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Comment on "Gauss-Bonnet inflation"

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Recently, an interesting inflationary scenario, named Gauss-Bonnet inflation, was proposed by Kanti *et al.* [Phys. Rev. D 92, 041302 (2015); Phys. Rev. D 92, 083524 (2015)]. In the model, there is no inflaton potential, but the inflaton couples to the Guass-Bonnet term. In the case of quadratic coupling, they find inflation occurs with a graceful exit. The scenario is attractive because of the natural setup. However, we show there exists a gradient instability in the tensor perturbations in this inflationary model. We further prove the no-go theorem for Gauss-Bonnet inflation without an inflaton potential.

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I. INTRODUCTION

It is believed that the most promising candidate for the ultimate unified theory is superstring theory. The effective action stemming from superstring theory contains higher order curvature terms. Indeed, in four dimensions, the Gauss-Bonnet (GB) term appears as a one-loop string correction [1]. Motivated by this fact, Einstein-scalar–Gauss-Bonnet theory is studied, and nonsingular cosmological solutions are found [2–5]. The solutions have a super inflation phase where the Hubble parameter increases, which indicates the violation of the weak energy condition. Subsequently, cosmological perturbations in this background are investigated [6–12]. In the process, the so-called gradient instability is found for the first time in [7,8]. Nowadays, this is known as a useful criterion for model selections in various extensions of general relativity [13]. By applying this criterion, it has been shown that nonsingular solutions are unstable, in general, because of the violation of the weak energy condition [6–8,10,11].

Recently, Kanti *et al.* [14,15] analyzed the Einstein-scalar–Gauss-Bonnet theory with a quadratic coupling function and found that the theory contains inflationary solutions where the de Sitter phase possesses a natural exit mechanism and is replaced by linearly expanding Milne phases. Remarkably, there is no inflaton potential in this model. They also claimed that only the quadratic coupling leads to these results among monomial coupling functions. In this sense, the model is simple and unique. Hence, it is worth investigating the inflationary scenario, named Gauss-Bonnet inflation.

In order for this GB inflation to be viable, its predictions must be compatible with current observational data [16]. Hence, we need to calculate perturbations and compare their predictions with observations. First, we calculate tensor perturbations in the inflationary background following previous results [7,8]. Unfortunately, we find gradient instability in tensor perturbations in GB inflation. Hence, the model is not viable phenomenologically. We also extend this result to more general coupling functions and establish the no-go theorem for GB inflation without an inflaton potential.

The organization of the paper is as follows. In Sec. II, we review the cosmological inflationary solutions obtained by Kanti *et al.* [14,15]. In Sec. III, we numerically examine the dynamics of tensor perturbations and find the instability of the GB inflation. In Sec. IV, we prove a no-go theorem for GB inflation without an inflaton potential. As a by-product, we also present the stability condition for GB inflation with an inflaton potential. The final section is devoted to the conclusions.

II. REVIEW OF GAUSS-BONNET INFLATION

In this section, we review the main results of the papers [14,15]. In particular, it is shown there exists a quasi-de Sitter phase in the Einstein-scalar–Gauss-Bonnet theory with a quadratic coupling function in spite of the absence of an inflaton potential.

The action considered there is GB gravity with a scalar field ϕ, which coupled nonminimally with a coupling function $f(\phi)$ to gravity via the GB term R^2_{GB},

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\nabla\phi)^2 + \frac{1}{8}f(\phi)R^2_{GB}\right], \quad (1)$$

where $\kappa^2 \equiv 8\pi G$ is the gravitational coupling constant, g is a determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, and the GB term is given by

$$R^2_{GB} = R^{\mu\nu\rho\lambda}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2. \quad (2)$$

Note that there is no inflaton potential $V(\phi)$. The variation of this action with respect to the metric tensor and scalar field gives us the field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + P_{\mu\alpha\nu\beta}\nabla^{\alpha\beta}f = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2, \quad (3)$$

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and

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\partial^\mu\phi]+\frac{1}{8}f'R^2_{GB}=0, \tag{4}$$

where we defined $f'\equiv df/d\phi$, κ^2 is set to unity, and $P_{\mu\alpha\nu\beta}$ is defined as

$$P_{\mu\alpha\nu\beta}=R_{\mu\alpha\nu\beta}+2g_{\mu[\beta}R_{\nu]\alpha}+2g_{\alpha[\nu}R_{\beta]\mu}+Rg_{\mu[\nu}g_{\beta]\alpha}. \tag{5}$$

For a homogeneous and isotropic flat spacetime, the metric is described by a scale factor $a(t)$ as

$$ds^2=-dt^2+a^2(t)\delta_{ij}dx^idx^j. \tag{6}$$

Now, the explicit form of the field equations reads

$$6H^2(1+H\dot{f})=\dot{\phi}^2, \tag{7}$$

$$2(1+H\dot{f})(H^2+\dot{H})+H^2(1+\ddot{f})=-\frac{1}{2}\dot{\phi}^2, \tag{8}$$

$$\ddot{\phi}+3H\dot{\phi}-3f'H^2(H^2+\dot{H})=0, \tag{9}$$

where $H\equiv\dot{a}/a$ is the Hubble parameter and a dot denotes a derivative with respect to time.

Concretely, it is needed to specify the function $f(\phi)$. Kanti *et al.* [14,15] found that the quadratic coupling function $f=\lambda\phi^2$ with a coupling constant λ gives rise to inflationary solutions. In the early time, that is, in the strong gravity regime, the GB term dominates over the Ricci scalar term. Indeed, the dynamics of the Universe does not show any significant change as long as the scalar field takes very large values or λ takes a very large value even if the Ricci term is neglected. Therefore, the Ricci term is neglected for a moment. With this approximation, the unity terms inside the brackets in Eqs. (7) and (8) can be neglected. Thus, we obtain the equation

$$\dot{H}+H^2\left(1-\frac{H^2}{H^2_{\rm dS}}\right)=0, \tag{10}$$

where $H^2_{\rm dS}=-5/24\lambda$ as we shown in [14]. From now on, we assume $\lambda<0$.

Apparently, there exists a solution with $\dot{H}=0$. For this case, Eq. (10) can be integrated as

$$a(t)=a_0\exp(H_{\rm dS}t). \tag{11}$$

From the solution (11), we clearly see that inflation can happen due to the GB term. Moreover, the scalar field can be solved as

$$\phi=\phi_0\exp\left(-\frac{5}{2}H_{\rm dS}t\right). \tag{12}$$

Even for general cases, we can analytically integrate Eq. (10) as

$$H=\frac{H_{\rm dS}}{\sqrt{1+\frac{2C}{5}H^2_{\rm dS}a^2}}, \tag{13}$$

where C is a constant of integration. As we can see, as the Universe expands, the Hubble parameter decreases. Hence, we have to tune $\nu^{-2}\equiv 2CH^2_{\rm dS}/5=C/12\lambda$ so that inflation occurs for a sufficiently long time. In this case, the scale factor can be obtained by analytically solving Eq. (13) using the change of variable, $a=\nu\tan\omega$. The resultant solution is given by

$$\sqrt{a^2+\nu^2}+\nu\log\left(\frac{\sqrt{a^2+\nu^2}-\nu}{a}\right)=\sqrt{\frac{5}{2C}}(t+t_0), \tag{14}$$

where t_0 is a constant of integration as we can see in [14,15].

Because of the implicit solution for the scale factor a in Eq. (14), the explicit solution for the scalar field cannot be found explicitly as a function of time. Yet, we can obtain it as a function of the scale factor,

$$\phi=C_0(\frac{2C}{5})^{5/4}\frac{(a^2+\nu^2)^{5/4}}{a^{5/2}}, \tag{15}$$

where C_0 is another constant of integration as in [14,15]. From this equation, we can see that the scalar field initially decreases. In the regime $a^2\gg\nu^2$, however, the scalar field becomes constant.

We have also numerically solved Eqs. (7), (8), and (9) without any approximation. The time evolution of the scale factor in the term of the number of e-foldings, $N=\log(a)$,

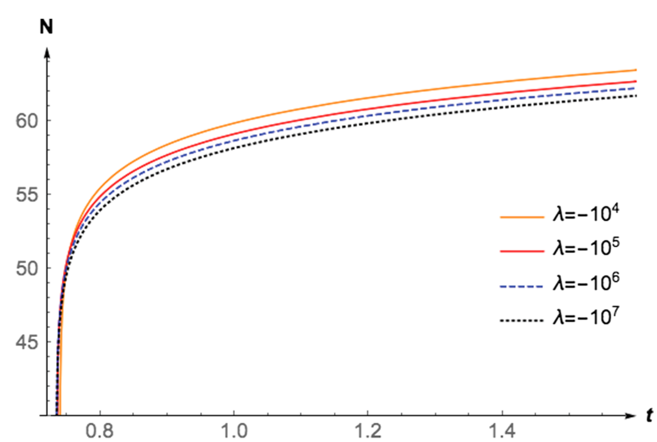

FIG. 1. Number of e-foldings: $N=\mathrm{Log}(a)$ is plotted as a function of time for various $\lambda<0$.

is plotted in Fig. 1. Here, we can see inflationary behavior and the subsequent decelerating expansion of the Universe. Notice that the initial inflationary period can be elongated by making ν (or λ) large.

III. INSTABILITY OF QUADRATIC COUPLING MODELS

In the previous section, it was shown that there exists inflation without an inflaton potential. Given the inflationary background, the next step is to calculate perturbations and check if the model is compatible with observational data. Unfortunately, however, the instability of the Gauss-Bonnet inflation is shown.

Here, we begin with tensor perturbations defined by

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j, \tag{16}$$

where $h^{ij}_{,j} = h^i_i = 0$. For the metric (16), we can deduce the quadratic action for the tensor perturbations as follows:

$$\begin{aligned} S = \frac{1}{8}\int d^4x a^3 \Big\{ &\Big[\dot{h}_{ij}\dot{h}^{ij} - \frac{1}{a^2}h_{ij,k}h^{ij,k} \\ &- (4\dot{H} + 6H^2 + \dot{\phi}^2)h_{ij}h^{ij}\Big] \\ &- \ddot{f}\Big[\frac{1}{a^2}h_{ij,k}h^{ij,k} + 2H^2 h_{ij}h^{ij}\Big] \\ &+ \dot{f}[-H\dot{h}_{ij}\dot{h}^{ij} + 4H(\dot{H} + H^2)h_{ij}h^{ij}]\Big\}, \end{aligned} \tag{17}$$

where we used the formula in the Appendix. Furthermore, using the background equations, we obtain

$$S = \frac{1}{8}\int d^4x a^3 \left[(1 + H\dot{f})\dot{h}_{ij}\dot{h}^{ij} - \frac{1}{a^2}(1 + \ddot{f})h_{ij,k}h^{ij,k}\right]. \tag{18}$$

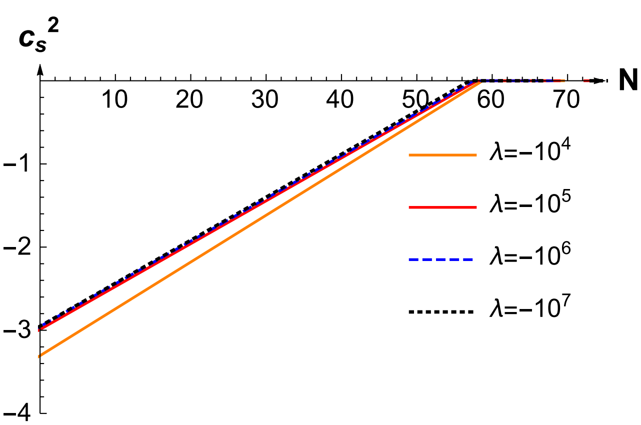

FIG. 2. The time evolution of the squared of the effective speed of sound c_s^2 of the model is plotted as a function of the number of e-foldings.

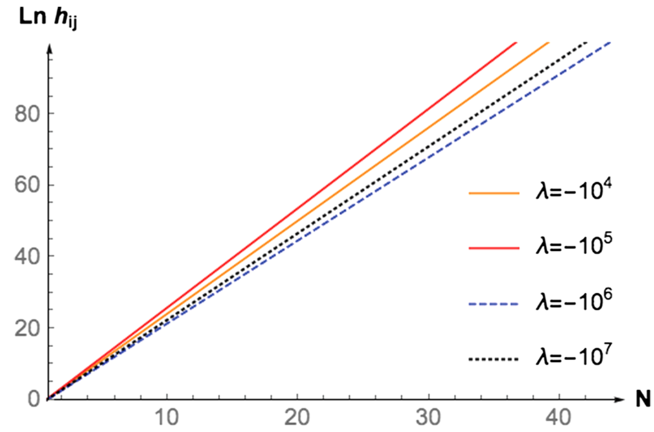

FIG. 3. The time evolution of the tensor perturbations h_{ij} is plotted as a function of the number of e-foldings. Here, we put $k = 1$.

This coincides with the action obtained in [8] with a simple replacement $f(\phi) = -\lambda\xi(\phi)/2$. Therefore, the equation for tensor perturbations can be found as follows:

$$\ddot{h}_{ij} + \left(3H + \frac{\dot{\alpha}}{\alpha}\right)\dot{h}_{ij} + \frac{k^2}{a^2}\frac{1 + \ddot{f}}{\alpha}h_{ij} = 0, \tag{19}$$

where $\alpha = (1 + H\dot{f})$ is defined and k is a wave number, as in [7] with $f(\phi) = -\lambda\xi(\phi)/2$. Note that the system contains a ghost if $\alpha < 0$. Since the existence of a ghost implies that the model is pathological, we must avoid this possibility. From Eq. (19), we can see that the stability of the system is determined by the last term, while the second term is a friction term. The model is stable for tensor perturbation when the squared effective speed of sound is positive,

$$c_s^2 \equiv \frac{1 + \ddot{f}}{\alpha} > 0. \tag{20}$$

Therefore, we can check the stability of tensor perturbations of the model by checking the value of c_s^2. From the equations of motion (7)–(9), we can get the time evolution for c_s^2 for several λ values can be seen in Fig. 2. Because c_s^2 is negative during inflation, we can conclude that the system is unstable under tensor perturbations.

In order to see the instability of the system more explicitly, we numerically solved the time evolution of the tensor perturbations. From Fig. 3, we can see that the tensor perturbations grow rapidly. Therefore, the system is unstable under tensor perturbations.

IV. NO-GO THEOREM FOR GAUSS-BONNET INFLATION

In the previous section, we have shown the instability of the GB theory for the specific coupling function

$f(\phi) = \lambda\phi^2$. In this section, we extend this result to more general coupling functions and prove a no-go theorem for GB inflation without an inflaton potential.

We start with the action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \frac{1}{8}f(\phi)R^2_{GB}\right], \tag{21}$$

where we have incorporated an inflaton potential into the model. The field equations for this action are similar to Eqs. (7)–(9). The Hamiltonian constraint reads

$$6H^2(1 + H\dot{f}) = \dot{\phi}^2 + 2V(\phi), \tag{22}$$

and the Einstein equation gives us the following evolution equation

$$2(1 + H\dot{f})(H^2 + \dot{H}) + H^2(1 + \ddot{f}) = -\frac{\dot{\phi}^2}{2} + V(\phi), \tag{23}$$

Finally, we obtain the scalar field equation,

$$\ddot{\phi} + 3H\dot{\phi} - 3f'H^2(H^2 + \dot{H}) + \frac{dV}{d\phi} = 0. \tag{24}$$

From the background field equations (22) and (23), we get the relation

$$\begin{aligned}(1 + \ddot{f}) &= \frac{2V}{H^2} - (1 + H\dot{f})\left(\frac{2\dot{H}}{H^2} + 5\right)\\ &= \frac{2V}{H^2} + \alpha(2\epsilon_H - 5),\end{aligned} \tag{25}$$

where we used α defined before and the slow-roll parameter $\epsilon_H = -\dot{H}/H^2$. Therefore, the final expression of the action can be written as follows:

$$S = \frac{1}{8}\int d^4x a^3\alpha\left[\dot{h}_{ij}\dot{h}^{ij} - \frac{1}{a^2}\left\{(2\epsilon_H - 5)\alpha + \frac{2V}{H^2\alpha}\right\}h_{ij,k}h^{ij,k}\right], \tag{26}$$

and the Hamiltonian is given by

$$H = \int d^4x\left[\frac{2\pi^{ij}\pi_{ij}}{a^3\alpha} + \left[\frac{Va}{4H^2} + \frac{1}{8}(2\epsilon_H - 5)a\alpha\right]h_{ij,k}h^{ij,k}\right]. \tag{27}$$

From the Hamiltonian in Eq. (27), we can see that the model will be stable if $\alpha > 0$ and

$$\left[\frac{V}{4H^2} + \frac{1}{8}(2\epsilon_H - 5)\alpha\right] > 0. \tag{28}$$

Now, let us consider models without an inflaton potential, namely $V = 0$. Suppose that quasi-de Sitter inflation occurs for some coupling function $f(\phi)$. Because of the quasi-de Sitter inflation, the slow-roll parameter satisfies the condition $\epsilon_H \ll 1$. Hence, we always have the inequality $2\epsilon_H - 5 < 0$. Thus, the condition (28) is satisfied only for the models $\alpha < 0$. However, that implies the existence of the ghost. Namely, inflationary solutions must be unstable or contain a ghost. In any case, the model is not phenomenologically allowed. Thus, we proved a no-go theorem for GB inflation without an inflaton potential. Therefore, the cosmological model with the GB term without an inflaton potential cannot be a viable model.

Even when an inflaton potential, V, is included, the stability condition (28) still remains useful. Indeed, it can be used to select viable models. In fact, because of the variety of the forms of potential functions, it is possible to find the stable cosmological model with the GB term.

V. CONCLUSION

We studied Einstein-scalar–Gauss-Bonnet theory with a nonminimal coupling function. In particular, we focused on inflation without an inflaton potential. In this model, in the case of quadratic coupling, inflation occurs and possesses a natural exit mechanism. The scenario is attractive because of the simple setup. This motivated us to examine the perturbative stability of the inflationary background solutions. We numerically solved the dynamics of tensor perturbations and found the gradient instability [8] in tensor perturbations in the inflationary model. We further extended this result and proved the no-go theorem for the GB inflation without an inflaton potential. Thus, we have shown that the GB inflation without an inflaton potential is not phenomenologically viable.

Of course, if we incorporate an inflaton potential into the model, there are stable inflationary solutions [17]. We have also given the stability criterion for this class of models. For stable models, we can discuss phenomenological predictions and compare them with observational data. In fact, at high energy, it is natural to consider the Gauss-Bonnet term as a correction [18]. Hence, it is still intriguing to study the Einstein-scalar–Gauss-Bonnet theory in the cosmological context.

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APPENDIX

Here, we list the necessary formulas to calculate the quadratic action for tensor perturbation in Chapter III of [8]:

$$
\begin{aligned}
R^{(2)0i}_{0j}R^{(0)0j}_{0i} &= -(H^2+\dot{H})\left[Hh^{ik}\dot{h}_{ik}+\frac{1}{4}\dot{h}^{ik}\dot{h}_{ik}+\frac{1}{2}h^{ik}\ddot{h}_{ik}\right]\\
R^{(2)ij}_{kl}R^{(0)kl}_{ij} &= -2H^2\left[\frac{1}{4a^2}h^{jk,m}h_{jk,m}+\frac{1}{4}\dot{h}_{ij}\dot{h}^{ij}+2Hh^{ij}\dot{h}_{ij}\right]\\
R^{(1)0i}_{0j}R^{(1)0j}_{0i} &= H^2\dot{h}^{ik}\dot{h}_{ik}+H\dot{h}^{ik}\ddot{h}_{ik}+\ddot{h}^{ik}\ddot{h}_{ik}\\
R^{(1)0i}_{jk}R^{(1)jk}_{0i} &= -\frac{1}{a^2}\dot{h}^{ik,j}\dot{h}_{ik,j}\\
R^{(1)ij}_{kl}R^{(1)kl}_{ij} &= \frac{1}{4a^2}\nabla^2h^{ik}\nabla^2h_{ik}-\frac{2H}{a^2}\dot{h}^{ik}\nabla^2h_{ik}+H^2\dot{h}^{ik}\dot{h}_{ik}\\
R^{(2)0}_{0}R^{(0)0}_{0} &= -3(H^2+\dot{H})\left[\frac{1}{4}\dot{h}^{ij}h_{ij}+Hh^{ij}\dot{h}_{ij}+\frac{1}{2}h^{ik}\ddot{h}_{ij}\right]\\
R^{(2)i}_{j}R^{(0)j}_{i} &= -(3H^2+\dot{H})\left[\frac{1}{2}h^{ik}\ddot{h}_{ik}+3Hh^{ik}\dot{h}_{ik}+\frac{1}{2}\dot{h}^{ik}\dot{h}_{ik}+\frac{1}{4a^2}h^{jk,m}h_{jk,m}\right]\\
R^{(1)i}_{j}R^{(1)j}_{i} &= \frac{1}{4}\ddot{h}^{ik}\ddot{h}_{ik}+\frac{1}{4a^2}\nabla^2h^{jk}\nabla^2h_{jk}+\frac{9}{4}H^2\dot{h}^{ik}\dot{h}_{ik}-\frac{1}{2a^2}\ddot{h}_{jk}\nabla^2h^{jk}+\frac{3}{2}H\ddot{h}^{ik}\dot{h}_{ik}-\frac{3H}{2a^2}\dot{h}_{jk}\nabla^2h^{jk}\\
R^{(0)}R^{(2)} &= -6(2H^2+\dot{H})\left[\frac{3}{4}\dot{h}^{jk}\dot{h}_{ik}+4Hh^{jk}\dot{h}_{jk}+h^{jk}\ddot{h}_{jk}+\frac{1}{4a^2}h^{jk,m}h_{jk,m}\right].
\end{aligned}
\tag{A1}
$$

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