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# Returns policy, new model introduction, and consumer welfare

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## **Abstract**

This paper investigates how economic outcomes differ for new model introduction between two extreme contracts: outright sales contract and returns policy contract with full credit. Under the outright sales contract, we show that the retailer's incentive to introduce a new model can lead to a reduction of consumer welfare when the retailer is moderately risk averse. The primary conclusion from this study is that the returns policy contract resolves this conflict between firms and consumers, and that the supply chain introduces the new model only when consumer surplus from the new model is higher than that from the old model.

*Keywords:* Economics; Returns policy; Supply chain; Consumer welfare

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## 1. Introduction

In many advanced economies, retail concentration is growing, and market power has accordingly been accumulated in dominant retailers in various product categories.<sup>1</sup> Against the background of recent structural changes in distribution channels and supply chains, this paper aims to investigate how the economic outcomes of new model introduction in a product category differ between two polar transaction contracts: the outright sales contract and the returns policy contract with full credit. In light of the ongoing power shift from manufacturers to retailers, we specifically consider a two-echelon supply chain consisting of a monopolistic manufacturer and a monopolistic retailer in which the retailer plays the role of channel leader to determine whether to introduce a new model into the retail market or to continue to sell the old model. The primary conclusion from this study is that the returns policy contract resolves the potential conflict of interest between the supply chain and consumers regarding the introduction of a new model. Under the outright sales contract, we will show that the retailer's incentive to introduce a new model, which can derive greater expected demand than an old model, can be at the expense of consumer welfare when the retailer is moderately risk averse. This means that the retailer chooses to market a new model even when consumer welfare is larger if the old model remains on sale. The outright sales contract, which predominates in many products, creates this conflict. On the other hand, the returns policy contract coordinates the interests of the supply chain and consumers on the introduction of the new model. This economic role of returns policy has not been documented in the previous literature.

The economic intuition behind this notable conclusion is as follows. Because the retailer rather than the manufacturer faces demand uncertainty in the introduction of the new model under the outright sales contract, the risk premium for that uncertainty is added to the price not at the wholesale stage but at the retail stage. Therefore, consumers pay the premium, while the retailer does not. In contrast, under the returns policy contract with full credit, the risk premium is included in the price at the wholesale stage, because the manufacturer, not the retailer, copes with the demand uncertainty. Therefore, both the retailer and consumers bear the premium for the new model. In summary, the returns policy enables the retailer to

make a decision on the introduction of the new model that is also favorable for consumers, leading to the protection of consumer benefit.

Examples of such product categories can be found in fashion goods such as apparel, shoes and handbags, where old models are aggressively replaced with new ones. In these categories, because retailers' clerks usually sell to customers face-to-face, richer information on consumers' needs tends to be gathered by retailers than by manufacturers. This accumulated information gives the retailer the power frequently to plan new fashion product models as the marketing channel leader, encouraging the manufacturer to produce the new model in the product category.<sup>2</sup> Along with this request for new models, retailers often exert their power by insisting on a returns policy or an allowance contract with manufacturers.

Previous research has identified a number of rationales for returns policies. In contrast to the setting of our model, past models usually assume that a manufacturer rather than a retailer initiates the returns schemes for the purpose of channel coordination. Among these, the most commonly cited reason is that they are a way of insuring retailers against the risk of unsold inventory. From such a point of view, return programs are interpreted as a device of sharing risk or channel coordinating activities along the supply chain. Flath and Nariu (1989) prove that resale price maintenance and manufacturer acceptance of returns have equivalent economic effects under a linear consumer demand function. In the same vein, allowing the demand function to be nonlinear, Nariu (1996) characterizes the condition that a liberal returns policy and resale price maintenance are equivalent. Lau and Lau (1999) explore how a manufacturer may design its pricing and return policy by considering not only manufacturing cost and retail price, but also the risk preference of both itself and the retailer, as well as demand uncertainty. They prove that a "liberal" return policy is not always beneficial to the retailer. Nariu and Ishigaki (2004) and Nariu (2008) demonstrate that the returns policy plays the role of convincing retailers that the manufacturer conveys correct demand state information when the manufacturer has superior information regarding the market condition.

Recent studies provide evidence that there is more to returns policies than simple insurance. Marvel and Peck (1995) show that return allowances encourage retailers to hold

higher stock by supporting the cost of inventories. They prove that the nature of uncertainty and the induced risk taking by the retailer critically influence the contract. Lau et al. (2000) develop the model in Marvel and Peck (1995) to obtain practical insights into the returns policy contract. Kandel (1996) identifies six main factors affecting the choice between outright sales and return policy, arguing that the book industry is consistent with the predictions of his model. Padmanabhan and Png (1997) also argue that risk sharing does not seem a plausible explanation for returns schemes. Manufacturers who allows returns are often much smaller and less diversified than the retailers who return to them unsold goods. Hence, they offer an alternative argument claiming that returns offer suppliers an opportunity to increase retailer competition. Subsequently, Wang (2004) reinforces the theoretical implication by correcting the structure of their model. In the light of recent e-commerce development, Choi et al. (2004) establish a model to investigate the optimal returns policy under the existence of the e-marketplace. Arya and Mittendorf (2004) show that returns policy is also useful to elicit retailer information on retail market conditions. Yao et al. (2005) consider a manufacture-retail supply chain consisting of a mix of a traditional retail channel and a direct channel. Under the assumption of an unknown ratio of customer demand in the direct channel and the coexisting retail channel, they estimate optimal order quantities and buyback prices under two cases: information sharing and non-information sharing between two players. Yue and Raghunathan (2007) study the impacts of full returns policy on supply chains with information asymmetry between a manufacturer and a retailer, identifying the condition that not only the retailer but also the manufacturer benefits from the policy. Marvel and Wang (2007) demonstrate that even a returns program offering less than full credit may induce a desirable degree of price dispersion at retail. Archibald et al. (2007) explore how startup companies who are cash restricted in the short term can control inventory costs through a returns policy, showing that they should be more risk averse than established companies until their capital accumulates to a certain level. Mukhopadhyay and Setaputra (2007) construct a model in which the manufacturer jointly determines the design quality of a product as well as an optimal returns program, exploring the relationship between design quality and returns policy. Using numerical methods in a Stackelberg game, Yao et al. (2008)

reveal several unique properties of the returns policy contract when price elasticity of demand is relatively high. They conclude that supply chain managers should consider price elasticity of demand when devising the contract. Brown et al. (2008) consider a multi-item returns policy called "pooled" returns policy under which the distributor can return any combination of the products up to a limited proportion of the total purchases across all products. They obtain the counter-intuitive result that the distributor may order less to the manufacturer under the pooled policy than under the "non-pooled" policy even though the pooled policy offers more flexibility.<sup>3</sup>

On the other hand, little research examines returns schemes from the viewpoint of consumer welfare rather than that of management. An exception, focusing on product reliability, is Chesnokova (2007), who demonstrates that returns programs not only decrease the retail price of products but also reduce their reliability. Therefore, only if the former effect outweighs the latter will consumer welfare increase with the allowance of returns. Assuming that wholesale price is exogenously fixed, Bose and Paul (2007) point to the possibility that a returns policy contract is not Pareto efficient with respect to a price-only contract and that it is suboptimal for the distribution channel as a whole.

Overall, our review of the literature suggests that no prior research has explored the influence of new model introduction on consumer welfare under a returns policy contract. Given the research strand, this paper attempts to address this issue. The remainder of the paper is structured as follows. Section 2 sketches the model structure, followed by the derivation of equilibrium outcomes under the outright sales contract (Section 3) and under the returns policy contract (Section 4). Section 5 examines consumer welfare based on the analytical results obtained in Sections 3 and 4. A possible extension of the model is illustrated in Section 6. Section 7 makes some concluding remarks.

## **2. Model structure**

### *2.1. Assumptions*

To get a clear picture of the model that we will construct, Table 1 presents a summary

of the variables that will be used. We consider a distribution channel, which we henceforth refer to as a supply chain, consisting of a monopolistic manufacturer and a monopolistic retailer, both of which deal in a specific product. The product is "perishable" in that all unsold units have no salvage value at the end of the multistage game. There are two models in the product category: a new model and an old model. Because the manufacturer incurs prohibitively high costs to produce both models simultaneously in its production line, it can produce only one model with marginal cost  $c$ . While the consumer demand function for the old model is known in advance to both channel members, the new model involves demand uncertainty. When the supply chain markets the new model, either a higher or a lower demand state obtains. The expected demand for the new model is larger on average than the demand for the old model.

[Table 1]

Suppose that the manufacturer is either risk averse or neutral. Its objective function,  $u^M$ , takes the constant absolute risk aversion (CARA) form with respect to profit,  $\pi^M$ :

$$u^M(\pi^M) = 1 - \exp(-R^M \pi^M), \quad (1)$$

where  $R^M$  represents the nonnegative risk coefficient of the manufacturer, and the intercept is set to unity so that  $u^M(0) = 0$  is met.<sup>4</sup> Equation (1) is approximated as:

$$E(u^M(\pi^M)) = 1 - \exp(-R^M (E(\pi^M) - R^M \text{var}(\pi^M)/2)). \quad (2)$$

See the Appendix for the derivation of Equation (2). Let  $CE^M$  denote the certainty equivalent of the expected profit for the manufacturer:

$$CE^M = E(\pi^M) - R^M \text{var}(\pi^M)/2. \quad (3)$$

In this supply chain, the manufacturer has to sell products once to the monopolistic retailer, and the retailer subsequently resells them to end consumers, implying that successive

monopoly arises within the distribution channel. The retailer is risk averse, and its objective function,  $u^R$ , also takes the CARA form with regard to the profit,  $\pi^R$ , similar to the manufacturer:

$$u^R(\pi^R) = 1 - \exp(-R^R \pi^R), \quad (4)$$

where  $R^R$  is the positive risk coefficient of the retailer. We may approximate the expected objective function as:

$$E(u^R(\pi^R)) = 1 - \exp(-R^R(E(\pi^R) - R^R \text{var}(\pi^R)/2)). \quad (5)$$

Let  $CE^R$  be the certainty equivalent of the retailer's profit:

$$CE^R = E(\pi^R) - R^R \text{var}(\pi^R)/2. \quad (6)$$

With respect to the demand system, assume that the inverse consumer demand function for the old model takes a linear form:

$$p = a - bQ, \quad (7)$$

where  $p$  represents the price of the product,  $Q$  is the quantity sold to consumers, and  $a$  and  $b$  are positive constants. To ensure that the supply chain wishes to handle the product, presume that  $a > c$ . On the other hand, demand functions for the new model under a high demand state (state  $H$ ) and a low demand state (state  $L$ ) after the uncertainty is resolved are:

$$\begin{aligned} p &= a_H - b_H Q \quad (\text{state } H), \\ p &= a_L - b_L Q \quad (\text{state } L), \end{aligned} \quad (8)$$

where  $a_H$  and  $a_L$  are positive primary demand and  $b_H$  and  $b_L$  are positive coefficients of



demand slope in each state.<sup>5</sup> Assume that  $a_H \geq a_L > 0$  (primary demand) and  $0 < b_H \leq b_L$  (sensitivity of demand to price) hold, indicating that demand at a given price is always greater in state  $H$  than in state  $L$ . In other words, the demand curve in the higher demand state lies outside of that in the lower demand state. Additionally, either  $a_H \neq a_L$  or  $b_H \neq b_L$ , which signifies that either price sensitivity or primary demand differs between the two states, excluding the possibility that the two states are exactly the same. Although neither the manufacturer nor the retailer knows the demand state when producing and transacting the product, both of them know the functional form of demand in either state (Equation (8)). The probability that the high demand state is realized is  $\mu$ , whereas the probability of the low state is  $1-\mu$ . Further presume that  $b_H$  and  $b_L$  satisfy the following equation:

$$\begin{aligned}\mu a_H + (1-\mu)a_L &> a, \\ \mu b_H + (1-\mu)b_L &< b.\end{aligned}\tag{9}$$

Inequality (9) implies that the new model draws more demand on average from consumers than the old model when the retail price is given, because the expected primary demand for the new model is larger than that for the old model, and the expected gradient of the demand slope for the new model is lower than that for the old model.

## 2.2. Timeline of events

We next delineate the timeline of a series of decisions made by the members comprising the supply chain. The timeline is described as follows.

Stage 1: The retailer determines which model it deals in and proposes a contract to a manufacturer to develop and produce the chosen model.

Stage 2: The manufacturer participates in the contract if the expected utility from its profit is positive. After accepting the contract, it produces the ordered model. Note that the manufacturer cannot simultaneously produce both the new and the old model because of prohibitively high production costs.

Stage 3: The manufacturer posts the wholesale price of the model,  $r$ , to the retailer.

Stage 4: Given the wholesale price,  $r$ , the retailer orders quantity  $q$  of the model.

Stage 5: The transaction of quantity  $q$  at the price  $r$  of the model between the manufacturer and the retailer is conducted.

Stage 6: Uncertainty over the demand state of the new model is resolved, and the demand function for the new model is revealed.

Stage 7: The retailer resells a quantity,  $Q$ , of the model to final consumers at a price,  $p$ .

Either: Stage 8: Returns policy contract: The retailer may return all unsold quantity,  $q-Q$ , for a rebate of  $r$  per unit because the manufacturer permits full credit for returns.

Or: Stage 8: Outright sales contract: The retailer may not return any unsold goods to the manufacturer.

Figure 1 illustrates the timing of the events described above. Along the timeline, the retailer and the manufacturer play the game under either the outright sales contract or the returns policy contract. Suppose that either of the two contracts is accepted as a business practice in the product category before the game begins. Using the subgame perfect equilibrium as the solution concept, we now solve the game under each of the two contracts.

[Figure 1]

### 3. Outright sales contract

Initially, supposing that the outright sales contract with no returns is accepted as commercial practice in the product category, we characterize decision making by the retailer and the manufacturer and then compare the equilibrium outcomes between new model introduction and retention of the old model.

#### 3.1. Retention of the old model

We first consider the case where the retailer orders the old model. Because both the manufacturer and the retailer know the form of the demand function, neither of them has to

cope with uncertainty. Thus, the retailer maximizes the following profit function,  $\pi^R$ , with respect to  $Q$  at Stage 7.<sup>6</sup>

$$\pi^R = (p - r)Q = (a - bQ - r)Q, \quad (10)$$

where  $r$  is the wholesale price. In the absence of uncertainty, the retailer will obviously order the same quantity as the sales quantity. Therefore, the manufacturer faces the following demand function from the retailer:

$$r = a - 2bQ = a - 2bq, \quad (11)$$

where  $q$  is order quantity. The manufacturer maximizes the next profit function,  $\pi^M$ , with respect to  $q$  at Stage 3:

$$\pi^M = (r - c)q = (a - 2bq - c)q, \quad (12)$$

Because the problem is simple double marginalization, the set of variables in equilibrium is immediately derived as below.

**Proposition 1.** *When the supply chain markets the old model under the outright sales contract, the set of equilibrium outcomes of variables is given as follows.*

$$Q^* = q^* = (a - c)/(4b), \quad r^* = (a + c)/2, \quad p^* = (3a + c)/4, \quad \pi^{R*} = (a - c)^2/(16b), \\ \pi^{M*} = (a - c)^2/(8b), \quad CS^* = (a - c)^2/(32b)$$

*The superscript, "\*", denotes the equilibrium for the old model. CS is the consumer surplus.*

### 3.2. New model introduction

Next, we consider the case where the retailer chooses to order the new model from the manufacturer. We solve the game backward along the timeline illustrated in Figure 1. First, notice that  $Q$  is not necessarily equal to  $q$ , because the retailer may have an incentive to sell a

lower volume at Stage 7 than that ordered at Stage 4 so as to drive up the retail price after observing the demand state. Because the retailer can determine  $Q$  based on the demand function revealed at Stage 6, we should solve the problem at Stage 7 for the realized state. First, suppose that the high demand state is realized. Using Equation (8), the problem for the retailer to maximize the profit in the high demand state,  $\pi_H^R$ , is written as:

$$\begin{aligned} \max_{Q_H} \quad & \pi_H^R = p_H Q_H - rq = (a_H - b_H Q_H) Q_H - rq \\ \text{s.t.} \quad & Q_H \leq q \end{aligned}$$

Solving the above problem yields the optimal sales quantity,  $Q_H$ , in each of the following two cases.

Case (i): If  $q > a_H / (2b_H)$ , the optimal sales quantity is:  $Q_H = a_H / (2b_H)$ , yielding retailer profit  $\pi_H^R = (a_H^2) / (4b_H) - rq$ .

Case (ii): If  $q \leq a_H / (2b_H)$ , the optimal sales quantity is:  $Q_H = q$ , indicating that it is optimal for the retailer to sell out the ordered quantity,  $q$ . The retailer's profit is:  $\pi_H^R = (a_H - b_H q - r)q$ .

Conversely, if the low demand state is realized at Stage 6, the problem to maximize the profit in the low state,  $\pi_L^R$ , is:

$$\begin{aligned} \max_{Q_L} \quad & \pi_L^R = p_L Q_L - rq = (a_L - b_L Q_L) Q_L - rq \\ \text{s.t.} \quad & Q_L \leq q \end{aligned}$$

Optimal  $Q_L$  is given in each of the following two cases.

Case (iii): If  $q \geq a_L / (2b_L)$ ,  $Q_L = a_L / (2b_L)$ . Hence, the retailer disposes of  $q - Q_L$  to drive up the retail price.  $\pi_L^R = (a_L^2) / (4b_L) - rq$ .

Case (iv): If  $q < a_L / (2b_L)$ ,  $Q_L = q$ . The retailer's profit is:  $\pi_L^R = (a_L - b_L q - r)q$ .

Because  $b_H < b_L$ , we should consider the following three choices at Stage 4 as a result: (I)  $q < a_L / (2b_L)$ , (II)  $a_L / (2b_L) \leq q \leq a_H / (2b_H)$ , and (III)  $a_H / (2b_H) < q$ . Choice (I) corresponds to Cases (ii) in the high state and (iv) in the low state, (II) corresponds to (ii) and (iii), and (III) corresponds to (i) and (iii), respectively. When the retailer chooses the order quantity,  $q$ , at Stage 4, the choice (III) among the three choices is obviously irrational for the retailer because unsold goods will be certainly left at Stage 8, given the decision. Therefore, the upper limit of the optimal order quantity for the retailer is  $a_H / (2b_H)$ . Because of space limitations, we henceforth concentrate on the situation where exogenous variables take values such that choice (II) dominates choice (I); i.e., choice (II) brings the retailer more profit than choice (I). As stated earlier, the primary purpose of this paper is to show that the returns policy contract resolves potential conflict regarding new model introduction that results under outright sales. This purpose is attainable even if we analyze only the case when choice (II) is dominant. Indeed, choice (II) is more likely to be preferable for the retailer than choice (I), because the latter means that the retailer orders a quantity less than the optimal sales quantity in the lower demand state. Hence, we assume that the order quantity satisfies:  $a_L / (2b_L) \leq q \leq a_H / (2b_H)$ .<sup>7</sup>

Because of demand uncertainty associated with the new model, the retailer maximizes the certainty equivalent,  $CE^R$ , of the profit with respect to  $q$  at Stage 4. Because the high and low demand states occur with the probabilities  $\mu$  and  $1-\mu$  respectively,  $CE^R$  is stated as follows by substituting the retailer's profit in cases (ii) and (iii) described above into Equation (6):

$$CE^R = \mu\pi_H^R + (1-\mu)\pi_L^R - R^R \text{var}(\pi^R) / 2 \\ = \mu q(a_H - b_H q - r) + (1-\mu)(a_L^2 - 4b_L r q) / (4b_L) - \left( \mu(1-\mu)(4b_L q(a_H - b_H q) - a_L^2)^2 / (32b_L^2) \right) R^R. \quad (13)$$

From  $dCE^R/dq=0$ , the retailer demand function that the manufacturer faces is given by:

$$r = \mu(a_H - 2b_H q) - \mu(1-\mu)(a_H - 2b_H q)(4b_L q(a_H - b_H q) - a_L^2) R^R / (4b_L). \quad (14)$$

We here assume that the second order condition is met; i.e.,  $d^2CE^R/dq^2 < 0$ .<sup>8</sup> Because the retailer orders a fixed quantity at the given wholesale price,  $r$ , based on the relationship of Equation (14) irrespective of the final demand state, and the manufacturer accepts no returns under the outright sales contract, the manufacturer faces no uncertainty. Therefore,  $\text{var}(\pi^M) = 0$ , meaning that the certainty equivalent written in Equation (3) is identical to that of the expected profit; i.e.,  $CE^M = E(\pi^M) = \pi^M$ . Thus, the manufacturer maximizes  $CE^M$  or  $\pi^M$  with respect to  $r$  given Equation (14) at Stage 3. For model tractability, we first remove  $r$  from  $CE^M$  by using Equation (14) and maximize  $CE^M$  with respect to  $q$  as follows without loss of generality.<sup>9</sup>

$$CE^M = E(\pi^M) = (r - c)q \\ = \left( \mu(a_H - 2b_H q) - \mu(1 - \mu)(a_H - 2b_H q)(4b_L q(a_H - b_H q) - a_L^2)R^R / (4b_L - c) \right) q. \quad (15)$$

Maximization of  $CE^M$  yields the following relationship:

$$R^R = \frac{4b_L(\mu a_H - 4\mu b_H q - c)}{\mu(1 - \mu)(8a_H^2 b_L q + 4b_H q(a_L^2 + 8b_H b_L q^2) - a_H(a_L^2 + 36b_H b_L q^2))}. \quad (16)$$

Suppose that the second-order condition of the maximization is satisfied.<sup>10</sup> Because Equation (16) includes no endogenous variable other than  $q$ , we may obtain the optimal order quantity in equilibrium by solving the equation for  $q$ .<sup>11</sup> Now we may summarize variables in equilibrium by using Equation (16).

**Proposition 2.**

*The variables in equilibrium of new model sales under the outright sales contract are calculated as follows. First,  $q$  in equilibrium, which is denoted by  $q^{**}$ , is derived by solving the next equation.*

$$R^R = \frac{4b_L(\mu a_H - 4\mu b_H q^{**} - c)}{\mu(1-\mu)(8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2}))}$$

Using  $q^{**}$ , we may present other variables and functions in equilibrium as follows.

$$E(Q^{**}) = \mu q^{**} + (1-\mu)a_L / (2b_L),$$

$$r^{**} = \mu a_H - 2\mu b_H q^{**} + \frac{(a_H - 2b_H q^{**})(c - \mu a_H + 4b_H \mu q^{**})(4b_L q^{**}(a_H - b_H q^{**}) - a_L^2)}{8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2})},$$

$$E(p^{**}) = \mu(a_H - b_H q^{**}) + (1-\mu)a_L / 2,$$

$$CE^{R^{**}} = \mu b_H q^{**2} + (1-\mu)a_L^2 / (4b_L) + (\mu(1-\mu)(16b_L^2 q^{**2}(a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{**2}) + 8a_L^2 b_H b_L q^{**2} - a_L^4) / (32b_L^2)) R^R,$$

$$CE^{M^{**}} = q^{**} \left( \mu a_H - 2\mu b_H q^{**} + \frac{(a_H - 2b_H q^{**})(c - \mu a_H + 4\mu b_H q^{**})(4b_L q^{**}(a_H - b_H q^{**}) - a_L^2)}{8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2})} - c \right),$$

$$CS^{**} = \mu b_H q^{**2} / 2 + (1-\mu)a_L^2 / (8b_L)$$

Superscript  $^{**}$  means that the variable represents the equilibrium when the new model is introduced under the outright sales contract. Note that only the quantity,  $q^{**}$ , among the endogenous variables is described in the implicit function of exogenous variables for simpler description of the equilibrium.

## 4. Returns policy contract

### 4.1. Retention of the old model

We next explore equilibrium under the returns policy contract. Similar to the outright sales case, we first consider the case where the retailer orders the old model, where no uncertainty is associated with the demand from consumers. Because the retailer orders an exact quantity from the manufacturer calculated from the consumer demand curve, no returns will happen even though the manufacturer is ready to accept unsold returns. Thus, the set of solutions under the returns policy contract is the same as that for the old model under the outright sales contract as follows.

**Proposition 3.** *The outcomes of selling the old model under the returns policy contract are*

exactly the same as Proposition 1 under the outright sales contract, because no returns will arise.

$$Q^* = q^* = (a - c)/(4b), \quad r^* = (a + c)/2, \quad p^* = (3a + c)/4, \quad \pi^{R*} = (a - c)^2/(16b),$$

$$\pi^{M*} = (a - c)^2/(8b), \quad CS^* = (a - c)^2/(32b)$$

#### 4.2. New model introduction

Next, we turn to the case when the new model is introduced. Under the returns policy contract, the retailer may return all unsold goods for a rebate of  $r$  per unit at Stage 8. We consider the following two cases, because the retailer cannot sell a larger quantity at Stage 7 than that ordered at Stage 4; i.e.,  $Q \leq q$ .

Case (i): If  $Q_K < q$  ( $K=H$  or  $L$ ), the retailer will return  $Q_K - q$  at Stage 8:

$$\begin{aligned} \pi_H^R &= (a_H - b_H Q_H)Q_H - rq + r(q - Q_H) = (a_H - b_H Q_H)Q_H - rQ_H, \\ \pi_L^R &= (a_L - b_L Q_L)Q_L - rq + r(q - Q_L) = (a_L - b_L Q_L)Q_L - rQ_L. \end{aligned} \quad (17)$$

Case (ii): If  $Q_K = q$ , the retailer sells out the ordered quantity:

$$\begin{aligned} \pi_H^R &= (a_H - b_H Q_H)Q_H - rQ_H, \\ \pi_L^R &= (a_L - b_L Q_L)Q_L - rQ_L. \end{aligned} \quad (18)$$

Equations (17) and (18) suggest that the profit function in either case is identical at Stage 8 under the returns policy contract. Given the profit functions, we solve the problem to determine  $Q_K$  at Stage 7. Maximization of the profit in each demand state in Equation (17) or (18) with respect to the sales quantity yields the optimal sales quantity in each state as:

$$\begin{aligned} Q_H &= (a_H - r)/(2b_H) \\ Q_L &= (a_L - r)/(2b_L). \end{aligned} \quad (19)$$

Working backward, we proceed to determine the order quantity,  $q$ , at Stage 4. Returns policy with full credit enables the retailer to order and stock the upper limit of the demand,



$Q_H$ , which may satisfy the demand in state  $H$ , in advance, and this is the optimal behavior for the retailer. Therefore, the retailer will order  $q \geq (a_H - r)/(2b_H)$  to avoid being out of stock at Stage 7. For model tractability, we here exclude the possibility that the retailer orders excess quantity that is greater than the upper limit sales quantity,  $q = (a_H - r)/(2b_H)$ , that will be sold in the higher demand state.<sup>12</sup> Hence, the order quantity at Stage 4 is:

$$q = (a_H - r)/(2b_H). \quad (20)$$

If the lower demand state is realized, the retailer returns the unsold stock,  $q - Q_L$ , to the manufacturer for a rebate of  $r$  per unit. It follows that the manufacturer bears the costs of producing  $q$  at Stage 2 regardless of the final demand state. Hence, given Equation (20), the profit for the manufacturer in the higher demand state,  $\pi_H^M$ , and in the lower demand state,  $\pi_L^M$ , is respectively written as:

$$\begin{aligned} \pi_H^M &= (r - c)q = (a_H - 2b_H q - c)q \\ \pi_L^M &= (r - c)q - r(q - Q_L) = rQ_L - cq = (a_H - 2b_H q)(a_L - a_H + 2b_H q)/(2b_L) - cq. \end{aligned} \quad (21)$$

Substituting Equation (21) into Equation (3), the certainty equivalent for the manufacturer,  $CE^M$ , is expressed as:

$$\begin{aligned} CE^M &= \mu\pi_H^M + (1 - \mu)\pi_L^M - R^M \text{var}(\pi^M)/2 \\ &= \mu(a_H - 2b_H q - c)q + (1 - \mu)((a_H - 2b_H q)(a_L - a_H + 2b_H q)/(2b_L) - cq) \\ &\quad - (\mu(1 - \mu)(a_H - 2b_H q)^2(a_H - a_L + 2(b_L - b_H)q)^2/(8b_L^2))R^M \end{aligned} \quad (22)$$

Maximization of  $CE^M$  with respect to  $q$  yields the following relationship:

$$R^M = \frac{2b_L(a_H(2(1 - \mu)b_H + \mu b_L) - (1 - \mu)a_L b_H - 4b_H((1 - \mu)b_H + \mu b_L)q - b_L c)}{\mu(1 - \mu)(a_H - 2b_H q)(a_H - a_L + 2(b_L - b_H)q)(a_H(b_L - 2b_H) + b_H(a_L - 4(b_L - b_H)q))}. \quad (23)$$

Presume that the second order condition of the maximization is met; i.e.,  $d^2CE^M/dq^2 < 0$ .<sup>13</sup> Because no endogenous variable other than  $q$  appears in Equation (23), we may obtain the optimal order quantity by solving the equation for  $q$ .<sup>14</sup> The set of solutions are summarized as the following proposition.

**Proposition 4.**

*When the supply chain introduces the new model and sells it to end consumers under the returns policy contract, the set of equilibrium solutions is calculated as follows. The equilibrium of the order quantity denoted by  $q^{***}$  is obtained by solving the following equation.*

$$R^M = \frac{2b_L(a_H(2(1-\mu)b_H + \mu b_L) - (1-\mu)a_L b_H - 4b_H((1-\mu)b_H + \mu b_L)q^{***} - b_L c)}{\mu(1-\mu)(a_H - 2b_H q^{***})(a_H - a_L + 2(b_L - b_H)q^{***})(a_H(b_L - 2b_H) + b_H(a_L - 4(b_L - b_H)q^{***}))}$$

Using  $q^{***}$ , other equilibrium variables and functions are calculated as follows.

$$\begin{aligned} r^{***} &= a_H - 2b_H q^{***}, \\ E(Q^{***}) &= \mu q^{***} + (1-\mu)(a_L - a_H + 2b_H q^{***})/(2b_L), \\ E(p^{***}) &= ((1+\mu)a_H + (1-\mu)a_L - 2b_H q^{***})/2, \\ CE^{R^{***}} &= \mu b_H q^{2^{***}} + (1-\mu)(a_L - a_H + 2b_H q^{***})^2/(4b_L) \\ &\quad - \mu(1-\mu)\left((a_H - a_L)^2 - 4b_H q^{***}(a_H - a_L + (b_L - b_H)q^{***})\right)^2/(32b_L^2)R^R, \\ CE^{M^{***}} &= \mu(a_H - 2b_H q^{***} - c)q^{***} + (1-\mu)(a_H - 2b_H q^{***})(a_L - a_H + 2b_H q^{***})/(2b_L) - cq^{***} \\ &\quad - \left(\mu(1-\mu)(a_H - 2b_H q^{***})^2(a_H - a_L + 2(b_L - b_H)q^{***})^2/(8b_L^2)\right)R^M, \\ E(CS^{***}) &= \mu b_H q^{***2}/2 + (1-\mu)(a_L - a_H + 2b_H q^{***})^2/(8b_L) \end{aligned}$$

Superscript "\*\*\*\*" signifies that the variable represents equilibrium when the new model is introduced under the returns policy contract. Note that only the quantity,  $q^{***}$ , among the endogenous variables is described in the implicit function of exogenous variables.

## 5. Consumer welfare analysis

In this section, we proceed to compare consumer welfare between the two models under each of the contract schemes, which is the central purpose of the current research.

Table 2 summarizes the equilibrium outcomes calculated earlier. First, the following proposition holds under the outright sales contract.

[Table 2]

**Proposition 5.** *Suppose that the following inequality holds.*

$$\begin{aligned}
& 4(4\mu b_H q^{**2} + (1-\mu)a_L^2/b_L) \\
& < (a-c)^2/b \\
& < 4(4\mu b_H q^{**2} + (1-\mu)a_L^2/b_L) \\
& \quad + (\mu(1-\mu)(16b_L^2 q^{**2}(a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{2**}) + 8a_L^2 b_H b_L q^{**2} - a_L^4)/(2b_L^2))R^R
\end{aligned} \tag{P1}$$

*Then, the retailer is moderately risk averse; i.e.,  $R^R$  takes a positive value. When Inequality (P1) holds, the retailer chooses to introduce the new model into the retail market under the outright sales contract, although the old model is better for consumers' welfare. This implies that conflict of interest on the introduction of the new model arises between the retailer and consumers as long as a set of exogenous variables satisfies Inequality (P1).*

**Proof.** See the Appendix.

Next, comparison of consumer welfare between each model under the returns policy contract leads us to the following proposition.

**Proposition 6.** *Under the returns policy contract, when the supply chain introduces the new model into the retail market, welfare for consumers is greater when they buy the new model than when they buy the old model. In this respect, the retailer and consumers share the same interest in whether to introduce the new model or not.*

**Proof.** See the Appendix.

It is worth highlighting that Propositions 5 and 6 suggest that the outcome of the returns policy contract is opposite to that of the outright sales contract. Looking at Table 2, we notice that the order (and production) quantity,  $q$ , in equilibrium under the outright sales contract depends on the retailer's degree of risk aversion,  $R^R$ , but does not depend on the

manufacturer's degree of risk aversion,  $R^M$ , whereas the quantity under the returns policy contract depends on  $R^M$  but does not depend on  $R^R$ . Furthermore, the table suggests that wholesale and expected retail prices under outright sales are dependent on  $R^R$  but are independent of  $R^M$ . By contrast, wholesale and retail prices under returns policy are subject to  $R^M$  but are uninfluenced by  $R^R$ . These two contrasting results imply the following logic behind our model. While the manufacturer faces no uncertainty and thus the wholesale price includes no risk premium under outright sales, the retailer passes on its risk premium to consumers at the retail market stage by including the premium in the retail price. Therefore, a decision regarding the choice of the model by the retailer is sometimes unfavorable to consumers. On the other hand, the returns policy enables the retailer to make a decision that is also favorable to consumers, because the retailer as well as consumers bears the risk premium added by the manufacturer when introducing the new model. For this reason, a returns program with full credit eliminates the conflict of interest between the supply chain and consumers concerning whether to introduce the new model or not.

## 6. Extension: multiple retailers and manufacturers

Our model has thus far considered a simple supply chain consisting of a monopolistic manufacturer and a monopolistic retailer. In reality, however, multiple retailers and manufacturers usually produce and sell a differentiated product in a competitive market environment. We may develop the model to allow the existence of multiple retailers and manufacturers as follows.

Suppose that  $n$  retailers and  $m$  manufacturers, which may develop and market a specific product, potentially exist. Because the first mover in the initial stage of the game in our model is not a manufacturer but a retailer, we consider the following sequential negotiations between retailers and manufacturers. First, retailer  $j$  plans a specific model of the product and proposes a contract to a manufacturer to develop the model. If the manufacturer accepts the contract, they play the game presented in Figure 1. If, on the other hand, the manufacturer declines the contract, the retailer will propose the product plan to another manufacturer.

Figure 2 illustrates an example of such negotiations.

[Figure 2]

To induce these sequential negotiations, we introduce asymmetry between manufacturers and asymmetry between retailers. We assume that a manufacturer incurs fixed production costs, which differ among manufacturers as:

$$\pi_i^M = (r - c)q - F_i, \quad (24)$$

where subscript  $i$  indexes the manufacturer and  $F_i$  represents the fixed cost for manufacturer  $i$ .<sup>15</sup>

As asymmetry between retailers, we assume that the demand function of a product planned by a retailer varies. The demand functions for the new model of the product planned by retailer  $j$  in high and low demand states are restated as:

$$\begin{aligned} p &= a_{Hj} - b_{Hj}Q \quad (\text{state } H), \\ p &= a_{Lj} - b_{Lj}Q \quad (\text{state } L), \end{aligned} \quad (25)$$

where subscript  $j$  indexes the retailer. Likewise, demand for the old model planned by retailer  $j$  is:

$$p = a_j - b_jQ. \quad (26)$$

With such arrangements, certainty equivalents of profit,  $CE^M$ , in the equilibrium presented in Table 2 will vary between manufacturers. As a consequence, sequential negotiations on product planning and marketing between manufacturers and retailers will take place. Even when the product planned by retailer  $j$  is not accepted by one manufacturer, another manufacturer with low fixed production cost may accept the contract at Stage 2 in Figure 1. On the other hand, even if manufacturer  $i$  cannot accept a product planned by a certain retailer, it may accept another product that was planned by another retailer and is likely to

enjoy larger demand.

Note that the model extension above still involves one-to-one correspondence between a manufacturer and a retailer for handling a product. If we further consider competition between retailers to deal in the same product, the model will become far more complicated because it indicates that other retailers as well as the retailer that has planned the product will handle the same product. Therefore, we would have to consider whether or not the retailer is willing to plan a model of the product and to propose a contract to a manufacturer even if the "idea" used for planning the product spills over to other rival retailers. For example, competition between multiple retailers for dealing in the same private brand (PB) that a specific retailer has planned is unrealistic. On the other hand, it would also be difficult to extend our model to consider competition between manufacturers. If manufacturers simultaneously compete to handle the same product in our model framework, competition arises at the stage when a manufacturer determines whether to close the contract rather than at the sales stage (e.g., Cournot or Bertrand competition) because only one manufacturer develops the product based on the product planning of a specific retailer. A further extension involving such competition is interesting, but we expect it to be analyzed in a future study, primarily because of its complexity. Recall that the primary purpose of the present study is to show that a returns policy plays a role in the resolution of conflict between supply chains and consumers, and we have achieved that purpose.

## **7. Concluding remarks**

Given the recent accumulation of market power by dominant retailers in advanced economies, this paper has investigated the economic outcomes of new model introduction when a monopolistic retailer, as the marketing channel leader, has the right to choose whether to introduce a new model with demand uncertainty. A conflict of interest on the introduction of a new model between the supply chain and consumers arises under the outright sales contract, which generally prevails in many product categories. This paper has shown that a returns policy contract may resolve this problem. By transferring risk associated with demand

uncertainty from the retailer to the manufacturer, the retailer can make a decision on the introduction of the new model that is more desirable for consumers, leading to the protection of consumer benefits. For example, consumers often find that an old model of fashion wear has already been replaced with a new one when visiting a clothing retailer, possibly because the retailer expects that the new model will sell more strongly. In such cases, returns policies serve to protect consumer welfare by mitigating the excessive incentive of the retailer to introduce a new model into the market at the expense of consumer benefits. This implication suggests that a regulator aiming to protect consumer benefits should not always prevent returns policies or similar allowances required by retailers. As we have demonstrated, the removal of risk from retailers under a returns policy removes uncertainty premiums from retail prices.

## **Footnotes**

1. Aalto-Setälä (2002) reviews growing concentration of retailers in European countries and the resulting power shift in distribution channels. Examining the bargaining power of retailers, Matsui (2009a) empirically reveals that larger retail stores receive significantly lower wholesale prices than do smaller ones even when supplier concentration is high.
2. Bose and Paul (2007) point out that apparel manufacturers such as Anna Klein, Finity, DKNY and Kids Wearhouse provide retailers with various return programs. Padmanabhan and Png (1997) also document the prevalence of returns policy contracts in products such as apparel goods, where retailers are larger than manufacturers. Returns systems are frequently employed in Japanese apparel distribution (Marvel and Peck, 1995). In particular, apparel sections of Japanese department stores have employed return systems in conjunction with manufacturers (Ariga et al., 1991; Ariga, 1995).
3. OR studies that aim to solve more practical problems such as inventory control or newsboy problems under returns policies also appear (e.g., Hahn et al., 2004; Lee, 2001; Mantrala and Raman, 1999).

4. While a CARA utility function is frequently used in risk management literature to assess various risks surrounding firms, the function also enables a researcher addressing a specific OR problem in the operations management context to derive quantitative solutions under uncertainty. The CARA utility function in operations management has been applied to a number of different situations: a dynamic competition game with a two-echelon supply chain consisting of two manufacturers and two retailers (Xiao and Choi, 2009), price–service competition with two supply chains consisting of one risk-neutral supplier and one risk-averse retailer (Xiao and Yang, 2008), supply chain decision making under a mixture of uncertainty of both price and production (Dalal and Alghalith, 2009), portfolio management of financial assets (Mitchell and Gelles, 2003; Buckley et al., 2008; Marín-Solano and Navas, 2009), a newsvendor problem (Wang et al., 2009), agricultural and environmental planning (Gómez-Limón et al., 2003; Flaten and Lien, 2007; Lien et al., 2007), and an auction system (Álvarez and Mazón, 2007). The above studies employ the CARA function to describe risk-averse behaviors of decision makers.
5. Operations management studies have investigated effects of the variation of the intercept of an inverse demand function on decisions within supply chains under uncertainty (e.g., Padmanabhan and Png, 1997; Yue and Raghunathan, 2007). On the other hand, research that has examined the effect of the sensitivity of demand to retail price also appears (e.g., Flath and Nariu, 2000; Fleshman and Willner, 2005; Matsui, 2009b). Given the settings in previous studies, we allow both primary demand and sensitivity to vary according to the demand state as in Equation (8) so as to discuss the effect of demand uncertainty on the model choice comprehensively.
6. When the retailer bears no risk, maximization of profit is equivalent to maximization of the certainty equivalent because variance of profit indicated by Equation (3) is 0. The same is true for the manufacturer.
7. Because  $q$  in equilibrium is described as an implicit function of exogenous variables later, this assumption signifies that exogenous variables take values that are consistent with  $a_L/(2b_L) \leq q \leq a_H/(2b_H)$ .
8. While the second-order derivative of the first term in Equation (13) with respect to  $q$  is



negative and that of the second term is 0, the sign of the third term is ambiguous. However, we may expect that the influence of the third term on the optimization is smaller than the other two terms, which represent the mean profit, because the third term represents the "risk premium" for the manufacturer as proved in Appendix. Indeed, if the variance of retailer's profit or the degree of risk aversion is so small that the third term can be approximated as 0,  $d^2CE^R/dq^2 < 0$  holds without any assumption.

9. Because Equation (14), which describes the relationship between  $r$  and  $q$ , is given to the manufacturer, maximization of  $CE^M$  with respect to  $q$  is equivalent to maximization with respect to  $r$ .
10. Also in this maximization, only the sign of the second-order derivative of the risk premium term in Equation (15) is ambiguous. If the retailer is almost risk neutral so that  $R^R$  may be approximated as 0, one may confirm that the second-order derivative is negative because the risk premium term disappears from the equation.
11. Although Equation (16) is a cubic equation on  $q$ , one may confirm that only one solution is a rational number and that the other two include complex numbers by solving the equation for  $q$  by using computational software. Hence, we here define the first rational solution as  $q^{**}$ .
12. As indicated by Equation (19), the retailer will not sell more than  $(a_H - r)/(2b_H)$  to consumers in either state. Because the manufacturer also knows this fact and recognizes that an order quantity greater than  $(a_H - r)/(2b_H)$  is an obvious over-ordering, the manufacturer will decline such an order.
13. The second-order derivative of the first term in Equation (22) with respect to  $q$  is negative, and that of the second term is 0; only that of the third term is ambiguous. Because the third term represents the risk premium for the manufacturer, we may expect that its influence on optimization is smaller than the other two terms, which represent the mean of the profit.
14. Although Equation (23) is a cubic equation on  $q$ , only one solution is a rational number, and the other two consist of complex numbers. Thus, we denote the rational solution as  $q^{***}$ .

15. Another possible asymmetry between manufacturers that we may introduce is the variation of the degree of risk aversion,  $R^M$ , because such assumption leads to variation of the certainty equivalent of profit of the potential manufacturers.

## Appendix

### Certainty equivalent under CARA utility function

This appendix presents a CARA utility function approximation process. First, let  $\pi$  denote profit, which is a random variable, and let  $\rho$  denote the risk premium for a risk-averse firm facing uncertainty. The risk premium generally signifies the difference between profit for the firm under uncertainty and a certain amount of profit that guarantees the firm the same expected utility as obtained from the uncertain profit. Accordingly,  $E\pi - \rho$  is defined as the certainty equivalent,  $CE$ , which compensates the firm for the expected utility from uncertain profit. The following equation describes the relationship between expected utility from uncertain profit and utility from certainty equivalent.

$$Eu(\pi) = u(E\pi - \rho), \quad (A1)$$

where  $u(\pi)$  represents the utility function of the firm and  $E$  is the expectation operator. If the firm is risk averse,  $\rho$  takes a positive value. Stated differently, the firm may bear less expected profit if a fixed amount of dollars, which is the expected profit minus risk premium, is guaranteed with certainty.

Because  $\rho$  takes a relatively small value, we may derive the first-order approximation of the right-hand side of Equation (A1) by using Taylor expansion around the expected profit as follows:

$$\begin{aligned} u(E\pi - \rho) &= u(E\pi) + u'(E\pi)(E\pi - \rho - E\pi) \\ &= u(E\pi) - \rho u'(E\pi) \end{aligned} \quad (A2)$$

On the other hand, because  $\pi$  may vary widely from its expected value, we derive both the first-order and second-order approximation terms of the left-hand side of Equation (A1) as follows:

$$Eu(\pi) = u(E\pi) + u'(E\pi)(\pi - E\pi) + u''(E\pi)(\pi - E\pi)^2 / 2. \quad (A3)$$

Taking the expected value of both sides of Equation (A3), we obtain:

$$Eu(\pi) = u(E\pi) + u''(E\pi)\text{var}(\pi) / 2. \quad (A4)$$

Substituting Equations (A2) and (A4) into Equation (A1) gives the risk premium,  $\rho$ , as:

$$\rho = -\{u''(E\pi) / u'(E\pi)\} \text{var}(\pi) / 2 = A_u \text{var}(\pi) / 2, \quad (A5)$$

where  $A_u$  represents the degree of absolute risk aversion and is defined as:

$$A_u \equiv -u''(E\pi) / u'(E\pi). \quad (A6)$$

Using Equation (A5), the certainty equivalent is calculated as:

$$CE = E\pi - \rho = E\pi - A_u \text{var}(\pi) / 2. \quad (A7)$$

Calculating Equation (A6) on the basis of Equation (1) yields  $A_u \equiv -u''(E\pi) / u'(E\pi) = R^M$ . Therefore, Equation (A7) is restated as  $CE^M = E(\pi^M) - R^M \text{var}(\pi^M) / 2$ , where superscript  $M$  denotes the manufacturer. This equation corresponds to Equation (3) in the main text.

### Proof of Proposition 5

Because the retailer initially determines whether to introduce the new model into the market in the game, we compare the certainty equivalents of the retailer's profit between new model introduction and retention of the old model to determine the condition that new model introduction is more preferable. The certainty equivalent for the retailer in the new model is:

$$CE^{R**} = \mu b_H q^{**2} + (1 - \mu) a_L^2 / (4b_L) + (\mu(1 - \mu) (16b_L^2 q^{**2} (a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{2**}) + 8a_L^2 b_H b_L q^{**2} - a_L^4) / (32b_L^2)) R^R. \quad (A8)$$

On the other hand, the profit for the retailer selling the old model is:

$$\pi^{R*} = (a - c)^2 / (16b). \quad (A9)$$

Therefore, the retailer introduces the new model into the market as long as the following inequality is met:

$$\begin{aligned}
& \pi^{R^*} < CE^{R^{**}} \\
& \Leftrightarrow \\
& (a-c)^2 / b \\
& < 4\left(4\mu b_H q^{**2} + (1-\mu)a_L^2 / b_L\right) \\
& + \left(\mu(1-\mu)\left(16b_L^2 q^{**2}\left(a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{2**}\right) + 8a_L^2 b_H b_L q^{**2} - a_L^4\right) / (2b_L^2)\right) R^R
\end{aligned} \tag{A10}$$

Next, we compare consumer welfare between the two models. The welfare of consumers when they purchase the old model is larger than that when they buy the new one if the next condition is met:

$$\begin{aligned}
& CS^{**} < CS^* \\
& \Leftrightarrow \mu b_H q^{**2} / 2 + (1-\mu)a_L^2 / (8b_L) < (a-c)^2 / (32b).
\end{aligned} \tag{A11}$$

Inequality (A11) is restated as:

$$4\left(4\mu b_H q^{**2} + (1-\mu)a_L^2 / b_L\right) < (a-c)^2 / b. \tag{A12}$$

Combining Inequalities (A10) and (A12) yields Inequality (P1). Thus, when Inequality (P1) holds, the retailer introduces the new model even though consumers prefer to buy the old model.

Next, we prove that  $R^R$  takes a positive value under Inequality (P1). First notice that when  $R^R=0$ , the following equation holds:

$$\begin{aligned}
& 4\left(4\mu b_H q^{**2} + (1-\mu)a_L^2 / b_L\right) \\
& = 4\left(4\mu b_H q^{**2} + (1-\mu)a_L^2 / b_L\right) \\
& + \left(\mu(1-\mu)\left(16b_L^2 q^{**2}\left(a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{2**}\right) + 8a_L^2 b_H b_L q^{**2} - a_L^4\right) / (2b_L^2)\right) R^R,
\end{aligned}$$

implying that the interval that satisfies Inequality (P1) disappears irrespective of the values of other exogenous variables. Therefore,  $R^R$  is unequal to 0 under Inequality (P1). For the proof of the existence of positive region of  $R^R$  consistent with Inequality (P1), it suffices to present a numerical example of a set of exogenous variables that satisfies the inequality. Suppose that exogenous variables take the following values.

$$(R^R, R^M, \mu, a, a_H, a_L, b, b_H, b_L, c) = (0.015, 0.07, 0.8, 50, 56, 40, 4, 2.5, 10, 1) \tag{A13}$$

Using Proposition 2, equilibrium endogenous variables and functions under new model introduction are calculated as:

$$(r^{**}, q^{**}, \pi^{M^{**}}, CE^{R^{**}}, CS^{**}) = (21.580, 3.126, 64.326, 46.355, 13.769), \tag{A14}$$

while those under old model retention based on Proposition 1 are:

$$(r^*, q^*, \pi^{M*}, \pi^{R*}, CS^*) = (25.5, 3.063, 75.031, 37.516, 18.758). \quad (A15)$$

Comparison of Equations (A14) and (A15) indicates that the retailer introduces the new model because it may obtain a higher certainty equivalent of the profit (i.e.,  $CE^{R**} > \pi^{R*}$ ), although consumer surplus is greater under old model retention than under new model introduction (i.e.,  $CS^{**} < CS^*$ ). This example implies that conflict of interest between firms and consumers arises. We may confirm that the above set of exogenous variables satisfies all prerequisites (e.g.,  $a_L / (2b_L) \leq q \leq a_H / (2b_H)$ ), and second-order conditions of optimizations) as well as Inequality (P1).  $\square$

### Proof of Proposition 6

The retailer prefers to introduce the new model when the next inequality is met:

$$\begin{aligned} CE^{R***} &> \pi^{R*} \\ \Leftrightarrow \mu b_H q^{***2} + (1-\mu)(a_L - a_H + 2b_H q^{***})^2 / (4b_L) \\ &\quad - \mu(1-\mu) \left( (a_H - a_L)^2 - 4b_H q^{***} (a_H - a_L + (b_L - b_H) q^{***}) \right)^2 / (32b_L^2) \Big) R^R \\ &> (a-c)^2 / (16b) \end{aligned} \quad (A16)$$

On the other hand, consumers' welfare on purchasing the new model is larger than purchasing the old model if:

$$\begin{aligned} E(CS^{***}) &> CS^* \\ \Leftrightarrow \mu b_H q^{***2} / 2 + (1-\mu)(a_L - a_H + 2b_H q^{***})^2 / (8b_L) &> (a-c)^2 / (32b) \end{aligned} \quad (A17)$$

One may immediately confirm that Inequalities (A16) and (A17) are identical if the third term of the left-hand side in Inequality (A16) is 0. However, because the retailer is risk averse and the third term is thus negative, Inequality (A16) is a stricter condition than Inequality (A17); i.e., Inequality (A17) is automatically satisfied when Inequality (A16) holds. Therefore, if the certainty equivalent of the retailer's profit when selling the new model is higher than the old model and the retailer thus introduces the new model into the retail market, consumer surplus from purchasing the new model is also greater than that from the old model.  $\square$

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**Fig. 1. Timeline of events**

Stage 1: The retailer determines which model it deals in and proposes a contract to a manufacturer to develop and produce the chosen model.



Stage 2: The manufacturer decides to participate in the contract if the expected utility from its profit is positive. If the manufacturer participates in the contract, it produces the ordered model.



Stage 3: The manufacturer posts the wholesale price of the model,  $r$ , to the retailer.



Stage 4: Given the wholesale price,  $r$ , the retailer orders quantity  $q$  of the model.



Stage 5: The transaction of quantity  $q$  at the price  $r$  of the model between the manufacturer and the retailer is conducted.



Stage 6: Uncertainty over the demand state of the new model is resolved, and the demand function for the new model is revealed.



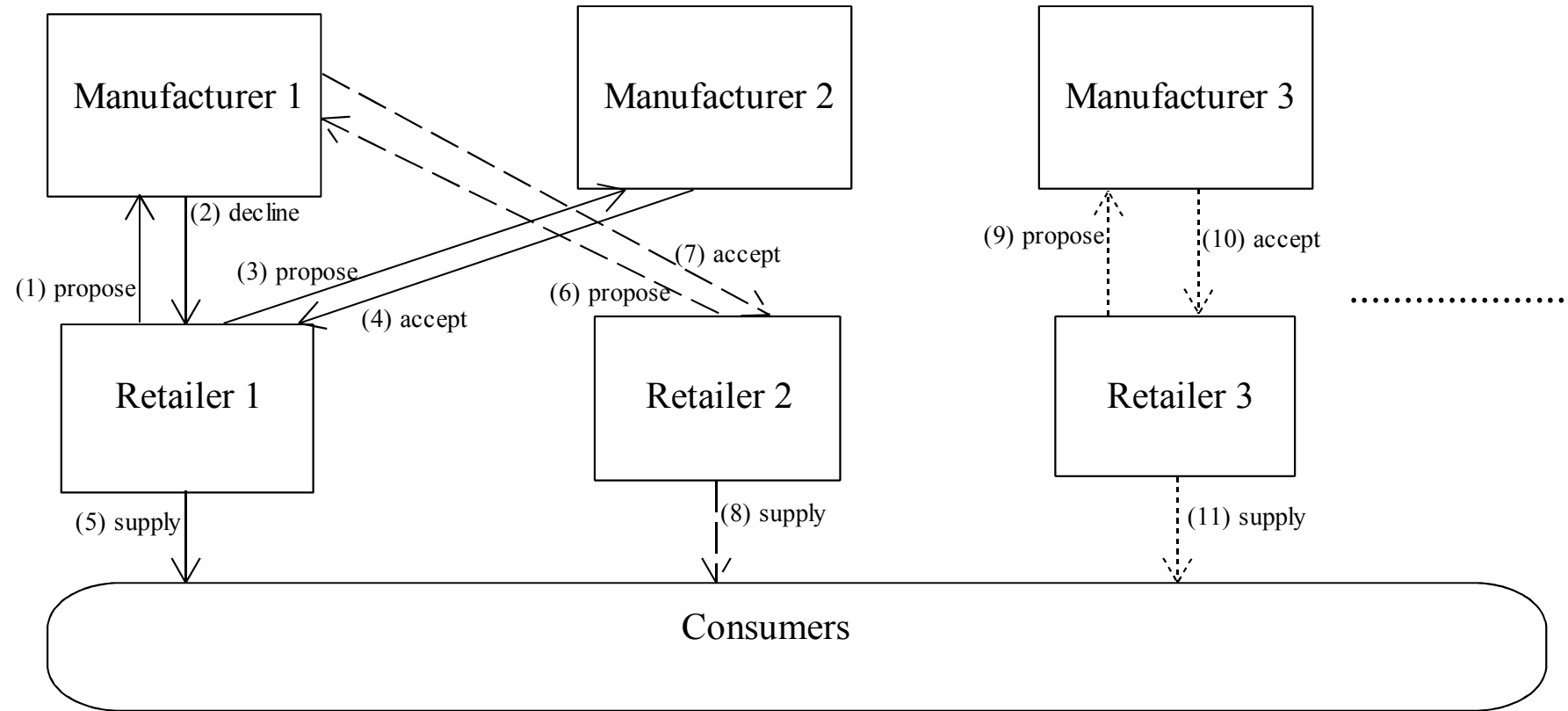
Stage 7: The retailer resells a quantity,  $Q$ , of the model to final consumers at a price,  $p$ .



Either: Stage 8: Returns policy contract: The retailer may return all unsold quantity,  $q-Q$ , for a rebate of  $r$  per unit under the returns policy contract because the manufacturer permits full credit for returns.

Or: Stage 8: Outright sales contract: The retailer may not return any unsold goods to the manufacturer.

**Fig. 2. Negotiations between multiple retailers and manufacturers**



Note: Numbers in parentheses denote the order of the timing of each event. If retailer  $j$  proposes a product plan to manufacturer  $i$  and the manufacturer accepts the proposition, they play the game presented in Figure 1.

**Table 1. Nomenclature**

$Q$	sales quantity from the retailer to consumers
$Q_H$	sales quantity when the higher demand state is realized
$Q_L$	sales quantity when the lower demand state is realized
$q$	order quantity from the retailer to the manufacturer
$r$	wholesale price
$p$	retail price
$p_H$	retail price when the higher demand state is realized
$p_L$	retail price when the lower demand state is realized
$R^R$	risk coefficient of the retailer
$R^M$	risk coefficient of the manufacturer
$\pi^R$	profit of the retailer
$\pi^M$	profit of the manufacturer
$\pi_H^R$	profit of the retailer in high demand state
$\pi_L^R$	profit of the retailer in low demand state
$\pi_H^M$	profit of the manufacturer in high demand state
$\pi_L^M$	profit of the manufacturer in low demand state
$u^R(\pi^R)$	objective function of the retailer
$u^M(\pi^M)$	objective function of the manufacturer
$CE^M$	certainty equivalent of the expected profit of the manufacturer
$CE^R$	certainty equivalent of the expected profit of the retailer
$a$	intercept of the inverse consumer demand function for the old model
$a_H$	intercept of the inverse consumer demand function for the new model in high demand state
$a_L$	intercept of the inverse consumer demand function for the new model in low demand state
$b$	slope of the inverse consumer demand function for the old model
$b_H$	slope of the inverse consumer demand function for the new model in high demand state
$b_L$	slope of the inverse consumer demand function for the new model in low demand state
$c$	marginal production cost of the manufacturer
$\mu$	probability that the high demand state is realized
*	superscript for equilibrium with old model sales
**	superscript for equilibrium with new model sales under outright sales contract
***	superscript for equilibrium with new model sales under returns policy contract

**Table 2. Summary of equilibrium**

	old model sales under outright sales or returns policy (Propositions 1 and 3)	new model sales under outright sales contract (Proposition 2)	new model sales under returns policy contract (Proposition 4)
$q$ : order quantity	$q^* = (a - c)/(4b)$	$R^R = \frac{4b_L(\mu a_H - 4\mu b_H q^{**} - c)}{\mu(1-\mu)(8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2}))}$	$R^M = \frac{2b_L(a_H(2(1-\mu)b_H + \mu b_L) - (1-\mu)a_L b_H - 4b_H((1-\mu)b_H + \mu b_L)q^{***} - b_L c)}{\mu(1-\mu)(a_H - 2b_H q^{***})(a_H - a_L + 2(b_L - b_H)q^{***})(a_H(b_L - 2b_H) + b_H(a_L - 4(b_L - b_H)q^{***}))}$
$Q$ : sales quantity	$Q^* = (a - c)/(4b)$	$E(Q^{**}) = \mu q^{**} + (1 - \mu)a_L / (2b_L)$	$E(Q^{***}) = \mu q^{***} + (1 - \mu)(a_L - a_H + 2b_H q^{***}) / (2b_L)$
$r$ : wholesale price	$r^* = (a + c)/2$	$r^{**} = \mu a_H - 2\mu b_H q^{**} + \frac{(a_H - 2b_H q^{**})(c - \mu a_H + 4\mu b_H q^{**})(4b_L q^{**}(a_H - b_H q^{**}) - a_L^2)}{8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2})}$	$r^{***} = a_H - 2b_H q^{***}$
$p$ : retail price	$p^* = (3a + c)/4$	$E(p^{**}) = \mu(a_H - b_H q^{**}) + (1 - \mu)a_L / 2$	$E(p^{***}) = ((1 + \mu)a_H + (1 - \mu)a_L - 2b_H q^{***}) / 2$
$CE^R$ or $\pi^R$ : (certainty equivalent of) profit of retailer	$\pi^{R*} = (a - c)^2 / (16b)$	$CE^{R**} = \mu b_H q^{**2} + (1 - \mu)a_L^2 / (4b_L) + (\mu(1 - \mu)(16b_L^2 q^{**2}(a_H^2 - 4a_H b_H q^{**} + 3b_H^2 q^{**2}) + 8a_L^2 b_H b_L q^{**2} - a_L^4) / (32b_L^2))R^R$	$CE^{R***} = \mu b_H q^{2***} + (1 - \mu)(a_L - a_H + 2b_H q^{***})^2 / (4b_L) - \mu(1 - \mu)((a_H - a_L)^2 - 4b_H q^{***}(a_H - a_L + (b_L - b_H)q^{***}))^2 / (32b_L^2)R^R$
$CE^M$ or $\pi^M$ : (certainty equivalent of) profit of manufacturer	$\pi^{M*} = (a - c)^2 / (8b)$	$CE^{M**} = q^{**}(\mu a_H - 2\mu b_H q^{**} + \frac{(a_H - 2b_H q^{**})(c - \mu a_H + 4\mu b_H q^{**})(4b_L q^{**}(a_H - b_H q^{**}) - a_L^2)}{8a_H^2 b_L q^{**} + 4b_H q^{**}(a_L^2 + 8b_H b_L q^{**2}) - a_H(a_L^2 + 36b_H b_L q^{**2})} - c)$	$CE^{M***} = \mu(a_H - 2b_H q^{***} - c)q^{***} + (1 - \mu)(a_H - 2b_H q^{***})(a_L - a_H + 2b_H q^{***}) / (2b_L) - c q^{***} - (\mu(1 - \mu)(a_H - 2b_H q^{***})^2(a_H - a_L + 2(b_L - b_H)q^{***})^2 / (8b_L^2))R^M$
$CS$ : consumer surplus	$CS^* = (a - c)^2 / (32b)$	$CS^{**} = \mu b_H q^{**2} / 2 + (1 - \mu)a_L^2 / (8b_L)$	$E(CS^{***}) = \mu b_H q^{***2} / 2 + (1 - \mu)(a_L - a_H + 2b_H q^{***})^2 / (8b_L)$

Note: Note that equilibrium order quantity,  $q^{**}$  or  $q^{***}$ , among the endogenous variables is described in the implicit function of exogenous variables.