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Optimal export policy with upstream price competition*

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Abstract

We present a third-market model with a vertical trading structure, in which upstream input suppliers engage in homogeneous price competition. We show that, under downstream Bertrand competition, a non-monotonic export policy may result. Specifically, the optimal policy of the exporting country can turn into a tax–subsidy–tax as the degree of product substitutability rises. We also confirm the conventional result for which the optimal policy is an export subsidy (tax) if there is Cournot (Bertrand) competition downstream, provided that the number of domestic suppliers is at an intermediate level. We further discuss bilateral policy interventions when both exporting countries offer a subsidy/tax to their domestic downstream firms. We show that a non-monotonic export policy (tax–subsidy–tax) can arise even in this extended setting.

Key words: Upstream price competition; Export subsidy/tax; Non-monotonic policy; Product substitutability

JEL classification: F12; F13; L13; D43

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1 Introduction

Recognizing the growing importance of vertical relations in international trade,[1] researchers investigated how the vertical structure affects a government's incentives related to export policies.[2] When upstream and downstream markets are imperfectly competitive, export subsidies/taxes on a downstream domestic firm have two kinds of rent-shifting effects: horizontal effects from downstream foreign rivals and vertical effects from upstream input suppliers. Thus, accounting for imperfectly competitive upstream markets, the conventional results on strategic export promotion can vary.

In line with this argument, Bernhofen (1997) indicates that if a monopoly input supplier exists outside the country, an export subsidy makes input demand less elastic and enables the supplier to set a higher input price, so the government's incentives to subsidize are weakened.[3] Although the study sheds light on the fact that a vertical (horizontal) rent-shifting effect tends to indicate an export tax (subsidy), it assumes a homogeneous Cournot duopoly in a third market. By contrast, Chou (2011) extends Bernhofen's model to a differentiated duopoly and

[1]Vertical relations are a major feature of contemporary international trade. As the fragmentation in production processes increases, each country specializes in producing particular intermediate inputs or in specific production stages, and vertical trading chains reach many countries (Hummels et al., 2001). Additionally, with progress in trade cost reduction and trade liberalization, the use of imported inputs has been expanding and the trade of inputs plays a central role in the goods trade (Ali and Dadush, 2011). For example, the World Trade Organization (2009, 2013) reports that trade in inputs (excluding fuel) accounted for 40% of total trade in 2008, and the share of inputs in non-fuel exports was over 50% during 2000–2011.

[2]Several authors study strategic trade policies under vertically related markets. Spencer and Jones (1991, 1992) examine the use of tariffs for final goods and of subsidies for inputs. The former (latter) focuses on export taxes (import subsidies) for inputs produced by domestic (foreign) vertically integrated firms. Bernhofen (1995) analyzes the role of anti-dumping tariffs in a reciprocal dumping model with input markets. Ishikawa and Spencer (1999) consider export policies in a more general setting, where upstream and downstream markets consist of many firms. Chang and Sugeta (2004) study optimal export policies in a case where an upstream monopolist and downstream firms engage in Nash bargaining. Hwang et al. (2007) focus on the role of the production function's returns to scale for a downstream monopolist.

[3]Ishikawa and Spencer (1999) also emphasize a similar rent-shift among upstream and downstream producers. That is, if an export subsidy not only shifts rents from foreign to domestic downstream firms, but also simultaneously shifts rents to foreign input suppliers, the government's incentives to subsidize are weakened.

shows that, despite Cournot rivalry, the optimal export policy can be a tax because, if the degree of product substitutability is small and the upstream monopolist has uniform pricing, the downstream competition is gentle and the vertical rent-shifting effect dominates the horizontal effect. Input market integration is also important. Kawabata (2010) shows that in a third-market model with a differentiated duopoly, if each exporter has an input supplier and the input market is integrated, the horizontal rent-shifting effect can dominate the vertical effect, and, thus, the government may offer an export subsidy, despite Bertrand rivalry.[4]

Past studies demonstrate that market structures, product substitutability, and input supplier behavior can influence the horizontal and vertical rent-shifting effects due to export policies.[5] However, they assume that the input market is a monopoly or Cournot oligopoly, which overlooks the implications of *upstream price competition*. Given that we do not observe Cournot industries very frequently,[6] it is important to study price competition in the upstream market.

Our purpose is to consider the implications of upstream price competition for the optimal export policy. To that aim, in a third market model with differentiated products, we incorporate Dastidar (1995)-type price competition into the input market and show that, if Bertrand rivalry exists downstream, the optimal export policy can be *tax–subsidy–tax*, depending on the degree of product substitutability. Under downstream Bertrand rivalry, the domestic firm's exports are U-shaped for product substitutability. Hence, raising the degree of substitutability when it is at a low level reduces the domestic firm's exports and worsens its competitive position, so it weakens the incentives for taxation. Conversely, a high degree of substitutability raises the

[4]In a similar setting, Kawabata (2012) emphasizes the role of cost asymmetries among downstream firms and examines export policies for both upstream and downstream firms.

[5]Some researchers study other policy issues in free trade areas (FTAs). Takauchi (2010) examines the interaction between export subsidy/tax and local content rate of FTA inputs. Kawabata et al. (2010) consider optimal import tariffs on both inputs and final goods in an FTA consisting of an importer and an exporter.

[6]For example, using annual data spanning the period from 1961 to 1990 for 70 Japanese manufacturing industries, Flath (2012) empirically shows that, although the Cournot specification is the most likely one for five industries, the Bertrand specification is the most likely one for 35 industries.

domestic firm's exports, and incentives for taxation are strengthened. The optimal export policy has a *tax–subsidy–tax* shape, so it becomes a tax when substitutability is low or high. When the actual level of substitutability is low, even if practitioners accidentally think it is high, the welfare loss may be small because the realized policy is a tax. In contrast, if the practitioners' estimate of the degree of product-substitutability is slightly inaccurate and substitutability is slightly higher than the estimated value, a policy contrary to the optimal one is implemented and the welfare loss may be considerable.[7] This implies that when implementing export policies, a case exists in which "*a large error is permissible, whereas a small error is not allowable*," indicating that the common belief that "great mistakes are impermissible" does not always hold. We believe that our result gives a new insight into trade policy.

We also show that, when the number of domestic input suppliers is at an intermediate level, the optimal export policy is a subsidy (tax) if downstream firms compete in a Cournot (Bertrand) fashion (Brander and Spencer, 1985; Eaton and Grossman, 1986). Because a larger number of domestic input suppliers can enhance welfare by increasing exports and demand for inputs, the incentives to subsidize become stronger as the number of domestic input suppliers rises. Under downstream Bertrand rivalry, since the incentive to tax is stronger than that in Cournot rivalry, a larger number of input suppliers than under Cournot rivalry are required to realize a subsidy and the threshold number of domestic input suppliers for which the optimal export policy is a subsidy is larger than the one under Cournot rivalry.

We further discuss a bilateral intervention, in which two identical exporting countries offer export subsidies or taxes to their domestic firms. We demonstrate that our main result—that the optimal export policy has a "tax–subsidy–tax" shape—is also true under this extension.

The rest of the paper is organized as follows: Sections 2 and 3 show the model and present the

[7] This type of welfare-loss can also appear in cases other than the "tax–subsidy–tax" Case. For detailed arguments, see Section 3.1.

main results. Section 4 provides some discussions of our results. Section 5 offers the conclusions.

2 Model

We consider a third-market model with upstream price competition. The downstream market consists of two final goods producers, firms H and F; firm i $(= H, F)$ is located in country i. We call country H (F) the Home (Foreign) country. Each firm i exports its product to the third market.[8] The demand and inverse demand functions in the third market are $q_i = \frac{(1-b)a - p_i + bp_j}{1-b^2}$ and $p_i = a - q_i - bq_j$, respectively, for $i, j = H, F$; $i \neq j$, where $a > 0$, p_i and q_i are the price and quantity supplied by firm i, and b $(\in [0, 1))$ measures the degree of product substitutability. The Home and Foreign products are perfect substitutes when $b = 1$, and are independently consumed when $b = 0$. We assume that firms have identical and linear production technology, where one unit of input is used to produce one unit of the final good. We also assume that any other production costs are normalized to zero, that is, the firm's production cost is the price of the purchased input, r. To focus on the government's incentives for choosing a policy, we examine the case where only the Home country subsidizes.[9] The Firms' profits are $\Pi_H \equiv (p_H - r + s_H)q_H$ and $\Pi_F \equiv (p_F - r)q_F$, where s_H is a per-unit subsidy/tax and s_H is a subsidy (tax) when it is positive (negative). In Section 4, we examine a bilateral intervention (i.e., the Foreign country also offers subsidies/taxes to its domestic firm).

In the upstream world market, there are n (≥ 2) symmetric input suppliers (hereafter called the *suppliers*). Each supplier k $(\in \{1, ..., n\})$ produces homogeneous inputs and offers them at a price of r_k. We denote supplier k's demand and the aggregate demand for inputs by q_k and Q $(= q_H + q_F)$, respectively. Since firm i purchases inputs from the supplier offering the lowest price, the demand for supplier k is $q_k = Q(r_{\min})/n_{\min}$ if the supplier offers the minimum price

[8] For simplicity, we omit trade costs in this analysis.

[9] Ishikawa and Spencer (1999) and Hwang et al. (2007) also consider a unilateral intervention.

$r_k = r_{\min}$, where $n_{\min}$ is the number of suppliers offering the minimum price; the demand is $q_k = 0$ if supplier k does not offer the minimum price. We assume that, for supplier k, the cost for producing inputs has a quadratic form and specify it as $(c/2)q_k^2$, where c (> 0) denotes the production efficiency. The profit of supplier k is $\pi_k = q_k r_k - (c/2)q_k^2$.

We assume that there are m $(\in [0, n])$ suppliers in the Home country and that the others are in a country other than the Home and Foreign countries. The welfare of the Home country is

$$W_H \equiv \Pi_H + m\pi_k - s_H q_H. \tag{1}$$

The game consists of three stages. In the first stage, the Home country government decides the level of s_H. In stage two, the input price r is determined through supplier price competition. In the final stage, each firm decides the price (quantity) of its product.[10] We solve the game by backward induction.

Since we assume that suppliers produce homogeneous inputs with a quadratic cost, there is a range of Nash equilibria (Dastidar, 1995). Thus, we need to employ some criterion for selecting equilibrium prices. We use the *payoff-dominance* criterion.[11] This approach is similar to that in Cabon-Dhersin and Drouhin (2014). Moreover, some studies focus on a collusive price to narrow the set of Nash equilibria (e.g., Dastidar, 2001; Gori et al., 2014). This criterion is

[10]We also assume, as in many studies examining the effects of trade policies in vertical oligopoly models, that downstream firms are the price-takers of inputs (e.g., Bernhofen, 1995, 1997; Chou, 2011; Hwang et al., 2007; Ishikawa and Spencer, 1999; Kawabada, 2010, 2012; Kawabata et al., 2010; Takauchi, 2010). However, this assumption is open to the criticism that downstream firms have market power in the final-good market, but no market power in the input market. As regards this criticism, Ishikawa and Spencer (1999) provide a discussion to justify the assumption when the number of downstream firms is small by introducing a positive integer "K" of identical downstream industries. According to Ishikawa and Spencer (1999), when all downstream industries purchase inputs from a single upstream industry, because the input demanded is K times the number of downstream firms, the single upstream industry is able to omit the monopsony power in the downstream firms if K is large. Based on this argument, they consider a representative downstream industry (i.e., $K = 1$) to simplify the analysis. For a more detailed discussion, see Ishikawa and Spencer (1999), pp. 204–205.

[11]In Section 4, we discuss the case with another equilibrium price selection mechanism.

also similar to that in our approach. We assume a collusive price higher than or equal to an upper bound of the set of Nash equilibria, whereas previous studies with collusive price criteria restrict the parameters so that the collusive price is in the set of Nash equilibria.

Finally, we explain how to select a price with the payoff dominance criterion in the set of Nash equilibria. We assume symmetric suppliers, so that in any pure-strategy Nash equilibrium in the upstream market, each supplier chooses the same price. We denote the set of prices each supplier chooses in the Nash equilibrium by $[\underline{r}, \bar{r}]$. In addition, we denote the collusive price that maximizes the suppliers' joint profits by r_{col}. Because we assume a range of parameters where $\bar{r} \leq r_{col}$, the input price each supplier selects must be equal to $\bar{r}$, which provides the highest profit for suppliers in the set of Nash equilibria.

3 Results

3.1 Downstream Bertrand

We first consider the case where the downstream exhibits Bertrand rivalry. In the third stage of the game, the FOC to maximize the profit of firm i $(= H, F)$, $\partial \Pi_i / \partial p_i = 0$, yields its exports $q_i(r, s_H)$ and total sales $Q(r, s_H)$.

In the second stage, the input price r is determined through Dastidar (1995)-type price competition.[12] In the pure-strategy Nash equilibria, two conditions, $\pi_k(n, r, s_H) = [Q(r, s_H)/n]r - (c/2)[Q(r, s_H)/n]^2 \geq 0$ and $\pi_k(n, r, s_H) \geq \pi_k(1, r, s_H) = Q(r, s_H)r - (c/2)[Q(r, s_H)]^2$, must be satisfied. The first is the condition that suppliers *do not raise* their prices and yields the lower bound $\underline{r} = \frac{(2a+s_H)c}{2[(2-b)(1+b)n+c]}$; the second is the condition that suppliers *do not reduce* their prices and gives the upper bound $\bar{r} = \frac{(2a+s_H)(1+n)c}{2[(2-b)(1+b)n+(1+n)c]}$. Thus, in equilibrium, the input price must lie between $\underline{r}$ and $\bar{r}$. Moreover, from the symmetry in suppliers, the collusive price r_{col} is given

[12] See Dastidar (1995), pp. 27–28. Moreover, Delbono and Lambertini (2016a, 2016b) employ Dastidar (1995)-type price competition. See also Cabon-Dhersin and Drouhin (2014), pp. 432–433, for equilibrium price selection.

by $r_{col} = \text{argmax}_r\, \pi_k(n, r, s_H) = \frac{[(2-b)(1+b)n+2c](2a+s_H)}{4[(2-b)(1+b)n+c]}$. These prices yield the following lemma.

Lemma 1. *(i) $r_{col} > \underline{r}$. (ii) $r_{col} \geq \bar{r}$ if and only if $c \leq c_B \equiv [(2-b)(1+b)n]/(n-1)$.*

Proof. (i) Simple algebra yields $r_{col} - \underline{r} = \frac{(2-b)(1+b)n(2a+s_H)}{4[(2-b)(1+b)n+c]} > 0$.

(ii) Since $r_{col} - \bar{r} = \frac{(2-b)(1+b)n(2a+s_H)[-c(n-1)+(2-b)(1+b)n]}{4[(2-b)(1+b)n+c][(2-b)(1+b)n+(1+n)c]}$, $r_{col} \geq \bar{r}$ iff $c \leq \frac{(2-b)(1+b)n}{n-1} \equiv c_B$.

Q.E.D.

Lemma 1 states that $r_{col} \in (\underline{r}, \bar{r})$ for $c > c_B$. Thus, the second stage input price is a collusive price, $r = r_{col}$, for $c > c_B$. However, if $r = r_{col}$, the conditions determining the sign of optimal export policy in the downstream Bertrand case (part 2 of Proposition 1) become highly complicated. To avoid unnecessary complexity in the analysis and to obtain clear-cut conditions, we assume $c \leq c_B$. With this restriction on c, since $\pi_k(n, r, s_H)$ is an increasing function for r in the interval $[\underline{r}, \bar{r}]$,[13] the second stage input price is $r = \bar{r}$.

Assumption 1. $0 < c \leq c_B$.

On the other hand, even if $c > c_B$, our main result (Proposition 1) does not qualitatively change. If $c > c_B$ (i.e., $r = r_{col}$), the optimal policy can non-monotonically change with respect to product substitutability. Particularly, in a case where $n = 2$, we obtain a similar result as in part 2 of Proposition 1, even though $r = r_{col}$ holds. We state this result as "Proposition 4" in Appendix A (See Fig. 6).

In the first stage, the Home government chooses s_H to maximize (1).[14] We use Kawabata's

[13] In a standard firms' profit maximization problem, it is required that the second order condition holds. By contrast, in our model, the profit maximization (or optimal input price setting) problem of suppliers does not rely on the differentiation of input price, so we do not need the standard second order condition of profit maximization. However, because the (second-stage) profit of suppliers is a concave function with respect to the input price r (i.e., $\partial^2 \pi_k(n, r, s_H)/\partial r^2 < 0$), the second order condition of a standard problem is also satisfied. For the proof, see Appendix B.

[14] The SOC for welfare maximization always holds, that is, $\partial^2 W_H(s_H)/\partial s_H^2 < 0$.

(2010) decomposition to express the FOC for welfare maximization as follows:

$$\frac{\partial W_H}{\partial s_H} = \overbrace{(p_H - r)\frac{\partial q_H}{\partial s_H}}^{\text{(i)}} + \overbrace{q_H\frac{\partial p_H}{\partial s_H}}^{\text{(ii)}} + \overbrace{\left[-\left(q_H - m\left(\frac{Q}{n}\right)\right)\frac{\partial r}{\partial s_H}\right]}^{\text{(iii)}}$$
$$+ \underbrace{\frac{m}{n}\left(r - c\left(\frac{Q}{n}\right)\right)\frac{\partial Q}{\partial s_H}}_{\text{(iv)}} = 0. \tag{2}$$

Equation (2) consists of the four effects of a subsidy:[15] (i) *the horizontal rent-shifting effect* on the Home country final product, which corresponds to the first term and is positive; (ii) *the terms of trade effect* for the Home country final product, which corresponds to the second term and is negative; (iii) *the rent extraction effect* from suppliers, which corresponds to the third term and is negative if $(q_H - m(Q/n)) > 0$; and, (iv) *the efficiency gain effect* from an increase in domestic input production, which corresponds to the forth term and is positive if $m > 0$.[16] Terms (i) and (ii) are the *horizontal* effects of the subsidy, whereas (iii) and (iv) are the *vertical* effects. Terms (iii) and (iv) are both increasing in the number of domestic suppliers. An increase in m weakens the negative effect (i.e., tax incentive) in the third term and strengthens the positive effect (i.e., subsidy incentive) in the fourth term: $\frac{\partial(\text{third term})}{\partial m} = \frac{c(1+n)(2a+s_H)}{2[(2-b)(1+b)n+(1+n)c]^2} > 0$ and $\frac{\partial(\text{fourth term})}{\partial m} = \frac{c(n-1)(2a+s_H)}{2[(2-b)(1+b)n+(1+n)c]^2} > 0$.

From (2), (A1), and (A2),[17] we calculate the optimal export policy as

$$s_H^B = \frac{2a(1-b)(2+b)n}{D}[2(2+b)cm - (b^3 + b^2 + c)n - c], \tag{3}$$

where $D \equiv 2[4(2-m+2n)+4b(1+n)-b^2(3+b)(1-m+n)]cn+(3+b)c^2(1+n)^2+8(1+b)(2-b^2)n^2 > 0$. The superscript "$B$" ("$C$") denotes the SPNE outcomes in the case of Bertrand (Cournot) rivalry. (Appendix D reports the SPNE outcomes.)

Let us first refer to a result in an existing study, which was obtained by a specific combination

[15] See Kawabata (2010), pp. 119–120.

[16] $(r - (c/n)Q) > 0$ holds.

[17] We present (A1) and (A2) in Appendix C.

of the number of suppliers in our model. When there is no domestic supplier and the upstream is a monopoly, that is, $m = 0$ and $n = 1$, from (3), the optimal export policy is a tax. This finding matches that of Chou (2011), who finds that even if there is an upstream monopolist other than the exporter, Eaton and Grossman's (1986) result does not change.

Remark 1. (Chou, 2011) *If $m = 0$ and $n = 1$, $s_H^B < 0$.*

If $m = 0$, the fourth term in (2) disappears, and, hence, the negative effects of the second and third terms dominate the positive effect of the first term.

Using (3), we establish the following proposition, the proof of which is given in Appendix A.

Proposition 1. *Suppose that the third-country downstream market has differentiated Bertrand rivalry and Assumption 1 holds. Then,*

1. *$s_H^B > 0$ if $m > m_B \equiv \frac{(b^3+b^2+c)n+c}{2(2+b)c}$, $s_H^B = 0$ if $m = m_B$, and $s_H^B < 0$ if $m < m_B$.*
2. *If $m \in (m_l, \min\{m_h, n\})$, then there exist two thresholds, b_1 and b_2, such that $s_H^B > 0$ for $b_1 < b < b_2$, and $s_H^B < 0$ for $0 \leq b < b_1$ or $b_2 < b < 1$. Here, $m_h = \min\{m_B\,|_{b=0}, m_B\,|_{b=1}\}$ and $m_l \equiv m_B\,|_{b=b_l}$, where b_l $(\in (0,1))$ minimizes m_B.*

Proposition 1 offers two important assertions. The first is that there is a threshold in the number of domestic suppliers that makes the optimal policy a subsidy. The second one demonstrates that the optimal policy can non-monotonically change, that is, follow a *tax–subsidy–tax* pattern, as the degree of substitutability between final goods increases. Fig. 1 illustrates the second part of Proposition 1 (See also Panel (b) in Fig. 2).

We explain the first assertion as follows: For a given number of suppliers n, an increase in the number of domestic suppliers m strengthens the positive effect of the fourth term in (2), whereas it can weaken the (negative) effect of the third term. This increases the motive to subsidize.[18]

[Figure 1 around here]

[Figure 2 around here]

To explain the logic behind the second assertion, for given c and n, let us consider four different sizes for m. Panels (a)–(d) in Fig. 2 illustrate the optimal export policy corresponding to each size of m. When m is small, the third term in (2) (tax incentive) is large and the fourth term (subsidy incentive) is small. Therefore, the optimal policy is a tax (see the "$m = 3$" case in Panel (a)). In contrast, when m is large, the magnitude of the third term (tax incentive) is small and that of the fourth term (subsidy incentive) is large. Therefore, the Home government considers domestic suppliers to be important and offers a subsidy to firm H to promote exports and domestic input production (see the "$m = 8$" case in Panel (d)). The limit case where $b = 1$ implies a homogeneous Bertrand competition in the downstream market, and hence, the firm's rent vanishes. Thus, the optimal policy approaches 0 as b approaches 1, regardless of the size of m.

When the value of m is intermediate, the role of b becomes more significant. In differentiated Bertrand rivalry, it is well-known that firms' output is U-shaped in b.[19] Hence, firm H's exports tend to increase as b exceeds a certain level. The positive effect of the fourth term depends on total sales (outputs), so it is also U-shaped in b. On the one hand, since the positive effect of the first term depends on the prices of both the input and the final product, it is not necessarily U-shaped in b. In contrast to the positive effect of the fourth term, the negative effects of the second and third terms can be inverted-U shaped in b. That is, if b affects the U-shape of the outputs of firm H, the tax-incentive decreases as b increases from a value below a certain level because the domestic firm's exports decrease. However, the incentive for taxation increases as

[18] $\partial s_H^B / \partial m > 0$. See Appendix E.

[19] We can immediately derive this characteristic by substituting $s_H = 0$ into q_H in (A1).

b exceeds a certain level because the domestic firm's exports increase in this case.

A rise in m reduces the magnitude of the third term. On the one hand, if m is not large relative to n, the magnitude of the third term is not necessarily small because the vertical rent-extraction can come from $n - m$ non-domestic suppliers. This implies that under a certain size of m, the change in the negative effects (second and third terms) due to an increase in b can dominate the change in the positive effects (first and fourth terms). When m is of an intermediate size, a reduced tax dominates if b has a smaller value. Within such a range of b, the optimal policy can change from a tax to a subsidy as b increases.[20] If b exceeds a certain level and enters the area where "an increase in the tax incentive," the optimal policy can change from a subsidy to a tax as b increases (see the "$m = 5$" case in Panel (b)).

In particular, the second part of Proposition 1 has a significant policy implication because the non-monotonicity in the optimal export policy implies that *a big mistake does not matter, but a small mistake can be fatal.* For example, in the case of "tax–subsidy–tax," the optimal taxes appear in two regions: one of lower b values and one of a higher b values. Thus, even if practitioners incorrectly recognize that "b is high" when its actual value is low, since they choose an export tax, the welfare loss may not be so large. In contrast, if there is a small gap between the practitioner's recognition of b and its actual value, he or she may unfortunately adopt a policy that is not recommended. This possibly yields a serious welfare-loss.

Such a welfare-loss also can appear in cases other than the "tax–subsidy–tax" case. This is because, when the optimal policy exhibits an inverted-U shaped region with respect to b, the same level of subsidy can result from two b values: a lower and a higher one. Panel (c) in Fig. 2 illustrates this situation. In Panel (c), for $b = b'$ and $b = b''$, the optimal policy has the same subsidy level. Suppose that the actual value of b is b'. Then, if the practitioners make a large

[20] In the case where "$m = 6$" (Panel (c) in Fig. 2), the effects of b are also important. However, in this case m is relatively large, and the subsidy incentive is larger than that in the case where $m = 5$ so the optimal policy becomes an export subsidy even if b is small.

mistake and estimate that $b = b''$, a welfare loss *does not* occur. However, if their estimate differs slightly from b', the level of the implemented subsidy is not at the optimal subsidy level and a welfare loss occurs.

3.2 Downstream Cournot

We next consider the case where downstream Cournot rivalry exists. To distinguish between the Bertrand and Cournot cases, we mark the variables in the Cournot case with a star ("$*$"). In the third stage of the game, the FOC for the profit maximization of firm i $(= H, F)$, $\partial \Pi_i / \partial q_i = 0$, yields the firm's exports $q_i^*(r, s_H)$ and total sales $Q^*(r, s_H)$. In the second-stage, the input price r^* is determined in a similar manner as in the previous section and yields the second-stage outcomes (see Appendix C, (A3)).

From (2), (A3), and (A4),[21] the optimal export policy in the Cournot case is

$$s_H^C = \frac{2a(2-b)n}{E}[b^2 n - (1-b)(1+n)c + 2(2-b)cm], \tag{4}$$

where $E \equiv 2\left[((1-b)(5+b)+3)(1+n) + ((4-b)b-4)m\right]cn + (3-2b)c^2(1+n)^2 + 8(2-b^2)n^2 > 0$. Here, we assume that $m < \min\{m_0, n\}$ (for m_0, see Appendix D).

Using (4), we obtain the following proposition, the proof of which is given in Appendix A.

Proposition 2. *Suppose that the third-country downstream market has differentiated Cournot rivalry. Then, (I) (i) If* $b > \widetilde{b}$*, or (ii)* $m > m_C$ *and* $b < \widetilde{b}$*, then* $s_H^C > 0$*. (II) If* $m = m_C$ *and* $b < \widetilde{b}$*, then* $s_H^C = 0$*. (III) If* $m < m_C$ *and* $b < \widetilde{b}$*, then* $s_H^C < 0$*. Here,* $m_C \equiv \frac{(1-b)(1+n)c - b^2 n}{2(2-b)c}$*,* $\widetilde{b} \equiv \frac{-(1+n)c + \sqrt{c(1+n)[c(1+n)+4n]}}{2n}$*, and* $0 < \widetilde{b} < 1$*.*

[Figure 3 around here]

Fig. 3 illustrates Proposition 2 in the b-m plane. We start by examining the case where

[21] Appendix C presents (A3) and (A4).

$m = 0$, in which the fourth term in (2) disappears and the vertical effect is equal to the "vertical rent-extraction effect" (tax incentive). When $m = 0$, the export policy depends on b. A smaller b corresponds to weaker competition between firms, and, thus, the horizontal rent-shifting effect (i.e., the positive effect of the first term in (2)) is weak. In this case, the vertical rent-extraction effect is dominant, so the optimal policy is a tax. Conversely, if b is close to unity, the competition between firms is keener and the horizontal rent-shifting effect is stronger. Then, the optimal policy becomes a subsidy. This result represents Chou's (2011) argument.

Remark 2. (Chou, 2011) *Suppose $m = 0$ and $n = 1$. If $b < (\geq) -c + \sqrt{c(2+c)}$, then $s_H^C < (\geq)\ 0$.*

Since an increase in m strengthens the positive effect of the fourth term and weakens the negative effect of the third term, the tax incentives can become weaker and the subsidy incentives can become stronger as m increases. Therefore, when m is large relative to n, the optimal policy becomes a subsidy (see Fig. 3). This corresponds to Kawabata's (2010) result, which is compatible with the specific combination of $m = 1$ and $n = 2$ in our model.

Remark 3. (Kawabata, 2010) *If $m = 1$ and $n = 2$, $s_H^C > 0$.*

Combining propositions 1 and 2, we establish the following proposition, the proof of which is given in Appendix A.

Proposition 3. *(I) Suppose $b > \widetilde{b}$. (i) If $m < m_B$, then $s_H^B < 0$ and $s_H^C > 0$; (ii) if $m > m_B$, then $s_H^B > 0$ and $s_H^C > 0$. (II) Suppose $b < \widetilde{b}$. (i) If $m < m_C$, then $s_H^B < 0$ and $s_H^C < 0$; (ii) if $m_C < m < m_B$, then $s_H^B < 0$ and $s_H^C > 0$; (iii) if $m > m_B$, then $s_H^B > 0$ and $s_H^C > 0$.*

Proposition 3 shows the conditions for which the conventional results hold (parts (i) in (I) and (ii) in (II)): when the number of domestic suppliers is intermediate, downstream Bertrand rivalry can yield a tax and Cournot rivalry can yield a subsidy in a vertical structure with upstream price competition (see also Fig. 3).

We reach this result, because in the Bertrand case, the threshold value of m that makes the optimal export policy a subsidy is larger than that in the Cournot case (i.e., $m_C \leq m_B$). In the case of downstream Bertrand rivalry, if there is no upstream sector, the negative effect of the second term in (2) (tax incentive) dominates the positive effect of the first term (subsidy incentive), and, thus, the optimal export policy is a tax (Eaton and Grossman, 1986). Additionally, if there is an upstream market and $m = 0$, because the vertical rent-extraction effect in the third term in (2) indicates a tax, the optimal policy is a tax for any level of product substitutability (Chou, 2011). In both the case of a downstream Bertrand and that of a Cournot competition, an increase in m weakens the negative effect of the third term (tax incentive) and strengthens the positive effect of the fourth term (subsidy incentive), but the tax incentives in the Bertrand case are stronger than those in the Cournot case. Therefore, $m_C \leq m_B$.

4 Discussion

Welfare comparison.

For a large m, the Home country welfare in the case of a downstream Bertrand competition can be higher than that in the case of a downstream Cournot competition. Panel (a) in Fig. 4 shows this in the b-m plane. We examine this result here.

[Figure 4 around here]

Let us first focus on the fact that a larger m improves the Home country's welfare, regardless of the competition mode in the downstream market (i.e., $\partial W_H^l/\partial m > 0$, $l = B, C$. See Appendix E). Since the incentives to subsidize are strengthened as m increases, s_H^l increases with m (i.e., $\partial s_H^l/\partial m > 0$). A higher subsidy (lower tax) shifts the input demand upward. This demand expansion allows suppliers to set a higher price, so an increase in m raises the input price ($\partial r^l/\partial m > 0$). Additionally, a higher subsidy raises total sales (or exports) because it increases the domestic firm's exports more than it reduces the foreign firm's exports (i.e., $\partial Q^l/\partial m > 0$). Hence, an increase in m promotes the domestic firm's exports, magnifies the input demand, raises the input price, and improves the welfare.

On the one hand, the competition is tougher in Bertrand rivalry rather than in Cournot rivalry, so the effects of export promotion (restriction) tend to be stronger in the Bertrand case. When m is small enough, the export policy is always a tax in the Bertrand case. In that case, since the suppliers' part of the profit (i.e., $m\pi_k$) is small and the export activity of firm H is dampened, the welfare level in the Bertrand case can be lower than that in the Cournot case. In contrast, when m is large enough and the export policy is a subsidy in the Bertrand case, since the suppliers' part of the profit is large and the effects of export promotion are also stronger, the welfare level can be higher than that in the Cournot case[22] (see Panel (a) in Fig. 4).

In contrast to the Home country, there is no input supplier in the Foreign country, and therefore, the Foreign welfare is equivalent to the profit of firm F: $W_F^l \equiv \Pi_F^l$, $l = B, C$ (see Appendix D). Panel (b) in Fig. 4 illustrates the welfare comparison for Foreign. For the most part, the welfare (i.e., profit of firm F) is higher in Cournot case than in Bertrand case ($W_F^B < W_F^C$). This is because, since the competition is tougher in the Bertrand case than that in the Cournot case, in the former, the production of final goods becomes more active, and the input demand is larger. This expansion of input demand can increase input price, so the profit of firm F tends to be lower in the Bertrand case.

When m is large enough, the Home country subsidizes its domestic firm even if the final-good market is a Bertrand competition (see Propositions 1 and 3). The subsidization of firm H lowers its product price, and also reduces firm F's product price through the strategic complementarity. In addition, since a larger b makes the market competition keener, the effect of an increase in

[22]However, when b is sufficiently close to 1, since the firms' profit is close to 0 in the Bertrand case, the welfare level in the Cournot case exceeds that in the Bertrand case.

exports due to a reduction in product price is strengthened. Hence, if both m and b are large enough, the Bertrand case can yield a higher profit (welfare) compared to the Cournot case ($W_F^B > W_F^C$; see Panel (b) in Fig. 4).

Foreign unilateral intervention.

In Section 3.1, we considered only the case of unilateral intervention from the Home country's perspective. However, our result can be easily applied to the case of the Foreign country's unilateral intervention.

The subgame outcomes in the second and third stages depend on the total number of input suppliers, n, but not on the number of the Home country's suppliers, m. In addition, the Home country's welfare only depends on the number of its domestic suppliers, m, but not on the number of the other country's suppliers. Hence, no matter how many suppliers the other country has, the optimal export policy does not change. Therefore, the outcomes in the case of unilateral intervention from the Foreign country's perspective can be obtained by assuming $m = 0$. From (3), we find that the optimal policy for the Foreign country is an export tax.

Bilateral intervention.

We examine whether Proposition 1 holds when the Foreign country offers export subsidies/taxes, s_F, to its domestic firm as in Home country by discussing the case where the Foreign country is active.

We assume that the Home and Foreign countries have the same number of suppliers, $m \leq n/2$, where $n \geq 2$. Let r_d be the input price under bilateral intervention. Firm i's profit is $\Pi_i \equiv (p_i - r_d + s_i)q_i$, $i = H, F$. Adopting a similar method for price setting as in the previous case, we obtain 3 types of input prices: the lower bound $\underline{r_d} = \frac{(2a+s_H+s_F)c}{2[(2-b)(1+b)n+c]}$, the upper bound $\overline{r_d} = \frac{(2a+s_H+s_F)(1+n)c}{2[(2-b)(1+b)n+(1+n)c]}$, and the collusive price $r_{d,col} = \frac{[(2-b)(1+b)n+2c](2a+s_H+s_F)}{4[(2-b)(1+b)n+c]}$. From these prices, we find that Lemma 1 holds in the same manner as it did before. We assume $c \leq c_B$

here as well. Thus, $r_d = \overline{r_d}$.

The welfare maximization problem in country i,[23] $\max_{s_i} W_i(s_i, s_j) = \max_{s_i} \{\Pi_i(s_i, s_j) + m\pi_k(n, s_H, s_F) - s_i q_i(s_i, sj)\}$, for $i, j = H, F$ and $i \neq j$, yields the optimal export policy:[24]

$$s_i^B = \frac{2a(1-b)n}{D_d}[2(2+b)cm - (b^3 + b^2 + c)n - c] \quad \text{for } i = H, F,$$

where $D_d \equiv [(1-m+n)(8-b-3b^2)+b(3+b)m]cn + 2(1+b)(4-2b-b^3)n^2 + c^2(1+n)^2 > 0$.

Although whether s_i^B becomes a tax depends on the expression "$2(2+b)cm-(b^3+b^2+c)n-c$," the latter is completely the same as in (3). As long as the exporting countries are identical, Proposition 1 also holds under bilateral intervention.

The reason why the plus or minus sign of the optimal export policy is the same in both the unilateral and bilateral intervention case is that the Home and Foreign countries are identical. Suppose that s_H (s_F) is the export policy of the Home (Foreign) country. Let s_H^{Bu} be the equilibrium of the unilateral intervention. By setting $s_F = 0$ in the objective function of Home in the bilateral intervention case, we obtain s_H^{Bu}. Hence, in the s_H-s_F plane, the equilibrium point of the unilateral intervention is $(s_H^{Bu}, 0)$ on the s_H axis. First, suppose $s_H^{Bu} > 0$. Let us also denote the equilibrium point of bilateral intervention by (s_H^{Bb}, s_F^{Bb}). Since the equilibrium appears at the intersection point of the 45-degree line (i.e., $s_F = s_H$) and the best response function of Home due to the symmetry between Home and Foreign, s_H^{Bb} lies to the right hand side of $(s_H^{Bu}, 0)$. Thus, $s_H^{Bb} > 0$. We next consider the case $s_H^{Bu} < 0$. Here, the equilibrium appears at the intersection point of the 45-degree line and the best response function of Home, so s_H^{Bb} lies to the left hand side of $(s_H^{Bu}, 0)$. Hence, $s_H^{Bb} < 0$. When $s_H^{Bu} = 0$, because the intersection point of the 45-degree line and the best response function of Home is the origin, $s_H^{Bb} = 0$.

[23] We present the second-stage outcomes—$q_i(s_i, s_j)$, $\Pi_i(s_i, s_j)$, and $\pi_k(n, s_H, s_F)$—in Appendix F.

[24] The best response function of each country has a positive slope, so export policies are strategic complements. See Appendix G.

Other criterion for selecting equilibrium prices.

In the main model, we employ the payoff-dominance criterion. Here, we show that our results may not robust if we use another criterion. Considering the argument on equilibrium refinements (i.e., the concept of the payoff-dominance criterion), since the lower bound of the equilibrium input price $\underline{r}$ makes the profit of input suppliers 0, $r = \underline{r}$ is not likely to arise. On the one hand, Delbono and Lambertini (2016a, 2016b) offer a model where $r = \underline{r}$ can hold for some cases. Now, in our model, suppose that "$r = \underline{r}$" occurs. All input suppliers have zero profit at $r = \underline{r}$, so the vertical effects (i.e., the third and fourth terms in (2)) disappear. Hence, there are only horizontal effects, and therefore, the optimal export policy is always a tax: employing $\underline{r}$, the optimal policy becomes

$$\underline{s}_H^B = -\frac{a(1-b)(2+b)n(c+b^2(1+b)n)}{((3+b)c+4(1+b)n)(c+2(2-b^2)n)} < 0.$$

This is the same as the result of Eaton and Grossman (1986), where there is no upstream supplier.

Integer constraints.

In the last part of this section, we discuss integer constraints on the number of input suppliers. That is, we show pairs of integers (n, m) such that Proposition 1 holds. We consider the (n, m) cases where $n = 2, 3, \ldots, 200$ and $m = 0, 1, \ldots, n$. Using numerical calculations, we obtain Fig. 5. The horizontal axis is the number of input suppliers and the vertical axis the number of the Home country's suppliers. Each dot in the figure represents an (n, m) pair for which the optimal export policy exhibits the "tax–subsidy–tax" pattern. From Fig. 5, we find that the number of such pairs increases as the number of input suppliers increases.

[Figure 5 around here]

5 Conclusion

This study incorporates Dastidar (1995)-type price competition in the intermediate input market into a standard export rivalry model with a vertical structure and investigates the nature of the optimal export policy. Although input suppliers have a quadratic cost and the equilibrium input price has a certain range, by adopting a similar approach as in Cabon-Dhersin and Drouhin (2014) (i.e., the payoff-dominance criterion), we narrow the range of input prices to a single equilibrium price and consider a differentiated Bertrand and Cournot competition in the downstream third market.

We first show that in the case of downstream Bertrand competition, the optimal export policy can exhibit the *tax–subsidy–tax* pattern, according to the degree of product substitutability. This non-monotonicity in the export policy yields the following policy implication: there exists a case where *a large mistake can be permissible, but a small mistake can be impermissible*. The optimal export policy becomes a tax when the degree of substitutability is either low or high. Hence, when the degree of substitutability is low, even if the practitioner accidentally estimates that its level is high, the realized policy is an export tax and the optimal and realized policies are consistent. Therefore, a large mistake is permissible. However, if the practitioner's estimate of product substitutability is slightly higher than its actual level, the realized policy be an export subsidy, which is the opposite of the optimal policy. Hence, a small mistake may be impermissible.

We also demonstrate that if the number of domestic input suppliers is of intermediate size, the conventional result holds, that is, the optimal export policy is a tax (subsidy) when exporters compete in a Bertrand (Cournot) fashion. A larger number of domestic input suppliers strengthens subsidy incentives; therefore, in both the Bertrand and Cournot cases, the optimal export policy becomes a subsidy when the number of domestic suppliers is large enough. Since the tax incentive in Bertrand competition is stronger than that in Cournot competition, to switch the optimal export policy from a tax to a subsidy in the case of downstream Bertrand competition, it is necessary to have a larger number of domestic suppliers than those in the Cournot case.

In this study, we considered upstream price competition in a standard third-market export-rivalry model. On the one hand, it may be possible to extend our model to a two-way trade environment. In such a situation, examining the role of upstream price competition may be an interesting consideration. However, this argument is beyond the scope of our analysis and is left for future work.

Appendices

Appendix A. Proofs and Proposition 4.

Proof of Proposition 1. From (3), setting $s_H^B = 0$ and solving for m, we have $m_B = [nb^3 + nb^2 + c(1+n)]/[2(2+b)c]$. Then, we obtain the first assertion.

Next, we consider the second assertion. Differentiating m_B with respect to b, we obtain

$$\frac{\partial m_B}{\partial b} = \frac{2nb^3 + 7nb^2 + 4nb - c(1+n)}{2c(2+b)^2}.$$

To prove the non-monotonicity of s_H^B, we show that m_B is a convex function and its first derivative with respect to b is negative at $b = 0$ and positive at $b = 1$. First, we have $\partial^2 m_B/\partial b^2 = [(4+12b+6b^2+b^3)n+c(1+n)]/[(2+b)^3 c] > 0$. Second, we have $\partial m_B/\partial b|_{b=0} = -c(1+n)/(8c) < 0$. Finally, we have $\partial m_B/\partial b|_{b=1} = (13n - cn - c)/(18c)$, which is a decreasing function of c. Since we assume $c \leq c_B$, $\partial m_B/\partial b|_{b=1}$ has a minimum at $c = c_B$. Substituting $c = c_B$, we have $\partial m_B/\partial b|_{b=1,\, c=c_B} = (11n - 15 - b - bn + b^2 + b^2 n)/[18(2-b)(1+b)] > 0$. The inequality holds since we assume $n \geq 2$. Hence, $\partial m_B/\partial b|_{b=1} > 0$.

From the above and the continuity of m_B, there exists a unique $b_l \equiv \operatorname{argmin}_b m_B$ in $(0, 1)$

such that for any $b < b_l$, m_B decreases with b, and for any $b > b_l$, m_B increases with b.

Here, we define $m_h = \min\{m_B|_{b=0}, m_B|_{b=1}\}$, where $m_B|_{b=0} > m_B|_{b=1}$ if $4n/(1+n) < c < c_B$, and $m_B|_{b=0} \leq m_B|_{b=1}$ if $0 < c \leq 4n/(1+n)$. Since $n - m_B|_{b=0} = (3n-1)/4 > 0$, we have $n > m_h$.

From the discussion above, we have some $m' \in (m_l, \min\{m_h, n\})$ such that $s_H^B > 0$ for $b_1 < b < b_2$, and $s_H^B < 0$ for $0 \leq b < b_1$ or $b_2 < b < 1$. Q.E.D.

Proof of Proposition 2. From (4), solving $s_H^C \leq (\geq)\ 0$ for m, we have $m \leq (\geq)\ m_C \equiv \frac{(1-b)(1+n)c-b^2n}{2(2-b)c}$. $m_C|_{b=0} = \frac{1+n}{4} > 0$, $\frac{\partial m_C}{\partial b} = -\frac{(4-b)bn+(1+n)c}{2(2-b)^2c} < 0$, and $m_C|_{b=1} = -\frac{n}{2c} < 0$, so $m_C = 0$ at some $b \in (0,1)$. Solving $m_C \geq 0$ for b, we obtain $b \leq \widetilde{b} \equiv \frac{-(1+n)c+\sqrt{c(1+n)[c(1+n)+4n]}}{2n} > 0$. Since $1-\widetilde{b} = \frac{(c+2n+cn)-\sqrt{c(1+n)[c(1+n)+4n]}}{2n}$ and $(c+2n+cn)^2 - \left(\sqrt{c(1+n)[c(1+n)+4n]}\right)^2 = 4n^2$, $0 < \widetilde{b} < 1$. Hence, if $b > \widetilde{b}$, then $s_H^C > 0$. When $b < \widetilde{b}$, $s_H^C < 0$ if $m < m_C$, and $s_H^C > 0$ if $m > m_C$. Q.E.D.

Proof of Proposition 3. Since $m_B > 0$, $m_B - m_C = \frac{b^2[(4+2b-b^2)n+(1+n)c]}{2(2-b)(2+b)c} \geq 0$. Further, we obtain $m_0 - m_B = \frac{(16-10b^2-3b^3+b^4)n+(6-b-3b^2)(1+n)c}{2(2-b)(2+b)c} > 0$. From these results and Propositions 1 and 2, we obtain Proposition 3. Q.E.D.

The collusive input price case: $r = r_{col}$ (i.e., $c > c_B$).

Proposition 4. *Suppose $c > c_B$ and $2 = n > m_l^{col} \equiv \min_b m_{col}$, where $m_{col} \equiv \frac{2+b+b^3+c}{2(2+b)(1+b)(2+b)}$. Then:*

1. *$s_H^{col} > 0$ if $m > m_{col}$, $s_H^{col} = 0$ if $m = m_{col}$, and $s_H^{col} < 0$ if $m < m_{col}$.*
2. *For $m'' \in (m_l^{col}, \min\{m_h^{col}, 2\})$, we have $s_H^{col} > 0$ for $b_1^{col} < b < b_2^{col}$, and $s_H^{col} < 0$ for $0 \leq b < b_1^{col}$ or $b_2^{col} < b < 1$, where $m_h^{col} \equiv m_{col}|_{b=1}$.*

Proof. For simplicity, we assume $n = 2$. From Lemma 1, we have $r_{col} < \bar{r}$ if $c > c_B$; we assume this range for c. Then, the equilibrium input price is $r = r_{col}$. Substituting it into W_H and solving the first-order condition $\partial W_H/\partial s_H = 0$ for s_H, we obtain the optimal export policy

$$s_H^{col} = \frac{2a(1-b)(2+b)(2+b+c-8m-8bm+2b^2m+b^3(1+2m))}{D_{col}},$$

where $D_{col} \equiv -60-28c-3c^2+16m-b(64+18c+c^2-8m)+b^2(33+10c-20m)+b^3(35+4c-10m)+b^4(4m-5)+b^5(2m-3)$.

Solving $s_H^{col}=0$ for m, we have

$$m = \frac{2+c+b+b^3}{2(2+b)(2+b-b^2)} \equiv m_{col}.$$

Thus, we have proven the first assertion.

To prove the second assertion, we show that m_{col} is a convex function and its first derivative with respect to b is negative at $b=0$ and positive at $b=1$.

First, we have

$$\frac{\partial^2 m_{col}}{\partial b^2} = \frac{24+60b+54b^2+41b^3+39b^4+15b^5-b^6+(20-9b^2+8b^3+6b^4)c}{(4+4b-b^2-b^3)^3}.$$

This function increases with c. Since we assume $c > c_B$, to calculate the minimum value of this function, we evaluate it at $c = c_B$. Thus, we have

$$\left.\frac{\partial^2 m_{col}}{\partial b^2}\right|_{c=c_B} = \frac{104-4b-18b^2+73b^3+24b^4-13b^5}{(1+b)^2(4-b^2)^3} > 0.$$

Hence, $\partial^2 m_{col}/\partial b^2 > 0$ always holds.

Next, we evaluate $\partial m_{col}/\partial b$ at $b=0$ and $b=1$. We have $\partial m_{col}/\partial b|_{b=0} = -(1+c)/8 < 0$ and $\partial m_{col}/\partial b|_{b=1} = (28+c)/72 > 0$. Moreover, we have $\partial m_{col}/\partial b|_{b=0} - \partial m_{col}/\partial b|_{b=1} = (c-2)/24$, which increases with c. Hence, evaluating it at $c=c_B$, we obtain its minimum value, $(1+b-b^2)/12$. Thus, we obtain $\partial m_{col}/\partial b|_{b=0} > \partial m_{col}/\partial b|_{b=1}$. From these results and the continuity of m_{col}, there exists a unique $b_{col} \equiv \operatorname{argmin}_b m_{col}$ in $(0,1)$ such that for any $b < b_{col}$, m_{col} decreases with b, and for any $b > b_{col}$, m_{col} increases with b.

Here, let $m_l^{col} \equiv \min_b m_{col}$ and $m_h^{col} = m_{col}|_{b=1}$. From the discussion above, if $m_l^{col} < 2$

$(= n)$, we have some $m'' \in (m_l^{col}, \min\{m_h^{col}, 2\})$ such that $s_H^{col} > 0$ for $b_1^{col} < b < b_2^{col}$, and $s_H^{col} < 0$ for $0 \leq b < b_1^{col}$ or $b_2^{col} < b < 1$.

Finally, we show that, at $c = 44/5$, three types of optimal policies occur: a tax at $m = 0$, a tax–subsidy–tax at $m = 1$, and a subsidy at $m = 2$. Q.E.D.

[Figure 6 around here]

Appendix B. Monotonicity of $\pi_k(n, r, s_H)$ in $[\underline{r}, \bar{r}]$.

The second-stage profit of supplier k when the input market is equally shared by n (≥ 2) identical suppliers is

$$\pi_k(n, r, s_H) = \frac{(2a + s_H)[(2-b)(1+b)n + 2c]r}{(2-b)^2(1+b)^2n^2} - \frac{2[(2-b)(1+b)n + c]r^2}{(2-b)^2(1+b)^2n^2} - \frac{(2a+s_H)^2c}{2(2-b)^2(1+b)^2n^2}.$$

Since $\partial^2\pi_k(n, r, s_H)/\partial r^2 = -4[(2-b)(1+b)n+c]/[(2-b)^2(1+b)^2n^2] < 0$ and $\partial\pi_k(n, r_{col}, s_H)/\partial r = 0$, r_{col} maximizes $\pi_k(n, r, s_H)$. As illustrated in the equation for $\pi_k(n, r, s_H)$, the coefficient of r^2 is negative, and, hence, $\pi_k(n, r, s_H)$ is an inverted-U shape function in the input price r. Furthermore, from the definition of $\underline{r}$ and Lemma 1, $\pi_k(n, \underline{r}, s_H) = 0$ and $r_{col} \geq \bar{r}$ for $c \leq c_B$. Therefore, in the range $[\underline{r}, \bar{r}]$, $\pi_k(n, r, s_H)$ is monotonically increasing in r if $c \leq c_B$.

Appendix C. Second-stage outcomes

The second-stage outcomes in the downstream Bertrand case are:

$$\left.\begin{aligned} p_H &= \tfrac{2a(2+b)[(1-b^2)n+(1+n)c]-[4(1+b)n+(1+n)c]s_H}{2(2+b)[(2-b)(1+b)+(1+n)c]};\ r = \tfrac{(2a+s_H)(1+n)c}{2[(2-b)(1+b)n+(1+n)c]}, \\ q_H &= \tfrac{2a(1-b)(2+b)n+[2n(2-b^2)+(1+n)c]s_H}{2(1-b)(2+b)[(2-b)(1+b)n+(1+n)c]};\ Q = \tfrac{(2a+s_H)n}{(2-b)(1+b)n+(1+n)c}. \end{aligned}\right\} \tag{A1}$$

The comparative statics results for (A1) are:

$$\left.\begin{aligned} \frac{\partial p_H}{\partial s_H} &= \tfrac{-[4(1+b)n+(1+n)c]}{2(2+b)[(2-b)(1+b)+(1+n)c]} < 0;\ \frac{\partial r}{\partial s_H} = \tfrac{(1+n)c}{2[(2-b)(1+b)n+(1+n)c]} > 0, \\ \frac{\partial q_H}{\partial s_H} &= \tfrac{2n(2-b^2)+(1+n)c}{2(1-b)(2+b)[(2-b)(1+b)n+(1+n)c]} > 0;\ \frac{\partial Q}{\partial s_H} = \tfrac{n}{(2-b)(1+b)n+(1+n)c} > 0. \end{aligned}\right\} \tag{A2}$$

The second-stage outcomes in the Cournot case are:

$$\left.\begin{aligned} p_H^* &= \tfrac{2a(2-b)[(1+n)c+n]-[2(2-b^2)n+(1-b)(1+n)c]s_H}{2(2-b)[(2+b)n+(1+n)c]};\ r^* = \tfrac{(2a+s_H)(1+n)c}{2[(2+b)n+(1+n)c]}, \\ q_H^* &= \tfrac{2a(2-b)n+[(1+n)c+4n]s_H}{2(2-b)[(2+b)n+(1+n)c]};\ Q^* = \tfrac{(2a+s_H)n}{(2+b)n+(1+n)c}. \end{aligned}\right\} \tag{A3}$$

The comparative statics results for (A3) are:

$$\left.\begin{aligned} \tfrac{\partial p_H^*}{\partial s_H} &= \tfrac{-[2(2-b^2)n+(1-b)(1+n)c]}{2(2-b)[(2+b)n+(1+n)c]} < 0;\ \ \tfrac{\partial r^*}{\partial s_H} = \tfrac{(1+n)c}{2[(2+b)n+(1+n)c]} > 0, \\ \tfrac{\partial q_H^*}{\partial s_H} &= \tfrac{(1+n)c+4n}{2(2-b)[(2+b)n+(1+n)c]} > 0;\ \ \tfrac{\partial Q^*}{\partial s_H} = \tfrac{n}{(2+b)n+(1+n)c} > 0. \end{aligned}\right\} \tag{A4}$$

Appendix D. SPNE outcomes

Bertrand case:

$$\begin{aligned} q_H^B &= \frac{a(2+b)n}{D}[2(2-b^2)n+c(1+2m+n)], \\ q_F^B &= \frac{an}{D}[2(4+2b-b^2)n+c(4(1-m+n)+b(1-2m+n))], \\ r^B &= \frac{ac(1+n)}{D}[(8+4b-3b^2-b^3)n+c(3+b)(1+n)];\ W_H^B = \frac{a^2n}{D}[(1-b)(2+b)^2n+2(3+b)cm]. \end{aligned}$$

The equilibrium profit of firm i is $\Pi_i^B = (1-b^2)\left(q_i^B\right)^2$, $i = H, F$. Supplier k's profit is $\pi_k^B = \frac{2n}{c(1+n)^2}\left(r^B\right)^2$, $k \in \{1, \ldots, n\}$.

Cournot case:

$$\begin{aligned} q_H^C &= \frac{a(2-b)n[4n+(1+2m+n)c]}{E}, \\ q_F^C &= \frac{an}{E}[2(4-2b-b^2)n+(4-3b)(1+n)c-2(2-b)cm], \\ r^C &= \frac{a(1+n)c}{E}[(8-4b-b^2)n+(3-2b)(1+n)c];\ W_H^C = \frac{a^2n[2(3-2b)cm+(2-b)^2n]}{E}. \end{aligned}$$

The equilibrium profit of firm i is $\Pi_i^C = \left(q_i^C\right)^2$. Supplier k's profit is $\pi_k^C = \frac{2n}{c(1+n)^2}\left(r^C\right)^2$. Here, to ensure a positive quantity, we assume

$$m < m_0 \equiv \frac{2(4-2b-b^2)n+(4-3b)(1+n)c}{2(2-b)c}\ (>0),$$

which is equivalent to $q_F^C > 0$.

Appendix E. Comparative statics results for SPNE outcomes

From the derivative of the outcomes, we obtain

$$\frac{\partial s_H^B}{\partial m} = \frac{4a(1-b)(2+b)^2cn(2-b)(1+b)(8+4b-3b^2-b^3)n^2}{D^2}$$
$$+\frac{4a(1-b)(2+b)^2cn[(14+9b-5b^2-2b^3)n(1+n)c+(3+b)(1+n)^2c^2]}{D^2} > 0,$$
$$\frac{\partial s_H^C}{\partial m} = \frac{4a(2-b)^2cn[(2+b)(8-4b-b^2)n^2+(14-5b-3b^2)n(1+n)c+(3-2b)(1+n)^2c^2]}{E^2} > 0,$$
$$\frac{\partial r^B}{\partial m} = \frac{2a(1-b)(2+b)^2c^2n(1+n)[(8+4b-3b^2-b^3)n+(3+b)c(1+n)]}{D^2} > 0,$$
$$\frac{\partial r^C}{\partial m} = \frac{2a(2-b)^2c^2n(1+n)[(8-4b-b^2)n+(3-2b)c(1+n)]}{E^2} > 0,$$

$$\frac{\partial Q^B}{\partial m} = \frac{4a(1-b)(2+b)^2cn^2[(8+4b-3b^2-b^3)n+(3+b)c(1+n)]}{D^2} > 0,$$
$$\frac{\partial Q^C}{\partial m} = \frac{4a(2-b)^2cn^2[(8-4b-b^2)n+(3-2b)c(n+1)]}{E^2} > 0,$$

$$\frac{\partial W_H^B}{\partial m} = \frac{2a^2cn[(8+4b-3b^2-b^3)n+(3+b)c(1+n)]^2}{D^2} > 0, \quad \text{and}$$
$$\frac{\partial W_H^C}{\partial m} = \frac{2a^2cn[(8-4b-b^2)n+(3-2b)c(1+n)]^2}{E^2} > 0.$$

Appendix F. Second-stage outcomes in the case of bilateral intervention

The second-stage exports, firm i's profit, and supplier k's profit are, respectively,

$$q_i(s_i,s_j) = \frac{2a(1-b)(2+b)n+c(1+n)(s_i-s_j)+2n((2-b^2)s_i-bs_j)}{2(1-b)(2+b)[(2-b)(1+b)n+(1+n)c]}, \ i \neq j,$$

$$\Pi_i(s_i,s_j) = (1-b^2)[q_i(s_i,s_j)]^2; \quad \pi_k(n,s_H,s_F) = \frac{(2a+s_H+s_F)^2nc}{2[(2-b)(1+b)n+(1+n)c]^2}.$$

Appendix G. Best response function for each country

The FOC for the welfare maximization of country i ($i = H, F$) yields the following best response function BR_i:

$$s_i = BR_i \equiv \frac{-2a(1-b)(2+b)n[b^2(1+b)n+c(1-2(2+b)m+n)]}{L}$$
$$+\frac{\left[(c(1+n)+2bn)(b^2(1+b)n+c(1+n))+2m(1-b)(2+b)^2cn\right]s_j}{L} \quad (i \neq j),$$

where $L \equiv 8(1+b)(2-b^2)n^2 + (3+b)c^2(1+n)^2 + 8cn(2-m+2n) - 2cnb^2(3+b)(1-m+n) + 8bcn(1+n) > 0$. From the above equation, $\partial BR_i/\partial s_j > 0$ $(i, j = H, F;\ i \neq j)$.

Since BR_i has a positive slope with respect to s_j, export policies are strategic complements.[25] The intuition is as follows: For example, suppose that the Home country reduces its export tax when the Foreign one raises its export tax. Since a lower (higher) export tax makes the product price lower (higher), a tax reduction in Home can reduce the price of the Home product. However, a rise in the Foreign product price also increases the price of the Home product through the strategic complementarity in the price competition, so a reduction in the Home export-tax is not effective. Therefore, because it would be more desirable to obtain a larger tax revenue by charging a higher export tax rate, the Home country raises its tax rate as the Foreign one does.

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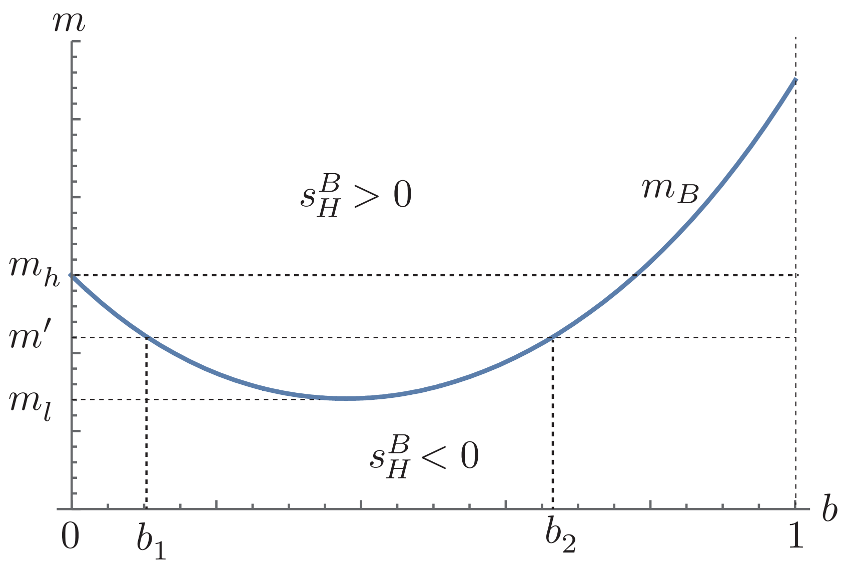
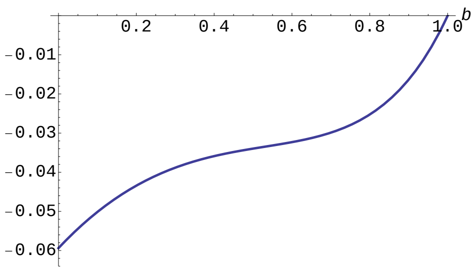
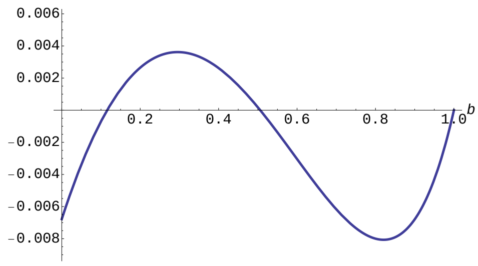


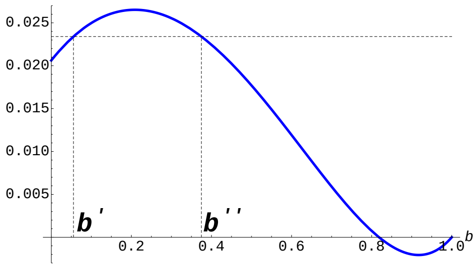
Figure 1: Area of *non-monotonic export policy* when $c \leq c_B$.



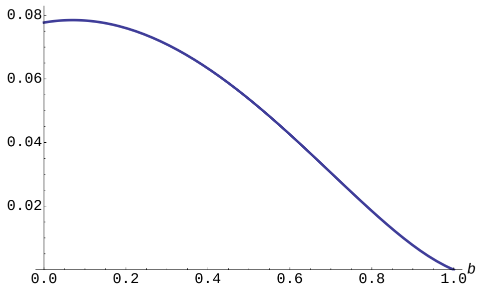
Panel (a): $m = 3$ (tax).



Panel (b): $m = 5$ (tax–subsidy–tax).



Panel (c): $m = 6$ (subsidy–tax).



Panel (d): $m = 8$ (subsidy).

Figure 2: Graph of s_H^B/a for b ($c = 1.9$ and $n = 20$).

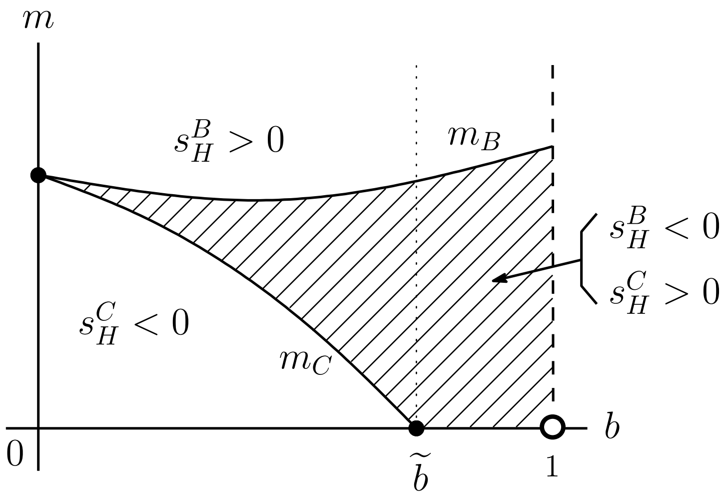
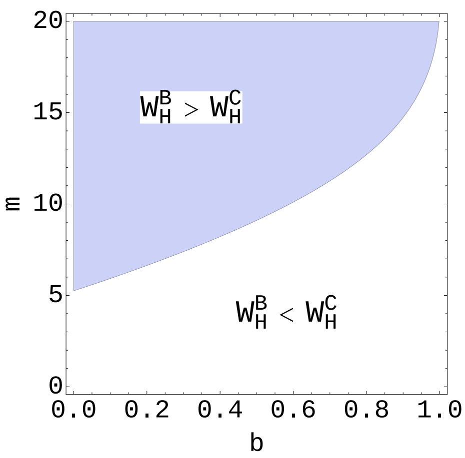
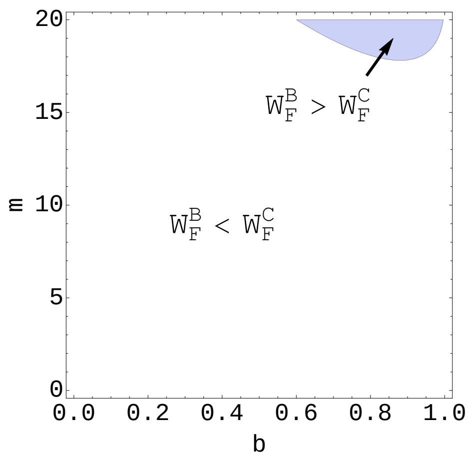


Figure 3: Area of "conventional results."



Panel (a): Home country.



Panel (b): Foreign country.

Figure 4: Welfare comparison ($a = 1$, $c = 1.9$, and $n = 20$).

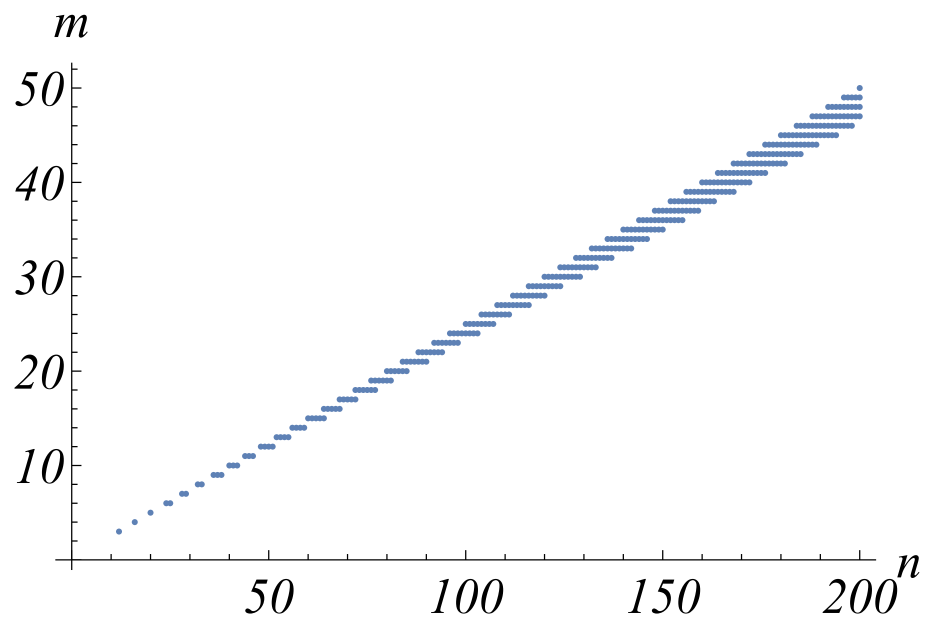


Figure 5: The (n, m) pairs for which the optimal export policy has a "tax–subsidy–tax" form.

Note: The numerical calculations were performed after substituting $a = 1$ and $c = 1.9$ in s_H^B (equation (3)).

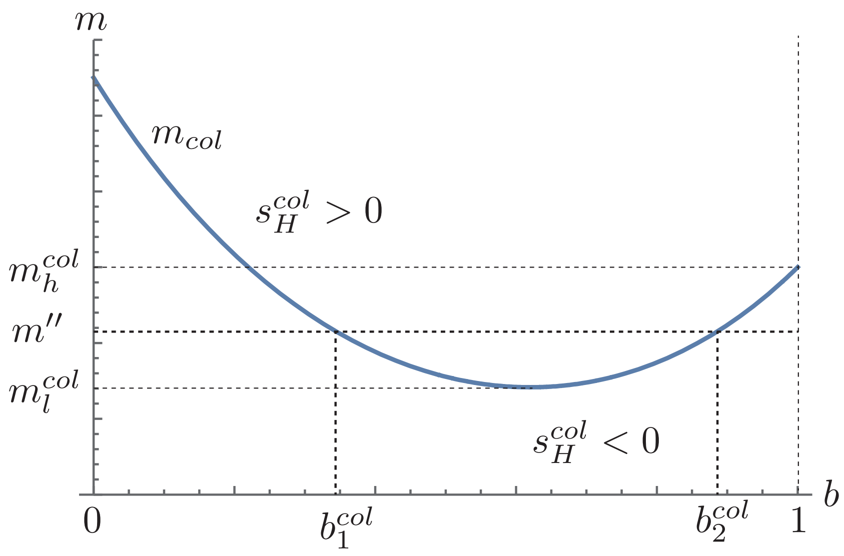


Figure 6: Area of *non-monotonic export policy* when $c > c_B$.