds, such that

$$fi(qp) = (fiq)p = fhp = fg. \tag{35}$$

Through a composition functor, endogenetic dynamics, ϕ is composed with the flow, f , and the exotic dynamics, ψ , is obtained.

A pseudo pull back functor can be approximated to a pull back functor, and a pull back is a special structure attribute to a category theory [46]. Although pull back is a generalized inverse image, it is rarely recognized as a case of the general notion. Given a pair of arrows, $f:A \rightarrow B$ and $g:C \rightarrow B$, a pull back can be expressed by the commutative diagram,

$$\begin{array}{ccc}
A \times_B C & \xrightarrow{\pi_2} & C \\
\pi_1 \downarrow & & \downarrow g \\
A & \xrightarrow{f} & B
\end{array}
\tag{36}$$

It satisfies that $f\pi_1 = g\pi_2$, and that given a pair of arrows, $p:X \rightarrow A$, $q:X \rightarrow C$ with an arbitrary object X such that $fp = gq$, there uniquely exists an arrow $k:X \rightarrow A \times_B C$ such that $\pi_2 k = q$ and $\pi_1 k = p$. If all symbols are regarded as meaningless symbols, the property mentioned here is just a commutative diagram under a specific condition (i.e., commutative diagram with respect to X). That is definition of pull back in a category theory.

Actually, in supposing that all objects are sets and arrows are maps, one can define

$$A \times_B C = \{ \langle a, c \rangle \mid f(a) = g(c), a \in A, c \in C \}. \quad (37)$$

Maps π_1 and π_2 are projection maps, and

$$\pi_1(\langle a, c \rangle) = a \quad (38a)$$

$$\pi_2(\langle a, c \rangle) = c. \quad (38b)$$

A pair of object (37) and arrows (38) is a “pull back for f and g ” in sets, since it satisfies the definition of pull back such as a commutative diagram under a specific condition.

Constructing a pull back is regarded as a functor, $C/B \rightarrow C/A$, such that an object, g in C/B is mapped to an object, π_1 . As a result, pull back can be regarded as an inverse image.

By utilizing a generalized inverse image, a pseudo pull back functor is defined. Given an object, $g: Y \rightarrow B$ in C/B , a pseudo pull-back functor maps it to an object, $f^*gf: P(A) \rightarrow A$ such that

$$\begin{array}{ccc} P(Y) & \xrightarrow{f} & Y \\ f^*gf \downarrow & & \downarrow g \\ A & \xrightarrow{f} & B \end{array} \quad (39)$$

where $f: P(Y) \rightarrow Y$ is the evident factorization of the restriction of f to $P(Y)$ ($x \in P(Y) \Rightarrow f(x) \in Y$), and

$$P(Y) = \{ x \in A \mid f(x) \in Y \}. \quad (40)$$

If $Y \subset B$, $P(Y) \subset A$, and if $B \subseteq Y$, then $P(Y) = A$. A map $f^*: B \rightarrow A$ is also defined by satisfying that $ff^*(x) = x$ for all x in $f(A)$, and is expressed as the definition (26). Then, f^* is a discontinuous map. If f is a one-to-one onto map, the diagram is commutative, and $ff^*gf = gf$.

Given an arrow, $s: Y \rightarrow Y'$ with $g: Y \rightarrow B$ and $h: Y' \rightarrow B$ such that $hs = g$, a pseudo pull back functor, $\langle \Pi_f \rangle$ maps it to an arrow, f^*sf . Diagrammatically, it is shown as,

$$\begin{array}{ccccc} & & f & & \\ & & \longrightarrow & & \\ P(Y) & \xrightarrow{\quad} & Y & \xrightarrow{s} & Y' \\ & \searrow f^*sf & \downarrow g & \searrow h & \\ & P(Y') & & & \\ & \nearrow f^*hf & \downarrow f & & \\ f^*gf \downarrow & & A & \xrightarrow{\quad} & B \end{array} \quad (41)$$

We here show that if the flow, $f:A \rightarrow B$ ($A = B$) is approximated into a one-to-one onto map, or $f^*f(x) = ff^*(x) = x$, then a pseudo pull-back pre-functor is a functor. Under the approximation, it is easy to see that the diagram (39) is a pull back diagram. Firstly, the diagram is commutative. Secondly,

$$\begin{array}{ccccc}
 X & & & & \\
 \downarrow p & \searrow k & & \searrow q & \\
 & P(Y) & \xrightarrow{f} & Y & \\
 & \downarrow f^*gf & & \downarrow g & \\
 & A & \xrightarrow{f} & B &
 \end{array} \quad (42)$$

In the diagram (42), suppose that the outer rectangle is commutative, or $fp = gq$. There exists $k:X \rightarrow P(Y)$ such that $fk = q$ because $k = f^*q$. At the same time,

$$f^*gfk = f^*gq = f^*fp = p. \quad (43)$$

In supposing that there is another $k':X \rightarrow P(Y)$ such that $fk' = q$ and $f^*gfk' = p$, one obtains that $k' = f^*fk' = f^*q = k$. Then, an arrow, $X \rightarrow P(Y)$ that commutes both triangles in diagram (42) is unique. As a result, the diagram (39) is a pull back diagram.

It is also easy to verify that f^*sf is uniquely determined to satisfy $f^*hff^*sf = f^*gf$ in the diagram (41). Since a pair of maps, $f:P(Y) \rightarrow Y$ and $f^*gf:P(Y) \rightarrow A$ constitutes a pull back ($gf = ff^*gf$) and $hs = g$ (from the definition of an arrow in C/B), the outer rectangle is commutative, and i.e.,

$$hsf = ff^*gf. \quad (44)$$

Because a pair of maps, $f:P(Y') \rightarrow Y'$ and $f^*hf:P(Y') \rightarrow A$ also constitutes a pull back, an arrow, $P(Y) \rightarrow P(Y')$ commutes both outer triangles in the diagram (41) is unique. In this case, a pre-functor, $\langle \Pi_f \rangle$ can be replaced by a functor, Π_f , satisfying

$$\Pi_f(r) \Pi_f(s) = f^*rff^*sf = f^*rsf = \Pi_f(rs) \quad (45)$$

$$\Pi_f(\text{id}_Y) = f^* \text{id}_Y f = f^*f = \text{id}_{P(Y)}. \quad (46)$$

Finally, under the approximation, one can obtain eq.(31) or an isomorphism between Intent and Extent perspectives. In the above chapter, we denote Intent perspective as $\text{Int}(X, \langle G \rangle Y)$ and Extent perspective as $\text{Ext}(FX, Y)$, and is approximated into the isomorphism, $\text{Int}(X, GY) \cong \text{Ext}(FX, Y)$.

In the two-cells system, Intent and Extent perspectives are expressed as C/A and C/B , respectively. Under the approximation, one obtains

$$C/B(\Sigma_f(X/A), Y/B) \cong C/A(X/A, \Pi_f(Y/B)). \quad (47)$$

The isomorphism (47) is easily verified by using a pull back diagram such as

$$\begin{array}{ccccc} X & & & & \\ & \searrow \psi & & \searrow \phi & \\ & P(Y) & \xrightarrow{f} & Y & \\ p \downarrow & \downarrow f^*gf & \downarrow f & \downarrow g & \\ & A & \xrightarrow{\quad} & B & \end{array} \quad (48)$$

Since a pair of maps, $f: P(Y) \rightarrow Y$ and $f^*gf: P(Y) \rightarrow A$ constitutes a pull-back, in supposing that outer rectangle is commutative (i.e., $g\phi = fp$), there exists a unique $\psi: X \rightarrow P(Y)$ such that

$$f\psi = \phi, \quad (49)$$

$$f^*gf\psi = p. \quad (50)$$

In this diagram, a map ψ in C/A (Intent perspective) is an arrow between two objects in C/A as

$$X/A = p: X \rightarrow A, \quad (51a)$$

$$\Pi_f(Y/B) = \Pi_f(g: Y \rightarrow B) = f^*gf: P(Y) \rightarrow A. \quad (51b)$$

A map $\phi: X \rightarrow Y$ is an arrow in C/B (Extent perspective) is an arrow between two objects in C/B as

$$\Sigma_f(X/A) = \Sigma_f(p: X \rightarrow A) = fp: X \rightarrow B, \quad (52a)$$

$$Y/B = g: Y \rightarrow B. \quad (52b)$$

As a result, it is easy to see that isomorphism (47) can be directly verified from the diagram (48).

Finally, the approximated consistency between Intent and Extent perspective expressed as eq.(31) is obtained as (49) in the pull back diagram (48). In remembering the condition of the formal model of an observational heterarchy (i)~(v), an observational heterarchy for the two-cells system satisfies (i)~(iii) by a pair of $C/B(\Sigma_f(X/A), Y/B)$ and $C/A(X/A, \Pi_f(Y/B))$, (iv) by the isomorphism (47) under the approximation. We, in addition, introduce the pre-adjunction and the commutative

diagram derived from a universal mapping property, so as to define time development of Intent and Extent perspectives (ψ and ϕ).

Pre-adjunction is derived from the adjunction. In the two-cells system, under the approximation, the following adjunction is obtained.

$$\begin{array}{ccc}
 X/A & \xrightarrow{\Sigma_f} & \Sigma_f(X/A) \\
 \phi \downarrow & & \downarrow \psi \\
 \Pi_f(Y/B) & \xleftarrow{\Pi_f} & Y/B
 \end{array} \quad (53)$$

Pre-adjunction is expressed as a commutative diagram such as

$$\begin{array}{ccc}
 X/A & \xrightarrow{\quad} & \Sigma_f(X/A) \\
 \phi \downarrow & & \downarrow \psi \\
 \langle \Pi_f \rangle(Y/B) & \xleftarrow{\quad} & Y/B
 \end{array} \quad (54)$$

How can one determine a pair of maps in the horizontal direction? Horizontal maps are operations with respect to objects in a comma category, and one obtains the following diagram;

$$\begin{array}{ccccc}
 & & \text{id}_X & & \\
 X & \xrightarrow{\quad} & X & & \\
 \phi \searrow & & f \downarrow & & \searrow \psi \\
 & P(Y) & \xrightarrow{\quad} & Y & \\
 \swarrow f^* & & \downarrow & & \swarrow \\
 A & \xleftarrow{\quad} & B & & \\
 & f & & &
 \end{array} \quad (55)$$

Because $\text{id}_X: X \rightarrow X$ can be defined from right to left, and a map $Y/B \rightarrow \langle \Pi_f \rangle(Y/B)$ constructs the inverse image derived from f , the diagram (54) can be replaced by the diagram as

$$\begin{array}{ccc}
 X & \xleftarrow{\quad} & X \\
 \phi \downarrow & \text{id}_X & \downarrow \psi \\
 P(Y) & \xrightarrow{f} & Y
 \end{array} \quad (56)$$

From the adjunction (53) a universal mapping property is obtained such as

$$\begin{array}{ccc}
 \Sigma_f(X/A) & & \Pi_f(\Sigma_f(X/A)) \\
 \downarrow \psi & \xrightarrow{\Pi_f} & \downarrow \Pi_f(\psi) \\
 Y/B & & \Pi_f(Y/B)
 \end{array}
 \begin{array}{c}
 \swarrow f^* \\
 X/A \\
 \searrow \phi
 \end{array}
 \quad (57)$$

Here we assume that the dynamics without the approximation is connected to the approximated dynamics. It implies that the commutative diagram (57) holds also with respect to $\langle \Pi_f \rangle$, and then

$$\begin{array}{ccc}
 \Sigma_f(X/A) & & \langle \Pi_f \rangle(\Sigma_f(X/A)) \\
 \downarrow \psi & \xrightarrow{\langle \Pi_f \rangle} & \downarrow \langle \Pi_f \rangle(\psi) \\
 Y/B & & \langle \Pi_f \rangle(Y/B)
 \end{array}
 \begin{array}{c}
 \swarrow f^* \\
 X/A \\
 \searrow \phi'
 \end{array}
 \quad (58)$$

A map ϕ' is no longer ϕ , and then the shift from ϕ to ϕ' implies time development, since the diagram (58) is not consistent with the diagram (56).

Finally, the dynamics of two-cells system called “active coupling” is expressed as

$$x_i^{t+1} = (1 - c)\phi^t(x_i^t) + \psi^t(x_j^t), \quad (59)$$

$$\phi^{t+1} = f^* \psi^t f^{*}, \quad (60)$$

$$\psi^{t+1} = f \phi^{t+1}, \quad (61)$$

$$\phi^0(x_i^t) = \alpha x_i^t (1 - x_i^t). \quad (62)$$

While usual coupling contains no time development with respect to dynamics (i.e.. no time shift with respect to ϕ and ψ such as eq.(27)), the dynamical system expressed as (59)-(62) is called active coupling. The eq.(59) is derived from eq.(30), eq.(60) from the diagram (58), and eq.(61) from the diagram (56). Pre-adjunction is obtained from the connection between the dynamical system without the approximation and one with the approximation. It leads that a pre-functor can be approximated to a functor, and that a universal mapping property is also obtained as the diagram (58). Remember that the approximation is defined by $ff^* = f^*f = \text{id}$. Therefore, by using the connection to the approximation, we can obtain the following equations instead of eq.(60) and (61),

$$\psi^{t+1} = f\phi^t f^*, \quad (63)$$

$$\phi^{t+1} = f^* \psi^{t+1}. \quad (64)$$

In the next section, we show some behaviors of active coupling, and use the system expressed by eq.(59), (62) and (63), (64). Because of the definition of pseudo-inverse, f^* , construction of f^* contains arbitrariness in the domain except for co-domain of f . Especially, all x such that $ff^*(x) \neq x$ depends on such arbitrariness. By contrast, all x such that $f^*f(x) \neq x$ is out of the arbitrary region, and inequality results from the discontinuity carried by f^* . That is why we use the dynamical expression with not ff^* but f^*f .

Finally, all conditions of an observational heterarchy, (i)~(v) are satisfied by the two-cells active coupling derived from the flow, $f:A \rightarrow B$.

3-2. Behaviors of active coupling

In this section we address the behavior of active coupling, and estimate whether a coupled map system called active coupling can achieve synchronization surrounded by perturbations, or whether perturbation enhances robust synchronization of two cells. Not only microscopic biochemical substrates but macroscopic populations in an ecosystem are perpetually influenced by stormy perturbations. As pointed out by Rosen [53, 54] and Varela [55], the perturbation influences not only state but also function, because the separation between state and function results just from the framework of a set theory. Therefore, the question mentioned first is replaced by, how can one formalize the influence of perturbation in a term of function. The answer to the question is yielded by observational heterarchy. As discussed later, usual coupled map system that we call passive coupling does not reveal the influence of perturbation in a term of function.

The system of active coupling is expressed by eq.(59), (62) and (63), (64). Compared with active coupling, we call the interacting system based on adjunction the passive coupling. The dynamical system defined by eqs. (27) and (29) is called the passive coupling, and that is a usual coupled map system. As mentioned before, active coupling can be approximated to passive coupling, where the approximation is defined by adjunction or the equivalence between Intent and Extent. The definition of approximation is consistent with usual description for dynamical behaviors of some biochemical substrates on which only Intent perspective is focused.

Firstly, we address the synchronization of the simplest passive coupling to compare with the active coupling. Such an analysis is evaluated by [56]. Since it follows eqs. (27), (29) and $f(x) = cx$, the time transition is expressed as a matrix form such as

$$\begin{pmatrix} x_0^{t+1} \\ x_1^{t+1} \end{pmatrix} = \begin{pmatrix} 1-c & c \\ c & 1-c \end{pmatrix} \begin{pmatrix} \phi(x_0^t) \\ \phi(x_1^t) \end{pmatrix} = \mathbf{M} \begin{pmatrix} \phi(x_0^t) \\ \phi(x_1^t) \end{pmatrix} \quad (65)$$

To evaluate stability of synchronized state of two cells, deviation from the synchronized state, z , is defined as

$$\delta x_0^t = x_0^t - z \quad (66a)$$

$$\delta x_1^t = x_1^t - z. \quad (66b)$$

The time transition of deviation is expressed as

$$\begin{pmatrix} \delta x_0^{t+1} + z \\ \delta x_1^{t+1} + z \end{pmatrix} = \mathbf{M} \begin{pmatrix} \phi(z) + \phi'(z) \delta x_0^t \\ \phi(z) + \phi'(z) \delta x_1^t \end{pmatrix} \quad (67)$$

where $\phi'(z)$ is differential coefficient on z and $\phi(z + \delta x_i^t) \sim \phi(z) + \phi'(z) \delta x_i^t$ with $i = 0, 1$. By canceling $(z, z) = \mathbf{M}(\phi(z), \phi(z))$, we obtain

$$\begin{pmatrix} \delta x_0^{t+1} \\ \delta x_1^{t+1} \end{pmatrix} = \phi'(z) \mathbf{M} \begin{pmatrix} \delta x_0^t \\ \delta x_1^t \end{pmatrix}. \quad (68)$$

Stability of eq.(68) is given by $\ln(|\phi'(z)\lambda|) < 0$ where λ is an eigenvalue of \mathbf{M} . It is well known that logistic map can be transformed into the tent map whose Lyapunov exponent is exactly $\ln 2$, where logistic parameter is 4.0. As a result, since one obtains that $\phi'(z) = 2$ and $\lambda = 1 - 2c$, 1, stability of synchronized states is expressed as

$$0.25 < c < 0.75. \quad (69)$$

Fig. 5A shows the diagram of bifurcation solution for $x_0^t - x_1^t$ against the coupling strength, c , where logistic parameter is 4.0. For each c , 10^4 iterations of the map, (65), are applied for a random initial state, and the first 10^3 steps are abandoned. It is clear to see that synchronization is stable in the range that $0.25 < c < 0.75$. Fig. 5B shows enlarged bifurcation diagram in the region, $0 < c < 0.25$. There are some windows in a chaotic region.

Consider the influence of perturbation in passive coupling. Firstly, it is implemented by non-monotonous (and/or non-linear) flow (i.e., coupling strength), $f:[0.0, 1.0] \rightarrow [0.0, 1.0]$. It is simply expressed as

$$\begin{aligned} f(x_i^t) &= c x_i^t ; & (0 < x_i^t < a) \\ (ac - bd)(x_i^t - a)/(a - b) + ac ; & (a < x_i^t < b) \\ (bd - c)(x_i^t - 1)/(b - 1) + c. & (b < x_i^t < 1). \end{aligned} \quad (70)$$

It reveals the simple fluctuated flow, since it is non-monotonous. The first and second crooked point is laid on the line, $f(x) = cx$ and $f(x) = dx$, respectively. Eq.(70) can be approximated to $f(x) = cx$. In this sense, we can utilize c as the coupling strength, and call apparent coupling strength. Due to the fluctuated coupling strength, the coupled map system, (65) is replaced by

$$x_0^{t+1} = (1 - c)\phi(x_0^t) + f(\phi(x_1^t)) \quad (71a)$$

$$x_1^{t+1} = (1 - c)\phi(x_1^t) + f(\phi(x_0^t)). \quad (71b)$$

As far as the Intent dynamics, $\phi(x_i^t)$, is expressed as a logistic map, the corresponding Extent dynamics, such as $f(\phi(x_i^t))$ is expressed as shown in Fig. 6. Only Extent dynamics is influenced by a non-monotonous map, f .

Here we calculate bifurcation of solutions against apparent coupling strength c in setting that $a = 0.733$, $b = 0.8$ and $d = 0.75c$. Since the coupling strength is no longer constant, it is approximated by that $f(x) \sim cx$, and the coupling strength is just an apparent coupling strength. Fig.7 shows the diagram of bifurcation solution for $x_0^t - x_1^t$ against the apparent coupling strength, c . For each c , 10^4 iterations of the map, (71), is applied for a random initial state, and the first 10^3 steps are abandoned. Due to the non-monotonous function of f , the efficient coupling strength is widely distributed beyond the apparent coupling strength. That is why stability of synchronization is lost, even if the apparent coupling strength is in the stable region of synchronization, $0.25 < c < 0.75$.

The effect of fluctuation is explicitly observed if the fluctuated flow is asymmetric such as

$$x_0^{t+1} = (1 - c)\phi(x_0^t) + f_0(\phi(x_1^t)) \quad (72a)$$

$$x_1^{t+1} = (1 - c)\phi(x_1^t) + f_1(\phi(x_0^t)), \quad (72b)$$

where $f_0 = f$ with $d=d_0$ and $f_1 = f$ with $d=d_1$, and all other parameters of f_0 are the same as those of f_1 . Fig.8 shows bifurcation solutions against apparent coupling strength. The above diagram is calculated, given $a = 0.733$, $b = 0.8$ and $d_0=0.75c$ and $d_1=0.76c$. The below one is calculated, given the same values of parameters except for $d_1=0.78c$. It is clear to see that the effect of asymmetry is

implemented only by the difference of d_0 and d_1 . The more asymmetric f_0 and f_1 are, the more instable synchronized state is. Due to the asymmetric fluctuation, synchronization disappears. It is concluded that stormy perturbation disturbs synchronization, in the framework of passive coupling.

Compared with passive coupling, the behavior of active coupling is much more complex. The active coupling to be used for simulating studies is expressed as

$$x_i^{t+1} = (1-c)\phi^t(x_i^t) + \psi^t(x_j^t), \quad (73a)$$

$$\psi^{t+1} = f\phi^t f^* f, \quad (73b)$$

$$\phi^{t+1} = f^* \psi^{t+1}, \quad (73c)$$

$$\phi^0(x_i^t) = \alpha x_i^t(1-x_i^t), \quad (73d)$$

with $i = 0, 1$, where $f: [0.0, 1.0] \rightarrow [0.0, 1.0]$ is expressed as eq.-(70), and all parameter setting is the same as one for the simulation as shown in Fig. 7, such that $a = 0.733$, $b = 0.8$ and $d = 0.75c$. Not only states but also maps are transformed step by step.

Because of a pseudo-inverse map, f^* , all calculations are destined to be approximation. We divide the interval, $[0.0, 1.0]$ into 10^4 cells, and each continuous map is defined as a map from cells to cells, such that $f(k/10^4) \bmod 1/10^4$ with $k=0, \dots, 10^4-1$. The f^* is constructed by utilizing each linear map connecting $(k/10^4, f(k/10^4))$ and $((k+1)/10^4, f((k+1)/10^4))$.

Due to the fluctuated flow, f , both intent and extent maps are transformed step by step. Although the initial Intent and Extent maps are the same as those of passive coupling as shown in Fig. 6, not only Extent but also Intent map are influenced by non-monotonous map, f (Fig. 9). Since f^* is a discontinuous map, the next Intent map, is also a discontinuous map, expressed by $\phi^{t+1} = f^* \psi^{t+1}$. In a logistic map, ϕ^0 , with a chaotic parameter, there is an instable fixed point. Compared with a logistic map, an instable fixed point is lost in the next Intent map, ϕ^1 . Due to such a mechanism, active coupling can give rise to robust synchronization.

The diagrams of bifurcation solution against the apparent coupling strength are shown in Fig. 10, where logistic parameter is 4.0 in the above, and 3.82 in the below, respectively. All parameters for a map, f , are the same as ones in Fig. 7. The synchronized region in active coupling is wider than ones in fluctuated passive coupling. Especially, synchronized state appears even in the chaotic region of passive coupling. Due to discontinuity, instable fixed points are given up and are replaced by many stable fixed points. As a result, solutions with finite period appear even in the chaotic region of passive coupling. There are some windows of periodic solutions in a chaotic region of passive coupling (Fig. 7). In this sense, windows are ubiquitously found in an active coupling, and then it gives rise to robust behavior.

By introducing asymmetric flow, f_0 and f_1 mentioned in eq.-(72), the active coupling is also modified such that

$$x_i^{t+1} = (1-c)\phi_i^t(x_i^t) + \psi_j^t(x_j^t), \quad (74a)$$

$$\psi_j^{t+1} = f_i \phi_i^t f_i^* f_i, \quad (74b)$$

$$\phi_i^{t+1} = f_i^* \psi_j^{t+1}, \quad (74c)$$

$$\phi_i^0(x_i^t) = \alpha x_i^t(1-x_i^t), \quad (74d)$$

if $i=0, j=1$, and vice versa. In this condition, bifurcation of solution against the apparent coupling strength is shown in Fig. 11. There is no explicit chaotic region, and there are ubiquitous windows of periodic solutions accompanied with synchronized states. It shows that active coupling leads to robust but emergently changed synchronization.

Damping to synchronization of active coupling is different from that of passive coupling. Fig. 12 shows riddle basin structures in a Cartesian space of initial conditions of two cells. From each initial condition, either eq.-(72) (for passive coupling) or eq.-(73) (for active coupling) is iterated T times. The co-ordinate representing initial condition is painted white if the synchronization ($x_0^t = x_1^t$) is achieved, and it is painted black otherwise, till T . In Fig. 12, the above and the below row represents basin structures of active and passive coupling, respectively, where logistic parameter $\alpha=4.0$ and from left to right, the apparent coupling strength, $c = 0.75, 0.80, 0.85$ and 0.90 . These coupling strengths are in the chaotic regions in terms of bifurcation solutions of synchronization. In passive coupling, there is no synchronization except for diagonal lines, where $T=1000$. Compared with passive coupling, synchronization can be achieved for active coupling in less iterations, where $T=50$.

Even in regions of periodic solutions, there is a difference between active and passive coupling with respect to damping time till the synchronization (Fig. 13). As well as Fig. 12, the above and the below row represents basin structures of active and passive coupling, respectively, where logistic parameter $\alpha=4.0$, and from left to right, the apparent coupling strength, $c = 0.65, 0.675, 0.70$ and 0.725 . In passive coupling, there is few synchronization can be achieved for all possible initial conditions with $T=65$. As $c = 0.675 \sim 0.725$, there is no synchronization till $T = 65$. By contrast, in active coupling, only a few steps ($T=5$) can reach the appearance of riddle basins of synchronization. Especially, the structure of the basins is much more complex than ones obtained in passive coupling, and shows a self-similar pattern. When a more steps proceeds ($T=7$), self-similar riddle basin of synchronization becomes much more clearly (Fig. 14).

In the coupled map system, one can estimate solutions of Intent dynamics, whether it is passive or active coupling. As for passive coupling, Intent dynamics is nothing but a logistic map, and the diagram of bifurcation solution is known very well as shown in Fig. 15. By contrast, in active coupling, bifurcation solution against logistic parameter ($3.0 < \alpha < 4.0$) is shown in Fig. 16. The apparent coupling strength is yielded by $c = 0.25$ (above) and $c = 0.05$ (below). Since not only

Extent but also Intent dynamics is influenced by the apparent coupling strength or the shape of a map, f , Intent dynamics is modified with respect to shape dependent on the apparent coupling strength (See Fig.9). It is clear to see that there is no chaotic solution over the parameter range. As it is shown in Fig. 9, Intent map is modified as a particular discontinuous map, of which flat peak region of logistic map remains and steep region including instable fixed point is removed. Iterated Intent map also shows such a structure consisting of flat undulating regions and no steep region near the fixed points. In some n -iterated Intent map contains a structure of which flat undulated region crosses the diagonal line, and that yields a stable periodic points. However the initial Intent map is a chaotic map, Intent map is finally modified to yield stable fixed points. Compared with passive coupling, it can be said that chaotic solutions are replaced by solutions with finite periods. Especially, for the apparent coupling strength, $c = 0.05$, Intent dynamics is modified more effectively than that of larger c . It results in replacement of instable fixed points by stable periodic points.

In taking all characters of active coupling mentioned above, it is significant to estimate how the active coupling behaves in the stormy perturbations. The stormy perturbation is here expressed as a non-monotonous flow, f , temporally changed. For this purpose, we modify the coupling system by introducing time-dependent flow, f^t , instead of f in eqs.(73), such that

$$f^t(x_i^t) = cx_i^t + c \sin(8m(t)\pi(x_i^t - a)), \quad (75)$$

where $0 < c$, $0 < a < 1$, and $0 < m(t) < 1$ is randomly chosen at each time step, t . The simulating studies are conducted with $a=0.4$. The time-dependent flow, eq.(75), yields the situation that the apparent coupling strength is perpetually changed along the time. It leads to the perpetual change of both Intent and Extent dynamics. With respect to time-dependent flow, we compare the active coupling with passive coupling such as eq.-(71) with f^t instead of f .

Fig. 17 shows time development of the passive coupling with time dependent flow, where the apparent coupling strength, $c=0.1$ and logistic parameter, $\alpha=4.0$. Especially for some time steps, pairs of Intent (left) and Extent map (right) are shown in the form of a return map. Only Extent dynamics is influenced by the time-dependent flow, and is perpetually changed. Two cells never been synchronized, and the difference between two states is not converged to be zero, and is fluctuated along the time. Since the apparent coupling strength is changeable

By contrast, in active coupling, both Intent and Extent dynamics are perpetually changed along the time, where $c=0.1$ and $\alpha=4.0$ (Fig. 18). By accumulating the discontinuous effect of pseudo-pullback functor, both Intent and Extent maps are changed as discontinuous maps in which they involve crenellated flat regions. It is clear to see that a state following such a map is trapped into the crenellated region, and it results in stable behavior. As a result, synchronized state in coupled map system are generated and maintained for a while. In Fig. 18, two states of two cells are

immediately converged into the synchronized states in a few steps from a random initial condition, and it is maintained for 400 steps. The stability of synchronized states results just from an apparent behavior, because the dynamics itself is perpetually changed, and synchronized behavior is retained. That is why we call such a behavior not a stable but a robust behavior. Even for the same initial condition as the case as shown in Fig. 18, given a different time series of time-dependent flow, f^t , the time series of two states are different from ones shown in Fig. 18 (Fig. 19). Although two cells are synchronized and it is retained just for a while, two states are drastically changed. As a result, two cells are frequently synchronized and are differentiated, alternatively.

Fig. 20 shows that the state space of two-cells system featuring time-dependent flow, where f^t is expressed as eq.-(75) with $a=0.4$. The co-ordinate represents a pair of cell-states, (x_0, x_1) , and 10^6 iterations are drawn. Left diagram shows the results of passive coupling, and right one shows the results of active coupling. In passive coupling, states of cells move over the most of domain, and the synchronization cannot be found, and that is demonstrated as a time series as shown in Fig. 17. In active coupling, a pair of states wanders among the synchronized states and particular attractors. These attractors are generated from the crenellated regions of discontinuous Intent dynamics. As shown in Fig. 19, a pair of two cells is synchronized for a while, and it is attracted into bimodal states and then it is attracted into synchronized states, again. Fig. 20 shows that non-synchronized states also reveal some basin structures.

We compare behavior of active coupling with one of passive coupling, with respect to the effect of stormy perturbations. Passive coupling consists only of Intent dynamics, where the concentration of substrate species is defined with neglecting that there are some individual substrates never been covered by Intent. In other words, there is no interface between macroscopic and microscopic perspectives. In the passive coupling, one can distinguish stable from instable attractors in terms of condition and/or parameter region, such as stable synchronization in eq.-(65) is determined by a coupling strength, $0.25 < c < 0.75$. In this sense, behavior retained functionally is expressed by a structure of dynamics, and then behaviors can be described in terms of stability theory.

While behavior of passive coupling is determined whether stable or instable, the stability of synchronization fails in collapse by stormy perturbation. We here estimate the effect of stormy perturbation by that coupling strength or diffusion constant is expressed as non-linear function. Especially, asymmetric flow breaks the stable synchronization, and it results in no synchronization all over the coupling strength regions. The effect of stormy perturbation is also estimated with respect to time-dependent non-monotonous flow. Under the passive coupling, the time-dependent flow inhibits synchronization, and then the difference of two cells is perpetually fluctuated along the time (Fig. 17).

By contrast, in active coupling, dynamics consists of both Intent and Extent dynamics, and the inconsistency between micro- and macroscopic perspectives is implemented in a model. It is such an inconsistency that can drive emergent and robust behavior. Since the inconsistency is implemented in the form of pre-functor that is expressed as a pseudo-pullback functor, both Intent and Extent maps are perpetually changed by the transformation as pinching Intent map between continuous and discontinuous maps. However Intent map is initialized as a non-linear chaotic continuous map, it results in discontinuous map in which there are some crenellated regions. In applying such a discontinuous map to an initial state repeatedly, states are trapped into quasi-periodic oscillation resulting from the crenellated regions, but are opened to instable behaviors. As a result, coupled two cells are often synchronized because of crenellated regions, but they are desynchronized again, and that is alternatively changed (Fig.18, 19).

In active coupling, it is difficult to distinguish stable from instable regions in terms of control (logistic) parameter and/or apparent coupling strength. The parameter region determined as chaotic region in passive coupling is changed into stable window in active coupling. In featuring time-dependent flow, a time series of two cells are alternatively changed between synchronization and non-synchronized states. It is remarked that synchronization can occur against stormy perturbation in spite of perpetual structural changes of dynamics. It is no longer stable but robust behavior. Finally, it can be said that the inconsistency between microscopic and macroscopic perspectives enhances robust and emergent behavior. In other words, the effect of microscopic attributes that are neglected in passive or usual coupled map system can give rise to robust synchronization against stormy perturbation. Deviation from synchronization is triggered from small intrinsic fluctuation involved in Intent dynamics. The intrinsic fluctuation results from the tiny crenellated regions in Intent and Extent dynamics.

4. Discussion and Conclusion

Heterarchy is an old and new concept in the science of complex system. It reveals robust and emergent behavior. Robustness is retaining a function accompanied with structural change, while stability results from invariant structure. That is why robustness and emergence are both sides of the same coin. The next two questions arise, (i) how one can describe heterarchy, and (ii) Is heterarchy a universal concept? We answer these two questions by proposing the notion of observational heterarchy, and a formal model based on pre-adjunction.

In this section, we address the relationship between observational heterarchy and a paradox. For example, take a paradox of sand pile [57]. If one removes one grain of sand from a sand pile, a sand pile retains. If one keeps on removing a grain of sand, then it results in a sand pile consists only of one grain. That is a paradox. When one introduces the notion of observational

heterarchy, one can find the evolvability from a grain of sand to a sand pile. Firstly, let us consider the cause of a paradox. Given a grain of sand, one does not regard it as a sand pile, and given a mass of sand grains, one regards it as a sand pile. In the former case, there is only one term, “a grain”, on one hand, and in the latter there are two terms, “a mass of grains” and a “sand pile”, on the other hand. Since both “sand pile” and “one sand grain” are concepts, each of them consists of Intent and Extent. In “sand pile”, the term, sand pile corresponds to Intent that is an attribute of concept, and the term, mass of grains corresponds to Extent to which “sand pile” can be applied. By contrast, a concept, “one sand grain” consists just of one term, a grain of sand. As far as a grain of sand is a material constituting sand pile, it corresponds to Extent of “one sand grain”. It reveals that there is no Intent for a concept, “one sand grain”. It is easy to see that a paradox results from a logical torsion from Extent (a grain of sand) of the concept, “one grain sand” to Intent (a sand pile) of the concept, “sand pile”, or from the mixture of Intent and Extent.

We can, however, find Extent as for the concept, “one sand grain”. It is a property of grain separated from earth. We can find the following sequence of concepts;

Concept	“one sand grain”		“sand pile”	
Extent	a grain of sand		mass of sand grains	
↓ ↑	↓ ↑	→	↓ ↑	(76)
Intent	grain separated from earth		a sand pile.	

Given a grain on the earth, one can find a grain if one distinguishes a grain from earth consisting of various grains. In this sense, cognitive action separating a grain from earth yields Intent of the concept, “one sand grain”. Distinction of Extent and Intent inevitably inherits cognition, observation and/or process of measurement.

In a sequence of pairs, (76), the consistency between Intent and Extent is not stable because the relationship between Intent and Extent involves logical brittleness. Given a mass of grains, one does not have to recognize a sand pile, and may recognize just a mass of grains that is different from sand pile with respect to concept. Similarly, given a grain on earth, one might ignore a grain and it might lead that all what is recognized is just earth. In other words, against logical brittleness, a concept, sand pile and a grain of sand holds robustly. That is why those concepts involve both robustness and evolvability.

If one finds inconsistency between Intent and Extent, and admits that consistency between them results just from an approximation, the paradox of sand pile can disappear. Whenever one accepts the origination of a concept of “a grain of sand” (i.e., origination of separating a grain from earth), one also has to accept the evolution and/or modification of the relationship between Intent and Extent. It leads to the emergence of a pair of Intent and Extent, constituting a concept, a sand

pile. One sees earth before, and finds a grain on earth after. We can experience such findings. Once one accepts the instance at which one recognizes a grain on earth, one also have to accept from a concept, one sand grain to another concept, sand pile. Evolution of concepts results from dynamical inconsistency between Intent and Extent.

The paradox of sand pile is as same as the paradox of consciousness, with respect to the structure. A population of neurons is just a population of neurons. The great population of neurons, however, leads to genesis of consciousness [58]. If one accepts inconsistency between Intent and Extent, such a paradox disappears. Firstly, one accepts a pair of Intent and Extent, as for a single neuron, where Extent of a neuron is a machinery cell processing an electric impulse and Intent is the basis or environment by which a specific material (cell) works as a neuron. In this sense, a neuron as a machine cannot be separated from its own environment of executing computation as a neuron, and such an environment is destined to be indefinite. It inherits inconsistency between Intent and Extent of a concept, “a neuron” that can give rise to evolution toward a concept such as “consciousness”.

The idea of observational heterarchy introduces dynamic inconsistency between what is observed and the consequence of observation. What is observed is usually regarded as a source, and the consequence of observation is regarded as a representation. Once a representation is made, it is considered that representation in description is separated from its source in a real world. However, a reality is not a source but a resource [59]. In retaining a representation, it needs perpetual provision of resources. Representation is not separated from resources, and then representation inherits indefiniteness and arbitrariness resulting from observation itself or interaction as observation [60]. That is why representation expressed as a pair of Intent and Extent inherits perpetual negotiation between Intent and Extent, and then representation itself appears as an internal observer.

Once one addresses the notion of heterarchy, it leads to the idea of observational heterarchy. Since an object becomes an object by separation of a figure and background, an object is expressed as a pair of Intent (figure) and Extent (background by which an object is expressed as a figure). The question, “Is this system a heterarchy?” is not a question with respect to fact, but a question with respect to epistemology. As far as any objects are expressed as a pair of Intent and Extent with inconsistency between them, one can find a heterarchy anywhere. That is why heterarchy is a universal structure, and the internal measurement inherited in a heterarchy is also universal. Finally, the idea of heterarchy (e.g., [1]) and the agent who makes the consequence of measurement the process of adjusting subsequent measurement (e.g., [2]) that are recently addressed in the field of complex systems are strongly relevant with each other. In other words, one can find internal measurement [9] and/or endo-observer [10], if one regards a system as complex system such as an observational heterarchy.

Rosen also proposes the notion of complexity, independent of the field of dynamical complex systems [53, 54]. He addresses the situation that perturbation influences not only a state but

also a map because the separation between actor and actant results just from measurement process. Given a map, $f:S \rightarrow S$, perturbation is usually expressed by, for x in S , $x+\delta x$. According to Rosen, we have to focus not only on $x+\delta x$ but also on $f+\delta f$. From this point of view, the next question arises, how a system, especially living system is robust in spite of perturbation. If it is possible to maintain a particular metabolism, f , against a perturbation, it is expressed as a pair of two maps, $\phi:S \rightarrow \text{Hom}(S, S) \Leftrightarrow f:S \rightarrow S$ [54]. In this sense, metabolic process against perturbation is expressed as $\phi(x+\delta x) = f$. However, such an equivalence does not hold, and instead of the equivalence, $S \rightarrow \text{Hom}(S, S) \Leftrightarrow X \times S \rightarrow S$ holds in a set theory [46, 47]. In other words, environment denoted by X that can make individualized f possible is necessary to determine a pair of robust metabolism. If one can determine the environment, the statement, $S \rightarrow \text{Hom}(S, S) \Leftrightarrow X \times S \rightarrow S$ is possible. The environment is, however, destined to be indefinite. It results in impossibility to prove the statement, $S \rightarrow \text{Hom}(S, S) \Leftrightarrow X \times S \rightarrow S$ by verifying the environment as X . One has to accept robust equivalence $S \rightarrow \text{Hom}(S, S) \Leftrightarrow X \times S \rightarrow S$ without verification. Conversely, the relationship between Intent, $S \rightarrow \text{Hom}(S, S)$ and Extent, $X \times S \rightarrow S$ has to inherit inconsistency. That is our interpretation of Rosen's complexity, and then it can lead to the idea of observational heterarchy.

As mentioned in above section, an observational heterarchy is expressed as a pair of Intent and Extent inheriting inconsistency between them. Since consistent relation between Intent and Extent in a formal sense is derived from adjoint functors, inconsistent one has to be derived from pre-functors carrying the mixture of a map and a functor. Since Intent and Extent can be compared to macroscopic and microscopic perspective, respectively, inconsistency that is usually neglected can be implemented as the engine driving evolution and robust behavior in the form of an observational heterarchy. As a result, perturbation can influence not only state like as $x+\delta x$ but also a map like $f+\delta f$, and that leads to robust behavior.

In human behavior, there are many robust behavior enhanced by fluctuation (e.g., [61, 62]), such as walking and balancing task. Although they can be explained by non-linear dynamics coupled with perturbation leading entrainment, one has to address the reason why robust behaviors enhanced by perturbation is universal and ubiquitous. In the framework of stability theory, structure of dynamics is thought to be an implicit attribute of the system. The change of structure such as $f+\delta f$ cannot be introduced in terms of the theory. The robust behavior enhanced by perturbation is interpreted by a specific structure of coupling between a specific dynamical system and perturbation. The coupling system (65) illustrates one of examples. Although a logistic parameter is settled for chaotic behavior, the coupling strength tunes interaction and leads to synchronization. It is the structure of dynamics (e.g., logistic map) that perfectly determines stable regions of coupling strength. The appearance of intermittency and/or oscillation enhanced by perturbation is also determined by the structure of non-linear dynamics in the perspective of stability theory [63]. For example, if a return map involves two different flat regions connected by steep regions, and even if it

is a chaotic dynamics, perturbation induces oscillation between flat regions. Since perturbation moves each state at the steep regions to a flat region, the state is trapped into an oscillation. That is why synchronization, oscillation and intermittency are robustly appeared by enhancement of perturbation due to specific structure of dynamics.

If one allows that a dynamics is invariant and is attribute to a system in its own right, the next question arises, how such specific dynamics are chosen as natural complex systems? Do they result from natural selection? There is no adequate solution in accepting natural selection by separating a dynamics from the origin and/or genesis of dynamics. Instead of the idea of such a separation, we should address the internal agent carrying the ability of choice, the agent who can adjust its own dynamics corresponding to perturbation, or who can adjust the way of measurement by consequences of measurement.

The perspective of observational heterarchy can yield the agent satisfying that condition in a natural way. In the framework of observational heterarchy, linear coupling of non-linear system (Intent map) is reduced to active coupling consisting of Intent and Extent maps, and perturbation is expressed as non-monotonous fluctuated map revealing the coupling strength. Such a perturbation can reduce modification of a dynamical system, and it reveals not only $x+\delta x$ but also perturbation with respect to a map, $f+\delta f$. Since a pseudo pull back functor can change each continuous dynamical map (Intent and Extent map) into a specific discontinuous map having flat regions in a term of a return map, due to the perturbation, a robust synchronization can occur while it is opened to emergent behavior. A pair of perturbation and a dynamical system as Intent dynamics can lead to modification of a pair of state and a map. Such an adjustment corresponding to perturbation is nothing but the ability of the agent carrying measurement. In this sense, robustness enhanced by a stormy perturbation can be revealed in the perspective of observational heterarchy.

This research was supported by the 21st century COE program from the JSPS.

References

- [1] E. Jen, *Complexity* **8**(2003)12.
- [2] S.A. Kauffman, *Investigation* (Oxford Univ. Press., 2002).
- [3] W.S. McCulloch, *Bull.Math.Biophys.* **7**(1945)69.
- [4] R.Kaeher and E. von Goldammer, *J.Molecular Electronics* **4**(1988)31.
- [5] N. Luhmann, *International Journal of General Systems* **8**(1982)131.
- [6] M. de May, *The Cognitive Paradigm, a Newly Explored Approach to the Study of Cognition Applied in an Analysis of Science and Scientific Knowledge*. (D. Reidel Pub. Co., 1982).
- [7] D. Stark, In: *The Biology of Business: Decoding the Natural Laws of Enterprise*, Clippinger, J.H. III, ed., pp.153-179, (Jossey-Bass Publishes, San Francisco, 1999).

- [8] A. Lehman and M. Girard, *Environment and Planning A* **34**(2002)1927.
- [9] K. Matsuno, *Protobiology: Physical Basis of Biology* (CRC Press, Boca Raton, MI, 1989).
- [10] O.E.Rossler, In: *Optimal Structures in Heterogeneous Reaction Systems*, pp. 123-138 (P.J. Plath, ed., New York: Springer-Verlag 1989).
- [11] Y.-P. Gunji, *Appl. Math. & Comp.* **61**(1994)231
- [12] Y.-P. Gunji, *Biosystems* **35**(1995)33.
- [13] Y.-P., Gunji, K. Ito, and H. Sadaoka, *Appl.Math.Comput.***79**(1996)43.
- [14] Y.-P. Gunji, Y-P. and S. Toyoda, *Physica D* **101**(1997)27.
- [15] Y.-P., Gunji, K. Ito and Y. Kusunoki, *Physica D* **110** (1997) 289.
- [16] K. Ito, and Y.-P. Gunji, *Physica D***102**(1997)275.
- [17] C. Lucas, Online [www.http://www.calresco.org/sosfaq.htm](http://www.calresco.org/sosfaq.htm)., 1997.
- [18] R.S. Sutton and A.G. Barto, *Reinforcement Learning: An Introduction* (MIT Press, Cambridge, 1998).
- [19] W. Fontana and L.W. Buss, *Bull.Math.Biol.***56**(1994)1.
- [20] J.L. Hartman, Garvik, B. and L. Hartwell, *Science* **291**(2001)1001.
- [21] P. Cariani, *Intellectica* **2**(1997)95.
- [22] K. Matsuno, *Ann. New York Acad. Sci.* **901**(2000)332.
- [23] O.E.Rossler, *Endo-Physiscs: The World as an Intyerface* (World Scientific, Singarpore, 1998).
- [24] F.W. Lawvere, In: *Lecture Notes in Mathematics: Category Theory, Homology Theory and their Applications II* pp.134-145, (P. Hilton ed., Springer-Verlag, New York, 1969).
- [25] Soto-Andrade and F.J. Varela, *Acta Applicandae Mathematicae* **2**(1984)1.
- [26] L.H. Kauffman, *Kybernetics* (to apper).
- [27] D. S. Scott, In: *Toposes, Algebraic Geometry and Logic* pp. 97-136, (F.W. Lawvere, ed., Springer-Verlag, New York, 1972).
- [28] C.A. Gunter, *Semantics of Programming Languages* (MIT Press, Cambridge, 1992).
- [29] D.S. Scott, In: *To H.B. Curry: essays in combinatory Logic, lambda calculus and Formalisms*(Academic Press, New York, 1980).
- [30] B.C. Smith, In: *Computationalism, New Directions*, pp. 23-58. (Scheutz, M. ed., MIT Press, Cambridge, 2002).
- [31] Y.-P. Gunji, K. M. Aono, S. Wakisaka and S. Tsuda, *Chaos,Solitons and Fractals*(to appear)
- [32] Y.-P. Gunji, M. Aono, K. Sasai, and S. Kamiya, *Chaos,Solitons and Fractals*(to appear)
- [33] H.R. Maturana, In: *A Theory of Living* (M. Zeleny ed., North Holland, New York, 1981).
- [34] Y.-P. Gunji and H. Higashi, *Physica D***156**(2001)283.
- [35] Y.-P. Gunji, M. Aono, M. and H. Hihashi, *Int. J. Comput. Anticipat. Systems* **8**(2001)303.
- [36] Ganter, B. and Wille, R. *Formal Concept Analysis: Mathematical Foundations*(Springer Verlag, Berlin, 1999).

- [37] Barwise, J. & Seligman, J. *Information Flow, The Logic of Distributed Systems* (Cambridge Univ. Press., Cambridge 1997).
- [38] Y.-P. Gunji, Y. Kusunoki and M. Aono, *Chaos, Solitons and Fractals* **13**(2002)261.
- [39] Y.-P. Gunji, *Theory of Life II: What is My Own Consciousness?* (Philosophical Press, Tokyo, 2003, in Japanese).
- [40] Y.-P. Gunji, T. Takahashi and M. Aono, *Chaos, Solitons & Fractals* (in press).
- [41] Y.-P. Gunji, T. H. Miyoshi, Takahashi, and M. Kamiura, *Chaos, Solitons & Fractals* (to appear).
- [42] M.H. Bickhard and L. Terveen, *Foundational Issues in Artificial Intelligence and Cognitive Science* (Elsevir Science Pub. New York, 1996).
- [43] J.L. McCauley, and J.I. Palmore, *Phys.Letters* **A15**:433.
- [44] K. Kaneko, *Prog.Theor.Phys.***72**(1984)480.
- [45] K. Kaneko and I. Tsuda, *Complex Systems: Chaos and Beyond* (Springer Verlag, Berlin, 2001).
- [46] S. Awody, 2002, *Category Theory*. <http://www.andrew.cmu.edu/course/80-317-713>.
- [47] S. MacLane, *Categories for the Working Mathematician* (Springer, New York, 1971).
- [48] J. Cairns and P.L. Foster, *Genetics*(1991)695.
- [49] J.A. Shapiro, *Genetica* **86**(1992)99.
- [50] J.A. Shapiro, *Science* **268**(1995)373.
- [51] J.A. Shapiro, *J. Biol. Phys.* **28** (2002) 745.
- [52] L. Deverney, *An Introduction to Chaotic Dynamical Systems*, (Benjamin/Cummings Comp. Boston, 1986).
- [53] R. Rosen, *Theoretical Biology and Complexity, Three Essays on the Natural Philosophy of Complex Systems* (Academic Press, Orlando, 1985).
- [54] R. Rosen, *Life Itself* (Columbia Univ. Press, New York, 1991).
- [55] F.J. Varela, *Principles of Biological Autonomy*.(North-Holland, Amsterdam, 1979).
- [56] H. Fujisaka and T. Yamada, *Prog. Theor. Phys.* **69**(1983)32.
- [57] M. Dummet, *Elements of Institutionism*, (Oxford univ. press, Oxford, 2000 (2nd)).
- [58] M. Tye, *Ten Problems of Consciousness* (The MIT press, Cambridge, 1995=1999 (3rd)).
- [59] L. Floridi, *Minds and Machine* **13**(2003)459.
- [60] M. H. Bickhard, In: *Caldron of Consciousness*, pp. 161-178, (R.D. Ellis and N. Newton eds., Benjamins, 2000).
- [61] A. Priplata, J. Niemi, M. Salen, J. Harry, L.A. Lipsitz, and J.J. Collins, *Phys.Rev.Lett.***89**(2002)8101.
- [62] J.L. Cabrera and J.G. Milton, *Phys.Rev.Lett.***89**(2002)8702.
- [63] E. Ott, *Chaos in Dynamical Systems* (Cambridge university Press, Cambridge, 1993).

Figure captions

Fig.1. Example of heterarchy illustrating inter-countries economic communication with the mixture of inter-layer operation and intra-layer operation. It consists of intra-layer operation such as a mapping a good to Yen or Dollar, inter-layer operation such as the exchange, and the mixture such that the exchange needs a cost. The mixture entails to infinite regression of re-definition of a system, and reveals impossibility to describe a heterarchy.

Fig. 2. Schematic diagrams of adjunction (above) and pre-adjunction (below). Adjunction verifies isomorphism between two subsets of different categories, C (intent) and D (extent) if there is no mixture between inter-levels operations (thick horizontal arrows) and intra-level operation (thin vertical arrow). Otherwise, it entails to a paradox. Schematic diagram of pre-adjunction and it reveals the model for heterarchy. The mixture enhances dynamical change of heterarchy.

Fig. 3. Schematic diagram of observational heterarchy illustrating description of a particular protein species. It consists of Intent and Extent perspective that inherits inconsistency between them.

Fig. 4. Pseudo-inverse map, f^* derived from continuous non-monotonous map, f .

Fig. 5. A. Diagram of bifurcation solution with respect to the deviation of synchronized state over all coupling strength, as for the passive coupling defined by eq.(68). Horizontal axis represents coupling strength. B. Enlarged diagram of A, focusing on the range, $0 < c < 0.25$.

Fig. 6. A pair of Intent and Extent maps in fluctuated passive coupling expressed as eq.(71). Left above diagram represents Intent map, $\phi(x_i^t)$ with $i=0,1$, and right above diagram represents Extent map, as $f(\phi(x_i^t))$. In the passive coupling, Extent map, $\psi(x)$ is expressed such that $\psi(x) = f(\phi(x))$. The diagram below represents a map of fluctuated flow from cell to cell, f .

Fig. 7. Diagram of bifurcation solution for $x_0^t - x_1^t$ against the apparent coupling strength, c , as for the fluctuated passive coupling defined by eq.(71).

Fig. 8. Diagram of bifurcation solution for $x_0^t - x_1^t$ against the apparent coupling strength, c , as for the “asymmetric” fluctuated passive coupling defined by eq.(72).

Fig. 9. A pair of Intent and Extent maps in fluctuated active coupling expressed as eq.(73). Left diagram represents Intent map, $\phi^l(x_i^t)$ with $i=0,1$, and right diagram represents Extent map, as $\psi^l(x_i^t)$.

Due to the effect of non-monotonous map, f , and discontinuous map, f^* , Intent map is replaced by discontinuous map, although initial Intent map is a continuous logistic map.

Fig. 10. Diagram of bifurcation solution for $x_0^t - x_1^t$ against the apparent coupling strength, c , as for the fluctuated active coupling defined by eq.(73). Logistic parameter is 4.0 (above) and 3.82 (below).

Fig. 11. Diagram of bifurcation solution for $x_0^t - x_1^t$ against the apparent coupling strength, c , as for the “asymmetric” fluctuated active coupling defined by eq.(74). Logistic parameter is 4.0 (above) and 3.82 (below).

Fig. 12. Riddle basins in a Cartesian space of a pair of initial states (x_0^0, x_1^0) . Each co-ordinate is represented by dot, if two states x_0^t and x_1^t are synchronized till T times iterations of applying eq.(73), and is represented by blank otherwise. The row above shows basin structures of active coupling, and the row below shows ones of passive coupling. The coupling (or apparent coupling) length is set in a chaotic region, and $c = 0.75, 0.80, 0.85, 0.90$ from left to right.

Fig. 13. Riddle basins in a Cartesian space of a pair of initial states (x_0^0, x_1^0) . Each co-ordinate is represented by dot, if two states x_0^t and x_1^t are synchronized till T times iterations of applying eq.(73), and is represented by blank otherwise. The row above shows basin structures of active coupling, and the row below shows ones of passive coupling. The coupling (or apparent coupling) length is set in the region of synchronization, and $c = 0.65, 0.675, 0.70, 0.725$ from left to right.

Fig. 14. Riddle basins in a Cartesian space of a pair of initial states (x_0^0, x_1^0) for active coupling, where $T=7$ and $c = 0.65, 0.675$ (above), $0.70, 0.725$ (below) from left to right.

Fig. 15. Bifurcation solutions of a logistic map against a logistic parameter, α , and it is Intent map in passive coupling.

Fig. 16. Bifurcation solution of Intent map, $\phi^l(x_i^t)$ with $i=0,1$ under active coupling against logistic parameter, α . The diagram above is obtained for $c = 0.25$, and one below for $c = 0.05$. There are no chaotic regions.

Fig. 17. A time series of two states, x_0^t and x_1^t , and the difference between them in passive coupling, where the flow from cell to cell is given as time-dependent flow, f^t as eq.(75) with $a = 0.4$ and $c = 0.1$. Pairs of return maps represent pair of Intent (left) and Extent (right) map. There is no synchronization, and the difference between two states is fluctuated along time.

Fig. 18. A time series of two states, x_0^t and x_1^t in active coupling, where the flow from cell to cell is given as time-dependent flow, f^t as eq.(75) with $a = 0.4$ and $c = 0.1$. Pairs of return maps represent pair of Intent (left) and Extent (right) map. Not only Extent but Intent map are modified as discontinuous map, and synchronization can be achieved in a few step.

Fig. 19. A time series of two states, x_0^t and x_1^t in active coupling, where the flow from cell to cell is given as time-dependent flow, f^t as eq.(75) with $a = 0.4$ and $c = 0.1$. The initial condition for the time dependent flow is different from the case shown in Fig. 18. Other parameter setting is the same as the case of Fig. 18. Pairs of return maps represent pair of Intent (left) and Extent (right) map. Although synchronization can be achieved and maintained for a while, it is disturbed and synchronization is obtained again. Time series is alternating changed in that manner.

Fig. 20. State space of (x_0^t, x_1^t) under passive coupling (left) and active coupling (right), where the flow from cell to cell is given as time-dependent flow expressed as eq.(75). $a = 0.4$ and $c = 0.1$. Other parameter setting is the same as the case of Fig. 18.

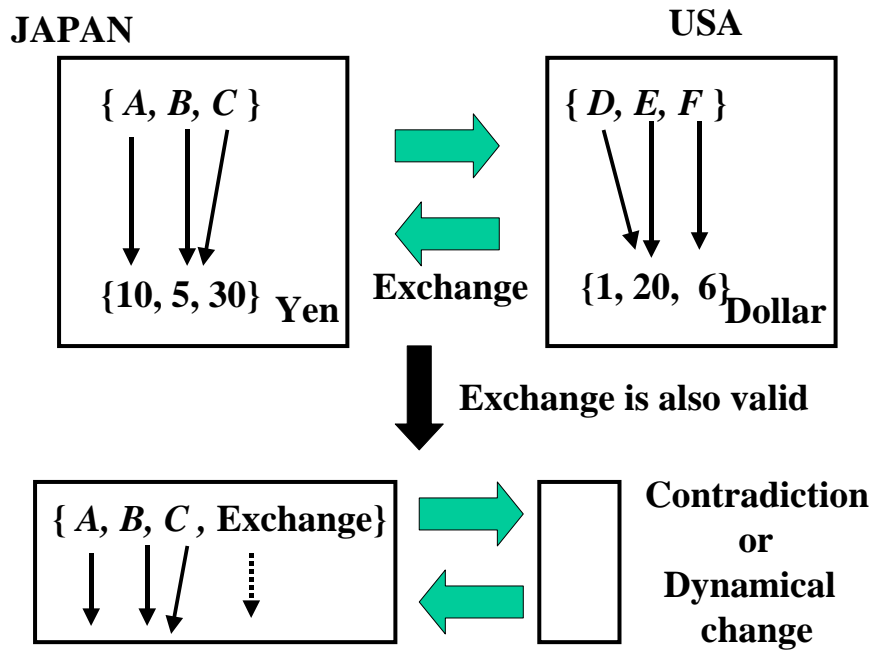


Fig. 1. Gunji and Kamiura

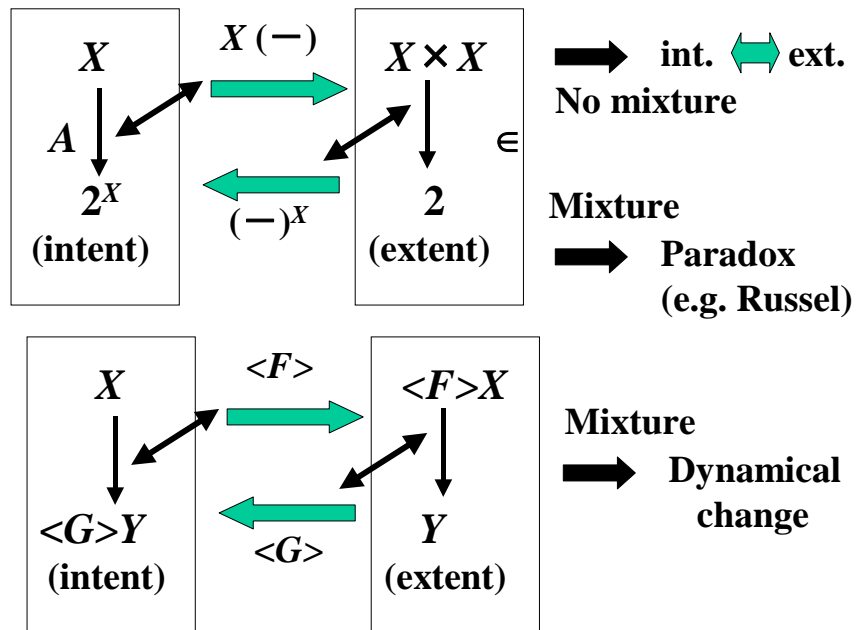


Fig. 2. Gunji and Kamiura

Observational Heterarchy as Incomplete Concept

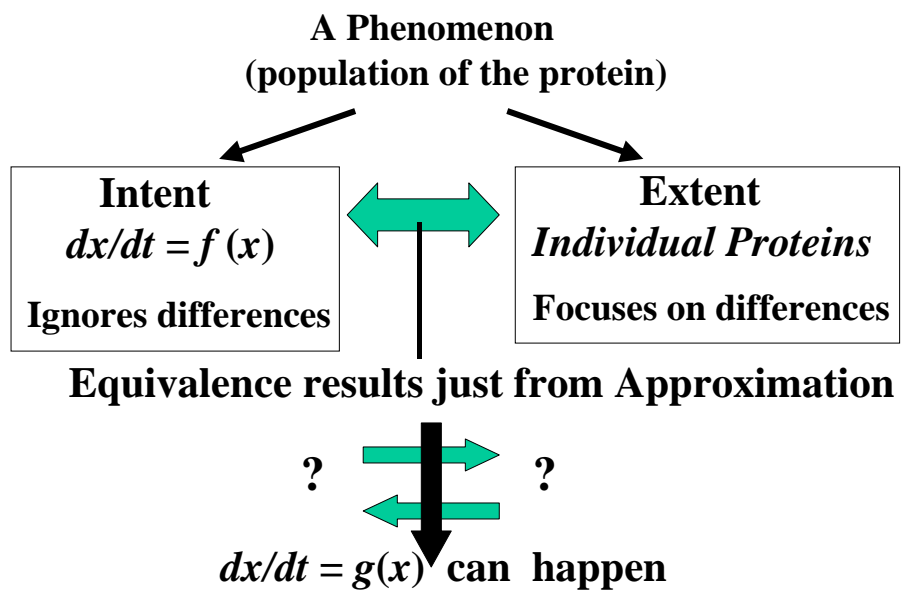
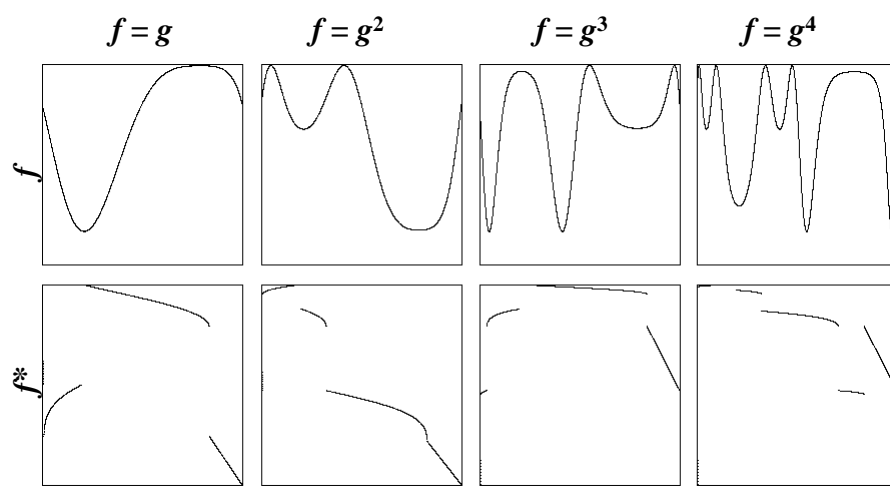


Fig. 3. Gunji and Kamiura



$$f^*(x) = y \text{ (} y \leq z, \forall z \in \{z \mid f(x) = z\} \text{)}$$

Fig. 4. Gunji and Kamiura

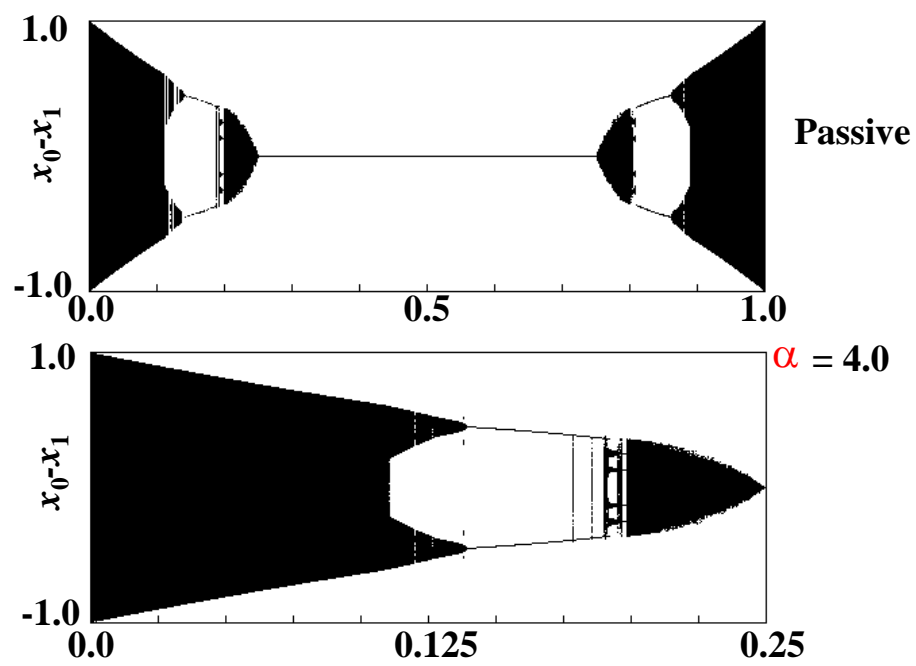


Fig. 5. Gunji and Kamiura

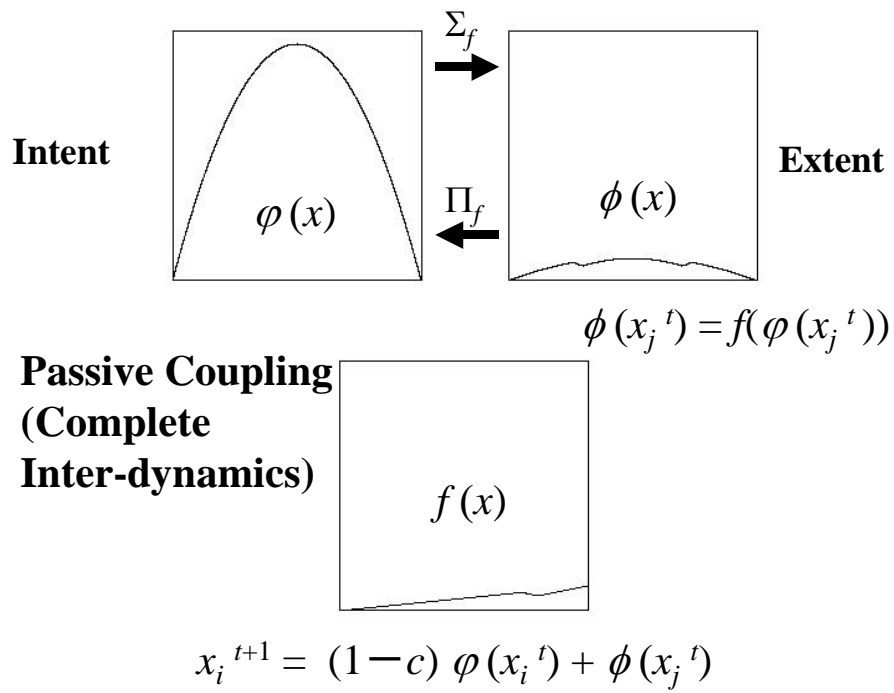


Fig. 6. Gunji and Kamiura

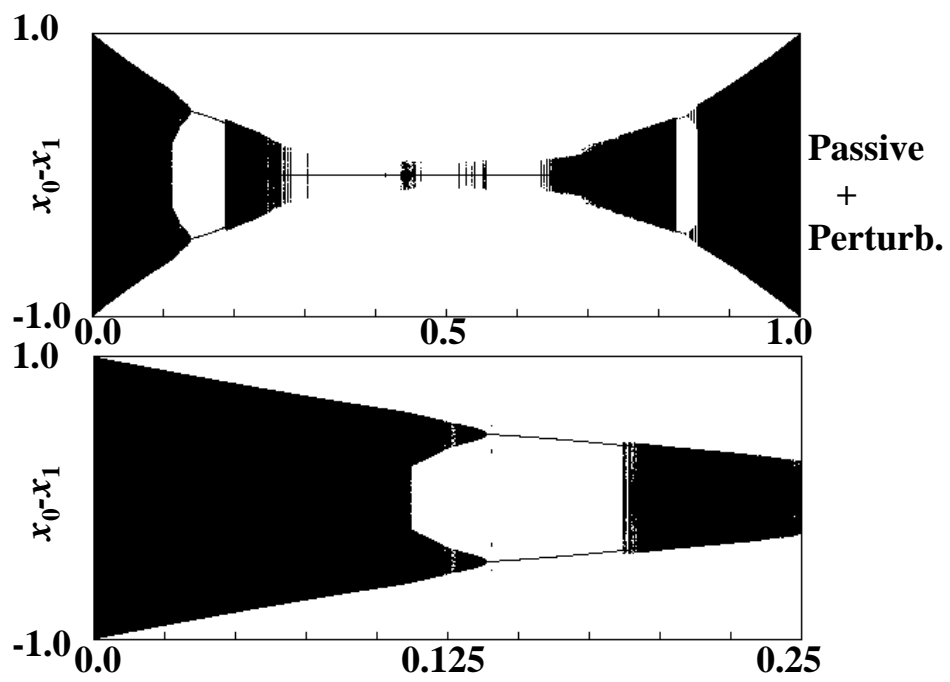


Fig. 7. Gunji and Kamiura

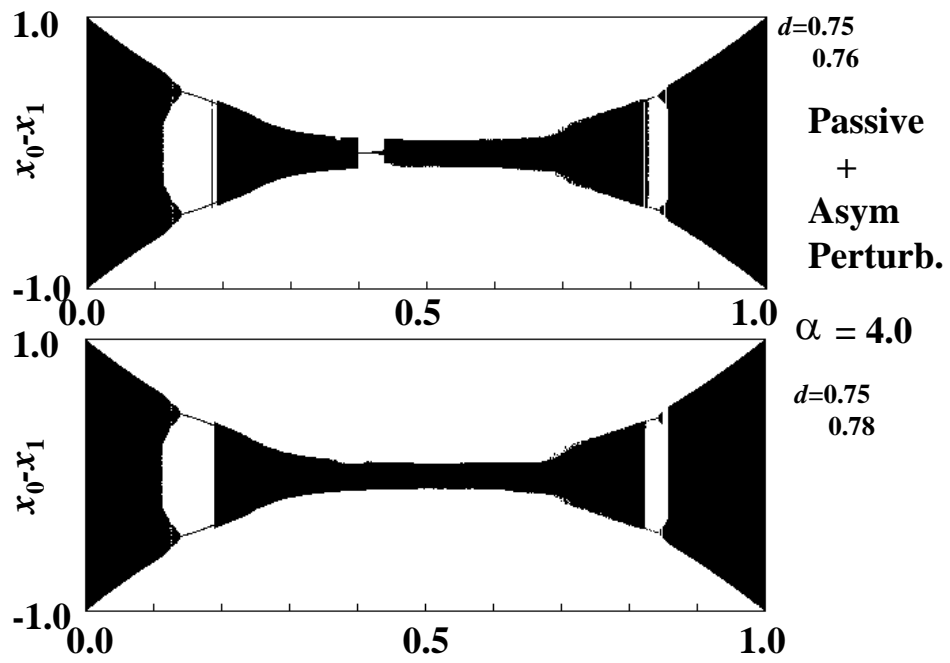


Fig. 8. Gunji and Kamiura

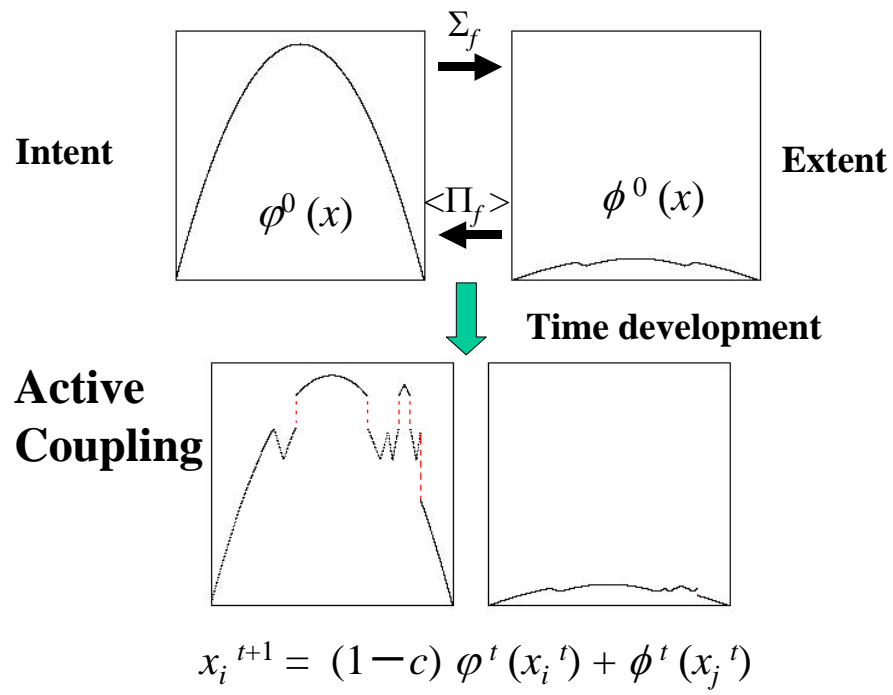


Fig. 9. Gunji and Kamiura

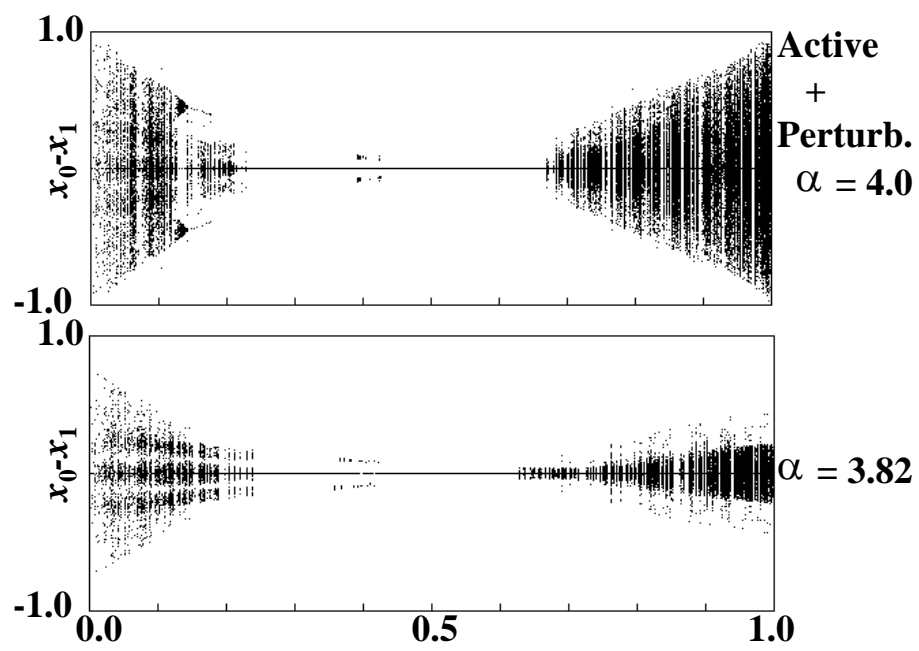


Fig. 10. Gunji and Kamiura

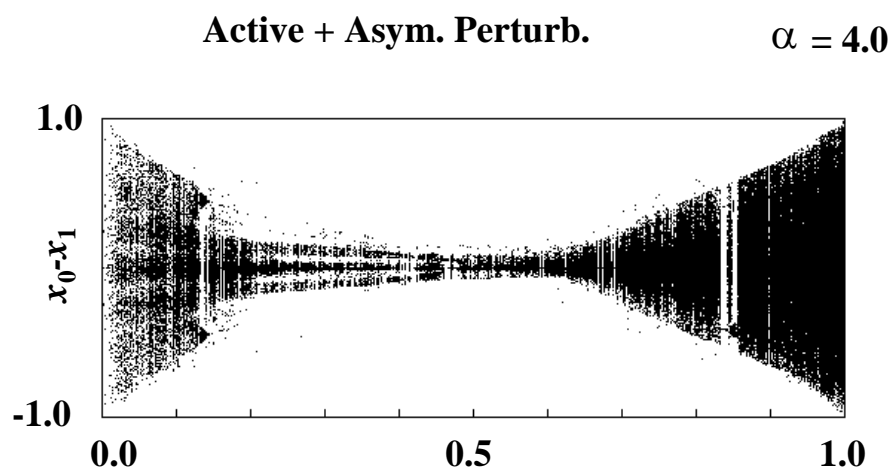
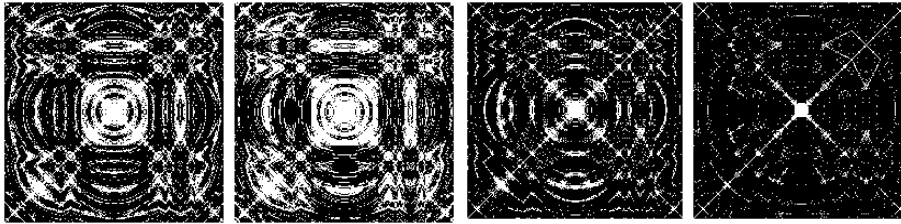
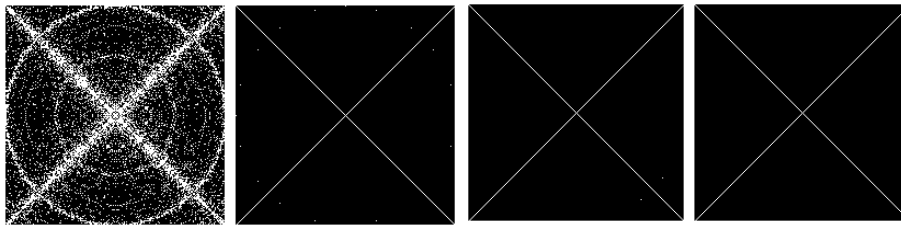


Fig. 11. Gunji and Kamiura

Active Coupling $T = 5$



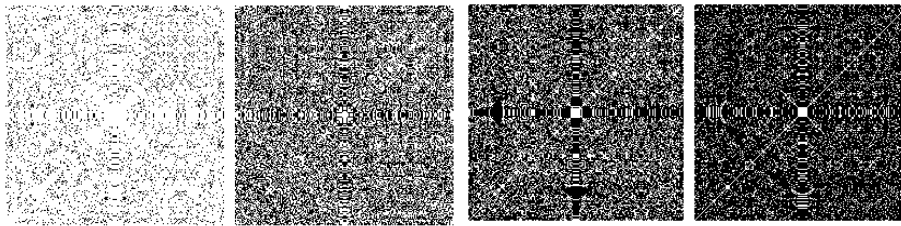
Passive Coupling $T = 65$



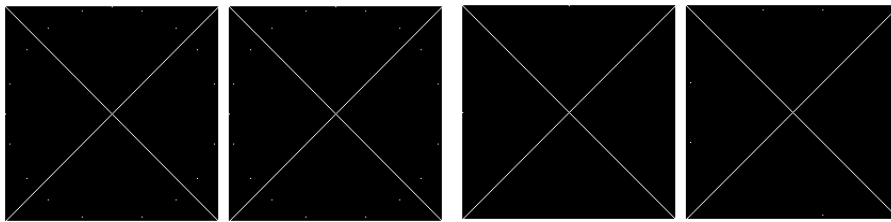
$\alpha = 4.0; \quad c = 0.65, 0.675, 0.70, 0.725$

Fig. 12 Gunji and Kamiura

Active Coupling



Passive Coupling



$T = 50$ (50~1000c) $\alpha = 4.0$; $c = 0.75, 0.80, 0.85, 0.90$

Fig. 13. Gunji and Kamiura

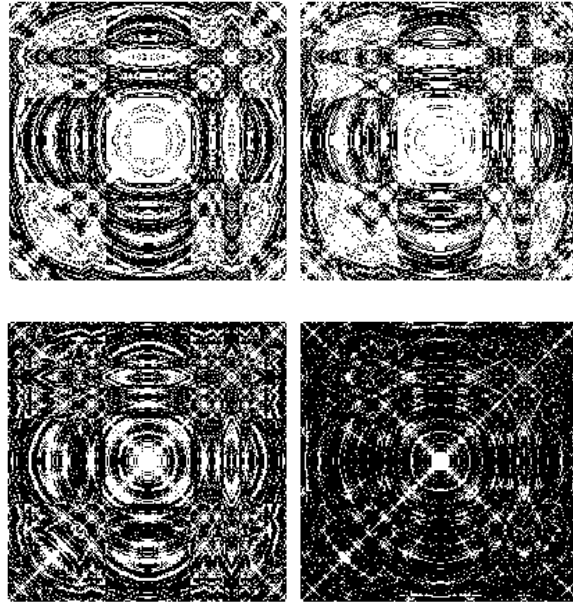


Fig. 14. Gunji and Kamiura

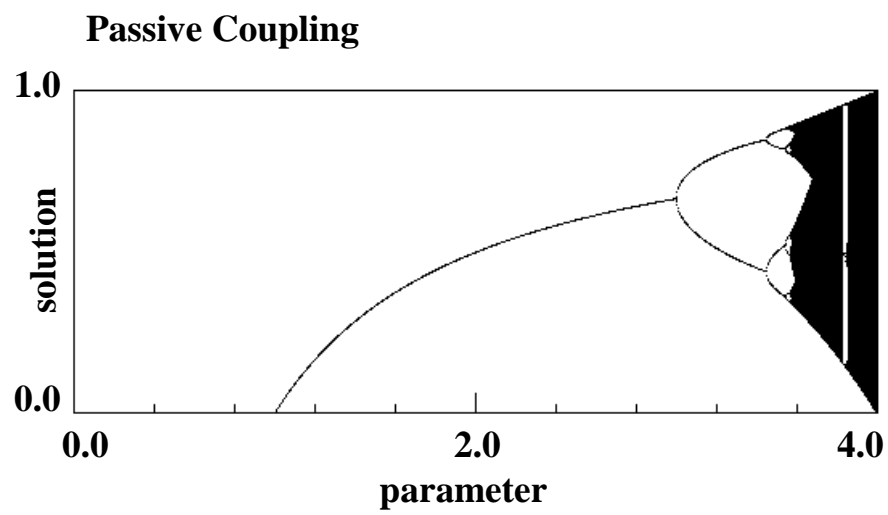


Fig. 15. Gunji and Kamiura

Active Coupling

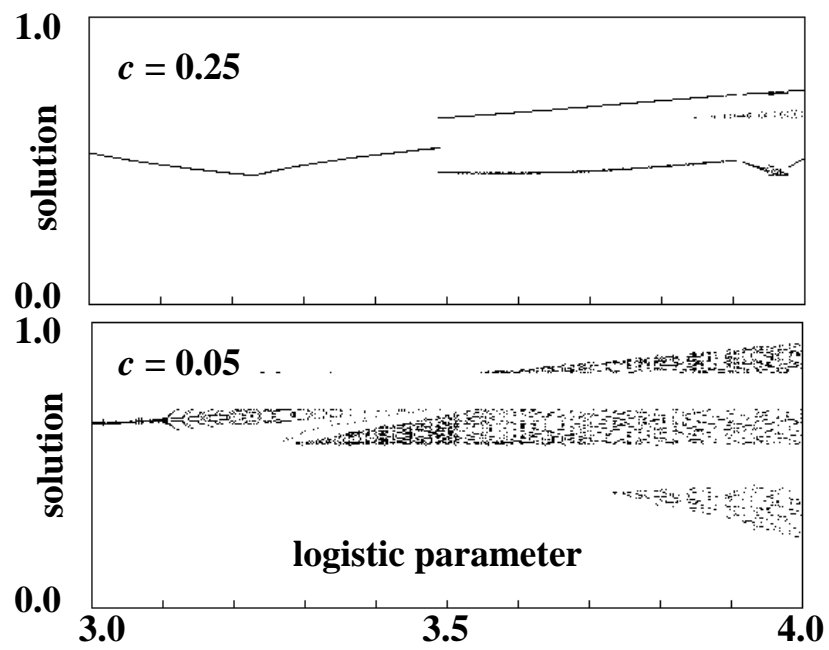


Fig. 16. Gunji and Kamiura

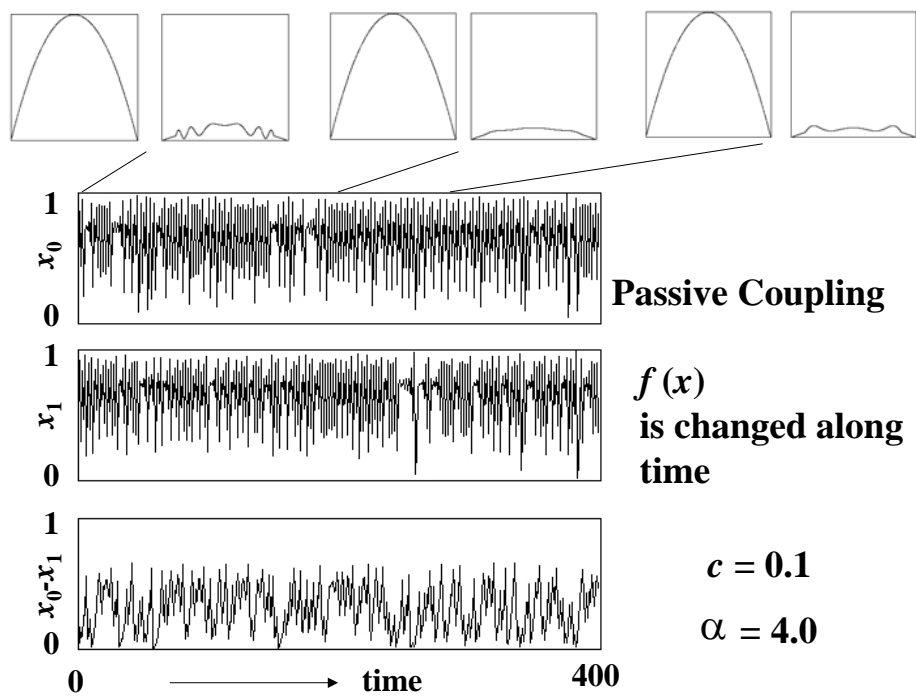


Fig. 17. Gunji and Kamiura

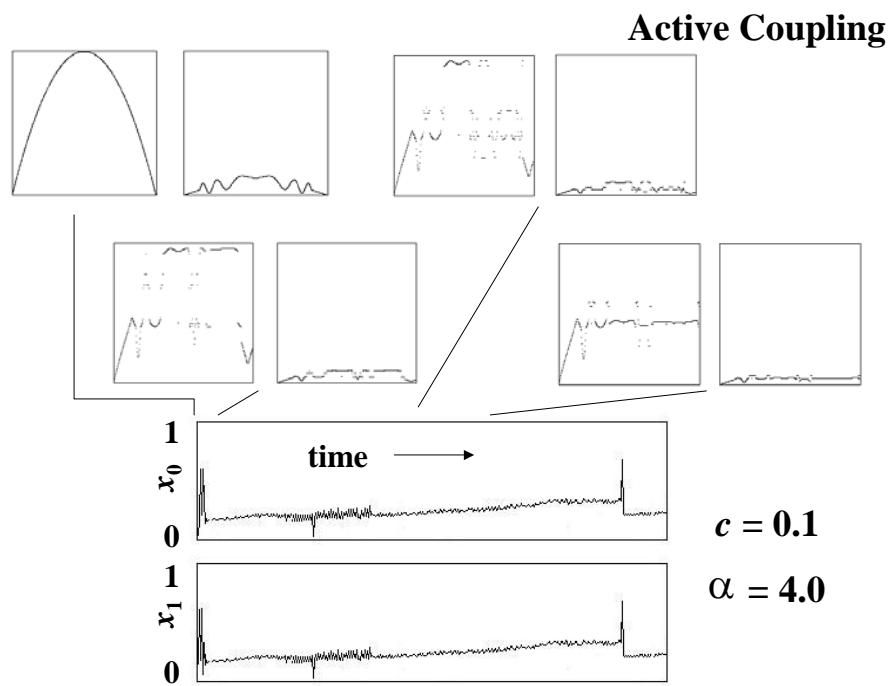


Fig. 18. Gunji and Kamiura

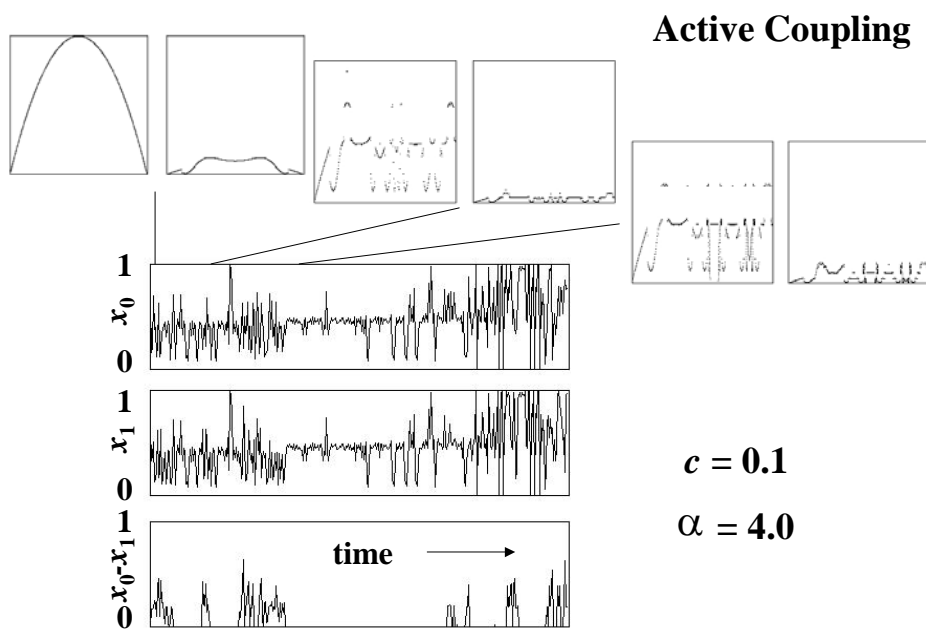


Fig. 19. Gunji and Kamiura

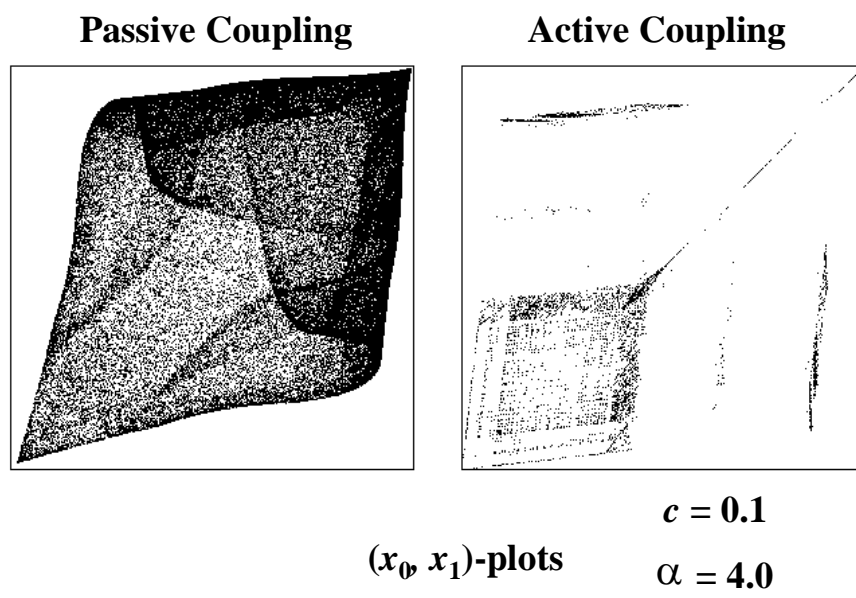


Fig. 20. Gunji and Kamiura