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## **ACOUSTICAL LETTER**

# Reduction of sound radiation by using extended radiation modes: Effects of added mass

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1. Introduction

The location of a vibration source within a machine is sometimes found to have a significant effect upon its radiated acoustic power. In practice, treatments based on vibration modal analysis are often applied because that approach is simple. However, it is known that the simple reduction of vibration cannot always reduce the radiated acoustic power [1,2] and that the sound radiation characteristics of a structure must be taken into account.

Radiation mode analysis has been developing since the early 1990s mainly in the field of Active Noise Control [3,4]. It is known to be a powerful tool for interpreting sound radiation since those modes are only dependent on geometrical information and are independent of a structure's surface vibration. Recently, radiation mode analysis has been applied to practical subjects [5,6].

Previously, it was suggested that the radiation mode concept can be extended to understand the relationship between the acoustic power and the driving force distribution. It was demonstrated that the radiated acoustic power can be reduced by moving the driving force location to the nodes of the extended radiation modes (force radiation modes) [7].

In order to reduce sound radiation, the addition of point masses to a structure has been studied [8,9]. In this paper, the modification of force radiation modes by attaching a mass to a structure is studied with the intention of moving a node to the driving force location and hence minimizing sound radiation.

#### 2. Force radiation modes ( $f_{rad}$ -modes)

The acoustic power can be calculated as the product of the radiation resistance matrix, R, and the vibration velocity on the boundary,  $v_e$ : i.e.,

$$W = \mathbf{v}_{\rm o}^{\rm H} \mathbf{R} \mathbf{v}_{\rm e}. \tag{1}$$

The eigenvectors of the radiation resistance matrix,  $\mathbf{R}$ , are the radiation modes.

In addition, the vibration velocities on the boundary can be expressed as the product of the structure's mobility matrix, T, and the driving force distribution,  $f_{\rm e}$ : i.e.,

$$v_{\rm e} = T f_{\rm e}. \tag{2}$$

By substituting Eq. (2) into Eq. (1), the driving forces can be made independent of both the sound field and the structure: i.e.,

$$W = f_{e}^{H} T^{H} R T f_{e} = f_{e}^{H} C f_{e}$$
(3)

where C is real, symmetric and positive definite. As a result of the latter properties, C possesses an eigenvalue/eigenvector decomposition that can be written as:

$$\boldsymbol{C} = \boldsymbol{M}^{\mathrm{T}} \boldsymbol{\Phi} \boldsymbol{M} \tag{4}$$

where M is the matrix of orthogonal eigenvectors which we define to be force radiation modes ( $f_{\text{rad}}$ -modes). Also,  $\Phi$  is a diagonal matrix whose elements,  $\phi_n$ , are the eigenvalues of C. Since the matrix C is positive definite, its eigenvalues are all positive and real. Thus the acoustic power can be written as:

$$W = f_{e}^{H} M^{T} \boldsymbol{\Phi} M f_{e} = d^{H} \boldsymbol{\Phi} d$$

$$= \sum_{n=1}^{N} W_{n} = \sum_{n=1}^{N} \phi_{n} |d_{n}|^{2}$$
(5)

where  $d = Mf_{\rm e}$  is the matrix of products of the  $f_{\rm rad}$ -modes and the driving force distribution. The total radiated acoustic power when expressed in this way is therefore composed of uncoupled modal radiation powers proportional to the eigenvalues,  $\phi_n$ . Therefore, when the driving force is located at the node of a  $f_{\rm rad}$ -mode, the sound power radiated by that mode is minimized.

#### 3. Numerical simulation

#### 3.1. Calculation model

Figure 1 shows the simply-supported baffled beam used to study the effect of an attached mass on the  $f_{\rm rad}$ -modes. The beam was assumed to be made of steel. The thickness was

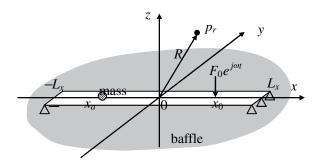


Fig. 1 Calculation model.

1 mm, the width was 20 mm, the length was  $300 \text{ mm} (= 2L_x)$  and the loss factor was 0.008.

#### 3.2. Calculation method

The beam, with attached mass at the point  $x_a$ , was driven at the point  $x_0$  by an external force of amplitude  $F_0$  and angular velocity,  $\omega$ . The beam response can be calculated as a superposition of the vibration velocity caused by the external force  $F_0$  and the inertial force of the mass  $F_a'$ . Thus the vibration velocity of the beam at  $x_b$  with an attached mass can be written as:

$$U_{b} = T_{0b} \cdot F_{0} + T_{ab} \cdot F'_{a}$$

$$= \left(T_{0b} - \frac{j\omega m_{a} T_{ab} T_{0a}}{1 + j\omega m_{a} T_{aa} T_{ab}}\right) \cdot F_{0}$$
(6)

where  $T_{\rm ab}$  is the transfer mobility between the points  $x_{\rm a}$  and  $x_{\rm b}$ ,  $T_{\rm 0b}$  is the mobility between the drive point,  $x_{\rm 0}$ , and the response point,  $x_{\rm b}$ ,  $T_{\rm aa}$  is the driving point mobility at  $x_{\rm a}$ , and  $m_{\rm a}$  is the mass of the attached mass. The mobility of the beam without the mass can be calculated as for an Euler–Bernoulli beam.

The far field sound pressure was calculated by means of a Rayleigh integral. The radius of the hemispherical recovery surfaces was chosen to be 10 m. The sound pressure was calculated at every 15 degrees of both elevation and horizontal angle. The total number of observation points was 64. The beam itself was segmented into 59 equal areas.

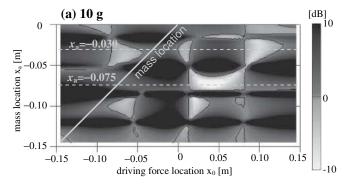
#### 4. Result and discussion

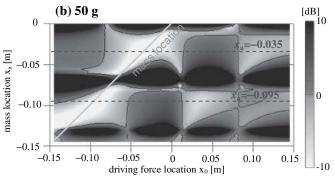
The radiated acoustic power was calculated when the beam was driven at a single point at 500 Hz, which is not a resonant frequency of the beam without the mass. A single mass is imagined to be placed on the beam.

Figure 2 shows the variation of the acoustic power radiated by the beam when the mass was attached to the beam. The horizontal axes of the figures show the location of the driving force and the vertical axes show the location of the mass. Figure 3 shows the first  $f_{\rm rad}$ -modes with/without a mass. In Figs. 2 and 3, (a) is the result of adding a 10 g mass and (b) is the result of adding a 50 g mass. The 10 g mass is 21% of the beam weight. The 50 g mass is 107% of the beam weight.

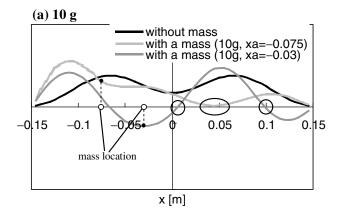
The dark areas in Fig. 2 show the regions in which the acoustic power is increased by attaching a mass. On the other hand, in the bright areas, the acoustic power is reduced by the attached mass.

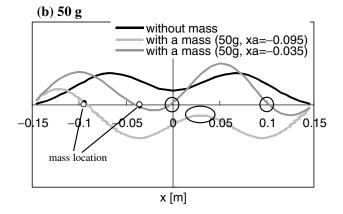
As shown in Fig. 2, some locations of the mass make the beam resonant at 500 Hz and the acoustic power consequently





**Fig. 2** Variation of the acoustic power of the beam at 500 Hz when the mass is attached: (a) 10 g, (b) 50 g.





**Fig. 3** The first  $f_{\text{rad}}$ -modes at 500 Hz with/without a mass: (a) 10 g, (b) 50 g.

increases: i.e., when the mass is located at  $x_a = -0.12$ , -0.085, -0.05, or -0.015, in Fig. 2(a), and at  $x_a = -0.13$ , -0.07 or 0 in Fig. 2(b).

As shown Figs. 2(a) and 3(a), when the 10 g mass was attached at  $x_a = -0.075$  or -0.03, the shape of the first  $f_{\rm rad}$  mode was changed and its nodes moved to x = 0.04 or x = 0, 0.01. Therefore, when the beam was driven at  $x_0 = 0$ , 0.04 or 0.1 the acoustic power was reduced by the addition of the mass.

As shown Figs. 2(b) and 3(b), when the 50 g mass was attached near the driving point, the acoustic power was significantly reduced since the location of the large mass becomes the node of the  $f_{\rm rad}$ -mode. The 50 g mass can also reduce the acoustic power even when the mass is attached some distance away from the driving point: e.g., at  $x_a = -0.095$  or -0.035. As to location of the driving point and the mass, the acoustic power is reduced when the driving point is between the nodes of the resonance which is caused by the attached mass and the mass should be attached to the beam where the mass does not excite the resonance. The result of adding a mass greater than 50 g is almost the same as that resulting from the adding the 50 g mass.

#### 5. Conclusion

In this paper, it was found that the radiated acoustic power can be reduced by attaching a mass located some distance from the driving point in order to shift a node of a  $f_{\rm rad}$ -mode close to the drive point.

It was also found that the mode shape indicates the mass location and driving point where the acoustic power is reduced by the mass. This method will be applied to more practical issues in future work.

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