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A Two-Sector Growth Model with Credit Market Imperfections and Production Externalities*

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Abstract

A two-sector dynamic general equilibrium model with financial constraints and production externalities is studied. Agents face idiosyncratic productivity shocks in each period. Agents who draw high productivity borrow resources in the financial market and become capital producers, whereas agents who draw low productivity become lenders. We analyze how the interaction between the extent of financial constraints and sector-specific production externalities affects the characterization of equilibria in a two sector economy.

Key Words: Two production sectors; financial constraints; production externalities; indeterminacy.

JEL Classification: E32, E44, O41.

Mathematical Subject Classification: 37N40, 39A30.

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1 Introduction

We investigate the local dynamics in the neighborhood of a steady state in an economy where agents face financial constraints and goods are produced in two sectors with sector-specific production externalities.

There are earlier works that investigate the local dynamics in an economy with financial constraints. Among others, Kiyotaki and Moore [16] and Cordoba and Ripoll [13, 14] demonstrate that equilibrium is unique and dampening cycles can occur in a collateral-constrained economy. In the model by Cordoba and Ripoll [14], a unique steady state is a saddle point under plausible parameter conditions. Woodford [24] and Barinci and Chéron [5] focus on an economy where capitalists and workers co-exist and show that indeterminacy can occur because workers face financial constraints.¹ Benhabib and Wang [10] and Liu and Wang [18] create a mechanism where financial constraints are the potential source of indeterminacy. In their models, the presence of a fixed cost directly or indirectly affecting financial constraints is a crucial factor for indeterminacy to occur. All the above-mentioned studies assume that an economy is endowed with an aggregate production sector in which only one final good is produced. In the current study, we consider an economy with two production sectors. We analyze how the interaction between the extent of financial constraints and the sector-specific production externalities affects the characterization of equilibria in a two-sector economy.²

Over the past twenty years, many researchers have investigated multiple equilibria or indeterminacy of equilibria in dynamic general equilibrium models.³ It is well known that indeterminacy causes self-fulfilling sunspot business fluctuations (Shell [22]; Azariadis [4]; Cass and Shell [12]; Woodford [23]). In this literature, production externalities have been an important feature of the model because they are a source of inefficiency that causes indeterminacy of equilibria. Among others, Benhabib and Nishimura [8] demonstrate that indeterminacy can easily occur in a model with two production sectors even though production externalities are small, provided that cap-

¹In contrast to Woodford [24], Barinci and Chéron [5] consider an economy with increasing-return-to-scale production externalities.

²We do not assume any fixed cost that affects financial constraints.

³See, for instance, Benhabib and Farmer [6, 7], Benhabib and Nishimura [8], Borlindrin and Rustichini [11], Benhabib et al. [9], Nishimura and Shimomura [19], Nishimura and Venditti [20, 21], and Dufourt et al. [15].

ital goods are labor intensive from the private perspective and capital intensive from the social perspective. In the current paper, a financial market is explicitly introduced in the two production sector model. In particular, agents face financial constraints and can borrow only up to a certain proportion of their own funds in our model. In each period, they receive an idiosyncratic productivity shock. Agents who draw higher productivity become borrowers (capital producers) and agents who draw lower productivity become lenders. In other words, borrowers and lenders endogenously appear in equilibrium.

Our main finding is that under the moderate parameter conditions, if financial constraints are severe, equilibrium is uniquely determined, with the steady state being a saddle point, whereas if the financial constraints are relaxed, equilibrium is indeterminate, with the steady state being totally stable.

The remainder of this paper proceeds as follows. In the next section, the model is presented in which the two production sectors exhibit sector-specific production externalities and agents face financial constraints. In section 3, we derive equilibrium and the dynamical system. In section 4, we characterize the dynamic property around the steady state and obtain the condition for the extent of financial constraints to produce multiple equilibria. Section 5 concludes the current study.

2 Model

A closed economy continues from time 0 to $+\infty$ in discrete time and consists of an infinitely lived representative firm and infinitely lived agents, whose population is equal to 1. In each period, the representative firm produces both consumption and intermediate goods. The intermediate goods are numeraire throughout the current analysis. The infinitely lived agents have potential investment opportunities to produce capital from the intermediate goods, but receive uninsured idiosyncratic productivity shocks in each period that affect the productivity in capital production.

2.1 Agents

2.1.1 Timing of events

Fig. 1 illustrates the timing of the events at time t . At the beginning of time t when the idiosyncratic productivity shock has not yet been realized, an agent earns the following incomes: a wage income, returns to her saving, and a lump-sum profit from the representative firm. The consumption good market at time t is opened at the beginning of the period and is closed before the idiosyncratic productivity shock is realized. Therefore, she must make a decision about consumption and saving at the beginning of time t without knowing her productivity in order to produce capital used at time $t + 1$. At the end of time t , the idiosyncratic productivity shock is realized. There are two saving methods: one is lending her savings in the financial market and the other is initiating an investment project. She optimally chooses one of the saving methods with knowing her productivity. Lending one unit of savings in the financial market at time t yields a claim to r_{t+1} units of intermediate goods at time $t + 1$, where r_{t+1} is the gross real interest rate. Purchasing one unit of intermediate goods at time t for capital production creates Φ_t units of capital used at time $t + 1$, which are sold at price, q_{t+1} , to the production sectors at time $t + 1$. Although she can borrow in the financial market when she starts to produce capital, she faces a financial constraint and can borrow up to a certain proportion of her own funds.

Productivity Φ_t is a random variable on a probability space $(\Omega, \mathcal{F}, P)$, where Ω is a sample space (for which one can assume $\Omega = [0, 1]$), $\mathcal{F}$ is a σ -algebra on Ω , and P is the probability measure. Φ_t is a function of $\omega_t \in \Omega$ and its support is $[0, \eta]$, where $\eta \in (0, \infty)$. The cumulative distribution function of Φ_t is denoted by $G(\Phi) := P(\{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \Phi\})$, which is time-invariant and continuously differentiable on the support, where $\{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \Phi\} \in \mathcal{F}$. $\Phi_0, \Phi_1, \dots$, are independent and identically distributed across both agents and time (the i.i.d. assumption). There is no insurance market for the productivity shocks, and thus, no one can insure against low productivity. Denote the history of ω_t by $\omega^{t-1} = \{\omega_0, \omega_1, \dots, \omega_{t-1}\}$. Then, we can define a probability space $(\Omega^t, \mathcal{F}^t, P^t)$ that is a Cartesian product of t copies of $(\Omega, \mathcal{F}, P)$, where ω^{t-1} is an element of Ω^t . Because the measure of the agent population is equal to one and because of the i.i.d. assumption, ω^{t-1} can denote an individual who experiences the history, $\omega^{t-1} = \{\omega_0, \omega_1, \dots, \omega_{t-1}\}$.

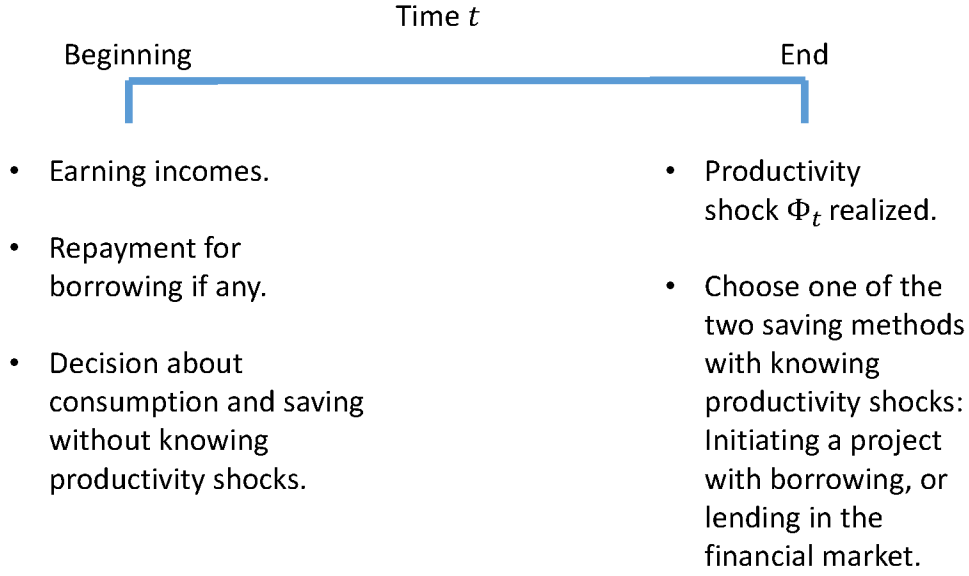


Fig. 1: Timing of events and an agent's behavior

2.1.2 Maximization problem

An agent solves a maximization problem for her lifetime utility given in the following:

$$\max E \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} c_{\tau}(\omega^{\tau-1}) \middle| \omega^{t-1} \right]$$

subject to

$$p_{\tau} c_{\tau}(\omega^{\tau-1}) + s_{\tau}(\omega^{\tau-1}) = q_{\tau} \Phi_{\tau-1}(\omega_{\tau-1}) x_{\tau-1}(\omega^{\tau-1}) + r_{\tau} b_{\tau-1}(\omega^{\tau-1}) + w_{\tau} + \pi_{\tau} \quad (1)$$

$$b_{\tau}(\omega^{\tau}) \geq -\lambda s_{\tau}(\omega^{\tau-1}) \quad (2)$$

$$x_{\tau}(\omega^{\tau}) \geq 0, \quad (3)$$

for $\tau \geq t$, where $\beta \in (0, 1)$ is the subjective discount factor, $c_{\tau}(\omega^{\tau-1})$ is consumption, and $E[.|\omega^{t-1}]$ is an expectation operator given the history, ω^{t-1} . In what follows, by using (2) and (3), we transform Eq. (1) into one budget constraint given by Eq. (6) below.

In (1), w_τ and π_τ are a wage income and a profit that is given from the representative firm in a lump-sum manner, respectively, and p_τ is the price of consumption goods. $s_\tau(\omega^{\tau-1}) := x_\tau(\omega^\tau) + b_\tau(\omega^\tau)$ is the agent's saving at time τ , where $x_\tau(\omega^\tau)$ is intermediate goods used for capital production and $b_\tau(\omega^\tau)$ is lending if $b_\tau(\omega^\tau) > 0$ and borrowing if $b_\tau(\omega^\tau) < 0$. A linear technology with respect to intermediate goods is assumed for capital production such as $\Phi_{\tau-1}(\omega_{\tau-1})x_{\tau-1}(\omega^{\tau-1})$, which is capital produced at time τ . Eq. (1) implies that when the agent makes a decision at time t about consumption, $c_t(\omega^{t-1})$, and saving, $s_t(\omega^{t-1})$, she does not know her productivity, $\Phi_t(\omega_t)$, as previously discussed. However, as noted from the expression $s_t(\omega^{t-1}) = x_t(\omega^t) + b_t(\omega^t)$, she knows $\Phi_t(\omega_t)$ when she makes a portfolio decision about investing in a capital production project, lending, and/or borrowing at time t . Eq. (1) is the flow budget constraint effective for $\tau \geq 1$. At time 0, the flow budget constraint is assumed to be given by $p_0c_0 + s_0 = q_0K_0 + w_0 + \pi_0$, where K_0 is the initial capital endowment that is common across agents.

Inequality (2) is the financial constraint the agent faces at time τ .⁴ Even though an agent is willing to borrow in the financial market, she can do so only up to a partial proportion of her savings, which is her own fund. In (2), $\lambda \in (0, \infty)$ is the extent of financial constraints: the smaller λ is, the more severe the financial constraint is. Inequality (2) can be rewritten as $b_\tau(\omega^\tau) \geq -\mu x_\tau(\omega^\tau)$, where $\mu = \lambda/(1 + \lambda) \in (0, 1)$. Because this constraint is more convenient than inequality (2), we use it henceforth. As μ goes to 1, the financial market approaches perfection, and as μ goes to zero, agents are unable to borrow in the financial market. The purchase of intermediate goods should be nonnegative, and thus, inequality (3) is imposed.

2.1.3 Optimal portfolio decision within a period

We define $\phi_t := r_{t+1}/q_{t+1}$. With knowing the productivity in capital production, agents who draw $\Phi_t > \phi_t$ optimally borrow up to the limit of the financial constraint and purchase intermediate goods for capital production, whereas agents who draw $\Phi_t \leq \phi_t$ lend all their savings in the financial market to acquire the gross interest,

⁴This type of financial constraints is employed by many researchers such as Aghion et al. [2], Aghion and Banerjee [1], and Aghion et al. [3].

r_{t+1} .⁵ Hence, ϕ_t is the cutoff for the productivity shocks that divide agents into lenders and borrowers (capital producers) at time t , and an agent's portfolio program is given by

$$x_t(\omega^t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) \leq \phi_t \\ \frac{s_t(\omega^{t-1})}{1-\mu} & \text{if } \Phi_t(\omega_t) > \phi_t, \end{cases} \quad (4)$$

and

$$b_t(\omega^t) = \begin{cases} s_t(\omega^{t-1}) & \text{if } \Phi_t(\omega_t) \leq \phi_t \\ -\frac{\mu}{1-\mu}s_t(\omega^{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \quad (5)$$

2.1.4 Euler equation

The portfolio program given by (4) and (5) rewrites the flow budget constraint (1) as

$$s_\tau(\omega^{\tau-1}) + p_\tau c_\tau(\omega^{\tau-1}) = R_\tau(\omega_{\tau-1})s_{\tau-1}(\omega^{\tau-2}) + w_\tau + \pi_\tau, \quad (6)$$

where $R_\tau(\omega_{\tau-1}) := \max\{r_\tau, (q_\tau \Phi_{\tau-1}(\omega_{\tau-1}) - r_\tau \mu)/(1 - \mu)\}$. The maximization of the agent's lifetime utility subject to (6) yields the Euler equation as follows:

$$p_{t+1} = \beta E [R_{t+1}(\omega_t) | \omega^{t-1}] p_t. \quad (7)$$

The necessary and sufficient optimality conditions for the lifetime utility maximization problem consist of the Euler equation (7) as well as the transversality condition $\lim_{\tau \rightarrow \infty} \beta^\tau E[s_{t+\tau}(\omega^{t+\tau-1})/p_{t+\tau} | \omega^{t-1}] = 0$.

2.2 Production

The representative firm produces both intermediate and consumption goods, being endowed with Cobb-Douglas technologies:

$$\bar{F}^1(l_t^1, k_t^1, \bar{l}_t^1, \bar{k}_t^1) = A(l_t^1)^{\alpha_L^1} (k_t^1)^{\alpha_K^1} (\bar{l}_t^1)^{a_L^1} (\bar{k}_t^1)^{a_K^1}$$

⁵The derivation of an optimal portfolio allocation of savings follows Kunieda and Shibata [17]. Although agents who draw $\Phi_t = \phi_t$ are indifferent between initiating a capital production project and lending in the financial market, it is assumed that they lend their savings in the financial market.

for intermediate goods, and

$$\bar{F}^2(l_t^2, k_t^2, \bar{l}_t^2, \bar{k}_t^2) = B(l_t^2)^{\alpha_L^2} (k_t^2)^{\alpha_K^2} (\bar{l}_t^2)^{a_L^2} (\bar{k}_t^2)^{a_K^2}$$

for consumption goods, where $\alpha_L^i, \alpha_K^i \in (0, 1)$, $\alpha_L^i + \alpha_K^i + a_L^i + a_K^i = 1$ for $i = 1, 2$, and $\Delta := \alpha_L^1 \alpha_K^2 - \alpha_L^2 \alpha_K^1 \neq 0$. A and B are the productivity parameters. In the production functions, l_t^i and k_t^i are labor and capital used for the production of each good, respectively, and $\bar{l}_t^i$ and $\bar{k}_t^i$ are the components of production externalities with respect to labor and capital, respectively. In equilibrium, it holds that $l_t^i = \bar{l}_t^i$ and $k_t^i = \bar{k}_t^i$, although $\bar{l}_t^i$ and $\bar{k}_t^i$ are exogenous when the firm solves the profit maximization problem. The firm solves the following maximization problem:

$$\max_{l_t^1, l_t^2, k_t^1, k_t^2} \Pi_t := \bar{F}^1(l_t^1, k_t^1, \bar{l}_t^1, \bar{k}_t^1) + p_t \bar{F}^2(l_t^2, k_t^2, \bar{l}_t^2, \bar{k}_t^2) + (1 - \delta)k_t - q_t k_t - w_t l_t, \quad (8)$$

where $\delta \in (0, 1)$ is the capital depreciation rate and $k_t = k_t^1 + k_t^2$ is the total capital in the economy. It is assumed that the remaining capital, $(1 - \delta)k_t$, after production at time t can be used as intermediate goods, and thus, the total intermediate goods, $\bar{F}^1(l_t^1, k_t^1, \bar{l}_t^1, \bar{k}_t^1) + (1 - \delta)k_t$, are sold to capital producers. The total labor supply is given by $l_t^1 + l_t^2 = l_t$, which is equal to the population of agents, i.e., $l_t = 1$. The first-order conditions for the profit maximization problem are given by

$$A\alpha_L^1 \left(\frac{k_t^1}{l_t^1} \right)^{1-\theta_1} = p_t B\alpha_L^2 \left(\frac{k_t^2}{l_t^2} \right)^{1-\theta_2} = w_t. \quad (9)$$

and

$$A\alpha_K^1 \left(\frac{k_t^1}{l_t^1} \right)^{-\theta_1} = p_t B\alpha_K^2 \left(\frac{k_t^2}{l_t^2} \right)^{-\theta_2} = q_t + \delta - 1, \quad (10)$$

where $\theta_i := \alpha_L^i + a_L^i$ for $i = 1, 2$. Note that we have used equilibrium conditions, $l_t^i = \bar{l}_t^i$ and $k_t^i = \bar{k}_t^i$, to obtain (9) and (10). Assumption 1 below is imposed so that the law of demand for each input is satisfied.

Assumption 1. $\theta_i \in (0, 1)$ for $i = 1, 2$.

Eqs. (9) and (10) yield

$$k_t^1 = \frac{\alpha_K^1 w_t}{\alpha_L^1 (q_t + \delta - 1)} l_t^1 \quad \text{and} \quad k_t^2 = \frac{\alpha_K^2 w_t}{\alpha_L^2 (q_t + \delta - 1)} l_t^2 \quad (11)$$

Eqs. (9) and (10) also yield

$$w_t = \Psi p_t^{\frac{1-\theta_1}{\theta_2-\theta_1}} =: w(p_t) \quad \text{and} \quad q_t + \delta - 1 = \Lambda p_t^{\frac{-\theta_1}{\theta_2-\theta_1}} =: v(p_t), \quad (12)$$

where

$$\Psi := [(A(\alpha_L^1)^{\theta_1}(\alpha_K^1)^{1-\theta_1})^{\theta_2-1} (B(\alpha_L^2)^{\theta_2}(\alpha_K^2)^{1-\theta_2})^{1-\theta_1}]^{1/(\theta_2-\theta_1)}$$

and

$$\Lambda := [(A(\alpha_L^1)^{\theta_1}(\alpha_K^1)^{1-\theta_1})^{\theta_2} (B(\alpha_L^2)^{\theta_2}(\alpha_K^2)^{1-\theta_2})^{-\theta_1}]^{1/(\theta_2-\theta_1)}.$$

As in Benhabib and Nishimura [8], it is said that if $\Delta = \alpha_L^1 \alpha_K^2 - \alpha_L^2 \alpha_K^1 > (<)0$, the intermediate goods are labor (capital) intensive from the private perspective and if $\theta_1 > (<)\theta_2$, the intermediate goods are labor (capital) intensive from the social perspective.

3 Equilibrium

A competitive equilibrium is expressed by sequences of prices: $\{w_t, q_t, p_t, r_{t+1}\}$ for all $t \geq 0$ and allocation: $\{k_t, k_t^1, k_t^2, l_t, l_t^1, l_t^2\}$ for all $t \geq 0$ and $\{c_t(\omega^{t-1}), s_t(\omega^{t-1}), x_t(\omega^t), b_t(\omega^t)\}$ for all $t \geq 0$, ω^t , and ω^{t-1} , so that (i) for each ω^t and ω^{t-1} , the consumer maximizes her lifetime utility from time t onward; (ii) the representative firm maximizes its profits in each period; and (iii) consumption and intermediate goods markets, a financial market, a capital market, and a labor market clear.⁶

3.1 Market clearing conditions

Because in each time, the total consumption is equal to the production of consumption goods, the consumption goods market clearing condition is given by

$$C_t := \int_{\Omega^t} c_t(\omega^{t-1}) dP^t(\omega^{t-1}) = F^2(k_t^2, l_t^2), \quad (13)$$

where $F^2(k_t^2, l_t^2) := \bar{F}^2(l_t^2, k_t^2, l_t^2, k_t^2)$. As seen in (4), the intermediate goods are purchased by agents who draw higher productivity, such that $\Phi_t(\omega_t) > \phi_t$. Therefore,

⁶To be accurate, c_0 is not subject to any history of the stochastic events and ω^{-1} is empty.

the intermediate goods market clearing condition is given by

$$\int_{\Omega^t \times (\Omega \setminus \Xi_t)} x_t(\omega^t) dP^{t+1}(\omega^t) = F^1(k_t^1, l_t^1) + (1 - \delta)k_t, \quad (14)$$

where $\Xi_t = \{\omega_t \in \Omega | \Phi_t(\omega_t) \leq \phi_t\}$ and $F^1(k_t^1, l_t^1) := \bar{F}^1(l_t^1, k_t^1, l_t^1, k_t^1)$. Because, in the financial market, all lending and borrowing are cancelled out, it follows that

$$\int_{\Omega^{t+1}} b_t(\omega^t) dP^{t+1}(\omega^t) = 0, \quad (15)$$

which is the financial market clearing condition. From (15), we obtain the following lemma.

Lemma 1. *The financial market clearing condition (15) is satisfied if and only if*

$$\int_{\Omega^t \times \Xi_t} s_t(\omega^{t-1}) dP^{t+1}(\omega^t) = \frac{\mu}{1 - \mu} \int_{\Omega^t \times (\Omega \setminus \Xi_t)} s_t(\omega^{t-1}) dP^{t+1}(\omega^t).$$

Proof. Inserting (5) into the financial market clearing condition (15) yields the desired equation. $\square$

Capital is supplied by agents who draw higher productivity such that $\Phi_t(\omega_t) > \phi_t$ and is demanded by the representative firm. Hence, the capital market clearing condition is given by

$$k_{t+1}^1 + k_{t+1}^2 = \int_{\Omega^t \times (\Omega \setminus \Xi_t)} \Phi_t(\omega_t) x_t(\omega^t) dP^{t+1}(\omega^t). \quad (16)$$

Finally, the labor market clearing condition is given by

$$l_t^1 + l_t^2 = l_t = 1. \quad (17)$$

3.2 Production in equilibrium

From (11), $k_t^1 + k_t^2 = k_t$, and $l_t^1 + l_t^2 = 1$, the production functions become as follows:

$$F^1(l_t^1, k_t^1) = -\frac{\alpha_L^2 v(p_t) k_t - \alpha_K^2 w(p_t)}{\Delta} \quad (18)$$

and

$$p_t F^2(l_t^2, k_t^2) = \frac{\alpha_L^1 v(p_t) k_t - \alpha_K^1 w(p_t)}{\Delta}. \quad (19)$$

From (18) and (19), the gross product, $Y_t = F^1(l_t^1, k_t^1) + p_t F^2(l_t^2, k_t^2)$, is obtained as follows:

$$Y_t = \frac{(\alpha_L^1 - \alpha_L^2) v(p_t) k_t + (\alpha_K^2 - \alpha_K^1) w(p_t)}{\Delta}. \quad (20)$$

3.3 Cutoff

The financial market clearing condition (15) yields a time-invariant cutoff, $\phi_t = \phi^*$, in equilibrium that divides agents into lenders and borrowers.

Proposition 1. *The cutoff, ϕ^* , is given by*

$$G(\phi^*) = \mu. \quad (21)$$

Proof. See the Appendix.

Because the cumulative distribution function is strictly increasing over the support, $\phi^* = G^{-1}(\mu)$ is uniquely determined. As μ increases, ϕ^* increases. This means that as the financial constraints are relaxed, the number of lenders increases and the number of capital producers decreases. Although the number of capital producers decreases, allocative inefficiency with respect to the use of intermediate goods is corrected. This is because the intermediate goods are intensively used by more highly productive agents.

3.4 Dynamical system

To aggregate the flow budget constraint (6) across all agents, we obtain Lemma 2 below, which follows from the financial market clearing condition (15).

Lemma 2.

$$\int_{\Omega^t} R_t(\omega_{t-1}) s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}) = q_t k_t \quad (22)$$

Proof. See the Appendix.

Lemma 2 implies that capital producers sell capital to the representative firm at price q_t . From the microeconomic perspective, the savings of agents who draw lower

productivity are lent out to agents who draw higher productivity, and the lenders are repaid with interest. Therefore, although the returns to individual savings vary depending upon the individual productivity, the total income from all agents' savings is eventually equal to the value of total capital in the economy.

The use of Lemma 2 aggregates the flow budget constraint (6) across all agents and obtains the relationship between the total demand for and the total supply of intermediate goods. The total funds available for capital production consist of capital producers' savings and their borrowing from lenders through the financial market, which is equal to the aggregate saving across all agents. The total funds are used to purchase the intermediate goods. Lemma 3 below describes this situation.

Lemma 3.

$$\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) = F^1(l_t^1, k_t^1) + (1 - \delta)k_t \quad (23)$$

Proof. See the Appendix.

The left-hand side of (23) is the total demand for intermediate goods, and the right-hand side is the total supply. As seen in (4), the intermediate goods are used by the more highly productive agents who draw $\Phi_t(\omega_t) > \phi^*$ to produce capital. Then, Lemma 3 and (4) with the i.i.d. assumption regarding the idiosyncratic productivity shocks yield capital k_{t+1} , as in Proposition 2.

Proposition 2.

$$k_{t+1} = \frac{H(\phi^*)}{1 - \mu} (F^1(l_t^1, k_t^1) + (1 - \delta)k_t), \quad (24)$$

where $H(\phi^*) = \int_{\phi^*}^{\eta} \Phi_t(\omega_t) dG(\Phi)$.

Proof. See the Appendix.

Define $M(\mu) := H(\phi^*)/(1 - \mu)$ in (24). Then, $M(\mu)$ can be considered as the aggregate productivity in the economy. By applying L'Hôpital's rule, one can prove that as $\mu \rightarrow 1$, i.e., as the financial market approaches perfection, it follows that $M(\mu) \rightarrow \eta$. This outcome means that only the agents who draw the highest productivity become capital producers, and the other agents become lenders when the financial market is perfect. In this case, allocative inefficiency regarding the intermediate goods is perfectly corrected, and the highest aggregate productivity is achieved. In contrast, we obtain $M(0) = H(0)$, which is equal to the mean of $\Phi_t(\omega_t)$. When

$\mu = 0$, there is no financial market and no agent can be a lender or a borrower. Instead, all agents become capital producers. The range of variation of $M(\mu)$ depends on the size of the support of the idiosyncratic productivity shocks. Regarding $M(\mu)$, Lemma 4 is formally obtained.

Lemma 4. *As μ increases from 0 to 1, the aggregate productivity, $M(\mu)$, also increases from $M(0)$ to η , where $M(0)$ is the mean of the idiosyncratic productivity shocks.*

Proof. See the Appendix.

Inserting (18) into (24) yields a dynamic equation with respect to capital as follows:

$$k_{t+1} = M(\mu) \left(1 - \delta - \frac{\alpha_L^2 v(p_t)}{\Delta} \right) k_t + \frac{M(\mu) \alpha_K^2}{\Delta} w(p_t). \quad (25)$$

$G(\phi^*) = \mu$ and $\phi^* = r_{t+1}/q_{t+1}$ are used to compute the expected return in Proposition 3 below.

Proposition 3.

$$E[R_{t+1}(\omega_t)|\omega^{t-1}] = q_{t+1}M(\mu). \quad (26)$$

Proof. See the Appendix.

Eqs. (12) and (26) rewrite (7) as follows:

$$\frac{p_{t+1}}{\Lambda p_{t+1}^{\frac{-\theta_1}{\theta_2 - \theta_1}} + 1 - \delta} = \beta M(\mu) p_t, \quad (27)$$

which is a dynamic equation with respect to the price of consumption goods.

3.5 Steady state

Assumption 2.

$$0 < \eta < \frac{1}{1 - \delta}$$

Assumption 2 implies that $(1 - \delta)M(\mu) < 1$ for all $\mu \in [0, 1)$. Under Assumption 2, $(1 - \delta)M(\mu)$ varies from $(1 - \delta)H(0)$ to $(1 - \delta)\eta$ as μ increases from 0 to 1. In the model with a perfect financial market, the aggregate productivity in capital production is

constant. In contrast, the current model allows for the aggregate productivity, $M(\mu)$, to vary from $H(0)$ to η , whose upper limit is $1/(1 - \delta)$.

Assumption 3. $\theta_2 > \theta_1$ and $\Delta = \alpha_L^1 \alpha_K^2 - \alpha_L^2 \alpha_K^1 > 0$.

In Benhabib and Nishimura's model, when the utility function is linear with respect to consumption, indeterminacy of equilibrium always occurs if the intermediate goods are capital intensive from the social perspective, i.e., $\theta_2 > \theta_1$, and labor intensive from the private perspective, i.e., $\Delta > 0$. Therefore, we exclusively examine the case in which $\theta_2 > \theta_1$ and $\Delta > 0$ to investigate whether indeterminacy occurs when the financial market is imperfect.

Under Assumption 2, (27) provides the consumption goods price, p^* , in the steady state, as follows:

$$p^* = \left(\frac{\beta \Lambda M(\mu)}{1 - (1 - \delta) \beta M(\mu)} \right)^{\frac{\theta_2 - \theta_1}{\theta_1}}. \quad (28)$$

Furthermore, under Assumption 2, (12), (25), and (28) yield the capital stock, k^* , in the steady state as follows:

$$k^* = \frac{\alpha_K^2 \Psi \Lambda^{\frac{1 - \theta_1}{\theta_1}} (\beta M(\mu))^{\frac{1}{\theta_1}}}{(\alpha_L^2 + \beta \Delta - (\Delta + \alpha_L^2) \beta (1 - \delta) M(\mu)) (1 - (1 - \delta) \beta M(\mu))^{\frac{1 - \theta_1}{\theta_1}}}. \quad (29)$$

To confirm that the economy produces both intermediate and consumption goods in the steady state, we obtain Lemma 5 below.

Lemma 5. *Under Assumptions 2 and 3, it holds that*

$$\frac{\alpha_K^1 w(p^*)}{\alpha_L^1 v(p^*)} < k^* < \frac{\alpha_K^2 w(p^*)}{\alpha_L^2 v(p^*)}.$$

Proof: See the Appendix.

From (18) and (19), Lemma 5 implies that the economy imperfectly specializes in production and consistently produces both intermediate and consumption goods in the steady state. By continuity, both intermediate and consumption goods are produced in the neighborhood of the steady state. Throughout the analysis, we exclusively focus on the case in which the economy produces both intermediate and consumption goods.

4 Local dynamics

From (25) and (27), the dynamical system with respect to k_t and p_t is given by

$$\begin{cases} k_{t+1} = J(k_t, p_t) \\ \frac{p_{t+1}}{\frac{-\theta_1}{\Lambda p_{t+1}^{\theta_2 - \theta_1} + 1 - \delta}} = \beta M(\mu) p_t, \end{cases} \quad (30)$$

where

$$J(Y_t, p_t) = M(\mu) \left(1 - \delta - \frac{\alpha_L^2 v(p_t)}{\Delta} \right) k_t + \frac{M(\mu) \alpha_K^2}{\Delta} w(p_t).$$

Note that the second equation is expressed by the consumption price only, because we assume that agents' period-wise utility is linear with respect to consumption. The linearization of the dynamical system (30) around the steady state is obtained as

$$\begin{pmatrix} k_{t+1} - k^* \\ p_{t+1} - p^* \end{pmatrix} = \begin{pmatrix} \frac{(\Delta + \alpha_L^2)(1 - \delta)\beta M(\mu) - \alpha_L^2}{\beta \Delta} & J_p(Y^*, p^*) \\ 0 & \frac{\theta_2 - \theta_1}{\theta_2 - \theta_1(1 - \delta)\beta M(\mu)} \end{pmatrix} \begin{pmatrix} k_t - k^* \\ p_t - p^* \end{pmatrix}, \quad (31)$$

where $J_p(k, p) := \partial J(k, p) / \partial p$. The eigenvalues, κ_1 and κ_2 , of this dynamical system are given by

$$\kappa_1 = \frac{(\Delta + \alpha_L^2)(1 - \delta)\beta M(\mu) - \alpha_L^2}{\beta \Delta} \quad (32)$$

and

$$\kappa_2 = \frac{\theta_2 - \theta_1}{\theta_2 - \theta_1(1 - \delta)\beta M(\mu)}. \quad (33)$$

To focus on a typical interesting case, Assumption 4 below is imposed in what follows.

Assumption 4. $1 < \alpha_L^2 / (\beta \Delta) < 1 / (1 - \beta)$.

The value of κ_1 is crucial for the determination of the dynamic property around the steady state, although it is easily shown that $\kappa_2 \in (0, 1)$, as proven in Lemma 6 in the Appendix. Fig. 2 illustrates the relationship between $M(\mu)$ and κ_1 under Assumption 4.

In Fig. 2, two critical values of $M(\mu)$ are defined: $M_1 := (\alpha_L^2 - \beta \Delta) / [(\Delta + \alpha_L^2)(1 - \delta)\beta]$ and $M_2 := \alpha_L^2 / [(\Delta + \alpha_L^2)(1 - \delta)\beta]$. We also define μ_1 and μ_2 , if any, such that $M(\mu_1) = M_1$ and $M(\mu_2) = M_2$. As noted in Fig. 2, M_1 gives $\kappa_1 = -1$ and M_2 gives $\kappa_1 = 0$. Note also that when $M(\mu) = 1 / (1 - \delta)$, we have $\kappa_1 = 1 - (1 - \beta)\alpha_L^2 / (\beta \Delta)$,

which is less than 1 and is greater than 0 from Assumption 4. The value of κ_1 varies depending on the extent of financial constraints.

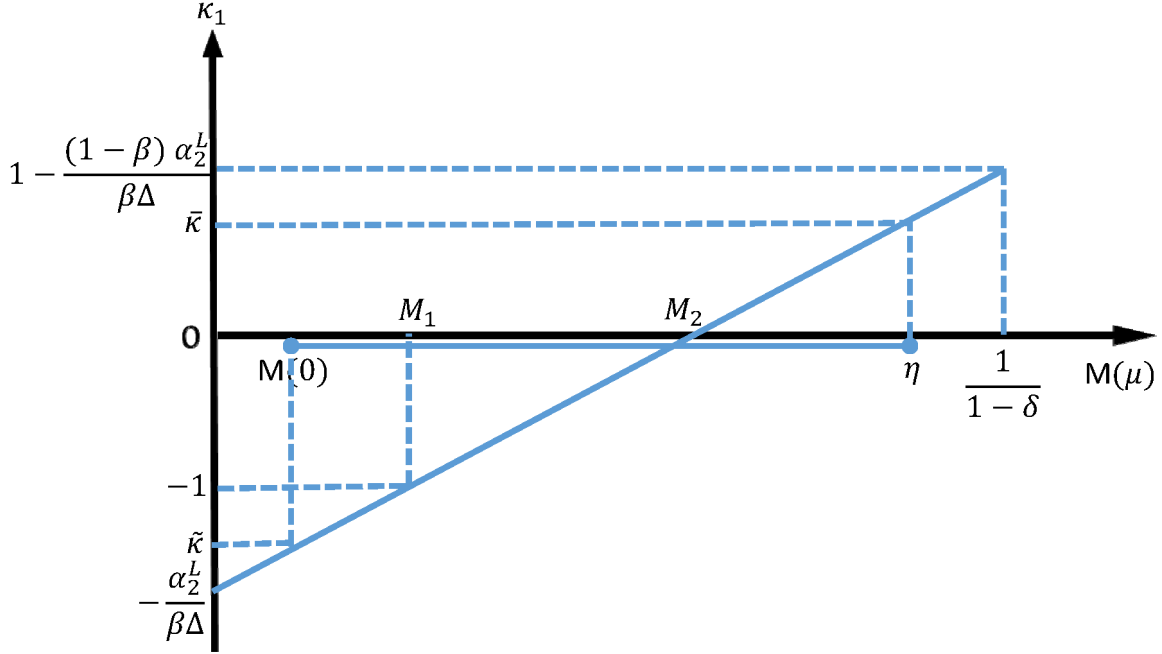


Fig. 2: The relationship between $M(\mu)$ and κ_1 . Notes: From Assumption 4, it follows that $1 - (1 - \beta)\alpha_2^L / (\beta\Delta) \in (0,1)$ and $-\alpha_2^L / (\beta\Delta) \in (-\infty, -1)$ on κ_1 axis.

Theorem 1. Consider the linearized dynamical system (31). Under Assumptions 1-4, suppose that the mean of the stochastic productivity shocks, $M(0)$, is smaller than M_1 and that the maximum, η , is greater than M_2 . Then, if the financial constraint is severe, the steady state is a saddle point and if the financial constraint is relaxed, the steady state is totally stable. More concretely, the following hold.

- If $\mu \in [0, \mu_1)$, the steady state is a saddle point.
- If $\mu \in (\mu_1, 1)$, the steady state is totally stable.

Proof. See the Appendix.

In the dynamical system given by (30), k_t is a state variable that is pre-determined at time t , and p_t is a jump variable. In Theorem 1 when the steady state is a saddle point, for any given initial capital, k_0 , there exist only an initial price of consumption

goods that yields a sequence $\{k_t, p_t\}_{t=0}^{\infty}$ in competitive equilibrium. Accordingly, in this case, equilibrium is uniquely determined. In contrast, when the steady state is totally stable, there exists a continuum of initial prices of consumption goods, each one of which produces a sequence $\{k_t, p_t\}_{t=0}^{\infty}$ in competitive equilibrium. In this case, equilibrium is indeterminate and multiple equilibria occur.

It is widely known that indeterminacy of equilibrium causes self-fulfilling sunspot business fluctuations (Shell [22]; Azariadis [4]; Cass and Shell [12]; Woodford [23]). Extrinsic random variables that may have impacts on agents' expectations without directly affecting economic fundamentals are called sunspots. If the resource allocation in equilibrium depends on the realization of a sunspot random variable, the equilibrium is called a sunspot equilibrium. When indeterminacy arises, extrinsic uncertainty randomizes multiple equilibria. Then, sunspot business fluctuations can occur as a rational expectations equilibrium. Note from Theorem 1 that when the financial constraint is severe, no self-fulfilling sunspot business fluctuations occur, whereas when the financial constraint is relaxed, self-fulfilling sunspot business fluctuations can occur.

5 Concluding remarks

A two-sector dynamic general equilibrium model in which agents face financial constraints and the production sectors exhibit externalities is presented. Whether production externalities cause indeterminacy of equilibria depends on the extent of financial constraints and the size of the support of the idiosyncratic productivity shocks. Under the moderate parameter conditions for a labor income share and sector-specific production externalities, if financial constraints are severe, equilibrium is unique. However, as financial constraints are relaxed, equilibrium is indeterminate, and thus, self-fulfilling sunspot business fluctuations can occur.

Recently, it has often been asserted that financial innovations that relax financial constraints destabilize economies. The outcomes from our analysis are consistent with this assertion. In our model, the relaxation of financial constraints magnifies the destabilization effect of production externalities.

Appendix

Proof of Proposition 1

It follows from Lemma 1 that

$$\int_{\Xi_t} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) dP(\omega_t) - \frac{\mu}{1-\mu} \int_{\Omega \setminus \Xi_t} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) dP(\omega_t) = 0. \quad (\text{A.1})$$

where $\Xi_t = \{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \phi_t\}$. (A.1) can be rewritten as

$$\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) \int_0^{\phi_t} dG(\Phi) - \frac{\mu}{1-\mu} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) \int_{\phi_t}^{\eta} dG(\Phi) = 0,$$

which is computed as

$$\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) \left[G(\phi_t) - \frac{\mu}{1-\mu} (1 - G(\phi_t)) \right] = 0.$$

Solving the last part, we obtain $G(\phi_t) = \mu$. $\square$

Proof of Lemma 2

Because $R_t(\omega_{t-1}) = \max\{r_t, (q_t \Phi_{t-1}(\omega_{t-1}) - r_t \mu) / (1 - \mu)\}$, it follows that

$$\begin{aligned} \int_{\Omega^t} R_t(\omega_{t-1}) s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}) &= \int_{\Omega^t} \max \left\{ r_t, \frac{q_t \Phi_{t-1}(\omega_{t-1}) - r_t \mu}{1 - \mu} \right\} \\ &\quad \times s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}) =: I_t \end{aligned}$$

Define $\Xi_{t-1} = \{\omega_{t-1} \in \Omega \mid \Phi_{t-1}(\omega_{t-1}) \leq \phi_{t-1}\}$, as in the proof of Proposition 1. Because $\phi_{t-1} = r_t / q_t$, I_t can be computed as follows:

$$\begin{aligned} I_t &= \int_{\Omega^{t-1} \times \Xi_{t-1}} r_t s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}) \\ &\quad + \int_{\Omega^{t-1} \times (\Omega \setminus \Xi_{t-1})} \frac{q_t \Phi_{t-1}(\omega_{t-1}) - r_t \mu}{1 - \mu} s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}) \\ &= \int_{\Omega^{t-1} \times (\Omega \setminus \Xi_{t-1})} \frac{q_t \Phi_{t-1}(\omega_{t-1})}{1 - \mu} s_{t-1}(\omega^{t-2}) dP^t(\omega^{t-1}), \end{aligned} \quad (\text{B.1})$$

where the second equality of (B.1) is obtained from Lemma 1. Agents who draw $\Phi_{t-1}(\omega_{t-1}) > \phi_{t-1}$ invest $x_{t-1}(\omega^{t-1}) = s_{t-1}(\omega^{t-2})/(1 - \mu)$, and thus, (B.1) becomes

$$\begin{aligned} I_t &= \int_{\Omega^{t-1} \times (\Omega \setminus \Xi_{t-1})} \frac{q_t \Phi_{t-1}(\omega_{t-1})}{1 - \mu} (1 - \mu) x_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1}) \\ &= q_t \int_{\Omega^{t-1} \times (\Omega \setminus \Xi_{t-1})} \Phi_{t-1} x_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1}). \end{aligned} \quad (\text{B.2})$$

Because $k_t = \int_{\Omega^{t-1} \times (\Omega \setminus \Xi_{t-1})} \Phi_{t-1} x_{t-1}(\omega^{t-1}) dP^t(\omega^{t-1})$, (B.2) becomes

$$I_t = q_t k_t. \quad \square$$

Proof of Lemma 3

By using Lemma 2 and because $\int_{\Omega^t} \pi_t dP^t(\omega^{t-1}) = \Pi_t$, the aggregation of (6) across all agents is obtained as follows:

$$\begin{aligned} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) &= \int_{\Omega^t} [R_t(\omega_{t-1}) s_{t-1}(\omega^{t-2}) + w_t + \pi_t \\ &\quad - p_t c_t(\omega^{t-1})] dP^t(\omega^{t-1}) \\ &= q_t k_t + w_t + \Pi_t - p_t C_t. \end{aligned} \quad (\text{C.1})$$

From (8), we have $F^1(l_t^1, k_t^1) + p_t F^2(l_t^2, k_t^2) + (1 - \delta)k_t = q_t k_t + w_t + \Pi_t$. Additionally, the consumption goods market clearing condition leads to $p_t F^2(l_t^2, k_t^2) = p_t C_t$. Therefore, (C.1) is transformed into

$$\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) = F^1(l_t^1, k_t^1) + (1 - \delta)k_t. \quad \square \quad (\text{C.2})$$

Proof of Proposition 2

Because k_{t+1} is produced by capital producers who draw an individual-specific productivity, $\Phi_t(\omega_t)$, that is greater than ϕ_t , Lemma 3 and the i.i.d. assumption compute

k_{t+1} as follows:

$$\begin{aligned}
k_{t+1} &= \int_{\Omega^t \times (\Omega \setminus \Xi_t)} \Phi_t(\omega_t) x_t(\omega^t) dP^{t+1}(\omega^t) \\
&= \int_{\Omega \setminus \Xi_t} \int_{\Omega^t} \Phi_t(\omega_t) \frac{s_t(\omega^{t-1})}{1-\mu} dP^t(\omega^{t-1}) dP(\omega_t) \\
&= \int_{\phi_t}^{\eta} \frac{\Phi_t(\omega_t)}{1-\mu} dG(\Phi) \int_{\Omega^t} s_t(\omega^{t-1}) dP(\omega^{t-1}) \\
&= \frac{H(\phi^*)}{1-\mu} (F^1(l_t^1, k_t^1) + (1-\delta)k_t),
\end{aligned}$$

where $H(\phi^*) = \int_{\phi^*}^{\eta} \Phi_t(\omega_t) dG(\Phi)$ because $\phi_t = \phi^*$ in equilibrium. $\square$.

Proof of Lemma 4

Obviously, $M(\mu)$ is continuous in $[0, 1)$. The inverse function theorem implies

$$\begin{aligned}
\frac{\partial M(\mu)}{\partial \mu} &= \frac{\partial}{\partial \mu} \left(\frac{H(\phi^*)}{1-\mu} \right) = \frac{-(1-\mu)\phi^*G'(\phi^*)(\partial\phi^*/\partial\mu) + H(\phi^*)}{(1-\mu)^2} \\
&= \frac{\int_{\phi^*}^h \Phi_t(\omega_t) dG(\Phi) - \phi^*(1-G(\phi^*))}{(1-\mu)^2} > 0.
\end{aligned}$$

Therefore, $M(\mu)$ is an increasing function with respect to μ in $[0, 1)$. It is straightforward to verify that $M(0) = H(0)$ is the mean of the idiosyncratic productivity shocks. By applying L'Hôpital's rule, we obtain $\lim_{\mu \rightarrow 1} M(\mu) = \lim_{\mu \rightarrow 1} G^{-1}(\mu)G^{-1'}(\mu)G'(\phi^*) = \eta$. For the last equality, we have used the inverse function theorem again. $\square$

Proof of Proposition 3

Because $\phi_t = r_{t+1}/q(p_{t+1})$ and $\phi_t = \phi^*$, it follows that

$$\begin{aligned}
E[R_{t+1}(\omega_t)|\omega^{t-1}] &= E \left[\max \left\{ r_{t+1}, \frac{q_{t+1}\Phi_t(\omega_t) - r_{t+1}\mu}{1 - \mu} \right\} \middle| \omega^{t-1} \right] \\
&= q_{t+1} E \left[\max \left\{ \phi_t, \frac{\Phi_t(\omega_t) - \phi_t\mu}{1 - \mu} \right\} \middle| \omega^{t-1} \right] \\
&= q_{t+1} \left[\int_0^{\phi^*} \phi^* dG(\Phi) + \int_{\phi^*}^{\eta} \frac{\Phi_t(\omega_t) - \phi^*\mu}{1 - \mu} dG(\Phi) \right] \\
&= q_{t+1} \left[\phi^* G(\phi^*) - \frac{\phi^*\mu}{1 - \mu} (1 - G(\phi^*)) + \frac{H(\phi^*)}{1 - \mu} \right] \\
&= q_{t+1} M(\mu).
\end{aligned}$$

To derive the last equality, Proposition 1 is applied. $\square$

Proof of Lemma 5

From Assumption 2, it follows that $(1 - \delta)M(\mu) < 1$. Then, under Assumption 3, from (12), (28), and (29), it follows that $\text{sign}\{k^* - \alpha_K^1 w(p^*)/(\alpha_L^1 v(p^*))\} = \text{sign}\{1 - (1 - \delta)\beta M(\mu) - \alpha_K^1 \beta(1 - (1 - \delta)M(\mu))\}$. Because $1 - (1 - \delta)\beta M(\mu) - \alpha_K^1 \beta(1 - (1 - \delta)M(\mu)) > (1 - \alpha_K^1 \beta)(1 - (1 - \delta)M(\mu)) > 0$, it follows that $\text{sign}\{k^* - \alpha_K^1 w(p^*)/(\alpha_L^1 v(p^*))\} > 0$. Additionally, it follows that $\text{sign}\{\alpha_K^2 w(p^*)/(\alpha_L^2 v(p^*)) - k^*\} = \text{sign}\{1 - (1 - \delta)M(\mu)\} > 0$. $\square$

Proof of Theorem 1

To prove Theorem 1, two lemmata are prepared.

Lemma 6. *Under Assumptions 1-3, it holds that $0 < \kappa_2 < 1$.*

Proof. It follows from Assumption 2 that $0 < (1 - \delta)\beta M(\mu) < 1$, which leads to $(\theta_2 - \theta_1)/(\theta_2 - \theta_1(1 - \delta)\beta M(\mu)) < 1$ from Assumptions 1-3. Additionally, it follows from Assumption 3 that $(\theta_2 - \theta_1)/(\theta_2 - \theta_1(1 - \delta)\beta M(\mu)) > 0$ $\square$

Lemma 7. *Under Assumptions 1-4, suppose that the mean of the stochastic productivity shocks, $M(0)$, is smaller than M_1 and that the maximum, η , is greater than M_2 .*

Then, as the value of μ increases from 0 to 1, the value of κ_1 increases in the ranges, as in the following.

- As μ increases in $[0, \mu_1)$, κ_1 increases in $[\tilde{\kappa}, -1)$.
- As μ increases in $[\mu_1, \mu_2)$, κ_1 increases in $[-1, 0)$.
- As μ increases in $[\mu_2, 1)$, κ_1 increases in $[0, \bar{\kappa})$,

where $\tilde{\kappa} := [(\Delta + \alpha_L^2)(1 - \delta)\beta M(0) - \alpha_L^2]/(\beta\Delta) \in (-\infty, -1)$ and $\bar{\kappa} < 1 - (1 - \beta)\alpha_L^2/(\beta\Delta) \in (0, 1)$, which is given when $M(\mu) = \eta$.

Proof. $M(\mu)$ is an increasing function with respect to μ . Then, Fig. 2 and (32) prove the claims. $\square$

Proof of Theorem 1: From Lemma 6, we have $|\kappa_2| < 1$. From Lemma 7, if $\mu \in [0, \mu_1)$, $|\kappa_1| > 1$, and if $\mu \in (\mu_1, 1)$, $|\kappa_1| < 1$. Therefore, the steady state is a saddle point if $\mu \in [0, \mu_1)$, and the steady state is totally stable if $\mu \in (\mu_1, 1)$. $\square$

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