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Finance and Economic Growth in a Dynamic Game*

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Abstract

We investigate how the relaxation of financial constraints affects economic growth in a dynamic game of the tragedy of the commons by introducing an imperfect financial market into Tornell and Velasco's (1992) model. It is shown that whereas the relaxation of financial constraints enhances economic growth if agents have access only to a common asset whose property rights are not secure, the relaxation of financial constraints reduces economic growth if agents can have access not only to a common asset but also to a private asset whose property rights are secure.

Keywords: dynamic game; interest groups; property rights; financial market imperfections; economic growth.

JEL Classification Numbers: C73; D92; O43

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1 Introduction

Engelbert Dockner, an incredibly good-natured scholar, has published many important works on dynamic games. The second author of the current paper was fortunate to write joint papers with him on dynamic games of capital accumulation.¹ Since Dockner has also recently worked in the field of finance, we investigate a dynamic game of capital accumulation by introducing a financial market.

The current paper belongs to the literature on the dynamic game of the tragedy of the commons. Since the pioneering work of Tornell and Velasco (1992), the dynamic game of the tragedy of the commons has been investigated by many researchers (e.g., Lane and Tornell, 1996; Tornell and Lane, 1999; Sorger, 2005; Long and Sorger, 2006). Those works consider dynamic games in continuous time (differential games). In those works, the tragedy of the commons can be mitigated if interest groups can hold not only a commonly accessible asset but also a private asset whose property rights are secure.

In this study, we consider a dynamic game in discrete time (a difference game), and introduce an imperfect financial market to investigate the effect that the relaxation of financial constraints has on economic growth, which the previous works do not take into consideration.

In the literature on finance and growth, many researchers have both theoretically and empirically demonstrated a positive relationship between the relaxation of financial constraints and economic growth (e.g., Greenwood and Jovanovic, 1990; Levine et al., 2000; Aghion et al., 2005; Levine, 2005). Recently, empirical studies have reported results indicating a negative relationship between the relaxation of financial constraints and economic growth in some cases (e.g., Loayza and Rancière, 2006; Saci et al., 2009; Arcand et al., 2015). The aim of this paper is to present a theoretical model in which the relaxation of financial constraints can impede economic growth.

In our model, there is a unit measure of infinitely lived firms. The corporate governance of these firms is so weak that interest groups exploit them. One can regard the

¹Dockner et al. (1999) and Dockner and Nishimura (2001, 2004, 2005), among others.

firms as commonly accessible assets in the context of the dynamic game of the tragedy of the commons. Interest groups clustering around a firm are in a game theoretic situation, and each of them appropriates the firm's product, while taking into consideration other interest groups' behaviors. Each firm receives uninsured idiosyncratic productivity shocks in each period. A firm that draws a lower productivity shock becomes a lender, and a firm that draws a higher productivity shock becomes a producer, borrowing up to the limit imposed by the financial constraints. Therefore, lending firms and borrowing firms endogenously appear in each period.

Our findings are as follows. If interest groups have access only to a common asset, the growth rate increases as the financial constraint is relaxed. However, if interest groups have access not only to a common asset but also to a private asset whose property rights are secure and rate of return of which is inferior to that of the common asset, the growth rate decreases as the financial constraint is relaxed. The latter result is consistent with recent empirical evidence that indicates a negative relationship between the relaxation of financial constraints and economic growth.

The paper proceeds as follows. In the next section, a difference game of capital accumulation is presented, and a Markov perfect equilibrium is derived. In section 3, we investigate how the growth rate is affected by the relaxation of financial constraints when there is only a commonly accessible asset, and in section 4, we add a private asset to the model and examine the effect of the relaxation of financial constraints on the growth rate. The conclusion of the paper is presented in section 5.

2 Model

A closed economy continues from period 0 to $+\infty$ in discrete time. A unit measure of infinitely lived firms populates the economy, and N infinitely lived agents cluster around each firm, where $N \geq 2$. That is, there are N agents per firm. Each of the agents is simultaneously a consumer and an interest group. An interest group is firm-specific, implying that it has a relationship with only one firm. Agents clustering around a

firm are in a game theoretic situation. An agent (interest group) decides how much of the firm's income it appropriates in each period, while taking into consideration other interest groups' behaviors. However, exploiting too much from the firm accelerates the opponent interest groups' exploitation, which leads to the exhaustion of the firm's funds. In each period, each firm earns income by producing general goods, which are used as both capital and consumption, or it turns over its funds in the financial market. The financial market is imperfect because of asymmetric information between lenders and borrowers, and thus, each firm can borrow only up to a certain proportion of its own funds.

The timing of events regarding firm $h \in \Omega$ in period τ (where Ω is the whole set of infinitely lived firms) is as follows. At the beginning of period τ , i) firm h acquires the return from lending or producing general goods, ii) a firm-specific productivity shock, Φ_τ^h , is realized, which is associated with general goods production in period $\tau + 1$, and iii) upon knowing Φ_τ^h , N firm-specific interest groups appropriate general goods from the firm and consume them. At the end of period τ , the firm makes a decision about the portfolio allocation of its savings, i.e., whether to invest in a project for general goods production or lend its funds in the financial market.

2.1 Agents as consumers

The lifetime utility of agent $i \in \{1, 2, \dots, N\}$, which is firm h 's interest group, in period t is given by

$$U_t^{i,h} := E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \ln c_\tau^{i,h} \right], \quad (1)$$

where $\beta \in (0, 1)$ is a subjective discount factor, $c_\tau^{i,h}$ is consumption, and $E_t(\cdot)$ is an expectation operator given the information about idiosyncratic productivity shocks until period t . Agent i maximizes its lifetime utility (1).

2.2 Firms

The resource constraint of firm h in period τ is given by

$$\sum_i^N c_\tau^{i,h} + s_\tau^h = y_\tau^h + r_\tau b_{\tau-1}^h =: I_\tau^h, \quad (2)$$

where b_τ^h is lending if positive and borrowing if negative, r_τ is the gross interest rate, y_t^h represents the general goods produced by the firm, $s_\tau^h := k_\tau^h + b_\tau^h$ represents the firm's savings, and I_τ^h is firm h 's income.² Equation (2) holds for $\tau \geq 1$. It is assumed that each firm is endowed with the same amount of funds at $\tau = 0$, such that $I_0^h = I_0$. Therefore, at $\tau = 0$, the resource constraint is given by $\sum_i^N c_0^{i,h} + s_0^h = I_0$.

Firm h is endowed with a linear production function, $y_\tau^h = A\Phi_{\tau-1}^h k_{\tau-1}^h$, in period τ , where $k_{\tau-1}^h$ is the capital allocated in period $\tau - 1$ and A is a productivity parameter for the production function. Capital depreciates entirely in one period, and one gestation period is required for production.³ $\Phi_{\tau-1}^h$ is a firm-specific period- τ productivity, which is a random variable realized in period $\tau - 1$. Although the firms can potentially become producers of general goods, they receive uninsured idiosyncratic productivity shocks in each period. The firms cannot insure their output against low productivity. $\Phi_0^h, \Phi_1^h, \dots$ are assumed to be independent and identically distributed across both time and firms (the i.i.d. assumption).⁴ Particularly, it is assumed that Φ^h has support over $[0, \eta]$, where $\eta \in \mathcal{R}_{++}$ is the maximum of the productivity shock. The cumulative distribution function of Φ^h is $G(\Phi^h)$.

As previously explained, the firms can borrow only up to the limit imposed by the following financial constraint:

$$b_\tau^h \geq -\lambda s_\tau^h \quad (3)$$

²The interest rate, r_τ , is endogenously determined, as will be explained later (refer to equation (16)).

³The proofs of the main results carry through even if a capital depreciation rate is introduced. For simplicity of exposition, we assume that capital fully depreciates in one period.

⁴The i.i.d. assumption for the productivity shocks can be interpreted in various ways. For instance, defective products, illness among workers, or a fire may accidentally reduce production.

and the non-negativity constraint of capital is

$$k_\tau^h \geq 0, \quad (4)$$

where $\lambda \in [0, \infty)$. The extent of financial constraints is measured by λ , with larger values implying more-relaxed constraints, as in the model of Aghion and Banerjee (2005).⁵⁶

We investigate the optimal portfolio allocation of firm h 's savings, s_t^h .⁷ Each firm starts a new investment project, borrowing in the financial market, or lends in the financial market. Upon knowing Φ_τ^h , the firm makes a portfolio allocation decision for its savings. A firm that draws a productivity shock such that $\Phi_\tau^h > r_{\tau+1}/A =: \phi_\tau$ becomes a producer, borrowing up to the limit of the financial constraint, whereas a firm that draws a productivity shock such that $\Phi_\tau^h \leq r_{\tau+1}/A$ lends its funds in the financial market. In each period, the firms are divided into borrowers and lenders by the cutoff, ϕ_τ . Therefore, from (2), (3), and (4), we obtain the optimal allocation of firm h 's savings as follows:

$$k_\tau^h = \begin{cases} 0 & \text{if } \Phi_\tau^h \leq \phi_\tau \\ \frac{s_\tau^h}{1-\mu} & \text{if } \Phi_\tau^h > \phi_\tau, \end{cases} \quad (5)$$

and

$$b_\tau^h = \begin{cases} s_\tau^h & \text{if } \Phi_\tau^h \leq \phi_\tau \\ -\frac{\mu s_\tau^h}{1-\mu} & \text{if } \Phi_\tau^h > \phi_\tau, \end{cases} \quad (6)$$

where $\mu := \lambda/(1 + \lambda)$. From (5), (6), and $y_\tau^h = A\Phi_{\tau-1}^h k_{\tau-1}^h$, the return in period τ from

⁵For the setting of financial constraints, see also Aghion et al. (1999) and Aghion et al. (2005).

⁶Lamoreaux (1994) examines insider lending in post-Revolution New England in the early nineteenth century and reports that insider lending resulted in the formation of investment clubs that pressured financial intermediaries to lend funds. Insider lending could promote economic growth resolving the asymmetric information between lenders and borrowers in a malfunctioning financial market. This implies that the value of λ might increase as the number of the interest groups increases. The result of the current paper will hold also for that case as in the working paper version of the paper (Kunieda and Nishimura, 2018). In the current paper, we assume λ to be constant for simplicity.

⁷The derivation of an optimal portfolio allocation of firm h 's savings follows Kunieda and Shibata (2016).

the firm's saving is intensively written as

$$R_\tau^h := \max \left\{ r_\tau, \frac{A\Phi_{\tau-1}^h - r_\tau\mu}{1 - \mu} \right\}. \quad (7)$$

2.3 Agents as interest groups

The flow budget constraint that agent i faces is obtained from (2), (5)-(7) as follows:

$$c_\tau^{i,h} + \sum_{j \neq i}^N c_\tau^{j,h} + s_\tau^h = R_\tau^h s_{\tau-1}^h = I_\tau^h. \quad (8)$$

Agent i maximizes lifetime utility (1), subject to the flow budget constraint (8), by choosing a sequence of controls, $\{c_t^{i,h}\}$, for $t = 0, 1, 2, \dots$ where $c_t^{i,h} \in [0, \infty)$.

The maximization problem defined here is an infinite-time-horizon dynamic game with N players.⁸ The state of the game in period t is given by firm h 's income, $I_t^h \in X$, where $X \subset \mathcal{R}_+$ is the state space of the game. The solution concept we use for the current dynamic game is Markov perfect equilibrium, in which each interest group reoptimizes its action in each period, while taking into consideration the policy functions of the other interest groups. We now define a Markovian strategy.

Definition 1. *Agent i 's Markovian strategy is given by $\{c_t^{i,h}(I_t^h)\}$, i.e., for each $t \geq 0$, agent i 's action, $c_t^{i,h}$, is a function of I_t^h such that $c_t^{i,h} : X \rightarrow [0, \infty)$.*

A Markov perfect equilibrium in the current dynamic game is defined below.

Definition 2. *A Markov perfect equilibrium is given by an N -tuple of strategies, $(\{\tilde{c}_t^{1,h}(I_t^h)\}, \{\tilde{c}_t^{2,h}(I_t^h)\}, \dots, \{\tilde{c}_t^{N,h}(I_t^h)\})$ for $t \geq 0$ such that for any $i \in \{1, 2, \dots, N\}$, given the Markovian strategies of other agents except for agent i , an optimal control path, $\{\tilde{c}_t^{i,h}\}$, exists, and it holds that $\tilde{c}_t^{i,h} = \tilde{c}_t^{i,h}(I_t^h)$ for all $t \geq 0$.*

We call $\tilde{c}_t^{i,h}(I_t^h)$ agent i 's policy function. In the current analysis, we consider symmetric Markovian strategies across the firm-specific interest groups that are linear with respect to the firm's income. Specifically, we guess a policy function such as

⁸Note that each firm optimally allocates its savings to k_τ^h and b_τ^h in each period. In this sense, one can consider that the return, R_τ^h , is determined by nature in this dynamic game.

$\tilde{c}_\tau^{j,h} = \alpha R_\tau^h s_{\tau-1}^h$, in which α is an interest group's appropriation rate, which is endogenously determined. Inserting this policy function into agent j 's consumption transforms (8) into

$$c_\tau^{i,h} + s_\tau^h = [1 - \alpha(N - 1)]R_\tau^h s_{\tau-1}^h. \quad (9)$$

Agent i maximizes its lifetime utility (1) subject to (9). The Euler equation is given by

$$\frac{1}{c_t^{i,h}} = \beta[1 - \alpha(N - 1)]R_{t+1}^h E_t \left(\frac{1}{c_{t+1}^{i,h}} \right). \quad (10)$$

Note that R_{t+1}^h is located outside of the expectation operator because Φ_t^h is known when agent i determines its appropriation and there is no aggregate uncertainty. Under the Markov perfect equilibrium, the optimal control path, $\{\tilde{c}_t^{i,h}\}$, exists if and only if $c_t^{i,h} = \tilde{c}_t^{i,h} = \alpha R_t^h s_{t-1}^h$ in (9) and (10) and the transversality condition, $\lim_{s \rightarrow \infty} \beta^s E_t [s_{t+s}^h / c_{t+s}^{i,h}] = 0$, is satisfied. Inserting $\tilde{c}_t^{i,h} = \alpha R_t^h s_{t-1}^h$ into (9) and (10) yields:

$$s_t^h = (1 - \alpha N)R_t^h s_{t-1}^h \quad (11)$$

and

$$s_t^h = \beta[1 - \alpha(N - 1)]R_t^h s_{t-1}^h, \quad (12)$$

respectively. From (11) and (12), we obtain that $\alpha = (1 - \beta)/[(1 - \beta)N + \beta]$.⁹ Note that α is an interest group's appropriation rate.

Lemma 1. *The policy function of agent i under the linear Markovian strategy is given by*

$$\tilde{c}_t^{i,h}(I_t^h) = \frac{1 - \beta}{(1 - \beta)N + \beta} I_t^h = \alpha I_t^h \quad (13)$$

and firm h 's savings under the Markov perfect equilibrium are given by

$$s_t^h = \frac{\beta}{(1 - \beta)N + \beta} I_t^h. \quad (14)$$

⁹ $\alpha N = \alpha(N)N$ is smaller than 1 and increasing in N .

Proof. The first part of the claims is obtained by inserting $\alpha = (1 - \beta)/[(1 - \beta)N + \beta]$ into the guessed policy function, $\tilde{c}_t^j = \alpha R_t^h s_{t-1}^h$. The second part is obtained by inserting $\alpha = (1 - \beta)/[(1 - \beta)N + \beta]$ into (11). $\square$.

Lemma 1 implies that given I_t^h , both each interest group's consumption and the firm's savings decrease as the number of interest groups increases. We can verify from (13) and (14) that the transversality condition, $\lim_{s \rightarrow \infty} \beta^s E_t \left[s_{t+s}^h / c_{t+s}^{i,h} \right] = 0$, is satisfied.

Remark 1. *Thus far, we have considered a dynamic game in which $N \geq 2$. However, note that (9) and (10) hold even though $N = 1$, in which case the model is reduced to a standard representative-agent growth model. Moreover, from (9), (10), and the transversality condition, one obtains (13) and (14) with $N = 1$. Therefore, we have all the results obtained in section 3 below regarding the total product and the growth rate for $N \geq 1$.*

3 Aggregation under Markov perfect equilibrium

3.1 Total product

Because the total product, $Y_t := \int_{h \in \Omega} A \Phi_{t-1}^h k_{t-1}^h dh$, is equal to the total savings, $S_t := \int_{h \in \Omega} s_t^h dh$, plus the total consumption, $C_t := \int_{h \in \Omega} \sum_i \tilde{c}_t^{i,h}(I_t^h) dh$, in the economy, it follows from (13) and (14) that $Y_t = \int_{h \in \Omega} A \Phi_{t-1}^h k_{t-1}^h dh = \int_{h \in \Omega} R_t^h s_{t-1}^h dh$. From (13) and (14), again, the total consumption, $C_t := \int_{h \in \Omega} \sum_i \tilde{c}_t^{i,h}(I_t^h) dh$, and savings, $S_t := \int_{h \in \Omega} s_t^h dh$, in the economy are given by

$$C_t = (1 - \tilde{\beta})Y_t \quad \text{and} \quad S_t = \tilde{\beta}Y_t, \quad (15)$$

where $\tilde{\beta} := \beta/[(1 - \beta)N + \beta]$.

The financial market clearing condition is given by $\int_{h \in \Omega} b_t^h dh = 0$. Given s_t^h , each firm becomes a lender with probability $G(\phi_t)$ and becomes a borrower with probability $1 - G(\phi_t)$. Therefore, the population of lenders is $G(\phi_t)$ and that of borrowers is $1 - G(\phi_t)$.

in period t . Thus, it follows from (6) and the i.i.d. assumption for productivity shocks that the financial market clearing condition, $\int_{h \in \Omega} b_t^h dh = 0$, becomes as follows:

$$S_t \times G(\phi_t) - \frac{\mu S_t}{1 - \mu} \times [1 - G(\phi_t)] = 0,$$

or equivalently,

$$G(\phi_t) = \mu. \quad (16)$$

Define a cutoff that satisfies (16) as $\phi^* = G^{-1}(\mu)$. From (5), a firm that draws a productivity shock, Φ_τ^h , greater than ϕ^* produces general goods, and thus, the total product in period $t + 1$ is given as follows:

$$Y_{t+1} = \frac{AH(\phi^*)}{1 - \mu} S_t = \frac{A\tilde{\beta}H(\phi^*)}{1 - \mu} Y_t, \quad (17)$$

where $H(\phi^*) = \int_{\phi^*}^{\eta} \Phi^h dG(\Phi^h)$. The second equality in (17) follows from (15). Because one unit measure of infinitely lived firms populates the economy, the per capita total product, $y_t := Y_t/N$, is obtained by substituting $Y_t = Ny_t$ into (17) as follows:

$$y_{t+1} = \frac{A\tilde{\beta}H(\phi^*)}{1 - \mu} y_t. \quad (18)$$

Remark 2. *It follows from (15) that the marginal propensity to save in the Markov perfect equilibrium is equal to $\tilde{\beta} := \beta/[(1 - \beta)N + \beta]$ whereas it is straightforward to show that the marginal propensity to save under a cooperative solution is equal to β . Note that β is strictly greater than $\tilde{\beta}$ when $N \geq 2$. This outcome means that given the total income, Y_t , in any period, over-consumption occurs in the economy in the Markov perfect equilibrium.*

3.2 Growth rate

In this section, we explicitly derive the growth rate in equilibrium. To do this, the distribution of Φ^h is specified as in Assumption 1.

Assumption 1. Φ^h is uniformly distributed over $[0, 1]$.

From Assumption 1 and (16), it follows that $H(\phi^*) = (1 - \mu^2)/2$.¹⁰

Inserting $H(\phi^*) = (1 - \mu^2)/2$ and $\tilde{\beta} := \beta/[(1 - \beta)N + \beta]$ into (18) computes the growth rate as follows:

$$y_{t+1}/y_t = \frac{A\beta(1 + \mu)}{2[(1 - \beta)N + \beta]} =: \Gamma(N, \mu). \quad (19)$$

Theorem 1. *Under Assumption 1, the following hold.*

- $\Gamma(N, \mu)$ decreases as N increases.
- $\Gamma(N, \mu)$ increases as μ increases.

Proof. The claims immediately follow from (19). $\square$

The negative impact of an increase in N on economic growth (the first result in Theorem 1) is derived because, given the total product Y_t , the total saving becomes smaller as the number of interest groups increases, as seen in (15).¹¹ This result is similar to that obtained by Tornell and Valasco (1992).

The positive impact of the relaxation of financial constraints on economic growth (the second result in Theorem 1) is obtained because as the financial constraint becomes more relaxed, more production resources are utilized by higher productivity firms and the allocative inefficiency regarding production resources is mitigated. As a result, economic growth is enhanced. A similar result is reported in the literature on finance and growth.

¹⁰We assume the uniform distribution for simplicity of exposition, as it significantly simplifies our analysis. We can investigate the growth rates in this and the next sections by applying more general distributions for productivity shocks (especially in the next section) without changing the main results.

¹¹One notes from (13) that as the number of interest groups increases, the appropriation rate, α , decreases, and thus, given I_t^h , each individual interest group's consumption decreases. However, the aggregate appropriation rate, which is given by $\alpha N = (1 - \tilde{\beta})$, increases as the number of interest groups increases. Therefore, given the total product Y_t , the total consumption becomes larger and the total saving becomes smaller as the number of interest groups increases, as seen in (15).

4 Introducing private assets

Thus far, we have investigated the dynamic game with a commonly accessible asset only. In this section, a private asset is introduced into the dynamic game, following Tornell and Velasco (1992). The property rights for the private asset are secure and the return from it is ensured when agents hold the asset. In the current paper, we consider the private asset as a foreign asset: agents can utilize investment opportunities in foreign countries with secure property rights to acquire a constant rate of return upon capital account liberalization. Note that each firm's portfolio decision is the same as in section 2 once its savings, s_t^h , are determined. Additionally, the process of aggregating variables is the same as in section 3, and thus, the cutoff, ϕ_t , is given by (16) in equilibrium. The difference is in that in the case of two assets, agents make separate decisions about appropriation and consumption. Therefore, the flow budget constraint (8) separates into the following two equations:

$$d_\tau^{i,h} + \sum_{j \neq i}^N d_\tau^{j,h} + s_\tau^h = R_\tau^h s_{\tau-1}^h = I_\tau^h. \quad (20)$$

and

$$f_\tau^{i,h} + c_\tau^{i,h} = \rho f_{\tau-1}^{i,h} + d_\tau^{i,h}, \quad (21)$$

where $f_\tau^{i,h} \geq 0$ is the stock of the private asset held by agent i in period τ , $d_\tau^{i,h}$ is the amount of appropriation, and ρ is the rate of return from the private asset.¹² From (16) with $\phi_t = r_{t+1}/A$, the equilibrium interest rate under Assumption 1 is given by $r_{t+1} = A\mu$. Furthermore, we impose the following assumption:

Assumption 2.

$$\frac{A(1+\mu)}{N} < \rho < A\mu. \quad (22)$$

Assumption 2 guarantees that the amount of appropriation and firm h 's savings in the Markov perfect equilibrium are positive under Assumption 1.

¹²It is assumed that agents cannot borrow from abroad, and thus, $f_\tau^{i,h} \geq 0$.

Remark 3. For ρ to exist, it must hold that $N \geq 3$.

Proof. Suppose that $N = 2$. For ρ to exist, it must hold that $A(1 + \mu)/2 < A\mu \iff \mu > 1$. This is a contradiction. $\square$

Because of Remark 3, we focus on the case in which $N \geq 3$ in what follows.¹³ The separation between appropriation and consumption revises the definitions of the Markovian strategy and the Markov perfect equilibrium as follows.

Definition 3. Agent i 's Markovian strategy is given by $\{d_t^{i,h}(I_t^h)\}$, i.e., for each $t \geq 0$, agent i 's action $d_t^{i,h}$ is a function of I_t^h such that $d_t^{i,h} : X \rightarrow [0, \infty)$.

Definition 4. A Markov perfect equilibrium is given by an N -tuple of strategies, $(\{\tilde{d}_t^{1,h}(I_t^h)\}, \{\tilde{d}_t^{2,h}(I_t^h)\}, \dots, \{\tilde{d}_t^{N,h}(I_t^h)\})$, for $t \geq 0$ such that for any $i \in \{1, 2, \dots, N\}$, given the Markovian strategies of other agents except for agent i , an optimal control path, $\{\tilde{d}_t^{i,h}, c_t^{i,h}\}$, exists, and it holds that $\tilde{d}_t^{i,h} = d_t^{i,h}(I_t^h)$ for all $t \geq 0$.

As in section 2, we guess a policy function such as $\tilde{d}_\tau^{j,h} = \theta_\tau R_\tau^h s_{\tau-1}^h$. Again, θ_τ is an interest group's appropriation rate, which is endogenously determined. In the case of two assets, the appropriation rate is subject to R_τ^h as seen later, and thus, θ_τ has a time subscript. This policy function transforms (20) into

$$d_\tau^{i,h} + s_\tau^h = [1 - \theta_\tau(N - 1)]R_\tau^h s_{\tau-1}^h, \quad (23)$$

where R_τ^h is the same as (7) because firm h 's portfolio decision is the same as (5) and (6). Agent i maximizes its lifetime utility:

$$U_t^{i,h} := E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \ln c_\tau^{i,h} \right], \quad (24)$$

subject to (21) and (23).

¹³In addition to Remark 3, for ρ to exist, the extent of financial constraints, μ , must be somewhat large. More concretely, it must hold that $\mu > 1/(N - 1)$. Therefore, if $\mu > 1/2$, ρ satisfying Assumption 2 can exist for all $N \geq 3$.

Lemma 2. *Suppose that Assumptions 1-2 hold. For $N \geq 3$, the policy function of agent i under the Markovian strategy with $\tilde{d}_\tau^{i,h} = \theta_\tau R_\tau^h s_{\tau-1}^h$ is given by*

$$\tilde{d}_t^{i,h}(I_t^h) = \frac{R_t^h - \rho}{R_t^h(N-1)} I_t^h =: \theta_t I_t^h. \quad (25)$$

Moreover, firm h 's savings and agent i 's consumption under the Markov perfect equilibrium are given by

$$\begin{aligned} s_t^h &= \frac{\rho N - R_t^h}{R_t^h(N-1)} I_t^h \\ &= \frac{\rho N}{N-1} s_{t-1}^h - \frac{1}{N-1} R_t^h s_{t-1}^h \end{aligned} \quad (26)$$

and

$$c_t^{i,h} = \rho(1 - \beta)(f_{t-1}^{i,h} + s_{t-1}^h), \quad (27)$$

respectively.

Proof. See the Appendix.

Note that the Markov perfect equilibrium obtained in Lemma 2 matches the interior equilibrium derived in Tornell and Valasco (1992).

Aggregating (26) across all firms by using $Y_t = \int_{h \in \Omega} R_t^h s_{t-1}^h dh$ yields

$$S_t = \frac{\rho N}{N-1} S_{t-1} - \frac{1}{N-1} Y_t. \quad (28)$$

By applying the first equality of (17), we obtain from (28) the following equation:

$$\frac{1 - \mu}{AH(\phi^*)} Y_{t+1} = \left(\frac{\rho N}{N-1} \right) \left(\frac{1 - \mu}{AH(\phi^*)} \right) Y_t - \left(\frac{1}{N-1} \right) Y_t. \quad (29)$$

Under Assumption 1, inserting $H(\phi^*)/(1 - \mu) = (1 + \mu)/2$ into (29) yields

$$y_{t+1}/y_t = \frac{2\rho N - A(1 + \mu)}{2(N-1)} =: \tilde{\Gamma}(N, \mu). \quad (30)$$

Theorem 2. *Under Assumptions 1 and 2, the following hold.*

- $\tilde{\Gamma}(N, \mu)$ *increases as N increases.*
- $\tilde{\Gamma}(N, \mu)$ *decreases as μ increases.*

Proof. See the Appendix.

The first result in Theorem 2 regarding the positive effect of an increase in the number of interest groups on economic growth is also obtained by Tornell and Valasco (1992) and Tornell and Lane (1999). When there are two assets, the no-arbitrage condition between the rate of return on the commonly accessible asset and the rate of return on the private asset holds, as (B.6) in the Appendix demonstrates, and accordingly, as the number of interest groups increases, each interest group must reduce its appropriation rate. The positive effect of the decrease in the appropriation rate on economic growth exceeds the negative effect of the increase in the number of interest groups in equilibrium. As a result, the growth rate increases.

The second result of Theorem 2 is new in that it shows that the relaxation of financial constraints is harmful to economic growth. The mechanism that causes this result is closely related to the voracity effect discussed in Tornell and Lane (1999). In the current model, as the financial constraint is relaxed, allocative inefficiency regarding production resources is mitigated and the aggregate productivity in the economy increases. At a microeconomic level, each firm can receive a higher return on average as the financial constraint is relaxed. Because of the higher return, each interest group perceives that the firm it exploits can afford more appropriations, and thus, the individual appropriation rate is increased. Then, the voracity effect works such that the increase in the aggregate appropriation rate is more than proportional to the increase in aggregate productivity. As a result, the growth rate decreases. This result is consistent with empirical evidence reported by Arcand et al. (2015) and others that indicates a negative relationship between the relaxation of financial constraints and economic growth in some cases.

5 Conclusion

We have investigated how the relaxation of financial constraints affects economic growth by introducing an imperfect financial market into Tornell and Velasco's (1992) dynamic game of the tragedy of the commons. When interest groups have access only to a common asset, whose property rights are not secure, the relaxation of financial constraints promotes economic growth. However, when interest groups can have access not only to a common asset but also to a private asset, whose property rights are secure, the relaxation of financial constraints impedes economic growth.

The effect of the relaxation of financial constraints when there are both common and private assets may be explained in the following way. If the return of a firm (which can be regarded as a commonly accessible asset) increases, on average, following the relaxation of financial constraints, each interest group increases the appropriation rate through the no-arbitrage condition between the two assets. Then, the increase in the aggregate appropriation rate surpasses the increase in aggregate productivity in the economy, which lowers the growth rate.

Appendix

Proof of Lemma 2

We set up the Lagrangian as follows:

$$\begin{aligned}
L_t^{i,h} := & E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \ln c_{\tau}^{i,h} \right] \\
& + E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \psi_{\tau}^{i,h} \left((1 - \theta_{\tau}(N-1)) R_{\tau}^h s_{\tau-1}^h - d_{\tau}^{i,h} - s_{\tau}^h \right) \right] \\
& + E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \zeta_{\tau}^{i,h} \left(\rho f_{\tau-1} + d_{\tau}^{i,h} - f_{\tau}^{i,h} - c_{\tau}^{i,h} \right) \right].
\end{aligned} \tag{B.1}$$

The first-order conditions are given by

$$\frac{1}{c_t^{i,h}} - \zeta_t^{i,h} = 0, \tag{B.2}$$

$$-\psi_t^{i,h} + \beta(1 - \theta_{t+1}(N - 1))R_{t+1}^h E_t[\psi_{t+1}^{i,h}] = 0, \quad (\text{B.3})$$

$$-\psi_t^{i,h} + \zeta_t^{i,h} = 0, \quad (\text{B.4})$$

and

$$-\zeta_t^{i,h} + \beta\rho E_t[\zeta_{t+1}^{i,h}] = 0. \quad (\text{B.5})$$

Under the Markov perfect equilibrium, the optimal control path, $\{\tilde{d}_t^{i,h}, c_t^{i,h}\}$, exists if and only if $d_t^{i,h} = \tilde{d}_t^{i,h} = \theta_t R_t^h s_{t-1}^h$ in (21) and (23), and the first-order conditions, (B.2)-(B.5) and the transversality conditions, $\lim_{s \rightarrow \infty} \beta^s E_t \left[s_{t+s}^h / c_{t+s}^{i,h} \right] = 0$ and $\lim_{s \rightarrow \infty} \beta^s E_t \left[f_{t+s}^{i,h} / c_{t+s}^{i,h} \right] = 0$, are satisfied.

From (B.3)-(B.5), we have

$$(1 - \theta_{t+1}(N - 1))R_{t+1}^h = \rho \quad (\text{B.6})$$

or, equivalently,

$$\theta_{t+1} = \frac{R_{t+1}^h - \rho}{R_{t+1}^h(N - 1)}. \quad (\text{B.7})$$

From (B.7), we obtain

$$\tilde{d}_t^{i,h}(I_t^h) = \frac{R_t^h - \rho}{R_t^h(N - 1)} I_t^h =: \theta_t I_t^h. \quad (\text{B.8})$$

This is equation (25). From (23), with $d_t^{i,h} = \theta_t I_t^{i,h} = \theta_t R_t^h s_{t-1}^h$, it follows that $s_t^h = (1 - \theta_t N) R_t^h s_{t-1}^h$. From this equation and (B.7), we obtain

$$s_t^h = \frac{\rho N - R_t^h}{R_t^h(N - 1)} I_t^h. \quad (\text{B.9})$$

This is equation (26). We must verify that $\tilde{d}_t^{i,h}(I_t^h) > 0$ and $s_t^h > 0$. Because $R_t^h - \rho \geq r_t - \rho = A\mu - \rho > 0$, from Assumption 2, we obtain $\tilde{d}_t^{i,h}(I_t^h) > 0$. Moreover, from Assumption 1, it follows that $R_t^h \leq (A - r_t\mu)/(1 - \mu) = A(1 + \mu)$. Therefore, we obtain $\rho N - R_t^h \geq \rho N - A(1 + \mu) > 0$ from Assumption 2. Finally, we derive agent i 's

consumption schedule in equilibrium. From (21), (23), and (B.6), it follows that

$$\frac{f_{t+1}^{i,h} + s_{t+1}^h}{c_{t+1}^{i,h}} + 1 = \rho \frac{f_t^{i,h} + s_t^h}{c_{t+1}^{i,h}}. \quad (\text{B.10})$$

Taking the expectation operator $E_t(\cdot)$ for both sides of (B.10) yields

$$E_t \left[\frac{f_{t+1}^{i,h} + s_{t+1}^h}{c_{t+1}^{i,h}} \right] + 1 = (f_t^{i,h} + s_t^h) \rho E_t \left[\frac{1}{c_{t+1}^{i,h}} \right]. \quad (\text{B.11})$$

By applying (B.2) and (B.5) to the right-hand side of (B.11), we obtain

$$\frac{f_t^{i,h} + s_t^h}{c_t^{i,h}} = \beta + \beta E_t \left[\frac{f_{t+1}^{i,h} + s_{t+1}^h}{c_{t+1}^{i,h}} \right]. \quad (\text{B.12})$$

From (B.12), it follows that

$$\begin{aligned} \frac{f_t^{i,h} + s_t^h}{c_t^{i,h}} &= \beta + \beta^2 + \dots + \beta^s + \beta^s E_t \left[\frac{f_{t+s}^{i,h} + s_{t+s}^h}{c_{t+s}^{i,h}} \right] \\ &= \frac{\beta}{1 - \beta} + \lim_{s \rightarrow \infty} \beta^s E_t \left[\frac{f_{t+s}^{i,h} + s_{t+s}^h}{c_{t+s}^{i,h}} \right] \end{aligned} \quad (\text{B.13})$$

From the transversality conditions, we have $\lim_{s \rightarrow \infty} \beta^s E_t \left[(f_{t+s}^{i,h} + s_{t+s}^h)/c_{t+s}^{i,h} \right] = 0$, and thus, $f_t^{i,h} + s_t^h = \beta c_t^{i,h}/(1 - \beta)$. This equation and (B.10) yield

$$c_t^{i,h} = \rho(1 - \beta)(f_{t-1}^{i,h} + s_{t-1}^h). \quad (\text{B.14})$$

This is agent i 's consumption schedule given by (27). Conversely, we verify that the transversality conditions hold. Suppose that agent i consumes following the schedule given by (B.14). Then, from (B.10) and (B.14), we obtain $f_t^{i,h} + s_t^h = \beta c_t^{i,h}/(1 - \beta)$. From the last equation and (B.13), it follows that

$$\lim_{s \rightarrow \infty} \beta^s E_t \left[\frac{f_{t+s}^{i,h} + s_{t+s}^h}{c_{t+s}^{i,h}} \right] = 0. \quad (\text{B.15})$$

In (B.15), because $f_{t+s}^{i,h} \geq 0$ and $s_{t+s}^h > 0$, it holds that $\lim_{s \rightarrow \infty} \beta^s E_t \left[s_{t+s}^h / c_{t+s}^{i,h} \right] = 0$ and $\lim_{s \rightarrow \infty} \beta^s E_t \left[f_{t+s}^{i,h} / c_{t+s}^{i,h} \right] = 0$. $\square$

Proof of Theorem 2

The second claim is obvious. For the first claim, define a function such that $f(x) = (2\rho x - A(1 + \mu))/(x - 1)$ in $x \in (0, \infty)$. Then, it follows from Assumption 2 that $\text{sign}(f'(x)) = \text{sign}(A(1 + \mu) - 2\rho) > 0$. The first claim follows from the last inequality. $\square$

References

- [1] Aghion P, Banerjee A (2005) Volatility and Growth. Oxford University Press, New York.
- [2] Aghion P, Banerjee A, Piketty T (1999) Dualism and macroeconomic volatility. Quarterly Journal of Economics 114(4): 1359-1397.
- [3] Aghion P, Howitt P, Mayer-Foulkes D (2005) The effect of financial development on convergence: theory and evidence. Quarterly Journal of Economics 120(1): 173-222.
- [4] Arcand JL, Berkes E, Panizza U (2015) Too much finance?. Journal of Economic Growth 20 (2) 105-148.
- [5] Dockner EJ, Plank M, Nishimura K (1999) Markov perfect equilibria for a class of capital accumulation games. Annals of Operations Research 89: 215-230.
- [6] Dockner EJ, Nishimura K (2001) Characterization of equilibrium strategies in a class of difference games. Journal of Difference Equations and Applications 7(6): 915-926.
- [7] Dockner EJ, Nishimura K (2004) Strategic growth. Journal of Difference Equations and Applications 10(5): 515-527.

- [8] Dockner EJ, Nishimura K (2005) Capital accumulation games with a non-concave production function. *Journal of Economic Behavior and Organization* 57(4): 408-420.
- [9] Greenwood J, Jovanovic B (1990) Financial development, growth, and the distribution of income. *Journal of Political Economy* 98(5): 1076-1107.
- [10] Kunieda T, Shibata A (2016) Asset bubbles, economic growth, and a self-fulfilling financial crisis. *Journal of Monetary Economics* 82: 70-84.
- [11] Kunieda T, Nishimura K (2018) Finance and economic growth in a dynamic game. Working paper. Kwansei Gakuin University.
- [12] Lamoreaux, NR. (1994) *Insider Lending. Banks, Personal Connections, and Economic Development in Industrial New England*. New York, NY: Cambridge University Press.
- [13] Lane PR, Tornell A (1996) Power, growth, and the voracity effect. *Journal of Economic Growth* 1(2): 213-241.
- [14] Levine R (2005) Finance and growth: theory and evidence. In: Aghion P, Durlauf SN (ed), *Handbook of economic growth*, vol.1A. Elsevier, Amsterdam, pp 865-934.
- [15] Levine R, Loayza N, Beck T (2000) Financial intermediation and growth: causality and causes. *Journal of Monetary Economics* 46(1): 31-77.
- [16] Loayza NV, Rancière R (2006) Financial development, financial fragility, and growth. *Journal of Money, Credit, and Banking* 38(4): 1051-1076.
- [17] Long NV, Sorger G (2006) Insecure property rights and growth: the role of appropriation costs, wealth effects, and heterogeneity. *Economic Theory* 28(3): 513-529.
- [18] Saci K, Giorgioni G, Holden K (2009) Does financial development affect growth?. *Applied Economics* 41(13): 1701-1707.

- [19] Sorger G (2005) A dynamic common property resource problem with amenity value and extraction costs. *International Journal of Economic Theory* 1(1): 3-19.
- [20] Tornell A, Velasco A (1992) The tragedy of the commons and economic growth: why does capital flow from poor to rich countries?. *Journal of Political Economy* 100(6): 1208-1231.
- [21] Tornell A, Lane PR (1999) The voracity effect. *American Economic Review* 89(1): 22-46.