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Experimental and Theoretical Evaluation of the Effect of Panel Geometry on the Failure of Corrugated Board Panel

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ABSTRACT

McKee's formula is widely used to predict the compression strength (CS) of corrugated boxes and panels. It can accurately estimate the compression strength of boxes that are within a practical size range, but recently, larger and smaller corrugated boxes than before have been extensively developed. Therefore, there is a need for a CS prediction formula that works beyond the application range of McKee's formula. Recent researches consider the failure mode as a combination of collapsing and buckling failure and remove the constraints and the assumptions associated with McKee's formula. This makes it possible to more accurately estimate the CS of boxes that are not covered by McKee's formula. Many CS formulae are derived logically from material mechanics, but doing so can make it difficult to account for various actual behaviors in detail up to when the box fails. Instead, by analyzing the behavior up to failure in detail, we explored relationships that could account for the CS consistently based on its behavior.

KEYWORDS

Panel, buckling, collapse, prediction of panel compression strength, compression displacement

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INTRODUCTION

Empty-box compression strength (CS) is a fundamental property in the design of corrugated boxes. During transport and storage, a corrugated box weakens because of external factors such as stacking load, humidity, vibration, and shock. Therefore, the designer must introduce a safety factor for a corrugated box to withstand actual use. Despite the introduction of approximate coefficients, e.g., safety factors, into the design of such boxes, accurately determining the empty-box CS remains a basic design element.

In the 1950s and 1960s, Kellicutt and Landt [1] and McKee et al. [2], respectively, developed formulae to estimate the CS of corrugated boxes on the basis of the basic physical properties of fiberboard or corrugated board. Because of their convenience and predictive accuracy, these formulae are still used widely.

McKee's formula was derived from experimental rule devised by the National Advisory Committee for Aeronautics [3] to predict the failure loads of isotropic flat plates. The CS of a panel is predicted by an exponential function of the buckling load and the edgewise compression strength (known as the ECT value) of the plate material:

$$P_z/P_{cr} = c (P_m/P_{cr})^b \quad (1)$$

where P_z is the CS of the plate per unit width, P_{cr} is the buckling load, P_m is the edgewise compression strength of the plate material, and b and c are constants.

Through rearrangement of Equation (1), the equation $P_z = c (P_m)^b (P_{cr})^{(1-b)}$ is obtained; this equation shows that the CS of the plate can be calculated by harmonizing the term P_m related to collapse with the term P_{cr} related to buckling. Many CS formulae for corrugated board panels, including McKee's formula, have been constructed with

collapse and buckling (i.e., deflection) terms.

McKee et al. [2] applied a load to a box, measured the load at the upper end of the panel at 1-inch intervals, and obtained a load intensity distribution with a downward convex shape, as shown in Fig. 1. This shape clarifies that the vicinity of the edge carries a larger load than the region near the center of the panel. Furthermore, McKee et al. suggested that the intensity near the edge was related to the ECT value and that the intensity near the center was related to the bending property, indicating that the contribution of collapse and buckling properties changes depending on the region of the panel.

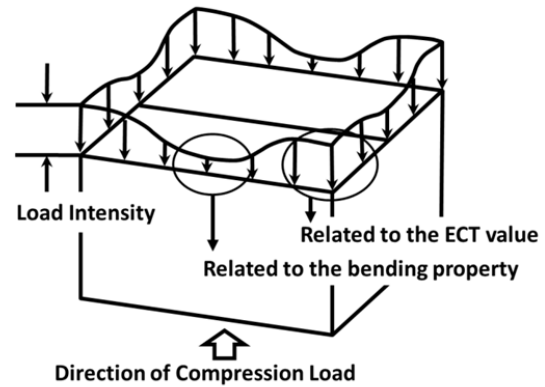


Fig. 1 Distribution of compression load intensity around box perimeter.

Compressing and crushing of corrugated boxes reveal various failure behaviors. Differences can be observed among various aspects such as the compression displacement at maximum load, the position at which buckling/collapse occurs, and the shape of the yield line. The failure behavior and the CS complement each other; that is, the failure behavior can explain the CS and vice versa. Therefore, analyzing the failure behavior in detail is considered an effective approach to explain the CS of corrugated boxes.

Peterson et al. [4] focused on the rotational behavior of the top and bottom of the panel edges of corrugated boxes and studied its influence on the CS. In the case of a regular slotted container (RSC),

whose upper and lower edges are easy to rotate, the center region of the panel bends easily and cannot carry a load effectively because the load is transferred to the vicinity of the left and right edges; consequently, the CS of the entire panel decreases. Conversely, in the case of a tube, whose upper and lower edges are difficult to rotate, the vicinity of the center of the panel will not deflect and the load distributes uniformly throughout the panel; consequently, the CS of the tube becomes higher than that of the RSC. Furthermore, Peterson et al. [4] found that the CS could be changed by modifying the rotational properties of the upper and lower edges and clarified the relationship between the rotational behavior and the CS.

Urbanik and Frank [5] suggested that the failure mode of a panel could be characterized as either collapsing failure or buckling failure according to the boundary term $U (= \sqrt{P_m/P_{cr}}$, known as the universal slenderness). Collapsing failure occurs when $U \leq 1$, in which case the CS increases in proportion to the panel width. Buckling failure occurs when $U > 1$, in which case the CS can be calculated by Equation (1). This approach enables the CS to be calculated more accurately than with McKee's formula by clarifying the boundary condition theoretically; McKee's formula does not consider the failure transition from collapse to buckling.

Ristinmaa et al. [6] proposed a CS formula by focusing on the shape of the yield-line curve when a carton board failed. They defined a region where a parabolic shaped yield line appeared in the vicinity of an edge as the corner region and defined a region where a horizontal yield line appeared away from an edge as the panel region. They showed that the CS could be calculated by summing the strength of the corner region and the panel region. The CS of the corner region is calculated on the basis of a short span compression test (SCT) value related to collapse. By contrast, the CS of the panel region is calculated on the basis of the SCT value and the bending stiffness related to buckling. Ristinmaa et

al. showed that the failure behavior of the panel was divided into two regions and that the physical properties contributing to CS varied depending on the region of the panel.

In the present study, we focus on the behavior of an actual corrugated board panel to the point of failure and explore how the failure behavior affects the CS as a fundamental consideration. We begin with compression tests using modeling corrugated board with different panel widths and heights, and we observe in detail how the CS changes with panel dimensions. Furthermore, focusing on the load–displacement curve, we build a picture of how the panel can fail. The results reveal that the failure behavior can clearly be divided into four stages according to the panel width. We analyze how the properties of collapse or buckling change according to each stage and define the boundary condition to divide each stage.

ANALYTICAL APPROACHES

In this research, we fixed the panel height to 0.075 m, 0.15 m, 0.30 m, or 0.50 m while varying the panel width from 0.05 m to 0.70 m and observed in detail the increase in CS with increasing panel width for each the panel heights.

The ability of a panel to support a load, i.e., its load-carrying capacity (LCC), varies with each part of the panel. The LCC is high in the vicinity of the edge and decreases with increasing distance from the edge. The ability of the whole panel to carry a load is considered to be obtained by summing the LCC of each part of the panel. Ristinmaa et al. estimated the CS of an entire panel by constructing a CS formula separately for two panel regions and summing the CS in each region because the region near the edge and the region far from the edge exhibit different failure behaviors.

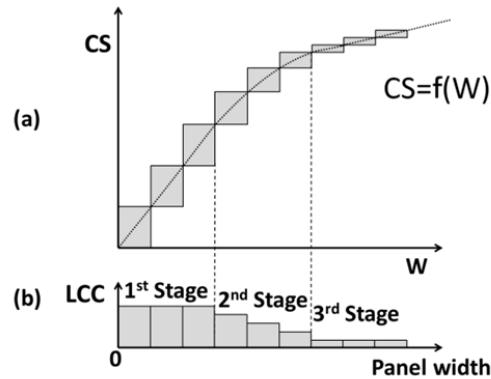


Fig. 2 Relation between LCC and compression strength (CS)
(a) CS - Panel width diagram.
(b) Change in LCC according to panel width.

In this research, to obtain the CS-panel width diagram (i.e., Fig.2 (a)), we initially experimentally observed the relation between panel width and CS. Assuming that the observed CS is the sum of each LCC of the part of the panel, we analyzed how the LCC varies with increasing panel width (Fig. 2(b)). Furthermore, by classifying the stage according to the characteristics of the change in the LCC and constructing the calculation method of the CS at each stage, we clarified the relation between the CS of the whole panel and the panel width.

MATERIALS AND TEST CONDITIONS

All experiments in this study were conducted with a single type of corrugated board whose characteristics are shown in Table 1.

Table1 Materials and test conditions

Corrugated board	
Flute type	C
Inner linerboard	160 g/m ²
Outer linerboard	160 g/m ²
Corrugating medium	120 g/m ²
Edgewise CS (TAPPI T-811)	4.78 k N/m
Supplier	Rengo Co., Ltd., Kawaguchi-city, Saitama, Japan
Compression test conditions	
Temperature	23° C
Humidity	50% RH
Compression rate	10 mm/min
Platen type	Floating

EXPERIMENTAL METHODS

CS of panels whose left and right edges are joined to other panels

We analyzed the CS of a panel whose left and right edges are joined to adjacent panels and to which flaps are attached at the top and bottom of the panel. These panels have the same geometrical features as the four-sided panels of an RSC. An actual RSC can be divided into an end panel with an inner flap and a side panel with an outer flap, thereby causing a time lag between the loading of an end panel and the loading of a side panel. Consequently, when the compression behavior of an RSC is measured, panels with different geometrical properties are measured simultaneously and the compression behavior of the panel cannot be observed accurately. Therefore, as shown in Fig. 3, a corrugated board structure was modeled such that the four panels had the same geometrical properties. The CS per panel was obtained by dividing the maximum compression load of the model corrugated board structure by four. The CS was measured five times, and the average value was taken as the CS of the panel.

Test sample sizes

Panel height: 0.075 m, 0.15 m, 0.30 m, 0.50 m

Panel width: 0.05 m, 0.075 m, 0.10 m, 0.125 m, 0.15 m, 0.20 m, 0.25 m, 0.30 m, 0.40 m, 0.50 m, 0.60 m, 0.70 m

In the model corrugated board structure shown in Fig. 3, the edge is not joined with a joint flap; however, one central part of the panel is joined with polypropylene tape. In a typical corrugated box, the panels are joined with a joint flap; however, the CS changes according to the joining method (e.g., tape joint and glue joint, among others). Furthermore, because the strength of the joint-flap portion affects the CS, we joined the central part of the panel with 50- μ m-thick polypropylene tape to eliminate these effects.

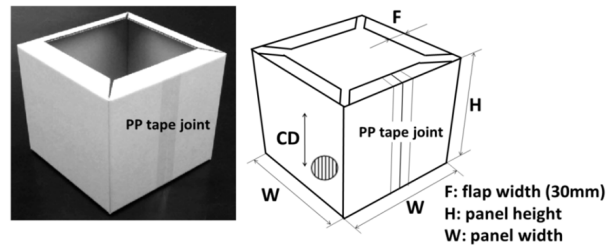


Fig. 3 Schematic of model corrugated board structure used to measure the CS of panels.

CS of panels whose left and right edges are free ends

We analyzed the CS of panels whose left and right edges were free ends and to which flaps were attached at the top and bottom of the panel. The maximum load of these panels observed after panel compression and buckling are considered a basic indicator of how much load can be withstood with bending. This indicator is considered to correspond to the LCC at the central part of the panel with infinitely long width because the central part of this panel is completely free from the influence of the vertical supporting effect of the edges; the central part of the panel is considered to have the same geometrical properties as the panels whose left and right edges are free ends. To obtain the bending property, hollow rectangular specimens with various heights and a fixed width to 0.20 m were used, as shown in Fig. 4. The heights of the tested samples are listed below.

Test sample sizes

Panel height: 0.075 m, 0.15 m, 0.30 m, 0.50 m

Panel width: 0.20 m (fixed)

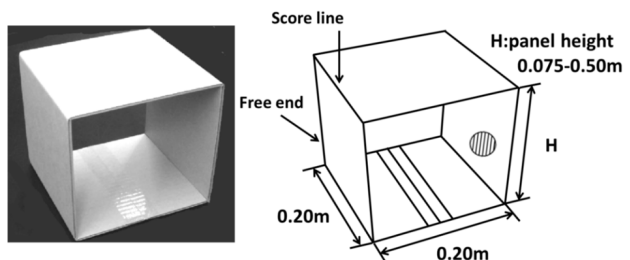


Fig. 4 Schematic of hollow rectangular specimen used to measure maximum load of panel.

RESULTS AND DISCUSSION

The CS measured with the model corrugated board structure in Fig. 3 is shown in Fig. 5. Table 2 shows the failure mode when the panels are broken down, along with the compression displacement at maximum load. Regarding the failure mode in Table 2, we define the failure mode of breaking down into a bellows shape from the upper and lower score line portions shown in Fig. 6(a) as “collapsing failure” and that of breaking down with the yield line shown in Fig. 6(b) as “buckling failure.” In addition, Fig. 7 shows how the compression displacement at the maximum load is determined in each case of collapsing failure and buckling failure.

Collapsing failure mode (first stage)

Hereafter, we focus on the panels of height 0.30 m for purposes of discussion. As shown in Fig. 5(c), the CSs increase in proportion to the panel width from 0.05 m to 0.20 m. In addition, all failure modes of these panels exhibit collapsing failure, as reported in Table 2. The compression load–compression displacement diagram of panels of width 0.20 m (Fig. 8(a)) shows that the compression load remains a certain constant level even when the compression displacement exceeds 10 mm, and it shows the typical collapsing failure. When the upper and lower score line parts break down in a bellows shape, the CS increases in proportion to the panel width because the bellows shaped score line width

Table 2 Failure mode and compression displacement at maximum load.

		Panel Width (m)											
		0.05	0.075	0.1	0.125	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7
Panel Height (m)	0.08	Failure mode	C	C	C	C	C	C	C	C	C	C	C
		Compression DISPL (mm)	7.9	9.3	9.4	9.4	9.2	9.7	9.8	13.1	11.9	12.0	12.1
		Standard deviation (mm)	2.5	0.4	0.2	0.4	0.7	0.6	0.7	3.7	0.4	0.5	0.4
	0.15	Failure mode	C	C	C	C	C	C	C	C/B	B	B	B
		Compression DISPL (mm)	9.2	10.2	10.6	10.1	10.3	9.2	8.4	9.0	8.7	4.5	4.3
		Standard deviation (mm)	1.0	0.5	0.8	0.8	2.3	1.2	0.8	0.1	2.2	0.1	0.1
	0.3	Failure mode	C	C	C	C	C	C	C/B	B	B	B	B
		Compression DISPL (mm)	9.7	12.4	12.0	10.6	11.0	10.5	8.0	5.7	5.5	4.9	5.0
		Standard deviation (mm)	1.3	1.0	1.3	0.8	2.4	1.5	1.3	1.7	0.5	0.3	0.1
	0.5	Failure mode	–	C	C	C	C	C	C	C/B	B	B	B
		Compression DISPL (mm)	–	10.7	11.0	11.5	11.8	11.7	10.3	9.4	5.3	5.3	5.2
		Standard deviation (mm)	–	0.4	0.2	0.6	0.8	1.0	0.5	2.0	0.6	0.1	0.1

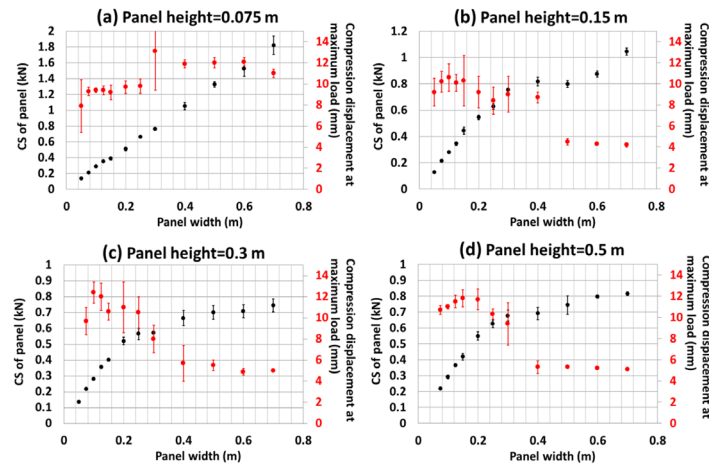


Fig.5 Graphs of CS and compression displacement at maximum load for panels of various dimensions.

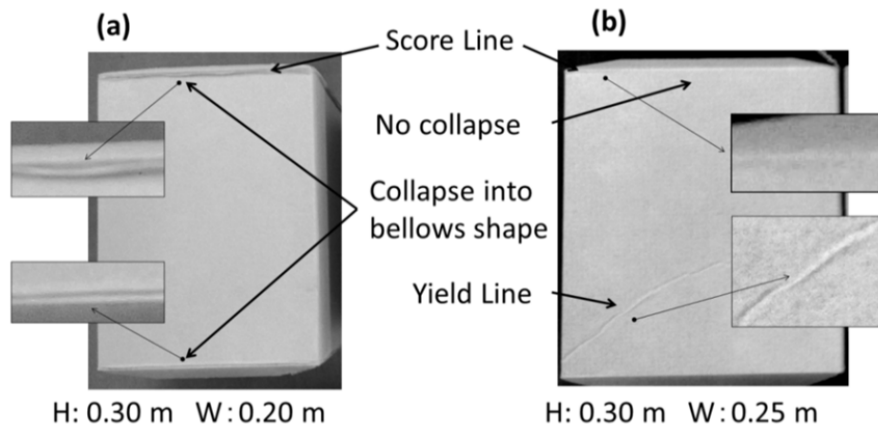


Fig. 6 Box failure modes: (a) collapsing failure and (b) buckling failure.

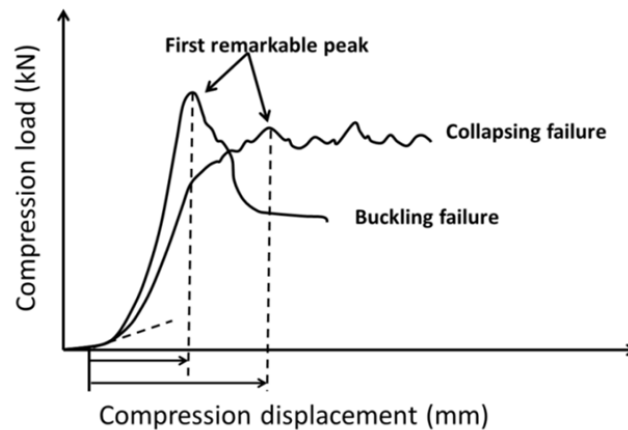


Fig. 7 Conceptual scheme for determining compression displacement at maximum compression load.

increases as the panel width increases. We define this panel width region as the first stage.

The relation between the panel CS and LCC in the first stage is shown conceptually in Fig. 9. As shown in Fig. 9(b), in the first stage, the LCC is the same at any position in the horizontal direction of the panel (point **a** to **c**). Therefore, the CS of the panel of width ω is the sum total of the LCC from points **a** to **c**. Furthermore, because points **a** to **c** have the same LCC, the CS-panel width diagram expresses a proportional relationship, as shown in Fig. 9(a).

Transition from collapsing failure to buckling failure mode (second stage)

As shown in Fig. 5(c), when the panel width exceeds 0.20 m, the CS no longer increases in proportion to the panel width. In addition, as shown in Table 2, when the panel width is 0.25 m, collapsing failure and buckling failure coexist. As shown in the load–compression displacement diagram in Fig. 8(b), the panels that break down before reaching the compression displacement of 10 mm and the panels that break down immediately even if compression displacement exceeds 10 mm coexist.

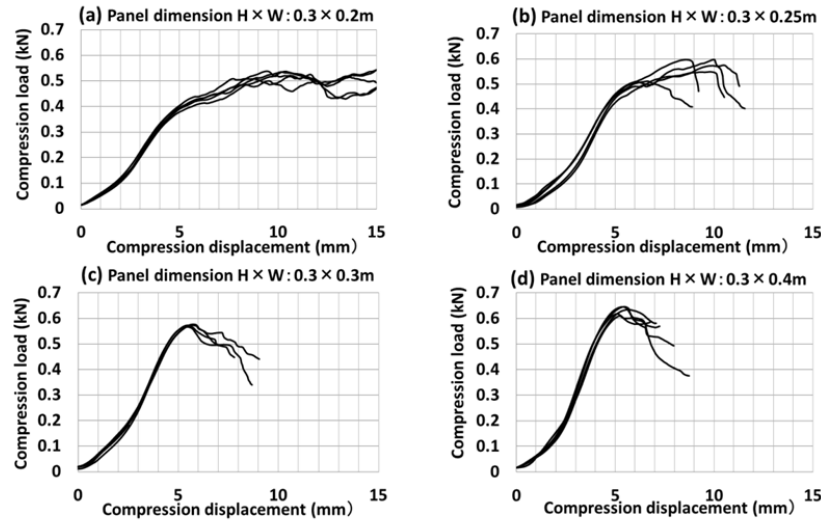


Fig.8 The compression load-compression displacement diagram of panels of height 0.30 m

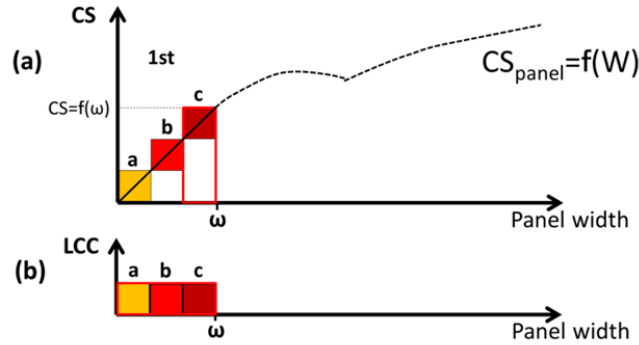


Fig. 9 Relation between CS and LCC in the first stage.
(CS increases in proportion to the panel width.)

Consequently, we infer that the panel of width of 0.25 m is in the transition process from collapsing failure to buckling failure.

We define a panel width region in which the failure mode gradually changes from collapsing to buckling; also, the compression displacement becomes shorter as the second stage. In the second stage, before reaching the CS of collapsing failure, the buckling triggers failure of the entire panel; thus, the LCCs of the entire panel correspond to the LCC at the time of buckling.

In the second stage, the LCCs of the entire panel are considered to gradually decrease as the compression displacement decreases from approximately 10 mm (at which collapsing failure occurs) to approximately 6 mm (at which buckling failure occurs). Fig. 10 shows conceptually the relation between panel CS and LCC in the second stage. In Fig. 10, points **d** to **g** correspond to the second stage. For a panel width ω_1 , regarding the LCC of point **g** in Fig. 10(b), we assume that points **a** to **c** of the first stage and **d** to **g** of the second stage have the same

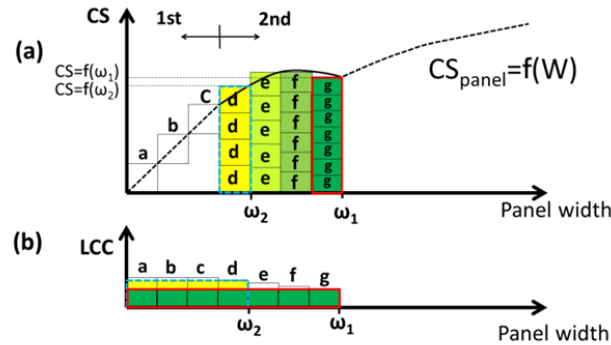


Fig. 10 Relation between CS and LCC in the second stage .
(failure mode shifts from collapse failure to buckling failure.)

LCC as that of point **g** (within the red-lined frame). Because points **a** to **f** begin to fail at the same time as point **g**, points **a** to **f** (which originally exhibit a greater LCC than that of point **g**) consequently exhibit the same LCC as that of point **g**. Therefore, the LCC up to ω_1 in Fig. 10(b) is the same as that of point **g** over the entire range of panel width. Likewise, for a panel width of ω_2 , we assume that points **a** to **c** of the first stage and **d** of the second stage have the same LCC as that of **d** (within the broken blue-lined frame).

Because the compression displacement at maximum load decreases as the panel width increases in the second stage, even those parts of the panel that originally had a higher LCC will fail before complete collapsing failure. Eventually, even if the panel width increases, the panel CS will not substantially increase much; in some cases, the CS might decrease.

Buckling failure mode (third and fourth stages)

As shown in Fig. 5(c), the CS of the panel of width 0.40 m increases again. As reported in Table 2, the failure mode of this panel is buckling failure and the compression displacement is 5.5 mm. Even if the panel width is increased further, the compression displacement does not decrease; it takes a

constant value of approximately 5 mm. We define the panel width region as for the 0.40-m-wide panel as the third stage. Because the compression displacement does not decrease and remains approximately 5 mm in the third stage, the LCC of the panel portion of the first and second stages also does not decrease further.

Although the panel portion added in the third stage also causes buckling failure, the panel portion close to the edge is supposed to be influenced by the vertical supporting effect of the edges and exhibits a higher LCC than the portion far from the edge. In the panel portion farther from the edge, we infer that the vertical supporting effect of the edge is eliminated and that the LCC of this panel portion becomes the same LCC as that of the panel whose left and right edges are free ends.

We define the panel width region where the vertical supporting effect is eliminated as the fourth stage. All LCCs of the fourth stage are considered to be the same as that of the panel whose left and right edges are free ends, even if the panel width is further increased. Therefore, in the fourth stage, we assume that the CS increases in proportion to the panel width as in the first stage.

Fig. 11 shows conceptually the relation between panel CS and LCC in the third stage. The LCC from points **a** to **g** shown in Fig. 11(b) have the same

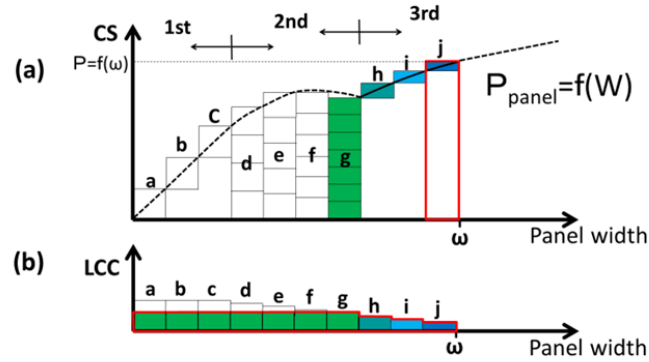


Fig. 11 Relation between CS and LCC in the third stage .

(LCC decreases gradually to the LCC of the panel whose left and right edges are free ends.)

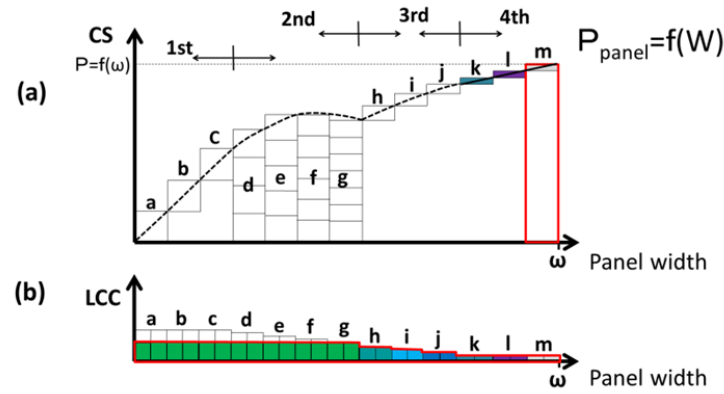


Fig. 12 Relation between CS and LCC in fourth stage.

(LCC is same as the LCC of the panel whose left and right edges are free ends.)

LCC as point **g**, which corresponds to the end of the second stage, as previously explained. When the position reaches point **h** in the third stage, the LCC gradually decreases with increasing panel width (the LCC: **h** > **i** > **j**).

As shown in Fig. 11(a), when the panel width is ω , we can obtain the CS (within the red-lined frame) by adding the LCC of the portion in which the panel width is increased (i.e., points **h**, **i**, and **j**) to the CS up to the second stage.

Fig. 12 shows conceptually the relation between panel CS and LCC in the fourth stage. In the fourth stage, much like in the first stage, the LCC after point **k** in Fig. 12(b) is constant. As shown in Fig. 12(a), the panel CS increases linearly with the same incremental gradient.

CS of panels whose left and right edges are free ends

Dividing the CS measured from the schematic of hollow rectangular specimens (Fig. 4) by 0.40 m gives the CS per unit meter (Table 3).

Matching CS formula to measured values

Fig. 13 represents a change in the LCC with increasing panel width and shows the relation between panel width and CS (the framed area) and the concept representing parameters determining its shape in the aforementioned four stages.

The panel width in the first stage is the region from W_0 to W_1 , where W_1 is considered the terminal point of collapsing failure, that is, the beginning point of buckling failure. The LCC in this region is constant at the level of P_1 , which is the collapse strength per unit width into a bellows shape.

The panel width in the second stage is the region from W_1 to W_2 , and W_2 can be considered as the point where the failure mode has completely shifted to buckling failure. P_2 can be considered

as the buckling strength, being affected somewhat by the vertical supporting effect of the edge. We assume that the LCC decreases linearly from P_1 to P_2 as the panel width increases. The CS of the entire panel in this stage is obtained by multiplying the panel width by the LCC at a certain width, and the LCC of the region from W_0 to W_1 of the first stage decreases from P_1 to P_2 as the panel width increases from W_1 to W_2 .

The panel width in the third stage is W_2 to W_3 , and W_3 is considered the point where the LCC has reached the LCC of the panel whose left and right edges are free ends; i.e., the point where the LCC has reached P_3 . We assume that the LCC decreases linearly from P_2 to P_3 . The CS of the entire panel is obtained by adding the CS up to the second stage to that of third stage.

The panel width in the fourth stage is the region above W_3 , and the LCC is constant at the level of P_3 .

In accordance with the aforementioned stages, we devised formulae to estimate the panel CS on the basis of summing the LCC.

Table 3 Maximum load per panel width of panels whose left and right edges are free ends.

	Panel height (m)			
	0.075	0.15	0.3	0.5
Maximum load per panel width (kN/m)	2.44	1.47	0.38	0.14
Standard deviation (kN/m)	0.20	0.06	0.01	0.01

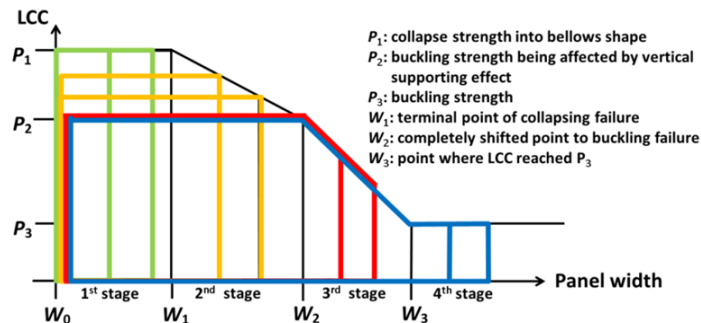


Fig. 13 Conceptual scheme for estimating CS of panels in terms of six parameters.

Panel width W from W_0 to W_1 (first stage)

CS:

$$P_{panel} = P_1 \times W$$

(2)

Panel width W from W_1 to W_2 (first stage + second stage)

CS:

$$P_{panel} = W^2 \frac{P_2 - P_1}{W_2 - W_1} + W \left\{ P_1 - W_1 \frac{P_2 - P_1}{W_2 - W_1} \right\}$$

(3)

Panel width W from W_2 to W_3 (first stage + second stage + third stage)

Because the CS when $W = W_2$ is $P_2 \times W_2$, the area of the trapezoidal part after W_2 may be added to this intensity:

CS:

$$\begin{aligned} P_{panel} &= P_2 \times W_2 + W^2 \frac{P_3 - P_2}{2(W_3 - W_2)} + W \left\{ P_2 - W_2 \frac{P_3 - P_2}{W_3 - W_2} \right\} - P_2 \times W_2 \\ &\quad + W_2^2 \frac{P_3 - P_2}{2(W_3 - W_2)} \\ &= W^2 \frac{P_3 - P_2}{2(W_3 - W_2)} + W \left\{ P_2 - W_2 \frac{P_3 - P_2}{W_3 - W_2} \right\} + W_2^2 \frac{P_3 - P_2}{2(W_3 - W_2)} \end{aligned}$$

(4)

Panel width W is W_4 or larger (first stage + second stage + third stage + fourth stage)

Because the area of the trapezoid from W_2 to W_3 when $W = W_3$ is $(P_3 + P_2)(W_3 - W_2)/2$, it is appropriate to add the strength $P_3(W - W_3)$ from W_3 onward to the intensity $P_2 W_2$ from W_0 to W_2 and the trapezoid area $(P_3 + P_2)(W_3 - W_2)/2$.

CS:

$$P_{panel} = P_2 \times W_2 + \frac{(P_3 + P_2)(W_3 - W_2)}{2} + P_3(W - W_3)$$

(5)

Note that $P_1 > P_2 > P_3$ and $W_3 > W_2 > W_1$.

P_1 , P_2 , P_3 , W_1 , W_2 , and W_3 for each panel height by minimizing the sum of the squares of the differences between the measured and estimated values; the results are given in Table 4. The correlation between the values estimated using the

obtained values of P_1 , P_2 , P_3 , W_1 , W_2 , and W_3 and the measured values is shown in Fig. 14. A comparison of the shape of the estimated panel width–CS diagram with that of measured panel width–CS diagram reveals that the shapes agree well. Therefore, we reason that the conceptual model can explain the actual failure behavior of the panel.

According to the results in Table 4, $2.60 \leq P_1 \leq 2.87$ kN/m, which is approximately 60% of the ECT value of 4.78 kN/m. Furthermore, P_1 is almost constant irrespective of the panel height. The value of P_1 is lower than the original ECT value because of the influence of the score line. Because the upper and lower score lines are weakened by folding, and because this part triggers collapsing failure into a bellows shape, P_1 is expected to decrease.

The value of P_2 is 1.48 kN/m at the panel of height 0.15 m, which is approximately 31% of the ECT value. It exceeds 1.70 kN/m when the panel height is 0.30 m or more, corresponding to 36% of the ECT value. We infer that the lower P_2 value of the lower 0.15-m-high panel is related to the shorter compression displacement at the time of buckling failure. The compression displacement is on the order of 4 mm at a panel 0.15 m high, whereas it is on the order of 5 mm for panels 0.30 m or higher, as shown in Table 2. That is, less compression displacement occurs at the buckling failure of the 0.15-m-high panel. We assume that a lower value of P_2 is observed because the panel fails even earlier before the buckling failure of panels 0.30 m high or higher. This observation of less compression displacement for shorter panels is related to the curvature of panel deflection, which increases even for a small amount of compression displacement.

Regarding the proposed formulae, conceptually, P_3 corresponds to the maximum load per unit width (in meters) of panels whose left and right edges are free ends. According to Euler's buckling law, the lower the panel, the higher the value of P_3 . As shown in Table 4, the P_3 value of a panel of

Table 4 Optimum solutions of each parameter by least squares method.

		Panel height (m)			
		0.075	0.15	0.3	0.5
P_1 (kN/m)	Collapse strength into bellows shape	2.60	2.86	2.78	2.87
P_2 (kN/m)	Buckling strength being affected by vertical supporting effect	–	1.48	1.76	1.73
P_3 (kN/m)	Buckling strength	–	1.48	0.19	0.35
W_1 (m)	Terminal point of collapsing failure	∞	0.187	0.176	0.179
W_2 (m)	Completely shifted point to buckling failure	–	0.54	0.32	0.40
W_3 (m)	Point where LCC reached P3	–	0.54	0.45	0.43

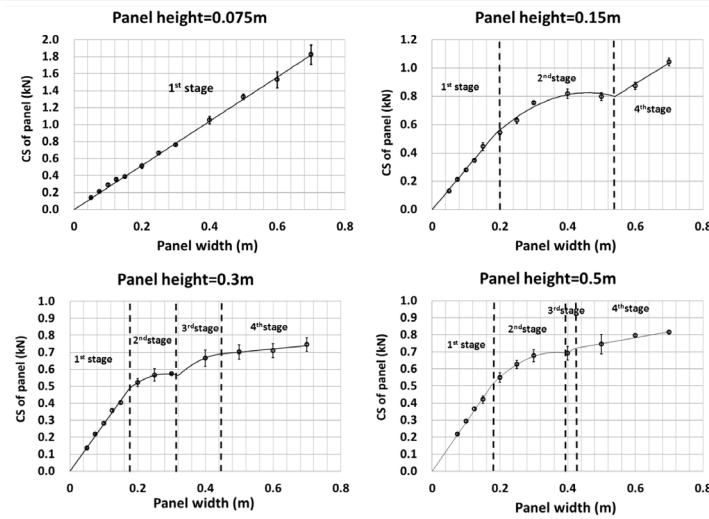


Fig. 14 Graphs of CS of panels with various dimensions. Solid lines show estimated values.

height 0.15 m is 1.48 kN/m. This value is consistent with a maximum load per unit width (in meters) of 1.47 kN/m for the panel whose left and right edges are free ends (Table 3). The P_3 values of panels 0.3 m and 0.5 m high are 0.19 kN/m and 0.35 kN/m, respectively. These values are much lower than the P_3 value of a 0.15-m-high panel but are not consistent with the values of the maximum load per unit width, 0.38 kN/m and 0.14 kN/m, respectively, reported in Table 3.

W_1 is found to be approximately 0.18 m regardless of the panel height. The region up to W_1 is considered to be the region where the panel experiences complete collapsing failure. The panel will not

deflect in the region 0.09 m from the left and right edges in the actual panel, regardless of the panel height, because of the vertical supporting effect of the edges, leading to certain collapsing failure.

Panel width W_2 is considered the panel width at the point of complete buckling failure. From Table 4, the 0.15-m-high panel exhibits a W_2 of 0.54 m, which is high compared with the W_2 values of the panels 0.30 m high or higher. Because the 0.15-m-high panel is short and resistant to buckling, we considered that a wider panel width is necessary to observe complete buckling. The 0.50-m-high panel also shows a relatively high W_2 of 0.40 m. These results are attributed to curvature due to deflection

of the panel in the vertical direction being alleviated in taller panels, resulting in the panels being less prone to buckling failure.

W_3 is considered to be the panel width at the point where the vertically supporting effect of the edge disappears. The 0.3-m- and 0.5-m-high panels show similar W_3 values of 0.45 m and 0.43 m, respectively. Assuming that the value of W_3 is constant regardless of the panel height, then the W_3 of the 0.15-m-high panel should also be approximately 0.44 m; however, the W_3 of the 0.15-m-high panel is 0.54 m, which is same as the W_2 value of the 0.15-m-high panel. We infer that the W_3 of the 0.15-m-high panel is originally approximately 0.44 m, but in this case W_2 may be greater than W_3 ; that is, the vertical supporting effect by the edge might disappear (corresponding to W_3) before complete buckling failure (corresponding to W_2). Therefore, we suppose that the value of W_3 is originally ~ 0.44 m and that the vertical supporting effect of the edge will disappear at the point 0.22 m from the edge in the actual panel.

All panels of height 0.075 m experience collapsing failure regardless of the panel width; thus, only the value of P_1 is obtained.

CONCLUSIONS

Focusing on the behavior up to failure when panels were compressed, we found that the failure behavior could clearly be divided into four stages according to the panel width. Six parameters are required to define each stage, and these parameters were determined by the least-squares method.

We inferred that parameter P_1 is related to the ECT value and that parameter P_3 is related to the maximum load of the panel whose left and right edges are free ends. In this study, we were unable to sufficiently correlate the other parameters to

the basic physical properties of corrugated board. However, we speculate that they are so related and could be estimated by further study.

Many formulae for predicting the CSs of corrugated boxes and panels have been constructed based on the theory of materials mechanics, including the formula due to Urbanik and Frank [5]. However, the manner in which actual corrugated board fails is complicated, and it is difficult to use such formulae to explain comprehensively such complicated behavior up to failure. In our research, by incorporating the behavior up to failure into a formula and correlating this formula with the physical properties of the corrugated board, we constructed a concept representing the shape of the panel-width–CS curve that accounted for the complicated phenomena of corrugated board failure. Regarding corrugated board failure, collapsing failure and buckling failure coexist and formulae have been proposed previously that consider both. In our research approach, we focus on the amount of compression displacement, which is the temporal difference between collapsing failure and buckling failure, and find that this difference changes the CS. For both collapsing failure and buckling failure, unless we consider either the amount of compression displacement at maximum load or the change in LCC due to the compression displacement, there is no formula that can accurately express panel behavior up to failure.

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