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A Low-Complexity LS Turbo Channel Estimation Technique for MU-MIMO Systems

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***Abstract*—Turbo channel estimation algorithms can improve estimation accuracy in multiuser (MU)-multi-input multi-output (MIMO) systems. However, the conventional least squares (LS) turbo channel estimation technique based on the maximum likelihood (ML) approach is difficult to be performed in large MU-MIMO systems since it requires a cubic complexity order $\mathcal{O}(\kappa^3 N_T^3 N_R^3)$ for a system constant κ and numbers of N_T-transmit and N_R-receive antennas. This is because the conventional technique computes inversion of a covariance matrix whose size is proportional to $\kappa N_T N_R$. We propose a new low-complexity LS estimation technique that requires a complexity order $\mathcal{O}(\kappa^3 N_T^3)$ only by utilizing algebraic property on the covariance matrix. Simulation results shown in this letter verify that the proposed technique significantly reduces processing time without sacrificing the estimation performance.**

***Index Terms*—Maximum likelihood (ML) estimation, turbo channel estimation, singular value decomposition (SVD).**

I. Introduction

Turbo receiver frameworks (e.g., [1], [2]) provide one of the most promising solutions to the multiuser interference (MUI) problem in multiuser (MU)-multiple-input multiple-output (MIMO) systems. Turbo channel estimation algorithms can improve estimation accuracy by jointly utilizing training sequence (TS) and soft replica of transmitted data [3]. Least squares (LS) channel estimation is the most basic technique since it can be extended to compressive estimation algorithms and/or subspace-based minimum mean square error (MMSE) techniques [4], [5]. However, the conventional LS channel estimation techniques [5], [6] based on the maximum likelihood (ML) approach (e.g., [7]) requires a cubic complexity order of the MIMO system size to consider spatial covariance matrices. It is, hence, difficult to be performed in MU-MIMO systems that require a large number of receive antennas. This letter proposes a new low-complexity LS turbo channel estimation technique without sacrificing the estimation performance.

After this Introduction, Section II shows the system model assumed in this paper. Section III proposes the new low-complexity MU-MIMO turbo LS channel estimation technique. Section IV verifies the effectiveness of the proposed technique via computer simulation results. Section V shows the concluding remarks.

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II. System Model

The same MU-MIMO system as that in [8] is assumed, where we refer to the system composed of U N_T-antenna users and an N_R-antenna base station as $\{U, N_T\} \times N_R$ MU-MIMO. As depicted in Fig. 1, the u-th transmitter modulates (Mod.) data sequence in phase shift keyed (PSK) symbols after interleaving (Π_u) the convolutional coded (CC) bits $c_u(i_c)$. At the slot timing l, the L_d-symbol data sequence $\mathbf{x}^u_{d,k'}(l)$ is transmitted together with L_t-symbol TS $\mathbf{x}^u_{t,k'}(l)$ using $k' = 1, \cdots, N_T$ antennas in a frequency selective fading channel whose channel impulse response (CIR) length is at most W symbols. The received signals can be described in an $N_R \times \tilde{L}_\mathrm{S}$ matrix, as $\mathbf{Y}(l) = \mathbf{H}(l)\mathbf{X}(l) + \mathbf{Z}$, where

$$\begin{aligned}\mathbf{X}(l) &= [\mathbf{X}_t(l), \mathbf{X}_d(l)] &\in \mathbb{C}^{WUN_T \times \tilde{L}_\mathrm{S}},\\ \mathbf{H}(l) &= [\mathbf{H}_1(l), \cdots, \mathbf{H}_{UN_T}(l)] &\in \mathbb{C}^{N_R \times WUN_T},\end{aligned}$$

with $\tilde{L}_\mathrm{S} = L_t + L_d + 2(W-1)$. The TS matrix $\mathbf{X}_t(l)$ is given by $[\mathbf{X}^\mathrm{T}_{t,1}(l), \cdots, \mathbf{X}^\mathrm{T}_{t,UN_T}(l)]^\mathrm{T}$, where $\mathbf{X}_{t,k}(l) = \mathtt{tplz}_W\left\{[\mathbf{x}^{u\,\mathrm{T}}_{t,k'}(l), \mathbf{0}^\mathrm{T}_{W-1}]\right\}$ for an index $k = k' + (u-1)N_T$ corresponding to the k'-th transmission (TX) stream of the u-th user. The operation $\mathtt{tplz}_W\{\mathbf{r}\}$ constructs a $W \times L_r$ Toeplitz matrix whose first row vector is $\mathbf{r} \in \mathbb{C}^{1 \times L_r}$. The data matrix $\mathbf{X}_d(l)$ can be defined similarly. The CIR matrix $\mathbf{H}(l)$ may follow the spatial channel model (SCM) [9], where the variance of the matrix Frobenius norm for the CIR sub-matrix is $\mathbb{E}[\|\mathbf{H}_k(l)\|^2] = \sigma^2_\mathbf{H}(\lceil k/N_T \rceil)$ with a constant $\sigma^2_\mathbf{H}(u)$. The operator $\lceil\cdot\rceil$ is the ceiling function. The n-th row vector in the noise matrix $\mathbf{Z}$ follows the Complex normal distribution $\mathcal{CN}(\mathbf{0}, \sigma^2_z \mathbf{I}_{\tilde{L}_\mathrm{S}})$.

As illustrated in Fig. 1, the receiver performs channel estimation (EST) by using the TS and the soft replica $\hat{\mathbf{x}}^u_{d,k}(l)$ of the transmitted symbols $\mathbf{x}^u_{d,k}(l)$. The soft replica $\hat{\mathbf{x}}^u_{d,k}(l)$ is generated from the *a priori* log-likelihood ratio (LLR) $\lambda^a_{\mathrm{EST},u}$ which is obtained after interleaving the *a posteriori* LLR $\lambda^p_{\mathrm{DEC},u}$ fed back from the decoder (CC^{-1}). Note that the EST is performed jointly over the receive (Rx) antennas in a turbo equalization (EQU) framework [8]. However, the channel decoding is performed per a user individually by means of the Bahl, Cocke, Jelinek and Raviv (BCJR) algorithm.

III. Channel Estimation

A. LS Turbo Channel Estimation

1) Problem: An LS turbo estimation based on the ML approach is formulated as $\hat{\mathbf{H}}(l) = \arg\min_\mathbf{H} \mathcal{L}_{td}(l, \mathbf{H})$. The

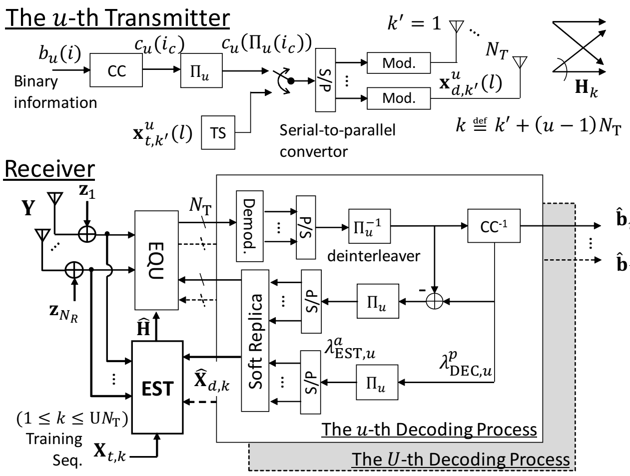

Fig. 1. A $\{U, N_T\} \times N_R$ MU-MIMO system.

joint log-likelihood function $\mathcal{L}_{td}(l, \mathbf{H})$ is, similarly to [3], [6], [7], defined by $\mathcal{L}_t(l, \mathbf{H}) + \mathcal{L}_d(l, \mathbf{H})$, where

$$\mathcal{L}_t(l, \mathbf{H}) = \frac{1}{\sigma_z^2}\|\mathbf{Y}_t(l) - \mathbf{H}\mathbf{X}_t(l)\|,$$
$$\mathcal{L}_d(l, \mathbf{H}) = \frac{1}{\sigma_z^2}\|\mathbf{Y}_d(l) - \mathbf{H}\hat{\mathbf{X}}_d(l)\|_{\boldsymbol{\Gamma}}^2$$

with $\hat{\mathbf{X}}_d(l)$ denoting the soft replica version of $\mathbf{X}_d(l)$. $\mathbf{Y}_t(l)$ and $\mathbf{Y}_d(l)$ are the received TS and data matrix corresponding to $\mathbf{X}_t(l)$ and $\mathbf{X}_d(l)$, respectively. The weighted matrix Frobenius norm is $\|\mathbf{M}\|_{\mathbf{A}}^2 = \mathtt{tr}\{\mathbf{M}^{\mathsf{H}}\mathbf{A}\mathbf{M}\}$ for conformable matrices $\mathbf{M}$ and $\mathbf{A}$. The spatial covariance matrix $\boldsymbol{\Gamma}$ [6] is given by $\boldsymbol{\Gamma} = \left(\mathbf{I}_{N_R} + \sum_{u=1}^{U} \frac{\Delta\hat{\sigma}_{d,u}^2}{\sigma_z^2}\hat{\mathbf{R}}_{\mathbf{H},u}\right)^{-1}$, where

$$\Delta\hat{\sigma}_{d,u}^2 = \frac{1}{L_d N_T}\sum_{k=1}^{N_T} \mathbb{E}[\|\mathbf{x}_{d,k}^u(l)\|^2] - \|\hat{\mathbf{x}}_{d,k}^u(l)\|^2 \tag{1}$$

and $\hat{\mathbf{R}}_{\mathbf{H},u} = \hat{\mathbf{H}}_u^{[i-1]}(\hat{\mathbf{H}}_u^{[i-1]})^{\mathsf{H}}$. The u-th user's channel estimate $\hat{\mathbf{H}}_u^{[i-1]}$ is obtained by the previous $[i-1]$-th iteration.

2) Exact LS Solution: For the sake of conciseness, the parameter l is omitted hereafter. The LS channel estimate can be obtained [6] via

$$\mathtt{vec}\{\hat{\mathbf{H}}\} = \mathfrak{R}_{\mathbf{XX}}^{-1} \cdot \mathtt{vec}\{\mathbf{R}_{\mathbf{YX}}\}, \tag{2}$$

where $\mathtt{vec}(\mathbf{A})$ is the vectorization operator to produce an $MN \times 1$ vector by stacking the columns of an $M \times N$ matrix $\mathbf{A}$. We define

$$\mathfrak{R}_{\mathbf{XX}} = \mathbf{R}_{\mathbf{XX}_t}^{\mathsf{T}} \otimes \mathbf{I}_{N_R} + \hat{\mathbf{R}}_{\mathbf{XX}_d}^{\mathsf{T}} \otimes \boldsymbol{\Gamma}, \tag{3}$$
$$\mathbf{R}_{\mathbf{YX}} = \mathbf{R}_{\mathbf{YX}_t} + \boldsymbol{\Gamma}\mathbf{R}_{\mathbf{YX}_d}, \tag{4}$$

where $\otimes$ denotes the Kronecker product and

$$\mathbf{R}_{\mathbf{XX}_t} = \mathbf{X}_t\mathbf{X}_t^{\mathsf{H}},\ \mathbf{R}_{\mathbf{YX}_t} = \mathbf{Y}_t\mathbf{X}_t^{\mathsf{H}},$$
$$\hat{\mathbf{R}}_{\mathbf{XX}_d} = \hat{\mathbf{X}}_d\hat{\mathbf{X}}_d^{\mathsf{H}},\ \mathbf{R}_{\mathbf{YX}_d} = \mathbf{Y}_d\hat{\mathbf{X}}_d^{\mathsf{H}}.$$

However, as discussed in [5], [6], the solution (2) requires the complexity order $\mathcal{O}((WUN_TN_R)^3)$ if we compute the matrix inversion by using the Gaussian elimination.

B. Approximated LS Turbo Channel Estimation

The LS estimation (2) can be performed with the complexity $\mathcal{O}((WUN_T)^3)$ if we assume the spatially uncorrelated channel approximation for $\forall u$ to compute the covariance matrix $\boldsymbol{\Gamma}$:

$$\hat{\mathbf{R}}_{\mathbf{H},u} \approx \mathbf{I}_{N_R}. \tag{5}$$

C. Low-Complexity LS Turbo Channel Estimation

1) Factorized Matrix Inversion: We can factorize (3) as

$$\mathfrak{R}_{\mathbf{XX}} = \left(\mathbf{R}_{\mathbf{XX}_t}^{\mathsf{T}/2} \otimes \mathbf{I}_{N_R}\right) \mathbf{J} \left(\mathbf{R}_{\mathbf{XX}_t}^{\mathsf{T}/2} \otimes \mathbf{I}_{N_R}\right)^{\mathsf{H}}, \tag{6}$$

where

$$\mathbf{J} = \mathbf{I}_{WUN_T} \otimes \mathbf{I}_{N_R} + \mathbf{Q} \otimes \boldsymbol{\Gamma}$$

with

$$\mathbf{Q} = \mathbf{R}_{\mathbf{XX}_t}^{-\mathsf{T}/2}\,\hat{\mathbf{R}}_{\mathbf{XX}_d}^{\mathsf{T}}\mathbf{R}_{\mathbf{XX}_t}^{-\frac{*}{2}}. \tag{7}$$

By using the singular value decomposition (SVD) for the matrices $\mathbf{Q}$ and $\boldsymbol{\Gamma}$:

$$\mathbf{Q} = \mathbf{U}_{\mathbf{Q}}\boldsymbol{\Sigma}_{\mathbf{Q}}\mathbf{U}_{\mathbf{Q}}^{\mathsf{H}}, \tag{8}$$
$$\boldsymbol{\Gamma} = \mathbf{U}_{\boldsymbol{\Gamma}}\boldsymbol{\Sigma}_{\boldsymbol{\Gamma}}\mathbf{U}_{\boldsymbol{\Gamma}}^{\mathsf{H}}, \tag{9}$$

the matrix $\mathbf{J}$ can be factorized, as

$$\mathbf{J} = (\mathbf{U}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}})\boldsymbol{\Sigma}_{\mathbf{J}}(\mathbf{U}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}})^{\mathsf{H}}, \tag{10}$$

where $\boldsymbol{\Sigma}_{\mathbf{J}} = \mathbf{I}_{WUN_TN_R} + \boldsymbol{\Sigma}_{\mathbf{Q}} \otimes \boldsymbol{\Sigma}_{\boldsymbol{\Gamma}}$ is a diagonal matrix. From (6) and (10), the matrix inverse of (3) is reduced to

$$\mathfrak{R}_{\mathbf{XX}}^{-1} = \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right) \boldsymbol{\Sigma}_{\mathbf{J}}^{-1} \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right)^{\mathsf{H}}, \tag{11}$$

where $\tilde{\mathbf{U}}_{\mathbf{Q}} = \mathbf{R}_{\mathbf{XX}_t}^{-\frac{*}{2}}\mathbf{U}_{\mathbf{Q}}$.

2) Proposed Solution: The computation that substitutes (11) into (2) after separately performing (11) still requires $\mathcal{O}((WUN_TN_R)^3)$. We can, however, perform the matrix inversion in (2) with $\mathcal{O}((WUN_T)^3 + N_R^3)$ by calculating its vector-wise factors. Specifically, by substituting (11) into (2),

$$\mathtt{vec}\{\hat{\mathbf{H}}\} = \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right) \boldsymbol{\Sigma}_{\mathbf{J}}^{-1} \left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right)^{\mathsf{H}} \mathtt{vec}\{\mathbf{R}_{\mathbf{YX}}\},$$

where we have

$$\left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right)^{\mathsf{H}} \mathtt{vec}\{\mathbf{R}_{\mathbf{YX}}\} = \mathtt{vec}\{\mathbf{U}_{\boldsymbol{\Gamma}}^{\mathsf{H}}\mathbf{R}_{\mathbf{YX}}\tilde{\mathbf{U}}_{\mathbf{Q}}^{*}\}. \tag{12}$$

This is because $(\mathbf{C}^{\mathsf{T}} \otimes \mathbf{A})\mathtt{vec}\{\mathbf{B}\} = \mathtt{vec}\{\mathbf{ABC}\}$ holds for conformable matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. The matrix version of (2) can, therefore, be written as

$$\hat{\mathbf{H}} = \mathtt{mat}_{N_R}\left[\left(\tilde{\mathbf{U}}_{\mathbf{Q}} \otimes \mathbf{U}_{\boldsymbol{\Gamma}}\right) \cdot \mathbf{v}\right]$$
$$= \mathbf{U}_{\boldsymbol{\Gamma}}\,\mathtt{mat}_{N_R}\{\mathbf{v}\}\,\tilde{\mathbf{U}}_{\mathbf{Q}}^{\mathsf{T}}, \tag{13}$$

where

$$\mathbf{v} = \mathtt{diag}\{\boldsymbol{\Sigma}_{\mathbf{J}}^{-1}\} \odot \mathtt{vec}\{\mathbf{U}_{\boldsymbol{\Gamma}}^{\mathsf{H}}\mathbf{R}_{\mathbf{YX}}\tilde{\mathbf{U}}_{\mathbf{Q}}^{*}\}. \tag{14}$$

The operation $\mathtt{mat}_N(\mathbf{a})$ forms an $N \times M$ matrix from the argument vector $\mathbf{a} \in \mathbb{C}^{NM\times 1}$, i.e., $\mathbf{a} = \mathtt{vec}\{\mathtt{mat}_N\{\mathbf{a}\}\}$. Moreover, the operator $\odot$ is the entry-wise vector multiplication.

The complexity order required to obtain (13) is dominated by $\mathcal{O}((WUN_T)^3 + N_R^3 + (WUN_T)^2L_{td}) = \mathcal{O}((WUN_T)^3)$ if $WUN_T \gg N_R$, where the details is shown in Table I. We note that Cholesky factorization $\mathbf{R}_{\mathbf{XX}_t}^{-1} = \mathbf{R}_{\mathbf{XX}_t}^{-1/2}\mathbf{R}_{\mathbf{XX}_t}^{-\mathsf{H}/2}$ can be performed off-line since the TS is known at the receiver.

TABLE I
DETAILS OF COMPUTATIONAL COMPLEXITY ORDER

Symbol	Complexity order	Eqn.
$\mathfrak{R}_{\mathbf{XX}}$	$\mathcal{O}((WUN_T)^2L_{td})$	(3)
$\mathbf{R}_{\mathbf{YX}}$	$\mathcal{O}(WUN_TN_RL_{td})$	(4)
$\mathbf{Q}$	$\mathcal{O}((WUN_T)^3)$	(7)
$\mathbf{U}_{\mathbf{Q}}, \mathbf{\Sigma}_{\mathbf{Q}}$	$\mathcal{O}((WUN_T)^3)$	(8)
$\mathbf{U}_{\mathbf{\Gamma}}, \mathbf{\Sigma}_{\mathbf{\Gamma}}$	$\mathcal{O}(N_R^3)$	(9)
$\mathbf{\Sigma}_{\mathbf{J}}^{-1}$	$\mathcal{O}(WUN_TN_R)$	(11)
$\mathbf{v}$	$\mathcal{O}((WUN_TN_R)^2)$	(14)

IV. NUMERICAL EXAMPLES

A. Simulation Setups

This paper assumes $\{2,4\} \times 12$ MIMO channels based on the SCM [9], where the antenna element spacing at the base station and the mobile station are set at 0.5 wavelength. The CIRs of the first and second users respectively follow the Pedestrian-B (PB) model [9] with a 3 km/h mobility (PB3) and the Vehicular-A (VA) model [9] with a 30 km/h mobility (VA30). The path positions of the PB and VA models are respectively set at $\{$1, 2.4, 6.6, 9.4, 17.1, 26.9$\}$ and $\{$1, 3.2, 6, 8.6, 13.1, 18.6$\}$ symbol timings assuming that a TX bandwidth is 7 MHz with a carrier frequency of 2 GHz. The CIR length W and the variance $\sigma_{\mathbf{H}}^2(u)$ are set at 31 and 1, respectively.

The TS vectors $\mathbf{x}_{t,k}^u$ are generated by using the first $L_t = 320$ bits of length 511 Gold sequences. The data sequence $\mathbf{x}_{d,k}^u$ is $L_d = 1024$ 8PSK-symbols, where the variance is $\mathbb{E}[\|\mathbf{x}_{d,k}^u\|^2]/L_d = 1$ for $\forall u, k$.

B. Receiver Performance

1) Channel Estimation Performance: Fig. 2(a) shows normalized MSE (NMSE) performances of the LS channel estimation techniques in the MU-MIMO channel realizations, where $\mathrm{NMSE} = \mathbb{E}[\|\hat{\mathbf{H}} - \mathbf{H}\|^2]/\mathbb{E}[\|\mathbf{H}\|^2]$ for channel estimates $\hat{\mathbf{H}}$. The analytical NMSE performance is given by $\mathrm{ANMSE}_{\bar{L}_{td}}(\sigma_z^2) = \sigma_z^2 WUN_TN_R/\{\bar{L}_{td}\mathbb{E}[\|\mathbf{H}\|^2]\}$ for length $\bar{L}_{td} = L_t + L_d$ reference signals [5]. As depicted in Fig. 2, the NMSE of the proposed technique is identical to that of the conventional exact LS technique and it follows the analytical NMSE performance in a high signal-to-noise ratio (SNR) regime, $\mathrm{SNR} \geq 16$ dB, where the turbo receiver can detect the data sequence correctly after six iterations.

2) Bit error rate (BER) Performance: Fig. 2(b) shows BER performance of the turbo receiver using the LS channel estimation algorithms, where *Genie* technique assumes known data sequences but only for the channel estimator in order to show a benchmark. The receiver using the approximated LS estimator exhibits BER deterioration by 0.5 dB at the $\mathrm{BER} = 10^{-5}$, compared to that of the exact LS tehcnique. This is because, as shown in Fig. 2(a), the approximated LS technique suffers from NMSE deterioration in a moderate SNR regime.

C. Convergence Property

Fig. 3 depicts NMSE convergence property over the LLR's accuracy, where the mutual information (MI) $\mathcal{I}_{\mathrm{EST},u}^a = \mathcal{I}(\lambda_{\mathrm{EST},u}^a; c_u)$ between the LLR $\lambda_{\mathrm{EST},u}^a$ and the coded bits c_u at the transmitter is defined by $\frac{1}{2}\sum_{\mathfrak{m}=\pm 1}\int_{-\infty}^{+\infty} \mathrm{P_r}(\lambda_{\mathrm{EST},u}^a|\mathfrak{m}) \log_2 \frac{\mathrm{P_r}(\lambda_{\mathrm{EST},u}^a|\mathfrak{m})}{\mathrm{P_r}(\lambda_{\mathrm{EST},u}^a)} d\lambda_{\mathrm{EST},u}^a$ with the conditional probability density $\mathrm{P_r}(\lambda_{\mathrm{EST},u}^a|\mathfrak{m})$ given $\mathfrak{m} = 1 - 2c_u$. For the sake of simplicity, this subsection assumes that all users take the same MI $\mathcal{I}_{\mathrm{EST},u}^a$. The average SNR is set at 18 dB. The analytical NMSE is given according to the reference signal length $\bar{L}_{td} \approx L_t + \gamma\hat{\sigma}_{d,u}^2 N_d$ with $\gamma = \sigma_z^2/(\sigma_z^2 + \Delta\hat{\sigma}_{d,u}^2 N_T \sigma_{\mathbf{H}}^2(u)/N_R)$, where $\Delta\hat{\sigma}_{d,u}^2$ defined by (1) is calculated from $\hat{\mathbf{x}}_{d,k}^u$ of the accuracy $\mathcal{I}_{\mathrm{EST},u}^a$.

As shown in Fig. 3(a), the NMSE performance of the proposed technique achieves exactly the same NMSE performance as that of the conventional estimator. The NMSE performance of the approximated LS channel estimation coincides with that of the conventional estimator at $\mathcal{I}_{\mathrm{EST},u}^a = 0$ and 1. This is because we may ignore the covariance matrix $\mathbf{\Gamma}$ when $\mathcal{I}_{\mathrm{EST},u}^a = 0$, i.e., when channel estimation is performed with the TS only. When $\mathcal{I}_{\mathrm{EST},u}^a = 1$, the approximation (5) is not used to compute $\mathbf{\Gamma}$ since $\Delta\hat{\sigma}_u^2 = 0$. However, the approximated estimator using (5) degrades the NMSE performance when $0 < \mathcal{I}_{\mathrm{EST},u}^a < 1$.

Hence, as shown in Fig. 3(b), the approximated estimator suffers from the NMSE deterioration in the first three iterations, which affects BER convergence performance. It is observed from Fig. 3(b) that the turbo receiver using the approximated estimator requires six iterations to achieve $\mathrm{BER} \leq 10^{-6}$ whereas five iterations are sufficient for the proposed technique at $\mathrm{SNR} = 18$ dB.

D. Processing Time

Fig. 4 shows the processing time of the LS channel estimation techniques, where the channel estimators implemented with Matlab R2017b are performed on a single core in a Xeon E5-2680v2 processor using 64 Gbyte RAM. As observed from Fig. 4, the processing time of the conventional LS estimator [5], [6] increases as the number of Rx antennas increases. It takes, on average, 100 sec to obtain an LS estimate matrix for a $\{2,4\} \times 64$ MIMO channel realization, although the Xeon processor of the clock frequency 2.8 GHz is utilized.

However, the processing time of the proposed technique is almost independent of the number of Rx antennas. This is because, as discussed in Section III, the complexity order required for the proposed technique is $\mathcal{O}((WUN_T)^3)$ when $N_R \ll WUN_T$. It reduces the processing time more than 99% in the $\{2,4\} \times 64$ MIMO receiver. The processing time required for the proposed technique is, however, five times greater than that of the approximated LS channel estimation due to the operations (7) and (8). Nevertheless, the increased processing time is very minor compared to that of the conventional LS channel estimation in the $\{2,4\} \times 8$ MU-MIMO receiver.

V. CONCLUSIONS

The conventional LS turbo channel estimation formulated as an ML problem requires the complexity order $\mathcal{O}((WUN_TN_R)^3)$, which makes it difficult to perform the ML-based estimation in an MU-MIMO system using a large number of Rx antennas. As a solution to the problem, this letter

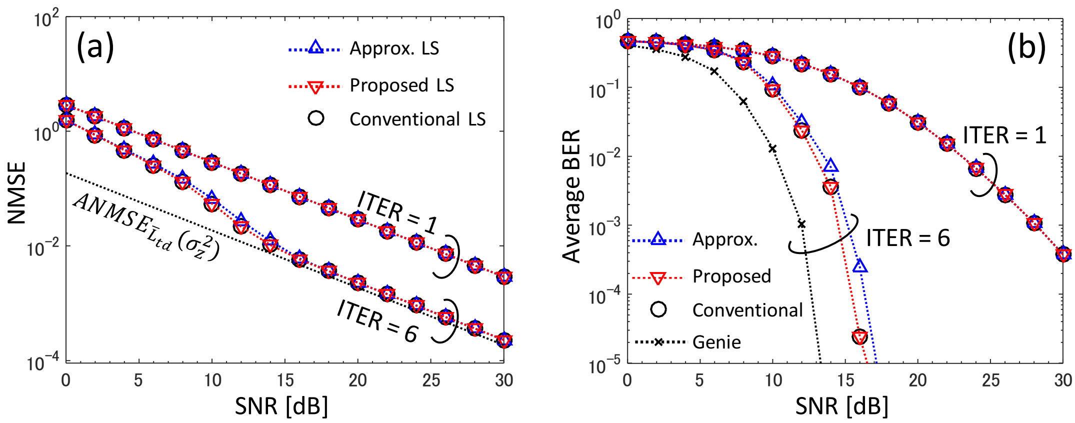

Fig. 2. NMSE (a) and BER (b) performances in the $\{2, 4\} \times 12$ MU-MIMO system, where the PB3 and VA30 models are assumed for the first and second users, respectively. The curve of $\mathrm{ANMSE}_{\bar{L}_{td}}(\sigma_z^2)$ shows the analytical NMSE performance. The generator polynomial of the CC is $(g_1, g_2) = (7, 5)_8$.

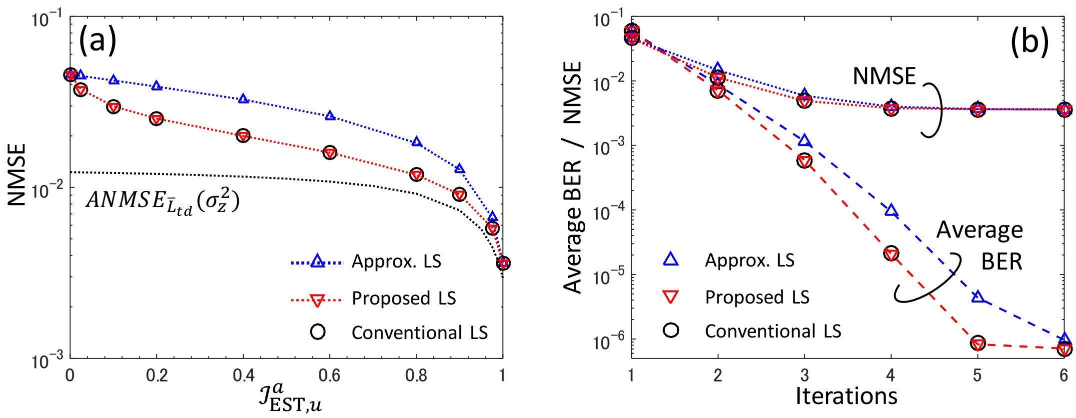

Fig. 3. Convergence properties at SNR = 18 dB: the NMSE over the MI (a), and the NMSE / BER performances over turbo iterations (b).

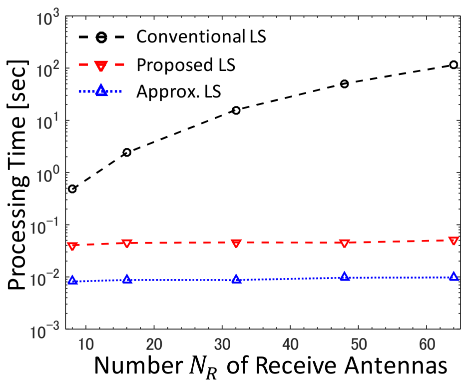

Fig. 4. Processing Time of LS estimators in the $\{2, 4\} \times N_R$ MU-MIMO.

has proposed a new low-complexity LS channel estimation technique by utilizing algebraic property on the covariance matrix of the reference signals. The simulation results presented in this letter verify that the proposed technique significantly reduces the processing time to $\mathcal{O}((WUN_T)^3)$ without sacrificing the estimation performance.

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