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# Weak Gravity Conjecture from Unitarity and Causality

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The weak gravity conjecture states that quantum gravity theories have to contain a charged state with a charge-to-mass ratio bigger than unity. By studying unitarity and causality constraints on higher derivative corrections to the charge-to-mass ratio of extremal back holes, we demonstrate that heavy extremal black holes can play the role of the required charged state under several assumptions. In particular, our argument is applicable when the higher-spin states Reggeizing graviton exchange are subdominant in the photon scattering. It covers (1) theories with light neutral bosons such as dilaton and moduli, and (2) UV completion where the photon and the graviton are accompanied by different sets of Regge states just like open string theory. Our result provides an existence proof of the weak gravity conjecture in a wide class of theories, including generic string theory setups with the dilaton or other moduli stabilized below the string scale.

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*Introduction.*—One of the greatest appeals of string theory is that it provides a consistent framework for constructing a variety of models for particle physics and cosmology while incorporating quantum gravity. The space of consistent string vacua is often known as the string landscape. This existence of a rich landscape does not however imply that anything goes. It has become increasingly clear that not every seemingly consistent quantum field theory (QFT) models can be consistently embedded into quantum gravity. Theories that are not ultraviolet (UV) completable when we turn on gravity are said to live in the swampland [1] (see also Ref. [2] for a review). Thus, identifying nontrivial ultraviolet constraints on QFTs can offer an interesting opportunity to probe the nature of quantum gravity phenomenologically.

Among the criteria that distinguish the landscape from the swampland, the weak gravity conjecture (WGC) [3] is arguably the most well-studied one. Its mild form states that quantum gravity theories have to contain *a charged state* with the charge-to-mass ratio $z$ bigger than unity. In $D=4$, this bound is given by

$$z=\frac{\sqrt{2}M_{\rm Pl}|q|}{m}\geq 1, \tag{1}$$



where $M_{\rm Pl}$ is the reduced Planck mass. This conjecture is motivated by black hole (BH) thought experiments and has passed various nontrivial checks in string theory examples [3]. Moreover, arguments based on holography [4–7], cosmic censorship [8–11], black holes and entropy consideration [9,12,13], dimensional reduction [14–18], and infrared consistency [19,20] have given further evidence for the conjecture. While these recent developments have significantly expanded our view of the WGC, it is fair to say that our understanding is still not complete and further studies toward a proof of the WGC are desired.

The purpose of this Letter is to provide an existence proof of the WGC in certain classes of theories, based on unitarity and causality. In particular we argue that even if there exists no particle satisfying the WGC bound [Eq. (1)], heavy extremal BHs play the role of the required charged state in the following two classes of theories: (1) theories with a parity-even light neutral scalar, such as dilaton and moduli, or a spin $s\geq 2$ light neutral particle [21]. Here "light" means lighter than the scale $\Lambda_{\rm QFT}$ where the quantum gravity effects come in and the QFT description breaks down. (2) UV completion where the photon and the graviton are accompanied by different sets of Regge states (just like open string theory), and those associated to the graviton are subdominant in the photon scattering. These two classes cover a wide variety of theories, including generic stringy setups, providing a strong evidence of the mild form of WGC. We focus on the $D=4$ case in this Letter, and relegate the extension to general spacetime dimension $D\geq 5$ to the Supplemental Material [22], which includes Refs. [23–32].

*Strategy.*—One might wonder whether our claim is trivial because the extremal charged BHs in the Einstein-Maxwell theory saturates the bound $z = 1$. However, it is not true because the BH solutions are modified by higher derivative corrections and so is the charge-to-mass ratio of extremal BHs accordingly [33].

Suppose that the theory is described by the photon and the graviton in the infrared. In $D = 4$ their general effective action up to four-derivative operators is as follows [34]:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_1}{4M_{\rm Pl}^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{\alpha_2}{4M_{\rm Pl}^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \frac{\alpha_3}{2M_{\rm Pl}^2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\right), \tag{2}$$

where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor and $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}/2$. Also we assumed parity invariance for simplicity. In general, we can add parity violating terms like $F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}\tilde{F}^{\rho\sigma}$, but they do not change the extremality condition at the leading order. Note that other four-derivative operators such as $R_{\mu\nu}^2$ are absorbed into the above three operators by field redefinition. The higher derivative operators modify black hole solutions, so that the charge-to-mass ratio of extremal BHs are corrected as [33]

$$z = \frac{\sqrt{2}M_{\rm Pl}|Q|}{M} = 1 + \frac{2}{5}\frac{(4\pi)^2}{Q^2}(2\alpha_1 - \alpha_3), \tag{3}$$

where $M$ and $Q$ are the mass and charge of the BH, respectively. This formula is applicable as long as the higher derivative corrections are small. More explicitly, it is applicable for sufficiently heavy BHs,

$$M^2 \sim Q^2M_{\rm Pl}^2 \gg \alpha_i M_{\rm Pl}^2, \tag{4}$$

because extremal BHs in the Einstein-Maxwell theory satisfy $R \sim M_{\rm Pl}^4/M^2$ and $F^2 \sim M_{\rm Pl}^6/M^2$.

An important observation made in Ref. [33] is that extremal BHs (in the mass range $M^2 \gg \alpha_i M_{\rm Pl}^2$) have the charge-to-mass ratio bigger than unity $z \geq 1$, *if the Wilson coefficients* $\alpha_i$ *satisfy the condition*,

$$2\alpha_1 - \alpha_3 \geq 0. \tag{5}$$

On the other hand, if $2\alpha_1 - \alpha_3 < 0$, the expectation is no longer valid that extremal BHs satisfy the WGC bound. In the following, we show that the bound [Eq. (5)] with a strict inequality indeed follows from unitarity and causality in the aforementioned two classes of setups.

*Unitarity constraints.*—We then summarize the unitarity constraints on the Wilson coefficients $\alpha_i$. For this purpose, let us clarify our setup by classifying possible sources of higher dimensional operators. Figure 1 shows a schematic picture of the particle contents we have in mind. First, we assume that the BH dynamics is controlled by photon and graviton in the infrared, and they are weakly coupled. We also assume a weakly coupled UV completion of gravity. There will be some high energy scale $\Lambda_{\rm QFT}$ where the ordinary QFT description breaks down. Generically, it is below the Planck scale $\Lambda_{\rm QFT} \ll M_{\rm Pl}$. For example, in string theory it is the string scale $\Lambda_{\rm QFT} \sim M_s$, beyond which we have to follow the dynamics of infinitely many local fields and thus the ordinary QFT description breaks down.

Below $\Lambda_{\rm QFT}$, there may exist massive particles, which we call light particles. Their contributions to higher dimensional operators are qualitatively different between the neutral and charged cases as we explain below.

(a) Light neutral bosons (ex. dilaton, axion, moduli): First, light neutral bosons may generate the effective interactions $\alpha_i$ at the tree-level. Let us consider, e.g., the dilaton $\phi$ and the axion $a$:

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m_\phi^2}{2}\phi^2 + \frac{\phi}{f_\phi}F_{\mu\nu}F^{\mu\nu}, \tag{6}$$

$$\mathcal{L}_a = -\frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{7}$$

where $m$ and $f$ are the mass and the decay constant, respectively. Integrating out the dilaton and axion, we obtain the tree-level effective couplings,

$$\alpha_1 = \frac{2M_{\rm Pl}^4}{m_\phi^2 f_\phi^2}, \qquad \alpha_2 = \frac{2M_{\rm Pl}^4}{m_a^2 f_a^2}. \tag{8}$$

More generally, the size of the effective couplings are estimated as

$$|\alpha_i| \gtrsim \mathcal{O}\left(\frac{M_{\rm Pl}^2}{m_i^2}\right), \tag{9}$$

which is indeed the case for the above examples if we assume $f \lesssim M_{\rm Pl}$. Also in the above examples, the Wilson coefficients enjoy positivity:

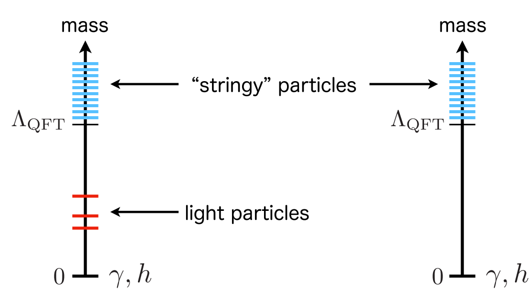


FIG. 1. A schematic picture of the particle spectrum: we assume that photon and graviton control the BH dynamics in the infrared. The massive spectrum below $\Lambda_{\rm QFT}$ may contain light particles (left) or may be empty (right).

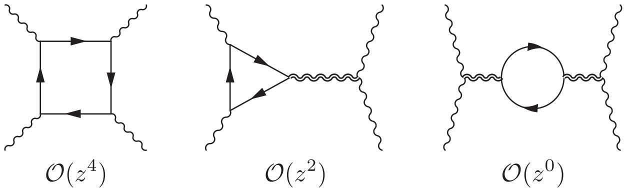


FIG. 2. Typical one-loop corrections to the $F^4$ terms: in the left figure the massive charged particle (solid line) induces four-point interactions of photon (wavy line) through the gauge coupling; hence it is proportional to $q^4 \propto z^4$. In the other two, the diagrams involve graviton (double wavy line).

$$\alpha_1 > 0, \qquad \alpha_2 > 0, \tag{10}$$

which is a consequence of unitarity. More generally, unitarity implies that $\alpha_1 > 0$ ($\alpha_2 > 0$) when photon is coupled to a parity-even (odd) neutral scalar or a spin $s \geq 2$ neutral particle with an arbitrary parity. See the Supplemental Material [22] for our derivation.

(b) Light charged bosons and fermions: In contrast to neutral bosons, charged bosons and fermions cannot generate the effective couplings $\alpha_i$ at the tree level; hence the leading contribution is at one loop. For example, the one-loop effective coupling generated by a minimally coupled massive charged particles are estimated as (cf., Fig. 2) [35].

$$\alpha_{1,2} = \max\{\mathcal{O}(z^4), \mathcal{O}(1)\}, \qquad \alpha_3 = \mathcal{O}(z^2), \tag{11}$$

where $z$ is the charge-to-mass ratio of the particle integrated out. Notice here that when the particle has a large charge-to-mass ratio $z \gg 1$, the Wilson coefficients enjoy $|\alpha_1|, |\alpha_2| \gg |\alpha_3| \gg 1$. Moreover, $\alpha_1$, $\alpha_2 > 0$ follows from unitarity for $z \gg 1$, where gravity is negligible compared to the electric force. On the other hand, we have $\alpha_i = \mathcal{O}(1)$ for $z \lesssim 1$. In this regime, as far as we know, no rigorous bound on $\alpha_i$ is known so far essentially because gravity is not negligible.

More generally, when the interaction between photon and the massive particle is stronger than the gravitational force, there exists the hierarchy $|\alpha_1|, |\alpha_2| \gg |\alpha_3|$ and the positivity of $\alpha_1$ and $\alpha_2$ follows from unitarity. If the two interactions are comparable, there is no known rigorous bound, but the induced effective interaction is very small $\alpha_i = \mathcal{O}(1)$ compared to other sources.

On top of these possible effects of light particles, there are higher derivative corrections from the UV completion of gravity, which we call the UV effects.

(c) UV effects: From the effective field theory (EFT) point of view, this effect is suppressed by the scale $\Lambda_{\rm QFT}$, where the quantum gravity effects come in and the ordinary QFT description breaks down. Generically, we have [36] the following:

$$\alpha_{1,2} = \mathcal{O}\left(\frac{M_{\rm Pl}^4}{\Lambda_{\rm QFT}^4}\right), \qquad \alpha_3 = \mathcal{O}\left(\frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2}\right), \tag{12}$$

which corresponds, e.g., to the $\alpha'$ corrections in string theory. In general it is difficult to fix the sign of this effect within the EFT framework without knowing the details of the UV completion of gravity. However, as we discuss in the Supplemental Material [22], $\alpha_1 > 0$ and $\alpha_2 > 0$ follow from unitarity as long as the higher-spin states Reggeizing graviton exchange are subdominant in the photon scattering. This may happen, e.g., when the photon and the graviton are accompanied by different sets of Regge states, just as in open string theory. The magnitude of the three effects (a)–(c) and the unitarity constraints on them are summarized in Table I. In particular, the loop effect (b) may be further classified into two, (b-1) and (b-2), by the size of interactions between the photon and the massive particle.

*WGC from unitarity.*—We now discuss implications on the WGC. See also Fig. 3 for a summary. One easy observation is that the inequality [Eq. (5)] is satisfied when the effect (b-1) dominates over the others because its contribution to the lhs of Eq. (5) is always positive. This is the case, e.g., when there exists a massive charged particle with $z \gg 1$. Since this particle trivially satisfies the WGC bound, this situation is not what we would like to explore [37]: we are interested in whether extremal BHs may play the role of the charged state required by the WGC in case there are no particles with $z \geq 1$. Also, the effect (b-2) is always subleading at least as long as $\Lambda_{\rm QFT} \lesssim M_{\rm Pl}$. Therefore, in nontrivial setups for our question, the loop effect (b) from light particles is always subleading.

Let us then focus on the tree-level effects (a) and (c) in the following. As we explained, $\alpha_1$ and $\alpha_2$ are well

TABLE I. Sources of higher derivative operators: The tree-level effect (a) from neutral bosons and the loop effect (b-1) give a positive contribution to $\alpha_1$ and $\alpha_2$ (if any). The same bounds apply to the UV effects (c) if the Regge states associated to the graviton are subdominant in photon scattering.

| | Magnitude | Unitarity |
|---|---|---|
| (a) Neutral bosons | $\lvert\alpha_i\rvert \gtrsim \mathcal{O}(M_{\rm Pl}^2/m^2)$ | $\alpha_1$, $\alpha_2 > 0$ |
| (b) Loop effects | | |
| (b-1) $z \gg 1$ | $\lvert\alpha_1\rvert, \lvert\alpha_2\rvert \gg \lvert\alpha_3\rvert \gg 1$ | $\alpha_1$, $\alpha_2 > 0$ |
| (b-2) $z = \mathcal{O}(1)$ | $\alpha_i = \mathcal{O}(1)$ | N.A. |
| (c) UV effects | $\alpha_{1,2} = \mathcal{O}(M_{\rm Pl}^4/\Lambda_{\rm QFT}^4)\alpha_3 = \mathcal{O}(M_{\rm Pl}^2/\Lambda_{\rm QFT}^2)$ | $\alpha_1$, $\alpha_2 > 0(\star)$ |

constrained by unitarity, but no rigorous bound on $\alpha_3$ is known so far. Since the inequality [Eq. (5)] involves $\alpha_3$, one might give up deriving it from unitarity. However, it is useful to recall that the $\alpha_3$ operator is significantly constrained by causality.

Causality constraints:The key is that $\alpha_3$ generates new photon-photon-graviton helicity amplitudes which do not exist in the Einstein-Maxwell theory. The photon-photon-graviton amplitudes in the setup [Eq. (2)] are schematically given by

$$\mathcal{M}(1^+,2^-,3^{\pm 2}) = \mathcal{M}(1^-,2^+,3^{\pm 2}) \sim \frac{E^2}{M_{\rm Pl}},$$
$$\mathcal{M}(1^+,2^+,3^{+2}) = \mathcal{M}(1^-,2^-,3^{-2}) \sim \alpha_3 \frac{E^4}{M_{\rm Pl}^3},$$
$$(\text{other helicity amplitudes}) = 0, \tag{13}$$

where $\mathcal{M}(1^+,2^+,3^{+2})$ stands for the scattering amplitude of two helicity plus photons and one helicity plus graviton (in the all incoming notation), for example. Also $E$ is a typical energy scale.

In Ref. [38], an interesting observation was made that the new helicity amplitudes lead to causality violation at the energy scale $E \sim M_{\rm Pl}/\alpha_3^{1/2}$, so that this scale has to be beyond the EFT cutoff. Moreover, it was argued that an infinite tower of massive higher-spin particles (just like string theory!) with the lightest particle at the scale $m \sim M_{\rm Pl}/\alpha_3^{1/2}$ is required to UV complete the EFT at the tree-level without causality violation (see also Refs. [39,40] for a holographic derivation based on the conformal bootstrap approach). In other words, the ordinary QFT description with a finite field content is not available beyond the scale $\sim M_{\rm Pl}/\alpha_3^{1/2}$, hence $\Lambda_{\rm QFT} \sim M_{\rm Pl}/\alpha_3^{1/2}$. Therefore, when $\alpha_3$ is nonzero, it is tightly bounded by the scale of gravitational Regge states as

$$\text{tree level}: \ |\alpha_3| \sim \frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2}. \tag{14}$$

Case (1): theories with light neutral bosons:We now find that in theories with light neutral bosons, the effect (a) dominates over the others and the Wilson coefficients enjoy [41] the following:

$$|\alpha_1|, |\alpha_2| \gg |\alpha_3| \tag{15}$$

as a consequence of causality. Since the effect (a) gives a positive contribution to $\alpha_1$ as a consequence of unitarity, the inequality [Eq. (5)] and thus the mild form of the WGC are satisfied. Recall that we need a parity-even neutral scalar or a spin $s \geq 2$ neutral particle to have nonzero $\alpha_1$. We therefore conclude that the mild form of WGC is satisfied by heavy extremal BHs even if there are no charged particles with $z \geq 1$, as long as the photon is coupled to a parity-even neutral scalar or a spin $s \geq 2$ neutral particle with a mass $m \ll \Lambda_{\rm QFT}$. The dilaton and moduli may play the role of this neutral particle (as long as they are not too heavy); hence this scenario is quite generic.

We also remark that our findings match well with the expectation from open-closed string duality [42]. In string theory, charged particles are generically associated to open strings. If their charge-to-mass ratios do not satisfy the WGC bound $z < 1$, the open string has to be long such that its lowest mode is heavy enough to make $z$ small. In this regime, it is more appropriate to interpret the open string loop as a tree-level exchange of closed strings, which naturally gives the tree-level effect (a) from light neutral particles such as dilaton and moduli.

Case (2): open string type UV completion:Then, what is the case without light neutral bosons? As mentioned, it is possible to give rigorous bounds on $\alpha_{1,2}$ if the photon and the graviton are accompanied by different sets of Regge states. As an illustrative example, let us consider open string theory: the Regge states associated to the photon and the graviton are the open and closed string states, respectively. Since the open string coupling $g_o$ is parametrically bigger than the closed string coupling $g_s$, $g_o \sim g_s^{1/2} \gg g_s$, the closed string effects are subdominant in the photon scattering. In particular, each sector contributes to the $F^4$ operators as [43] this:

$$[\alpha_{1,2}]_{\rm open} \sim \frac{M_{\rm Pl}^2}{g_s M_s^2}, \qquad [\alpha_{1,2}]_{\rm closed} \sim \frac{M_{\rm Pl}^2}{M_s^2}, \tag{16}$$

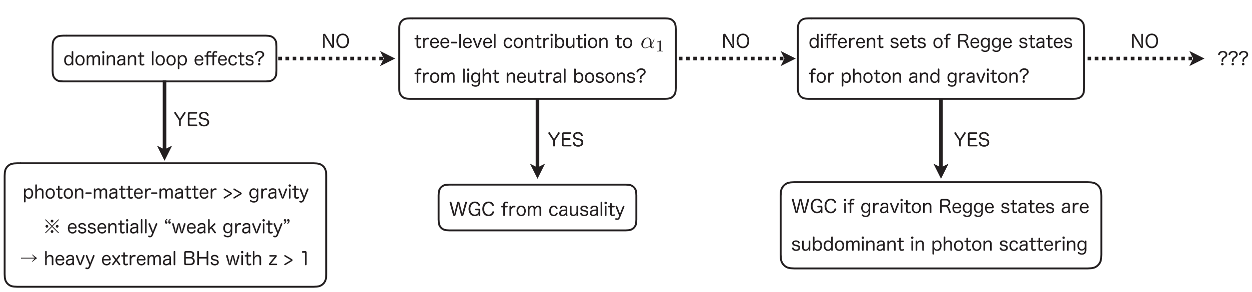


FIG. 3. A flow chart for our derivation of the WGC from unitarity: each step explains which conditions are necessary besides unitarity to show that heavy extremal BHs have the charge-to-mass ratio $z > 1$ and thus the mild form of WGC is satisfied.

and then unitarity implies

$$\alpha_{1,2} \simeq [\alpha_{1,2}]_{\rm open} > 0. \tag{17}$$

As an example, the positivity of $\alpha_1$ can explicitly be seen in the photon scattering of type-I superstring, where infinitely many higher-spin open string states contribute to the effective coupling $\alpha_{1,2}$ (see also the Supplemental Material [22]). Also recall that the graviton has to be accompanied by an infinite tower of higher-spin particles, i.e., the Regge states, with the mass scale $m \sim M_{\rm Pl}/\alpha_3^{1/2}$ if $\alpha_3$ is nonzero. Indeed, in the bosonic string we have,

$$\text{bosonic string:}\ \alpha_3 \sim \frac{M_{\rm Pl}^2}{M_s^2}. \tag{18}$$

Note that $\alpha_3$ is prohibited in $\mathcal{N} \geq 1$ supersymmetric (SUSY) theories because it generates the helicity amplitudes $\mathcal{M}(1^+, 2^+, 3^{+2})$ and $\mathcal{M}(1^-, 2^-, 3^{-2})$ incompatible with the SUSY Ward-Takahashi identity (see, e.g., Ref. [44]):

$$\text{SUSY:}\ \alpha_3 = 0. \tag{19}$$

Therefore, both in SUSY and non-SUSY cases, $\alpha_3$ is suppressed compared with the open string contributions to $\alpha_1$. Clearly, we have

$$\alpha_1 + \frac{1}{2}\alpha_3 \simeq [\alpha_1]_{\rm open} > 0. \tag{20}$$

More generally, the mass scale of the Regge states associated to the graviton is specified by the value of $\alpha_3$ (if nonzero) and their contribution to $\alpha_{1,2}$ is of the same order. If the photon is accompanied by another set of Regge states and these effects are dominant in the photon scattering, unitarity implies the inequality [Eq. (5)] and thus the mild form of the WGC is satisfied.

*Conclusion.*—In this Letter, based on unitarity and causality, we demonstrated that heavy extremal BHs have the charge-to-mass ratio bigger than unity $z > 1$ under some assumptions. The coverage of our argument is summarized by the flow chart in Fig. 3. This provides an existence proof of the mild form of WGC in a wide class of theories, including generic stringy setups with dilaton or moduli stabilized below the string scale.

As a concluding remark, we present several promising future directions. First, while our proof already has wide applicabilities, it would be desirable to relax further the assumptions in the present Letter. A nontrivial example for the UV completion not covered by our argument is the heterotic superstring (with the stabilization scale $\gtrsim M_s$): since both the photon and the graviton are from the closed string, we cannot directly apply our unitarity argument for $\alpha_1 > 0$ and $\alpha_2 > 0$. Nevertheless, from the explicit calculation of scattering amplitudes [45], we know that $\alpha_1$ and $\alpha_2$ are positive. Also, $\alpha_3 = 0$ because of SUSY. Hence, the inequality [Eq. (5)] is satisfied. Here we would like to remark that this observation is applicable as long as the tree-level scattering accommodates the same structure. For example, it is applicable to the heterotic superstring without spacetime SUSY [46], where spacetime SUSY is broken by an unconventional GSO projection, but the tree-level vertices of the bosonic sector are the same as the ordinary $E_8 \times E_8$ heterotic superstring. It would be interesting to find out how the UV consistencies of string theory lead to the right sign.

Another important direction is to extend our argument to other swampland conjectures. For example, it was conjectured in Ref. [47] that any nonsupersymmetric AdS vacuum must be unstable (see Refs. [48–51] for the application to the particle physics). Since the near-horizon geometry of an extremal BH is AdS, this conjecture is well motivated by our result showing that a decay process of extremal BHs is kinematically allowed. Further studies in this direction will be encouraged. We believe that our Letter provides a foundation for such future studies in the swampland program and for deepening our understanding of the quantum gravity landscape.

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[21] This is the same setup considered in Ref. [13] to motivate the WGC from an entropy perspective. However, in the Supplemental Material [22], we point out a loophole in their argument. Also, as we shall see, our unitarity argument can put stronger constraints in this setup and furthermore has wider applicabilities.
[22] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.051601 for derivation of the unitarity bounds on the Wilson coefficients (Sec. I), extension of our argument to general spacetime dimension (Sec.II), implications from unitarity and causality on BH entropy corrections (Sec. III) and comments on Ref. [13] (Sec. IV).
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[34] Even though we consider a single $U(1)$ for simplicity, generalization to the multiple $U(1)$ case is straightforward. In particular, it trivially follows from our result that there exists heavy extremal BHs with $z > 1$ in any charge direction under our assumptions.
[35] The running of coupling constants are included in the $\mathcal{O}(1)$ effect, which is valid in the perturbative regime.
[36] One would expect a hierarchy $|\alpha_1|, |\alpha_2| \gg |\alpha_3|$, but it is not a general statement. In this estimate, we assumed that there is a single scale $\Lambda_{\rm QFT}$ and other dimensionless constants are $\mathcal{O}(1)$, which corresponds to assuming $m \sim f$ in Eqs. (6)–(7). The estimate changes, e.g., when $m \ll f \sim M_{\rm Pl}$. As we shall see, another ingredient such as causality or symmetry is necessary to have $|\alpha_1|, |\alpha_2| \gg |\alpha_3|$ in general. Our point here is simply that the Wilson coefficients $\alpha_i$ are suppressed by $\Lambda_{\rm QFT}$.
[37] Note that extremal BHs satisfy the WGC bound even in this situation.
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[41] In contrast to $\alpha_{1,2}$, $\alpha_3$ has a cubic coupling of two photons and a graviton, so that it can be thought of as a fundamental vertex which cannot be generated by tree-level exchange. The role of the causality bound [Eq. (14)] here is to guarantee that $\alpha_3$ is subdominant to the effect (a) of light neutral particles generating $\alpha_{1,2}$.
[42] We thank Cumrun Vafa for sharing this observation with us.
[43] Here we assumed that the scale of compactification and volume of the cycles on which the brane wrapped are $\mathcal{O}(M_s)$. The same hierarchy is expected to hold as long as there is no unusual hierarchy between them.
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