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# Subjective Spacetime Derived from a Causal Histories Approach

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The internal description of spacetime can reveal ambiguity regarding an observer's perception of the present, where an observer can refer to the present as if he were outside spacetime while actually existing in the present. This ambiguity can be expressed as the compatibility between an element and a set, and is here called  $a/\{a\}$ -compatibility. We describe a causal set as a lattice and a causal history as a quotient lattice, and implement the  $a/\{a\}$ -compatibility in the framework of a causal histories approach. This leads to a perpetual change of a pair of causal set and causal history, and can be used to describe subjective spacetime including the déjà vu experience and/or schizophrenic time.

## 1. Introduction

The internal description of spacetime has been explored since the entire spacetime viewed by an observer outside the universe is unphysical. For the entire spacetime, measurement is expressed as a transformation from objective spacetime to the internal observer's description. Instead of Lorentian spacetime, a causal set has been introduced to make a theory in principle causal, and has been used in quantum gravity approaches [1], [2], [3]. The internal description of spacetime was originally proposed in the context of pursuing a program for the quantization of gravity based on causal structures [23, 24]. A causal set is defined as a partially ordered set, and is connected to quantum theory by mapping an acausal set in a partially ordered set to a tensor product of Hilbert space [4], [5].

In [6], the measurement is defined as a transformation from the causal event in a partially ordered set to the causal history in a collection of sets. In terms of category theory, that is a functor from a general category to a category of sets [7]. A collection of all functors, which is a functor category, can constitute a topos, which is described using Heyting algebra [8, 25]. As long as the objective spacetime or the causal set is fixed, a functor category can be regarded as an evolving set

of measurement, and the properties of Heyting algebra are argued with respect to the global properties of the causal set such as black holes [6].

Although the way in which subjective spacetime can be described has also been explored by physicists [9], it has been unclear how to bridge subjective spacetime and objective (physical) spacetime, notwithstanding the problems of how to measure subjective time and describe periods of subjective time such as the past and future introduced in a causal histories approach. In particular, the framework used for the internal description of spacetime in physics is the same framework of time that was proposed by the philosopher McTaggart about one hundred years ago [10]. There are two series used in [10], the B-series that is defined as an ordered set of events and the A-series that is defined as a local series of past-present-future. McTaggart defines that the present or A-series moves in the B-series, and that once an event is indicated as the present, the past and future of the A-series arise. Although the B- and A-series can correspond to a causal set and causal history, respectively, McTaggart discovered a contradiction regarding the A-series in the B-series. As a consequence, many philosophical controversies arose [11], [12].

On the one hand, the mapping from the B-series (causal set) to the A-series (causal past and future) has been well defined and is regarded as the standard measurement in physics [6]. On the other hand, it has also been concluded that this mapping is impossible in the case described in [10]. Solving this problem could be the key to bridge subjective and objective spacetime. And we consider this problem to result from the position of the observer. From the viewpoint of McTaggart, whenever an event is indicated as the present, the subject is located in the very event in the B-series. On the other hand, from the viewpoint of causal past, an event in the B-series (a causal set) is viewed by an observer outside the B-series, and this constitutes the internal description “viewed by the external observer”. Although here we follow McTaggart’s stance, we resolve the contradiction in the A-series, and attain the dynamical duality of the A- and B-series, regarded as subjective internal spacetime. We will show that subjective internal spacetime can explain the various kinds of subjective time reported in cognitive science, including the *déjà vu* experience [17] and schizophrenic time.

The measurement itself does not affect the given spacetime in the causal histories approach. A non-trivial past-future relation results only from a particular non-trivial causal set, and that leads to the consequence that a lattice which is a partially ordered set closed with respect to binary operations is trivial [6]. The problem is that the causal set assumed is usually too simple, and is inconsistent with general relativity. Since a logical structure is dependent on the structure of the causal set, the causal set had to be non-lattice to make the logical structure non-trivial. Recently, however, Martin and Panangaden have shown that a bicontinuous poset equipped with interval topology is a causal set consistent with general relativity [13], and that a bicontinuous poset is isomorphic to the globally hyperbolic spacetime favored by Penrose [26, 27]. This idea also results from the invariant causal set. The torsion can be also solved by introducing dynamical duality of the

A- and B-series.

This paper is organized as follows. In section 2, we review the causal set as a partially ordered set, and introduce a lattice and a quotient lattice. In [6], the measurement is defined as a functor from a partially ordered set to a category of sets. Here, we restrict the causal set as a lattice and thereby also restrict the measurement [18]. Congruence and a quotient lattice are introduced, and the measurement is expressed as a congruence. This is our framework for the measurement in the causal set, and is referred to as a generalized causal histories approach. In section 3, we apply McTaggart's stance to the measurement in the causal set. We strictly distinguish the earlier/later distinction from the past/future distinction with respect to the position of the observer. In this sense, causal past and future can correspond to earlier causal events and later ones, respectively. When the observer sits at a particular event in the causal set, it is expressed as the ambiguity of existence and indication of the event, which is referred to here as  $a/\{a\}$ -compatibility. In section 4, we formally describe  $a/\{a\}$ -compatibility in the generalized causal histories approach. This is a way in which a subject can be embedded in a causal set, thereby leading to the perspective of dynamical duality of the A- and B-series. In section 5, we show that the dynamical duality of the A- and B-series can explain the various kinds of subjective time, including déjà vu. Various kinds of subjective time can result from the perpetual negotiation between the A- and B-series. In section 6, we argue the significance of subjectivity in spacetime and discuss future work. The negotiation resulting from  $a/\{a\}$ -compatibility can originate intrinsic perturbation, and that can be regarded as the origin of non-linearity.

## 2. Generalized causal histories

A causal set consisting of events is defined as a partially ordered set. A partially ordered set  $P$  is a set with a partial order  $\leq \subseteq P \times P$ , where for any  $a, b, c \in P$ , (i)  $a \leq a$ , (ii)  $a \leq b, b \leq a \Leftrightarrow a = b$ , (iii)  $a \leq b, b \leq c \Rightarrow a \leq c$  [1], [2], [3], [6]. This allows the existence of an acausal set (anti-chain) in which any two elements  $a, b \in P$ , satisfy neither  $a \leq b$  nor  $b \leq a$ . Given a partially ordered set  $P$ , for a subset of  $S \subseteq P$ ,  $a$  is the upper bound of  $S$ , if for any  $x \in S$   $x \leq a$ . The least upper bound is called a join, and is represented by  $\vee S$ . In particular, if  $S$  is a two-element set  $\{x, y\}$ , the join is represented by  $x \vee y$ . The greatest lower bound (meet),  $\wedge S$  and  $x \wedge y$  are dually defined. A lattice is a partially ordered set such that for any two-element set, it is closed under the join and meet. In particular, a lattice  $L$  is complete if and only if for any subset  $S$  of  $L$ ,  $\vee S$  and  $\wedge S$  exist in  $L$  [18].

In [13], a physically reasonable description of spacetime known as globally hyperbolic spacetime is proved to be a bicontinuous partially ordered set. Due to the continuity, it features completion and approximation that can lead to interpolation. Here, we consider that requiring both continuity and completion is strong a requirement, and focus only on the completion. This is why the

causal set is defined as a complete lattice.

The internal description of the causal histories is defined as a collection of the operator as a measurement by which the notion of past is constructed by an observer, such as  $Past(x) = \{y \text{ in the causal set} \mid y \leq x\}$  [6]. The operator  $Past$  is a map that assigns a set for an element of a partially ordered set, and this kind of map is generalized as a transformation in terms of both elements and order, known as a functor. By means of the transformation, an element in a partially ordered set is mapped to a set, and the order is mapped to the map between sets. Since the composition of maps also follow the transitive law, the transformation preserves the structure in terms of partial order. A collection of all transformations results in a category with a special structure, a subobject classifier called a topos, that implies Heyting algebra [6], [8].

It is clear that the internal description of the causal history is the observer's own categorization of a causal set. In other words, the observer defines a particular criterion by which a causal set is classified into a collection of sets. Since we introduce a complete lattice as a causal set, more useful properties are accessible such as equivalence class, congruence and quotient lattice. Given a lattice  $L$ , let an equivalence relation on  $L$  be  $\theta = \{\langle x, y \rangle \in L \times L\}$  such that, for  $\forall x, y, z \in L$ , (i)  $\langle x, x \rangle \in \theta$ , (ii)  $\langle x, y \rangle \in \theta \Leftrightarrow \langle y, x \rangle \in \theta$ , and (iii)  $\langle x, y \rangle \in \theta, \langle y, z \rangle \in \theta \Rightarrow \langle x, z \rangle \in \theta$ . The notation  $\langle x, y \rangle \in \theta$  can be also expressed as  $x \theta y$  or  $x \equiv y \pmod{\theta}$ . An equivalence relation  $\theta$  is a congruence on  $L$  if, for all  $a, b, c, d \in L$ ,  $(a \equiv b \pmod{\theta} \text{ and } c \equiv d \pmod{\theta}) \Rightarrow (a \vee c \equiv b \vee d \pmod{\theta} \text{ and } a \wedge c \equiv b \wedge d \pmod{\theta})$ . Let  $\theta$  be a congruence on a lattice  $L$ , and a set  $L/\theta$  is defined by  $L/\theta = \{[a]_\theta \mid a \in L\}$  with  $[a]_\theta = \{b \in L \mid b \equiv a \pmod{\theta}\}$ . If meet ( $\wedge$ ) and join ( $\vee$ ) are defined by

$$[a]_\theta \wedge [b]_\theta = [a \wedge b]_\theta, \quad [a]_\theta \vee [b]_\theta = [a \vee b]_\theta, \quad (1)$$

we call  $\langle L/\theta, \wedge, \vee \rangle$  the quotient lattice of  $L$  modulo  $\theta$  [18]. If  $\theta$  is a congruence, meet and join are well defined. Actually, if  $x \in [a]_\theta, y \in [b]_\theta$  then one obtains  $x \equiv a \pmod{\theta}$  and  $y \equiv b \pmod{\theta}$ . Since  $\theta$  is a congruence,  $x \wedge y \equiv a \wedge b \pmod{\theta}$  and  $x \vee y \equiv a \vee b \pmod{\theta}$ , and that implies  $x \wedge y \in [a \wedge b]_\theta$  and  $x \vee y \in [a \vee b]_\theta$ .

In a causal histories approach, given a complete lattice, the observer's measurement process is described as a congruence that maps a causal event to its equivalence class in which events are equivalent to each other with respect to the congruence. An observer can indicate an element  $x$  in the complete lattice  $L$  as the position of the present in spacetime. When the past is expressed as the down set  $\downarrow x = \{y \in L \mid y \leq x\}$ , the equivalence relation  $\theta$  is derived from  $\downarrow x$  such that  $\theta = \{\langle x, y \rangle \in L \times L \mid \exists z \in \downarrow x (z \vee x = z \vee y)\}$ . The map  $F_\theta: L \rightarrow L/\theta$  that is defined by, for any  $x \in L$ ,  $F_\theta(x) = [x]_\theta$  is the observer's measurement process of categorization of events in a causal set. In particular, if a complete lattice  $L$  is distributive (i.e., for any  $a, b, c \in L$ ,  $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ ),  $\downarrow x$  is also one of the equivalence classes, and the equivalence class derived from  $\downarrow x$  is congruence for any  $x$  in  $L$ .

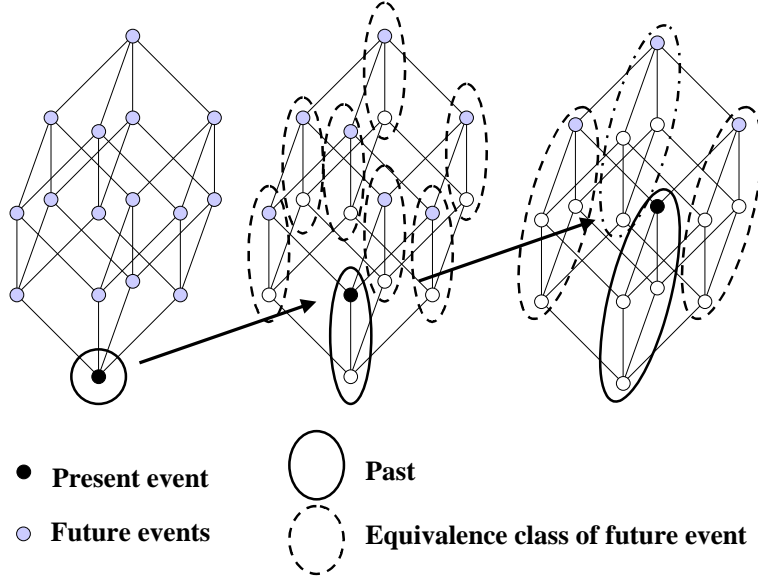


Fig. 1. The present moving on a causal set as a lattice. The arrow indicates the motion of the present. The present  $x$  induces a causal past  $\downarrow x$  and derives a congruence  $\theta$  as the measurement. The congruence  $\theta$  leads to an equivalence class  $[x]_\theta$  (indicated by dashed or solid loops). The structure consisting of  $[x]_\theta$ , which is a quotient lattice  $L/\theta$ , is a generalized causal past and future.

Fig. 1 shows the present moving on a causal set as a lattice,  $L$ . In other words, it is assumed that an observer sits at a particular event that is the present. When the present  $x$  moves in a causal set, the causal past  $\downarrow x$  and causal future  $\uparrow x = \{y \in L \mid x \leq y\}$  are also changed. The equivalence class  $[x]_\theta$  for  $x$  in  $L$  is obtained from the equivalence relation  $\theta$  derived by  $\downarrow x$ . Each equivalence class constitutes either the causal future or the causal past. In fact, for any  $z \in \uparrow x$ ,

$$[z]_\theta = \downarrow z - \bigcup_{y \in \uparrow x, y < z} \downarrow y, \quad (2)$$

and  $\bigcup_{z \in \uparrow x} [z]_\theta = L$  [14]. This also implies  $L/\theta \cong \uparrow x$ . In this sense, the equivalence classes constitute the causal history induced by an observer.

In Fig. 1, since a lattice is distributive, the equivalence class derived by  $\downarrow x$  is a congruence. However, the equivalence relation derived by  $\downarrow x$  is not always a congruence. To utilize the properties of the congruence, we define the causal past and causal future as the equivalence class derived by a congruence. Given a complete lattice  $L$ , for any  $x \in L$  that is regarded as the present, a congruence  $\theta \subseteq L \times L$  is defined as the measurement and a collection of the equivalence class or the quotient lattice  $L/\theta$  is called a generalized causal history.

### 3. McTaggart's view of causal histories

In the above section, we reviewed the causal histories approach based on the transformation from objective spacetime to subjective spacetime, where the transformation is well defined. The same approach for describing time was proposed by McTaggart about one hundred years ago [10]. Since he ignored spatial variation and focused only on simultaneous spacetime points, termed “moments”, his objective spacetime implied only objective time expressed as a totally ordered set (i.e., for any element of the set  $a, b$ , either  $a \leq b$  or  $b \leq a$ ). He calls objective time the B-series, and subjective time, past-present-future, the A-series. As a natural extension of McTaggart's ideas, if the velocity of observation propagation is introduced as the speed of light, the B-series must be expressed as a lattice. Notwithstanding the fact that the same framework as in the causal histories approach is used, McTaggart concluded that the transformation from the B- to A-series is erroneous.

The B-series is expressed, for example, as  $\dots a \leq b \leq c \leq d \leq e \dots$ , and the A-series moves in the B-series, if “ $b$  is present”,  $a$  is past, and  $c, d$  and  $e$  are future. Compared with the causal histories approach, we call each element of the B-series an event, and define that  $Past(a) = \{x \text{ in B-series} \mid x < a\}$  and  $Future(a) = \{x \text{ in B-series} \mid a < x\}$ . This implies that if  $y$  is in  $Past(a)$  and  $z$  in  $Future(a)$ , then  $a$  is the present,  $y$  is past and  $z$  is future. In other words, if an event  $a$  is indicated as the present, an A-series such as  $Past(a)$ - $a$ - $Future(a)$  appears. It is easy to see that there can also be a mapping from the B-series to the A-series in McTaggart's view.

In the A-series, the present, past and future are incompatible determinations. When the A-series moves in the B-series and we say that the event is in the present; however, a contradiction arises. Given a B-series such as  $\dots a \leq b \leq c \leq d \leq e \dots$ , if  $b$  is the present,  $c$  is future. After that, the position of the present moves, and the situation in which  $c$  is the present arises, which leads to  $b$  being past. Therefore, we obtain the situation in which  $b$  is both the present and past (and  $c$  is both future and present). Analogously,  $b$  is also regarded as future. Co-existence of the present, past and future is inconsistent with the incompatibility. It sounds as though the contradiction resulted from confusing the aspect of the A-series with the tense. If the present moves from  $b$  to  $c$ , the statement “ $b$  is the present” changes to “ $b$  has been the present”. Is the contradiction resolved by introducing this difference in tense? McTaggart says no. If one can say that “ $b$  has been the present”, there exists a future in which the event  $b$  is regarded as past, for example,  $c$ . Although this means that  $c$  is future, “has been” is only distinguished from “is” by being existent in the past. Only because  $c$  is present, is  $b$  existent in the past. Finally, we obtain the situation in which  $c$  is both the present and future. This is inconsistent with the incompatibility of the past, present and future.

If McTaggart's view is right, the measurement process from a partially ordered set to a category of sets (including our generalized causal histories approach) is also erroneous. Strictly



speaking, the measurement process defined in the causal histories approach is different from McTaggart's A-series. McTaggart distinguishes the earlier/later relation in the B-series from the past/future relation in the A-series, and says on the one hand that the event  $a$  being earlier than the other event  $b$  is permanent, and that the event  $a$  being past is dependent on what is present. Herein, we point out some essential differences between earlier/later of the B-series and past/future of the A-series that were not given in McTaggart's sense.

Recall our notation  $Past(a)$ - $a$ - $Future(a)$ . This denotes that the present is indicated by an event  $a$  that is an element of a partially ordered set, and that the past or future is the name of a set. Then, the present is different from both past and future with respect to the set-theoretical or logical property. One can define the set of earlier/later in the same manner such that  $Earlier(a) = \{x \text{ in B-series} \mid x < a\}$  and  $Later(a) = \{x \text{ in B-series} \mid a < x\}$  as the *Past* and *Future*, although there is an essential difference between Earlier/Later and Past/Future. In an A-series, the present "becomes" past, and future "becomes" present. This entails that the assimilation of an element with a set is admitted in the A-series; however, the mixture of an element and a set implies Russell's paradox. The A-series needs the assimilation of  $a$  with  $\{a\}$  to avoid a contradiction. This is why it is possible to say that the present becomes past and the future becomes present.

Here, we show the difference between Earlier/Later and Past/Future in terms of the position of the observer. In the Earlier/Later relation, an observer sits outside the B-series, and indicates an event  $a$  by means of which  $Earlier(a)$  and  $Later(a)$  are defined. Only by means of the position of the observer is "indicating  $a$ " distinguished from " $a$ " (existence of  $a$ ). Given  $\dots a \leq b \leq c \dots$ , if  $b$  is indicated, then  $a$  is earlier than  $b$ . Even if  $c$  is indicated, however, it never implies that " $b$  becomes earlier" since the perspective indicated by  $b$  (and  $Earlier(b)$ ) is totally different from the one indicated by  $c$  (and  $Earlier(c)$ ). The event  $b$  defined in the perspective indicated by  $b$  never belongs to  $Earlier(c)$  indicated by  $c$ . In this sense "becoming" never makes sense to the observer sitting outside the B-series. In contrast, if an observer sits at an event, "becoming" makes sense. The statement "an event  $a$  is the present" holds when the event  $a$  is being experienced by an observer now. The observer must sit at the event  $a$  in the B-series. The existence of an event  $a$  cannot be distinguished from the event  $a$  indicated by the observer, only because the observer sits at an event  $a$ . Since the most primitive indication is made by surrounding a letter with brackets, we call this the assimilation of existence and indication of the  $a/\{a\}$ -compatibility. In other words, the internal description of spacetime or the description in the B-series needs  $a/\{a\}$ -compatibility that allows "present ( $a$ ) becomes past ( $\{a\}$ )".

The  $a/\{a\}$ -compatibility is an expression for the subject. The subject can experience an event and can assimilate the existence of  $a$  by indicating  $a$ . Finally, if one gives up the concept of the subject in describing spacetime, an observer will always be outside spacetime, and the Past/Future relation is always replaced by the Earlier/Later relation. In the causal histories approach, the internal

description excludes the subject. That is why  $Past(a)-a-Future(a)$  is the same as the Earlier/Later relation in McTaggart's approach and why the measurement can be well defined. Readers may feel that there are differences between philosophy and physics, and may think that the subjectivity and/or  $a/\{a\}$ -compatibility cannot be described by physics. We consider that  $a/\{a\}$ -compatibility is already used in mathematics. Such a usage can be extended and applied to the causal histories approach, and that constitutes the introduction of the subject in spacetime.

#### 4. Subject in spacetime

First, we show an example of  $a/\{a\}$ -compatibility according to a lattice theory that is found in Dedekind-MacNeille completion of a lattice. A partially ordered set  $P$  is given, as shown in Fig. 2. If an element  $x$  is connected with the other element  $y$  by a line, and  $x$  is drawn lower than  $y$ , it means that  $x \leq y$ . It is easy to see that  $P$  is not a complete lattice since there is no  $c \wedge d$ . One of the ways in which some elements can be added to  $P$  and a complete lattice can be obtained is known as the Dedekind-MacNeille completion. Given a partially ordered set  $P$ , it is defined by  $\mathbf{DM}(P) = \{A \subseteq P \mid A^{ul} = A\}$ , where  $A^u = \{x \in P \mid x \geq s, \forall s \in A\}$  and  $A^l = \{y \in P \mid s \geq y, \forall s \in A\}$ . An element of  $\mathbf{DM}(P)$  is collected as the following;  $\{a\}^{ul} = \{a, c, d\}^l = \{a\}$  and  $\{a, b\}^{ul} = \{c, d\}^l = \{a, b\}$ . When all elements of  $\mathbf{DM}(P)$  have been collected, we obtain a lattice  $\langle \mathbf{DM}(P), \subseteq \rangle$  in which partial ordering of  $\mathbf{DM}(P)$  is defined by the inclusion. Finally, the completed  $P$  is obtained as an abstract lattice that is isomorphic to  $\langle \mathbf{DM}(P), \subseteq \rangle$ .

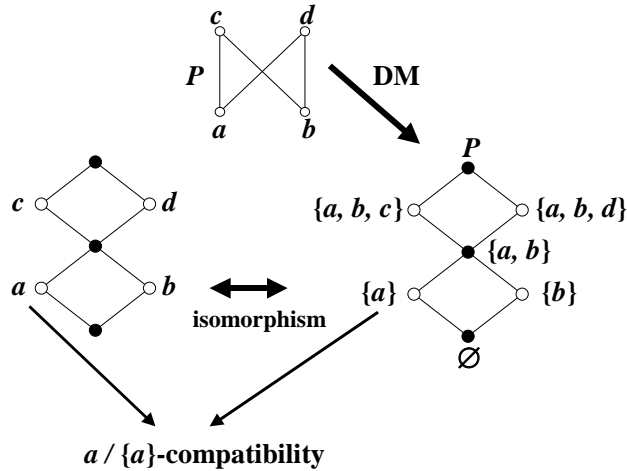


Fig. 2. The  $a/\{a\}$ -compatibility found in the Dedekind-MacNeille completion in a lattice theory. Given a partially ordered set, the completion is revealed by assimilating an element with a set. See text for details.

Why could such a completion be obtained? If two lattices, the completed  $P$  (bottom left in Fig. 2) and  $\mathbf{DM}(P)$  (bottom right in Fig. 2), are given, one can verify isomorphism notwithstanding the difference between an element  $a$  and a set  $\{a\}$ . The issue we intend to tackle is different from such verification. Before obtaining a completed  $P$ , the structure of a complete lattice is revealed by assimilating  $P$ -level (whose elements are elements of  $P$ ) and the power set of the  $P$ -level (whose elements are subsets of  $P$ ), and that leads to a particular thinking that completion of  $P$  by the power set of  $P$ -level. The  $a/\{a\}$ -compatibility is found in the instance of inventing  $\mathbf{DM}(P)$ . Although McTaggart assimilated  $a$  with  $\{a\}$ , he never explicitly defined the connection between  $a$  and  $\{a\}$ , which can lead to avoidance of Russell's paradox. We consider that  $a/\{a\}$ -compatibility is the most primitive implicit operation hidden in mathematics and other formal languages.

We implement the subject in a generalized causal histories approach mentioned in section 2. First we apply McTaggart's idea to an ill-defined relation between the B- and A-series in the generalized causal histories approach. The B-series is defined by a complete lattice  $L$ , and the measurement process is defined by a congruence  $\theta \subseteq L \times L$ . The A-series is defined by a quotient lattice  $L/\theta$ . In this framework, McTaggart's concept of the present, future and past co-existing, is implemented as the following.

Recall the definitions,  $Past(a) = \{x \text{ in B-series} \mid x < a\}$  and  $Future(a) = \{x \text{ in B-series} \mid a < x\}$ , where  $a$  is the present. We mentioned that past and future is the name of a set and the present is the name of an element in this definition. As we take an A-series moving in a B-series into consideration, we are faced with the  $a/\{a\}$ -compatibility such that one cannot determine whether past is a name of a set or a name of an element. In a generalized causal histories approach,  $F_\theta: L \rightarrow L/\theta$  maps an element of a (finite) lattice (B-series) to an equivalent set of its quotient lattice (A-series) such that for any  $x$  in  $L$ ,  $F_\theta(x) = [x]_\theta = \{y \in L \mid x \equiv y \pmod{\theta}\}$ . Actually, this kind of map that assigns the equivalence class is equivalent to the equivalence relation, and that is expressed as

$$\theta: L \times L \rightarrow \{1, 0\} \quad \Leftrightarrow \quad \Theta: L \rightarrow \text{hom}(L, \{1, 0\}), \quad (3)$$

where  $\theta$  is a map derived directly from the equivalence relation  $\theta$  (here, it is a congruence) such that for any  $\langle x, y \rangle \in L \times L$ ,  $\theta(\langle x, y \rangle) = 1$  if  $\langle x, y \rangle \in \theta$ , and  $\theta(\langle x, y \rangle) = 0$ , otherwise. This is why this map is also designated by  $\theta$ . The term  $\text{hom}(X, Y)$  represents a set of all maps from  $X$  to  $Y$ . For any  $x \in L$ ,  $\Theta(x) \in \text{hom}(L, \{1, 0\})$  is a map that for any  $y \in L$   $\Theta(x)(y) = 1$  if  $\langle x, y \rangle \in \theta$ , and  $\Theta(x)(y) = 0$  otherwise. Therefore, if one defines a map  $F: L \rightarrow \wp(L)$ , where  $\wp(L)$  is a power-set of  $L$ , for any  $x \in L$ ,  $F(x)$  is a subset of  $L$ , and one can define that  $\Theta(x)(y) = 1$  if  $y \in F(x)$ , and  $\Theta(x)(y) = 0$  otherwise. By the equivalence (3) between  $\Theta$  and  $\theta$ , we finally obtain the equivalence relation.

The ambiguity of a set and an element or the  $a/\{a\}$ -compatibility in past (or future) can be

expressed as the ambiguity of the representative of the equivalence class. In  $[x]_\theta$  the representative  $x$  represents not only an element of the set but also the name of the set. If one enhances the ambiguity of the role of element and the role of the name of the set, one can replace a set  $[x]_\theta$  by an element  $x$ , in losing the definition of  $\theta$ . Imagine the situation in which any set  $[x]_\theta$  in  $L/\theta$  is replaced by the representative  $x$ , and a collection of the representatives  $R$  is given, where the greatest element in  $[x]_\theta$  can be chosen as the representative, but other representatives can be also chosen. Note that  $R$  is a lattice isomorphic to  $L/\theta$ . Since the definition of  $\theta$  is lost, it is impossible to collect all elements of  $[x]_\theta$ . Instead of correctly re-constructing  $[x]_\theta$ , one arbitrarily has to define  $\Gamma: R \rightarrow \wp(L)$ , where for any  $x, y$  ( $x \neq y$ ) in  $R$   $\Gamma(x) \cap \Gamma(y) = \emptyset$ . Given  $R$ , one can obtain  $\Gamma(x)$  for any  $x \in R \subseteq L$ , and a new partially ordered set as  $\cup_{x \in R} \Gamma(x)$ . In other words, if the  $a/\{a\}$ -compatibility is admitted in the generalized causal histories approach, one has to accept that objective spacetime (B-series) expressed as a complete lattice is reconstructed in losing the definition of  $\theta$  after one obtains  $L/\theta$  (A-series)  $\cong R$ .

The subjectivity as the  $a/\{a\}$ -compatibility leads to the dual change of both the B- and A-series. The problem, however, remains. Generally a partially ordered set such as  $\cup_{x \in R} \Gamma(x)$  is not a lattice. It is inconsistent with objective spacetime (B-series) as a complete lattice. For this problem, the other side of the  $a/\{a\}$ -compatibility can play an essential role in constructing a complete lattice. McTaggart shows the negative aspect of the  $a/\{a\}$ -compatibility as the discrepancy between the A- and B-series; however, we demonstrate the positive aspect of the  $a/\{a\}$ -compatibility by means of the Dedekind-MacNeille completion. We enhance the positive aspect by using  $\Gamma(x)$  again.

The assimilation of  $x$  with  $[x]_\theta$  makes us introduce  $\Gamma(x)$ . Therefore,  $x$  in  $\Gamma(x)$  implies both a set and an element implicitly. To describe such an ambiguity, we define  $\langle \Gamma \rangle(*)$  in which both a set and an element can be substituted for  $*$ , and  $\langle \Gamma \rangle$  is called a skeleton [14]. The map  $\Gamma: R \rightarrow \wp(L)$  is isomorphic to  $\Theta: R \rightarrow \text{hom}(L, \{1, 0\})$ , and  $\Gamma(x)$  is introduced to construct and/or mimic the equivalence class  $[x]_\theta$ . In the definition of  $L/\theta$ ,  $[x \vee y]_\theta = [x]_\theta \vee [y]_\theta$  and  $[x \wedge y]_\theta = [x]_\theta \wedge [y]_\theta$  are well defined since  $\theta$  is a congruence. Then,  $\Gamma(a) \wedge \Gamma(b)$  and  $\Gamma(a) \vee \Gamma(b)$  are also expected to satisfy these equations ( $\Gamma(a) \wedge \Gamma(b) = \Gamma(a \wedge b)$ ), which leads to the situation in which each  $\Gamma(a)$  is expected to be the equivalence class derived by a congruence.

This means that both  $\langle \Gamma \rangle: P \rightarrow \wp(L)$  and  $\langle \Gamma \rangle: \wp(P) \rightarrow \wp(L)$ , where  $P = \cup_{x \in R} \Gamma(x)$ . If an element  $a$  is substituted for  $*$  ( $\forall a \in P$ ),

$$\langle \Gamma \rangle(a) = \Gamma(b), \quad a \in \Gamma(b), b \in R, \quad (4)$$

which is well defined. For any two-element set  $S = \{a, b\}$ , it is defined that

$$\langle \Gamma \rangle(S) = \langle \Gamma \rangle(a \wedge b) = \Gamma(c), \quad a \wedge b \in \Gamma(c), c \in R, \quad (5)$$

if  $a \wedge b$  exists in  $P$ .

If the following statement holds, such that if  $\langle \Gamma \rangle(x) = \langle \Gamma \rangle(y)$ ,  $\langle \Gamma \rangle(s) = \langle \Gamma \rangle(t)$  then  $\langle \Gamma \rangle(x) \wedge \langle \Gamma \rangle(s) = \langle \Gamma \rangle(y) \wedge \langle \Gamma \rangle(t)$  (join also holds dually), one can find a quotient lattice consisting of elements  $\langle \Gamma \rangle(x)$ , and meet (join, dually) is defined by  $\langle \Gamma \rangle(x) \wedge \langle \Gamma \rangle(y) = \langle \Gamma \rangle(x \wedge y)$ . Actually, if  $\langle \Gamma \rangle(x) = \Gamma(p)$  ( $x \in \Gamma(p)$ ) and  $\langle \Gamma \rangle(y) = \Gamma(q)$  ( $y \in \Gamma(q)$ ),  $\langle \Gamma \rangle(x) = \langle \Gamma \rangle(p)$  and  $\langle \Gamma \rangle(y) = \langle \Gamma \rangle(q)$ , and then  $\langle \Gamma \rangle(x) \wedge \langle \Gamma \rangle(y) = \langle \Gamma \rangle(p) \wedge \langle \Gamma \rangle(q)$ , and  $\langle \Gamma \rangle(x \wedge y) = \langle \Gamma \rangle(p \wedge q)$ .

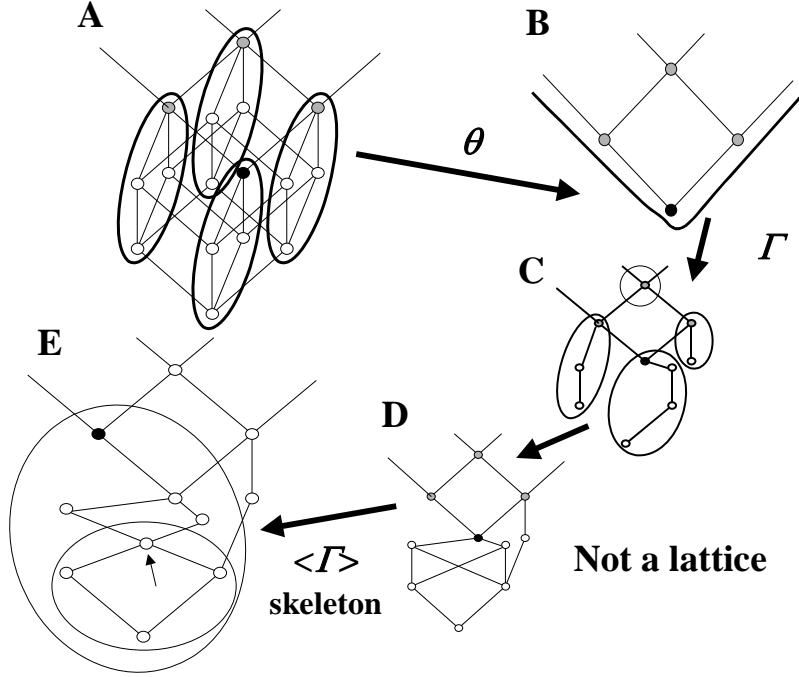


Fig. 3. Dynamical dualities of objective spacetime (B-series) and subjective spacetime (A-series). A. Objective spacetime (causal set) in the form of a complete lattice. The equivalence classes derived by a congruence  $\theta$  are represented by loops. B. The quotient lattice (the generalized causal history) shown by the representatives. C. A collection of  $\Gamma(x)$  for any  $x$  in the set of the representatives. D. The ordering of  $\cup_{x \in R} \Gamma(x)$ . This shows that the obtained partially ordered set is not a lattice. E. Due to  $\langle \Gamma \rangle$  featuring in the  $a/\{a\}$ -compatibility, a fresh element (indicated by an arrow) is generated to complete a partially ordered set.

In contrast, if there is no  $a \wedge b$  in  $\cup_{x \in R} \Gamma(x)$ , a new symbol  $s$  is introduced such that

$$\langle \Gamma \rangle(\{a, b\}) = \langle \Gamma \rangle(a \wedge b) = \Gamma(s) = \{s\}. \quad (6)$$

This implies that  $a \wedge b \in I(s) = \{s\}$ . Then,  $s$  can play a role in the element of  $a \wedge b$ . In other words,  $\langle I \rangle$  can generate  $a \wedge b$  for  $\{a, b\}$  in the name of  $s$  if there is no  $a \wedge b$  in  $\cup_{x \in R} I(x)$ , and that leads to the completion of  $\cup_{x \in R} I(x)$ . Due to  $\langle I \rangle$ , a fresh element  $s$  that can play a role in  $a \wedge b$  is generated, and can be coincide with the existing element in  $L$ . Generating a fresh element results from choice of a subset without meet or join, and the choice is accompanied with randomness. The outside intervenes in the interaction between A- and B-series.

Fig. 3 shows the dynamical duality of the causal set (B-series) and the causal histories (A-series). Now, let a large finite set be  $X$ , and consider the collection of all subsets of  $X$  that is denoted by  $\wp(X)$ . Here,  $X$  is a collection of all possible events, and a complete lattice  $L$  that is a subset of  $\wp(X)$  is a possible B-series. When a causal set is defined by a complete lattice  $L$  that is a subset of  $\wp(X)$ , the order is defined by inclusion.

Given a causal set as a complete lattice  $L$  that corresponds to objective spacetime and the B-series, in the complete lattice, an element is chosen as the present, and the congruence  $\theta$  including the present event  $p$  is defined as the measurement process to make the past  $\downarrow p$  be one of the equivalence classes of  $\theta$ . The congruence  $\theta$  derives the equivalence classes represented by loops (Fig. 3A). When the equivalence class is represented only by the representative, the quotient lattice  $L/\theta$  is shown as Fig. 3B. The measurement process from the causal set to the causal history (causal past and future) can correspond to the transformation from Fig. 3A to Fig. 3B if McTaggart's claim is neglected. Since we take McTaggart's claim as the  $a/\{a\}$ -compatibility, it is assumed that  $[x]_\theta$  is compatible with the representative  $x$ . Thus, from  $x$ , re-construction of  $[x]_\theta$  is attempted. Such an attempt is carried out using a map  $I$ , and that gives rise to  $\cup_{x \in R} I(x)$ , where  $R$  is a collection of the representatives of  $L/\theta$ . Each  $I(x)$  for any  $x$  in  $R$  is surrounded by a loop in Fig. 3C.

Since the representative of  $[x]_\theta$  is here chosen as the greatest element of  $[x]_\theta$ , elements less than  $x$  are collected as  $I(x)$ , from a set of possible events  $X$ . The line representing the order in each loop in Fig. 3C implies the order only in each  $I(x)$ . In Fig. 3C, the order of elements is re-estimated over a whole partially ordered set. Since each element of  $\cup_{x \in R} I(x)$  is a subset of  $X$ , the order defined by inclusion can be determined for each pair of elements. This can result in a partially ordered set that is not a lattice (Fig. 3D). Finally, the positive side of the  $a/\{a\}$ -compatibility that is expressed as  $\langle I \rangle$  plays a role in completing a partially ordered set. A fresh element indicated by an arrow in Fig. 3E is generated due to  $\langle I \rangle$ . Note that the fresh element resulting from the completion is separated from  $X$  since it is generated only from the requisite (6) for the fresh element. The fresh element is the unknown in principle. It plays an essential role in subjective time, as discussed later. As a result, the causal set or B-series is transformed from that shown in Fig. 3A to that shown in Fig. 3E. Dually, the causal histories or A-series is also transformed.

One of possible time evolutions of A- and B-series resulting from the negotiation is described as follows. (1) From a given lattice (B-series), one element is chosen as the present. (2)

After down set of the present is determined, the congruence is derived from the down set. It generates a quotient lattice (A-series). (3) For each element  $a$  of A-series, some elements lower than  $a$  are randomly chosen from a given lattice  $L$ . It is partially ordered set  $P$ . (4) Due to  $\langle I \rangle$ , it results in completion of  $P$ , and that is newly generated B-series. The procedure (1)-(4) are iterated.

## 5. Subjective time

Since we accept McTaggart's claim that the causal set (B-series) cannot be connected with the causal history (A-series) in a consistent manner, we also accept the mediator connecting the B-series with the A-series, which carries the  $a/\{a\}$ -compatibility. Finally, it can give rise to the situation in which both the B- and A-series are dynamically changed, where the positive aspect of the  $a/\{a\}$ -compatibility can contribute to the perpetual completion of the causal set. In other words, the measurement process can modify the causal set or objective spacetime that is assumed to be invariant. Markopoulou also pointed out that it is possible to consider the dynamical change of the causal set. If it is indeed possible, we do not have to say that the non-trivial causal history results only from the non-trivial causal set. The non-trivial structure of spacetime, whether it is objective or subjective, can result from the perpetual change of spacetime. Although it looks as if our scheme could weaken the objectivity that is destined to be invariant, the objectivity may be subjective-objective in its own right. In this way, a pair of the causal set and the causal history is a subjective-objective/subjective-subjective pair.

A whole structure featuring dynamical duality of objective spacetime (causal set) and the causal history can reveal the abstract brain structure responsible for generating subjective time. If one takes a pair of the causal set and causal history as a pair of subjective-objective and subjective-subjective, the structural change of the causal set can also be a model for subjective time generated in a brain. In [9], [15], the geometrical model for subjective time that is determinant of the past-present and future. This model covers not only normal subjective time but also schizophrenic experiences involving time, such as "ever lasting now", "fragmented now" and "standing still (no present)". Our model can also explain these examples of schizophrenic time, as will be discussed later.

Here, we also examine an example of abnormal experience of time that is not psychosis and concentrate on the déjà vu experience that involves sensing the pre-existence of an event [16], [17]. We also explain why the déjà vu experience is so important when considering subjectivity. In paying attention to subjectivity, one is faced with the issue of free will with which decision making is not determined a priori. This is strongly relevant to the notion of evolution and development, and is consistent with our model in which even objective spacetime can be dynamically changed. One of the difficulties is considering the development of a subject who can accept an object that was

previously unknown. If a fresh object is accepted without hesitation, it means that it was previously known, on the one hand. If there is no token or premise with which to comprehend a fresh object, it seems to be impossible that the subject would be able to accept the fresh object, on the other hand. Thus, it means a fresh object is new thing, for example, at a conscious level, and is an old one at the subconscious level. The sense of the old and new or new pre-existence is, therefore, strongly connected with the evolution and development of consciousness. We think that the déjà vu experience (the simultaneous perception of old and new) necessarily results from this development.

Recently, in [17], Brown reviewed reports and research on déjà vu in an extensive survey of the literature. More than one hundred years ago, déjà vu was defined as “the feeling that the present moment in its completeness has been experienced before” in [19], and recently a similar definition has been proposed that is “a feeling of already having lived through an event that is occurring ostensibly for the first time” [20]. These definitions suggest that retrieval and familiarity are two independent memory functions, and that déjà vu can result from dysfunction in which the two functions are confused [21]. Familiarity is activated in the absence of retrieval, which leads to déjà vu. From a neurological perspective, the delay in perception between the two hemispheres is a candidate for the cause of such dysfunction regarding familiarity. The pre-seizure electrical disturbance in patients suffering from temporal lobe epilepsies can cause delay in the transmission of information through the right hemisphere, which leads to déjà vu [22].

We think, however, that the confusion between retrieval and familiarity is insufficient to explain déjà vu, since the familiarity is not just familiarity but familiarity of the past. Although it sounds as if the retrieval could correspond to the B-series and the familiarity could correspond to the A-series in our framework, the sense of past/present/future including the familiarity of the past holds in the A-series located in the B-series. The A-series itself cannot reveal the sense of past/present/future when independently separated from the B-series, and the A-series itself implies a specific relative order. Two kinds of order slide with each other, and that can give rise to the sense of past-present-future, the existence of the present with  $a/\{a\}$ -compatibility, and the familiarity of the past. Without the order of past-present-future one cannot think about either the familiarity or the familiarity of the past. The familiarity of the past cannot be understood in connection with the existence of the present until it can be explained with the  $a/\{a\}$ -compatibility resulting from dysfunction of sliding of two kinds of order (A- and B-series).

In fact, the  $a/\{a\}$ -compatibility can explain the déjà vu experience. In Fig. 3, we show that the  $a/\{a\}$ -compatibility contributes to the completion of a partially ordered set. Via translation into the A-series (Fig. 3B), the partially ordered set (Fig. 3C) is obtained but it is not closed with respect to join and meet. Due to the  $a/\{a\}$ -compatibility, a fresh element satisfying the requisite,  $I(a \wedge b) = I(s)$ , is generated. Although the element  $s$  is generated satisfying  $a \wedge b = s$ , the concrete element is not determined. In the above section, given a set  $X$ , a lattice is expressed as a subset of the power-set



of  $X$ , first. Although all possible elements of a lattice are expressed as a subset of  $X$ , the fresh element is not chosen as a subset of  $X$  and is independent of  $X$ . For example, the fresh element  $S$  is not expressed as  $\{a, b\}$  even if  $\{a\} \leq S \leq \{a, b, c\}$ , since  $S$  is just formally determined to satisfy the requisites. Functionally  $S$  is the same as  $\{a, b\}$ , but is different from  $\{a, b\}$  with respect to the structure.

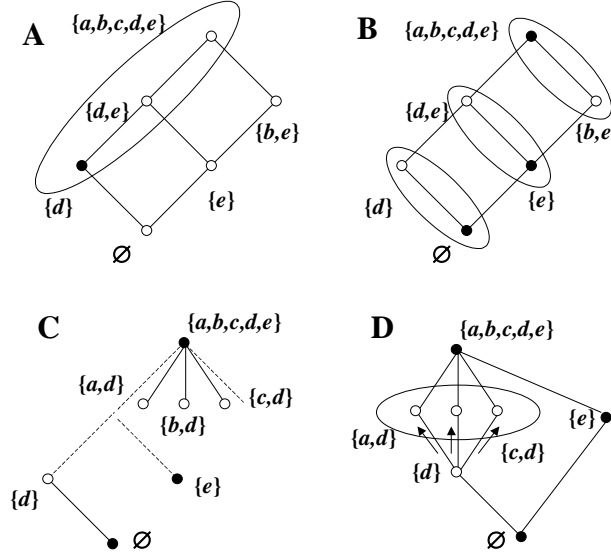


Fig. 4. Dynamical change of spacetime revealing schizophrenic subjective time “fragmented now”. A. B-series including  $\{d\} \leq \{d, e\} \leq \{a, b, c, d, e\}$ , where the present is located at  $\{d\}$ . B. The equivalence classes (each is surrounded by a loop) reduced from the congruence. C. Re-constructed partially ordered set derived from A-series. D. B-series re-constructed as a lattice. Since the next present was assumed to be only one in A, the present is split into three elements  $\{a, d\}$ ,  $\{b, d\}$  and  $\{c, d\}$ .

Here, we consider the implication of  $\{a\} \leq S \leq \{a, b, c\}$  in the context of subjective time. Let  $\{a\} \leq S \leq \{a, b, c\}$  be the B-series, and imagine that  $\{a, b, c\}$  is the present. What is  $S$ ? Since  $S \leq \{a, b, c\}$ , one can know  $S$  is the past but has no name. In other words, while an element expressed as a subset of  $X$  can be known as a particular event,  $S$  cannot be identified with any event. This means that the past has never been the present. In this sense we call  $S$  the lost past. Although the lost past is also used as the tag for the past, one cannot estimate how far it is from the present. Recall that  $\{a\} \leq S \leq \{a, b, c\}$ . If  $\{a, b, c\}$  is the present,  $\{a\}$  is earlier than the lost past, and is far away from the present. Although  $\{a\}$  has been the present, it can be regarded as what “had” been the present. This

is our formal expression for déjà vu. In Fig. 3E, a black element represents the present and the outer loop represents a set of the past. The element indicated by an arrow is the fresh element and the lost past. A set of elements surrounded by the inner loop represents the down set of the lost past. Even if these elements have been the present just now, they are regarded as what had been the present because they are earlier than the lost past. Such a phenomenon is experienced as déjà vu.

Other examples of schizophrenic subjective time can also be explained using our dynamic spacetime. Since both objective spacetime (B-series) and subjective spacetime (A-series) are dynamically changed in the brain, the local geometry including the present can be changed. Imagine that each possible event is expressed as a subset of  $\{a, b, c, d, e\}$ , and the order is an inclusion relation. Imagine a lattice including  $\{d\} \leq \{d, e\} \leq \{a, b, c, d, e\}$  as the B-series, as shown in Fig. 4A, where  $\{d\}$  is the present, and the event  $\{d, e\}$  is expected as the next present and is only one. Since  $\{d\}$  is the present, the future is  $\uparrow(\{d\}) = \{\{d\}, \{d, e\}, \{a, b, c, d, e\}\}$  (surrounded by a loop in Fig. 4A), and the past  $\downarrow(\{d\}) = \{\emptyset, \{d\}\}$  can derive a congruence  $\theta$  by which an equivalence relation  $[z]_\theta = [z]_\theta = \downarrow z - \bigcup_{y \in \uparrow x, y < z} \downarrow y$  can be defined for any  $z$  in  $\uparrow(\{d\})$ . Actually,  $[\{d\}]_\theta = \{\emptyset, \{d\}\}$ ,  $[\{d, e\}]_\theta = \{\{e\}, \{d, e\}\}$ , and  $[\{a, b, c, d, e\}]_\theta = \{\{b, e\}, \{a, b, c, d, e\}\}$ . Assume that each representative of the equivalence class is chosen by  $\emptyset$  from  $[\{d\}]_\theta$ ,  $\{e\}$  from  $[\{d, e\}]_\theta$  and  $\{a, b, c, d, e\}$  from  $[\{a, b, c, d, e\}]_\theta$  as shown as a black element in Fig. 4B. Via the A-series such as  $\{\emptyset, \{e\}, \{a, b, c, d, e\}\}$  (a lattice), a partially ordered set that is not a lattice can be obtained by adapting a specific map  $\Gamma$  (Fig. 4C). Finally elements are ordered over the whole set, and that gives rise to a complete lattice (Fig. 4D).

In Fig. 4A, the present is located at  $\{d\}$ , and the next present is expected as a unique next upper element  $\{d, e\}$ . Although a subject expects the next present that is the location at which the subject sits to be unique, three elements  $\{a, d\}$ ,  $\{b, d\}$  and  $\{c, d\}$  are given since the B-series was reconstructed (Fig. 4D). The subject cannot choose one of events, but he himself is split into three events. That is the three presents and a “fragmented now”.

Fig. 5 shows the dynamical change of the B-series revealing the ever-lasting now, where each element is expressed as a subset of  $\{a, b, c, d\}$ . Given the B-series shown in Fig. 5A, the equivalence classes are obtained (Fig. 5B) by the congruence, where  $\{a\}$  is the present. It is possible to obtain a re-constructed partially ordered set that is not a lattice (Fig. 5C). In this case, the  $a/\{a\}$  compatibility can contribute to completion and the fresh element that is not a subset of  $\{a, b, c, d\}$  is obtained in a similar manner to the lost past. The fresh element is not the past, but is a candidate of the next present. Although the next state was previously the named event that was  $\{a, b, c\}$  as a subset of  $X$  (Fig. 5A), the next state is now unknown and is not an event. Since the present is located at any “event”, a subject cannot be located at the unknown (Fig. 5D). This is why a subject cannot move from “the present” and there is nothing but the ever-lasting now. In our framework, the ever-lasting now is expressed as a standstill state since spacetime is perpetually changed.

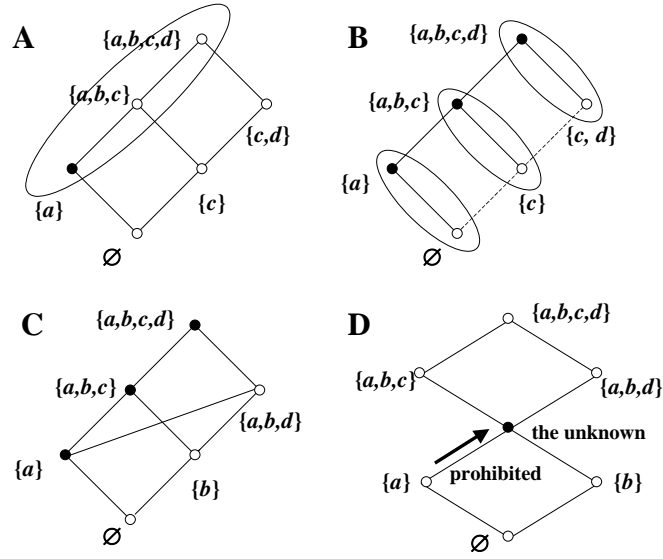


Fig. 5. Dynamical change of spacetime revealing the “ever-lasting now”.  
A. B-series. B. Equivalence classes reduced from the congruence. C.  
Re-constructed partially ordered set that is not a lattice. D.  
Re-constructed lattice containing the unknown event. Thus, a subject  
cannot move to the unknown and continues to stay in the present.

Since both objective and subjective spacetime are perpetually changed, the local geometry of spacetime is modified, which can result in schizophrenic time. Indeed, the  $a/\{a\}$ -compatibility can generate a fresh lost event that cannot be interpreted as a previously existing event. Due to the lost event, there is inaccessibility to a particular past and/or future. This can result in the déjà vu experience and the ever-lasting now.

Recently subjective time is objectively estimated in the field of neuroscience, since Libet [28] It is experimented whether the subjective time really slows down during a frightening event, and it is found that the subjective feeling is not a function of increased time resolution during the event but a function of recollection of memories [30-33]. It suggests that B-series is explicitly recollected and reconstructed during ongoing now, A-series). It is also found that voluntary action reduces the duration between the cause and effect [34,35]. It may be explained by the reconstruction of B-series influenced by active A-series. Once the subject is adapted to the delayed condition with respect to the cause-effect relation, he perceives the effect before perceiving its cause [36-39]. The reversal of cause-effect can be also explained by reconstruction of B-series.

## 6. Conclusion

We mentioned that the causal histories approach used in physics [6] has the same framework of time used in the discourse of philosophy [10], and that physics concentrates on mapping from the causal set to the causal history, which is regarded as ill-defined mapping in philosophy. What is the difference between the two approaches in the same framework? Since the causal history past-present-future (A-series in [10]) moves in the causal set (B-series in [10]), the present was the future and will be the past. If one accepts that the present is indicated in a causal set, the present corresponds to an element of a partially ordered set, but the past (and the future) corresponds to a set. Thus, the moving present reveals the ambiguity of the present as a set and as an element. We call this ambiguity the  $a/\{a\}$ -compatibility, and show that if one neglects the  $a/\{a\}$ -compatibility, then the mapping from a causal set to a causal history is well defined and otherwise the mapping reveals a contradiction. This is the difference between the two approaches.

The next question arises as to whether or not the  $a/\{a\}$ -compatibility hidden in the moving present is an issue to be solved. We think that the  $a/\{a\}$ -compatibility is an essential feature of the moving present, and that the  $a/\{a\}$ -compatibility rather contributes to subjective time or internally described spacetime. Since we generalize the causal history approach using a pair of a lattice (B-series) and a quotient lattice (A-series) derived by the congruence as the measurement, an event in the causal set is mapped to a set of the equivalence class of the congruence. The  $a/\{a\}$ -compatibility is expressed as the ambiguity of the representative of the equivalence class. On the one hand, the representative is also an element of a lattice, and on the other hand, the representative is the name of the equivalence class. Thus, the  $a/\{a\}$ -compatibility is expressed as the operation by which a set of the equivalence class  $[x]_\theta$  can be re-constructed from the representative  $x$ . Since it is impossible to complete such an operation, the lattice (B-series) is re-constructed with modification. Although re-constructing the B-series sometimes fails to complete a partially ordered set, the  $a/\{a\}$ -compatibility also contributes to the completion by generating a fresh element. Thus, a pair of the lattice (causal set; B-series) and the corresponding quotient lattice (causal history; A-series) is destined to be perpetually changed, and this reveals the internally described subjective spacetime.

Due to the perpetual changing of a causal set, the local geometry of the events is changed, and this can give rise to conspicuous subjective time. In particular, since the completion of a partially ordered set results from the  $a/\{a\}$ -compatibility, a newly generated element in our framework has neither address nor name in the causal set (i.e., B-series). This means that there exists a particular element which can prohibit the present from approaching the next event and which can result in the earlier event being labeled as the lost-past. Thus, this can explain a schizophrenic time experience and the déjà vu experience. In cognitive science, abnormal perception of time such as déjà vu has been explained by the dysfunction of confusing retrieval and familiarity. The sense of

familiarity is, however, dependent on the operation of comparison. The degree of success of the negotiation between two things can give rise to the sense of familiarity, where the two things are nothing but the A- and B-series. Since both causal set and causal history are perpetually changed, spacetime in our framework can be regarded as subjective-objective spacetime and the causal history is labeled as the subjective-subjective matter. Thus, the dynamical duality of the A- and B-series can be regarded as a model for the way in which the brain perceives time.

The causal histories approach is strongly relevant to quantum mechanics. An anti-chain in a partially ordered set is called an acausal set that is mapped to a tensor product of Hilbert space. The question arises as to whether a composite system expressed as a tensor product of Hilbert space can reveal time development in a consistent manner. This is replaced by the question, what is the mechanism that keeps each pair of acausal sets a complete pair? That is a future problem, but it will also be re-constructed in a framework in a pair of a lattice and a quotient lattice.

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