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Majorana neutrino masses in the scenario of gauge–Higgs unification

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In this paper we consider possible mechanisms to generate small Majorana neutrino masses for active neutrinos in the scenario of gauge–Higgs unification, a candidate for physics beyond the standard model. We stress that it is non-trivial to find a gauge-invariant operator, responsible for the Majorana masses, which is the counterpart of the well-known $SU(2)_L \times U(1)_Y$ invariant higher-mass-dimensional ($d = 5$) operator. As the first possibility we discuss the seesaw mechanism by assigning leptonic fields to the adjoint representation of the gauge group, so that a $d = 5$ gauge-invariant operator can be formed. It turns out that the mechanism leading to the small Majorana masses is the admixture of the Type I and Type III seesaw mechanisms. As the second possibility, we consider the case where the relevant operator has $d = 7$, by introducing a matter scalar belonging to the fundamental representation of the gauge group. Reflecting the fact that the mass dimension of the operator is higher than usually expected, the Majorana masses are generated by a "double seesaw mechanism."
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Subject Index   B40, B43, B54

1. Introduction

The standard model (SM) possesses a few serious theoretical problems. A well-known important problem is that of gauge hierarchy. The attempts to solve this problem have led to representative scenarios of physics beyond the standard model (BSM). The most well-studied scenario is supersymmetry, whose concrete realization is minimal supersymmetric standard model (MSSM). In this paper we focus on the scenario of gauge–Higgs unification (GHU), where the Higgs boson is originally a gauge boson. To be precise, the Higgs field is identified with the (Kaluza–Klein (KK) zero mode of) an extra-dimensional component of a higher-dimensional gauge field [1–4]. A nice feature of this scenario is that, by virtue of the higher-dimensional local gauge symmetry, the quantum correction to the Higgs mass is UV-finite, thus opening a new avenue for the solution of the hierarchy problem [5].

Another basic theoretical problem in the standard model is that there is no principle to restrict Higgs interactions, such as Yukawa couplings. Namely, there is no guiding principle to determine the quark and lepton masses theoretically. From this viewpoint, again the GHU scenario is hopeful: the GHU scenario may provide a natural mechanism to restrict Higgs interactions, relying on the gauge principle. Let us note that in GHU, Yukawa couplings are originally gauge couplings since the Higgs field is originally a gauge field.

Neutrino masses are expected to play special roles in the investigation of the viability of the various BSM scenarios. First, it should be noticed that if neutrinos are assumed to be Majorana fermions, the neutrino mass matrix (in the basis of weak eigenstates) is directly determined by the observed neutrino mass eigenvalues, generation mixing angles, and (physical) CP phases, some of them having

already been fixed (with some errors) or restricted experimentally. Thus it is possible to compare the prediction of each BSM scenario with such determined mass matrices. This is in contrast to the case of quark mass matrices; here, because of the freedom of unitary transformations in the sector of right-handed quarks, even though we know all of the observables mentioned above, the mass matrices cannot be uniquely fixed.

It should also be noticed that the mass matrices of the lepton sector show very characteristic features: neutrino mass eigenvalues are remarkably small compared to those of quarks and charged leptons. Also impressive is that (two of the) generation mixing angles in the Maki–Nakagawa–Sakata matrix are considerably greater than the corresponding angles in the Kobayashi–Maskawa matrix. These interesting features may also have their origin in the Majorana nature of neutrinos. Let us recall that only neutrinos, being electrically neutral, can be Majorana fermions without contradicting charge conservation.

Based on these observations as the first step, in this paper we study systematically how small Majorana neutrino masses can be realized in the GHU scenario. In the literature, the most popular mechanism for realizing small neutrino masses is the seesaw mechanism [6–8]. In this paper, we will propose possible models to realize this idea concretely in the framework of GHU.

What is special for the GHU scenario in the discussion of Majorana neutrino masses? We may easily note that to realize the aforementioned mechanisms for inducing small Majorana neutrino masses in GHU is a little challenging. First, since the Higgs field is originally a gauge field belonging to the adjoint representation of the gauge group, it is non-trivial to form a gauge-invariant operator with mass dimension $d = 5$ (from a four-dimensional (4D) point of view) corresponding to the well-discussed $SU(2)_L \times U(1)_Y$ invariant operator, $(\phi^\dagger L)^2$, where L is a left-handed lepton doublet and ϕ is the Higgs doublet. Also, the Yukawa coupling coming from the covariant derivative of higher-dimensional gauge theory usually preserves fermion number, and to break the lepton number is a non-trivial task, though if we extend our discussion to the grand GHU [9–11], the gauge interactions there may lead to the violation of baryon and/or lepton number.

2. Seesaw mechanism in the GHU scenario

We discuss how the seesaw mechanism is realized in the GHU scenario, by taking the minimal unified electro-weak GHU model, i.e. the 5D $SU(3)$ model [12,13]. The extra dimension is assumed to be an orbifold S^1/Z_2 in order to break $SU(3)$ into the gauge group of the SM and also to realize a chiral theory. Let us note that in GHU the gauge group of the SM should inevitably be enlarged, and the simplest choice is $SU(3)$. The Higgs field behaves as an octet, the adjoint representation of $SU(3)$. Then, in this model, assigning leptonic fields in an $SU(3)$ triplet will be unrealistic. First, the charge assignment in this model is such that the fields in the triplet all have fractional charges, being identified with those of quark fields. (The situation will change if the gauge group has an additional $U(1)$ factor [14,15].) Second, the $d = 5$ operator $(A_y L)^2$ (L: lepton triplet, A_y: the fifth component of the 5D gauge field) contained in $(D_y L)^2$ (D_y denotes the gauge covariant derivative), which should be the counterpart of $(\phi^\dagger L)^2$, clearly cannot be gauge invariant.

For these reasons, we assign lepton fields to an $SU(3)$ octet Ψ, whose component fields have integer charges. Also, by taking this choice of representation, we can immediately find a $d = 5$ operator $\mathrm{Tr}\{[A_y, \Psi]^2\}$ stemming from a gauge-invariant operator $\mathrm{Tr}\{(D_y\Psi)^2\}$ (with spinor indices being omitted for brevity), responsible for the Majorana mass of ν_L. One may wonder why we do not introduce an $SU(3)$ singlet field to be identified with ν_R. Let us note that such a ν_R cannot form a Dirac mass with ν_L through the vacuum expectation value (VEV) of A_y, since ν_R and ν_L belong

to different representations of gauge group and therefore cannot communicate with each another through Yukawa coupling. The octet Ψ possesses both the $SU(2)_L$ doublet containing ν_L and the $SU(2)_L$ singlet containing ν_R in a single representation.

In order to complete the seesaw mechanism, in addition to the Dirac mass term mentioned above, the Majorana mass term for ν_R is needed. At first glance the Majorana mass term seems to be provided by a gauge-invariant operator $\mathrm{Tr}\,\Psi^2$. Unfortunately, the story is not so straightforward. First, although the adjoint representation is a real representation and therefore it seems to be natural to assign Majorana particles to this representation, it is known that we cannot regard each component of Ψ as an ordinary 4D Majorana field, satisfying the Majorana condition $\psi^c = \psi$ with $\psi^c = C(\bar\psi)^t$ ($C = i\gamma^0\gamma^2$) being 4D charge conjugation, even though the number of components of the spinor is the same as in the case of 4D space-time. It may be worth noting the fact that there does not exist a Majorana spinor in 5D space-time. In fact, even if we try to form a Majorana mass term for a generic four-component spinor ψ, $\overline{\psi^c}\psi$, the mass term is known to be non-invariant under the 5D Lorentz transformation, which connects 4D space-time coordinates with the extra space coordinate.

Interestingly, we realize that if we add γ_5 to the mass term to form $\overline{\psi^c}\gamma_5\psi$, the modified mass term turns out to be invariant under the full 5D Lorentz transformations. So, the linear combination $\psi + \gamma_5\psi^c$ seems to be a self-conjugate spinor, correctly transforming under the 5D Lorentz transformation. Unfortunately this is not the case, since $\gamma_5(\gamma_5\psi^c)^c = -\psi$. However, this in turn means that once we form an eight-component spinor ψ_{SM},

$$\psi_{\mathrm{SM}} = \begin{pmatrix} \psi \\ \gamma_5\psi^c \end{pmatrix}, \tag{1}$$

it is self-conjugate in the following sense:

$$\psi_{\mathrm{SM}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\gamma_5(\psi_{\mathrm{SM}})^c. \tag{2}$$

ψ_{SM} represents a "symplectic Majonara" spinor [16,17]. Just as the 5D SUSY gauge theory can be naturally obtained from the SUSY (pure) Yang–Mills theory in 6D space-time by naive dimensional reduction, it may be useful to construct a Lagrangian for ψ_{SM}, as if the space-time is 6D, and then perform a naive dimensional reduction to 5D. Adopting the following basis for 6D gamma matrices,

$$\Gamma^\mu = \gamma^\mu \otimes \sigma_1,\ \Gamma^5 = i\gamma_5 \otimes \sigma_1,\ \Gamma^6 = -iI_4 \otimes \sigma_2 \tag{3}$$

(with the 6D chiral operator being given by $\Gamma_7 = I_4 \otimes \sigma_3$), the symplectic Majorana condition reads

$$\psi_{\mathrm{SM}} = \Gamma^5\Gamma^6\Gamma^2(\psi_{\mathrm{SM}})^*. \tag{4}$$

In 5D space-time with the S^1/Z_2 orbifold as the extra dimension, the Z_2 transformation is a sort of chiral transformation from the 4D point of view, and hence ψ and ψ^c should have opposite Z_2 parities. Thus, the 4D Majorana spinor is not compatible with the orbifolding. (This is a reflection of the fact that in 4D space-time there is no Majorana–Weyl spinor.) For the symplectic Majorana spinor, the Z_2 transformation can be modified into that in the orbifold T^2/Z_2, the extra dimension of 6D space-time:

$$Z_2:\ \psi_{\mathrm{SM}}(x^\mu, y)\ \rightarrow\ P^{-1}(-i)\Gamma^5\Gamma^6\psi_{\mathrm{SM}}(x^\mu, -y)P, \tag{5}$$

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where y is the extra-dimensional coordinate and the 3×3 matrix P defines the Z_2-parities of each component of the fundamental representation of SU(3), i.e. triplet, as

$$P = \mathrm{diag}(1, 1, -1). \tag{6}$$

Since $-i\Gamma^5\Gamma^6 = \gamma_5 \otimes \sigma_3$, ψ and ψ^c are now allowed to have opposite 4D chiralities, as they should. The transformation $-i\Gamma^5\Gamma^6$ is a rotation of an angle π in the 2D extra dimension. So, it should be equivalent to ordinary Z_2 transformation after the dimensional reduction to the 5D space-time. In fact, it is easy to check that the bilinear form $\overline{\psi_{\mathrm{SM}}}\Gamma^M\psi_{\mathrm{SM}}$ for $M = \mu$ ($\mu = 0$–3), 5, the transformation in Eq. (5) is equivalent to the transformation without Γ^6.

If the 4D spinor ψ of Eq. (1) only has the left-handed Weyl spinor ψ_L, for instance as the result of the orbifolding mentioned above,

$$\psi_{\mathrm{SM}} = \left(\frac{\psi_L}{(\psi_L)^c} \right) = \begin{pmatrix} 0 \\ \bar{\eta}_{\dot\alpha} \\ \eta^\alpha \\ 0 \end{pmatrix} \quad (\alpha, \dot\alpha = 1, 2), \tag{7}$$

which just reduces to the four-component 4D Majorana spinor ψ_{M}:

$$\psi_{\mathrm{M}} = \psi_L + (\psi_L)^c = \begin{pmatrix} \eta^\alpha \\ \bar{\eta}_{\dot\alpha} \end{pmatrix}. \tag{8}$$

Then the mass term for ψ_{SM} just reduces to the 4D Majorana mass term for ψ_{M}:

$$M\overline{\psi_{\mathrm{SM}}}\psi_{\mathrm{SM}} = M\overline{\psi_{\mathrm{M}}}\psi_{\mathrm{M}}, \tag{9}$$

where $\overline{\psi_{\mathrm{SM}}} = \psi_{\mathrm{SM}}^\dagger \Gamma^0$, while $\overline{\psi_{\mathrm{M}}} = \psi_{\mathrm{M}}^\dagger \gamma^0$.

Now we are ready to discuss our model more concretely. The SU(3) octet Ψ contains symplectic Majorana spinors ψ_{SM}^a ($a = 1$–8) as its component fields:

$$\Psi = \psi_{\mathrm{SM}}^a \frac{\lambda_a}{2}, \quad \psi_{\mathrm{SM}}^a = \begin{pmatrix} \psi^a \\ \gamma_5(\psi^a)^c \end{pmatrix}, \tag{10}$$

where λ_a ($a = 1$–8) are Gell-Mann matrices. The free Lagrangian for Ψ with Majorana mass M, after naive dimensional reduction into 5D space-time, is given as

$$\mathcal{L}_{\mathrm{free}} = \mathrm{Tr}\left\{ \bar{\Psi} \left(\sum_{M=0\text{–}3,5} i\partial_M \Gamma^M - M \right) \Psi \right\} \tag{11}$$

Let us note that the condition in Eq. (4) is compatible with 5D Lorentz transformation, but not compatible with the Lorentz transformation connecting a sixth (extra space) coordinate with 5D coordinates, reflecting the fact that in 6D space-time there does not exist a Majorana spinor. So, Eq. (11) is invariant only under 5D Lorentz transformation, which is sufficient for our purpose. The gauge interaction of Ψ is described by

$$\mathcal{L}_{\mathrm{int}} = 2g\,\mathrm{Tr}\left\{ \bar{\Psi} \sum_{M=0\text{–}3,5} A_M \Gamma^M \frac{1+\Gamma^7}{2} \Psi \right\}. \tag{12}$$

As the result of the orbifolding, the sector of the KK zero mode is given as follows (we show only the part with +1 eigenvalue of Γ_7, $\Psi^{(+)}$):

$$\Psi^{(+)} = \Psi_L^{(+)} + \Psi_R^{(+)},$$

$$\Psi_L^{(+)} = \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} 0 & 0 & \tilde{e}^+ \\ \hline 0 & 0 & \tilde{\nu} \\ \hline e^- & \nu & 0 \end{array} \right)_L, \quad \Psi_R^{(+)} = \left(\begin{array}{c|c|c} \frac{N_\gamma}{\sqrt{3}} & \frac{\tilde{E}^+}{\sqrt{2}} & 0 \\ \hline \frac{E^-}{\sqrt{2}} & -\frac{N_\gamma}{2\sqrt{3}} - \frac{N_Z}{2} & 0 \\ \hline 0 & 0 & -\frac{N_\gamma}{2\sqrt{3}} + \frac{N_Z}{2} \end{array} \right)_R, \quad (13)$$

where N_γ, N_Z are associated with the generators, which are identical to those for the neutral gauge bosons γ, Z, and hence both are mixtures of the SU(2) singlet (associated with λ_8), corresponding to ν_R, and triplet (associated with λ_3) leptons.

Also relevant is the KK zero mode of the extra-dimensional component of the gauge field, A_y:

$$A_y = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c|c} 0 & 0 & \phi^+ \\ \hline 0 & 0 & \phi^0 \\ \hline \phi^- & \phi^{0*} & 0 \end{array} \right), \quad (14)$$

where (ϕ^+, ϕ^0) is nothing but the Higgs doublet in the SM, whose VEV, $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$, spontaneously breaks the gauge symmetry of the SM through the Hosotani mechanism [2–4].

Now let us move to the discussion of how the seesaw mechanism is realized in this model. For that purpose, we restrict our discussion to electrically neutral leptons. Our task is to realize the small Majorana mass for ν_L, belonging to the SU(2) doublet $L = (\nu, e^-)_L$, through the seesaw mechanism. For the mechanism to work, the "exotic" left-handed doublet $\tilde{L} = (\tilde{\nu}, \tilde{e}^+)_L$ is redundant, as it does not exist in the standard model. Also, if it remains in the low-energy effective theory, it will form a gauge-invariant Dirac mass term with the doublet (ν, e^-) of our interest,

$$M \left\{ \left(\begin{array}{cc} \overline{\nu_L} & \overline{e_L^-} \end{array} \right) \left(\begin{array}{c} (\tilde{\nu}_L)^c \\ (\tilde{e}_L^+)^c \end{array} \right) + h.c. \right\}. \quad (15)$$

The coefficient M, supposed to be the mass scale of the ν_R Majorana mass, is assumed to be much larger than the weak scale M_W for the seesaw mechanism to work, and hence ν_L will decouple from our low-energy world. A possible way out of this problem is to introduce a brane-localized SU(2) doublet $L_b = (\nu_b, e_b^+)_R$ to form a brane-localized Dirac mass term at one of the fixed points of the orbifold (where the gauge symmetry is reduced to that of the SM by the orbifolding),

$$M_b \left\{ \left(\begin{array}{cc} \bar{\nu}_{bR} & \bar{e}^+_{bR} \end{array} \right) \left(\begin{array}{c} \tilde{\nu}_L \\ \tilde{e}_L^+ \end{array} \right) + h.c. \right\}. \quad (16)$$

The "brane-localized mass" M_b is assumed to be much larger than the Majorana mass M, $M_b \gg M$. In an extreme limit, $M_b \to \infty$, $\tilde{\nu}$ is completely decoupled from the theory forming a Dirac mass with ν_b, thus leaving ν alone as a massless state. By the way, all the fields appearing in Eqs. (13)–(16) should be understood to be 4D fields with proper mass dimension and kinetic terms after the dimensional reduction to 4D space-time.

After the spontaneous gauge symmetry breaking due to $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$, ν_L belonging to the SU(2) doublet L forms a Dirac mass term of order $gv \sim M_W$ with a right-handed neutral lepton in Eq. (13), behaving as either a singlet or a triplet of SU(2)$_L$ ($2 \times 2 = 1 + 3$). It turns out that the partner of ν_L to form the Dirac mass is N_Z (not N_γ). This is basically because the VEV of A_y is electrically neutral

and hence $[Q, \langle A_y \rangle] = 0$, with $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ being the charge operator associated with N_γ. $N_{\gamma R}$, being isolated from other states, obtains a Majorana mass M by itself from the Majorana mass term $-M\mathrm{Tr}(\bar{\Psi}\Psi)$ in Eq. (11).

From these lessons, we learn that though we have five neutral leptonic states to start with, ν_L, $\tilde{\nu}_L$, ν_{bR}, $N_{\gamma R}$, N_{ZR}, in the limit $M_b \to \infty$, the Majorana mass of ν_L is effectively determined by the diagonalization of the mass matrix $M_{2\times2}$ in the basis of subsystem $(\nu_L, (N_{ZR})^c)$:

$$\mathcal{L}_m = -\frac{1}{2} \left(\begin{array}{cc} \overline{(\nu_L)^c} & \overline{N_{ZR}} \end{array} \right) M_{2\times2} \left(\begin{array}{c} \nu_L \\ (N_{ZR})^c \end{array} \right), \quad M_{2\times2} = \left(\begin{array}{cc} 0 & \sqrt{2}M_W \\ \sqrt{2}M_W & M \end{array} \right). \tag{17}$$

The mass eigenvalues (their absolute values) are well approximated under $M_W \ll M$ to be M and $\frac{2M_W^2}{M}$. The smaller mass $\frac{2M_W^2}{M}$ is the Majorana mass for the mass eigenstate, which is nearly ν_L.

This completes the seesaw mechanism. Though not shown here, we have investigated the diagonalization of the full 5×5 mass matrix and have confirmed that under the condition $M_W \ll M \ll M_b$ we obtain the approximate result, identical to the one mentioned above.

The physical reason for getting the small Majorana mass for ν_L is the decoupling of N_Z due to its large Majorana mass. An important remark here is that the state N_Z is a mixture of SU(2) singlet and triplet states. Namely, the seesaw mechanism operating in this model is the admixture of two types of seesaw mechanism, i.e. Type I [6–8] and Type III [18].

3. Majorana neutrino masses due to higher-mass-dimensional operator

As was discussed in the introduction, in GHU it seems to be impossible to form a gauge-invariant operator corresponding to the dimension $d = 5$ and $SU(2)_L \times U(1)_Y$ invariant operator, $(\phi^\dagger L)^2$, when the lepton doublet L is assigned as a member of the fundamental representation of the gauge group. Actually, this is based on our implicit assumption that only the extra-dimensional component of the gauge field A_y is the field developing the VEV, which breaks the gauge symmetry of the standard model. Once we relax this constraint and allow the introduction of a scalar field, which also develops a VEV, the situation will change. If the VEV of the introduced scalar is a singlet concerning the gauge group of the SM, it has nothing to do with the weak scale M_W and hence the gauge hierarchy problem. By introducing such a scalar field, together with the lepton multiplet and A_y, it will become possible to form a gauge-invariant operator with mass dimension d higher than 5, typically 7.

As an example to make this idea concrete, we discuss a 5D SU(4) unified electro-weak GHU model. In this model, the Higgs doublet is contained in the triplet representation of the subgroup SU(3), in contrast to the case of the SU(3) model in the previous section, where the Higgs doublet is a member of the SU(3) octet. As a result, in this model the predicted weak mixing angle is a desirable one, $\sin^2 \theta_W = 1/4$ [19]. The fields responsible for the Majorana neutrino mass generation are denoted as follows:

$$A_y = \left(\begin{array}{ccc|c} -\frac{2}{\sqrt{6}}a_Z & w^+ & w^{++} & \phi^0 \\ w^- & -\frac{1}{\sqrt{2}}a_\gamma + \frac{1}{\sqrt{6}}a_Z & \tilde{w}^+ & \phi^- \\ w^{--} & \tilde{w}^- & \frac{1}{\sqrt{2}}a_\gamma + \frac{1}{\sqrt{6}}a_Z & s^+ \\ \hline \phi^{0*} & \phi^+ & s^- & a_{Z'} \end{array} \right), \quad \psi = \left(\begin{array}{c} \nu_L \\ e_L^- \\ e_L^+ \\ \hline \nu_R \end{array} \right), \quad \Phi = \left(\begin{array}{c} \hat{\phi}^0 \\ \hat{\phi}^- \\ \hat{\phi}^+ \\ \hline \hat{s}^0 \end{array} \right), \chi^0.$$

$$\tag{18}$$

We have introduced a scalar field Φ. Leptons have now been assigned to the fundamental representation of SU(4), whose chirality is tentative and will be fixed by the orbifolding discussed below. We also introduce a gauge singlet fermion χ^0.

The Lagrangian, relevant for the neutrino mass, is

$$\mathcal{L} = g\bar{\psi}A_y\psi + \epsilon(y)M_b\bar{\psi}\psi + \alpha(\bar{\psi}\Phi\chi^0 + h.c.) + \frac{1}{2}M\{\bar{\chi}^0\gamma_5(\chi^0)^c + h.c.\}, \tag{19}$$

where $\epsilon(y)$, with y being the coordinate of the extra space assumed to be an orbifold S^1/Z_2, is the "sign function": $\epsilon(y) = 1$ and -1 for positive and negative y, respectively, and the "Z_2-odd" bulk mass term $\epsilon(y)M_b\bar{\psi}\psi$ causes exponential suppression of the Yukawa coupling $f \simeq g(\pi RM_b)e^{-\pi RM_b}$ (R: the radius of S^1), which is desirable in order to get a small Dirac mass, relevant for the lighter (first or second) generation.

The breaking $SU(4) \to SU(3) \to SU(2)_L \times U(1)_Y$ due to the orbifolding is realized by adopting the following assignment of the Z_2 parities at two fixed points of the orbifold S^1/Z_2 for the fundamental representation of $SU(4)$: $P = \text{diag}(++, ++, +-, --)$.

The remaining KK zero modes as the result of the orbifolding are

$$A_y = \begin{pmatrix} 0 & 0 & 0 & \phi^0 \\ 0 & 0 & 0 & \phi^- \\ 0 & 0 & 0 & 0 \\ \hline \phi^{0*} & \phi^+ & 0 & 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} \nu_L \\ e_L^- \\ 0 \\ \hline \nu_R \end{pmatrix}, \quad \Phi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline \hat{s}^0 \end{pmatrix}, \quad \chi_L^0. \tag{20}$$

We have put the overall phase as -1 for Φ in its Z_2 transformation, so that only $\hat{s}^0$ has the even parity $(+, +)$.

The higher-mass-dimensional ($d = 7$) gauge-invariant operator relevant for the Majorana mass of ν_L is known to be

$$\left(\bar{\psi}A_y\Phi\right)^2. \tag{21}$$

It is easy to confirm that this operator does contribute to the Majorana mass after A_y and Φ are replaced by their VEV: $\langle\phi^0\rangle = v$, $\langle\hat{s}^0\rangle = V$.
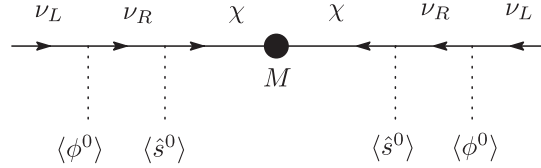
Here some comments are in order on the issue of how the scalar field Φ can develop its non-zero VEV, V. One possibility is just to add a gauge-invariant potential term for Φ, $-\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$ ($\mu^2, \lambda > 0$). We notice, however, that this may cause a problem, since the mass-squared term for Φ is not protected by any symmetry and the radiatively induced VEV, V, may be quite large (unless we perform some fine-tuning). This in turn may cause too-large spontaneous symmetry breaking of $SU(4)$ (though the breaking of $SU(4)$ itself is realized by orbifolding, as was discussed above), leading to a too-large positive mass-squared for the Higgs doublet, belonging to the "broken generator" of $SU(4)$, through the term $(D_M\Phi)^\dagger(D^M\Phi)$. Thus, realizing the spontaneous breaking of the SM gauge symmetry and getting a small Higgs mass becomes non-trivial.

Hence, a desirable alternative choice would be to embed the Φ field as part of A_y by adopting a larger gauge group, so that the potential of Φ becomes under control thanks to the higher-dimensional gauge symmetry. Two VEVs, V and v, both being generated by the Hosotani mechanism [2–4], may be naturally comparable in their orders of magnitude.

After the spontaneous breaking, the mass terms relevant for ν_L can be read off from the Lagrangian in Eq. (19), and are summarized here in a form using a mass matrix:

$$-\frac{1}{2} \begin{pmatrix} \overline{(\nu_L)^c} & \overline{\nu_R} & \overline{(\chi_L^0)^c} \end{pmatrix} \begin{pmatrix} 0 & fv & 0 \\ fv & 0 & \alpha V \\ 0 & \alpha V & M \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \\ \chi_L^0 \end{pmatrix} + h.c. \tag{22}$$

Fig. 1. The diagram contributing to Eq. (24).

We assume a hierarchical structure, $fv \ll \alpha V \ll M$. $fv \ll \alpha V$ is achieved by the exponentially suppressed small Yukawa coupling f, and the mass scale M can be much larger than v and V, since it is a singlet with respect to the SM gauge symmetry. Then, the diagonalization of the 3×3 mass matrix is straightforward. Namely, by an orthogonal rotation in the 2×2 submatrix for the lower two components with the small angle of $\mathcal{O}(\alpha V/M)$, we get the approximate form

$$\begin{pmatrix} 0 & fv & 0 \\ fv & -\frac{(\alpha V)^2}{M} & 0 \\ 0 & 0 & M \end{pmatrix}. \tag{23}$$

We now immediately get three approximated mass eigenvalues (their absolute values), M, $\frac{(\alpha V)^2}{M}$, and

$$\frac{(fv)^2}{\left(\frac{(\alpha V)^2}{M}\right)}, \tag{24}$$

which is identified with the Majorana mass of ν_L. In this derivation we have made the additional assumption $(fv)M \ll (\alpha V)^2$. Here are two steps of a seesaw-like mechanisms, which are seen schematically in Fig. 1. Figure 1 clearly shows that the operator giving rise to the Majorana mass is the one given in Eq. (21). A similar mechanism to get the small Majorana neutrino mass has been discussed in Ref. [20].

4.   Summary and discussion

In this paper we have considered possible mechanisms to generate small Majorana neutrino masses for (active) left-handed neutrinos in the scenario of gauge–Higgs unification, one of the attractive scenarios of physics beyond the standard model. A specific feature of this scenario in the construction of the Majorana mass term is that it is non-trivial to find an operator, responsible for the Majorana neutrino masses, which is a counterpart of the $SU(2)_L \times U(1)_Y$ invariant higher-mass-dimensional $d = 5$ (from 4D point of view) operator $(\phi^\dagger L)^2$ (L: left-handed lepton doublet, ϕ: Higgs doublet). For instance, in the minimal unified electro-weak SU(3) GHU model [12,13], we cannot get a gauge-invariant operator by just replacing the lepton doublet L by a triplet field, the fundamental representation of SU(3), since the Higgs field in this scenario, corresponding to ϕ, is A_y (the extra-dimensional component of the gauge field), which of course belongs to the adjoint representation of the gauge group.

 As the first possible mechanism to generate small Majorana neutrino masses we discussed the see-saw mechanism [6–8]. Leptonic matter fields are assigned to the adjoint representation of SU(3), i.e. the SU(3) octet, so that the component fields have integer charges and a $d = 5$ operator $\text{Tr}\{[A_y, \Psi]^2\}$ stemming from a gauge-invariant operator $\text{Tr}\{(D_y\Psi)^2\}$ can be formed. Though the Majorana spinor

seems to fit naturally to the octet, i.e. the real representation of the gauge group, it has been known that in 5D space-time there does not exist a Majorana spinor. Thus we formulated the Lagrangian for leptons by use of the symplectic Majorana spinor [16,17], which has eight components and naturally fits 6D space-time. Interestingly, in our model the partner of ν_L to form a Dirac mass is the admixture of SU(2)$_L$ singlet (corresponding to ν_R) and triplet fermions. Thus the mechanism operating in this model based on the GHU scenario turns out to be the admixture of Type I [6–8] and Type III [18] seesaw mechanisms.

As the second possibility we considered the case where Majorana neutrino masses are generated in a form of higher-mass-dimensional ($d > 5$) gauge-invariant operator. We argued that once the implicit constraint that the VEV to break the gauge symmetry should be given only by the VEV of A_y is relaxed, introducing a matter scalar belonging to the fundamental representation of the gauge group (together with an additional singlet fermionic field), we can form a higher-mass-dimensional ($d = 7$) gauge-invariant operator, responsible for the Majorana neutrino masses. Reflecting the fact that the relevant operator has a mass dimension than usually expected, the Majorana neutrino masses are generated by, say, the double seesaw mechanism.

References

[1] N. S. Manton, Nucl. Phys. B **158**, 141 (1979).
[2] Y. Hosotani, Phys. Lett. B **126**, 309 (1983).
[3] Y. Hosotani, Phys. Lett. B **129**, 193 (1983).
[4] Y. Hosotani, Ann. Phys. **190**, 233 (1989).
[5] H. Hatanaka, T. Inami, and C. S. Lim, Mod. Phys. Lett. A **13**, 2601 (1998).
[6] T. Yanagida, Proc. Workshop on the Unified Theory and the Baryon Number in the Universe, KEK Report 79-18.
[7] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, eds. P. Van Nieuwenhuizen et al. (North Holland, Amsterdam, 1979).
[8] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[9] C. S. Lim and N. Maru, Phys. Lett. B **653**, 320 (2007).
[10] K. Kojima, K. Takenaga, and T. Yamashita, Phys. Rev. D **84**, 051701 (2011).
[11] Y. Hosotani and N. Yamatsu, Prog. Theor. Exp. Phys. **2015**, 111B01 (2015).
[12] M. Kubo, C. S. Lim, and H. Yamashita, Mod. Phys. Lett. A **17**, 2249 (2002).
[13] C. A. Scrucca, M. Serone, and L. Silvestrini, Nucl. Phys. B **669**, 128 (2003).
[14] Y. Adachi, N. Kurahashi, and N. Maru, arXiv:1404.4281 [hep-ph] [Search INSPIRE].
[15] N. Maru, N. Okada, and S. Okada, Phys. Rev. D **96**, 115023 (2017).
[16] T. Kugo and P. Townsend, Nucl. Phys. B **221**, 357 (1983).
[17] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D **58**, 065002 (1998).
[18] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. C **44**, 441 (1989).
[19] K. Hasegawa, C. S. Lim, and N. Maru, J. Phys. Soc. Jpn. **85**, 074101 (2016).
[20] Y. Hosotani and N. Yamatsu, Prog. Theor. Exp. Phys. **2017**, 091B01 (2017).