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ACOUSTICAL LETTER

Basic study on relationship between airborne sound transmission and structure-borne sound radiation of a finite elastic plate

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1. Introduction

In architectural acoustics, the sound radiation from a solid body caused by a sound-induced vibration is known as airborne sound transmission and that caused by a force-excited vibration is known as structure-borne sound radiation. They are different in the type of excitation, but are essentially similar phenomena that they both involve radiation induced by the vibration of a solid body. However, airborne sound transmission and structure-borne sound radiation have been treated as different issues. There are not many studies directly discussing the relationship between these phenomena, for example, there are few studies dealing with the relationship between the impact sound and sound transmission loss of a floor [1,2].

If a general relationship between airborne sound transmission and structure-borne sound radiation exists and can be expressed in a simple form without the physical properties of the vibrating body, such as an elastic plate, it will be useful for understanding these phenomena and can also be applied for engineering purposes such as transmission/radiation prediction. For example, one may estimate the characteristics of the airborne sound transmission of a wall simply by measuring the characteristics of its structure-borne sound radiation, or vice versa, in the situation where the structure and physical properties of the wall are unknown. Therefore, a study has been conducted on an infinite elastic plate so far [3]. In that study, a transmission coefficient for random-incidence sound and the radiated sound power under point force excitation were introduced as evaluation indices for airborne sound transmission and structure-borne sound radiation, respectively, and a conversion function that relates the two problems was derived. The conversion function for the infinite elastic plate was found to be a function of the specific impedance and the wave number, and does not include any elastic plate parameters.

In this study, based on the above idea, a theoretical analysis of a finite elastic plate under simplified conditions is attempted, and a conversion function for the finite elastic plate is derived through the same method. Moreover, the generality of the relationship between airborne sound transmission and structure-borne sound radiation is confirmed by comparing the conversion function for the finite elastic plate with that for the infinite elastic plate.

2. Theoretical analysis

As described above, in the case of an infinite elastic plate, it is theoretically clarified that the relationship between airborne sound transmission and structure-borne sound radiation can be expressed by the following equation via the conversion function:

$$\begin{aligned} \tau_f(\omega) &= \varepsilon_{inf}(\omega)W(\omega), \\ \therefore\ \varepsilon_{inf}(\omega) &= \tau_f(\omega)/W(\omega), \end{aligned} \tag{1}$$

where ω is the angular frequency, τ_f is the transmission coefficient for random-incidence sound, W is the radiated sound power under point force excitation, and ε_{inf} is the conversion function for the infinite elastic plate. ε_{inf} is given by the following equation, and does not depend on any elastic plate parameters:

$$\varepsilon_{inf}(\omega) = \frac{52\pi\rho_0 c_0}{k_0^2}, \tag{2}$$

where ρ_0 is the air density, c_0 is the speed of sound in air, and $k_0 = \omega/c_0$ is the acoustic wavenumber in the air.

In this study, as shown in Fig. 1, it is assumed that a single rectangular plate simply supported at its four sides is installed inside a square duct and that the surface of the duct other than the plate is acoustically soft, that is, perfectly reflective. τ_f and W are derived under such conditions. For such a problem, a baffled finite plate has been traditionally used; however, it is very difficult to obtain a strict analytical solution for the boundary values on the plate surface. The present model easily gives strict solutions for the boundary values on the plate surface using the modal expansion technique [4]. Assuming that a similar relation as Eq. (1) holds between the τ_f and W as well as for the finite elastic plate, the conversion function for the finite elastic plate ε_f is analytically derived. In addition, ε_f and ε_{inf} are compared and discussed.

The analysis is outlined below (for the details, refer to Ref. [4]). First, the pressure of an oblique incident plane wave
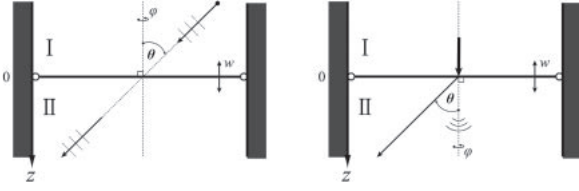
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Fig. 1 Single rectangular plate installed inside a square duct in the case of airborne sound transmission. Left: sound transmission of an oblique incident plane sound wave; right: sound radiation by point force excitation at the centre of the plate. I and II signify the regions of the sound field.

of unit amplitude, p_i, is described in terms of the angle of incidence θ (elevation angle) and φ (azimuth angle) as $p_i(x, y, z) = \exp[ik_0(x \sin\theta \cos\varphi + y \sin\theta \sin\varphi + z \cos\theta)]$. The equation of motion of the plate is written in terms of the flexural rigidity D, density ρ, thickness h, and displacement $w(x, y)$ of the plate as $D\nabla^4 w(x, y) - \rho h \omega^2 w(x, y) = 2p_i(x, y, 0)$, where $D = E(1 - i\eta)h/12(1 - \nu^2)$ with E Young's modulus, η the loss factor, and ν Poisson's ratio. The sound field around the plate ϕ is expressed by a series of plane waves propagating in the $+z$ and $-z$ directions and the mode function of the plate vibration $\psi_{mn}(x, y)$ as follows:

$$\phi(x, y, z) = \sum_{mn}[\phi_{mn}^{+} \exp(ik_{gmn}z) + \phi_{mn}^{-} \exp(-ik_{gmn}z)]\psi_{mn}(x, y), \tag{3}$$

where ϕ_{mn}^{+} and ϕ_{mn}^{-} are the amplitudes of the plane waves. k_{gmn} is a constant determined by the side lengths of the plate a and b and the acoustic wavenumber in the air k_0. The subscripts m and n denote the number of the mode of the plate.

Here the mode function of the plate vibration used for expansion is

$$\psi_{mn}(x, y) = \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \tag{4}$$

Secondly, the displacement of the plate w and the incident plane wave p_i are written as follows in terms of ψ_{mn} with the coefficients W_{mn} and I_{mn} representing the amplitudes of each component:

$$w(x, y) = \sum_{mn} W_{mn}\psi_{mn}(x, y) \tag{5}$$

$$p_i(x, y, z) = \sum_{mn} I_{mn} \exp(ik_0 z \cos\theta)\psi_{mn}(x, y) \tag{6}$$

where W_{mn} is an unknown quantity and I_{mn} is a constant determined by the side lengths of the plate a and b, the angles of incidence θ and φ and the acoustic wavenumber in the air k_0.

Using these expressions, simultaneous equations consisting of the equation of motion of the plate and the equation for the sound field to be solved are obtained. By solving the equations, the pressure amplitude of the sound emitted into Region II, P_{2mn}^{+}, which propagates in the $+z$ direction, is

$$P_{2mn}^{+} = \frac{Q_{mn}}{2 - \dfrac{k_{gmn}\rho h(\omega_{mn}^2 - \omega^2)}{i\rho_0\omega^2}}, \tag{7}$$

where Q_{mn} is the amplitude of the (m, n) component, ω_{mn} is the characteristic angular frequency of the plate.

2.1. Transmission coefficient for random-incidence sound

The transmission coefficient for random-incidence sound τ_f of the square plate in Fig. 1 is now obtained from Eq. (7) as follows:

$$\tau_\theta(\varphi, \theta) = \frac{k_0}{\cos\theta}\left(\frac{\rho_0 c_0 \omega}{\rho h}\right)^2 \sum_{mn} \frac{\mathrm{Re}(k_{gmn})|I_{mn}|^2}{|k_{gmn}|^2(\omega_{mn}^2 - \omega^2)^2}$$

$$\therefore \tau_f(\omega) = \frac{\displaystyle\int_0^{78}\int_0^{45} \tau_\theta(\omega)\cos\theta\sin\theta\,d\varphi\,d\theta}{\displaystyle\int_0^{78}\int_0^{45} \cos\theta\sin\theta\,d\varphi\,d\theta}, \tag{8}$$

where τ_θ is the transmission coefficient for oblique-incidence sound. Note that the integration for the azimuth angle is from 0 to 45° because of the symmetry of the model.

2.2. Radiated sound power under point force excitation

The radiated sound power under point force excitation W is expressed in the same way by the following equation:

$$W(\omega) = \frac{ab}{8\rho_0\omega} \sum_{mn} \mathrm{Re}(k_{gmn})|p_{2mn}^{+}|^2. \tag{9}$$

3. Results and discussion

In the case of a gypsum board, the calculated transmission loss for random-incidence sound R_f, the radiated sound power under point force excitation W, and the conversion function for the finite elastic plate ε_f are shown in Fig. 2 in comparison with those for an infinite gypsum plate. To see their behaviour in a wider frequency range of the mass-control region, the plate is assumed to be a square of side 4 m.

Both R_f and W have peaks and dips due to the characteristic vibration, which is caused by the finiteness of the plate. However, both of them show the coincidence effect. On the whole, R_f increases with the frequency, which corresponds to the mass law for random-incidence sound, and W remains around 60 dB. Therefore, it can be stated that R_f and W for the finite elastic plate show similar tendencies as those for the infinite elastic plate.

In the mass control region, ε_f fluctuates up and down around a slightly higher value than ε_{inf}, with an almost constant difference due to the characteristic vibration of the finite plate. In the coincidence region, the value corresponding to the centre of the fluctuation becomes smaller. Despite these quantitative differences, the tendency of ε_f shows rough agreement with that of ε_{inf}. Similar results are obtained for other materials: Fig. 3 shows ε_f for a concrete plate, and Fig. 4 shows ε_f for a wooden plate (both 4 m square). In the case of concrete, the peaks are more significant and consequently the discrepancy with the infinite plate becomes large. However, in both cases, the behaviour of the conversion factor is qualitatively similar to that in the infinite case.
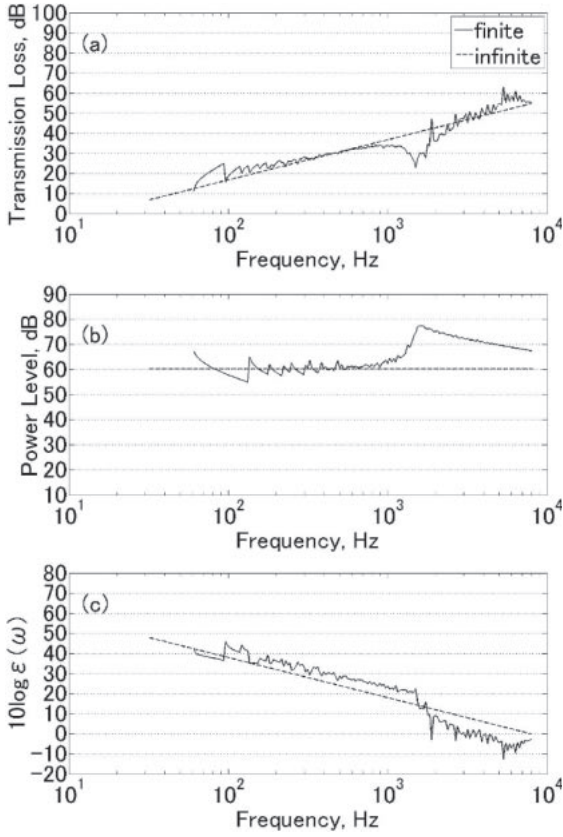
Fig. 2 (a) R_f, (b) W, (c) ε_f for a finite elastic plate in the case of a gypsum board in comparison with those for an infinite plate. (In the finite case, side lengths $a = b = 4$ m, thickness $h = 0.025$ m, density of the plate $\rho = 650$ kg/m^3, Young's modulus $E = 1.8 \times 10^9$ N/m^2, Poisson's ratio $\nu = 0.2$, loss factor $\eta = 0.03$.)

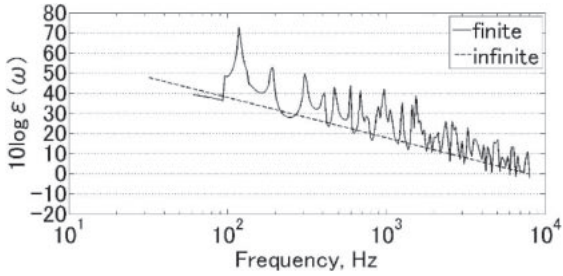**Fig. 3** ε_f for a finite elastic plate made of concrete (side lengths $a = b = 4$ m, thickness $h = 0.25$ m, density of the plate $\rho = 2{,}400$ kg/m^3, Young's modulus $E = 2.6 \times 10^{10}$ N/m^2, Poisson's ratio $\nu = 0.2$, loss factor $\eta = 0.005$).

Fig. 4 ε_f for a finite elastic plate made of wood (side lengths $a = b = 4$ m, thickness $h = 0.018$ m, density of the plate $\rho = 600$ kg/m^3, Young's modulus $E = 6.0 \times 10^9$ N/m^2, Poisson's ratio $\nu = 0.2$, loss factor $\eta = 0.01$).

Fig. 5 Comparison of the values of ε_f for finite elastic plates made of gypsum board of side 1 m, 4 m, and 8 m. In each graph, the value for an infinite plate is shown for reference. The physical properties of the panels are the same as those in Fig. 2.

Figure 5 shows the effect of the plate size. The frequency range of the mass-controlled region changes with the plate size. Therefore, a comparison is made in different frequency ranges. However, for all sizes a similar tendency to that mentioned above is observed. Also, the trend of the difference between finite and infinite cases does not change significantly, which may be a subject of further investigation in future works.
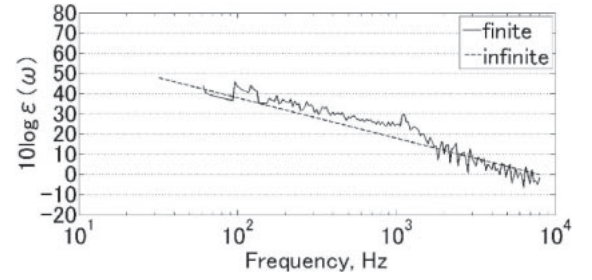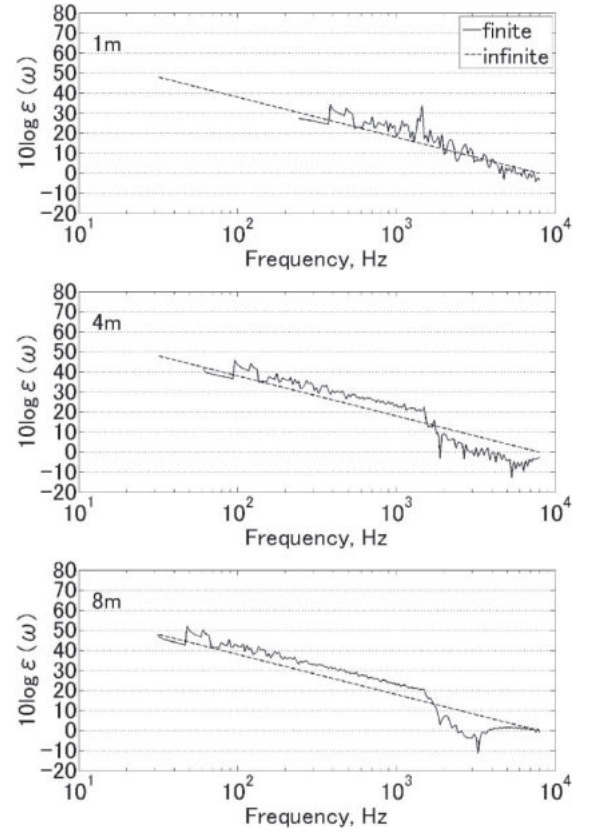
From the above, even in the case of a finite elastic plate, it is inferred that there is a conversion function having a similar tendency and qualitative behaviour to that for the infinite elastic plate, although there is a quantitative discrepancy.

4. Concluding remarks

In order to gain a physical insight into the relationship between the airborne sound transmission and structure-borne sound radiation of a finite elastic plate, this study was carried out using the finite elastic plate model under simplified

conditions. The transmission coefficient for random-incidence sound, the radiated sound power under point force excitation, and the conversion function for the finite elastic plate that relates the two problems were theoretically derived. Moreover, the obtained conversion function for the finite elastic plate was compared with that for the infinite elastic plate.

The results suggest that there is also a conversion function in the case of a finite elastic plate, which roughly agrees qualitatively with the tendency of the conversion function for an infinite elastic plate. However, in the case of a finite elastic plate, the conversion function fluctuates strongly due to the characteristic vibration owing to the finiteness of the plate. In addition, the value corresponding to the centre of the fluctuation is slightly higher than that of the conversion function for the infinite elastic plate in the mass control region, and becomes relatively lower in the coincidence region. The reason for these discrepancies should be the subject of future studies.

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