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Freemium Competition Among Ad-Sponsored Platforms*

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Abstract

This paper studies competition between ad-sponsored platforms that strategically determine business models. In addition to basic services including annoying advertisements, each platform decides whether to introduce an ad-free premium service (i.e., a freemium business model). Freemium platforms encounter a trade-off between increasing the number of premium users for the subscription-based revenues and increasing the number of basic users for the ad-sponsored revenues. I characterize how the freemium platforms should segment their users into basic and premium services. Moreover, I show that the equilibrium business model choice depends on the extent of fixed costs for introducing a premium service. When the fixed cost is at an intermediate level, asymmetric equilibria may arise: i.e., only one platform introduces the premium service. Competing platforms may have an incentive to coordinate their choices toward asymmetric market structures; however, these structures can be harmful to both consumers and advertisers.

Keywords: *freemium, two-sided markets, indirect network externalities, advertising*

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1 Introduction

Many ad-sponsored platforms, such as Spotify, Deezer, Pandora, YouTube, Flickr, and many smartphone applications, offer basic (or free) services including annoying advertisements as well as ad-free premium services.[1] For example, Spotify is a music-streaming service providing both free and premium services. In the free service, some advertisements are inserted between songs, but if users upgrade to the premium service by paying \$9.99/month, they can eliminate these annoying advertisements. On October 28, 2015, Google introduced a new ad-free membership named YouTube Red, which allows users to enjoy all YouTube videos without advertisements for \$9.99/month. The above business model can be considered a form of *freemium*.

In reality, not all ad-sponsored platforms adopt the freemium model. For instance, many mobile applications rely on revenue from in-app advertisements.[2] The video-streaming platform, Dailymotion, is largely financed by ad revenue. A possible factor determining the business model is the extent of fixed costs for introducing an ad-free paid service additionally. For example, when introducing an ad-free premium service, a computer program/system must be built to prevent any advertisements from being displayed to their premium users. In another instance, if basic services are provided for free, platforms must newly consolidate a payment system for their paid users. These initial investment costs influence the platforms' decision on whether to introduce ad-free premium services. Many mobile applications are developed by small firms (or even individuals) that appear to have insufficient funds to prepare for their initial fixed costs,[3] whereas streaming services are typically operated by relatively large companies that are well equipped to incur the related fixed costs.

From the perspective of strategic relationships among competing platforms, however, it is not clear if all platforms introduce premium services even when the fixed costs are not too high. If all of them adopt the freemium business model, fierce competition may take place not only between their basic services but also between their premium services. In contrast, different business model

[1] Wilbur (2008) empirically shows that the typical viewer experiences disutility in the presence of advertising. Ghose and Han (2014) also find that the demand for mobile applications decreases with the presence of an in-app advertisement option.

[2] According to "Distribution of worldwide mobile application revenues in 2017, by channel" (https://www.statista.com/statistics/273120/share-of-worldwide-mobile-app-revenues-by-channel/), in mobile application markets, ad revenue accounts for more than 50% share of the total revenue.

[3] According to "Top Mobile App Development Companies" (https://clutch.co/directory/mobile-application-developers), e.g., 5,153 out of 15,397 firms have less than 10 employees. Last visited October 20, 2019.

choices may serve to ease the intense competition by inducing the competing platforms to target different user segments. Therefore, the purpose of this paper is to examine the strategic decision of ad-sponsored platforms on whether to adopt the freemium model or not. In addition, I also address whether the resulting market structure in equilibrium is beneficial to consumers and advertisers.

To this end, I develop a model of competition between two ad-sponsored platforms, which contains the following three features: (a) users are heterogeneous with respect to their utility losses from advertising; (b) platforms can provide both an ad-sponsored service (i.e., basic service) and an ad-free service (i.e., premium service) simultaneously; and (c) platforms endogenously determine whether to introduce the premium services or not.

Feature (a) creates users' endogenous decision-making for the basic or premium service. That is, if a platform decides to use the freemium model (i.e., features [b] and [c]), there exists an interesting trade-off between increasing the number of premium users for subscription-based revenues and increasing the number of basic users for ad-sponsored revenues, which is crucially related to the strategy on how to segment users into basic and premium services.

In this regard, this paper provides the characterization of the equilibrium strategy of freemium platforms. How to segment users into basic and premium services significantly depends on the distribution of users' disutility from advertisements. In equilibrium, the price discount for the basic service is set to be equal to the advertising revenue that the platform obtains from a unit of basic users. That is, the advertising revenues are fully utilized to discount the price of the basic service. Therefore, the equilibrium price of the basic service can take a negative value (i.e., *freemium*) if the platform can gain a lot from advertising revenues. This is typical of two-sided pricing when indirect network externalities across two sides are divergent.[4]

Moreover, I demonstrate the equilibrium business model choices. Adopting the freemium model is the dominant strategy when the fixed cost of introducing a premium service is small enough. Interestingly, when the fixed cost is at an intermediate level, asymmetric equilibria can arise, in which only one platform introduces the premium service. The asymmetric market structures enable the competing platforms to target different user segments than each other, which mitigates

[4]In media markets, especially, consumers are more likely to be subsidized because advertisements are a nuisance to consumers. In this context, Anderson and Peitz (2017) use the aggregative game approach to show that advertiser and consumer interests are opposed (they call this a media seesaw) when consumers dislike advertising and media platforms are fully ad-financed. It is also interesting to reexamine the media seesaw principle in the framework of freemium media platforms.

the platform competition. The freemium platform targets two polar user segments who are either tolerant or allergic to advertisements, whereas the ad-sponsored platform effectively deals with the remaining users with midrange preferences for advertisements.

From a different viewpoint, the lower competition under asymmetric market structures incentivizes the platforms to coordinately choose different business models with a monetary transfer that satisfies the incentive compatibility constraints for them. However, I also find that such coordination is detrimental to consumers and advertisers. These findings indicate the important caveat that anticompetitive coordination may emerge.

2 Related Literature

This paper is related to the literature on two-sided markets, in which platforms facilitate interactions between two groups of users (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2009; Weyl, 2010). A distinctive feature of two-sided markets is the existence of indirect network externalities, which can be positive in the examples of video-game consoles, credit cards, and shopping malls, where an increase in the number of users on one side improves the willingness to pay on the other side, and vice versa. In contrast, negative indirect network externalities can be experienced in media platforms, where subscribers feel annoyed by advertisements while advertisers are attracted to platforms with large numbers of subscribers.

Numerous studies explore ad-sponsored platforms such as television, radio, magazines, and newspapers (e.g., Anderson and Coate, 2005; Armstrong, 2006; Crampes et al., 2009; Godes et al., 2009; Reisinger, 2012; von Ehrlich and Greiner, 2013). Choi (2006) and Peitz and Valletti (2008) compare two different symmetric regimes: i.e., (i) two competing platforms are free-to-air broadcasters or (ii) they are both pay-TV channels. Choi (2006) investigates broadcast competition with free entry under both regimes (i) and (ii), and then compares those outcomes with the socially optimal one. Peitz and Valletti (2008) study the location choices of broadcasters in a Hotelling linear market with single-homing consumers.[5] They show that when the disutility consumers experience from advertisements is relatively high, platforms provide advertising-free programs. However, these

[5] By contrast, Anderson et al. (2016) consider multihoming media consumers in investigating the consequences of relaxing the assumption of single-homing consumers. The present paper assumes single-homing consumers, although some recent studies emphasize that accounting for multihoming substantially changes the nature of competition (e.g., Ambrus et al., 2016; Athey et al., 2018), which is beyond the scope of this paper.

two papers consider only symmetric platform competition.

Dietl et al. (2013) investigate asymmetric competition between pay-TV and free-to-air TV companies. More recently, Calvano and Polo (2019) also focus on asymmetric business models with a more general setting, in which competing broadcasting stations endogenously determine both the amount of advertising and the subscription fee. They show that, even if viewers (and advertisers) are assumed to be homogeneous, there exist asymmetric equilibria where one station has more advertisements and has a lower subscription price. However, these studies assume that each platform provides only one service.

Thomes (2013) considers a model where a monopoly platform provides *both* free-of-charge and flat-rate services. He shows that free-of-charge services are more likely to be adopted if users are highly tolerant of advertisements and that the monopolist launches both services when the parameter for nuisance costs from advertisements is at an intermediate level. In his model, the nuisance cost parameter is assumed to be identical for all users, as in Choi (2006), Peitz and Valletti (2008), and Dietl et al. (2013). Unlike those existing studies, the present paper assumes that users are heterogeneous with respect to their utility losses from advertising, which creates the users' endogenous decisions for the basic or premium services. In addition, this paper also addresses media platform competition.

Prasad et al. (2003) and Tåg (2009) incorporate the heterogeneity of users' disutility from advertisements, as with this paper. In particular, they examine the incentive for a monopoly firm to introduce an option that allows consumers to pay to remove advertisements (i.e., premium services), in addition to the ad-sponsored basic service.[6] They show that the introduction of such a payment option would be profitable when consumers receive sufficiently high disutility from advertisements. However, they address only a monopoly platform model. In addition, users' types for nuisance from advertisements are assumed to follow a uniform distribution function. By contrast, in this paper, I examine a platform competition model where each platform endogenously determines its business model. Furthermore, the current paper uses general distribution functions to represent users' types for disutility from advertisements.

[6]Carroni and Paolini (2017) also consider the heterogeneity of users' disutility. However, the focus of their paper is very different. They study a three-sided market among copyright owners (e.g., artists), advertisers, and users. They focus on the monopolistic platform's decision between freemium and premium models. They find a possible conflict between platform and copyright owners such that, even if copyright owners strictly prefer the premium model, the platform would find it optimal to choose the freemium model.

The closest paper is Sato (2019) who develops a model of menu pricing to consumers by advertising platforms, which is also related to the literature on second-degree price discrimination in two-sided markets (e.g., Choi et al., 2015; Jeon et al., 2016). He shows that, even though a monopoly platform can choose any menu of price-advertisement pairs, the optimal monopoly menu pricing consists of only two services: i.e., basic ad-supported and premium ad-free services. Sato (2019) also confirms the robustness of his main result on freemium as optimal menu pricing under duopoly platform competition. However, asymmetric market structures are not examined, unlike in the present paper.

3 Model

3.1 Platforms

I consider two competing ad-sponsored platforms ($i = 1, 2$). Each platform provides a basic service. The basic service includes some advertisements, which can create disutility for users. At the same time, platforms can earn revenue from such advertisements.

In addition, each platform has an option to introduce a premium service that includes no advertisement. Thus, they can also gain revenue from usage fees for the premium service. I assume that a platform incurs an additional fixed cost of Φ to introduce the premium service.[7] This additional fixed cost can be interpreted in several ways. For example, when introducing an ad-free premium service, platforms need to build a computer program/system that prevents any advertisements from being displayed to their premium users. In addition, if basic services are provided for free, platforms must newly consolidate a payment system for users. A fixed cost of Φ is necessary for those prior investments.

Let s_i be the strategy of platform i about whether to introduce the premium service or not.[8] I use $s_i = FP$ when platform i introduces the premium service (where FP means *freemium platform*) and $s_i = ASP$ when not introducing the premium service (where ASP means *ad-sponsored platform*).

[7] One can consider that a platform incurs a fixed cost of Φ_B to introduce the basic service only and incurs a fixed cost of $\Phi_B + \Phi_P$ to introduce both basic and premium services. For simplicity, without loss of generality, I denote the difference of the fixed costs by Φ.

[8] This paper implicitly assumes that platforms cannot employ a fully subscription-based business model. Apple Music and Amazon Prime Video are examples of the fully subscription-based model. However, when those services started, their parent companies were already big multiproduct firms with multiple revenue sources and huge management vitalities. The main focus of this paper is on the competition among ad-sponsored platforms.

Therefore, profit function of platform i is given by

$$\pi_i = \begin{cases} p_i^P \cdot d_i^P + p_i^B \cdot d_i^B + r_i \cdot a_i - \Phi & \text{if } s_i = FP, \\ p_i^B \cdot d_i^B + r_i \cdot a_i & \text{if } s_i = ASP, \end{cases} \tag{1}$$

where p_i^B and p_i^P denote the prices for the basic and premium services of platform i, d_i^B and d_i^P denote the number of users for each service of platform i, r_i represents the lump-sum fee for running an advertisement on platform i, and a_i is the number of advertisements on platform i.

3.2 Users

I assume that users enjoy the stand-alone benefit v from joining a platform, which is large enough for every consumer to join a platform. The benefits that a user derives from basic and premium services of platform i are respectively given by $u_i^B = v - p_i^B - \delta a_i$ and $u_i^P = v - p_i^P$, where $\delta \in [0, \Delta]$ represents the disutility from advertisements.[9] In this paper, users are assumed to be heterogeneous with respect to this disutility. Users' type δ follows cumulative distribution function $F(\delta)$ with a continuously differentiable positive density function $f(\delta)$. I use δ_μ to denote the expectation of δ, that is, $\delta_\mu \equiv \int_0^\Delta \delta f(\delta) d\delta$.

To analyze the users' choices of platform, I use a Hotelling model of product differentiation. Users have heterogeneous tastes for each platform, denoted by $x \in [0, 1]$, which are distributed uniformly on the unit interval with a unit density. Platforms 1 and 2 are differentiated along with a unit interval $[0, 1]$, with platform 1 located at 0 and platform 2 at 1. Each user incurs a constant proportional disutility per unit length, denoted by t.

In total, utility functions of users who enjoy the basic and premium services of platform i are given as follows.

$$\begin{aligned} U_1^P(x,\delta) &= v - p_1^P - tx & \qquad U_2^P(x,\delta) &= v - p_2^P - t(1-x) \\ U_1^B(x,\delta) &= v - p_1^B - \delta a_1 - tx & \qquad U_2^B(x,\delta) &= v - p_2^B - \delta a_2 - t(1-x) \end{aligned} \tag{2}$$

[9] This type of utility function of users is used in Dietl et al. (2013). Here, the stand-alone benefit v is assumed to be the same for both the basic and premium services. In reality, however, a quality gap may exist between the basic and premium services. In Section 5.2, I explore how an exogenous quality difference affects the strategies of platforms.

Every user selects one of the above four possible options that generates the largest utility.[10]

3.3 Advertisers

I adopt the approach used in Peitz and Valletti (2008). Advertisers are assumed to be heterogeneous with respect to the benefit derived from a unit of basic users, denoted by $\alpha \in [0, A]$. The profit of an advertiser of type α that places its advertisement on platform i is given by $z_i(\alpha) = \alpha d_i^B - r_i$.[11] The type of advertisers is distributed according to a cumulative distribution function $H(\alpha)$ with a continuously differentiable positive density $h(\alpha)$. I assume that advertisers can advertise on both platforms (i.e., multihoming) if it is profitable.[12] That is, the advertisers run their advertisements on platform i if their type α is large enough to satisfy $\alpha > r_i/d_i^B$. Thus, the number of advertisements is given by $a_i = 1 - H\left(r_i/d_i^B\right)$. I derive the inverse demand function of advertisers on platform i as $r_i = H^{-1}(1 - a_i)d_i^B$.

3.4 Timing of the game

I consider the following three-stage game:

1. Each platform chooses between two business models: freemium model ($s_i = FP$) and ad-sponsored model (i.e., $s_i = ASP$). When choosing the freemium business model, the platform incurs an additional fixed cost of Φ.

2. Platforms select their pricing strategies. If platform i did not introduce the premium service (i.e., $s_i = ASP$), it chooses the amount of advertisements a_i and the price for its basic service

[10] In this paper, users are assumed to subscribe to either platform (i.e., single-homing). In practice, however, consumers' multihoming behaviors are observed in several platforms. It would be a valuable direction for future research to incorporate multihoming into the model because some recent studies emphasize that accounting for multihoming substantially changes the nature of competition (e.g., Ambrus et al., 2016; Anderson et al., 2016; Athey et al., 2018).

[11] The revenue of each advertiser depends heavily on the number of basic users. In this paper, all basic users are assumed to generate the same *average revenue per user*, regardless of the differences in their degree of dislike for advertisements. I provide the following economic explanation behind this assumption. Users may encounter both valuable and nuisance advertisements. It is natural to consider whether each user's degree of dislike for advertisements is independent of the probability of viewing valuable advertisements that make the users purchase the goods of the advertisers. Therefore, I assume that all basic users generate the same revenue for all advertisers.

[12] It is commonly observed in the real world that advertisers often place their advertisements on several media platforms. Therefore, this assumption is not unrealistic. Indeed, this assumption has been employed often in recent theoretical studies on two-sided markets (e.g., Hagiu, 2009; Rasch and Wenzel, 2014; Maruyama and Zennyo, 2015). However, I can consider the case in which advertisers negotiate with each platform independently. Further analysis of the bargaining between platforms and advertisers remains a task for future work.

p_i^B. Otherwise, if platform i introduced the premium service (i.e., $s_i = FP$), it chooses the price for premium service p_i^P, as well as a_i and p_i^B.

3. Advertisers choose whether to run their advertisements on each platform. At the same time, every user decides which service as well as which platform to join.

Note that all proofs in the Appendix are formally obtained under the above timing with users' *simultaneous* decisions: Every user determines which service as well as which platform to join *simultaneously* in Stage 3.

For expositional brevity, in the next section, only when both platforms choose the same business model (i.e., Sections 4.1 and 4.2), I will consider the following alternative timing with users' *sequential* decisions: At the beginning of Stage 3, users do not know their own type of disutility δ before choosing the platform they use. At this time, they only know that their types will be realized according to a cumulative distribution function $F(\delta)$ with a continuously differentiable positive density $f(\delta)$ after their platform choices. Based on this expectation, they first determine which platform to join, and then start to use the basic service of the platform they chose. After using it for a while, each user learns his/her own type δ. Finally, depending on the realized value of δ, each user decides whether to upgrade to the premium service of the platform.[13]

It is worth emphasizing that whether simultaneous or sequential timing does not alter the platforms' strategies in Stage 2 when the same business model was chosen in Stage 1. The analysis with sequential timing yields clear and neat expositions, which make it easier for us to understand the intuition for each finding. On the contrary, however, when competing platforms choose different business models in Stage 1, the results depend on which timing is adopted. For example, let me consider a case in which platform 1 provides the ad-sponsored basic service only, whereas platform 2 chooses the freemium business model. In this case, under the *sequential* timing setting, even if a user who has joined platform 1 realizes that his/her type δ is very high, he/she cannot switch to the premium service of platform 2. However, he/she must be able to do so in reality. To capture this kind of switching behavior well, *simultaneous* timing is more appropriate. This is one of the

[13] This *sequential* timing is not necessarily unrealistic assumption. For instance, most people may start by enjoying the basic services of YouTube for free. Users learn how valuable or annoying the advertisements are when an advertisement suddenly appears before the video. Therefore, it is quite a natural assumption that users who have never watched YouTube do not know how valuable or annoying the advertisements are. The same is equally true of examples other than YouTube, including smartphone applications.

reasons why I use *simultaneous* timing in the formal proofs in the Appendix.

4 Equilibrium

In this section, I derive the subgame-perfect Nash equilibrium by backward induction. Depending on the platforms' choice in Stage 1, there are four possible subgames: (i) neither platform introduces the premium service; (ii) both platforms introduce the premium services; (iii) only platform 1 introduces the premium service while platform 2 does not; and (iv) only platform 2 introduces the premium service while platform 1 does not. Because cases (iii) and (iv) are equivalent, I first obtain the equilibrium of each subgame: (i), (ii), and (iii). After that, by comparing them, I derive the subgame-perfect Nash equilibrium.

4.1 Neither platform introduces premium service

Consider the case of $(s_1, s_2) = (ASP, ASP)$. As discussed above, I use the *sequential* timing for the sake of readability. See the Appendix for the formal proof.

Users determine which platform to join based on their expected utilities. A type of user who is indifferent between joining platforms 1 and 2 is as follows.

$$E[u_1] = E[u_2] \iff v - p_1^B - \delta_\mu a_1 - tx = v - p_2^B - \delta_\mu a_2 - t(1-x) \tag{3}$$

$$\iff x = \frac{1}{2} - \frac{(p_1 + \delta_\mu a_1) - (p_2 + \delta_\mu a_2)}{2t} \tag{4}$$

Therefore, the demand of platform i is written by

$$d_i^B = \frac{1}{2} - \frac{(p_i^B + \delta_\mu a_i) - (p_j^B + \delta_\mu a_j)}{2t} \tag{5}$$

I can derive the profit of platform i as follows.

$$\pi_i = p_i^B d_i^B + r_i a_i = p_i^B d_i^B + H^{-1}(1-a_i) d_i^B \cdot a_i \tag{6}$$

I denote by $\rho(a_i) \equiv a_i H^{-1}(1-a_i)$ the advertising revenue per user, as in Peitz and Valletti (2008). For an interior solution, I assume that $\rho''(a) < 0$ and $\lim_{a \to +0} \rho'(a) > \delta_\mu$. Thus, the profit of platform i is rewritten by $\pi_i = \left\{p_i^B + \rho(a_i)\right\} d_i^B$.

The first-order conditions with respect to p_i^B and a_i are characterized by

$$\frac{\partial \pi_i}{\partial p_i^B} = d_i^B + \left\{p_i^B + \rho(a_i)\right\} \frac{\partial d_i^B}{\partial p_i^B} = 0, \tag{7}$$

$$\frac{\partial \pi_i}{\partial a_i} = \rho'(a_i) \cdot d_i^B + \left\{p_i^B + \rho(a_i)\right\} \frac{\partial d_i^B}{\partial a_i} = 0, \tag{8}$$

where $\frac{\partial d_i^B}{\partial p_i^B} = -\frac{1}{2t}$ and $\frac{\partial d_i^B}{\partial a_i} = -\frac{\delta_\mu}{2t}$. Note that equations (7) and (8) are the same as equations (5) and (6) in Peitz and Valletti (2008), respectively. Solving those first-order conditions, I obtain the qualitatively same result with theirs.

Proposition 1 (Peitz and Valletti (2008))**.** *When neither platform introduces the premium services, in equilibrium, the amount of advertisements, a^*, and the price for basic services, p_B^*, are determined by the following system:*

$$\begin{cases} \rho'(a^*) = \delta_\mu \\ p_B^* = t - \rho(a^*) \end{cases} \tag{9}$$

Then, the equilibrium profit is $\pi^* = t/2$.

I use superscript '$*$' to denote the equilibrium outcome when neither platform introduces the premium service. The total profit of platforms, consumer surplus, and advertiser surplus are computed as follows.

$$\Pi^* = 2\pi^* = t \tag{10}$$

$$CS^* = v - \frac{5}{4}t + \rho(a^*) - \delta_\mu a^* \tag{11}$$

$$AS^* = \int_{H^{-1}(1-a^*)}^{A} \{1 - H(\alpha)\} d\alpha \tag{12}$$

4.2 Both platforms introduce premium services

Consider the case of $(s_1, s_2) = (FP, FP)$. As in the previous case, users determine which platform to join based on their expected utilities. Thus, let me obtain the expected utility of joining platform

i. Solving equation $u_i^P = u_i^B$ yields

$$\delta = \frac{p_i^P - p_i^B}{a_i} \equiv \delta_i, \tag{13}$$

which represents the type of user that is indifferent between the basic and premium services of platform i. This δ_i can also be interpreted as the amount of money required to remove a unit of advertisements if a user upgrades to the premium service of platform i. In other words, basic users of platform i could receive a discount of δ_i per unit of advertisements. Henceforth, δ_i will be expressed in two ways: i.e., threshold type of users and discount per advertisement.

Note that, for tractability, I use δ_i as a strategic variable instead of p_i^B because it is mathematically equivalent to the profit-maximization problem with respect to (p_i^P, p_i^B, a_i).[14] Therefore, I consider that platform i maximizes its profit with respect to (p_i^P, a_i, δ_i) in stage two.

Users who are tolerant enough of advertisements to satisfy $\delta < \delta_i$ will remain basic users. Other users who do not like advertisements enough to satisfy $\delta \geq \delta_i$ will upgrade to the premium service. Suppose that users who upgrade to the premium service pay an additional fee $p_i^P - p_i^B$. Users' expected utility from choosing platform i is given by

$$E[u_i] = \int_0^{\delta_i} u_i^B f(\delta) d\delta + \int_{\delta_i}^{\Delta} u_i^P f(\delta) d\delta = v - p_i^P + a_i \int_0^{\delta_i} F(\delta) d\delta \tag{14}$$

A type of consumer who is indifferent between both platforms is characterized by

$$E[u_1] - tx = E[u_2] - t(1-x) \iff x = \frac{1}{2} + \frac{E[u_1] - E[u_2]}{2t}. \tag{15}$$

Thus, the total demand for platform i is given by $D_i = \frac{1}{2} + \frac{E[u_i] - E[u_j]}{2t}$.

Users who join platform i decide whether to upgrade or not after realizing their own type of disutility of advertisements. Users with $\delta \geq \delta_i$ upgrade to the premium service and users with $\delta < \delta_i$ continue as basic users. Thus, the numbers of users of basic and premium services of platform i are given by $d_i^B = F(\delta_i) D_i$ and $d_i^P = \{1 - F(\delta_i)\} D_i$. Therefore, I can derive the profit function of

[14] If both p_i^B and a_i are strategic variables, p_i^B would depend largely on a_i. However, by using δ_i instead of p_i^B, I can eliminate such interdependencies between strategy variables in the first-order conditions.

platform i as follows.

$$\pi_i = \left[p_i^P\{1 - F(\delta_i)\} + \{p_i^B + a_i H^{-1}(1 - a_i)\}F(\delta_i)\right] D_i - \Phi \tag{16}$$

Every user of platform i would become the premium user with probability $1 - F(\delta_i)$ and the basic user with probability $F(\delta_i)$. All premium users pay p_i^P. All basic users pay p_i^B and also generate advertising revenue, which is given by $a_i H^{-1}(1 - a_i) = \rho(a_i)$. In addition, after replacing the price for the basic service by $p_i^B = p_i^P - \delta_i a_i$, the profit of platform i can be rewritten by

$$\pi_i = \left[p_i^P + \{\rho(a_i) - \delta_i a_i\}F(\delta_i)\right] D_i - \Phi. \tag{17}$$

I denote by $R_i \equiv p_i^P + \{\rho(a_i) - \delta_i a_i\}F(\delta_i)$ the net revenue that platform i earns from a user. Once I consider that all users pay a price for the premium service p_i^P as the base, the term $\rho(a_i) - \delta_i a_i$ in equation (17) is the additional revenue that platform i gains from a basic user, where $\rho(a_i)$ is the ad-sponsored revenue generated from a basic user, and $\delta_i a_i$ is the price discount for a basic user. The first-order conditions with respect to p_i^P, a_i, and δ_i are characterized by

$$\frac{\partial \pi_i}{\partial p_i^P} = D_i - \frac{R_i}{2t} = 0, \tag{18}$$

$$\frac{\partial \pi_i}{\partial a_i} = \{\rho'(a_i) - \delta_i\}F(\delta_i)D_i + \frac{R_i}{2t}\int_0^{\delta_i} F(\delta)d\delta = 0, \tag{19}$$

$$\frac{\partial \pi_i}{\partial \delta_i} = \left[\{\rho(a_i) - \delta_i a_i\}f(\delta_i) - a_i F(\delta_i)\right] D_i + \frac{R_i}{2t} \cdot a_i F(\delta_i) = 0, \tag{20}$$

which can be reduced as follows.

$$\begin{cases} D_i = \dfrac{R_i}{2t} \\ \{\rho'(a_i) - \delta_i\}F(\delta_i) + \displaystyle\int_0^{\delta_i} F(\delta)d\delta = 0 \\ \{\rho(a_i) - \delta_i a_i\}f(\delta_i) = 0 \end{cases} \tag{21}$$

The condition $\rho(a_i) = \delta_i a_i$ must be satisfied in equilibrium because $f(\delta)$ is positive for all δ.

From $p_i^B = p_i^P - \delta_i a_i$, I have the following proposition.

Lemma 1. *When both platforms introduce the premium services, all advertising revenues are used to discount the price of basic services. That is,* $p_i^P - p_i^B = \delta_i a_i = \rho(a_i)$ *holds in equilibrium.*

Lemma 1 implies that the platform chooses equilibrium strategies such that the advertising revenue per basic user and the price discount for a basic user balance out. In other words, the additional revenue from a basic user reaches zero, that is, $\rho(a_i) - \delta_i a_i = 0$. A similar result was obtained in Section 4.1 and Peitz and Valletti (2008), such that the price for users is discounted by the amount of advertising revenue.

Simplifying equation $\rho(a_i) = \delta_i a_i$ yields the following relationship.

$$\rho(a_i) = \delta_i a_i \iff a_i = 1 - H(\delta_i) \tag{22}$$

With equation (22), I obtain the derivative of $\rho(a_i)$ as follows.

$$\rho'(a_i) = H^{-1}(1 - a_i) - \frac{a_i}{h(1 - a_i)} = \delta_i - \frac{1 - H(\delta_i)}{h(H(\delta_i))} \tag{23}$$

Substituting equation (23) into equation (21), I can derive the following lemma that characterizes the equilibrium threshold between basic and premium users, denoted by δ^{**}, where I use superscript '$**$' to denote the equilibrium outcome when both platforms introduce the premium services.

Lemma 2. *When both platforms introduce the premium services, the equilibrium threshold between basic and premium users is determined to satisfy the following condition.*[15]

$$\int_0^{\delta^{**}} F(\delta) d\delta = \frac{1 - H(\delta^{**})}{h(H(\delta^{**}))} \cdot F(\delta^{**}) \tag{24}$$

Equation (24) shows how the platforms segment their users into basic and premium services in equilibrium. It depends crucially on the distribution functions $F(\cdot)$ and $H(\cdot)$. Any other parameters, including the transportation cost and the stand-alone benefit of the platform, do not affect the equilibrium segmentation of users. This independence stems from the multihoming assumption

[15] If the random variable δ follows a uniform distribution on a unit interval $[0, 1]$, it follows that $H(\delta) = \delta$. The right-hand side of equation (24) takes the form of the hazard function.

for advertisers as well as the specification of the Hotelling model. First, with the multihoming assumption, there is no need for platforms to compete in the advertiser side. Advertisers' willingness to pay depends on the number of basic users on the platform, which is crucially related to the share of basic users to premium users, due to the Hotelling specification that the total demand is fixed. Therefore, the equilibrium segmentation δ^{**} is influenced by two distribution functions $F(\cdot)$ and $H(\cdot)$, not by any other parameters.

Note that equation (24) is derived from the first-order condition with respect to the amount of advertisements, a_i. The left-hand side is equivalent to $\partial E[u_i]/\partial a_i$ and the right-hand side is equivalent to $\partial R_i/\partial a_i$. Therefore, the equilibrium threshold between basic and premium users would be determined so that the effects of a marginal change in the number of advertisements on $E[u_i]$ and on R_i are equal.

For example, suppose that the domain of a random variable δ is $[0, 1]$ and that the advertiser's type α is uniformly distributed on a unit interval $[0, 1]$.[16] Then, equation (24) can be rewritten by

$$\int_0^{\delta^{**}} F(\delta)d\delta = (1 - \delta^{**})F(\delta^{**}). \tag{25}$$

Both the left- and right-hand sides in equation (25) are shown as the gray and light-gray areas in Figure 1, respectively. This implies that the equilibrium δ^{**} is determined where the areas colored in gray and light-gray are equal.

Substituting δ^{**} into equation (22) yields the equilibrium number of advertisers, that is, $a^{**} = 1 - H(\delta^{**})$. From Proposition 1, it holds that $\rho(a^{**}) = \delta^{**}a^{**}$. I can rewrite equation (18) and then derive the equilibrium price for the premium service, p_P^{**}. Finally, the equilibrium price for the basic service can be computed as $p_B^{**} = p_P^{**} - \rho(a^{**})$. The following proposition summarizes the preceding analysis.[17]

Proposition 2. *When both platforms introduce premium services, in equilibrium, the amount of advertisements is given by $a^{**} = 1 - H(\delta^{**})$, where δ^{**} satisfies equation (24). The equilibrium prices for basic and premium service are given by $p_P^{**} = t$ and $p_B^{**} = t - \rho(a^{**})$, respectively. The corresponding demands for basic and premium services are respectively given by $d_B^{**} = F(\delta^{**})/2$*

[16]With this simplification, the uniqueness of the equilibrium is always guaranteed.

[17]Note that, also in the formal proof in the Appendix, I focus on the symmetric equilibrium only. Thus, there might exist asymmetric equilibria.

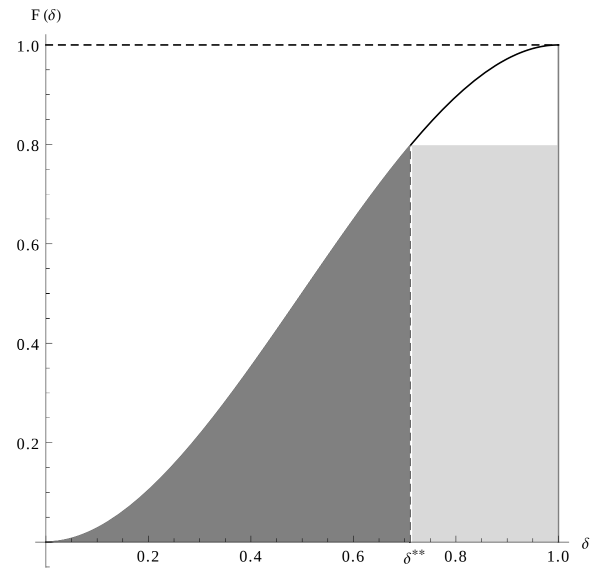


Figure 1: Graphical illustration of equation (25).

and $d_P^{**} = \{1 - F(\delta^{**})\}/2$. *Thus, the equilibrium profit of platforms is given by* $\pi^{**} = t/2 - \Phi$.

The following two conditions are required to guarantee the second-order conditions and the uniqueness of the equilibrium to be satisfied, respectively.[18]

$$\frac{f(\delta^{**})}{F(\delta^{**})} < \frac{2h(H(\delta^{**}))}{1 - H(\delta^{**})} + h'(H(\delta^{**})) \tag{26}$$

$$\left[1 + \frac{1 - H(\delta^{**})}{h(H(\delta^{**}))} \cdot h'(H(\delta^{**}))\right]\left[1 - \frac{h(\delta^{**})}{h(H(\delta^{**}))}\right] \leq 0 \tag{27}$$

Consider that δ and α follow a uniform distribution, i.e., $\delta \sim U[0, \Delta]$ and $\alpha \sim U[0, A]$, respectively. Then, the second-order condition (26) can be rewritten as $A < 3\delta^{**}$, which strictly holds because $\delta^{**} = 2A/3$ in this case. The second-order condition would be satisfied unless distribution functions are highly peaked. In addition, equation (27) is a sufficient condition, which is satisfied when the type of advertisers α follows a uniform distribution (i.e., $\alpha \sim U[0, A]$).

Proposition 2 implies that, when the discount for the basic service is large enough to satisfy $t < \rho(a^{**})$, the price for the basic service takes a negative value. In other words, when ad revenues account for a significant portion of total revenues, freemium platforms should subsidize their basic users in an effort to keep the best balance between basic and premium users. This result is consistent

[18] See supplementary online Appendix for proofs.

with that of Sato (2019).

If monetary subsidies (i.e., negative prices) are difficult to implement for any reason (e.g., adverse selection and opportunistic behaviors), tying other goods to their basic services can be a potential strategy (e.g., Amelio and Jullien, 2012). Even when facing nonnegative price constraints, platforms can implement a substantially negative price by tying or bundling other goods/services for free.[19] If neither offering subsidies nor tying other goods is possible, then freemium platforms must charge a pair of prices different from the optimal one. It implies that the attractiveness of the freemium strategy would decline.[20]

The equilibrium price for the premium service is equal to the degree of product differentiation t, as in the standard Hotelling model where two firms producing horizontally differentiated goods compete in prices in the linear market. Under the timing with users' *sequential* decisions, users can be interpreted to be preliminarily homogeneous with respect to the advertisements supplied by a certain platform. In other words, they are heterogeneous in their valuation for platforms only, which is the reason why the competition can be the same as the standard Hotelling competition.[21]

However, it is worth emphasizing that, even under the timing with users' *simultaneous* decisions, the equilibrium in Proposition 2 remains unchanged. Therefore, even if users are originally heterogeneous not only in their taste for platforms, but also in losses from advertisements, the equilibrium price for premium services is set at the Hotelling price. See the Appendix for details.

Moreover, the losses associated with the price discount for basic users are covered by the advertising revenue (i.e., $\rho(a_i^{**}) = \delta_i^{**} a_i^{**}$). This implies that the profits of platforms are not affected by the advertising revenue. Therefore, the model is based on standard Hotelling competition. As a result, each platform enjoys the profit $t/2$ in equilibrium, which equals that derived in the standard Hotelling model.[22]

[19] For example, Choi and Jeon (2016) study the leverage effects caused by tying in two-sided markets.

[20] In the model, remarkably, it makes the analysis tractable to endogenize pricing for basic services as well as pricing for premium services. In other words, if fixing $p_i^B = 0$, it is difficult to solve the first-order conditions and derive the equilibrium. The technical reason is the following. If the prices for basic services are exogenously fixed (to zero), Lemma 1 does not hold. Consequently, the result of Lemma 2 would be expressed in a different, and more complex, formulation. As a result, it is difficult to derive the equilibrium strategy, as in Proposition 2.

[21] Due to this reason, neither platform has any incentive to deviate from the symmetric equilibrium. See d'Aspremont et al. (1979) for details.

[22] Sato (2019) obtains the same result, which stems from the facts that every consumer necessarily adopts one service and that any increase in revenues per user is competed away. This property is also called *revenue neutrality property*. See Armstrong (2006) for details.

The total profit of platforms, consumer surplus, and advertiser surplus are computed as follows.

$$\Pi^{**} = 2\pi^{*} = t - 2\Phi \tag{28}$$

$$CS^{**} = v - \frac{5}{4}t + a^{**}\int_0^{\delta^{**}} F(\delta)d\delta \tag{29}$$

$$AS^{**} = F(\delta^{**})\int_{\delta^{**}}^{A}\{1 - H(\alpha)\}d\alpha \tag{30}$$

4.3 Only one platform introduces premium service

Here, I investigate an asymmetric competition in which one platform introduces the premium service while the other does not. Let me consider that, without any loss of generality, only platform 1 introduces the premium service, i.e., $(s_1, s_2) = (FP, ASP)$. Unlike the symmetric cases, I here consider the *simultaneous* timing because considering the *sequential* timing alters the equilibrium outcome. Thus, users simultaneously choose one of the three possible options: platform 1's premium service, platform 1's basic service, and platform 2's basic service. Solving the game by backward induction, I derive the equilibrium strategies as follows.

Proposition 3. *Suppose that only platform 1 introduces the premium service. The equilibrium threshold between basic and premium users on platform 1 is equal to δ^{**}. In addition, the equilibrium amounts of advertisements that both platforms choose do not depend on whether the rival introduces premium service or not (i.e., $a_1 = a^{**}$ and $a_2 = a^{*}$). The equilibrium prices can be computed as follows:*

$$\begin{cases} p_1^P = t + \dfrac{a^{**}\int_0^{\delta^{**}} F(\delta)d\delta - \rho(a^{*}) + \delta_\mu a^{*}}{3} \equiv p_P^{FP} \\[2ex] p_1^B = p_1^P - \rho(a^{**}) \equiv p_B^{FP} \\[2ex] p_2^B = t - \dfrac{a^{**}\int_0^{\delta^{**}} F(\delta)d\delta - \rho(a^{*}) + \delta_\mu a^{*}}{3} - \rho(a^{*}) \equiv p_B^{ASP} \end{cases} \tag{31}$$

Then, the equilibrium profit of each platform is as follows.

$$\begin{cases} \pi_1 = \dfrac{\left\{3t + a^{**}\int_0^{\delta^{**}} F(\delta)d\delta - \rho(a^*) + \delta_\mu a^*\right\}\left\{3t + a^{**}\int_0^{\delta^{**}} F(\delta)d\delta + 5\rho(a^*) + \delta_\mu a^*\right\}}{18t} - \Phi \equiv \pi^{FP} \\ \pi_2 = \dfrac{\left\{3t - a^{**}\int_0^{\delta^{**}} F(\delta)d\delta + \rho(a^*) - \delta_\mu a^*\right\}\left\{3t - a^{**}\int_0^{\delta^{**}} F(\delta)d\delta - 5\rho(a^*) - \delta_\mu a^*\right\}}{18t} \equiv \pi^{ASP} \end{cases} \tag{32}$$

I use superscript FP to denote the equilibrium outcomes for the *freemium platform* that introduces the premium service, and ASP for the *ad-sponsored platform* that does not do so. All outcomes are symmetric when only platform 2 introduces the premium service, i.e., $(s_1, s_2) = (ASP, FP)$.

Proposition 3 implies that the prices for basic services are more likely to be negative (i.e., freemium) when ad revenues account for a significant portion of total revenues, as in Proposition 2. In particular, even the ad-sponsored platform 2 may subsidize its users, which is compensated by advertising revenues, but not from premium users. This is a well-known result in the literature on two-sided markets: i.e., platforms tend to subsidize consumers in a side that exerts greater indirect network externalities.

More interestingly, both the equilibrium threshold type of users and the equilibrium amount of advertisements for each platform do not depend on whether the rival introduces premium service or not. Only user prices are being adjusted in this asymmetric competition. In particular, platform 1 that chooses the freemium model sets $\delta_1 = \delta^{**}$ and $a_1 = a^{**}$, which are equivalent to those derived in the competition where both platforms introduce premium services. The ad-sponsored platform 2 chooses $a_2 = a^*$, which is identical to the number of advertisements obtained in the competition where neither platform introduces the premium service. These correspondences seem to come from the multihoming assumption for advertisers. In the present model, as assumed in the literature, each advertiser can advertise on both platforms if it is profitable, which implies that the platforms do not need to compete for advertisers. Therefore, the optimal amount of advertisements is determined according to their own decisions about the business model irrespective of the rival's choice.

4.4 Business model choice

I examine the platforms' incentives to introduce premium services. I use $BR_i(s_j)$ to denote the platform i's best response strategy when platform j chooses s_j.

Lemma 3. *Suppose that the rival platform does not introduce the premium service. If the fixed cost of introducing premium service is low enough to satisfy $\Phi < \pi^{FP} - t/2 \equiv \Phi'$, then the best response strategy is to introduce the premium service. Formally,*

$$BR_i(ASP) = \begin{cases} FP & \text{if } 0 \leq \Phi \leq \Phi' \\ ASP & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2. \tag{33}$$

Lemma 4. *Suppose that the rival platform introduces the premium service. If the fixed cost of introducing premium service is low enough to satisfy $\Phi < t/2 - \pi^{ASP} \equiv \Phi''$, then the best response strategy is to introduce the premium service. Formally,*

$$BR_i(FP) = \begin{cases} FP & \text{if } 0 \leq \Phi \leq \Phi'' \\ ASP & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2. \tag{34}$$

The order between the two thresholds Φ' and Φ'' is determined as follows.

$$\begin{aligned} \Phi' - \Phi'' &= \left(\pi^{FP} - \frac{t}{2}\right) - \left(\frac{t}{2} - \pi^{ASP}\right) = \pi^{FP} + \pi^{ASP} - t \\ &= \frac{\left(\int_0^{\delta^{**}} F(\delta)d\delta - \{\rho(a^*) - \delta_\mu a^*\}\right)\left(\int_0^{\delta^{**}} F(\delta)d\delta + 5\rho(a^*) + \delta_\mu a^*\right)}{9t} > 0 \\ &\iff \int_0^{\delta^{**}} F(\delta)d\delta > \rho(a^*) - \delta_\mu a^* \end{aligned} \tag{35}$$

Note that both left- and right-hand sides of inequality (35) are greater than zero. I have the following proposition regarding the equilibrium business models.

Proposition 4. *If inequality* (35) *holds, then the equilibrium business models are as follow:*

$$(s_1, s_2) = \begin{cases} (FP, FP) & \text{if } 0 \leq \Phi \leq \Phi'' \\ (FP, ASP) \ \& \ (ASP, FP) & \text{if } \Phi'' < \Phi \leq \Phi' \\ (ASP, ASP) & \text{if } \Phi' < \Phi \end{cases} \tag{36}$$

Otherwise, if inequality (35) *is not satisfied, then the equilibrium business models are as follow:*

$$(s_1, s_2) = \begin{cases} (FP, FP) & \text{if } 0 \leq \Phi \leq \Phi' \\ (FP, FP) \ \& \ (ASP, ASP) & \text{if } \Phi' < \Phi \leq \Phi'' \\ (ASP, ASP) & \text{if } \Phi'' < \Phi \end{cases} \tag{37}$$

Proposition 4 shows that three kinds of market structures can arise depending on the extent of the fixed cost. First, if the fixed cost of introducing a premium service is small enough, then choosing the freemium business model is the dominant strategy for the platforms. Introducing premium services enables platforms to manipulate more strategic variables, which would be preferable to being ad-sponsored platforms, as indicated by the strategy space of ad-sponsored platforms being a subset of the strategy space of freemium platforms.

Second, asymmetric equilibria, in which only one platform introduces the premium service, can arise if inequality (35) is satisfied and the fixed cost is at an intermediate level. This result may explain the present situation of the video-streaming service market: Dailymotion adopts the fully ad-sponsored business model whereas YouTube adopts the freemium one.[23] By choosing different business models, platforms can effectively lessen competition. On the one hand, the freemium platform pursues the users with extreme preferences for advertisements (users who are either tolerant or allergic to advertisements). In particular, it offers basic service with many advertisements, which can increase the willingness to pay of ad-allergic users for upgrading to the premium service. On the other hand, the ad-sponsored platform effectively deals with the remaining users with midrange preferences who are paid little attention by the rival freemium platform.[24]

[23] See Table 1 in Sato (2019).

[24] Several literature have also pointed out the possibility of asymmetric business model choices in different settings (e.g., Maruyama and Zennyo, 2013, 2015, 2017; Adner et al., 2019).

Third, when the fixed cost is too high compared to the expected profits, both platforms opt for the ad-sponsored business model. This result would be close to the current situation of the mobile application market: i.e., many mobile applications rely on revenues from in-app advertisements.[25]

I also note that when δ and α follow a uniform distribution, respectively (i.e., $\delta \sim U[0,\Delta]$ and $\alpha \sim U[0,A]$), inequality in (35) is always satisfied.

Corollary 1. *If $\delta \sim U[0,\Delta]$ and $\alpha \sim U[0,A]$, inequality (35) always holds: every market structure can be an equilibrium outcome, depending on the fixed cost of introducing premium services.*

5 Discussion and Policy Implications

In this section, I first conduct a welfare analysis to provide some policy implications in Section 5.1. Then, in Section 5.2, I discuss how an exogenous quality difference between basic and premium services affects the equilibrium strategies of freemium platforms.

5.1 Welfare analysis

I here consider both distribution functions F and H to be uniform distribution in $[0,1]$ for simplicity. That is, $\delta \sim U[0,1]$ and $\alpha \sim U[0,1]$. In this case, I have $\delta_\mu = \frac{1}{2}$, $a^* = \frac{1}{4}$, $\delta^{**} = \frac{2}{3}$, and $a^{**} = \frac{1}{3}$. I further assume that $t > 491/1296$ and $0 \leq \Phi < (2455 + 642816t + 1679616t^2)/(3359232t)$ to ensure an interior equilibrium such that all surpluses (Π, CS, AS, and SW) are positive.

As shown in Corollary 1, in this case, every market structure can be an equilibrium outcome depending on the fixed cost of introducing premium services. For each market structure, platform surplus (i.e., the joint profit of platforms), consumer surplus, advertiser surplus (i.e., the aggregate profit of advertisers), and social welfare can be computed and summarized in Table 1.

First, let me compare platform surplus among the three market structures.

Proposition 5. *The total profit of platforms is ordered as follows:*

$$\begin{cases} \Pi(FP,FP) = \Pi(ASP,ASP) < \Pi(FP,ASP) & \text{if } \Phi = 0, \\ \Pi(FP,FP) < \Pi(ASP,ASP) \leq \Pi(FP,ASP) & \text{if } 0 < \Phi \leq \Phi_\Pi, \\ \Pi(FP,FP) < \Pi(FP,ASP) < \Pi(ASP,ASP) & \text{otherwise}, \end{cases} \tag{38}$$

[25] See Footnote 2 for evidence.

Table 1: Total profit of platforms, consumer surplus, advertiser surplus, and social welfare.

$s = (s_1, s_2)$	(ASP, ASP)	(FP, ASP)	(FP, FP)
$\Pi(s)$	t	$t + \frac{2455}{1679616t} - \Phi$	$t - 2\Phi$
$CS(s)$	$v - \frac{5}{4}t + \frac{1}{16}$	$v - \frac{5}{4}t + \frac{709+458784t}{6718464t}$	$v - \frac{5}{4}t + \frac{2}{27}$
$AS(s)$	$\frac{1}{16}$	$\frac{76464t-167}{2239488t}$	$\frac{1}{27}$
$SW(s)$	$v - \frac{t}{4} + \frac{1}{8}$	$v - \frac{t}{4} + \frac{2507+172044t}{1679616t} - \Phi$	$v - \frac{t}{4} + \frac{1}{9} - 2\Phi$

where $\Phi_\Pi \equiv \frac{2455}{1679616t}$.

For sufficiently small fixed costs, even though Proposition 4 showed that introducing the premium service is the dominant strategy for competing platforms, Proposition 5 shows that platform surplus is largest in the asymmetric market structure. If both platforms can coordinate their business model choices, it is profitable to select different business models from each other and to make a monetary transfer T from the freemium platform to the other that satisfies $\pi^{FP} - T > t/2$ and $\pi^{ASP} + T > t/2$.

In contrast, when the fixed cost is sufficiently large, the platform surplus under the asymmetric market structures is lower than the one under the ad-sponsored market structure, but is never lower than the platform surplus in the freemium market structure.

The result of Proposition 5 can be interpreted from a different perspective. Condition $\Phi \leq \Phi_\Pi$ is equivalent to $t < \bar{t} \equiv \frac{2455}{1679616\Phi}$. That is, if market competition is particularly intense (i.e., $t < \bar{t}$), the asymmetric business model choices with a side payment can be the best collusive outcome for platforms. Otherwise, if both platforms are highly differentiated (i.e., $t > \bar{t}$), the total profit of platforms increases with the number of platforms that adopt the ad-sponsored business model.

Next, I compare consumer surplus and advertiser surplus.

Proposition 6. *Consumer and advertiser surpluses are ordered as follows:*

$$CS(ASP, ASP) < CS(FP, ASP) < CS(FP, FP) \tag{39}$$

$$AS(FP, ASP) < AS(FP, FP) < AS(ASP, ASP) \tag{40}$$

This proposition shows that consumer surplus will be largest under the freemium market structure and be lowest under the ad-sponsored market structure. When both platforms introduce their premium services, every consumer can choose the most suitable option from a wider range of choice sets, which would simultaneously lead to fierce platform competition. In contrast, the lack of ad-free premium services forces consumers who dislike advertisements to endure them, which reduces consumer surplus.

Proposition 6 also shows that both consumer and advertiser surpluses are lower under the asymmetric market structure than under the freemium market structure. Along with Proposition 5, this finding offers a caution that, if the intensity of platform competition is high enough to satisfy $t < \bar{t}$, platforms have an incentive to coordinate their business model choices toward the asymmetric market structure, which can worsen both consumer and advertiser surpluses.

Finally, I compare social welfare among the three market structures.

Proposition 7. *Social welfare is ordered as follows:*

$$\begin{cases} SW(FP, ASP) < SW(FP, FP) < SW(ASP, ASP) & \text{if} \quad 0 \leq \Phi < \Phi_W \\ SW(FP, FP) < SW(FP, ASP) < SW(ASP, ASP) & \text{if} \quad \Phi > \Phi_W \end{cases} \tag{41}$$

where $\Phi_W \equiv \frac{14580t-2507}{1679616t}$.

Social welfare will be largest under the ad-sponsored market structure, irrespective of the extent of fixed cost Φ, which is not realized in equilibrium unless the fixed cost of introducing a premium service is too high. In other words, a sufficiently high fixed cost would lead the resulting market structure to the first best one.

Moreover, it is noteworthy that the threshold value Φ_W is strictly positive for all t, but is negligibly small.[26] Let me suppose that the fixed cost is small, but nonnegligible (i.e., $\Phi_W < \Phi < \Phi''$). In equilibrium, as shown in Proposition 4, both competing platforms choose the freemium business model, which leads to the smallest social welfare. In this regard, if the platforms coordinate their business model choices towards the asymmetric market structure, then social welfare, although not the first best, can be improved. However, this increase in social welfare stems from the steep increase in the platforms' joint profits which arises at the expenses of advertisers and final users, as inequalities (39) and (40) demonstrate that users and advertisers always prefer the symmetric

[26]Threshold Φ_W increases in $t > 491/1296$. Further, it can be computed that $\lim_{t\to\infty} \Phi_W = 0.00868056$.

(FP, FP) to the asymmetric (FP, ASP). Therefore, even though the interplatform coordination can be beneficial to social welfare, competition authorities should be cautious in their evaluation of it in terms of consumer protection.

5.2 Quality differences between basic and premium services

Heretofore, I have assumed that both basic and premium services have the same quality (i.e., stand-alone benefit). However, premium products feature higher quality than basic products in reality. For instance, Spotify and Deezer Premium allow users to select directly a song as well as to listen to music offline. These features are not available in the ad-based Spotify/Deezer.

Thus, I here investigate how the quality difference between basic and premium services affects the platforms' strategies. To this end, I modify users' utility functions in equation (2) as shown below:

$$
\begin{aligned}
U_1^P(x,\delta) &= v + q - p_1^P - tx, & U_2^P(x,\delta) &= v + q - p_2^P - t(1-x), \\
U_1^B(x,\delta) &= v - p_1^B - \delta a_1 - tx, & U_2^B(x,\delta) &= v - p_2^B - \delta a_2 - t(1-x),
\end{aligned} \tag{42}
$$

where I use q to denote an exogenous quality difference.

For simplicity, let me focus on the symmetric equilibrium. That is, I assume both that competing platforms adopt the freemium business model and that the quality difference is identical across the platforms.[27] Solving the game backward, I derive the equilibrium strategy of the platforms.

Proposition 8. *Consider an exogenous quality difference between basic and premium services. When both platforms introduce the premium services, the equilibrium threshold $\tilde{\delta}$ is given by $\tilde{\delta} = H^{-1}(1-\tilde{a}) - q/\tilde{a}$, where $\tilde{a}$ satisfies the following condition:*

$$\left(\frac{q}{\tilde{a}} - \frac{\tilde{a}}{h(1-\tilde{a})}\right) F\left(H^{-1}(1-\tilde{a}) - \frac{q}{\tilde{a}}\right) + \int_0^{H^{-1}(1-\tilde{a})-q/\tilde{a}} F(\delta)d\delta = 0 \tag{43}$$

Then, the equilibrium prices for basic and premium services are given by $\tilde{p}_B = t - \tilde{\delta}\tilde{a} - q$ and $\tilde{p}_P = t$, respectively.

[27] In the present model with platform competition, it is difficult to examine endogenous quality choices by competing platforms. Technically, in doing so, we need to solve the pricing subgame with different quality choices, implying that we cannot simply assume the symmetric equilibrium. In this regard, Carroni and Paolini (2019) investigate the optimal quality choice by monopoly platform.

Note that, when $q = 0$, the equilibrium strategy stated above is perfectly equivalent to that of Proposition 2. When the two distribution functions $F(\cdot)$ and $H(\cdot)$ follow a uniform distribution along a unit interval, Proposition 8 can be rewritten as follows.

Corollary 2. *Assume that $\delta \sim U[0,1]$ and $\alpha \sim U[0,1]$. The equilibrium strategies are given by*

$$\tilde{a} = \frac{1+\sqrt{1+12q}}{6}, \quad \tilde{\delta} = \frac{2(1-12q+\sqrt{1+12q})}{3(1+\sqrt{1+12q})}, \quad \tilde{p}_B = t - \frac{1-3q+\sqrt{1+12q}}{9}, \quad \tilde{p}_P = t \ . \quad (44)$$

Corollary 2 implies that an exogenous increase in the quality differentiation induces the platforms to more aggressively discount the price for basic services (i.e., $d(\tilde{p}_P - \tilde{p}_B)/dq > 0$) in an effort to prevent an excessive number of users from upgrading to the premium service. In so doing, along with this price discount, the platforms also increase the number of advertisements (i.e., $d\tilde{a}/dq > 0$). Eventually, however, the threshold value declines (i.e., $d\tilde{\delta}/dq < 0$), implying that the larger number of users will upgrade to premium services in pursuit of higher quality.

6 Conclusion

This paper demonstrates the consequence of endogenous business model choices by ad-sponsored platforms, and then provides several discussions about its welfare implications.

I show that adopting the freemium model is the dominant strategy for the platforms when the fixed cost for introducing premium services is small enough. By contrast, when the fixed cost is too high, both platforms do not adopt the freemium model. Of special interest is when the fixed cost is at an intermediate level. In this case, asymmetric equilibria, in which only one platform introduces the premium service, can arise. By choosing different business models, competing platforms target different user segments each other, which can successfully mitigate the platform competition.

The comparison among all possible market structures raises an anticompetitive concern: If the platform competition is particularly intense, competing platforms benefit from coordinating their business model choices toward asymmetric ones with a side payment. Interplatform coordination can heighten social welfare. However, this increase in social welfare stems from the steep increase in the platforms' joint profits, which arises at the expenses of advertisers and final users. In the present static model, the coordination requires a side payment. In repeated interactions, however, it might be possible for competing platforms to maintain their coordination even without side payment. Therefore, in terms of consumer protection, competition authorities should be cautious in the evaluation of the interplatform coordination on their business model choices.

I conclude by mentioning the limitations of this paper and discussing potential avenues for future research. First, this paper does not consider any *direct* network externalities among users. In some cases (e.g., YouTube, Flickr), one of the reasons that people join the platform is the existence of *user-generated content* (e.g., Albuquerque et al., 2012). To incorporate this issue, it might be required to build a model where each user endogenously decides whether to generate contents or enjoy them, or both.[28]

Second, although I have discussed how an exogenous quality difference between basic and premium services affects the equilibrium strategy of freemium platforms in Section 5.2, much further analyses would be required for future research. A possible way is to endogenize the platforms' quality decisions.[29] Furthermore, it will be very interesting to explore when free products should be differentiated in the quality dimension instead of the inclusion of nuisance advertisements.

Third, it would be important to investigate how users' multihoming behaviors alter the result derived, as discussed in Section 3.2. In practice, competing platforms seem to provide some overlapping functionalities. Ideally, the utility of multihoming should be expressed in a more general form that accounts for the overlapping functionalities. This extension would change the nature of competition, as pointed out by recent studies (e.g., Ambrus et al., 2016; Athey et al., 2018), which is beyond the scope of this paper.

Finally, the paper does not consider the use of ad-blockers, which has been investigated in the literature (e.g., Anderson and Gans, 2011). Indeed, how the existence of ad-blockers affects the strategies of freemium platforms is an important question.

[28] Choi and Zennyo (2019) investigate the users' endogenous side decisions, in which every user is not assigned to a side *ex ante*, and then endogenously chooses which one of the two sides to join. Gao (2018) analyzes a general model for two-sided markets in which a consumer may appear on different sides of the market. Those models could be applied for future studies on this issue.

[29] Although most studies on competition in two-sided markets have considered horizontal differentiation to obtain the symmetric equilibrium pricing, some recent studies investigate vertical differentiation (e.g., Gabszewicz and Wauthy, 2014; Zennyo, 2016).

Appendix A: Proofs

Proof of Proposition 1

I prove that the strategies stated in Proposition 1 constitutes the equilibrium. First, I characterize the threshold type of users (x, δ) who are indifferent between two platforms.

$$u_1^B = u_2^B \iff x = \frac{1}{2} - \frac{p_1^B - p_2^B}{2t} + \frac{\delta(a_2 - a_1)}{2t} \equiv x_B(\delta) \tag{A.1}$$

Thus, the demand for platform 1 and 2 can be written by

$$d_1^B = \frac{1}{2} - \frac{p_1^B - p_2^B}{2t} + \int_0^\Delta \frac{\delta(a_2 - a_1)}{2t} f(\delta) d\delta = \frac{1}{2} - \frac{p_1^B - p_2^B}{2t} + \frac{(a_2 - a_1)}{2t}\delta_\mu, \tag{A.2}$$

$$d_2^B = 1 - d_1^B = \frac{1}{2} + \frac{p_1^B - p_2^B}{2t} - \frac{(a_2 - a_1)}{2t}\delta_\mu, \tag{A.3}$$

which are equivalent to equation (5). Therefore, I can obtain the same equilibrium strategy as stated in Proposition 1. □

Proof of Proposition 2

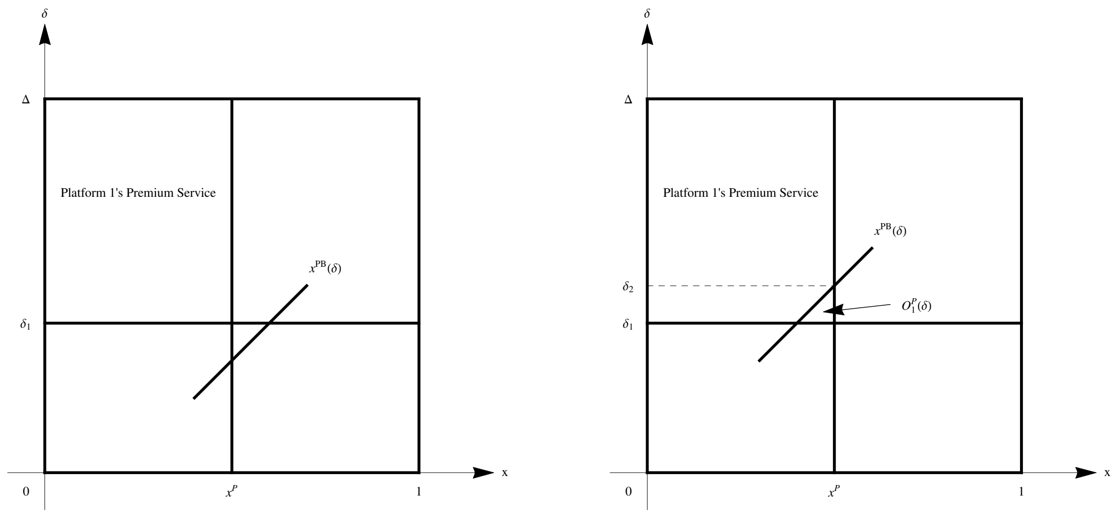
I prove that the strategies of Proposition 2 constitutes the equilibrium. First, I characterize the threshold types of users (x, δ), who are indifferent between any two of four possible options.

$$\begin{cases} u_i^P = u_i^B \iff \delta = \frac{p_i^P - p_i^B}{a_i} \equiv \delta_i \quad \text{for} \quad i = 1, 2 \\ u_1^P = u_2^P \iff x = \frac{1}{2} - \frac{p_1^P - p_2^P}{2t} \equiv x^P \\ u_1^B = u_2^B \iff x = \frac{1}{2} - \frac{p_1^P - p_2^P}{2t} + \frac{(\delta_1 - \delta)a_1 - (\delta_2 - \delta)a_2}{2t} \equiv x^B(\delta) \\ u_1^P = u_2^B \iff x = \frac{1}{2} - \frac{p_1^P - p_2^P}{2t} + \frac{\delta a_2}{2t} \equiv x^{PB}(\delta) \\ u_1^B = u_2^P \iff x = \frac{1}{2} - \frac{p_1^P - p_2^P}{2t} - \frac{\delta a_1}{2t} \equiv x^{BP}(\delta) \end{cases} \tag{A.4}$$

Thus, the demand function for the premium service of platform 1 is given by

$$d_1^P = \begin{cases} \{1 - F(\delta_1)\}x^P & \text{if } x^{PB}(\delta_1) > x^P \iff \delta_1 > \delta_2, \\ \{1 - F(\delta_1)\}x^P - O_1^P & \text{otherwise,} \end{cases} \tag{A.5}$$

where $O_1^P = \int_{\delta_1}^{\delta_2} \left\{x^P - x^{PB}(\delta)\right\} d\delta$. Figure A.1 shows that, when $x^{PB}(\delta_1) > x^P$ holds, the range of



(i) $x^{PB}(\delta_1) > x^P$ (ii) $x^{PB}(\delta_1) < x^P$

Figure A. 1: Demand for the premium service of platform 1

types of users who choose the premium service of platform 1 is depicted by the upper left rectangle in Panel (i). Otherwise, when $x^{PB}(\delta_1) < x^P$, some users (represented by O_1^P) are deprived by the basic service of platform 2, which is shown in Panel (ii).

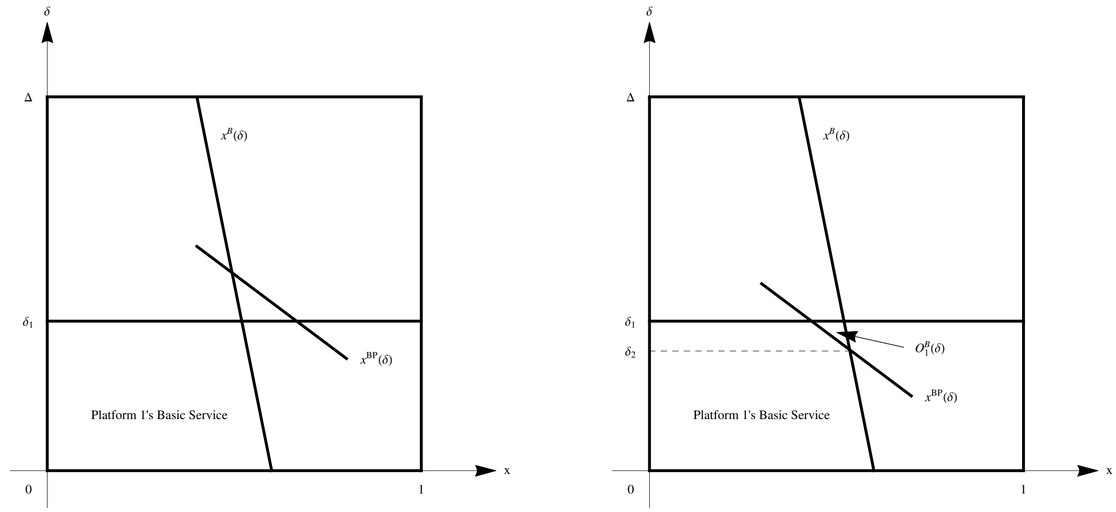
Similarly, the demand function for the basic service of platform 1 is given by

$$d_1^B = \begin{cases} \int_0^{\delta_1} x^B(\delta) f(\delta) d\delta & \text{if } x^{BP}(\delta_1) > x^B(\delta_1) \iff \delta_1 < \delta_2, \\ \int_0^{\delta_1} x^B(\delta) f(\delta) d\delta - O_1^B & \text{otherwise,} \end{cases} \tag{A.6}$$

where $O_1^B = \int_{\delta_2}^{\delta_1} \left\{ x^B(\delta) - x^{BP}(\delta) \right\} f(\delta) d\delta$. Figure A.2 shows that, when $x^{BP}(\delta_1) > x^B(\delta_1)$ holds, the range of types of users who choose the basic service of platform 1 is depicted by the left bottom rectangle in Panel (i). Otherwise, when $x^{BP}(\delta_1) < x^B(\delta_1)$, some users (represented by O_1^B) are deprived by the premium service of platform 2, which is shown in Panel (ii).

Therefore, the profit of platform 1 can be given by

$$\begin{aligned} \pi_1 =& p_1^P d_1^P + p_1^B d_1^B + r_1 a_1 = p_1^P d_1^P + \{p_1^P + \rho(a_1) - \delta_1 a_1\} d_1^B \qquad \text{(A.7)} \\ =& p_1^P \{1 - F(\delta_1)\} x^P + \{p_1^P + \rho(a_1) - \delta_1 a_1\} \left[F(\delta_1) x^P + \frac{\delta_1 a_1 - \delta_2 a_2}{2t} F(\delta_1) - \frac{a_1 - a_2}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta \right] - O_1, \end{aligned}$$



(i) $x^{BP}(\delta_1) > x^B(\delta_1)$ (ii) $x^{BP}(\delta_1) < x^B(\delta_1)$

Figure A. 2: Demand for the basic service of platform 1

where

$$O_1 = \begin{cases} p_1^P O_1^P & \text{if } \delta_1 \geq \delta_2, \\ \{p_1^P + \rho(a_1) - \delta_1 a_1\} O_1^B & \text{otherwise.} \end{cases} \tag{A.8}$$

The first-order conditions are computed as follows.

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1^P} =& x^P + \left(p_1^P + \{\rho(a_1) - \delta_1 a_1\} F(\delta)\right) \frac{\partial x^P}{\partial p_1^P} \\ &+ \frac{\delta_1 a_1 - \delta_2 a_2}{2t} F(\delta_1) - \frac{a_1 - a_2}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta - \frac{\partial O_1}{\partial p_1^P} = 0 \end{aligned} \tag{A.9}$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial a_1} =& \{\rho'(a_1) - \delta_1\} \left[F(\delta_1) x^P + \frac{\delta_1 a_1 - \delta_2 a_2}{2t} F(\delta_1) - \frac{a_1 - a_2}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta \right] \\ &+ \{p_1^P + \rho(a_1) - \delta_1 a_1\} \left[\frac{\delta_1 F(\delta_1)}{2t} - \frac{1}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta \right] - \frac{\partial O_1}{\partial a_1} = 0 \end{aligned} \tag{A.10}$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial \delta_1} =& - f(\delta_1) p_1^P x^P - a_1 \left[F(\delta_1) x^P + \frac{\delta_1 a_1 - \delta_2 a_2}{2t} F(\delta_1) - \frac{a_1 - a_2}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta \right] \\ &+ \{p_1^P + \rho(a_1) - \delta_1 a_1\} \left[f(\delta_1) x^P + \frac{a_1 F(\delta_1)}{2t} + \frac{\delta_1 a_1 - \delta_2 a_2}{2t} f(\delta_1) - \frac{a_1 - a_2}{2t} \delta_1 f(\delta_1) \right] \end{aligned} \tag{A.11}$$

In the same way, I can obtain the first-order conditions for platform 2. Then, assuming the symmetric equilibrium, the first-order conditions can be rewritten as follows.

$$\begin{cases} \frac{1}{2} - \frac{1}{2t}\left(p_i^P + \{\rho(a_i) - \delta_i a_i\}F(\delta_i)\right) = 0 \\ \{\rho'(a_i) - \delta_i\}\frac{F(\delta_i)}{2} + \{p_i^P + \rho(a_i) - \delta_i a_i\}\frac{1}{2t}\left(\delta_i F(\delta_i) - \int_0^{\delta_i}\delta f(\delta)d\delta\right) = 0 \\ -\frac{f(\delta_i)}{2}p_i^P - a_i\frac{F(\delta_i)}{2} + \{p_i^P + \rho(a_i) - \delta_i a_i\}\frac{1}{2t}\{tf(\delta_i) + a_i F(\delta_i)\} = 0 \end{cases} \tag{A.12}$$

It can be easily confirmed that $p_1^P = t$ and $\rho(a_i) = \delta_i a_i$ satisfy the first and third line of equation (A.12). Further, simplifying the second line of equation (A.12) yields equation (24) as follows.

$$\{\rho'(a_i) - \delta_i\}\frac{F(\delta_i)}{2} + t \cdot \frac{1}{2t}\left(\delta_i F(\delta_i) - \int_0^{\delta_i}\delta f(\delta)d\delta\right) = 0$$

$$\iff \{\rho'(a_i) - \delta_i\}F(\delta_i) + \int_0^{\delta_i} F(\delta)d\delta = 0 \iff \int_0^{\delta^{**}} F(\delta)d\delta = \frac{1 - H(\delta^{**})}{h(H(\delta^{**}))} \cdot F(\delta^{**}) \tag{A.13}$$

□

Proof of Proposition 3

Users select one of the three possible options: platform 1's premium service, platform 1's basic service, and platform 2's basic service. The corresponding utilities are given by

$$U_1^P(x,\delta) = v - p_1^P - tx, \quad U_1^B(x,\delta) = v - p_1^B - \delta a_1 - tx, \quad U_2^B(x,\delta) = v - p_2^B - \delta a_2 - t(1-x). \tag{A.14}$$

First, solving $u_1^P = u_1^B$ yields $\delta = \frac{p_1^P - p_1^B}{a_1} \equiv \delta_1$. Second, a type of user (x, δ) who is indifferent between the two basic services is given by

$$u_1^B = u_2^B \iff x = \frac{1}{2} - \frac{(p_1^B + \delta a_1) - (p_2^B + \delta a_2)}{2t} \equiv x^B(\delta). \tag{A.15}$$

Third, a type of user (x, δ) who is indifferent between platform 1's premium service and platform 2's basic service is given by

$$u_1^P = u_2^B \iff x = \frac{1}{2} - \frac{p_1^P - p_2^B}{2t} + \frac{\delta a_2}{2t} \equiv x^{PB}(\delta). \tag{A.16}$$

Here, solving equation $x^B(\delta) = x^{PB}(\delta)$, I obtain $\delta = \delta_1$. Therefore, the partition of users can

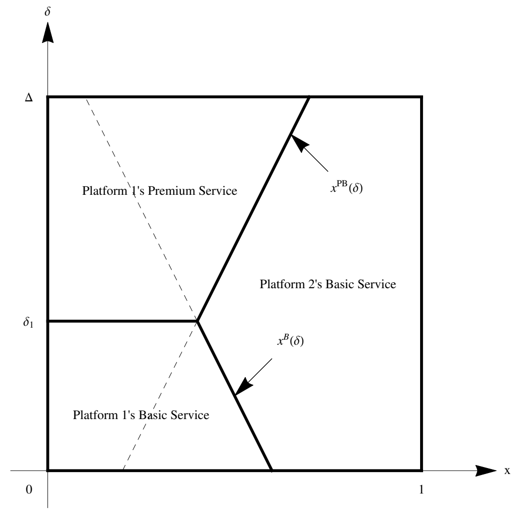


Figure A. 3: Partition of users in the asymmetric competition

be illustrated in Figure A.3.

Thus, the demand function for each service is written as follows.

$$d_1^P = \int_{\delta_1}^{\Delta} x^{PB}(\delta) f(\delta) d\delta \left(\frac{1}{2} - \frac{p_1^P - p_2^B}{2t}\right) \{1 - F(\delta_1)\} + \frac{a_2}{2t} \int_{\delta_1}^{\Delta} \delta f(\delta) d\delta \tag{A.17}$$

$$d_1^B = \int_0^{\delta_1} x^B(\delta) f(\delta) d\delta \left(\frac{1}{2} - \frac{p_1^P - p_2^B}{2t}\right) F(\delta_1) + \frac{a_1}{2t} \int_0^{\delta_1} F(\delta) d\delta + \frac{a_2}{2t} \int_0^{\delta_1} \delta f(\delta) d\delta \tag{A.18}$$

$$d_2^B = 1 - d_1^P - d_2 B\frac{1}{2} + \frac{p_1^P - p_2^B}{2t} - \frac{a_1}{2t} \int_0^{\delta_1} F(\delta) d\delta - \frac{a_2}{2t} \delta_\mu \tag{A.19}$$

The profit of platform 1 can be rewritten by $\pi_1 = p_1^P d_1^P + p_1^B d_1^B + r_1 a_1 = p_1^P D_1 + \{\rho(a_1) - \delta_1 a_1\} d_1^B$, where $D_1 = d_1^P + d_1^B$. The first-order conditions with respect to p_1^P, a_1, and δ_1 are as follows.

$$\frac{\partial \pi_1}{\partial p_1^P} = D_1 + p_1^P \frac{\partial D_1}{\partial p_1^P} + \{\rho(a_1) - \delta_1 a_1\} \frac{\partial d_1^B}{\partial p_1^P} = 0$$

$$\iff \quad 2tD_1 = p_1^P + \{\rho(a_1) - \delta_1 a_1\} F(\delta_1) \tag{A.20}$$

$$\frac{\partial \pi_1}{\partial a_1} = p_1^P \frac{\partial D_1}{\partial a_1} + \{\rho'(a_1) - \delta_1\} d_1^B + \{\rho(a_1) - \delta_1 a_1\} \frac{\partial d_1^B}{\partial a_1} = 0$$

$$\iff \quad 2td_1^B \{\delta_1 - \rho'(a_1)\} = \left(p_1^P + \{\rho(a_1) - \delta_1 a_1\}\right) \int_0^{\delta_1} F(\delta) d\delta \tag{A.21}$$

$$\frac{\partial \pi_1}{\partial \delta_1} = p_1^P \frac{\partial D_1}{\partial \delta_1} - a_1 d_1^B + \{\rho(a_1) - \delta_1 a_1\} \frac{\partial d_1^B}{\partial \delta_1} = 0$$

$$\iff \quad 2td_1^B a_1 = p_1^P a_1 F(\delta_1) + \{\rho(a_1) - \delta_1 a_1\} \{a_1 F(\delta_1) + a_2 \delta_1 f(\delta_1)\} \tag{A.22}$$

Similarly, the profit of platform 2 can be given by $\pi_2 = p_2^B d_2^B + r_2 a_2 = \{p_2^B + \rho(a_2)\} d_2^B$, and the first-order conditions with respect to p_2^B and a_2 are as follows.

$$\frac{\partial \pi_2}{\partial p_2^B} = d_2^B + \{p_2^B + \rho(a_2)\}\frac{\partial d_2^B}{\partial p_2^B} = 0 \qquad \Longleftrightarrow \qquad 2td_2^B = p_2^B + \rho(a_2) \tag{A.23}$$

$$\frac{\partial \pi_2}{\partial a_2} = \rho'(a_2) d_2^B + \{p_2^B + \rho(a_2)\}\frac{\partial d_2^B}{\partial a_2} = 0 \qquad \Longleftrightarrow \qquad 2td_2^B = \{p_2^B + \rho(a_2)\}\frac{\rho'(a_2)}{\delta_\mu} \tag{A.24}$$

Solving equations (A.20) and (A.23) with respect to p_1^P and p_2^B yields

$$\begin{cases} p_1^P(\delta_1, a_1, a_2) = t + \dfrac{a_1 \int_0^{\delta_1} F(\delta) d\delta}{3} + \dfrac{\delta_\mu a_2}{3} - \dfrac{2F(\delta_1)\{\rho(a_1) - \delta_1 a_1\} + \rho(a_2)}{3}, \\ p_2^B(\delta_1, a_1, a_2) = t - \dfrac{a_1 \int_0^{\delta_1} F(\delta) d\delta}{3} - \dfrac{\delta_\mu a_2}{3} - \dfrac{F(\delta_1)\{\rho(a_1) - \delta_1 a_1\} + 2\rho(a_2)}{3}. \end{cases} \tag{A.25}$$

Moreover, from equations (A.23) and (A.24), it should hold that $\rho'(a_2) = \delta_\mu$, leading to $a_2 = a^*$.

Lastly, I can prove that $\delta_1 = \delta^{**}$ and $a_1 = a^{**}$ satisfy these equations as follows. Here, since $\rho(a^{**}) = \delta^{**} a^{**}$ holds, equations (A.21) and (A.22) are rewritten by

$$\begin{cases} 2td_1^B\{\delta^{**} - \rho'(a^{**})\} = p_1^P \int_0^{\delta^{**}} F(\delta) d\delta \\ 2td_1^B = p_1^P F(\delta^{**}) \end{cases}$$

$$\Longleftrightarrow p_1^P \int_0^{\delta^{**}} F(\delta) d\delta = p_1^P F(\delta^{**})\{\delta^{**} - \rho'(a^{**})\} \Longleftrightarrow \int_0^{\delta^{**}} F(\delta) d\delta = \frac{1 - H(\delta^{**})}{h(H(\delta^{**}))} \cdot F(\delta^{**}), \tag{A.26}$$

Equation (A.26) is equivalent to equation (24), which means that $\delta_1 = \delta^{**}$ and $a_1 = a^{**}$ constitute the equilibrium. Therefore, the equilibrium prices are as follows.

$$\begin{cases} p_1^P = t + \dfrac{a^{**} \int_0^{\delta^{**}} F(\delta) d\delta - \rho(a^*) + \delta_\mu a^*}{3} \\ p_2^B = t - \dfrac{a^{**} \int_0^{\delta^{**}} F(\delta) d\delta - \rho(a^*) + \delta_\mu a^*}{3} - \rho(a^*) \end{cases} \tag{A.27}$$

$\square$

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Appendix B (Supplementary): Second-Order Condition and Uniqueness of the Equilibrium

Second-order condition:

Suppose that $\sigma = (\sigma_i, \sigma_j) = ((p_i^P, a_i, \delta_i), (p_j^P, a_j, \delta_j))$. To derive the second-order conditions for profit maximization, I investigate the conditions under which the following 3×3 matrix $\Psi_i(\sigma^{**})$ is negative definite.

$$\Psi_i(\sigma^{**}) = \begin{pmatrix} \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^{P^2}} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^P \partial \delta_i} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^P \partial a_i} \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial \delta_i \partial p_i^P} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial \delta_i^2} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial \delta_i \partial a_i} \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial a_i \partial p_i^P} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial a_i \partial \delta_i} & \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial a_i^2} \end{pmatrix} \tag{B.1}$$

Here, I can calculate all elements of $\Psi_i(\sigma^{**})$ as follows:

$$\begin{cases} \frac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^{P^2}} = -\frac{1}{t} \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial \delta_i^2} = -\frac{a^{**} f(\delta^{**})}{2} - \frac{\{a^{**} F(\delta^{**})\}^2}{t} \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial a_i^2} = \frac{\rho''(a^{**}) F(\delta^{**})}{2} - \frac{1}{t} \left(\int_{-\infty}^{\delta^{**}} F(\delta) d\delta \right)^2 \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^P \partial \delta_i} = \frac{a^{**} F(\delta^{**})}{t} \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial p_i^P \partial a_i} = \frac{1}{t} \int_{-\infty}^{\delta^{**}} F(\delta) d\delta \\ \dfrac{\partial^2 \pi_i(\sigma^{**})}{\partial \delta_i \partial a_i} = -\int_{-\infty}^{\delta^{**}} F(\delta) d\delta \cdot \left(\frac{f(\delta^{**})}{2F(\delta^{**})} + \frac{a^{**} F(\delta^{**})}{t} \right) \end{cases} \tag{B.2}$$

Using these, I obtain every northwest principal minor determinant of matrix $\Psi_i(\sigma^{**})$ as follows:

$$det\Psi_i^1 = det \left(\frac{\partial^2 \pi_i}{\partial p_i^{P^2}} \right) = -\frac{1}{t} < 0 \tag{B.3}$$

$$
\begin{aligned}
det\Psi_i^2 =& det\begin{pmatrix} \dfrac{\partial^2\pi_i}{\partial p_i^{P2}} & \dfrac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i} \\ \dfrac{\partial^2\pi_i}{\partial\delta_i\partial p_i^P} & \dfrac{\partial^2\pi_i}{\partial\delta_i^2} \end{pmatrix} = det\begin{pmatrix} -\dfrac{1}{t} & \dfrac{a^{**}F(\delta^{**})}{t} \\ \dfrac{a^{**}F(\delta^{**})}{t} & -\dfrac{a^{**}f(\delta^{**})}{2}-\dfrac{\{a^{**}F(\delta^{**})\}^2}{t} \end{pmatrix} \\
=& \frac{a^{**}f(\delta^{**})}{2t}+\frac{\{a^{**}F(\delta^{**})\}^2}{t^2}-\frac{\{a^{**}F(\delta^{**})\}^2}{t^2}=\frac{a^{**}f(\delta^{**})}{2t}>0 \qquad \text{(B.4)}
\end{aligned}
$$

$$
\begin{aligned}
det\Psi_i^3 =& det\Psi_i = det\begin{pmatrix} \dfrac{\partial^2\pi_i}{\partial p_i^{P2}} & \dfrac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i} & \dfrac{\partial^2\pi_i}{\partial p_i^P\partial a_i} \\ \dfrac{\partial^2\pi_i}{\partial\delta_i\partial p_i^P} & \dfrac{\partial^2\pi_i}{\partial\delta_i^2} & \dfrac{\partial^2\pi_i}{\partial\delta_i\partial a_i} \\ \dfrac{\partial^2\pi_i}{\partial a_i\partial p_i^P} & \dfrac{\partial^2\pi_i}{\partial a_i\partial\delta_i} & \dfrac{\partial^2\pi_i}{\partial a_i^2} \end{pmatrix} \\
=& \frac{\partial^2\pi_i}{\partial a_i^2}\cdot det\begin{pmatrix} \dfrac{\partial^2\pi_i}{\partial p_i^{P2}} & \dfrac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i} \\ \dfrac{\partial^2\pi_i}{\partial\delta_i\partial p_i^P} & \dfrac{\partial^2\pi_i}{\partial\delta_i^2} \end{pmatrix} + 2\frac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i}\cdot\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}\cdot\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i} \\
& -\frac{\partial^2\pi_i}{\partial p_i^{P2}}\cdot\left(\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}\right)^2-\frac{\partial^2\pi_i}{\partial\delta_i^2}\cdot\left(\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}\right)^2 \\
=& \frac{\partial^2\pi_i}{\partial a_i^2}\cdot det\begin{pmatrix} \dfrac{\partial^2\pi_i}{\partial p_i^{P2}} & \dfrac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i} \\ \dfrac{\partial^2\pi_i}{\partial\delta_i\partial p_i^P} & \dfrac{\partial^2\pi_i}{\partial\delta_i^2} \end{pmatrix} + \frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}\left(\frac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i}\cdot\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}-\frac{\partial^2\pi_i}{\partial p_i^{P2}}\cdot\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}\right) \\
& +\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}\left(\frac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i}\cdot\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}-\frac{\partial^2\pi_i}{\partial\delta_i^2}\cdot\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}\right) \qquad \text{(B.5)}
\end{aligned}
$$

Thus, I obtain the following results:

$$
\begin{aligned}
&\frac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i}\cdot\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}-\frac{\partial^2\pi_i}{\partial p_i^{P2}}\cdot\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i} \\
&\quad=\frac{a^{**}F(\delta^{**})}{t}\cdot\frac{\int_{-\infty}^{\delta^{**}}F(\delta)d\delta}{t}-\left(-\frac{1}{t}\right)\cdot\left\{-\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\right\}\left\{\frac{f(\delta^{**})}{2F(\delta^{**})}+\frac{a^{**}F(\delta^{**})}{t}\right\} \\
&\quad=-\frac{f(\delta^{**})}{2t}\cdot\frac{\int_{-\infty}^{\delta^{**}}F(\delta)d\delta}{F(\delta^{**})} \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2\pi_i}{\partial p_i^P\partial\delta_i}\cdot\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}-\frac{\partial^2\pi_i}{\partial\delta_i^2}\cdot\frac{\partial^2\pi_i}{\partial p_i^P\partial a_i}\\
&\quad=\frac{a^{**}F(\delta^{**})}{t}\cdot\left\{-\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\right\}\left\{\frac{f(\delta^{**})}{2F(\delta^{**})}+\frac{a^{**}F(\delta^{**})}{t}\right\}\\
&\quad\quad-\left\{-\frac{a^{**}f(\delta^{**})}{2}-\frac{(a^{**}F(\delta^{**}))^2}{t}\right\}\cdot\frac{1}{t}\int_{-\infty}^{\delta^{**}}F(\delta)d\delta=0
\end{aligned}\tag{B.7}
$$

By using these results, I can rewrite equation (B.5) as follows:

$$
\begin{aligned}
det\Psi_i=&\frac{\partial^2\pi_i}{\partial a_i^2}\cdot det\Psi_i^2+\frac{\partial^2\pi_i}{\partial\delta_i\partial a_i}\left(-\frac{f(\delta^{**})}{2t}\cdot\frac{\int_{-\infty}^{\delta^{**}}F(\delta)d\delta}{F(\delta^{**})}\right)\\
=&\left[\frac{\rho''(a^{**})F(\delta^{**})}{2}-\frac{1}{t}\left(\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\right)^2\right]\cdot\frac{a^{**}f(\delta^{**})}{2t}\\
&+\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\left[\frac{f(\delta^{**})}{2F(\delta^{**})}+\frac{a^{**}F(\delta^{**})}{t}\right]\cdot\frac{f(\delta^{**})}{2t}\cdot\frac{\int_{-\infty}^{\delta^{**}}F(\delta)d\delta}{F(\delta^{**})}\\
=&\frac{a^{**}f(\delta^{**})F(\delta^{**})\rho''(a^{**})}{4t}\\
&+\frac{1}{t}\left(\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\right)^2\left(-\frac{a^{**}f(\delta^{**})}{2t}+\left[\frac{f(\delta^{**})}{2F(\delta^{**})}+\frac{a^{**}F(\delta^{**})}{t}\right]\cdot\frac{f(\delta^{**})}{2F(\delta^{**})}\right)\\
=&\frac{a^{**}f(\delta^{**})F(\delta^{**})\rho''(a^{**})}{4t}+\frac{1}{t}\left(\int_{-\infty}^{\delta^{**}}F(\delta)d\delta\right)^2\left(\frac{f(\delta^{**})}{2F(\delta^{**})}\right)^2\\
=&\frac{a^{**}f(\delta^{**})F(\delta^{**})\rho''(a^{**})}{4t}+\frac{1}{t}\left[\frac{\int_{-\infty}^{\delta^{**}}F(\delta)d\delta}{F(\delta^{**})}\right]^2\left[\frac{f(\delta^{**})}{2}\right]^2
\end{aligned}\tag{B.8}
$$

Here, $a^{**}\rho''(a^{**})$ can be computed as follows:

$$
\rho''(a_i)=-\frac{2}{h(1-a_i)}-\frac{a_ih'(1-a_i)}{\{h(1-a_i)\}^2}\tag{B.9}
$$

$$
\rho''(a^{**})=-\frac{2}{h(H(\delta^{**}))}-\frac{\{1-H(\delta^{**})\}h'(H(\delta^{**}))}{\{h(H(\delta^{**}))\}^2}\quad\text{(because } a^{**}=1-H(\delta^{**})\text{)}\tag{B.10}
$$

$$
a^{**}\rho''(a^{**})=-\frac{2\{1-H(\delta^{**})\}}{h(H(\delta^{**}))}-\frac{\{1-H(\delta^{**})\}^2h'(H(\delta^{**}))}{\{h(H(\delta^{**}))\}^2}\tag{B.11}
$$

This result simplifies the preceding calculation as follows:

$$
det\Psi_i=-\frac{f(\delta^{**})F(\delta^{**})}{4t}\left[\frac{2\{1-H(\delta^{**})\}}{h(H(\delta^{**}))}+\left(\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\right)^2h'(H(\delta^{**}))\right]+\frac{1}{t}\left[\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\right]^2\left[\frac{f(\delta^{**})}{2}\right]^2\tag{B.12}
$$

Therefore, I need the following conditions for $det\Psi_i < 0$:

$$
\begin{aligned}
det\Psi_i < 0 \iff & \frac{1}{t}\left[\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\right]^2\left[\frac{f(\delta^{**})}{2}\right]^2 \\
& < \frac{f(\delta^{**})F(\delta^{**})}{4t}\left[\frac{2\{1-H(\delta^{**})\}}{h(H(\delta^{**}))}+\left(\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\right)^2 h'(H(\delta^{**}))\right] && \text{(B.13)} \\
\iff & \frac{1}{t}\left[\frac{f(\delta^{**})}{2}\right]^2 < \frac{f(\delta^{**})F(\delta^{**})}{4t}\left[\frac{2h(H(\delta^{**}))}{1-H(\delta^{**})}+h'(H(\delta^{**}))\right] && \text{(B.14)} \\
\iff & \frac{f(\delta^{**})}{F(\delta^{**})} < \frac{2h(H(\delta^{**}))}{1-H(\delta^{**})}+h'(H(\delta^{**})), && \text{(B.15)}
\end{aligned}
$$

which are the second-order conditions for profit maximization. □

Uniqueness of the equilibrium (sufficient condition):

For a uniqueness of equilibrium, I prove that the following equation has a unique solution in δ.

$$
\frac{F(\delta)}{\int_{-\infty}^{\delta} F(t)dt} = \frac{h(H(\delta))}{1-H(\delta)} \tag{B.16}
$$

To this end, I define the following function $\xi(\delta)$ for $\delta \in \mathbb{R}$.

$$
\xi(\delta) := \frac{1-H(\delta)}{h(H(\delta))}F(\delta) - \int_{-\infty}^{\delta} F(t)dt \tag{B.17}
$$

That is, I prove that the equation $\xi(\delta) = 0$ has a unique solution δ^{**}. I obtain the following result:

$$
\begin{aligned}
& \lim_{\delta\to-\infty} \xi(\delta) = 0 && \text{(B.18)} \\
& \lim_{\delta\to+\infty} \xi(\delta) = -\int_{-\infty}^{\infty} F(t)dt < 0 && \text{(B.19)} \\
& \xi'(\delta) = -h(\delta)\frac{h(H(\delta))+\{1-H(\delta)\}h'(H(\delta))}{\{h(H(\delta))\}^2}\cdot F(\delta) + \frac{1-H(\delta)}{h(H(\delta))}\cdot f(\delta) - F(\delta) && \text{(B.20)} \\
& \lim_{\delta\to-\infty} \xi'(\delta) = \frac{1}{h(0)}\lim_{\delta\to-\infty} f(\delta) > 0 && \text{(B.21)}
\end{aligned}
$$

From equations (B.18), (B.19), and (B.21), I can state that $\xi'(\delta^{**}) < 0$ for all δ^{**} is a sufficient condition for the uniqueness of δ^{**} because $\xi(\cdot)$ is a continuous function. The reason is that, if there exist multiple solutions δ^{**}, it is necessary that there exists at least one δ^{**} such that it satisfies $\xi'(\delta^{**}) > 0$. Therefore, the equation $\xi(\delta) = 0$ has only one solution δ^{**} if and only if $\xi'(\delta^{**}) < 0$.

By using the second-order condition (26), which is equivalent to

$$\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot f(\delta^{**}) < 2F(\delta^{**}) + \frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot h'(H(\delta^{**}))\cdot F(\delta^{**}), \tag{B.22}$$

I can derive the sufficient condition for $\xi'(\delta^{**}) < 0$ as follows:

$$\begin{aligned}
\xi'(\delta^{**}) =& - h(\delta^{**})\frac{h(H(\delta^{**})) + \{1-H(\delta^{**})\}h'(H(\delta^{**}))}{\{h(H(\delta^{**}))\}^2}\cdot F(\delta^{**}) \\
&+ \frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot f(\delta^{**}) - F(\delta^{**}) && \text{(B.23)} \\
<& - h(\delta^{**})\frac{h(H(\delta^{**})) + \{1-H(\delta^{**})\}h'(H(\delta^{**}))}{\{h(H(\delta^{**}))\}^2}\cdot F(\delta^{**}) \\
&+ F(\delta^{**}) + \frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot h'(H(\delta^{**}))\cdot F(\delta^{**}) && \text{(B.24)} \\
=& F(\delta^{**})\left[1+\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot h'(H(\delta^{**}))\right]\left[1-\frac{h(\delta^{**})}{h(H(\delta^{**}))}\right] \le 0 && \text{(B.25)} \\
\Longleftrightarrow& \left[1+\frac{1-H(\delta^{**})}{h(H(\delta^{**}))}\cdot h'(H(\delta^{**}))\right]\left[1-\frac{h(\delta^{**})}{h(H(\delta^{**}))}\right] \le 0 && \text{(B.26)}
\end{aligned}$$

The inequality (B.26) is a sufficient condition for $\xi'(\delta^{**}) < 0$, which in turn is also a sufficient condition for the uniqueness of equilibrium δ^{**}. □

Appendix C (Supplementary): Quality difference between basic and premium services

Consider that both competing platforms adopt the freemium business model. Here, for sake of tractability, I solve the game backwards under the *sequential* timing. As in the analysis without quality differentiation, the equilibrium strategies are equivalent both under the *sequential* and *simultaneous* timings. The subsequent analysis is quite similar to the analysis of the model with no quality difference.

Users determine which platform to join based on their expected utilities. First, solving equation $u_i^P = u_i^B$ yields $\delta = (p_i^P - p_i^B - q)/a_i \equiv \delta_i$, which represents the type of user that is indifferent between the basic and premium services of platform i. Users who are tolerant enough of advertisements to satisfy $\delta < \delta_i$ will remain as basic users. Other users who do not like advertisements enough to satisfy $\delta \geq \delta_i$ will upgrade to the premium service.

Thus, users' expected utility from choosing platform i is given by

$$E[u_i] = \int_0^{\delta_i} u_i^B f(\delta) d\delta + \int_{\delta_i}^{\Delta} u_i^P f(\delta) d\delta = v + q - p_i^P + a_i \int_0^{\delta_i} F(\delta) d\delta \tag{C.1}$$

A type of consumer who is indifferent between both platforms is characterized by

$$E[u_1] - tx = E[u_2] - t(1-x) \iff x = \frac{1}{2} + \frac{E[u_1] - E[u_2]}{2t}, \tag{C.2}$$

implying that the total demand for platform i is given by $D_i = \frac{1}{2} + \frac{E[u_i] - E[u_j]}{2t}$. Then, the numbers of users of basic and premium services of platform i are given by $d_i^B = F(\delta_i) D_i$ and $d_i^P = \{1 - F(\delta_i)\} D_i$.

In a similar vein, I can derive the profit function of platform i as follows:

$$\pi_i = \left(p_i^P + \{\rho(a_i) - \delta_i a_i - q\} F(\delta_i)\right) D_i, \tag{C.3}$$

where I use $R_i \equiv p_i^P + \{\rho(a_i) - \delta_i a_i\} F(\delta_i)$ to denote the net revenue that platform i earns from a unit of users.

The first-order conditions with respect to p_i^P, a_i, and δ_i are characterized by

$$\frac{\partial \pi_i}{\partial p_i^P} = D_i - \frac{R_i}{2t} = 0, \tag{C.4}$$

$$\frac{\partial \pi_i}{\partial a_i} = \{\rho'(a_i) - \delta_i\}F(\delta_i)D_i + \frac{R_i}{2t}\int_0^{\delta_i} F(\delta)d\delta = 0, \tag{C.5}$$

$$\frac{\partial \pi_i}{\partial \delta_i} = \left(\{\rho(a_i) - \delta_i a_i - q\}f(\delta_i) - a_i F(\delta_i)\right) D_i + \frac{R_i}{2t} \cdot a_i F(\delta_i) = 0, \tag{C.6}$$

which can be simplified as follows.

$$\begin{cases} D_i = \dfrac{R_i}{2t} \\ \{\rho'(a_i) - \delta_i\}F(\delta_i) + \displaystyle\int_0^{\delta_i} F(\delta)d\delta = 0 \\ \{\rho(a_i) - \delta_i a_i - q\}f(\delta_i) = 0 \end{cases} \tag{C.7}$$

Condition $\rho(a_i) = \delta_i a_i - q$ must be satisfied in equilibrium because $f(\delta)$ is positive for all δ. This result corresponds to that of Lemma 1. Simplifying $\rho(a_i) = \delta_i a_i - q$ implies $\delta_i = H^{-1}(1-a_i) - q/a_i$. By using this and the second line of equation (C.7), I derive the following condition that the equilibrium amount of advertisements $\tilde{a}$ should satisfy:

$$\left(\frac{q}{\tilde{a}} - \frac{\tilde{a}}{h(1-\tilde{a})}\right) F\left(H^{-1}(1-\tilde{a}) - \frac{q}{\tilde{a}}\right) + \int_0^{H^{-1}(1-\tilde{a}) - \frac{q}{\tilde{a}}} F(\delta)d\delta = 0 \tag{C.8}$$

Then, using $\tilde{a}$, the equilibrium threshold value is represented as $\tilde{q} = H^{-1}(1-\tilde{a}) - q/\tilde{a}$.

Moreover, using equation (C.4) and assuming the symmetric equilibrium, I derive the equilibrium price for the premium service as $\tilde{p}_P = t$. Finally, the equilibrium price for the basic service can be computed as $\tilde{p}_B = t - \tilde{\delta}\tilde{a} - q$.