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On the relationship between the normal incidence airborne sound-excited and the structurally-excited sound radiation from a wall: a theoretical trial with simplified models.

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Abstract

Sound transmission and sound radiation by force excitation of a plate are both basic and important problems in acoustics. Both the sound transmission and force excited sound radiation are basically the same phenomena, i.e., sound radiation from a vibrating plate, and the difference is the type of the excitation force, i.e., surrounding sound field or mechanical force. However, their relationship has not been explicitly discussed. If the relationship is known, it should be useful for comprehensive understanding of those phenomena, and to gain insight into relevant problems. In this study, simplified theoretical analyses to find a possibility to obtain a relationship between the sound transmission and force-excited sound radiation of a single-leaf plate of infinite extent are carried out with simplified models. As a result, it is suggested that there is a conversion factor which relates the sound transmission and force-excited sound radiation, and its behavior is discussed through numerical examples. Even though the present analyses are approximated and simple, this result suggests that there is a possibility, with further sophisticated studies, for practical applications.

1. Introduction

Sound transmission of a wall, which is a sound radiation from the sound-induced vibration of a plate, is one of the important problem and has long been discussed by many researchers. The most important problem with sound transmission in actual engineering situations is the sound insulation in buildings or other constructions. Its performance is a basic requirement for all kinds of buildings to realize a comfortable environment. Particularly in recent years, considering the increasing collective housings and overcrowding urban areas, offering better sound insulation performance is of paramount importance. Sound insulation has long been studied by many researchers as the sound transmission of a wall, which is one of the important and fundamental problems in architectural acoustics, therefore, there have been considerable knowledge published so far [1, 2], and recently as the applications of those studies, various new type wall structures have been proposed and studied [3].

The sound insulation performance of a wall is usually evaluated by its sound reduction index (transmission loss), which is the reciprocal of the sound transmission coefficient represented in dB scale. This is in general measured in an experimental facility, i.e., coupled reverberation chambers. However, in an actual building, the sound insulation performance of a wall is governed by many other factors and quite complex problem [1]. Therefore, it is usually measured as an averaged sound pressure level difference between rooms [4]. Also recently

available is the measurement in-situ by the intensity technique [5], which is useful to understand complex phenomena in actual buildings.

On the other hand, the sound radiation by structurally-excited wall is also a fundamental problem in acoustics and has long been studied intensively: As the most fundamental problem, an elastic plate with point force excitation in various cases are studied in building acoustics and underwater acoustics [6,7].

Sound transmission and force-excited sound radiation a wall are both fundamental and important problem in acoustics. Although they are both a sound radiation from a wall – only difference is the type of the excitation force: a sound wave for the sound transmission and a mechanical force for the sound radiation. To the authors' knowledge, there are only a few attempt to discuss their relationship: e.g., Ver derived the approximated formula of the relationship between the floor impact noise level and the floor's sound transmission loss by statistical energy method [8]. However, the same formula is only valid for a building floor and does not have generality, i.e., it does not discuss the general relationship between sound- and structurally-excited radiations of general wall.

Thus, considering the above background, in order to find a possibility to obtain a general relationship between the sound transmission and the force-excited sound radiation, as a basic trial, an analytical study is made with simplified models. As introduced above, both the sound transmission and radiation in actual engineering situation are quite complicated problems, in this study we start from the most simplified case: a single-leaf elastic plate of infinite extent, with normal sound incidence for sound transmission and point force excitation for structurally-excited radiation. By this simplification, in the present study we expect to find the relationship in a simple and explicit closed form through approximation analyses. As explained later in detail, we assume that the sound transmission and the sound radiation are related in a linear equation with a conversion factor. The target of the present work is to find the possibility of existence of such a conversion factor and linear relationship. This study is a trial and the first step. Therefore, to the first approximation, we use the existing simplified models. The numerical examples of the results are given and discussions are made with them, through which the possibility is also discussed.

2. Theoretical considerations

2.1 Assumptions in the analyses

In order to obtain the relationship between the sound transmission and the force-excited sound radiation of a wall, to the first approximation, the following simplification is assumed in the analyses. For the sound transmission, the classical mass-law model is employed for the sake of simplicity. For the force-excited sound radiation, two models are employed: One is the simple piston vibration model. The other is the model with a point force excited elastic plate of infinite extent. These models are fundamental and quite simple, but, as the purpose of this study is to search a possibility of obtain the relationship between the sound transmission and the force-excited sound radiation, it is assumed to be useful for this purpose. Regarding the two models for the force-excited sound radiation analyses, two cases – with and without the acoustic loading – are considered.

Considering that the reduction index (sound transmission loss) is based on the ratio of the input sound power P_i and output (transmitted) sound power P_r behind the wall, it is necessary to establish the relationship between the sound transmission and the force-excited sound radiation that a similar power ratio should be obtained for the force-excited sound radiation.

The input and output power ratio for the sound transmission is the ratio of incident power to the transmitted power, which is the transmission coefficient τ , and here it is defined as P_t .

As for the force-excited sound radiation, the ratio of the input power P_i and output power, i.e., radiated sound power P_{rad} , is defined as $P_{wr} = P_i / P_{rad}$. In order to obtain P_{wr} , it is necessary to obtain “input power to the wall”, P_i . Therefore, later in the analysis, the excitation force is converted into the input power and it is used to obtain P_{wr} .

In this study, it is assumed that there is a relationship between P_t and P_{wr} , such as $P_t = \varphi P_{wr}$ with a conversion factor φ , which is to obtain in this theoretical consideration.

2.2 The input and output power ratio for the sound transmission, P_t

As the present purpose is to obtain a simple and explicit form, we start the analyses by using the classical approximated simple solutions. According to the classical mass-law [9], as mentioned above, the ratio $P_t = P_r / P_i$ is obtained as:

$$P_t = \tau = \frac{1}{1 + \left(\frac{\omega m}{2\rho_0 c_0} \right)^2}, \quad (1)$$

where ω is the angular frequency, m is the surface density of the wall, and $\rho_0 c_0$ is the characteristic impedance of the air. Though this is basically restricted for the normal incidence of a plane wave, it is well-known that this solution can be useful to give an insight into other cases. Therefore, to the first approximation, this equation is employed in the followings.

2.3 The input and output power ratio for the force-excited sound radiation, P_{wr}

For the force-excited sound radiation, first of all, it is needed to obtain the input power P_i . In this procedure, the excitation force is converted to the input power by considering the following converting formula:

$$P_i = \frac{1}{2} \frac{|F|^2}{8\sqrt{Dm}} = \frac{1}{2} \frac{|F|^2}{Z_p}, \quad (2)$$

where $D = Eh^3/[12(1-\nu)^2]$ is the flexural rigidity of the planar elastic plate (with E the Young's modulus, h the thickness and ν the Poisson's ratio), and $Z_p = 8(Dm)^{1/2}$ is its excitation point impedance [10]. This formula for converting the excitation force into the input power can be obtained by analogy with the effective power consumed in an electric circuit.

Before going into the input and output power ratio, the derivation of the radiated sound power from a force-excited wall is briefly introduced below.

For a piston plate of infinite extent under a vibration caused by a force (Fig. 1 (a)), from the equation of motion of the plate (including the sound pressures on the surfaces):

$$F + p_1 - p_2 = m \frac{dv}{dt} = -i\omega m v, \quad (3)$$

with F the force per unit area, v the vibrating velocity and $p_{1,2}$ are the surface pressures, and the equation of continuity at the surfaces of the plate, the radiated sound power P_{rad} is easily obtained as follows:

$$P_{rad} = \frac{1}{2} \rho_0 c_0 \frac{|F|^2}{(2\rho_0 c_0)^2 + (m\omega)^2}. \quad (4)$$

When the acoustic loading (namely, $p_{1,2}$ in the equation of the motion) is neglected, $(2\rho_0 c_0)^2$ is removed from the denominator.

For a point-force excited elastic plate of infinite extent (Fig. 1 (b)), the derivation is somewhat more complicated, but employing the standard procedure presented in various reference books, e.g., [6,7], it is not very difficult. The equation of motion (including the sound pressures on the surfaces):

$$D \frac{d^4 w(r)}{dr^4} - m c_0^2 w(r) = p_1 - p_2 + \frac{F}{2\pi r}, \quad (5)$$

with the $w(r)$ the vibration displacement. This should be solved with coupling with wave equation of the sound field. Using a Hankel transform, this becomes a simple algebraic form such as follows:

$$\left[\frac{2\rho_0 \omega^2}{\sqrt{k^2 - k_0^2}} (Dk^4 - m\omega^2) \right] W(k) = \frac{F}{2\pi}. \quad (6)$$

Solving this to obtain the transformed displacement in the wavenumber space, the radiated sound power is given by the following [11,12]:

$$\Pi(\omega) = \pi \rho_0 \omega^3 \int_0^{k_0} \frac{|W(k)|^2}{\sqrt{k_0^2 - k^2}} k dk. \quad (7)$$

This leads, with some approximations, to a closed form solution of the radiated pressure, and with applying $\omega c^2 = m c_0^4 / D$ and assuming $\omega_c \gg \omega$, the following simplified formula for the radiated sound pressure is obtained:

$$p(\theta) \cong \frac{2i\rho_0 \omega^2 F}{\frac{2\rho_0 c_0 \omega}{\cos \theta} - im\omega^2} \frac{e^{ik_0 r}}{4\pi r}, \quad (8)$$

Integrating the radial intensity, $|p(r, \theta)|^2 / 2\rho_0 c_0$, over a hemisphere of radius r , we can finally obtain an approximated solution of the radiated sound power as follows:

$$P_{rad} = \frac{1}{4\pi \rho_0 c_0} \frac{|F|^2}{\left(\frac{m}{\rho_0}\right)^2} \left(1 - \frac{\tan^{-1} \frac{m\omega}{2\rho_0 c_0}}{\frac{m\omega}{2\rho_0 c_0}} \right). \quad (9)$$

When the acoustic loading is neglected, the third term of the above equation should be eliminated. As known this approach is the most simplified one and restricted to a simple cases, it reflects the fundamentals of the phenomenon and useful to gain insight into the present problem in this study.

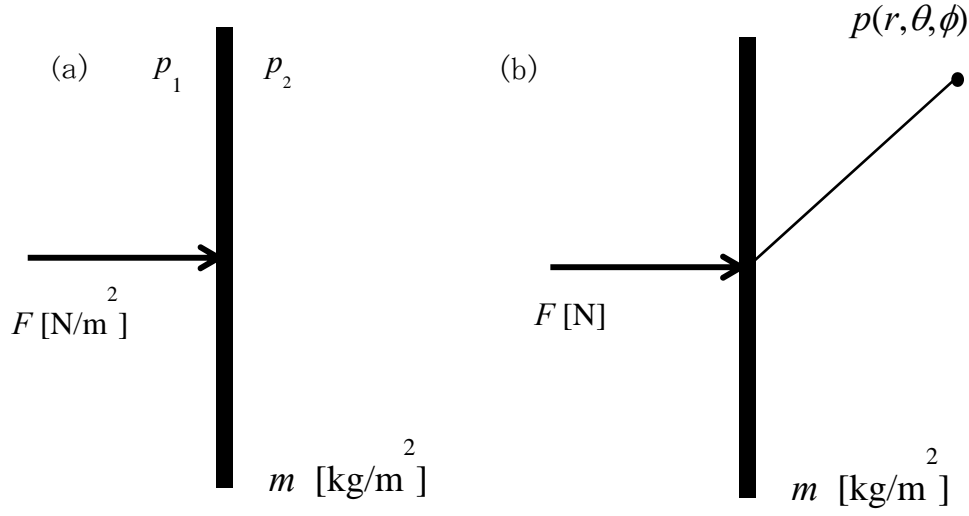


Fig. 1 Models considered in this study. (a) a piston (of infinite extent) model with excitation force, (b) an elastic plate (of infinite extent) with a point force excitation.

By using the above equations, the input and output power ratio $P_{wr} = P_i / P_{rad}$ is now obtained in each model as follows:

Model A: Piston model without the acoustic loading:

$$P_{wr} = \frac{\omega m}{\rho_0 c_0} \quad (10)$$

Model B: Piston model with the acoustic loading:

$$P_{wr} = 2 \sqrt{1 + \left(\frac{\omega m}{2 \rho_0 c_0} \right)^2} \quad (11)$$

Model C: Point-force excited infinite elastic plate without acoustic loading:

$$P_{wr} = \frac{2 \pi \rho_0 c_0}{Z_p} \left(\frac{m}{\rho_0} \right)^2 \quad (12)$$

Model D: Point-force excited infinite elastic plate with acoustic loading:

$$P_{wr} = \frac{2 \pi \rho_0 c_0}{Z_p} \left(\frac{m}{\rho_0} \right)^2 \frac{1}{\left(\tan^{-1} \frac{\omega m}{2 \rho_0 c_0} \right) \left(1 - \frac{\omega m}{2 \rho_0 c_0} \right)} \quad (13)$$

Note that in the models C and D, the approximation for the frequencies lower than the coincidence frequency is used as the basis of this derivation [6].

2.4 The conversion factor, φ

Substituting the above P_t and P_{wr} into $P_t = \varphi P_{wr}$, the conversion factor φ is obtained for each case as follows:

Model A: Piston model without the acoustic loading:

$$\varphi = \frac{\omega m}{4\rho_0 c_0} \quad (14)$$

Model B: Piston model with the acoustic loading:

$$\varphi = \frac{\omega^2 m^2}{2(4\rho_0^2 c_0^2 + \omega^2 m^2)} \sqrt{1 + \left(\frac{\omega m}{2\rho_0 c_0} \right)^2} \quad (15)$$

Model C: Point-force excited infinite elastic plate without acoustic loading:

$$\varphi = \frac{Z_p \omega^2}{8\pi\rho_0 c_0^3} \quad (16)$$

Model D: Point-force excited infinite elastic plate with acoustic loading:

$$\varphi = \frac{Z_p \omega^2}{8\pi\rho_0 c_0^3} \left(1 - \frac{\tan^{-1} \frac{\omega m}{2\rho_0 c_0}}{\frac{\omega m}{2\rho_0 c_0}} \right) \quad (17)$$

In the following, both P_{wr} and φ are shown with taking $10 \log P_{wr}$ and $10 \log \varphi$ in dB. These results suggest a possibility of existence of a general relationship between sound- and structurally-excited radiation from a plate.

3. Numerical examples and discussions

3.1 Numerical examples in the case of common building materials assumed

Figure 2 shows the results of P_{wr} for each model, A...D, in the case that a gypsum board (Density 600 kg/m³, thickness 0.012 m, surface density 7.2 kg/m², Young's modulus 6.18*10⁹ N/m², and Poisson's ratio 0.03) is assumed. It is observed from this figure that the values obtained from the piston models (A and B) are increasing proportionally with the frequency, whereas those obtained from the point-force excited infinite elastic plate models (C and D) take almost constant value in all frequency ranges. The effect of the acoustic loading appears at low frequencies in both models, though it is more significant in the elastic plate model.

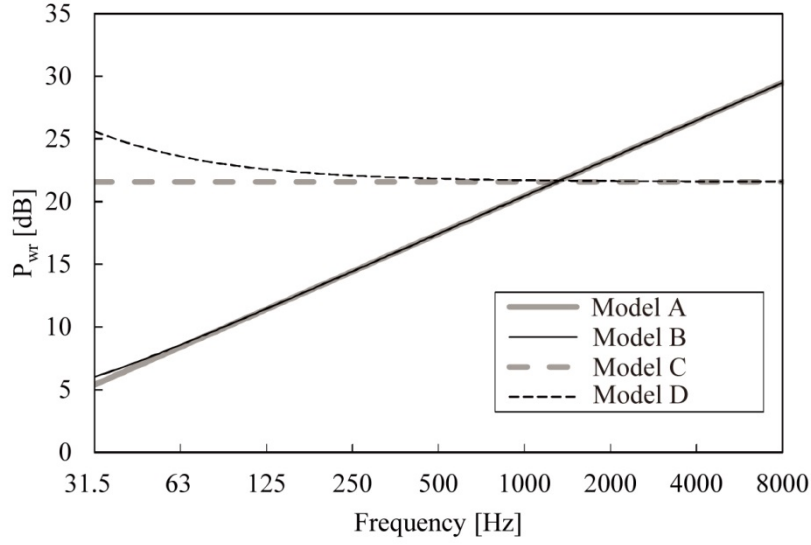


Fig. 2 Calculated results of the input and output power ratio P_{wr} , for each model, A...D, in the case that a gypsum board is assumed. The properties of the wall are set to typical values of a gypsum board (see text).

Figure 3 shows the results of the conversion factor φ for each model, A...D, in the case that a gypsum board is assumed. In both the piston and elastic infinite plate model, the conversion factor φ increases proportionally with the frequency, but the slope of the graph is different: the piston models (A and B) give steeper slope. This difference is attributed to the dependence of the radiated power on the frequency of the models. Again, the effect of the acoustic loading appears at low frequencies and it is more significant in the elastic infinite plate model (Model D).

Considering the fact that the piston vibration of a panel is less realistic, the point-force excited elastic infinite plate model can be rather closer to the actual phenomena in real situations. Therefore, it would be more realistic to use the elastic infinite plate model and better used for discussions.

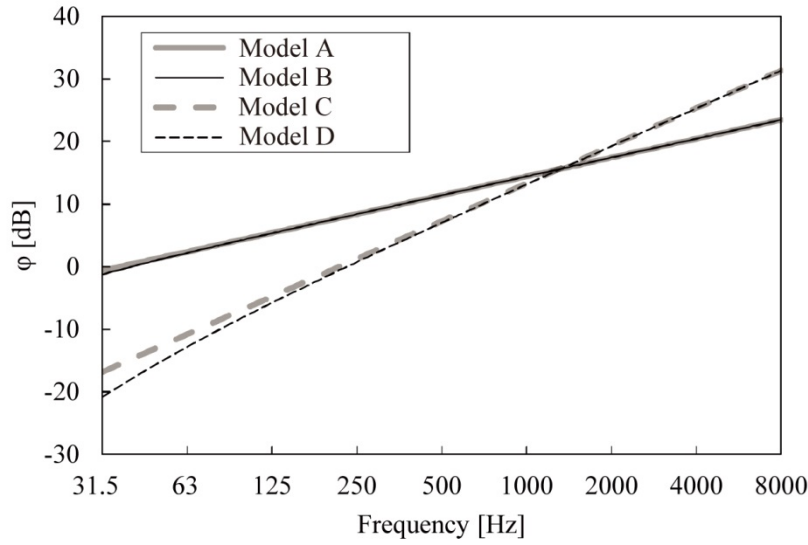


Fig. 3 Calculated results of the input and output power ratio ϕ , for each model, (a)...(d), in the case that a gypsum board is assumed. The properties of the wall are set to typical values of a gypsum board (see text).

Figure 4 shows the results of P_{wr} for each model, A...D, in the case that a concrete wall (Density 2300 kg/m^3 , thickness 0.15 m , surface density 345 kg/m^2 , Young's modulus $20 \cdot 10^9 \text{ N/m}^2$, and Poisson's ratio 0.167) is assumed. In this case, as in the case of a gypsum board, a similar tendency is observed: the values increase proportionally with the frequency in the piston models (A and B), but are almost constant in the point-force excited elastic infinite plate models (C and D). However, the effect of the acoustic loading is very small in this case for all models. This is because the concrete wall is quite heavy and is little affected by the radiated pressure, so that the acoustic loading is not significant.

Figure 5 shows the results of ϕ for each model, A...D, in the case that a concrete wall is assumed. Again, in this case, both the piston and the elastic infinite plate models show an increasing proportional to the frequency. Also the effect of the acoustic loading in this case is almost negligible. A similar discussion can be obtained as in the case of a gypsum board.

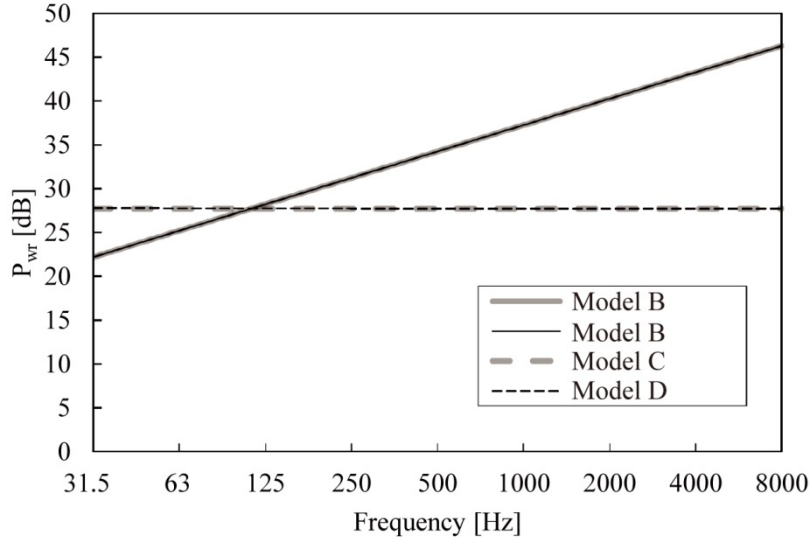


Fig. 4 Calculated results of the input and output power ratio P_{wr} , for each model, A...D, in the case that a concrete wall is assumed. The properties of the wall are set to typical values of a gypsum board (see text). Note that Models A and B, C and D are overlapped each other.

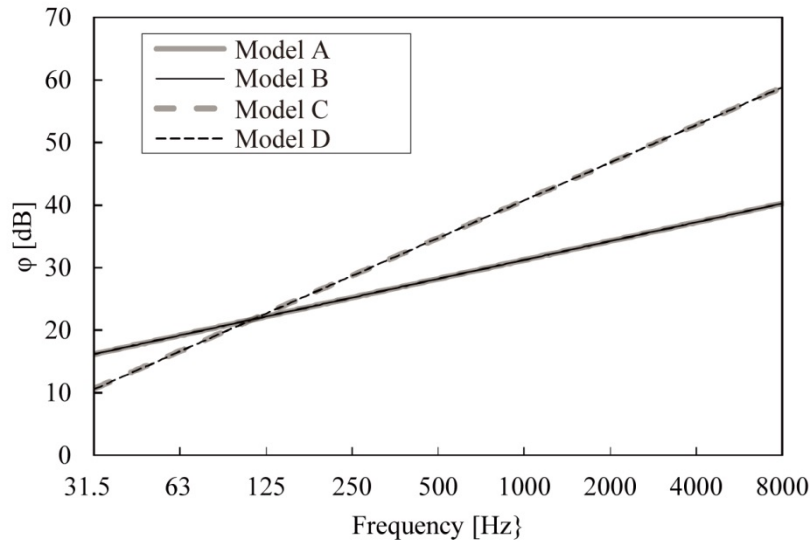


Fig. 5 Calculated results of the input and output power ratio ϕ , for each model, A...D, in the case that a concrete wall is assumed. The properties of the wall are set to typical values of a gypsum board (see text). Note that Models A and B, C and D are overlapped each other.

3.2 Effect of parameters of the wall

In this section, the effect of the wall parameters on the input and output powers P_{wr} and the conversion factor ϕ are discussed with the numerical results. Here, the following values are used as the reference values for the wall parameters: density 1000 kg/m^3 , Young's modulus $10 \cdot 10^{10} \text{ N/m}^2$, Poisson's ratio 0.2, and first the effect of the wall thickness h is discussed.

Figures 6 and 7 shows the results of P_{wr} and ϕ , respectively, calculated by the piston model with the acoustic loading (Model B). Both P_{wr} and ϕ change with the thickness, but they both increase and are proportional to the frequency. The shape of the curve does not change even if

the thickness changes: they only shift in parallel by the change in thickness. Both increase by 3 dB with doubling the thickness.

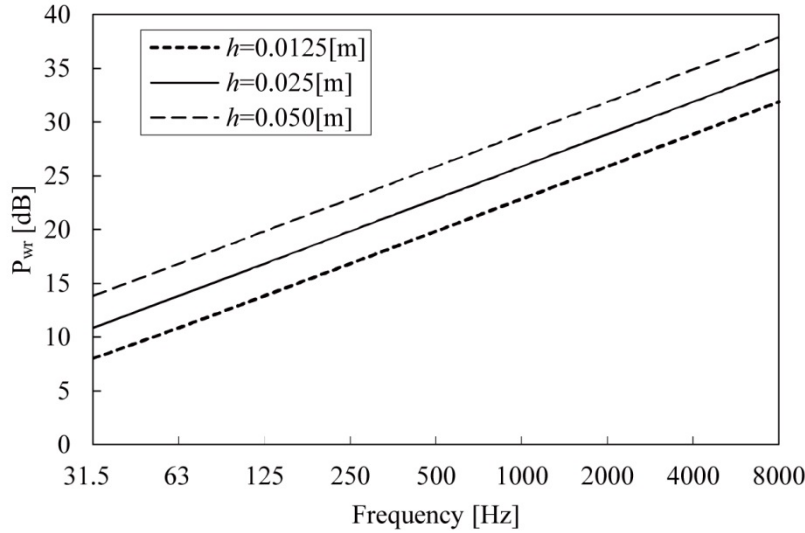


Fig. 6 Effect of the wall thickness on P_{wr} calculated by the piston model with the acoustic loading (Model B).

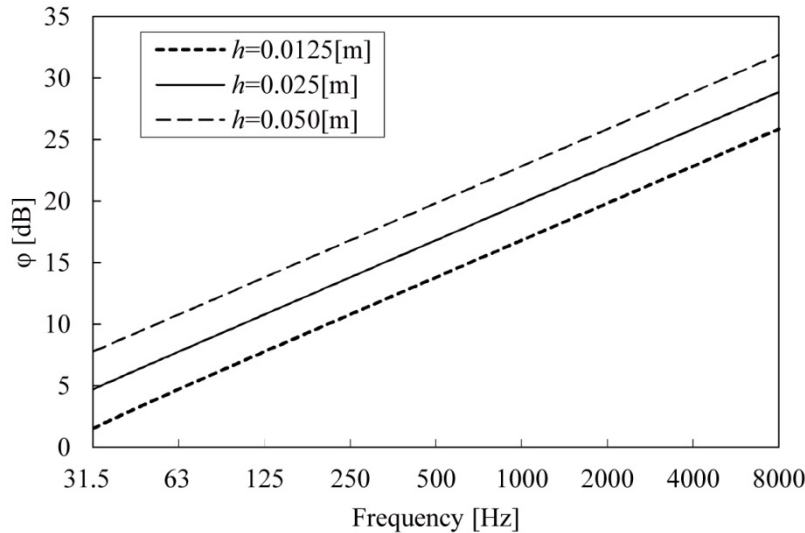


Fig. 7 Effect of the wall thickness on ϕ calculated by the piston model with the acoustic loading (Model B).

Figures 8 and 9 shows the results of P_{wr} and ϕ , respectively, calculated by the point-force excited elastic infinite plate model with the acoustic loading (Model D). The value of P_{wr} does not change and shows almost constant value throughout all frequency ranges, but a small effect of the acoustic loading is observed at low frequencies. This is because, in this model, the thickness is included only in the acoustic loading term. On the contrary, the value of ϕ change with the thickness, but it increases and is proportional to the frequency. This tendency does not change even if the thickness changes, therefore, the curve moves in parallel with the thickness. Both increase by 6 dB with doubling the thickness.

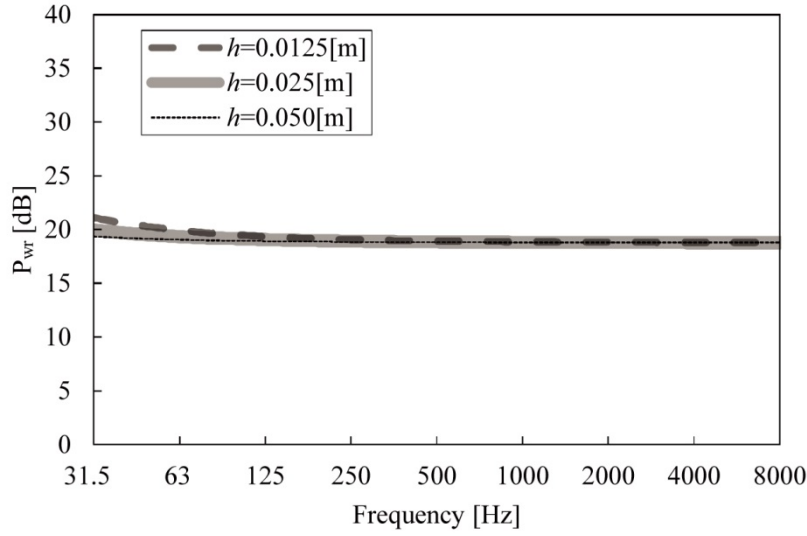


Fig. 8 Effect of the wall thickness on P_{wr} calculated by the elastic infinite plate model with the acoustic loading (Model D).

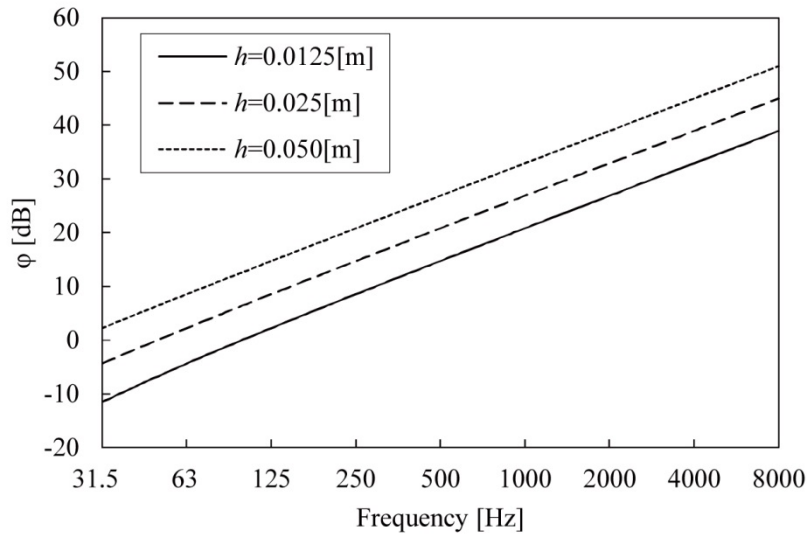


Fig. 9 Effect of the wall thickness on ϕ calculated by the elastic infinite plate model with the acoustic loading (Model D).

Next, in the case of another set of the wall parameters (density 1000 kg/m^3 , thickness 0.025 m , surface density 25 kg/m^2 , and Poisson's ratio 0.2), the effect of Young's modulus is discussed. As the piston models do not include Young's modulus, the discussion here is only made through the results by the point-force excited elastic infinite plate model.

Figures 10 and 11 shows the results of P_{wr} and ϕ , respectively, calculated by the point-force excited elastic infinite plate model with the acoustic loading (Model D). It is observed from Fig. 10 that P_{wr} shifts in parallel but the curve shape does not change with Young's modulus. When Young's modulus becomes 10 times larger, P_{wr} increases about 5 dB. On the contrary, from Fig. 11, it is observed that the curve of ϕ does not change with Young's modulus and shifts in parallel. Similar to P_{wr} , ϕ increases 5 dB when Young's modulus becomes 10 times larger.

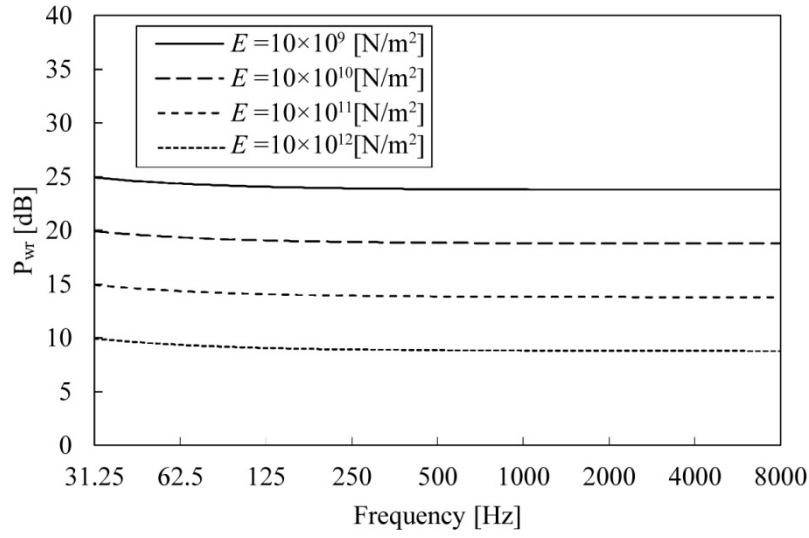


Fig. 10 Effect of Young's modulus on P_{wr} calculated by the elastic infinite plate model with the acoustic loading (Model D).

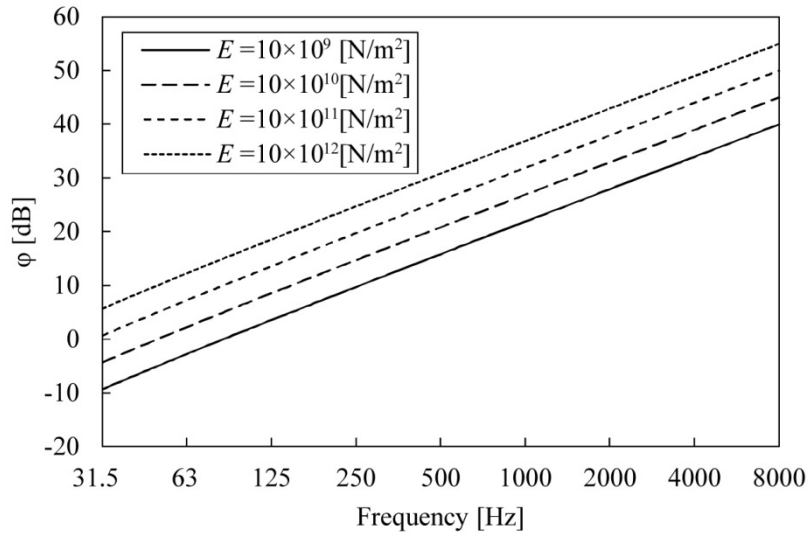


Fig. 11 Effect of Young's modulus on φ calculated by the elastic infinite plate model with the acoustic loading (Model D).

4. Concluding remarks

In this study, the relationship between the sound transmission and the force-excited sound radiation of a wall is studied theoretically with simplified theoretical models as a trial. The analyses suggest that it can be found that there is a possibility to obtain a conversion factor φ to relate them. Even though the models employed here are very much simplified ones, piston vibration and point-force excited elastic infinite plate, this result suggests that there is a possibility to find a more general relationship between the sound transmission and the force-

excited sound radiation. In order to find it more sophisticated theoretical analyses, as well as, experimental studies will be needed, which is the next step of our study.

The conversion factor obtained in this study is dependent on the wall properties. This is because it includes the point impedance of a wall Z_p , which includes the physical properties of the wall. However, it can be possible to remove Z_p from the parameter φ , and then it is expected that a more simple and general relation can be obtained. For applying this idea to practical situations, there are many complex problems which should be tackled. These are the target of the future studies.

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