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# When and what wholesale and retail prices should be set in multi-channel supply chains?

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## ABSTRACT

This paper investigates the optimal timing and level of wholesale and retail prices set in multi-channel supply chains, where a manufacturer produces and sells products to retailers that compete to resell the products, by applying the framework of an observable delay game devised in noncooperative game theory. We assume that one manufacturer and two retailers, which constitute a two-echelon supply chain, can select not only the levels of wholesale and retail prices, respectively, but also the timing of pricing. Our analysis of a dynamic game composed of discrete periods provides two useful conclusions for operational decision support. First, the manufacturer must simultaneously set its wholesale prices for products that are sold to separate retailers at the same time. Second, in contrast to the simultaneous price setting by the manufacturer, the retailers must sequentially set respective retail prices at different times; thus, the retailers should stagger their timings for setting retail prices. We formally demonstrate that these timings of pricing decisions by the manufacturer and the retailers constitute a subgame perfect Nash equilibrium in the dynamic noncooperative game played by the three supply chain parties. Consequently, these conclusions can be used as practical guidelines for supply chain members choosing optimal timing of pricing.

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## 1. Introduction

Nowadays, the management of multi-channel supply chains has become an increasingly critical issue for firms involved in supply chains in a variety of industries. In particular, the prevalence of information and communication technologies (ICT) among both consumers and firms has enabled potential customers to compare prices of a specific product more easily between multiple retailers with lower search costs by referring to online information. In addition, manufacturing firms need to distribute their products to multiple physical retailers to cover wide geographical areas where their potential consumers are dispersed. When a firm employs a multi-channel supply chain system, the timing of pricing in separate supply chains is a crucially important problem, because the pricing timing can trigger subsequent strategic reactions by other supply chain members by influencing their pricing decisions.

In the operational research and management science (OR/MS) literature, the question of "what" wholesale and retail prices a manufacturer and a retailer should respectively choose for the purpose of supply chain coordination has commanded significant

attention and been discussed extensively from both academic and practical perspectives. However, research investigating "when" wholesale and retail prices should be determined is missing in the existing literature, even though this is a critical and practical issue for manufacturers and retailers in multi-channel supply chains. Our model constructed in this paper provides an answer to this question: we investigate the optimal timing of wholesale and retail pricing set in multi-channel supply chains, where a manufacturer produces and wholesales products to multiple retailers that compete to sell the products by introducing the framework of an observable delay game devised in the literature of noncooperative game theory (e.g., Amir & Stepanova, 2006; Hamilton & Slutsky, 1990; van Damme & Hurkens, 1996, 1999, 2004). We assume that one manufacturer and two retailers constituting a two-echelon supply chain can choose not only the levels of wholesale and retail prices, respectively, but also the timing of pricing. Our analysis of a dynamic game composed of discrete periods provides the following two useful conclusions for operational decision support. First, the manufacturer must simultaneously set its wholesale prices of products that are sold to separate retailers at the same time. Otherwise, the manufacturer may face the risk that the equilibrium determining a stable sequence of pricing by retailers disappears. Second, in contrast to the simultaneous price setting

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by the manufacturer, the retailers must sequentially set respective retail prices at different times; thus, the retailers should stagger their timings of setting retail prices. In other words, they must never decide retail prices simultaneously. We formally demonstrate that these pricing decision timings by the manufacturer and retailers constitute a subgame perfect Nash equilibrium (SPNE) in the dynamic noncooperative game played by the three supply chain parties. Consequently, these conclusions can be used as practical guidelines for supply chain members choosing optimal timing of pricing.

The basic logic behind the results summarized above is as follows. Existing game theory literature has shown that a second-mover advantage arises in price competition under very general conditions; i.e., a player involved in price competition is inclined to set the price later (e.g., see Gal-Or, 1985). Due to the second-mover advantage, simultaneous moves by players tend to become unstable in the equilibrium of price competition. Indeed, the literature demonstrates that total profits earned by competitors under price competition are higher when they play a sequential move game than when they play a simultaneous move game (van Damme & Hurkens, 2004). As a result, sequential, rather than simultaneous, decisions by the retailers to set retail prices occur in equilibrium. On the other hand, the manufacturer has an incentive to set its wholesale prices at the earliest timing to ensure optimal wholesale prices by predicting subsequent strategic responses by the retailers.

One unique feature of an observable delay game model is the consideration of all possible orders of decision-making by all players in the game even if the orders are complicated. Because one manufacturer and two retailers are involved in the observable delay game in our model, such possible orders include the situation in which first, a manufacturer announces a price to one retailer; second, that retailer announces its retail price; and third, the manufacturer announces a price to the competing retailer. Such a complicated decision sequence can occur when a manufacturer sells a product first through specific retailers and subsequently through other retailers because the manufacturer takes a strategy of gradually developing its distribution channels. As a real business case, Gabrielsson, Kirpalani, and Luostarinen (2002) detail the case of channel development strategy conducted by Compaq, which sold personal computers (PCs) to a wide range of consumers and held the largest market share in the 1990s. Specifically, Compaq used an indirect sales channel strategy, consisting of authorized dealers (resellers), when it entered the European PC market. In 1984, Compaq had 330 international dealers in 13 countries worldwide and three foreign sales subsidiaries, all of which were located in Europe (United Kingdom, France, and Germany). In 1990, there were 1600 European dealers located in 21 countries and 13 European sales subsidiaries, meaning that Compaq gradually increased the number of retailers who dealt in its PCs. A manufacturer such as Compaq in this case needs to first determine a wholesale price of a product applied to a retailer, which deals in the product and hence sets its retail price in an earlier period, and subsequently determine a wholesale price applied to another retailer, which starts to deal in the product in a later period. In addition, if a manufacturer has sold a product through some retailers and subsequently starts to sell the product to other retailers because of gradual development of distribution channels in such real business environments, the latter retailers cannot help allowing the manufacturer to announce the wholesale price to the former retailers earlier. Therefore, while optimal timing of pricing is a generally important issue for supply chain members, the investigation of the timing is essential, especially when a manufacturer gradually develops new distribution channels.

Another feature associated with the framework of the observable delay game is that each player chooses one of the discrete

periods as its decision timing, as well as the decision variable itself. Such a timing decision is particularly important when a manufacturer markets a new product. For example, if a manufacturer markets a new product as early as possible, the manufacturer sets the wholesale price of the product in a particular period, and then, after observing the wholesale price, retailers set retail prices in the following period. In addition, if a manufacturer accelerates the timing of releasing a new product into a market before a rival manufacturer releases a similar product, the product launched earlier may command more attention from media and consumers, thereby improving profit. By contrast, if a manufacturer postpones marketing a new product for some reason, the manufacturer needs to set the wholesale price of its product later, and accordingly, retailers must postpone setting retail prices in the period following the observation of the wholesale price. There are several real-life cases in which manufacturing firms have strategically accelerated or delayed the release of new products. A typical recent case in which a company accelerated the timing of releasing a new product is found in the smartphone market. Samsung Electronics Co. and Apple Inc., which supply the Galaxy Note and iPhone, respectively, hold a large share of the global smartphone market. While Apple launched the iPhone 5, 5s/5c, and 6/6Plus models in each September of the three years between 2012 and 2014, Samsung released Galaxy Note II, 3, and 4 around the end of September or early October in each of the same three years, just after each of the new generations of iPhone was launched in each year. In 2015, however, Samsung released the Galaxy Note 5 in August, before Apple released the iPhone 6s/6sPlus in September. Cheng (2015) and Lee (2015) point out that Samsung strategically moved forward the autumn launch of Galaxy Note 5 to mid-August to command attention from the media and consumers before the release of iPhone 6s/6sPlus, which can be interpreted as a strategic early launch of a new model to compete with an archrival. Another recent case is found in the smartwatch market. Apple released its iWatch in September 2014, whereas the test model of iWatch had been shown to the public prior to the release, which indicates a delay in the marketing of the product (Phillips, 2014).<sup>1</sup> These cases indicate that the optimal decision timing regarding the setting of the discrete pricing periods assumed in the framework of the observable delay game is important, especially when a manufacturing firm develops its distribution channels or markets a new product, because the firm may intentionally adjust the timing to make its decision.

To the best of the author's knowledge, no previous research has applied the observable delay game framework to the decision problem of wholesale and retail prices in multi-channel supply chains. Hence, we address the pricing problem from a different viewpoint compared with the existing OR literature. Consequently, the application of the observable delay game to the pricing problem in multi-channel supply chains is also an original contribution of this study.

The remainder of the paper is structured as follows. Section 2 provides a review of the OR/MS literature related to game-theoretic models for supply chain management. Basic settings of our model are outlined in Section 3. In Section 4, we investigate the strategic behaviors undertaken by supply chain members and identify the relevant equilibrium that determines the optimal choice of both timing and level of the prices. Section 5 draws managerial implications based on the equilibrium outcomes. The final section concludes our paper.

<sup>1</sup> Pan (2017) provides several business cases in which manufacturers strategically control the timing of releasing new products.

## 2. Literature review

Thus far, a considerable number of OR studies have applied a game-theoretic approach to examine supply chain management problems under various economic environments (e.g., Anderson & Bao, 2010; Atkins & Liang, 2010; Biswas, Avittathur, & Chatterjee, 2016; Chen & Xiao, 2009; Chen, Liang, Yao, & Sun, 2017; Choi, 1991; Giovanni & Zaccour, 2014; Groznik & Heese, 2010; Huang & Swaminathan, 2009; Ingene & Parry, 1995, 1998; Jeuland & Shugan, 1983; Jørgensen & Zaccour, 2014; Kopel & Löffler, 2008; Kurata, Yao, & Liu, 2007; Li, Lin, Xu, & Swain, 2015; Matsui, 2012, 2016, 2017; McGuire & Staelin, 1983; Moorthy, 1988; Parlar & Weng, 2006; Qing, Deng, & Wang, 2017; Rajagopalan & Xia, 2012; Xiao & Choi, 2009; Xiao & Yang, 2008; Xiao & Qi, 2010; Xie, Zhou, Wei, & Zhao, 2010; Yan, Xiong, Chu, Li, & Xiong, 2018; Yan, Zhao, & Lan, 2017; Yang & Zhou, 2006; Yao, Leung, & Lai, 2008; Zhang, Bell, Cai, & Chen, 2010). McGuire and Staelin (1983) construct a game-theoretic model of two manufacturers who sell competing brands through retailers. Assuming that the manufacturers encounter price competition, they demonstrate that vertical separation is the optimal strategy for a manufacturer compared with vertical integration if the manufacturer adopts a two-part tariff contract for a retailer who exclusively resells the product. The rationale behind their result is that the delegation of the decision on pricing to a retailer through vertical separation moderates the reaction of the other competitor involving a manufacturer and a retailer when the strategic variable for firms is the price. Moorthy (1988) shows that a condition for the result in McGuire and Staelin (1983) is that the prices chosen by firms are strategic complements. His results show that a key factor for the vertical separation between a manufacturer and a retailer to take place in equilibrium is not the substitutability between the manufacturers' products but the nature of the strategic dependencies between competing firms. Ingene and Parry (1995) consider a two-echelon supply chain, in which one manufacturer and two retailers behave noncooperatively. They explore channel coordination by a manufacturer that sells its product through competing retailers and that is required to treat the retailers equally. They show that an appropriately controlled quantity-discount schedule enables the multi-channel system to achieve the same profits as earned by a vertically integrated system. They also demonstrate the existence of a schedule of two-part tariffs that replicate the results of a vertically integrated system. Following Ingene and Parry's (1995) game-theoretic model on multi-channel management, we also assume a two-echelon supply chain composed of one manufacturer and two retailers. Parlar and Weng (2006) investigate how the coordination of production and pricing decisions improves the position of a firm in a duopolistic price competition. They show that coordination by a firm's marketing and production departments enables the firm to increase its profitability, especially when market conditions are unfavorable. Yao, Leung, and Lai (2008) examine a revenue-sharing contract for the coordination of a supply chain involving one manufacturer and two competing retailers that face stochastic demand before a selling season. They find that providing a revenue-sharing contract by the manufacturer can achieve better performance than a contract that stipulates only the price. Moreover, they show that the profits generated under the revenue-sharing contract are different between supply chain members due to the impact of demand variability. Anderson and Bao (2010) also investigate price competition by comparing the two organizational forms of vertical integration and vertical separation. Assuming that a fixed amount of demand is allocated to oligopolistic firms as their underlying market shares, they demonstrate that the coefficient of the variation of the market share determines whether vertically separated channels outperform integrated channels. Considering both economies of scale and the intensity of competition, Atkins and Liang (2010) generalize

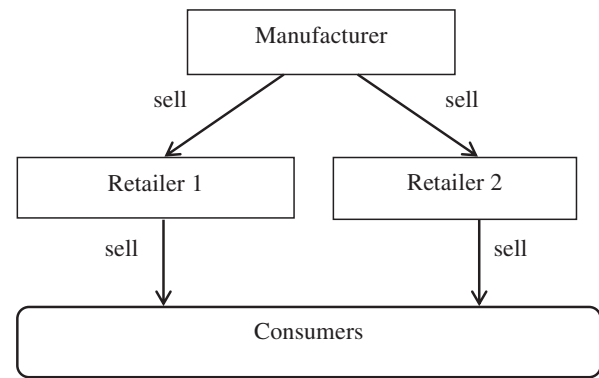


Fig. 1. Supply chain structure.

Note: Consumers perceive that products are horizontally differentiated between the two retailers.

the result of McGuire and Staelin (1983). They find that a primary factor that determines equilibrium channel structures is competitive intensity, even under the presence of scale economies. Overall, the above studies aim to identify the optimal wholesale and retail prices under cooperative or noncooperative environments, which can be described by game theory. This line of OR/MS research, which constructs game-theoretic supply chain management models, is most closely associated with this paper.

Another line of research is related closely to the present paper; i.e., the game theory literature investigating the endogenous timing of decision-making. In a noncooperative game, whether players make their decisions simultaneously or sequentially is of importance because a different order of moves often leads to substantially different results. For instance, Gal-Or (1985) shows that when two identical players make sequential decisions in a non-cooperative game, the player making its decision first (second) achieves higher payoff than the player who makes its decision second (first) if the slopes of the players' reaction functions are downward (upward). Stated differently, there occur first- (second-) mover advantages if the strategy choices of players are negatively (positively) related. Because the decision timing accompanies this type of advantage, it is recognized that simultaneity and sequentiality of moves, as well as the order of moves by the players in the sequentiality case, need to be endogenously chosen by players (e.g., see Hamilton & Slutsky, 1990; van Damme & Hurkens, 1996, 1999, 2004). Stated differently, the order of moves needs to reflect players' inherent incentives in the absence of an exogenously given timing structure in a noncooperative game.

Despite the significant volume of OR/MS research on pricing in supply chains from a game-theoretic perspective, a review of the literature suggests that previous work incorporating the choice of optimal decision timing into the pricing issue in multiple retail channels is lacking, even though the timing of setting wholesale and retail prices is a crucial issue for supply chain management. This paper addresses the problem of the endogenous sequence of moves in pricing by employing the framework of the observable delay game (e.g., Hamilton & Slutsky, 1990; van Damme & Hurkens, 1999). Therefore, it is worth noting that this paper is the first to apply the observable delay game framework to the wholesale- and retail-pricing problem in multi-channel supply chain management.

## 3. Model description

In this section, we describe our model. Table 1 lists the notations used in the model. We follow previous major multi-channel management studies (e.g., Ingene & Parry, 1995, 1998) in assuming the supply chain structure in our model outlined below. Suppose that a manufacturer produces two products, denoted as Product 1

**Table 1**  
Notations.

$p$	retail price
$q$	quantity
$r$	wholesale price
$c$	marginal production cost
$a$	positive constant greater than $c$
$A$	$a - c$
$b$	positive constant
$\theta$	substitutability of products
$i$	index of the retailer or the product; $i = 1$ or $2$
$j$	index of the retailer or the product that is different from $i$ ; $(i, j) = (1, 2)$ or $(2, 1)$
$t(x)$	period when the price denoted by $x$ is chosen
$\Pi$	profit for the manufacturer
$\pi$	profit for a retailer
$\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix}$	combination of timing strategies where the manufacturer respectively sets the wholesale prices of Product $i$ and Product $j$ in periods of $t(r_i)$ and $t(r_j)$ , Retailer $i$ sets the retail price of Product $i$ in period of $t(p_i)$ , and Retailer $j$ sets the retail price of Product $j$ in period of $t(p_j)$

and Product 2, at a variable cost of  $c_1$  and  $c_2$  per unit, respectively, and wholesales the products to two retailers, as illustrated in Fig. 1.<sup>2</sup> The retailers respectively resell the two products to end consumers. We define the retailer that deals with Product 1 as Retailer 1 and the retailer that deals with Product 2 as Retailer 2. The manufacturer determines the levels of the wholesale prices of Products 1 and 2, which are denoted by  $r_1$  and  $r_2$ , and the timing of setting the wholesale prices.<sup>3</sup> Retailers 1 and 2 choose the levels of the retail prices, which are denoted by  $p_1$  and  $p_2$ , and the timings of pricing.

Next, we assume that the inverse demand function for the products is:

$$p_i = a_i - b_i q_i - \theta q_j \quad (i, j) = (1, 2), (2, 1), \quad (1)$$

where  $p$  and  $q$  are the retail price and quantity. Henceforth,  $i$  represents either 1 or 2. Meanwhile,  $(i, j)$  represents either  $(1, 2)$  or  $(2, 1)$  when both variables are simultaneously present. Therefore, subscripts  $i$  and  $j$  attached to  $p$  and  $q$  signify Product  $i$  and Product  $j$ , respectively.<sup>4</sup> The parameter  $\theta$  measures the degree of substitution between the products. Because Eq. (1) suggests that the two markets become independent as  $\theta$  decreases, a lower value of  $\theta$  means that consumers perceive that products are more horizontally differentiated between the retailers. The parameters  $a_i (> c_i)$  and  $b_i$  represent the demand base and the price sensitivity of Product  $i$ ,

respectively. We assume that the exogenous parameters satisfy the following two inequalities:

$$b_i > \theta, \quad (i = 1, 2) \quad (2)$$

$$(a_i - c_i)/(a_j - c_j) < 2b_i/\theta - \theta/b_j. \quad (i, j) = (1, 2), (2, 1) \quad (3)$$

Inequality (2) indicates that the price sensitivity for own product is higher than that for the other product, which is considered a natural assumption. Meanwhile, Inequality (3) ensures that the firm sells products to both the two retailers in equilibrium.<sup>5</sup>

The inverse demand function of Eq. (1) can be restated as the following demand function:

$$q_i = (b_j(a_i - p_i) - \theta(a_j - p_j))/(b_i b_j - \theta^2) \quad (i, j) = (1, 2), (2, 1). \quad (4)$$

Based on the settings described above, we construct an observable delay game model composed of two stages. In stage one, the manufacturer and the two retailers announce the period when they will choose prices and make a commitment to the choice before actually doing so. In stage two, after the announcement, the manufacturer and the retailers choose their prices while knowing when the other players will choose their prices. Because there are four control variables,  $r_1$ ,  $r_2$ ,  $p_1$ , and  $p_2$ , we assume that the second stage consists of four periods.<sup>6</sup> Namely, the manufacturer determines the wholesale prices,  $r_1$  and  $r_2$ , and Retailer  $i$  ( $i = 1, 2$ ) determines the retail price,  $p_i$ , in one of these four periods. All of the four prices are observable to the three firms.

To indicate the timing of pricing, let  $t(x)$  denote the timing of choosing the price of  $x$ , which corresponds to either  $r_1$ ,  $r_2$ ,  $p_1$ , or  $p_2$ . Because there are four periods in which the manufacturer and the retailers choose to determine each price, the four variables can take timings of 1, 2, 3, or 4. Specifically, in the first stage of the game, the manufacturer chooses  $t(r_1)$  and  $t(r_2)$  from  $\{1, 2, 3, 4\}$ , Retailer 1 chooses  $t(p_1)$  from  $\{1, 2, 3, 4\}$ , and Retailer 2 chooses  $t(p_2)$

<sup>2</sup> For tractability of the model, we assume the existence of only two retailers to examine the incentives of retailers in a horizontal relationship, following typical game-theoretic multi-channel management models presented in Ingene and Parry (1995, 1998). Notice that this assumption includes cases in which not only are two different products wholesaled but also the same products are wholesaled. Specifically, we may consider the case where the two products are regarded as undifferentiated and thus identical at the manufacturer level by additionally assuming that  $c_1 = c_2$  holds. Even if the two retailers deal in the same products in this case, consumers will perceive that the two products are differentiated at the retail level as long as the retailers are distantly located (Hotelling, 1929).

<sup>3</sup> This assumption indicates that the possibility of wholesale price discrimination is taken into consideration in our model. Garcia and Janssen (2017) state that laws pertaining to the price discrimination are "rarely enforced when it comes to wholesale price discrimination as it may be difficult to prove that competition is harmed or (as wholesale prices are not generally observed) it may be difficult to establish that a manufacturer effectively applies wholesale price discrimination." Consistent with this statement, previous empirical studies overall detect wholesale price discrimination. Villas-Boas (2009) provides empirical evidence that manufacturers sell packaged coffee at different wholesale prices to supermarket chains in Germany. Moreover, Coloma (2003) finds that geographic wholesale price discrimination prevails in the Argentine gasoline market. Chen and Hwang (2014) refer to other empirical studies that detect wholesale price discrimination. Because the literature has provided substantial empirical evidence of the prevalence of wholesale price discrimination, our model allows for the possibility of wholesale price discrimination.

<sup>4</sup> The inverse demand function of Eq. (1) can be derived from the consumer behavior that maximizes utility subject to a budget constraint if the utility function for a consumer is formulated as  $a_1 q_1 + a_2 q_2 - (b_1 q_1^2 + b_2 q_2^2 + 2\theta q_1 q_2)/2$ . For more detail of the derivation process, see Ingene and Parry (2004) (Chapter 11).

<sup>5</sup> If this inequality was not met, the manufacturer would wish to distribute products to only one retailer in equilibrium so as to avoid too fierce competition between the two competing retailers that causes the optimal level of  $q_1$  or  $q_2$  to fall to 0. We exclude the situation where the manufacturer distributes products to only one retailer, because such a situation indicates that the supply chain need not consider the optimal sequence of pricing timing and is thus outside the scope of this paper.

<sup>6</sup> Lu (2006) models an observable delay game in which three control variables are chosen in three periods. Given that there are four strategic variables, i.e.,  $r_1$ ,  $r_2$ ,  $p_1$ , and  $p_2$ , in our model, we employ the setting of four discrete periods to ensure consistency with previous research, such as Lu (2006). This setting is relaxed in Section 5 so that the number of periods can be more than four, in one of which each firm sets its selling price.

from {1, 2, 3, 4}. Following previous major game-theoretic distribution channel and supply chain management studies (e.g., [Atkins & Liang, 2010](#); [Ingene & Parry, 1995, 1998](#); [Jeuland & Shugan, 1983](#); [McGuire & Staelin, 1983](#)), we assume that the timing variables must satisfy  $t(r_i) < t(p_i)$ , such that Retailer  $i$  sets the retail price of Product  $i$  after observing the wholesale price of the product set by the manufacturer.

Because an essential assumption for our observable delay game model is the perfect observability of not only prices but also the timing of pricing as described above, we elaborate on why this specific assumption is employed in our model in the context of supply chain management research. First, all prices, including wholesale prices and retail prices, are observable to all supply chain parties, which implies that a retailer knows the wholesale price announced to another competing retailer. This assumption has been employed in a substantial number of previous studies that construct supply chain management models in the OR literature. More specifically, there are a number of previous OR papers that examine the "power structure" in supply chains (e.g., [Edirisinghe, Bichescu, & Shi, 2011](#); [Huang, Ke, & Wang, 2016](#); [Luo, Chen, Chen, & Wang, 2017](#); [Pan, Lai, Leung, & Xiao, 2010](#); [Wei, Zhao, & Li, 2013](#); [Wu, Chen, & Hsieh, 2012](#); [Yu, Cheong, & Sun, 2017](#)). In these papers, a channel member which makes its decision in the first move is regarded as exerting more power over other supply chain members. Based on this notion, the line of research on the power structure examines how the difference in the sequence of decision moves of supply chain members leads to their different profits. Such studies that consider a variety of sequences assume that wholesale prices are observable and thus each retailer knows all the wholesale prices.<sup>7</sup> Second, the decision timing is also observable to all supply chain members; that is, every supply chain member knows when other members will make their price choices in stage two. This assumption has also been employed in important previous studies that construct game-theoretic supply chain management models. For example, previous research that examines the power structure of the sequence of moves between the supply chain members mentioned above assumes that the manufacturer and the retailers choose their prices while knowing when the other players will choose their prices (e.g., [Wu et al., 2012](#)). In summary, we follow previous OR studies to employ the assumption that each supply chain member observes the timing of pricing as well as prices themselves.

#### 4. Results

Based on [Eq. \(4\)](#), profit for Retailer  $i$ ,  $\pi_i$ , and profit for the manufacturer,  $\Pi$ , are stated as:

$$\begin{aligned}\pi_i &= (p_i - r_i)q_i \\ &= (p_i - r_i)(b_j(a_i - p_i) - \theta(a_j - p_j)) / (b_i b_j - \theta^2) \\ (i, j) &= (1, 2), (2, 1)\end{aligned}\quad (5)$$

$$\Pi = \sum_{i=1}^2 (r_i - c_i)q_i. \quad (6)$$

Using [Eqs. \(5\) and \(6\)](#), we derive the respective profits for the manufacturer and the retailers by the timing strategy at stage one

<sup>7</sup> Specifically, the SSS model in [Wu et al. \(2012, p. 268\)](#), in which there are one upstream manufacturer and two downstream retailers, considers the following sequence: (1) the manufacturer sets the wholesale prices for the two retailers, (2) Retailer 1 chooses its retail price knowing both wholesale prices, and (3) Retailer 2 chooses its retail price knowing both wholesale prices. Moreover, the MSN model in [Huang et al. \(2016, p. 15\)](#) also assumes that there are one upstream manufacturer and two downstream retailers. They consider the following sequence: (1) the manufacturer sets the wholesale prices, (2) the two retailers choose their retail prices knowing both of the two wholesale prices.

of this game. Because stage two consists of four periods at which each of the four control variables,  $r_1$ ,  $r_2$ ,  $p_1$ , and  $p_2$ , is set, we need to consider  $4^2 \times 4 \times 4 = 256$  combinations of the sequence of timings so as to calculate all possible payoffs.<sup>8</sup> However, the constraints of  $t(r_1) < t(p_1)$  and  $t(r_2) < t(p_2)$  indicate that it suffices to investigate only 36 combinations of the sequence of feasible timings. For example,  $t(r_i)$  cannot equal 4, because then there would be no candidates for periods for the Retailer  $i$  to set its retail price. The following proposition shows the equilibrium outcomes by the combination of timing strategies. (All proofs are provided in the [Appendix](#).)

**Proposition 1.** Equilibrium profits for the manufacturer and the retailers, wholesale prices, and retail prices depend on the combination of timing strategies represented by  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix}$  as the following four cases demonstrate.

$$\begin{aligned}\text{Case (I) when } \begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} &= \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}, \\ &\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 4 & 4 \end{pmatrix}, \\ &\begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 1 & 3 \\ 4 & 4 \end{pmatrix}, \Pi = \Pi^*, \pi_i = \pi_i^*, \pi_j = \pi_j^*, \\ &r_i = r_i^*, r_j = r_j^*, p_i = p_i^* \text{ and } p_j = p_j^*, \text{ where :}\end{aligned}$$

$$\Pi^* = ((b_1 A_2^2 + b_2 A_1^2)B_1 - 2\theta b_1 b_2 A_1 A_2) / (4B_0 B_2),$$

$$\pi_i^* = b_j(\theta b_i A_j - B_1 A_i)^2 / (4B_0 B_2^2),$$

$$\pi_j^* = b_i(\theta b_j A_i - B_1 A_j)^2 / (4B_0 B_2^2),$$

$$r_i^* = (a_i + c_i)/2, r_j^* = (a_j + c_j)/2,$$

$$p_i^* = (3a_i + c_i)/4 - \theta(2b_i A_j + \theta A_i) / (4B_2),$$

$$p_j^* = (3a_j + c_j)/4 - \theta(2b_j A_i + \theta A_j) / (4B_2).$$

$$\begin{aligned}\text{Case (II) when } \begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} &= \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}, \\ &\begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, \\ &\Pi = \Pi^{**}, \pi_i = \pi_i^{**}, \pi_j = \pi_j^{**}, r_i = r_i^{**}, r_j = r_j^{**}, p_i = p_i^{**}, \\ &\text{and } p_j = p_j^{**}, \text{ where :}\end{aligned}$$

$$\Pi^{**} = (b_i^2 B_3 A_j^2 + B_1 A_i (A_i B_1 - 2\theta b_i A_j)) / (16b_i B_0 B_1),$$

$$\pi_i^{**} = (\theta b_i A_j - B_1 A_i)^2 / (32b_i B_0 B_1),$$

$$\pi_j^{**} = (b_i B_3 A_j - \theta B_1 A_i)^2 / (64b_i B_0 B_1^2),$$

$$r_i^{**} = (a_i + c_i)/2, r_j^{**} = (a_j + c_j)/2,$$

$$p_i^{**} = (3a_i + c_i)/4 - b_i \theta A_j / (4B_1),$$

<sup>8</sup> For a detailed explanation of this, see [Table 1](#) in [Lu \(2006, pp. 58–59\)](#).

$$p_j^{**} = (3a_j + c_j)/4 - \theta(B_1A_i + b_i\theta A_j)/(8b_iB_1).$$

$$\text{Case (III) when } \begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix},$$

$$\Pi = \Pi^{***}, \pi_i = \pi_i^{***}, \pi_j = \pi_j^{***}, r_i = r_i^{***}, r_j = r_j^{***},$$

$$p_i = p_i^{***}, \text{ and } p_j = p_j^{***}, \text{ where:}$$

$$\Pi^{***} = (2A_iB_1^2(2b_i(b_jA_i - \theta A_j) - \theta^2A_i) + b_i^2(16b_i^2b_j^2 - 19\theta^2b_ib_j + 5\theta^4)A_j^2)/(4b_iB_0B_1B_4),$$

$$\pi_i^{***} = (B_2^2(B_1A_i - \theta b_iA_j)^2)/(2b_iB_0B_1B_4^2),$$

$$\pi_j^{***} = (b_i(16b_i^2b_j^2 - 17\theta^2b_ib_j + 4\theta^4)A_j - 3\theta B_1^2A_i^2)/(4b_iB_0B_1^2B_4^2),$$

$$r_i^{***} = (a_i + c_i)/2 + \theta^2(b_i\theta A_j - B_1A_i)/(2B_1B_4),$$

$$r_j^{***} = (a_j + c_j)/2 + \theta(B_1A_i - b_i\theta A_j)/(b_iB_4),$$

$$p_i^{***} = (3a_i + c_i)/4 - \theta(2b_i(8b_ib_j - 3\theta^2)A_j - \theta B_1A_i)/(4B_1B_4),$$

$$p_j^{***} = (3a_j + c_j)/4 - \theta(2B_1^2A_i + b_i\theta(12b_ib_j - 5\theta^2)A_j)/(4b_iB_1B_4).$$

$$\text{Case (IV) when } \begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix},$$

$$\Pi = \Pi^{****}, \pi_i = \pi_i^{****}, \pi_j = \pi_j^{****}, r_i = r_i^{****}, r_j = r_j^{****},$$

$$p_i = p_i^{****}, \text{ and } p_j = p_j^{****}, \text{ where:}$$

$$\Pi^{****} = (B_3^2A_i(2b_i(b_jA_i - \theta A_j) - \theta^2A_i) + b_i^2(32b_i^2b_j^2 - 50\theta^2b_ib_j + 19\theta^4)A_j^2)/(16b_iB_0B_5),$$

$$\pi_i^{****} = B_3B_1^2(B_1A_i - \theta b_iA_j)^2/(4b_iB_0B_5^2),$$

$$\pi_j^{****} = (b_i(16b_i^2b_j^2 - 23\theta^2b_ib_j + 8\theta^4)A_j - \theta B_3(3b_ib_j - 2\theta^2)A_i^2)/(16b_iB_0B_5^2),$$

$$r_i^{****} = (a_i + c_i)/2 - \theta^2(B_1A_i - \theta b_iA_j)/(4B_5),$$

$$r_j^{****} = (a_j + c_j)/2 + \theta B_3(B_1A_i - \theta b_iA_j)/(4b_iB_5),$$

$$p_i^{****} = (3a_i + c_i)/4 - \theta(b_i(8b_ib_j - 5\theta^2)A_j - \theta(3b_ib_j - 2\theta^2)A_i)/(4B_5),$$

$$p_j^{****} = (3a_j + c_j)/4 - \theta(2b_i\theta(3b_ib_j - 2\theta^2)A_j + B_0B_3A_i)/(4b_iB_5).$$

The symbols used in the values respectively denote the following:  $B_0 \equiv b_1b_2 - \theta^2$ ,  $B_1 \equiv 2b_1b_2 - \theta^2$ ,  $B_2 \equiv 4b_1b_2 - \theta^2$ ,  $B_3 \equiv 4b_1b_2 - 3\theta^2$ ,  $B_4 \equiv 16b_1b_2 - 7\theta^2$ ,  $B_5 \equiv 16b_1^2b_2^2 - 21\theta^2b_1b_2 + 7\theta^4$ . Moreover,  $A_i \equiv a_i - c_i$  ( $i = 1, 2$ ) and  $(i, j) = (1, 2)$  or  $(2, 1)$  in all the cases.

The next corollary immediately follows from Proposition 1.

**Corollary 1.** The equilibrium profits shown in Proposition 1 satisfy the following inequalities.

$$\Pi^* > \Pi^{**} > \Pi^{***} > \Pi^{****}$$

$$\pi_i^* < \pi_i^{**} < \pi_i^{***}, \pi_i^* < \pi_i^{****} < \pi_i^{****}$$

$$\pi_j^* < \pi_j^{**}$$

Proposition 1 and Corollary 1 indicate that first-mover advantage for the manufacturer arises in the following way. First, the manufacturer sets the wholesale prices of both the products before the retail prices are set in Cases (I) and (II) in the proposition. Next, the manufacturer sets the wholesale price of Product  $j$  simultaneously with the timing of Retailer  $i$  setting the retail price of Product  $i$  in Case (III). Lastly, the manufacturer sets the wholesale price of Product  $j$  only after Retailer  $i$  sets the retail price of Product  $i$  in Case (IV). Therefore, the manufacturer sets the wholesale prices at the earliest timing in Case (I) or Case (II), at the second earliest timing in Case (III), and at the last timing in Case (IV). Moreover,  $\Pi^* > \Pi^{**} > \Pi^{***} > \Pi^{****}$  in Corollary 1 suggests that the profit of the manufacturer is highest when the cases are ordered as (I), (II), (III), and (IV). These two facts indicate that the first-mover advantage concerning wholesale price setting for the manufacturer arises.<sup>9</sup>

In addition to Corollary 1, Proposition 1 leads to the following corollary regarding retailer's behavior.

**Corollary 2.** (i) Profit of Retailer  $i$  is higher (lower) when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$  than when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}$ ,

if  $A_i/A_j < X_{ij}$  or  $Y_{ij} < A_i/A_j$  ( $X_{ij} < A_i/A_j < Y_{ij}$ ),

$$\text{where } X_{ij} \equiv \left( b_ib_j(2b_ib_j - \theta^2) - (b_ib_j - \theta^2) \right. \\ \left. \times \sqrt{2b_ib_j(2b_ib_j - \theta^2)} \right) / (b_j\theta(3b_ib_j - 2\theta^2)),$$

$$Y_{ij} \equiv \left( b_ib_j(2b_ib_j - \theta^2) + (b_ib_j - \theta^2) \right. \\ \left. \times \sqrt{2b_ib_j(2b_ib_j - \theta^2)} \right) / (b_j\theta(3b_ib_j - 2\theta^2)).$$

<sup>9</sup> Even if the periods of decisions are different, profits are the same as long as the sequence of the decisions by the manufacturer and retailers is the same. For example, the profits in Case (II) in Proposition 1 are the same between the three different strategies of  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 3)$ ,  $(1, 1, 2, 4)$ , and  $(1, 1, 3, 4)$  because the sequence of decisions does not differ. Moreover, note that the meaning of the number of asterisks attached to payoffs and prices as superscripts in Proposition 1 (i.e., \*, \*\*, \*\*\*, or \*\*\*\*) is different from the meaning of the number of periods denoted by  $t(\cdot)$  (i.e., 1, 2, 3, or 4). For example, Corollary 1 indicates that Retailer  $i$  has the maximum profit of  $\pi_i^{****}$  in Case (IV) of Proposition 1, in which the combination of timing strategies is  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 3, 2, 4)$ . However, this combination of strategies does not constitute the SPNE of the whole game, because Corollary 1 also implies that the manufacturer has an incentive to deviate from this status so as to increase its profit. Therefore, the combinations of strategies in the SPNE, which will be shown in Proposition 2, are derived based on the orders of profits of the manufacturer ( $\Pi^*$ ,  $\Pi^{**}$ ,  $\Pi^{***}$ , or  $\Pi^{****}$ ) and the retailers ( $\pi_i^*$ ,  $\pi_i^{**}$ ,  $\pi_i^{***}$ , or  $\pi_i^{****}$ ) classified by Cases (I)–(IV) in Proposition 1.

(ii) Profit of Retailer  $i$  is higher (lower) when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$  than when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ ,

if  $A_i/A_j < V_{ij}$  or  $W_{ij} < A_i/A_j$  ( $V_{ij} < A_i/A_j < W_{ij}$ ),

where  $V_{ij} = (2560b_i^4b_j^4\theta - 3472b_i^3b_j^3\theta^3 + 1556b_i^2b_j^2\theta^5 - 230b_ib_j\theta^7 - 8\theta(64b_i^3b_j^3 - 108b_i^2b_j^2\theta^2 + 51b_ib_j\theta^4 - 7\theta^6)\sqrt{2b_ib_j(2b_ib_j - \theta^2)}) / (2b_j(1536b_i^4b_j^4 - 2064b_i^3b_j^3\theta^2 + 760b_i^2b_j^2\theta^4 + 7b_ib_j\theta^6 - 32\theta^8))$ , and  $W_{ij} = (2560b_i^4b_j^4\theta - 3472b_i^3b_j^3\theta^3 + 1556b_i^2b_j^2\theta^5 - 230b_ib_j\theta^7 + 8\theta(64b_i^3b_j^3 - 108b_i^2b_j^2\theta^2 + 51b_ib_j\theta^4 - 7\theta^6) \times \sqrt{2b_ib_j(2b_ib_j - \theta^2)}) / (2b_j(1536b_i^4b_j^4 - 2064b_i^3b_j^3\theta^2 + 760b_i^2b_j^2\theta^4 + 7b_ib_j\theta^6 - 32\theta^8))$ .

Note that  $A_i = a_i - c_i$  ( $i = 1, 2$ ) and  $(i, j) = (1, 2)$  or  $(2, 1)$ .

Using the relationships relating to profits in Corollaries 1 and 2, we can identify the optimal timing strategies for the manufacturer and for the retailers. At stage one, the manufacturer determines  $\{t(r_1), t(r_2)\}$  from  $\{1, 2, 3, 4\}$ , which means that the number of timing strategies from which the manufacturer can choose is  $4^2 = 16$ . Meanwhile, Retailer  $i$  chooses  $t(p_i)$  from  $\{1, 2, 3, 4\}$ . Therefore, all payoffs resulting from combinations of timing strategies chosen by the three players are completely described by a third-order tensor, which is composed of  $16 \times 4 \times 4 = 256$  cells. As discussed, however, there are only 36 feasible combinations of the sequence of timings because the condition  $t(r_i) < t(p_i)$  ( $i = 1, 2$ ) must be satisfied. As it is difficult to illustrate all payoffs for the three players in the form of the third-order tensor, we fix the timing strategy of  $(t(r_1), t(r_2))$  undertaken by the manufacturer. We then break down the tensor into matrices that represent the payoffs by the combination of timing strategies chosen by the two retailers. Table 2 illustrates the payoff matrices of the timing game at stage one. Note that we draw diagonal lines in the cells to indicate where the timing strategies do not satisfy either constraints  $t(r_1) < t(p_1)$  or  $t(r_2) < t(p_2)$ .

In order to identify the optimal timing strategy, a payoff is enclosed by a circle resulting from the optimal strategy as shown in Table 2. Namely, a circle enclosing the left variable in a parenthesis in a cell in the table means that the manufacturer chooses its optimal timing strategy. Similarly, a circle enclosing the middle variable in a parenthesis means that Retailer 1 chooses its optimal strategy, and a circle enclosing the right variable in a parenthesis means that Retailer 2 chooses its optimal strategy. A solid-line circle means that the strategy is always optimal, while a dotted-line circle means that the strategy is sometimes optimal depending on the values of the exogenous parameters.<sup>10</sup> Hence, the cell where all of the three payoffs in one parenthesis are enclosed by circles can constitute the Nash equilibrium of the timing game in stage one. Because we formulate a noncooperative dynamic complete information game in this paper, we employ a SPNE as the solution concept of the entire game composed of two stages. Referring to Table 2, we present the following proposition that identifies the equilibrium.

<sup>10</sup> Corollary 2 is used to mark the dotted-line circles. For example, comparing the timing strategies  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 1, 2, 3)$  and  $(1, 1, 4, 3)$  in Panel (i) of Table 2, Corollary 2 suggests that if  $A_1/A_2 < X_{12}$  or  $Y_{12} < A_1/A_2$ , then Retailer 1 prefers  $t(p_1) = 2$  to 4 and  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 1, 2, 3)$  thus constitutes the SPNE. Otherwise, Retailer 1 prefers  $t(p_1) = 4$  to 2 and  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 1, 4, 3)$  constitutes the SPNE. Therefore, the payoffs of Retailer 1 in the cells of  $(1, 1, 2, 3)$  and  $(1, 1, 4, 3)$  are enclosed by dotted lines.

**Proposition 2.** (i) The following combinations of timing strategies always constitute a SPNE irrespective of the values of the exogenous parameters.

$$\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}$$

- (ii)  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$  constitute a SPNE when  $A_i/A_j \leq X_{ij}$  or  $Y_{ij} \leq A_i/A_j$ .
- (iii)  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$  constitute a SPNE when  $X_{ji} \leq A_j/A_i \leq Y_{ji}$ .
- (iv)  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$  constitute a SPNE when  $V_{ji} \leq A_j/A_i \leq W_{ji}$ .

Additionally, when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ ,  $\pi_i > \pi_j$  holds if  $A_i/A_j < \frac{b_i(\theta^3 - 4\sqrt{4b_ib_j - 2\theta^2}(b_ib_j - \theta^2))}{(4b_ib_j - 3\theta^2)(2b_ib_j - \theta^2)}$  or  $\frac{b_i(\theta^3 + 4\sqrt{4b_ib_j - 2\theta^2}(b_ib_j - \theta^2))}{(4b_ib_j - 3\theta^2)(2b_ib_j - \theta^2)} < A_i/A_j$ . Note that  $(i, j) = (1, 2)$  or  $(2, 1)$ . See Corollary 2 for the definitions of  $A_i, A_j, X_{ij}, Y_{ij}, V_{ij}$ , and  $W_{ij}$ .

Observe that all of the combinations of timing strategies shown in Proposition 2 fall into either Case (I) or (II) in Proposition 1.

## 5. Generalization: multiple finite periods case

Up to the previous section, the number of periods in stage two was fixed at four, which coincides with the number of control variables in our model. More realistically, however, there are an infinite number of pricing timings for a firm because a firm generally makes decisions not in discrete periods but based on a continuous timeline. This means that the number of periods for decisions should not be limited to four, but rather can take a sufficient positive number of periods in the context of our model. Following the previous literature relating to the observable delay game (e.g., Lu, 2006), we extend the model in this section by modifying the assumption regarding the number of periods so that the number is greater than four, in each of which a supply chain member sets its selling price. Under this general setting, we obtain the following lemma.

**Lemma 1.** The sequence  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (3, 3, 4, 4)$ , which always constitutes a SPNE under the assumption of four periods in Proposition 2, no longer constitutes a SPNE under the general situation where the number of periods is a general finite integer greater than four.

While Lemma 1 suggests that the combination  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (3, 3, 4, 4)$  does not constitute the SPNE, the other combinations of timings that constitute a SPNE when the number of periods is four as shown in Proposition 2 remain to constitute a SPNE under the general situation where the number of periods is greater than four.<sup>11</sup> Eventually, we reach the last proposition in the present research.

<sup>11</sup> This fact can be confirmed by showing that neither the manufacturer nor a retailer has an incentive to deviate from the timing strategy that can constitute the SPNE shown in Proposition 2, except for  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (3, 3, 4, 4)$ .

**Table 2**  
Payoff matrices.

Panel (i) $(t(r_1), t(r_2)) = (1, 1)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : 1	
Timing 2	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
strategy of 3	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Retailer 1 4	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (ii) $(t(r_1), t(r_2)) = (1, 2)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : 1	
Timing 2	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
strategy of 3	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Retailer 1 4	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (iii) $(t(r_1), t(r_2)) = (1, 3)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : 1	
Timing 2	$(\Pi, \pi_1, \pi_2)$
strategy of 3	$(\Pi, \pi_1, \pi_2)$
Retailer 1 4	$(\Pi, \pi_1, \pi_2)$
Panel (iv) $(t(r_1), t(r_2)) = (2, 1)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : 1	
Timing 2	
strategy of 3	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Retailer 1 4	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (v) $(t(r_1), t(r_2)) = (2, 2)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : 1	
Timing 2	
strategy of 3	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Retailer 1 4	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$

(continued on next page)

Table 2 (continued)

Panel (vi) $(t(r_1), t(r_2)) = (2, 3)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : Timing strategy of Retailer 1	1 2 3 4
	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (vii) $(t(r_1), t(r_2)) = (3, 1)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : Timing strategy of Retailer 1	1 2 3 4
	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (viii) $(t(r_1), t(r_2)) = (3, 2)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : Timing strategy of Retailer 1	1 2 3 4
	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$
Panel (ix) $(t(r_1), t(r_2)) = (3, 3)$	
$t(p_2)$ : Timing strategy of Retailer 2	
	1 2 3 4
$t(p_1)$ : Timing strategy of Retailer 1	1 2 3 4
	$(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$ $(\Pi, \pi_1, \pi_2)$

Notes: The left, middle, and right variables in parentheses respectively represent the profits for the manufacturer, Retailer 1, and Retailer 2 from the combination of timing strategies. Using Corollaries 1 and 2, we identify the best responses for the manufacturer and the two retailers. The profit enclosed by a circle indicates the best response for each of the manufacturer or the retailers. Hence, the cell where all of the three payoffs in one set of parentheses are enclosed by circles constitutes the Nash equilibrium of the timing game at stage one. The cells with diagonal lines are the combinations of strategies which are never realized because  $t(r_i) < t(p_i)$  ( $i = 1, 2$ ) must be satisfied in this timing game. See Proposition 1 for the values of the profits in each cell.

### Proposition 3.

- (i) The following relationship always holds in the combinations of the sequence of timing that constitutes the SPNE, irrespective of the values of the parameters:

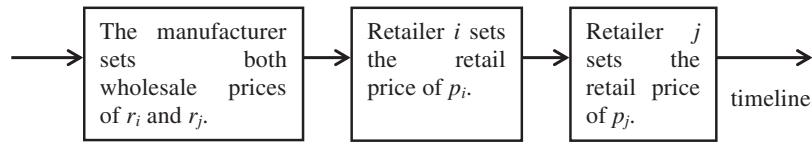
$$t(r_i) = t(r_j) < t(p_i) < t(p_j) \quad ((i, j) = (1, 2) \text{ or } (2, 1)).$$

- (ii) The following relationship holds in the combinations of the sequence of timing that can constitute a SPNE in some cases, in which the values of the exogenous parameters fall into a certain region:

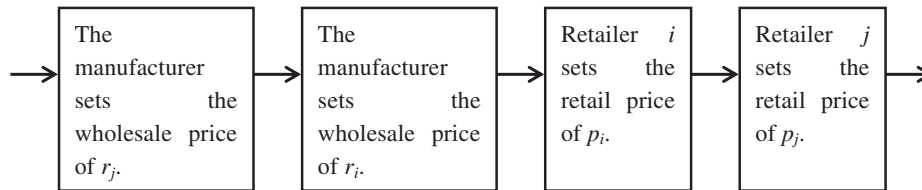
$$t(r_j) \leq t(r_i) < t(p_i) < t(p_j) \quad ((i, j) = (1, 2) \text{ or } (2, 1)).$$

Proposition 3 is the most central result in this paper; therefore, they provide a useful operational implication. To illustrate this implication, Fig. 2 describes the timelines of optimal sequences of pricing decisions by the manufacturer and the retailers. Timeline (i) in the figure shows the sequence of decisions that always constitutes the equilibrium, while Timeline (ii) shows the sequence of decisions that can constitute the equilibrium depending on the values of the exogenous parameters. In particular, Timeline (i) proposes a decision guideline concerning desirable pricing timing for supply chain members. Namely, the manufacturer must first set the wholesale prices of  $r_i$  and  $r_j$ , Retailer  $i$  then sets the retail price  $p_i$ , and Retailer  $j$  finally sets the retail price  $p_j$ . By setting prices

(i) always SPNE



(ii) sometimes SPNE



**Fig. 2.** Equilibrium sequence of pricing decisions in timeline.

Note: "(i) always SPNE" represents that the sequence of decisions on the timeline constitutes the SPNE and is thus stable irrespective of the values of the parameters. Meanwhile, "(ii) sometimes SPNE" represents that the sequence of decisions constitutes the SPNE only when the values of the exogenous parameters fall into a certain region. Note that  $(i, j) = (1, 2)$  or  $(2, 1)$ .

along this timeline, the three supply chain members attain a stable equilibrium, in which the profit for each member is maximized.

Note that the two separate cases in Proposition 3 further suggest the following implication for the manufacturer. If the manufacturer does not set both wholesale prices at the same time so that  $t(r_i) = t(r_j)$  holds, the manufacturer may face the risk that the Nash equilibrium disappears in the timing game in stage one because a SPNE exists only when the exogenous parameter values fall into a certain region in this case. Actually, Proposition 2 shows that the combination of  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (2, 1, 3, 4)$  might not constitute a SPNE when the values of the exogenous parameters fall into a certain region. Consequently, the manufacturer should simultaneously set its wholesale prices of products sold to separate retailers to avoid the risk that the equilibrium determining a stable pricing sequence disappears.

## 6. Conclusion and discussion

This paper examines the optimal timing of wholesale and retail pricing in two-echelon multi-channel supply chains. Introducing the framework of the observable delay game, we derive managerial implications concerning the timing of when wholesale and retail prices should be set. First, the manufacturer must simultaneously set its wholesale prices of products that are sold to separate retailers at the same time. Second, the retailers must sequentially set respective retail prices at different times; namely, the retailers must stagger their timings of setting retail prices.

The mechanism that leads to our results can be explained from a game-theoretic perspective. Specifically, the major results are associated with the incentive for a manufacturer to synchronize the wholesale pricing applied to multiple retailers and with the incentive for retailers to stagger the timing of retail pricing. As briefly discussed in the first section, the rationale for why a sequential move game occurs among the retailers is explicable by the concept of second-mover advantage arising in price competition. That is, previous game-theoretic work has shown that when price competition between two firms occurs, the firm that determines its price later generates higher profit than the firm that determines its price earlier (e.g., Gal-Or, 1985). This advantage of a later decision is called the second-mover advantage. Due to retailers' incentives to obtain this second-mover advantage, simultaneous price setting becomes unstable and the retailers tend to prefer sequential price setting. Moreover, van Damme and Hurkens (2004, p. 405) demon-

strate that when two competing firms set prices sequentially, both the first mover and the second mover achieve higher profits than when the firms set prices simultaneously, irrespective of which firm is the first mover in the sequential case. Therefore, multiple equilibria, in which one of the two retailers is the first mover and the other is the second mover, arise in the timing game of the model in this paper. This is the fundamental rationale for why the sequential move game among the retailers always occurs in the SPNE in our model. On the other hand, the manufacturer has an incentive to determine its wholesale prices for both channels as early as possible, because the manufacturer can set its prices at the optimal level by predicting the future reactions of the retailers through backward induction, thus mitigating inefficient double marginalization of the products.<sup>12</sup> To sum up, the main results in this paper are consistent with these theoretical OR insights gained in the literature, which underpins the robustness of the implications derived from the model.

Finally, it should be noted that there can be other possible factors in real markets that potentially affect the timing of decisions made by supply chain members. For example, because we assume that the second stage in our model consists of multiple periods, a firm setting its price in a later period may obtain a learning effect. If we also consider the learning effect for a firm buying a product later as a factor in the model, the late mover in the game will have an additional positive payoff from the learning effect, which increases the incentive for a retailer to set its price in a later period. Here, remember that one of the major results in this paper is that the retailers sequentially set respective retail prices in different periods in equilibrium; that is, the retailers stagger their timings of setting retail prices. This major result does not change even if we consider the learning effect of the late mover, because this increased late-mover advantage resulting from the learning effect makes the state in which the two retailers simultaneously make their decisions more unstable. In other words, the learning effect induces a retailer to set its price later and hence to dislike the simultaneous price setting. To sum up, the major finding derived from our model that the two retailers set prices not simultaneously but rather sequentially in equilibrium is robust, even under the presence of the learning effect.

<sup>12</sup> See Spengler (1950) for details regarding how double marginalization negatively impacts overall supply chain profit.

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## Appendix

**Proof of Proposition 1.** We use a standard process of backward induction to derive the prices and payoffs in the SPNE at stage two by the combination of timing strategies.

**Case. (I)-(i):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}$

Because the two retailers set the retail prices simultaneously at the last move in this case, we first maximize profits for the two retailers in Eq. (5) by solving  $\partial\pi_i/\partial p_i = \partial\pi_j/\partial p_j = 0$ , yielding:

$$p_i = (b_i(2b_j(a_i + r_i) - \theta(a_j - r_j)) - \theta^2 a_i) / (4b_i b_j - \theta^2),$$

$$(i, j) = (1, 2), (2, 1). \quad (\text{A1})$$

Note that the second-order conditions for all maximization problems in this appendix are satisfied because all profit functions are concave and quadratic with respect to the strategic variable (price). Therefore, we henceforth omit writing the second-order conditions.

Next, we analyze the manufacturer's decision on the wholesale prices. We substitute Eq. (A1) into Eq. (6) and maximize the manufacturer profit by solving  $\partial\Pi/\partial r_i = \partial\Pi/\partial r_j = 0$ , yielding:

$$r_i = (a_i + c_i)/2 \quad (i, j) = (1, 2), (2, 1). \quad (\text{A2})$$

Substituting Eq. (A2) into Eq. (A1), we obtain the equilibrium retail prices  $p_i^*$  and  $p_j^*$  in this proposition. Finally, we substitute  $p_i^*, p_j^*$ , and Eq. (A2) into Eqs. (5) and (6), obtaining  $\Pi$  and  $\pi_i$  in equilibrium as  $\Pi^*$  and  $\pi_i^*$  ((i, j) = (1, 2), (2, 1)).

**Case. (I)-(ii):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 1 & 3 \\ 4 & 4 \end{pmatrix}$

Using Eq. (5), we maximize  $\pi_i$  and  $\pi_j$  with respect to  $p_i$  and  $p_j$  respectively by solving  $\partial\pi_i/\partial p_i = \partial\pi_j/\partial p_j = 0$ , yielding:

$$p_i = (b_i(2b_j(a_i + r_i) - \theta(a_j - r_j)) - \theta^2 a_i) / (4b_i b_j - \theta^2)$$

$$p_j = (b_j(2b_i(a_j + r_j) - \theta(a_i - r_i)) - \theta^2 a_j) / (4b_i b_j - \theta^2). \quad (\text{A3})$$

We put Eq. (A3) into the manufacturer's profit in Eq. (6) and maximize it with respect to  $r_j$  by solving  $\partial\Pi/\partial r_j = 0$ , yielding:

$$r_j = \frac{a_j + c_j}{2} - \frac{b_j \theta (a_i + c_i - 2r_i)}{2(2b_i b_j - \theta^2)}. \quad (\text{A4})$$

Inserting Eq. (A4) into Eq. (A3), we have:

$$p_i = a_i - \frac{b_i b_j (a_i + c_i - 2r_i)}{2(2b_i b_j - \theta^2)} - \frac{b_i (2b_j (a_i - c_i) + \theta(a_j - c_j))}{2(4b_i b_j - \theta^2)}, \quad (\text{A5})$$

$$p_j = a_j - \frac{b_j \theta (a_i + c_i - 2r_i)}{2(2b_i b_j - \theta^2)} - \frac{b_j (2b_i (a_j - c_j) + \theta(a_i - c_i))}{2(4b_i b_j - \theta^2)}. \quad (\text{A6})$$

After substituting Eqs. (A4), (A5), and (A6) into the manufacturer's profit in Eq. (6), we maximize it with respect to  $r_i$  by solving  $\partial\Pi/\partial r_i = 0$ , yielding:

$$r_i = (a_i + c_i)/2. \quad (\text{A7})$$

Substituting Eq. (A7) into (A4), (A5), and (A6), we obtain  $p_i^*, p_j^*$ , and:

$$r_j = (a_j + c_j)/2. \quad (\text{A8})$$

Inserting  $p_i^*, p_j^*$ , Eqs. (A7) and (A8) into Eqs. (5) and (6), we obtain the equilibrium manufacturer's profit of  $\Pi^*$  and retailer's profits of  $\pi_i^*$  and  $\pi_j^*$ .

**Case. (II)-(i):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}, \text{ or } \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}$

Using Eq. (5), we first maximize the profit of Retailer  $j$  by solving  $\partial\pi_j/\partial p_j = 0$ , yielding:

$$p_j = (b_j(a_j + r_j) - \theta(a_i - p_i)) / (2b_i). \quad (\text{A9})$$

After substituting Eq. (A9) into  $\pi_i$  represented by Eq. (5), we maximize  $\pi_i$  with respect to  $p_i$  by solving  $\partial\pi_i/\partial p_i = 0$ , yielding:

$$p_i = \frac{a_i + r_i}{2} - \frac{b_i \theta (a_j - r_j)}{2(2b_i b_j - \theta^2)}. \quad (\text{A10})$$

Replacing  $p_i$  in Eq. (A9) with Eq. (A10) yields:

$$p_j = \frac{a_j + r_j}{2} - \frac{\theta^2 (a_j - r_j)}{4(2b_i b_j - \theta^2)} - \frac{\theta(a_i - r_i)}{4b_i}. \quad (\text{A11})$$

We next substitute Eqs. (A10) and (A11) into the manufacturer's profit of Eq. (6) and maximize it with respect to the two wholesale prices by solving  $\partial\Pi/\partial r_i = \partial\Pi/\partial r_j = 0$ , obtaining:

$$r_i = (a_i + c_i)/2, r_j = (a_j + c_j)/2. \quad (\text{A12})$$

Substituting Eq. (A12) into Eqs. (A10) and (A11) yields the equilibrium retail prices as  $p_i^{**}$  and  $p_j^{**}$  in this proposition. Finally, we substitute  $p_i^{**}, p_j^{**}$ , and Eq. (A12) into Eqs. (5) and (6), yielding  $\Pi^{**}, \pi_i^{**}$  and  $\pi_j^{**}$ .

**Case. (II)-(ii):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Using Eq. (5), we maximize the profit of Retailer  $j$  by solving  $\partial\pi_j/\partial p_j = 0$ , yielding:

$$p_j = (b_j(a_j + r_j) - \theta(a_i - p_i)) / (2b_i). \quad (\text{A13})$$

After substituting Eq. (A13) into  $\pi_i$  represented by Eq. (5), we maximize  $\pi_i$  with respect to  $p_i$  by solving  $\partial\pi_i/\partial p_i = 0$ , yielding:

$$p_i = \frac{a_i + r_i}{2} - \frac{b_i \theta (a_j - r_j)}{2(2b_i b_j - \theta^2)}. \quad (\text{A14})$$

Replacing  $p_i$  in Eq. (A13) with Eq. (A14) yields:

$$p_j = \frac{a_j + r_j}{2} - \frac{\theta^2 (a_j - r_j)}{4(2b_i b_j - \theta^2)} - \frac{\theta(a_i - r_i)}{4b_i}. \quad (\text{A15})$$

We next substitute Eqs. (A14) and (A15) into the manufacturer's profit of Eq. (6) and maximize it with respect to  $r_j$  by solving  $\partial\Pi/\partial r_j = 0$ , obtaining:

$$r_j = \frac{a_j + c_j}{2} - \frac{\theta(a_i + c_i - 2r_i)}{6b_i} - \frac{b_j \theta (a_i + c_i - 2r_i)}{3(4b_i b_j - 3\theta^2)}. \quad (\text{A16})$$

We replace  $r_j$  in Eqs. (A14) and (A15) with Eq. (A16), yielding the retail prices as the functions of  $r_i$ . Next, we further substitute

these retail prices and Eq. (A16) into the manufacturer's profit of Eq. (6) and maximize it with respect to  $r_i$  by solving  $\partial \Pi / \partial r_i = 0$ , yielding:

$$r_i = (a_i + c_i) / 2. \quad (\text{A17})$$

Substituting Eq. (A17) into Eq. (A16) gives:

$$r_j = (a_j + c_j) / 2. \quad (\text{A18})$$

Substituting Eqs. (A17) and (A18) into Eqs. (A14) and (A15) yields the equilibrium retail prices of  $p_i^{**}$  and  $p_j^{**}$ . Substituting  $p_i^{**}$ ,  $p_j^{**}$ , Eqs. (A17) and (A18) into Eqs. (5) and (6) gives the equilibrium manufacturer's profit of  $\Pi^{**}$  and retailer profits of  $\pi_i^{**}$  and  $\pi_j^{**}$ .

**Case. (II)-(iii):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$

Using Eq. (5), we maximize the profit of Retailer  $j$  by solving  $\partial \pi_j / \partial p_j = 0$ , yielding:

$$p_j = (b_i(a_j + r_j) - \theta(a_i - p_i)) / (2b_i). \quad (\text{A19})$$

After substituting Eq. (A19) into  $\pi_i$  of Eq. (5), we maximize  $\pi_i$  with respect to  $p_i$  by solving  $\partial \pi_i / \partial p_i = 0$ , yielding:

$$p_i = \frac{a_i + r_i}{2} - \frac{b_i \theta (a_j - r_j)}{2(2b_i b_j - \theta^2)}. \quad (\text{A20})$$

Replacing  $p_i$  in Eq. (A19) with Eq. (A20) gives:

$$p_j = \frac{a_j + r_j}{2} - \frac{\theta^2 (a_j - r_j)}{4(2b_i b_j - \theta^2)} - \frac{\theta (a_i - r_i)}{4b_i}. \quad (\text{A21})$$

After substituting Eqs. (A20) and (A21) into  $\Pi$  of Eq. (6), we maximize it with respect to  $r_i$  by solving  $\partial \Pi / \partial r_i = 0$ , yielding:

$$r_i = \frac{a_i + c_i}{2} - \frac{b_i \theta (a_j + c_j - 2r_j)}{2(2b_i b_j - \theta^2)}. \quad (\text{A22})$$

We replace  $r_i$  in Eqs. (A20) and (A21) with Eq. (A22), yielding the retail prices as functions of  $r_j$ . We then substitute these retail prices and Eq. (A22) into the manufacturer's profit of Eq. (6) and maximize it with respect to  $r_j$  by solving  $\partial \Pi / \partial r_j = 0$ , obtaining:

$$r_j = (a_j + c_j) / 2. \quad (\text{A23})$$

Substituting Eq. (A23) into Eq. (A22), we have:

$$r_i = (a_i + c_i) / 2. \quad (\text{A24})$$

Inserting Eqs. (A23) and (A24) into Eqs. (A20) and (A21) yields equilibrium retail prices as  $p_i^{**}$  and  $p_j^{**}$ . Furthermore, substituting  $p_i^{**}$ ,  $p_j^{**}$ , Eqs. (A23) and (A24) into Eqs. (5) and (6) yields the equilibrium manufacturer's profit of  $\Pi^{**}$  and retailers' profits of  $\pi_i^{**}$  and  $\pi_j^{**}$ .

**Case. (III):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}$ ,  
or  $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

We maximize the profit of Retailer  $j$  represented by Eq. (5) by solving  $\partial \pi_j / \partial p_j = 0$ , yielding:

$$p_j = (b_i(a_j + r_j) - \theta(a_i - p_i)) / (2b_i). \quad (\text{A25})$$

After substituting Eq. (A25) into  $\Pi$  in Eq. (6) and  $\pi_i$  of Eq. (5), we respectively maximize  $\Pi$  and  $\pi_i$  with respect to  $r_j$  and  $p_i$  by

solving  $\partial \Pi / \partial r_j = \partial \pi_i / \partial p_i = 0$ , yielding:

$$r_j = \frac{b_i(4b_i b_j - \theta^2)a_j + (2b_i b_j - \theta^2)(2b_i c_j - \theta(a_i + 2c_i - 3r_i))}{b_i(8b_i b_j - 5\theta^2)}, \quad (\text{A26})$$

$$p_i = \frac{b_i(4b_j(a_i + r_i) - \theta(a_j - c_j)) - \theta^2(3a_i + c_i + r_i)}{8b_i b_j - 5\theta^2}. \quad (\text{A27})$$

Substituting Eqs. (A26) and (A27) into Eq. (A25), we obtain:

$$p_j = \frac{9a_j + c_j}{10} - \frac{\theta(3a_i + c_i - 4r_i)}{10b_i} - \frac{3b_j(2b_i(a_j - c_j) + \theta(a_i + 2c_i - 3r_i))}{5(8b_i b_j - 5\theta^2)}. \quad (\text{A28})$$

We insert Eqs. (A26), (A27), and (A28) into  $\Pi$  in Eq. (6) and maximize it with respect to  $r_i$  by solving  $\partial \Pi / \partial r_i = 0$ , yielding:

$$r_i = \frac{a_i + c_i}{2} + \frac{\theta^2(b_i \theta (a_j - c_j) - (2b_i b_j - \theta^2)(a_i - c_i))}{2(2b_i b_j - \theta^2)(16b_i b_j - 7\theta^2)}. \quad (\text{A29})$$

Inserting Eq. (A29) into Eqs. (A26), (A27), and (A28) yields the equilibrium retail prices of  $p_i^{***}$  and  $p_j^{***}$  in this proposition and:

$$r_j = \frac{a_j + c_j}{2} + \frac{\theta((2b_i b_j - \theta^2)(a_i - c_i) - b_i \theta (a_j - c_j))}{b_i(16b_i b_j - 7\theta^2)}. \quad (\text{A30})$$

Substituting  $p_i^{***}$ ,  $p_j^{***}$ , Eqs. (A29) and (A30) into Eqs. (5) and (6), we have the equilibrium manufacturer's profit of  $\Pi^{***}$  and retailers' profits of  $\pi_i^{***}$  and  $\pi_j^{***}$ .

**Case. (IV):** when  $\begin{pmatrix} t(r_i) & t(r_j) \\ t(p_i) & t(p_j) \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

Using Eq. (5), we maximize the profit of Retailer  $j$  by solving  $\partial \pi_j / \partial p_j = 0$ , yielding:

$$p_j = (b_i(a_j + r_j) - \theta(a_i - p_i)) / (2b_i). \quad (\text{A31})$$

After substituting Eq. (A31) into  $\Pi$ , represented by Eq. (6), we maximize  $\Pi$  with respect to  $r_j$  by solving  $\partial \Pi / \partial r_j = 0$ , yielding:

$$r_j = \frac{a_j + c_j}{2} - \frac{\theta(a_i + c_i - p_i - r_i)}{2b_i}. \quad (\text{A32})$$

Replacing  $r_j$  in Eq. (A31) with Eq. (A32), we have:

$$p_j = \frac{3a_j + c_j}{4} - \frac{\theta(3a_i + c_i - 3p_i - r_i)}{4b_i}. \quad (\text{A33})$$

Substituting Eqs. (A32) and (A33) into  $\pi_i$  in Eq. (5) and maximizing it with respect to  $p_i$ , we have:

$$p_i = \frac{3a_i + c_i + 2r_i}{6} - \frac{b_i(3\theta(a_j - c_j) - 4b_j(a_i - c_i))}{6(4b_i b_j - 3\theta^2)}. \quad (\text{A34})$$

Substituting Eq. (A34) into Eqs. (A32) and (A33) gives:

$$r_j = \frac{7a_j + 5c_j}{12} - \frac{\theta(3a_i + 5c_i - 8r_i)}{12b_i} - \frac{b_j(b_i(a_j - c_j) - \theta(a_i - c_i))}{3(4b_i b_j - 3\theta^2)} \quad (\text{A35})$$

$$p_j = \frac{7a_j + c_j}{8} - \frac{\theta(3a_i + c_i - 4r_i)}{8b_i} - \frac{b_j(b_i(a_j - c_j) - \theta(a_i - c_i))}{2(4b_i b_j - 3\theta^2)}. \quad (\text{A36})$$

After substituting Eqs. (A34), (A35), and (A36) into  $\Pi$  as represented by Eq. (6), we maximize  $\Pi$  with respect to  $r_i$  by solving  $\partial\Pi/\partial r_i = 0$ , yielding:

$$r_i = \frac{a_i + c_i}{2} - \frac{\theta^2((2b_i b_j - \theta^2)(a_i - c_i) - \theta b_i(a_j - c_j))}{4(16b_i^2 b_j^2 - 21\theta^2 b_i b_j + 7\theta^4)}. \quad (\text{A37})$$

Inserting Eq. (A37) into Eq. (A35), we have:

$$r_j = \frac{a_j + c_j}{2} + \frac{\theta(4b_i b_j - 3\theta^2)((2b_i b_j - \theta^2)(a_i - c_i) - \theta b_i(a_j - c_j))}{4b_i(16b_i^2 b_j^2 - 21\theta^2 b_i b_j + 7\theta^4)}. \quad (\text{A38})$$

Substituting Eq. (A37) into Eqs. (A34) and (A36) yields the equilibrium retail prices  $p_i^{***}$  and  $p_j^{***}$  in this proposition. Finally, inserting  $p_i^{***}$  and  $p_j^{***}$ , and Eqs. (A37) and (A38) into Eqs. (5) and (6) yields the equilibrium profits as:  $\Pi = \Pi^{***}$ ,  $\pi_i = \pi_i^{***}$ , and  $\pi_j = \pi_j^{***}$ .

The equilibrium profits and prices shown in Cases (I)–(IV) of this proposition by the combination of timing strategies summarize all the cases examined in this proof.  $\square$

**Proof of Corollary 1.** In this proof, upper case notations are defined as:  $A_i \equiv a_i - c_i$  ( $i=1, 2$ ),  $B_0 \equiv b_1 b_2 - \theta^2$ ,  $B_1 \equiv 2b_1 b_2 - \theta^2$ ,  $B_2 \equiv 4b_1 b_2 - \theta^2$ ,  $B_3 \equiv 4b_1 b_2 - 3\theta^2$ ,  $B_4 \equiv 16b_1 b_2 - 7\theta^2$ , and  $B_5 \equiv 16b_1^2 b_2^2 - 21\theta^2 b_1 b_2 + 7\theta^4$ . Initially, we confirm that all of these variables in upper cases are positive. First, the assumption of Inequality (2) ( $b_i > \theta$ ,  $i=1, 2$ ) obviously guarantees that  $B_0 > 0$ ,  $B_1 > 0$ ,  $B_2 > 0$ ,  $B_3 > 0$ , and  $B_4 > 0$  hold. Second,  $B_5 \equiv 16b_1^2 b_2^2 - 21\theta^2 b_1 b_2 + 7\theta^4 = 16(b_1 b_2 - 21\theta^2/32)^2 + 7\theta^4/64 > 0$ .

Proposition 1 suggests that the following two inequalities hold.

$$\Pi^* - \Pi^{**} = \theta^2(B_1 A_i - b_i \theta A_j)^2 / (16b_i B_0 B_1 B_2) > 0$$

$$\Pi^{**} - \Pi^{***} = \theta^2(B_1 A_i - b_i \theta A_j)^2 / (16b_i B_0 B_1 B_4) > 0$$

Next, the following equation holds.

$$\Pi^{***} - \Pi^{****} = \theta^2(B_1 A_i - b_i \theta A_j)^2 (32b_i^2 b_j^2 - 32b_i b_j \theta^2 + 7\theta^4) / (16b_i B_0 B_1 B_4 B_5)$$

Note that  $32b_i^2 b_j^2 - 32b_i b_j \theta^2 + 7\theta^4 = 32(b_i b_j - \theta^2/2)^2 - \theta^4$ . The assumption of Inequality (2) ( $b_i > \theta$ ,  $i=1, 2$ ) indicates that  $b_i b_j > \theta^2$  holds. Therefore,  $32b_i^2 b_j^2 - 32b_i b_j \theta^2 + 7\theta^4$  is convex with respect to  $b_i b_j$  and hence takes its lowest value when  $b_i b_j = \theta^2$ . Substituting  $b_i b_j = \theta^2$  into  $32b_i^2 b_j^2 - 32b_i b_j \theta^2 + 7\theta^4$  yields  $7\theta^4$ , which is positive. Hence,  $\Pi^{***} - \Pi^{****} > 0$  holds.

Next, we examine the order of profits of retailers. First, the following inequality holds.

$$\pi_i^* - \pi_i^{**} = -\theta^4(B_1 A_i - b_i \theta A_j)^2 / (32b_i B_0 B_1 B_2^2) < 0$$

Second, the following equation holds.

$$\pi_i^{**} - \pi_i^{***} = -\theta^2(96b_i^3 b_j^3 - 185b_i^2 b_j^2 \theta^2 + 118b_i b_j \theta^4 - 25\theta^6) \times (B_1 A_i - b_i \theta A_j)^2 / (32b_i B_0 B_1 B_5^2)$$

Note that  $96b_i^3 b_j^3 - 185b_i^2 b_j^2 \theta^2 + 118b_i b_j \theta^4 - 25\theta^6 = \theta^6(96(b_i b_j / \theta^2)^3 - 185(b_i b_j / \theta^2)^2 + 118(b_i b_j / \theta^2) - 25)$  holds. The factor of  $96(b_i b_j / \theta^2)^3 - 185(b_i b_j / \theta^2)^2 + 118(b_i b_j / \theta^2) - 25$  in this equation is a cubic function with respect to  $b_i b_j / \theta^2$ . We confirm through simple computation that the function has two local extremums with respect to  $b_i b_j / \theta^2$  when  $b_i b_j / \theta^2$  is approximately 0.588 or 0.696. Because the assumption of Inequality (2) ( $b_i > \theta$ ,  $i=1, 2$ ) indicates that  $b_i b_j / \theta^2 > 1$  holds, this cubic function has

its global minimum when  $b_i b_j / \theta^2 = 1$ . Substituting  $b_i b_j / \theta^2 = 1$  into  $96(b_i b_j / \theta^2)^3 - 185(b_i b_j / \theta^2)^2 + 118(b_i b_j / \theta^2) - 25$  gives 4, which is positive. Hence,  $\pi_i^{**} - \pi_i^{***} < 0$  holds.

Third, the following equation holds.

$$\pi_i^* - \pi_i^{***} = -\theta^2(192b_i^3 b_j^3 - 130b_i^2 b_j^2 \theta^2 + 17b_i b_j \theta^4 + 2\theta^6) \times (B_1 A_i - b_i \theta A_j)^2 / (4b_i B_0 B_1 B_2^2 B_4^2)$$

Note that  $192b_i^3 b_j^3 - 130b_i^2 b_j^2 \theta^2 + 17b_i b_j \theta^4 + 2\theta^6 = \theta^6(192(b_i b_j / \theta^2)^3 - 130(b_i b_j / \theta^2)^2 + 17(b_i b_j / \theta^2) + 2)$  holds. The factor of  $192(b_i b_j / \theta^2)^3 - 130(b_i b_j / \theta^2)^2 + 17(b_i b_j / \theta^2) + 2$  in this equation is a cubic function with respect to  $b_i b_j / \theta^2$ . We confirm after simple computation that the function has local extremums with respect to  $b_i b_j / \theta^2$  when  $b_i b_j / \theta^2$  is approximately 0.079 or 0.372. Because the assumption of Inequality (2) ( $b_i > \theta$ ,  $i=1, 2$ ) indicates that  $b_i b_j / \theta^2 > 1$  holds, this cubic function has its global minimum when  $b_i b_j / \theta^2 = 1$ . Substituting  $b_i b_j / \theta^2 = 1$  into  $192(b_i b_j / \theta^2)^3 - 130(b_i b_j / \theta^2)^2 + 17(b_i b_j / \theta^2) + 2$  gives 81, which is positive. Hence,  $\pi_i^* - \pi_i^{***} < 0$  holds.

Fourth, the following equation holds.

$$\pi_i^{***} - \pi_i^{****} = -\theta^4(512b_i^4 b_j^4 - 1208b_i^3 b_j^3 \theta^2 + 1034b_i^2 b_j^2 \theta^4 - 378b_i b_j \theta^6 + 49\theta^8) \times (B_1 A_i - b_i \theta A_j)^2 / (4b_i B_0 B_1 B_4^2 B_5^2)$$

Note that  $512b_i^4 b_j^4 - 1208b_i^3 b_j^3 \theta^2 + 1034b_i^2 b_j^2 \theta^4 - 378b_i b_j \theta^6 + 49\theta^8 = \theta^8(512(b_i b_j / \theta^2)^4 - 1208(b_i b_j / \theta^2)^3 + 1034(b_i b_j / \theta^2)^2 - 378(b_i b_j / \theta^2) + 49)$  holds. The factor of  $512(b_i b_j / \theta^2)^4 - 1208(b_i b_j / \theta^2)^3 + 1034(b_i b_j / \theta^2)^2 - 378(b_i b_j / \theta^2) + 49$  in this equation is a quartic function with respect to  $b_i b_j / \theta^2$ . We confirm through simple computation that the function has local extremums with respect to  $b_i b_j / \theta^2$  when  $b_i b_j / \theta^2$  is approximately 0.397, 0.608, or 0.765. Because the assumption of Inequality (2) ( $b_i > \theta$ ,  $i=1, 2$ ) indicates that  $b_i b_j / \theta^2 > 1$  holds, this quartic function has its global minimum when  $b_i b_j / \theta^2 = 1$ . Substituting  $b_i b_j / \theta^2 = 1$  into  $512(b_i b_j / \theta^2)^4 - 1208(b_i b_j / \theta^2)^3 + 1034(b_i b_j / \theta^2)^2 - 378(b_i b_j / \theta^2) + 49$  gives 9, which is positive. Hence,  $\pi_i^{***} - \pi_i^{****} < 0$  holds.

Finally, the following equation holds.

$$\pi_j^* - \pi_j^{**} = \theta^3(B_1 A_i - b_i \theta A_j)(\theta B_1(8b_i b_j - \theta^2)A_i - b_i(32b_i^2 b_j^2 - 32b_i b_j \theta^2 + 7\theta^4)A_j) / (64b_i B_0 B_1^2 B_2^2)$$

The assumption of Inequality (3) in our model suggests that the following condition must be satisfied.

$$b_i \theta A_j / (2b_i b_j - \theta^2) < A_i < (2b_i / \theta - \theta / b_j) A_j$$

Because  $\pi_j^* - \pi_j^{**}$  is a quadratic function with respect to  $A_i$  and the coefficient on  $A_i^2$  is positive,  $\pi_j^* - \pi_j^{**}$  is convex with respect to  $A_i$ . Moreover,  $\pi_j^* - \pi_j^{**} = 0$  when  $A_i = b_i \theta A_j / (2b_i b_j - \theta^2)$ , and  $\pi_j^* - \pi_j^{**} = -\theta^4(b_i b_j - \theta^2) A_j^2 / (64b_i b_j^2 (2b_i b_j - \theta^2)^2) < 0$  when  $A_i = (2b_i / \theta - \theta / b_j) A_j$ . These two values indicate that  $\pi_j^* - \pi_j^{**} < 0$  always holds within the domain of definitions of the parameters.  $\square$

**Proof of Corollary 2.** We substitute  $(i, j) = (1, 2)$  into  $\pi_i^{**}$  and  $(i, j) = (2, 1)$  into  $\pi_j^{**}$  in Case (II) of Proposition 1, calculating  $\pi_i^{**} - \pi_j^{**} = (\theta b_1 A_2 - B_1 A_1)^2 / (32b_1 B_0 B_1) - (b_2 B_3 A_1 - \theta B_1 A_2)^2 / (64b_2 B_0 B_1^2)$ . If this value is positive, the profit of Retailer 1 under the strategies  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 1, 2, 3)$  is higher than that under the strategies  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 1, 4, 3)$ .

$(\theta b_1 A_2 - B_1 A_1)^2 / (32b_1 B_0 B_1) - (b_2 B_3 A_1 - \theta B_1 A_2)^2 / (64b_2 B_0 B_1^2) > 0$  is restated as  $A_1 / A_2 < (b_1 b_2 (2b_1 b_2 - \theta^2) - (b_1 b_2 - \theta^2) \sqrt{2b_1 b_2 (2b_1 b_2 - \theta^2)}) / (b_2 \theta (3b_1 b_2 - 2\theta^2))$  or

$(b_1b_2(2b_1b_2 - \theta^2) + (b_1b_2 - \theta^2)\sqrt{2b_1b_2(2b_1b_2 - \theta^2)})/(b_2\theta(3b_1b_2 - 2\theta^2)) < A_1/A_2$ . The symmetry of this inequality between Retailers 1 and 2 proves (i) of this corollary.

Next, we substitute  $(i, j) = (1, 2)$  into  $\pi_i^{***}$  in Case (III) and  $(i, j) = (2, 1)$  into  $\pi_j^{**}$  in Case (II), calculating  $\pi_i^{***} - \pi_j^{**} = (B_2^2(B_1A_1 - \theta b_1A_2)^2)/(2b_1B_0B_1B_4^2) - (b_2B_3A_1 - \theta b_1A_2)^2/(64b_2B_0B_1^2)$ . If this value is positive, the profit of Retailer 1 under the strategies  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 2, 2, 3)$  is higher than that under the strategies  $(t(r_1), t(r_2), t(p_1), t(p_2)) = (1, 2, 4, 3)$ .  $(B_2^2(B_1A_1 - \theta b_1A_2)^2)/(2b_1B_4^2B_0B_1) - (b_2B_3A_1 - \theta b_1A_2)^2/(64b_2B_0B_1^2) > 0$  is restated as:  $A_1/A_2 < (2560b_1^4b_2^4\theta - 3472b_1^3b_2^3\theta^3 + 1556b_1^2b_2^2\theta^5 - 230b_1b_2\theta^7 - 8\theta(64b_1^3b_2^3 - 108b_1^2b_2^2\theta^2 + 51b_1b_2\theta^4 - 7\theta^6)\sqrt{2b_1b_2(2b_1b_2 - \theta^2)})/(2(1536b_1^4b_2^5 - 2064b_1^3b_2^4\theta^2 + 760b_1^2b_2^3\theta^4 + 7b_1b_2^2\theta^6 - 32b_2\theta^8))$  or  $(2560b_1^4b_2^4\theta - 3472b_1^3b_2^3\theta^3 + 1556b_1^2b_2^2\theta^5 - 230b_1b_2\theta^7 + 8\theta(64b_1^3b_2^3 - 108b_1^2b_2^2\theta^2 + 51b_1b_2\theta^4 - 7\theta^6)\sqrt{2b_1b_2(2b_1b_2 - \theta^2)})/(2(1536b_1^4b_2^5 - 2064b_1^3b_2^4\theta^2 + 760b_1^2b_2^3\theta^4 + 7b_1b_2^2\theta^6 - 32b_2\theta^8)) < A_1/A_2$ . The symmetry of this inequality between Retailers 1 and 2 proves (ii) of the corollary.  $\square$

**Proof of Proposition 2.** Table 2 suggests that the cells in which all of the three payoffs are circled with a solid line are:  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 4), (2, 2, 3, 4),$  or  $(3, 3, 4, 4)$  ( $(i, j) = (1, 2), (2, 1)$ ), which correspond to (i) in this proposition. Meanwhile, the cells in which some payoffs are circled with a solid line and the others are circled with a dotted line include:  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 3), (1, 1, 3, 4),$  or  $(2, 1, 3, 4)$  ( $(i, j) = (1, 2), (2, 1)$ ), which correspond to (ii)–(iv) in this proposition. Corollary 1 and (i) in Corollary 2 prove that  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 3)$  or  $(1, 1, 3, 4)$  respectively constitute a SPNE if the conditions of the inequalities in (ii) or (iii) in this proposition are satisfied because no supply chain member has an incentive to change its timing strategy. Likewise, Corollary 1 and (ii) in Corollary 2 prove that  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (2, 1, 3, 4)$  constitute a SPNE if the inequality in (iv) in this proposition is satisfied. Finally, restatement of  $\pi_i > \pi_j$  under strategies  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 4), (2, 2, 3, 4), (1, 1, 2, 3), (1, 1, 3, 4),$  and  $(2, 1, 3, 4)$  yields  $A_i/A_j < b_i(\theta^3 - 4\sqrt{4b_1b_j - 2\theta^2}(b_1b_j - \theta^2))/(4b_1b_j - 3\theta^2)(2b_1b_j - \theta^2)$  or  $b_i(\theta^3 + 4\sqrt{4b_1b_j - 2\theta^2}(b_1b_j - \theta^2))/(4b_1b_j - 3\theta^2)(2b_1b_j - \theta^2) < A_i/A_j$ .  $\square$

**Proof of Lemma 1.** When  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (3, 3, 4, 4)$ , the profit of Retailer  $j$  is  $\pi_j^*$ . By changing its pricing timing from  $t(p_j) = 4$  to  $t(p_j) = 5$  when the number of periods is more than four, Retailer  $j$  increases its profit from  $\pi_j^*$  to  $\pi_j^{**}$  because the combination of the timing changes from  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (3, 3, 4, 4)$  to  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (3, 3, 4, 5)$  and the resulting sequence of pricing corresponds to Case (II) in Proposition 1. This means that Retailer  $j$  has an incentive to deviate from the strategy that  $t(p_j) = 4$ , indicating that  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (3, 3, 4, 4)$  no longer constitutes the Nash equilibrium of the timing game in stage one. As a result,  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (3, 3, 4, 4)$  does not constitute the SPNE of the whole dynamic game when the number of periods is set at more than four.  $\square$

**Proof of Proposition 3.** From Proposition 2 and Lemma 1, the combinations of timings that always constitute the SPNE under the general setting where there are more than four periods include:  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 4)$  and  $(2, 2, 3, 4)$  ( $(i, j) = (1, 2)$  or  $(2, 1)$ ). Both of the combinations satisfy  $t(r_i) = t(r_j) < t(p_i) < t(p_j)$ . Meanwhile, the proposition and the lemma suggest that the combinations of timings that can constitute the SPNE under the general setting include:  $(t(r_i), t(r_j), t(p_i), t(p_j)) = (1, 1, 2, 3), (1, 1, 3, 4),$  and

$(2, 1, 3, 4)$ , all of which satisfy  $t(r_j) \leq t(r_i) < t(p_i) < t(p_j)$  ( $(i, j) = (1, 2)$  or  $(2, 1)$ ).  $\square$

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