Exploring the string axiverse and parity violation in gravity with gravitational waves

Yoshida, Daiske

Soda, Jiro

(Citation)
International Journal of Modern Physics D,27(9):1850096-1850096

(Issue Date)
2018-07

(Resource Type)
journal article

(Version)
Accepted Manuscript

International Journal of Modern Physics D

EXPLORING THE STRING AXIVERSE AND PARITY VIOLATION IN GRAVITY WITH GRAVITATIONAL WAVES

DAISKE YOSHIDA

*Physics Department, Kobe University, Kobe, 657-8501, Japan**

JIRO SODA

Physics Department, Kobe University, Kobe, 657-8501, Japan†

Received Day Month Year
Revised Day Month Year

We show that the parametric resonance of gravitational waves occurs due to the axion coherent oscillation and the circular polarization of gravitational waves is induced by the Chern-Simons coupling. However, we have never observed these signatures in the data of gravitational waves. Using this fact, we give stringent constraints on the Chern-Simons coupling constant ℓ and the abundance of the light string axion. In particular, we improved the current bound $\ell \leq 10^8$ km by many orders of magnitude.

Keywords: Gravitational wave; Modified gravity; Dark matter

PACS numbers: 04.30.Db, 04.30.Nk, 04.30.Tv, 04.50.Kd, 04.80.Cc, 95.35.+d

1. Introduction

The direct detection of the gravitational waves (GWs), GW150914, has opened a new window for observing the universe.[1] Thus, we can utilize GWs for exploring not only astronomy but also fundamental physics. One of important issues in fundamental physics is to establish quantum theory of gravity. As is well known, string theory is a promising candidate of quantum theory of gravity. Hence, it would be worth studying GWs to extract a hint for profound understanding of string theory. In this letter, we will focus on the string axiverse[2] which is one of fundamental aspects of string theory.

Remarkably, the mass of string axions can take the values in the broad range from 10^{-33} eV to 10^{-10} eV.[2,3] In order to explore this string axiverse, we can use the cosmic microwave background radiations (CMB),[4] the large scale structure of

*dice-k.yoshida@stu.kobe-u.ac.jp
†jiro@phys.sci.kobe-u.ac.jp

the universe,[5] pulsar timing arrays,[6–8] interferometer detectors,[9,10] the dynamics of binary system,[11] black hole physics[12,13] and nuclear spin precession.[14] The present work intends to add an item to this list.

One of the important aspects of string axiverse is that string axions can be the dark component of the universe. Indeed, recently, the possibility that the axion can replace the cold dark matter (CDM) has been intensively studied.[15,16] As is well known, supersymmetric particles, the so-called neutralinos, have been regarded as the most promising candidate for the CDM. However, there was no signature of supersymmetry at the LHC. Moreover, although the CDM works quite well on large scales, there exist problems on small scales. Hence, it is worth investigating the axion dark matter instead of the CDM. In particular, the axion with the ultralight mass 10^{-22} eV can resolve the small scale problem of the cold dark matter[17](see[18] for earlier history). In fact, the axion with any mass can behave as the cold dark matter on large scales because of the coherent oscillation.

In the presence of the axion dark matter, it is natural to consider the Chern-Simons terms both in the gauge sector and in the gravity sector.[19,20] In particular, the gravitational Chern-Simons term induces a coupling between the GWs and the axion.[21,22] It should be noted that the Chern-Simons coupling provides a natural mechanism for parity violation in gravity provided nontrivial profile of the axion field. In fact, the parity violation in gravity is also phenomenologically intriguing.[23–25] Another key ingredient of the axion dark matter is the coherent oscillation of the axion field. Since the axion is coherently oscillating, the occurrence of parametric resonance due to the Chern-Simons coupling can be expected.[26]

In this letter, we examine this possibility and find a new way to explore the string axiverse. It should be noted that the detectable frequency range of GWs with ground based interferometers is from 1 Hz to 10^4 Hz, which corresponds to the axion mass range from 10^{-14} eV to 10^{-10} eV. The GWs in this relevant frequency range can undergo the coherent oscillation of the axion in the center of the Galaxy where the density of the dark matter is about 10^3 times higher than the edge of the galaxy where we live. During the journey, the GWs can be enhanced by the resonance. Since the resonance process violates parity, there should be parity-violation in GWs. Thus, if the chiral gravitational waves are observed,[27] that indicates the existence of the axion dark matter. When we do not observe the chiral gravitational waves, we can constrain the Chern-Simons coupling constant or the abundance of the string axion in the relevant mass range. Hence, it is worth investigating the mechanism in detail.

2. Model

The model we consider is the dynamical Chern-Simons gravity coupled with the axion.[21] We adopt the natural unit $c = \hbar = 1$ and the conventions in.[22] We choose the coordinate $x^\mu \equiv (x^0, x^i) = (\eta, x^i)$ with the conformal time η and $x^i = (x^1, x^2, x^3) = (x, y, z)$. Then, the action is given by

$$S = S_{\rm EH} + S_{\rm CS} + S_\Phi \ , \tag{1}$$

where the Einstein-Hilbert action reads

$$S_{\rm EH} \equiv \kappa \int_{\mathcal{V}} dx^4 \sqrt{-g} R. \tag{2}$$

Here, κ is proportional to the inverse of Newton constant, $\kappa \equiv (16\pi G)^{-1}$ and R is a Ricci scalar. The Chern-Simons term $S_{\rm CS}$ is given by

$$S_{\rm CS} = \frac{1}{4}\alpha \int_{\mathcal{V}} dx^4 \sqrt{-g}\, \Phi \tilde{R}R \ , \tag{3}$$

where Φ is the axion field, α is a coupling constant, and $\tilde{R}R$ is the Pontryagin density defined as

$$\tilde{R}R \equiv \tilde{R}^{\alpha}_{\ \beta}{}^{\gamma\delta} R^{\beta}{}_{\alpha\gamma\delta} \quad \text{and} \quad \tilde{R}^{\alpha}_{\ \beta}{}^{\gamma\delta} \equiv \frac{1}{2}\epsilon^{\gamma\delta\rho\sigma} R^{\alpha}{}_{\beta\rho\sigma}. \tag{4}$$

Note that, since the axion field Φ have the dimension of the mass, the Chern-Simons coupling constant α have the dimension of length. Hence, it is convenient to express the coupling constant as

$$\alpha = \sqrt{\frac{\kappa}{2}}\ell^2 \ , \tag{5}$$

where ℓ has the dimension of the length.[28] The current limit coming from the Solar System test is $\ell \leq 10^8$ km.[29] The future experiment might improve the contraint on ℓ by sixth order stronger than the Solar System constraint.[30] The action S_Φ of the axion field Φ is given by

$$S_\Phi \equiv -\frac{1}{2} \int_{\mathcal{V}} dx^4 \sqrt{-g} \left[g^{\mu\nu} \left(\nabla_\mu \Phi\right) \left(\nabla_\nu \Phi\right) + 2V(\Phi) \right] \tag{6}$$

where $V(\Phi)$ is the potential of the axion field.

From the action, we obtain the equations for the metric field

$$G_{\mu\nu} + \frac{\alpha}{\kappa} C_{\mu\nu} = \frac{1}{2\kappa} T_{\mu\nu} \ , \tag{7}$$

where $G_{\mu\nu}$ is the Einstein tensor and $C_{\mu\nu}$ is defined as

$$C^{\mu\nu} \equiv (\nabla_\alpha \Phi) \epsilon^{\alpha\beta\gamma(\mu} \nabla_\gamma R^{\nu)}{}_{\beta} + (\nabla_\alpha \nabla_\beta \Phi) \tilde{R}^{\beta(\mu\nu)\alpha} \ . \tag{8}$$

The trace of this tensor identically vanishes. The energy momentum tensor $T_{\mu\nu}$ becomes

$$T_{\mu\nu} = \left[(\nabla_\mu \Phi)(\nabla_\nu \Phi) - \frac{1}{2} g_{\mu\nu} (\nabla_\sigma \Phi)(\nabla^\sigma \Phi) - g_{\mu\nu} V(\Phi) \right] \ . \tag{9}$$

The equation of motion for the axion field is the modified Klein-Gordon equation given by

$$\Box \Phi - \frac{dV(\Phi)}{d\Phi} = -\frac{\alpha}{4} \tilde{R}R \ , \tag{10}$$

where $\Box$ is the d'Alembertian operator defined by $\Box\Phi \equiv \nabla_a \nabla^a \Phi$.

3. Setup

The potential of the axion field is given by

$$V(\Phi) \equiv \frac{1}{2} m^2 \Phi^2 \ , \tag{11}$$

where m is the mass of the axion. First of all, we must solve the equations of motion in the homogeneous background spacetime

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= a^2(\eta) \left(-d\eta^2 + \delta_{ij} dx^i dx^j \right) \ . \end{aligned} \tag{12}$$

The axion depends only on the conformal time η, that is, $\Phi(x^\mu) \equiv \Phi(\eta)$. Thus, we have the modified Klein-Gordon equation

$$\Phi'' + 2\frac{a'}{a}\Phi' + a^2 m^2 \Phi = 0 \ , \tag{13}$$

where the prime denotes the derivative with respect to the conformal time η. These equations can be solved numerically. However, since the time scale of the expansion of the universe is much longer than the oscillation time scale for the mass range $m \simeq 10^{-14} \sim 10^{-10}$ eV, we can ignore the cosmic expansion in the Klein-Gordon equation. Hence, the scale factor can be put as $a(\eta) = 1$. Now, we have the solution as

$$\Phi = \Phi_0 \cos(m\eta) \ , \tag{14}$$

where Φ_0 is determined by the energy density of the axion field as

$$\Phi_0 = \frac{\sqrt{2\rho}}{m} \tag{15}$$

$$\simeq 2.1 \times 10^7 \text{ eV} \times \left(\frac{10^{-10}\text{ eV}}{m} \right) \sqrt{\frac{\rho}{0.3 \text{ GeV/cm}^3}} \ . \tag{16}$$

Here, we normalized by the energy density of the dark matter in the halo of the Galaxy. Of course, in general, the axions do not need to dominate the dark matter component.

Now, we move on to the analysis of equation of motion for GWs. The GWs can be described by the metric

$$ds^2 \simeq -d\eta^2 + \delta_{ij} dx^i dx^j + h_{ij} dx^i dx^j \ , \tag{17}$$

where the spatial tensor h_{ij} is transverse and traceless. We work in the Fourier space and consider GWs with the wave number vector $\boldsymbol{k} = (k^1, k^2, k^3)$. We can define the unit vector $\boldsymbol{n} \equiv \boldsymbol{k}/k$ with $k \equiv |\boldsymbol{k}|$. Then, the transverse condition is rewritten as $n^i h_{ij} = 0$ in the Fourier space. The perturbed gravitational field can be expressed as

$$h_{ij}(\eta, \boldsymbol{k}) = h_+(\eta, \boldsymbol{k})\, e^{+}_{ij}(\boldsymbol{n}) + h_\times(\eta, \boldsymbol{k})\, e^{\times}_{ij}(\boldsymbol{n}) \ , \tag{18}$$

where $h_+(\eta, \boldsymbol{k})$ and $h_\times(\eta, \boldsymbol{k})$ are linear polarization modes. Here, the polarization tensors, $e^{+}_{ij}(\boldsymbol{n})$ and $e^{\times}_{ij}(\boldsymbol{n})$, are defined by

$$e^{+}_{ij}(\boldsymbol{n}) \equiv u_i u_j - v_i v_j \quad \text{and} \quad e^{\times}_{ij}(\boldsymbol{n}) \equiv u_i v_j + v_i u_j . \tag{19}$$

where the two orthogonal unit vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$, satisfy the relation, $\boldsymbol{n} = \boldsymbol{u} \times \boldsymbol{v}$. In order to discuss parity violation, it is useful to define circular polarization tensors

$$e^{\mathrm{R}}_{ij}(\boldsymbol{n}) = \frac{1}{\sqrt{2}} \left(e^{+}_{ij}(\boldsymbol{n}) + i e^{\times}_{ij}(\boldsymbol{n}) \right) \ , \tag{20}$$

$$e^{\mathrm{L}}_{ij}(\boldsymbol{n}) = \frac{1}{\sqrt{2}} \left(e^{+}_{ij}(\boldsymbol{n}) - i e^{\times}_{ij}(\boldsymbol{n}) \right) \ . \tag{21}$$

Then, the perturbed gravitational field h_{ij} is expressed by

$$h_{ij} = h_{\mathrm{R}} \ e^{\mathrm{R}}_{ij}(\boldsymbol{n}) + h_{\mathrm{L}} \ e^{\mathrm{L}}_{ij}(\boldsymbol{n}) \ , \tag{22}$$

where the circular polarization modes are defined by

$$h_{\mathrm{R}} \equiv \ \frac{1}{\sqrt{2}} \left(h_+ - i h_\times \right) \ , \quad h_{\mathrm{L}} \equiv \ \frac{1}{\sqrt{2}} \left(h_+ + i h_\times \right) . \tag{23}$$

By using these expressions and the relation

$$i \epsilon_{ilm} n_l e^{\mathrm{R/L}}_{mj}(\boldsymbol{n}) = \pm e^{\mathrm{R/L}}_{ij}(\boldsymbol{n}) \ , \tag{24}$$

the gravitational wave equations can be diagonalized as

$$h''_{\mathrm{A}} + \frac{\epsilon_{\mathrm{A}} \delta \ \cos(m\eta)}{1 + \epsilon_{\mathrm{A}} \frac{k}{m} \delta \ \sin(m\eta)} k \, h'_{\mathrm{A}} + k^2 h_{\mathrm{A}} = 0 \ , \tag{25}$$

where we defined the dimensionless parameter δ as

$$\delta \equiv \ \frac{\alpha}{\kappa} m^2 \Phi_0 \ . \tag{26}$$

The parameter δ characterize the behavior of the gravitational wave resonance. The capital latin index represents the each parity state $A = \mathrm{R}$ or L and ϵ_{A} is defined by

$$\epsilon_{\mathrm{A}} \equiv \ \begin{cases} 1 & \mathrm{A} = \mathrm{R} \ , \\ -1 & \mathrm{A} = \mathrm{L} \ . \end{cases} \tag{27}$$

In the following, we will show Eq.(25) exhibits the parametric resonance.

4. Resonant amplification of GWs

Suppose that the gravitational waves from some violent astrophysical events go through our galaxy to reach us. Thus, GWs are affcted by the coherent oscillation of the axion dark matter.

We numerically solved Eq.(25) and plotted the results. In Fig.1, we plotted the time evolution of the amplitude of each mode at the resonance frequency. We see the growth rate depends on the chirality, which stems from the parity violation. In Fig.2, the time evolution of the degree of the circular polarization

$$\text{parity}(\eta) \equiv \frac{|h_{\rm R}|^2 - |h_{\rm L}|^2}{|h_{\rm R}|^2 + |h_{\rm L}|^2}. \tag{28}$$

is plotted. We can see the parity violation clearly from Fig.2.

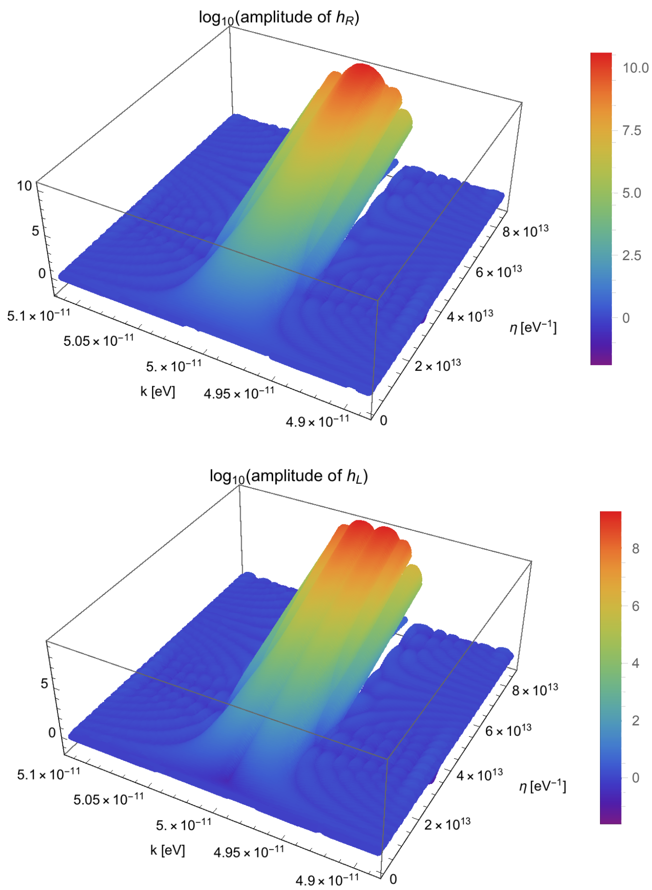

Fig. 1. The growth of the amplitude of GWs is plotted for $\ell = 10^8\,\text{km}$, $m = 10^{-10}\,\text{eV}$, $\rho = 0.3 \times 10^6\,\text{GeV/cm}^3$.

We can also give analytical estimates. From Eq.(25), we find the first resonance wave-number $k_{\rm r}$ is given by $k_{\rm r} = m/2$. Here, its frequency $f_{\rm r}$ can be converted into

$$f_{\rm r} = \frac{k_{\rm r}}{2\pi} \simeq 1.2 \times 10^4\ \text{Hz} \left(\frac{m}{10^{-10}\ \text{eV}}\right). \tag{29}$$

Note that this value lies in the detectable range by the ground based interferometer detectors for GWs. Following the standard analysis of the parametric resonance, we

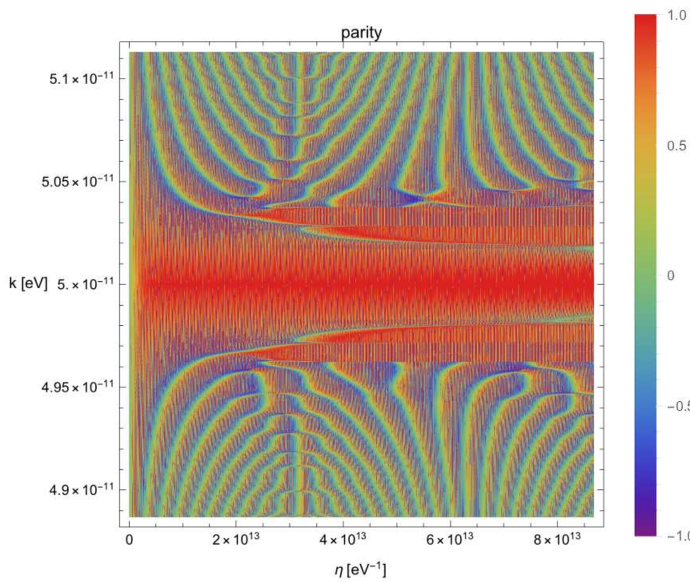

Fig. 2. The growth of the parity-violation is plotted for $\ell = 10^8\,\mathrm{km},\ m = 10^{-10}\,\mathrm{eV},\ \rho = 0.3 \times 10^6\,\mathrm{GeV/cm^3}$.

obtain the width of the resonance as

$$\frac{m}{2} - \frac{m}{8}\delta \lesssim k_{\mathrm{r}} \lesssim \frac{m}{2} + \frac{m}{8}\delta. \tag{30}$$

In the case of the black hole binary system, gravitational waves are superposition of waves with various frequencies. Among them only waves with the resonant frequency will be enhanced. Hence, the gravitational waves will have an almost monochromatic frequency. Since the resonance peak is very sharp, if we detect this signal, we can determine the mass of the axion dark matter. We can also calculate the growth rate of GWs due to the resonance by the axion oscillation. The maximum growth rate $\Gamma_{\max}$ is given by

$$\Gamma_{\max} = \frac{m}{8}\delta \tag{31}$$

$$\simeq 2.8 \times 10^{-16}\ \mathrm{eV} \times \left(\frac{m}{10^{-10}\ \mathrm{eV}}\right)^2 \left(\frac{\ell}{10^8\ \mathrm{km}}\right)^2 \sqrt{\frac{\rho}{0.3\ \mathrm{GeV/cm^3}}}\ . \tag{32}$$

Thus, the time $t_{\times 10}$ when the amplitude become ten times bigger is estimated as

$$\begin{aligned} t_{\times 10} \simeq\ & 8.1 \times 10^{15}\ \mathrm{eV}^{-1} \\ & \times \left(\frac{10^{-10}\ \mathrm{eV}}{m}\right)^2 \left(\frac{10^8\ \mathrm{km}}{\ell}\right)^2 \sqrt{\frac{0.3\ \mathrm{GeV/cm^3}}{\rho}}. \end{aligned} \tag{33}$$

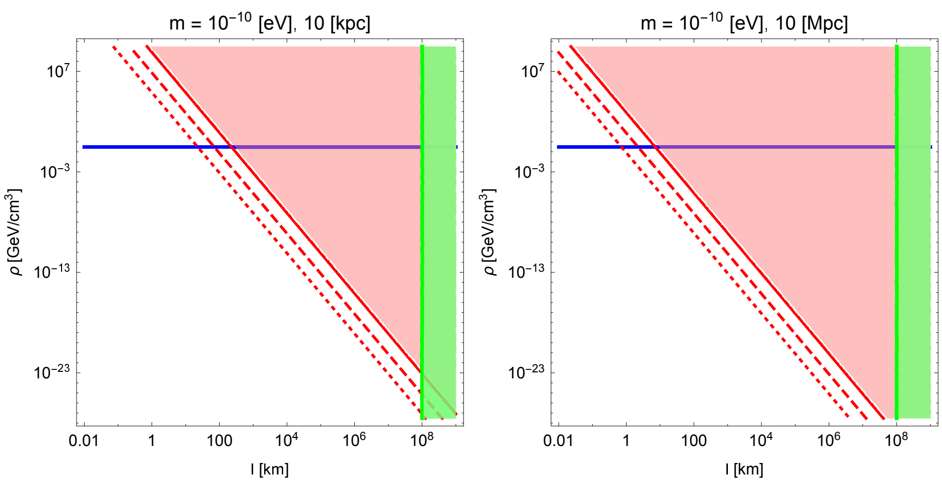

Fig. 3. The constraint on the coupling constant l and the density of the axion dark matter ρ for $m = 10^{-10}$ eV is shown. The left one shows the excluded region by the gravitaitonal wave travelling 10 kpc in the axion dark matter, and the right one shows the excluded region for 10 Mpc propagation in the axion dark matter. The blue line show the local dark matter density $0.3\,\mathrm{GeV/cm^3}$. The green region is excluded by the observation.[29] The red line represents the ten times enhancement of gravitational waves. The red dashed line represents the 0.1 times enhancement. The red dotted line represents the 0.01 times enhancement. Since these sinatures have never been observed, the upper parameter regions of these lines are excluded.

If the distance from the source to the earth is 10 kpc, the amplitude of GWs can be significantly enhanced. However, there was no such signal in the past data. This allows us to give constraints on the coupling ℓ and the density ρ. In Fig. 4, we the constraint on the coupling constant l and the density of the axion dark matter ρ. Thus, we obtain the stringent constraint on the Chern-Simons coupling in the presence of the axion dark mater.

For any axion mass from 10^{-10} eV up to 10^{-14} eV, we obtain the similar constraint. There are two ways to use the results we have obtained. If the current upper limit $\ell \sim 10^8$ km is assumed, the abundance should be constrained. If we assume the axion is the dark matter, then we obtain the stronger constraint than the current one $\ell \leq 10^8$ km.

5. Conclusion

We studied the string axiverse and parity violation in gravity sector by considering propagation of GWs in the axion dark matter with the mass range from 10^{-14} eV to 10^{-10} eV. It turned out that the axion coherent oscillation induces the parametric resonance of GWs due to the Chern-Simons term resulting in the circular polarization of GWs. Thus, the observation of GWs can strongly constrain the coupling constant of Chern-Simons term and/or the abundance of the light axions.

In the core of our Galaxy, the dark matter density is higher than that near the sun by about $10^3 \sim 10^4$. Therefore, it is possible to have more stringent constraint. If the sizable amount of the light axion exists, the circularly polarized GWs might be observed. If this happens, it would be a great discovery. If not, as the detection events increase, the constraints on the Chern-Simons coupling and/or the abundance of the light axion become tight. In this sense, GWs can explore the string axiverse and parity violation in gravity.

There are many directions to be studied. We can discuss primordial gravitational waves in the view of string axiverse. It would be interesting to extend the present analysis to the gauge Chern-Simons term studied in.[31] Moreover, the axion-photon conversion analysis in[32] should be extended by taking into account the axion coherent oscillation. We leave these issues for future work.

Acknowledgments

We would like to thank A. Ito, and R. Kato for useful discussions. D.Y. was supported by Grant-in-Aid for JSPS Research Fellow and JSPS KAKENHI Grant Number JP17J00490. J.S. was in part supported by JSPS KAKENHI Grant Numbers JP17H02894 and JP17K18778, and MEXT KAKENHI Grant Numbers JP15H05895 and JP17H06359.

References

1. Virgo, LIGO Scientific Collaboration (B. P. Abbott *et al.*), *Phys. Rev. Lett.* **116** (2016) 061102, `arXiv:1602.03837 [gr-qc]`.
2. A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, *Phys. Rev.* **D81** (2010) 123530, `arXiv:0905.4720 [hep-th]`.
3. P. Svrcek and E. Witten, *JHEP* **06** (2006) 051, `arXiv:hep-th/0605206 [hep-th]`.
4. R. Hlozek, D. J. E. Marsh and D. Grin (2017) `arXiv:1708.05681 [astro-ph.CO]`.
5. D. J. E. Marsh and P. G. Ferreira, *Phys. Rev.* **D82** (2010) 103528, `arXiv:1009.3501 [hep-ph]`.
6. A. Khmelnitsky and V. Rubakov, *JCAP* **1402** (2014) 019, `arXiv:1309.5888 [astro-ph.CO]`.
7. N. K. Porayko and K. A. Postnov, *Phys. Rev.* **D90** (2014) 062008, `arXiv:1408.4670 [astro-ph.CO]`.
8. A. Aoki and J. Soda, *Phys. Rev.* **D93** (2016) 083503, `arXiv:1601.03904 [hep-ph]`.
9. A. Aoki and J. Soda, *Int. J. Mod. Phys.* **D26** (2016) 1750063, `arXiv:1608.05933 [astro-ph.CO]`.

10. A. Aoki and J. Soda, *Phys. Rev.* **D96** (2017) 023534, arXiv:1703.03589 [astro-ph.CO].
11. D. Blas, D. L. Nacir and S. Sibiryakov, *Phys. Rev. Lett.* **118** (Jun 2017) 261102, arXiv:1612.06789 [hep-ph].
12. A. Arvanitaki and S. Dubovsky, *Phys. Rev.* **D83** (Feb 2011) 044026, arXiv:1004.3558 [hep-th].
13. H. Yoshino and H. Kodama, *Class. Quant. Grav.* **32** (2015) 214001, arXiv:1505.00714 [gr-qc].
14. C. Abel *et al.* (2017) arXiv:1708.06367 [hep-ph].
15. D. J. E. Marsh, *Phys. Rept.* **643** (2016) 1, arXiv:1510.07633 [astro-ph.CO].
16. L. Hui, J. P. Ostriker, S. Tremaine and E. Witten, *Phys. Rev.* **D95** (2017) 043541, arXiv:1610.08297 [astro-ph.CO].
17. W. Hu, R. Barkana and A. Gruzinov, *Phys. Rev. Lett.* **85** (2000) 1158, arXiv:astro-ph/0003365 [astro-ph].
18. J.-W. Lee (2017) arXiv:1704.05057 [astro-ph.CO].
19. B. A. Campbell, M. J. Duncan, N. Kaloper and K. A. Olive, *Nucl. Phys.* **B351** (Mar 1991) 778.
20. A. Lue, L.-M. Wang and M. Kamionkowski, *Phys. Rev. Lett.* **83** (1999) 1506, arXiv:astro-ph/9812088 [astro-ph].
21. R. Jackiw and S. Y. Pi, *Phys. Rev.* **D68** (2003) 104012, arXiv:gr-qc/0308071 [gr-qc].
22. S. Alexander and N. Yunes, *Phys. Rept.* **480** (2009) 1, arXiv:0907.2562 [hep-th].
23. M. Satoh, S. Kanno and J. Soda, *Phys. Rev.* **D77** (2008) 023526, arXiv:0706.3585 [astro-ph].
24. C. R. Contaldi, J. Magueijo and L. Smolin, *Phys. Rev. Lett.* **101** (2008) 141101, arXiv:0806.3082 [astro-ph].
25. T. Takahashi and J. Soda, *Phys. Rev. Lett.* **102** (2009) 231301, arXiv:0904.0554 [hep-th].
26. J. Soda and D. Yoshida, *Galaxies* **5** (2017) 96.
27. N. Seto and A. Taruya, *Phys. Rev. Lett.* **99** (2007) 121101, arXiv:0707.0535 [astro-ph].
28. M. Okounkova, L. C. Stein, M. A. Scheel and D. A. Hemberger, *Phys. Rev.* **D96** (2017) 044020, arXiv:1705.07924 [gr-qc].
29. Y. Ali-Haimoud and Y. Chen, *Phys. Rev.* **D84** (2011) 124033, arXiv:1110.5329 [astro-ph.HE].
30. K. Yagi, N. Yunes and T. Tanaka, *Phys. Rev. Lett.* **109** (2012) 251105, arXiv:1208.5102 [gr-qc], [Erratum: Phys. Rev. Lett.116,no.16,169902(2016)].
31. D. Yoshida and J. Soda (2017) arXiv:1704.04169 [gr-qc].
32. E. Masaki, A. Aoki and J. Soda, *Phys. Rev.* **D96** (2017) 043519, arXiv:1702.08843 [astro-ph.CO].