Tax-Expenditure Policy and Piecemeal Welfare Economics

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Tax-Expenditure Policy and Piecemeal Welfare Economics

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Preface

In modern mixed economies, the role of the public sector is getting more and more important. Especially, tax-expenditure measures are considerably influential on the welfare of nations. In the present thesis we investigate desirable tax-expenditure policy recommendations, in principle taking the standpoint of "budget incidence" and based on piecemeal welfare economics.

The thesis consists of five chapters, which can be classified into three parts. The first part includes chapter 1, in which we clarify the existence of the problem we consider and show the characteristics of the thesis. The problems we face can be roughly divided into two matters: efficiency of resource allocation and equity of income distribution. The second part, which contains Chapter 2 and Chapter 3, examines the problem of efficiency. Chapter 4 and Chapter 5 consist in the third part, which investigates the latter aspects of redistributional function of tax-expenditure measures.

In the research leading to this thesis, the author owes much to Professor Tetsuya Kishimoto for helpful instructions and useful discussion. The earlier drafts of this thesis have been corrected and improved by helpful instructions of, and useful discussion with Professor Kishimoto. Professors Mitsuo Saito, Kazuo Ogawa, Tatsuo Hatta, Kazuo Nakamura, Koji Shinjo, Yasuhide Tanaka and Douglas Wohlers also made many valuable comments. Furthermore, the author should thank the discussants of his reports at the Western and Annual Meeting of Japan

Society of Economics and Econometrics. Needless to say, remaining errors are the author's responsibility.

Kazuhiko Mikami

Contents

Chapter 1

Public Policy by means of Tax-Expenditure Measures: An Overview

Abstract: The public sector with tax-expenditure measures faces the problem of efficiency of resource allocation and equity of income distribution. The present thesis considers this problem, mainly based on the thought of 'budget incidence' and piecemeal approach.

1. The Problems faced by the Public Sector

In a mixed economy like most of the advanced industrialized countries, public policies taken by a government by means of tax-expenditure measures are considerably important. It greatly concerns the behavior of economic agents, and then affects the efficiency of resource allocation and the equity of income distribution among people. These two matters, efficiency and equity, are often referred to as the basic viewpoints in carrying out a tax-expenditure policy(*1).

When we think of the economic efficiency in relation with tax-expenditure policy, we should note that there can be two kinds of "inefficiency." One is tax-induced distortion within the private sector, and the other is inefficiency caused by an inappropriate resource allocation between the private sector and the public sector.

In a laissez faire market economy, since consumers and producers face the same relative prices, the marginal rate of substitution (MRS) of a consumer comes to coincide with the marginal rate of transformation (MRT) of a producer, and consequently the Pareto optimum resource allocation is established. In an economy with commodity taxes, however, such a mechanism towards Pareto optimum does not work, because consumer prices generally diverge from producer prices by an amount of the commodity taxes and then there occurs tax-induced distortion, excess burden or deadweight loss due to taxation.

Excess burden can be explained diagramatically as in Figure 1-1. For simplicity we consider two private goods, leisure x^0 and consumption good x^1. The amount of leisure consumed is measured on the horizontal axis, so that x^0 is negative. Total endowment of leisure is represented by the length OG. The budget line with no commodity taxes is shown by OA. Then, the initial consumption point is at A and the consumer enjoys the utility level u^A. Suppose now that a commodity tax is imposed on consumption goods and leisure is left untaxed. Then, since the budget line rotates to OB, the consumption point moves to B and the utility level is reduced from u^A to u^B. If we let leisure be the numeraire good, the value of gradient DE/OD represents the original price, p^1, and DB/OD shows the price with tax t^1, $q^1=p^1+t^1$. Therefore, the current tax revenue is represented by the length EB, or alternatively OF. Now, if the same tax revenue were collected by poll taxes, the budget line would be FB, which has the same gradient as the initial budget line OA, and the consumer could enjoy the utility level u^C, which is higher than u^B. It is clear that the difference between u^B and u^C comes from the method of collecting tax revenue- that is, a non-distortionary poll tax or distortionary discriminate commodity taxes. Thus, we refer to this difference as excess burden due to commodity taxes(*2).

Thus, it would be natural to consider that a commodity tax which minimizes the excess burden is desirable. Such questions of finding desirable tax structure have been investigated through at least two branches of study, although those two are closely related to each other. One is a study on a second best solution of commodity taxes by Corlett-Hague(1953), Lipsey-Lancaster (1956) and Green(1961). Although the framework of the above analyses are different from one another, essentially they state that the goods which are complementary with the untaxed good should be taxed more heavily than the goods which are substitutable for the untaxed good. The other is a series of articles on optimum taxation, including the pioneering work of Ramsey(1927), which was followed by Samuelson(1986) and Diamond-Mirrlees(1971a,b). Though there are several patterns of presenting the optimal tax rule, a representative one requires that the total substitution effects of a proportional change in all tax rates should be proportional to demand. Excellent surveys on this area were made by

Sandmo(1976) and Mirrlees(1976).

On the other hand, traditionally there has been a belief that a uniform commodity tax is optimal. The relation between the Ramsey type optimum tax theory and the optimality of uniform taxation has been examined by Dixit(1970), Sandmo(1974), Atkinson-Stiglitz(1972), Sadka (1977) and Deaton(1979). They proved that if all the commodities have the same compensated elasticities with respect to the wage rate, or if a uniform tax on all the commodities maximizes the labor supply, then a uniform tax is optimal.

Let us next turn to the other kind of inefficiency: an inappropriate resource allocation between the private and public sectors. On the optimum provision of public goods, we have already had a famous rule in Samuelson(1954, 1955), which requires an equalization of sum of the MRSs to MRT. We should notice, however, that this well-known condition implicitly assumes availability of non-distortionary taxes, like lump-sum taxes. Unless such non-distortionary taxes are usable, Samuelson's rule is no longer true, since the financing cost includes tax-induced distortion as well as tax revenue itself. Such a problem of optimal provision of public goods under distortionary taxes has been inspected by Pigou(1947), Stiglitz-Dasgupta(1971), Atkinson-Stern(1974) and Wildasin(1979, 1984). They pointed out that normally the distortionary taxes increase the marginal cost of public goods provision, although marginal cost under distortionary taxes can be smaller than that under lump-sum taxes, depending on the inferiority of taxed commodities or the choice of untaxed goods.

In line with efficiency aspects, redistribution through tax-expenditure policy is also quite an important role of the public sector. Indeed, there are many types of consumers in the real economy and they are different in some respects, such as ability to work or preference over goods they consume. Therefore, we should propose tax-expenditure policies taking account of such differences among people.

Introducing an excellent concept of "distributional characteristic" or "covariance between the commodity and the social marginal utility of income," redistributional functions of linear commodity taxes have been investigated by Feldstein(1972a,b), Diamond(1975), Atkinson-Stiglitz(1976,1980), Boadway(1976), etc. Although the formations of modeling the economy are to some extent different among those articles, essentially they pointed out that, under the assumption of decreasing social marginal utility of income, luxury commodities should be taxed more heavily than necessities. On the other hand, public goods have similar distributional functions to commodity taxes. Assuming many different consumers and many different kinds of public goods, King(1986) concluded that public goods which are necessities should take precedence over luxurious public goods.

Pursuit of efficiency often conflicts with an improvement in the state of distribution. Such a tradeoff will also be examined in this thesis.

2. "Differential Incidence" vs "Budget Incidence"

Traditionally, in analyzing the positive effects of taxation, we have two different concepts of tax incidence, that is, "differential incidence" and "budget incidence."(*3) The former assumes that one kind of tax is substituted for another so that total revenue and expenditures are held constant, while the latter considers the combined effects of tax and expenditure changes when one sort of tax is varied. Almost all of the existing articles on tax theory are based on the former concept. By taking the standpoint of differential incidence, we can substantially neglect the expenditure aspects, which enables us to focus our mind on characterizing desirable tax structure. There should be no doubt that it has contributed greatly to a simplification of the analyses. However, if we think of the real tax policy, the value of public expenditure is in general varied by a tax reform, and therefore studies based on the thought of budget incidence should be given more importance. Especially, as shown in the articles on the optimum provision of public good under a second best situation by Stiglitz-Dasgupta(1971), Atkinson-Stern(1974) and Wildasin(1979, 1984) there often exists interrelations between commodity taxes and the optimum amount of public good. This kind of investigation can be made only by adapting the idea of budget incidence. Hence, in principle, we take a standpoint of budget incidence in this thesis and examine tax policies and expenditure policies synthetically. (*4)

3. Optimum vs Piecemeal

Lipsey-Lancaster(1956), the basic work on the theory of second best, maintained that, even though we know the necessary conditions for welfare maximum, there are in general no sufficient conditions for an increase in welfare. Further, they insisted that finding sufficient conditions for an increase in welfare are more important than detecting necessary conditions for a welfare maximum if policy recommendations are to be made in the real economy.

Succeeding to Lipsey-Lancaster(1956), Feldstein (1976) applied such thought of piecemeal welfare economics to a more limiting area of taxation. He criticized the optimal taxation approach because such a tax design is a guide for tax policy in the Garden of Eden- that is, societies with no taxes. Alternatively, he advocated optimal tax reform, which takes the existing tax system as its starting point and considers the welfare effects of small change in tax rates.

Indeed, the piecemeal approach has some advantages over the optimum approach. First of all, sufficient conditions are easier to use for carrying out actual policies than are necessary conditions. Secondly, those sufficient conditions consist of local or present economic information, and therefore they are easy to observe. On the other hand, necessary conditions in optimum taxation are made up with informations at the second best optimal circumstances, so that we need global information which is often very difficult to obtain. Hence, we will take the standpoint of the theory of tax reform throughout the present thesis, although we should not make light of the significance and contribution of optimum taxation theory.

Footnotes

(*1) Allocation, distribution and stabilization are referred to as principal functions of the public sector. The present thesis concerns the first two functions, and does not mention stabilization.

(*2) This explanation of excess burden is of the equivalent variation (EV) type, and it was examined by Kay(1980) and further refined by Stutzer(1982). Another way of defining deadweight loss based on compensated variation (CV) can be seen in Diamond-McFadden(1974).

(*3) See Musgrave-Musgrave(1982), chapter 12.

(*4) Chapter 4 is exceptionally based on the differential incidence.

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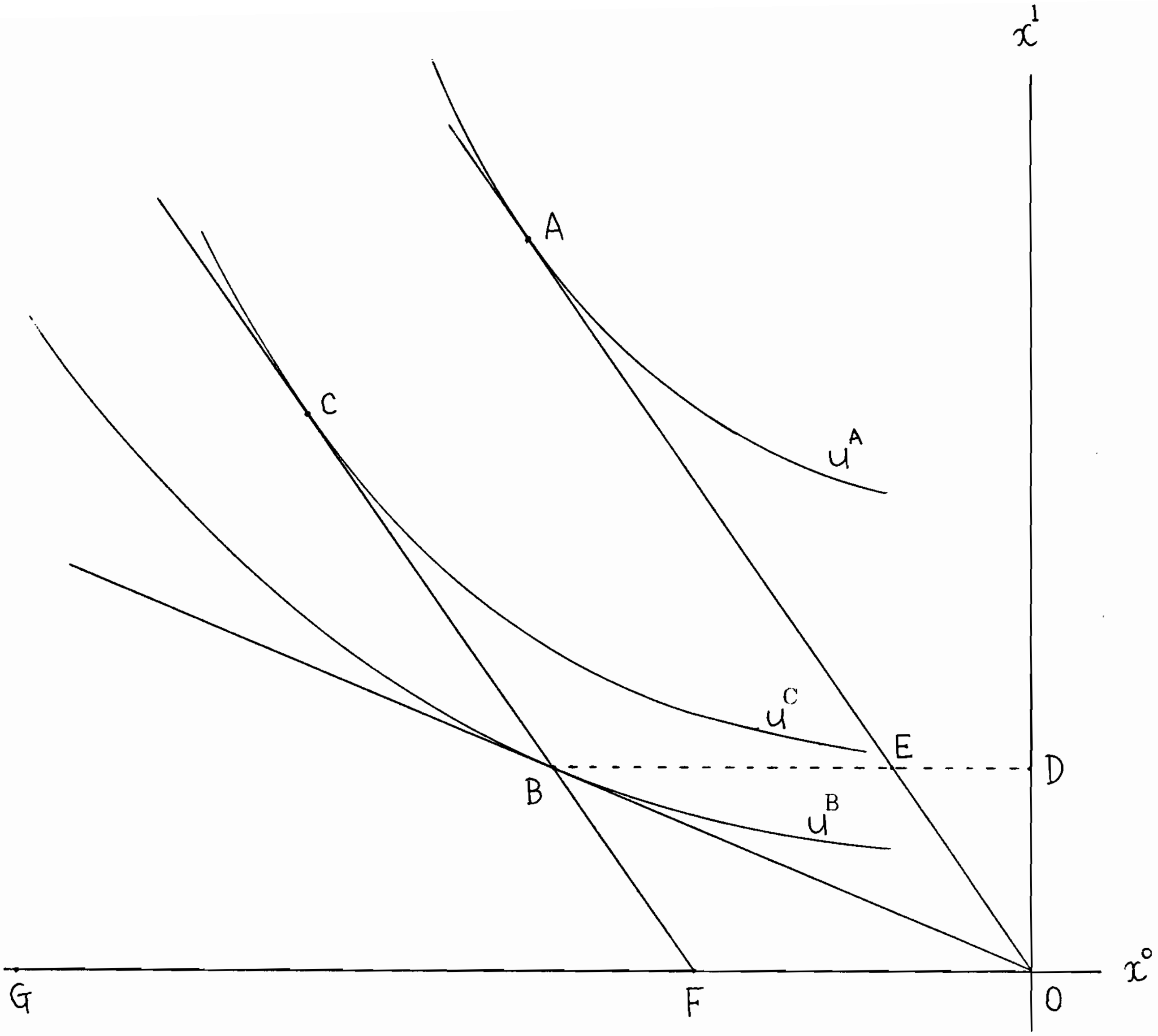

Figure 1-1

Chapter 2

Tax Reform with Variable Public Expenditure(*)

Abstract: This chapter presents sufficient conditions for a tax reform to be welfare improving, admitting public expenditure to be varied endogeneously. Our analysis corresponds to the concept of "budget incidence" in another context, not the "differential incidence" followed by most of the existing works on optimal taxation and tax reform.

1. Introduction

The optimality of uniform commodity taxes has been investigated by Dixit(1970), Atkinson-Stiglitz(1972), Sandmo(1974), Sadka(1977) and Deaton(1979) in the framework of the Ramsey problem; i.e., leisure is untaxed and the government has to collect a fixed amount of tax revenue. One of their conclusions is that uniform commodity taxes are optimal if and only if all commodities have the same compensated elasticities with respect to the wage, or equivalently, if uniform commodity taxes maximize the supply of labor. One obvious circumstance under which this necessary and sufficient condition is satisfied is when the labor supply is constant. On the other hand, based on the piecemeal approach having been advocated by Lipsey-Lancaster(1956) and Feldstein (1976), Hatta(1977) showed that changing commodity tax rates towards uniformity improves the welfare under substitutability conditions, assuming that labor supply is constant and that government revenue is returned to the consumer in a lump-sum manner.

Practically, however, government revenue is spent on public goods and then the change in tax revenue followed by a tax reform varies the amount of public goods, which influences the demand patterns for commodities and consequently affects the consumers' welfare. The purpose of this chapter is to find out sufficient conditions for a tax reform to be welfare improving, assuming that tax revenue, which has been spent on a public good, can be varied after the tax reform[*1].

Two types of tax reform will be examined; changing only the highest or the lowest tax rate, and changing all tax rates simultaneously towards a certain target. Once we allow the government revenue to vary at the time of tax reform, we should take account of two types of efficiency: the tax-induced distortion and the state of resource allocation between the private and public sectors. Indeed, an improvement in welfare will prove to relate to those two matters.

The model of the economy is exposed in the next section. Before we proceed to the main analyses, we show some preliminary lemmas in section 3. Sections 4 and 5 present sufficient conditions for tax reform to be welfare improving. Those conditions concern a kind of substitutability between commodities, and the positions of tax rates compared with ν, an index reflecting the desirability of present magnitude of the public sector. Section 6 considers the meaning of ν in detail. Finally, some remarks are given in section 7.

In principle, we use superscripts to index goods, and subscripts to show derivatives.

2. The Model

Consider an economy with many homogeneous consumers. A representative consumer has a preference over n private commodities $x' = (x^1 \cdots x^n)$ and one public good g(*2). All commodities are consumed in a positive quantity, and this consumer supplies a constant amount of labor, which is the only factor of production. For simplicity, let his utility function $u(\cdot)$ be additively separable in commodities and public good(*3):

$$u(x,g) = f(x) + h(g) \tag{1}$$

where $f_i = \partial f/\partial x^i > 0$, $i=1 \cdots n$, and $h_g = \partial h/\partial g > 0$. The corresponding expenditure function $m(\cdot)$ can be written as $m(q, u-h(g))$, where q is the consumer price vector and u is his utility level. His budget constraint is then written as

$$m(q, u-h(g)) = y \tag{2}$$

where y is the amount of his labor supply, letting his wage rate be chosen as a numeraire.

The producers have constant cost type technology and are competitive. Then, the production possibility frontier can be expressed as

$$p' z + rg = y \tag{3}$$

where p, z and r are the producer price vector of commodities, product vector of commodities and the producer price of public good, respectively.

The government imposes ad valorem commodity taxes on commodities, $\tau' = (\tau^1 \cdots \tau^n)$, and spends the revenue on the public good. Therefore, consumer prices can be related to producer prices as

$$q' = (e+\tau)' P \tag{4}$$

where e is an n dimensional unit vector, and P is the diagonal matrix whose diagonal elements are the producer prices. We assume that $1+\tau^j > 0$ holds for all j, i.e. subsidy rates cannot exceed one hundred percent.

In equilibrium, demand equals supply in every market:

$$c(q,u-h(g)) = z \tag{5}$$

where $c(\cdot)$ is the vector of the compensated demand function. Then, (3) can be written as

$$p' c(q,u-h(g)) + rg = y. \tag{6}$$

(2), (4) and (6) represent an equilibrium of the economy (*4). The endogeneous variables are u, g and q, while tax rates are given exogeneously.

3. Preliminaries

In advance of the main analyses, we show some important lemmas.

We denote the uncompensated demand function by $x(q,y)$, and then x_q shows the gross substitution matrix. The following lemma is used to prove lemma 2.

Lemma 1: The following equation holds for any scalar $a \neq -1$;

$$p' x_q = \frac{1}{1+a}(ae-\tau)' Px_q - \frac{1}{1+a}x' . \tag{7}$$

Proof: The consumer's budget constraint (2) can be rewritten in terms of an uncompensated demand function as

$$q' x(q,y) = y. \tag{8}$$

Differentiating (8) with respect to q yields

$$q' x_q + x' = 0. \tag{9}$$

Then, for any $a \neq 1$, we have

$$\begin{aligned}(1+a)p' x_q &= (e+ae)' Px_q - (e+\tau)' Px_q - x' \\ &= (ae-\tau)' Px_q - x' . \end{aligned} \tag{10}$$

Multiplying $1/(1+a)$ by both sides of (10) yields (7).‖

Next we show another lemma, which will be the basic equation of analyzing the welfare effect of a tax reform.

Lemma 2: The welfare effect of a tax reform can be given, for any $a \neq -1$, by

$$du = -D^{-1}\{k^1 \cdot (ae-\tau)' Px_q + k^2 \cdot (a-\nu)x' \}Pd\tau \tag{11}$$

where $D=rm_u > 0$, $k^1=m_u h_g/(1+a)$, $k^2=r/(1+a)$ and $\nu=(m_u h_g$

$-r)/r$ (*5).

<u>Proof</u>: Substituting (4) for q in (2) and (6) and totally differentiating them with respect to u, g and τ, we get

$$du = -D^{-1}\{p' c_q \cdot m_u h_g + c'(r - p' c_u h_g)\} P d\tau . \qquad (12)$$

Since an equation $c_u = x_y m_u$ holds(*6), (12) can be rewritten, by replacing c with x, as

$$du = -D^{-1}\{(p' c_q - x' p' x_y) m_u h_g + r x'\} P d\tau$$

$$= -D^{-1}(p' x_q \cdot m_u h_g + r x') P \, d\tau \qquad (13)$$

where the second equation follows from the Slutsky decomposition. By the Lemma 1, (13) can be further rewritten as

$$du = -D^{-1}\{\frac{m_u h_g}{1+a}(ae-\tau)' P x_q + \frac{r}{1+a}(-\frac{m_u h_g}{r} + 1 + a) x'\} P d\tau$$

$$= -D^{-1}\{k^1 (a-\tau)' P x_q + k^2 (a - \frac{m_u h_g - 1}{r}) x'\} P d\tau$$

which reduces to (11) by using the notation ν. ‖

Note that the sign of k^1 and k^2 depends on the tax reform we consider. Let us assume that $\nu > -1$ for convenience.

4. Changing the Extreme Tax Rate

This section considers such reforms as changing the highest or the lowest tax rate. The following proposition gives a sufficient condition for such tax reforms to be welfare improving.

<u>Proposition 1</u>: Suppose that (a) the commodities on which the highest (lowest) tax rate is <u>not</u> imposed are grossly substitutable for all the commodities which share the highest(lowest) tax rate, (b) the highest(lowest) tax rate is higher(lower) than ν. Then a reduction (an increase) in the highest(lowest) tax rate improves the welfare.

<u>Proof</u>: (We will only prove the case of reducing the highest tax rate, since the effect of increasing the lowest tax rate can be analyzed in a similar way(*7).)

Index the commodities as

$$\tau^1 \leqq \cdots \leqq \tau^m < \tau^{m+1} = \cdots = \tau^n \tag{14}$$

and consider the following tax reform:

$$d\tau^j = \begin{cases} 0 & \text{for } j=1\cdots m \\ -1 & \text{for } j=m+1\cdots n \end{cases}. \tag{15}$$

Then, substituting (15) for $d\tau$ and letting $a=\tau^n$ in (11), we have

$$du=D^{-1}\{k^1 \sum_{i=1}^{m} \sum_{j=m+1}^{n} (\tau^n-\tau^i)p^i x^i{}_j p^j + k^2(\tau^n-\nu) \sum_{j=m+1}^{n} p^j x^j\} \tag{16}$$

where $k^1=m_u h_g/(1+\tau^n)>0$ and $k^2=r/(1+\tau^n)>0$. Therefore, the RHS of (16) is positive under the conditions (a) and (b). ‖

We find from (16) that the welfare effect of reducing the highest tax rate can be decomposed into two terms. The first term in the brace on the RHS of (16) shows that lessening the divergence between the highest tax rate and the other tax rates improves the welfare under substitutability conditions. This result is consistent with the existing work by Hatta(1977), which maintains that reforming commodity tax rates towards uniformity is welfare improving(*8). On the other hand, the second term in the same brace concerns the state of resource allocation between the private and public sectors. In an economy without public good, like Hatta(1977), the target level towards which commodity tax rates are reformed is arbitrary. Once public good comes to be determined endogeneously, however, a certain target of tax reform is required in order to adjust the amount of public good to a more proper level. Since the number ν plays a significant role in carrying out tax reforms according to Proposition 1, we will inspect the meaning of ν later in section 6.

5. Changing All Tax Rates

Next, we will consider the other type of tax reform that changes all tax rates simultaneously towards ν. Formally, this way of reforming tax rates can be written as

$$d\tau = (\nu - \tau). \qquad (17)$$

We call the tax reform (17) the "proportional change in all tax rates towards ν ." The following proposition shows that such reforms will improve the welfare under a type of substitutability condition.

Proposition 2: Suppose that the gross substitution matrix x_q is quasi negative definite(*9). Then the proportional change in all tax rates towards ν improves the welfare.

Proof: Substituting (17) for $d\tau$ and letting $a=\nu$ in (11), we get

$$du = -D^{-1}k^1 b' x_q b > 0 \tag{18}$$

where $k^1 = m_u h_g/(1+\nu) > 0$ and $b=P(\nu - \tau)$. Thereafter, the RHS of (18) is positive under the quasi negative definiteness of the gross substitution matrix.‖

By the Slutsky decomposition, the quadratic form $b' x_q b$ on the RHS of (18) can be broke up as

$$b' x_q b = b' c_q b - b' c_u c' b. \tag{19}$$

Therefore, if the substitution effect is dominant compared with the income effect in the economy in question, $b' x_q b$ will be negative and then a proportional change in all tax rates towards ν will improve the welfare.

If b happens to be proportional to q, i.e. $b=\alpha q$ for any $\alpha \neq 0$, the first term on the RHS of (19) vanishes by the homogeneity property of the compensated demand function and we cannot expect the dominance of the substitution effect anymore. However, substituting $b=\alpha q$ for b in $b' c_u c' b$, and noting that $q' c_u = 1$ and $q' c = y$, we have

$$b' x_q b = -b' c_u c' b = -\alpha^2 y < 0. \tag{20}$$

Hence, even in such a particular case, $b' x_q b$ is still negative and then the welfare can be improved by carrying out the tax reform proposed in Proposition 2.

6. Resource Allocation between the Private and Public Sectors

Let us give an intuitive interpretation of how the number ν plays the role of an index of the present state of resource allocation between the private and public sectors. First of all, $m_u h_g$ represents how much money can be saved when one more unit of public good is supplied exogeneously, leaving the consumer's utility level unchanged. In this sense, $m_u h_g$ shows the marginal benefit of public good evaluated by consumer prices. On the other hand, r is the unit cost of production. Therefore, ν is the marginal net benefit per cost.

Suppose now that commodities are taxed so heavily that we have already had too much of a public good. Then, the marginal net benefit of a public good would be low, mainly because of decreasing marginal utility of the public good. Consequently, the value of ν is relatively small. Then, since our propositions suggest in general that tax rates should be made closer to ν , the going tax rates will be reduced in line with our propositions. This results in a decrease in the amount of public good. On the contrary, if the current tax rates are at a low level and we do not have enough public good in the economy, the value of ν would be relatively large for the similar reason. Then, according to the suggestions of our proposition, the going tax rates will be raised towards ν, and hence the amount of public good will be increased. Thus, the amount of public good will be adjusted to more proper level by carrying out tax reform in accordance with our propositions.

Next, let us consider the optimum uniform tax rate. Without any abbreviation, ν can be written as

$$\nu = \frac{m_u((1+\tau)' P, u-h(g))h_g(g)-r}{r}. \tag{21}$$

Since the amount of g is endogeneously determined if τ is chosen exogeneously by the government, g can be regarded as a function of τ, $g(\tau)$. Substituting $g(\tau)$ for g in (21), we can express (21) as

$$\nu = \phi(\tau). \tag{22}$$

(22) implies that the value of ν is determined responding to the <u>current</u> tax rates.

Next, consider the following maximization problem:

$$\max_{\{\tau, g\}} v((1+\tau)' P, y) + h(g) \tag{23}$$

$$\text{s.t.} \quad p' x((1+\tau)' P, y) + rg - y \leqq 0$$

where $v(\cdot)$ is the indirect utility function corresponding to the original utility function (1). The first order condition for τ is given by

$$(\mu \cdot p' x_q + \lambda \cdot x')P = 0 \qquad (24)$$

where μ is the Lagrangean multiplier and $\lambda = v_y$, the marginal utility of income. Referring to (9) and using the relation $m_u \lambda = 1$(*10), (24) can be rewritten as

$$\{(m_u \mu - 1)p' x_q - \tau' P x_q\}P = 0. \qquad (25)$$

To show that uniform taxes can be optimal, let us substitute $\tau = \nu e$ for τ in (25) to get

$$\{(m_u \mu - 1 - \nu)p' x_q\}P = 0. \qquad (26)$$

We find from (26) that taxes with a uniform rate at the level

$$\nu = m_u \mu - 1 \qquad (27)$$

are optimal under the given amount of public good. Let us denote ν which satisfies (27) by ν^*.

Next, the first order condition for g is given by

$$h_g - \mu r = 0. \qquad (28)$$

Solving (28) for μ and substituting it for μ in (27) yield

$$\nu = \frac{m_u h_g - r}{r} = \phi(\nu). \qquad (29)$$

That is, the optimum uniform rate is determined such that ν is a fixed point of function $\phi(\cdot)$. We denote ν which satisfies (29) by ν^{**}. When all tax rates are made uniform at the level of ν^{**}, Pareto optimum is attained, i.e. there is no tax-induced distortions and resource allocation between the private and public sectors are totally appropriate.

7. Concluding Remarks

Let us now relate our analyses with the existing work by Hatta(1977). (12) can be easily rewritten as

$$du = -\left(\frac{h_g}{r} \cdot p' c_q + \frac{r - p' c_u h_g}{r m_u} \cdot c'\right) P d\tau . \tag{30}$$

The term $p' c_u h_g$ represents how many resources can be saved by supplying another unit of public good, leaving the utility level unchanged. Hence, this term can be regarded as the benefit, evaluated by producer prices. On the other hand, r is the unit cost of public good. Therefore, $r-p' c_u h_g$ as a whole shows the net cost of producing one more unit of public good. Suppose now that $r-p' c_u h_g=0$, i.e. public good is supplied in an optimal amount in the sense that marginal cost equals marginal benefit. Then, (30) is reduced to

$$\frac{du}{d\tau} = -\frac{1}{p' c_u} \cdot p' c_q P \tag{31}$$

which is exactly the same expression as eq.(10) in Hatta(1977). Therefore, we can say that if the public good is supplied in an optimal amount, our results are reduced to those of Hatta(1977).

Finally, we should note that the tax reform proposed in our propositions can be carried out only if we know the current level of ν ; the information of the optimal level ν^{**} is not required. This is different from the policy recommendations based on the optimum approach. In addition, we find that repetition of our piecemeal policy eventually brings the economy to Pareto optimum.

Footnotes

(*) An earlier draft of this chapter was reported at the Kinki-region Meeting of the Japan Association of Public Finance on 11 July 1987 at the Osaka Prefectural Gymnasium, Osaka.

(*1) This type of analysis corresponds to the idea of "budget incidence" or "benefit taxation." On the other hand, most existing articles on optimal taxation and tax reform, including Hatta(1977), is of the "differential incidence" type. See Musgrave(1976) and Musgrave-Musgrave(1982).

(*2) Though the aspects of joint consumption of public good is not taken into account effectively in a single consumer economy, we still consider g as a public good for the reason that it is furnished by the government. Hence, we could also call g the rationed good.

(*3) Atkinson-Stern(1974) assumed this type of separability.

(*4) The budget balance equation of the government τ' $Pc(q,u-h(g))=rg$ is implied by (2) and (6).

(*5) The term $m_u h_g$ can be interpreted as the marginal rate of substitution between the private income and the public good. Hence, by making an inquiry survey, for example, we would be able to get the information on the magnitude of ν .

(*6) Differentiating the identity $c(q,u)=x(q,m(q,u))$ with respect to u yields the equation.

(*7) In proving the case of increasing the lowest tax rate, it suffices to modify (14) and (15) as

$$\tau^1 = \cdots = \tau^m < \tau^{m+1} \leqq \cdots \leqq \tau^n$$

and

$$d\tau^j = \begin{cases} 1 & \text{for } j=1\cdots m \\ 0 & \text{for } j=m+1\cdots n \end{cases}$$

respectively, and then let $a=t^1$. This results in

$$du=-D^{-1}\{k^1\sum_{i=1}^{m}\sum_{j=m+1}^{n}(\tau^1-\tau^j)p^jx^j{}_ip^i+k^2(\tau^1-\nu)\sum_{i=1}^{m}p^ix^i\}.$$

where $k^1=m_uh_g/(1+\tau^1)>0$ and $k^2=r/(1+\tau^1)>0$.

(*8) Strictly speaking, Hatta(1977) assumes substitutability in terms of a compensated demand function, different from gross substitutability used here. From the standpoint of policy recommendations, this difference would not be so significant as long as substitution effects are relatively dominant compared with income effects. Or rather, the fact that gross substitution terms are easier to estimate could be an advantage of our sufficient conditions.

(*9) For any vector $h \neq 0$, a matrix B, which may or may not be symmetrical, is called negative quasi definite if $h'Bh<0$.

(*10) Differentiating the identity $m(q,v(q,y))=y$ with respect to y yields the equation.

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Chapter 3

Analyses on Tax-Expenditure Policy: Value Added Tax and Earmarked Commodity Tax

Abstract: Provision of public goods is greatly influential in considering desirable tax policies. We link the tax to the expenditure, and consider the following two issues from an efficiency point of view: One is the problem of an increase in VAT rate, and the other is the classical issue of earmarked commodity taxes.

1. Introduction

Provision of public goods is greatly influential in considering the desirable direction of tax reform. Although most articles on the theory of tax reform neglect the expenditure side, in reality tax reform often results in a change in government revenue and consequently in construction of public goods. Change in quantity of public goods alters the demand for private goods, including taxed goods. Then, change in quantity of taxed goods changes the amount of government revenue once again. Taking account of such interrelations between public goods supply and private goods demand, this chapter considers two issues concerning normative tax policies from an efficiency point of view. One is the problem of altering value added tax (henceforce referred to VAT) rate, and the other is on the desirability of earmarked commodity taxes. A rise in the VAT rate or earmarked commodity tax rate varies, or in general expands the government revenue. Then a question arises: which public good should the additional revenue be spent on? To respond to this problem, we should take into account two matters. First of all, the public good which is absolutely scarce in the economy ought to be enlarged. Secondly, supply of such kind of public good is recommended to be increased that rectifies the tax induced distortions in demand for private goods, including leisure, which have been generated by imposing VAT or earmarked commodity taxes.

The basic model is exposed in the next section. The problem of VAT and earmarked commodity taxes are discussed in section 4 and 5, respectively. In advance of these main examinations, a preliminary analysis is made in section 3, which enables us to treat the two different problems synthetically. Some important concepts, including the definition of substitutability and complementarity between public goods and private goods, are also shown in this section. Finally, the results of the analyses on VAT and earmarked commodity taxes are interpreted in section 6.

In principle, we use superscripts to index goods, and subscripts to show derivatives. Especially, derivatives with respect to public goods are indicated by subscripts with parentheses. For instance, $c_{(i)}$ means the derivative of compensated demand vector for private goods c with respect to the ith public good.

2. The Model

Consider an economy with many homogeneous consumers. A representative consumer has a preference over n+1 private goods ($x^0 \cdots x^n$) and m public goods ($g^1 \cdots g^m$). The 0th private good is leisure, and then $x^0 < 0$. The consumer's budget constraint at equilibrium is expressed, by using the expenditure function $m(\cdot)$, as

$$m(q,g,u) = 0 \tag{1}$$

where q, g and u show the n+1 dimensional vector of the consumer prices of the private goods, the m dimensional vector of the quantities of the public goods and the consumer's utility level, respectively.

By differentiating $m(\cdot)$ with respect to g^i, the ith element of g, we get the shadow price of the ith public good s^i: $s^i(q,g,u) = -m_{(i)}(\cdot)$.

Producers have constant cost technology and are competitive, so that the production possibility frontier at equilibrium is given by

$$p'z + r'g = 0 \tag{2}$$

where p and z represent the n+1 dimensional vector of the producer prices and the products of the private goods, respectively, and r is the m dimensional vector of the producer prices of the public goods.

Private goods are divided into taxed goods and untaxed goods. Let T denote the set of superscripts of taxed goods. Commodities in group T are taxed uniformly. Then, consumer prices can be related to producer prices as

$$\begin{cases} q^j = (1+\tau)p^j & \text{for } j \in T \\ q^j = p^j & \text{for } j \notin T \end{cases}. \tag{3}$$

The government spends the tax revenue on the provision of public goods.

In equilibrium, demand equals supply in every market. Then, (2) can be written as

$$p' c(q,g,u) + r' g = 0 \tag{4}$$

where $c(\cdot)$ is the n+1 dimensional vector of a compensated demand function for the private goods. An equilibrium of the economy can be summarized by (1), (3) and (4), where u, q and one of the elements of g (e.g. g^i) are determined endogeneously according to τ and the rest of the elements of g which are chosen by the government.

3. Preliminary Analysis

Taking account of (3), total differentials of (1) and (4) with respect to u, g^i and τ yield

$$u_\tau{}^i = -(D^i)^{-1}\{p^{*\prime} c^* \cdot (p' c_{(i)}+r^i)+s^i \cdot p' c_q{}^* p^*\} \tag{5}$$

where $u_\tau{}^i = du/d\tau$. In this equation, superscript i shows that g^i is varied endogeneously, p^* and c^* denote the price and quantity vector of taxed goods respectively, $c_q{}^*$ is the substitution matrix except the jth columns for $j \in T$, and

$$D^i = m_u \cdot (p' c_{(i)} + r^i) + s^i \cdot p' c_u . \qquad (6)$$

Let either g^a or g^b, the ath or the bth element of g, be varied endogeneously followed by a change in τ. Then, from (5), we get

$$u_\tau{}^a - u_\tau{}^b = k\left(\frac{p' c_{(b)} + r^b}{s^b} - \frac{p' c_{(a)} + r^a}{s^a} \right) \qquad (7)$$

where

$$k = s^a s^b (D^a D^b)^{-1} (p' c_u \cdot p^{*\prime} c^* - m_u \cdot p' c_q{}^* p^*). \qquad (8)$$

The following lemma will play an important role in deriving main results in later sections.

<u>Lemma 1</u>: The following equation holds:

$$u_\tau{}^a - u_\tau{}^b = k\{(\eta^a - \eta^b) - \theta \sum_{i \in T} (\varepsilon^a{}_j - \varepsilon^b{}_j)\} \qquad (9)$$

where $\theta = \tau/(1+\tau)$, $\eta^i = (s^i - r^i)/s^i$, $\varepsilon^i{}_j = s^i{}_j (q^j/s^i)$, $i=a,b$, and

$$k = -m_u s^a s^b (D^a D^b)^{-1} p' x_q{}^* p^* \qquad (10)$$

in which $x_q{}^*$ is the gross substitution matrix except the jth column for $j \in T$.

<u>Proof</u>: Since

$$p' c_{(i)} + r^i = q' c_{(i)} - \tau p^{*\prime} c^*{}_{(i)} + r^i$$
$$= -(s^i - r^i) - \tau \sum_{j \in T} c^j{}_{(i)} p^j \qquad (11)$$

and

$$c^j{}_{(i)} = m_{j(i)} = m_{(i)j} = -s^i{}_j \qquad (12)$$

we have

$$\frac{p' c_{(i)} + r^i}{s^i} = -\eta^i + \theta \sum_{j \in T} \varepsilon^i{}_j . \qquad (13)$$

By (13), (7) can be rewritten as (9).

Next, since $c_u = x_y m_u$ holds(*1), from (8) we have

$$k = -m_u s^a s^b (D^a D^b)^{-1} p' \ (c_q{}^* - x_y \cdot c^{*\prime}) p^* \tag{14}$$

which can be rewritten as (10) by using the Slutsky decomposition.‖

The sign of η^i, which we call the net benefit rate of the ith public good, depends on the magnitude of the shadow price and the unit cost of the public good. Suppose that s^i is larger(smaller) in magnitude than r^i and therefore η^i is positive(negative). Then, by increasing(decreasing) the amount of g^i, we can improve the welfare, on condition that lump-sum taxes are available. In this sense, η^i can be an index of undersupply and oversupply of the ith public good in the economy. Formally written as:

<u>Definition 1</u>: The ith public good is said to be undersupplied(oversupplied) if η^i is positive(negative). Furthermore, the ath public good is said to be more undersupplied (more oversupplied) than the bth public good is if $\eta^a > \eta^b$ ($\eta^a < \eta^b$) holds.‖

The shadow price s^i can be regarded as the marginal valuation of g^i. Suppose now that first the price of x^j had increased and then the compensated demand for x^j decreased. At the same time, if the marginal valuation of g^i increased (decreased), g^i is considered to be a substitute (complement) of x^j(*2). Therefore:

Definition 2: The ith public good is said to be substitutable for (complementary with) the jth private good if $\varepsilon^{i}{}_{j}$ is positive(negative). Furthermore, the ath public good is said to be more substitutable for (more complementary with) the jth private good than the bth public good is if $\varepsilon^{a}{}_{j} > \varepsilon^{b}{}_{j}$ ($\varepsilon^{a}{}_{j} < \varepsilon^{b}{}_{j}$) holds.‖

The following lemma specifies the sign of k in (9).

Lemma 2: If raising τ increases the government revenue, then k in (9) is positive.

Proof: Taking account of (3), total differentials of (1) and (4) yield

$$g^{i}{}_{\tau} = (D^{i})^{-1}(-m_{u} \cdot p' x_{q}{}^{*}p^{*}) \tag{15}$$

where $g^{i}{}_{\tau} = dg^{i}/d\tau$. Then we have

$$g^{a}{}_{\tau} \cdot g^{b}{}_{\tau} = (D^{a}D^{b})^{-1}(-m_{u} \cdot p' x_{q}{}^{*}p^{*})^{2} \tag{16}$$

which is positive by assumption. Therefore, we get

$$D^{a}D^{b} > 0. \tag{17}$$

Next, the consumer's budget constraint at an equilibrium can be written, in terms of an uncompensated demand function, as

$$q' x(q,g,y) = y \tag{18}$$

where y=0. Taking account of (3), differential of (18) with respect to τ yields

$$p' x_{q}{}^{*}p^{*} + \tau p^{*\prime} x_{q}{}^{*}p^{*} + p^{*\prime} x^{*} = 0. \tag{19}$$

Defining the government revenue function G(·) as

$$G(\tau) = \tau p^{*\prime} x^{*}(q,g,u) \tag{20}$$

and differentiating it with respect to τ , we obtain

$$G_{\tau} = \tau p^{*\prime} x_q{}^* p^* + p^{*\prime} x^* \quad (21)$$

which is positive by the assumption. From (19) and (21), we have

$$p' x_q{}^* p^* = -G_{\tau} < 0. \quad (22)$$

Since s^a, s^b and m_u are positive, (17) and (22) imply that $k > 0$. ‖

4. Financing Public Goods by Raising the VAT Rate

A new commodity tax system named the "consumption tax," substantially a kind of VAT, has been carried out since 1 April 1989 in Japan. Under this tax rule, all of the commodities, in principle, are subject to three percent taxation at an ad valorem rate.

Although the initial rate was set at three percent, probably it will be lifted in the near future. Actually, VAT rates in the other countries have been raised several times and consequently they are now in much higher level than they were. (See Table 3-1.) Now the question arises: on what kind of public goods, or government expenditure items, should be spent the additional VAT revenue obtained by raising its rate? On this point, Japanese government seems to consider that the expected shortage of budget for the social security items like medical care or annuity can be made up by an introduction of a VAT and by raising its rate in the future(*3)(*4). In fact, in northern European countries, especially Denmark and Sweden, the VAT rate has been raised by more than ten percent, which can be related to some extent with an extension of welfare items.

This section investigates the desirability of spending the additional VAT revenue on welfare items from an efficiency point of view. We let all the commodities be taxed at the rate τ, while leisure be untaxed by the normalization of tax rates.

The following lemma is used later to prove the first proposition.

Lemma 3: The following equation holds:

$$\sum_{j=0}^{n} \varepsilon^{i}_{j} = 1 \text{ for } i=a,b. \tag{23}$$

Proof: Since $s^i = -m_{(i)} = -q' c_{(i)}$, we have

$$\sum_{j=0}^{n} \varepsilon^{i}_{j} = \frac{\sum_{j=0}^{n} q^j s^i_j}{s^i} = \frac{-\sum_{j=0}^{n} q^j c^i_{(i)}}{s^i} = 1 \tag{24}$$

where the second equality follows from (12).‖

The following proposition presents a sufficient condition for paying the additional VAT revenue for a public good to be more welfare improving than paying it for the other public good.

Proposition 1: Suppose that raising the VAT rate increases the government revenue. Then, spending the increased VAT revenue on g^a is more welfare improving than spending it on g^b if (a) g^a is more undersupplied than g^b is, and (b) g^a is more substitutable for leisure than g^b is.

Proof: Let $T=\{1,2\cdots n\}$. Then, from (9) we have

$$u_{\tau}{}^{a}-u_{\tau}{}^{b} = k\{(\eta^{a}-\eta^{b})-\theta\sum_{j=1}^{n}(\varepsilon^{a}{}_{j}-\varepsilon^{b}{}_{j})\}. \tag{25}$$

By making use of (23), (25) is rewritten as

$$u_{\tau}{}^{a}-u_{\tau}{}^{b} = k\{(\eta^{a}-\eta^{b})+\theta(\varepsilon^{a}{}_{0}-\varepsilon^{b}{}_{0})\}. \tag{26}$$

Lemma 2 ensures the positivity of k on the RHS of (26). Both the terms in the first and the second parenthesis on the RHS of (26) are positive under the assumption (a) and (b). Then, since θ is positive, the RHS of (26) proves to be positive.‖

From Proposition 1, we find that the increased VAT revenue should be spent on the public good which are substitutable for leisure, or complementary with labor, as long as we leave the absolute scarceness of the public goods out of consideration. Take annuities, a major item in the social security, as an example. Suppose that the price of leisure, i.e. the wage rate, has been raised and then the supply of labor increased. In this case, the marginal valuation of annuity seems to be reduced because of the increased earnings, and therefore the annuities can be complementary with leisurey. Hence, we cannot consider social security as an appropriate item on which the increased VAT revenue will be spent, even though the government plans to earmark VAT revenue for welfare items. Instead, among the major expenditure items in the general government budget which are listed in Table 3-2, public works, small and medium businesses, energy, etc. could be better items. This is because they would be at least less complementary with labor than the social security(*5).

5. Efficiency of Earmarked Commodity Taxes

Earmarked taxes have been justified in view of what is called the "benefit principle" traditionally used in the theory of public finance(*6). The benefit principle insists that an equitable tax system is one under which each tax payer contributes in line with the benefits which he or she receives from public services. Therefore, where imposition of direct charges for public services is desirable but too costly, a tax on a complementary commodity may be used in lieu of charges. Gasoline taxes for tidying up a road is a familiar example.

This section reconsiders the desirability of earmarked commodity taxes in the light of modern tax theory. Especially, we attempt to justify the earmarked commodity taxes from an efficiency standpoint. For simplicity, we assume that only one private good is taxed, and the other goods are untaxed.

The following proposition suggests an appropriate public good on which the increased earmarked tax revenue is spent.

Proposition 2: Suppose that only the kth private good is taxed and that raising the tax rate increases the government revenue. Then, spending the increased tax revenue on g^a is more welfare improving than on g^b if (a) g^a is more undersupplied than g^b is, and (b) g^a is more complementary with the kth private good than g^b is.

Proof: Let T={k}. Then, from (9), we have

$$u_\tau{}^a - u_\tau{}^b = k\{(\eta^a - \eta^b) - \theta(\varepsilon^a{}_k - \varepsilon^b{}_k)\}. \quad (27)$$

Lemma 2 assures the positivity of k. The term in the first parenthesis on the RHS of (27) is positive by the assumption (a), and the term in the second parenthesis is negative by the assumption (b). Then, since θ is positive, the RHS of (27) turns out to be positive.∥

Let us now recall the example of gasoline taxes and the paved roads. Suppose that the tax rate on gasoline had been raised and the demand for gasoline decreased. Then, the demand for automobiles, a close complement of gasoline, will decrease and consequently the marginal valuation of paved roads will decrease. Therefore, the road can be considered to be complementary with gasoline. Hence, Proposition 2, especially condition (b), will support the desirability of traditional earmarked commodity taxes, like gasoline taxes, from an efficiency point of view.

6. Concluding Remarks

Proposition 1 and 2 can be interpreted as follows. In section 4, we assumed that all the private goods except leisure are taxed. This implies that the relative price of leisure is low for consumers, and then the leisure is consumed more under a VAT system than otherwise. Therefore, substitutable public good for leisure should be increased to depress the overconsumption of leisure. On the other hand, only one private good is assumed to be taxed in section 5. In this case, the taxed good is consumed less compared with the untaxed case. Thereafter, we should furnish a larger amount of the complementary public good with the taxed good to cope with the underconsumption of the taxed good.

Footnotes

(*1) Differentiating the identity $c(q,g,u)=x(q,g,m(q,g,u))$ with respect to u yields this equation.

(*2) For example, paved roads and automobiles are considered to be complements. Suppose that the price of automobiles had increased and then the compensated demand for automobiles decreased. In this case, the marginal valuation of paved roads will decrease because the utility of paved roads is mostly enjoyed by driving cars.

(*3) Let us cite a supporting statement for this fact. "We should try to find out a tax system or structure which can meet elastically and stably the expected increase in expenditure required to cope with the aging society and … ." (Usui(1987), p.25.) Apparently, they are describing an introduction of a VAT and an increase in its rate in Japan. Another assertion of former Prime Minister S.Uno is as follows: "… Uno defended the consumption tax that was introduced April 1 as a necessity to prepare for the welfare of Japan's rapidly aging society." (The Japan Times, Thursday 6 July 1989.)

(*4) According to the simulation by Tamaoka(1989), by pulling up the going three percent rate to five percent, approximately an additional four trillion yen tax revenue can be expected.

(*5) Indeed there are some untaxed commodities under the going VAT system. The existence of such untaxed commodities creates substantially the same effect as leisure did in the text. Then, basically the public good which is strongly substitutable for the untaxed commodity is recommended to be increased. Under the going tax system in Japan, part of medical care and education, etc. are untaxed. See Miyaji(1988).

(*6) See, for instance, Musgrave-Musgrave(1982), Chapter 11.

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Table 3-1

percentage VAT rates in main countries(*)

country	date of introduction	rate at introduction	rate in 1988
(EC nations)			
Denmark	1967	10.0	22.0
France	1968	13.6	18.6
Germany,Fed.Rep.of	1968	10.0	14.0
Netherlands	1969	12.0	20.0
Luxembourg	1970	8.0	12.0
Belgium	1971	18.0	19.0
Ireland	1972	16.4	25.0
Italy	1973	12.0	18.0
U.K.	1973	10.0	15.0
(others)			
Sweden	1969	11.1	23.5
Norway	1970	20.0	20.0
Austria	1973	8.0	20.0

(*) based on Alan(1988)

Table 3-2

government expenditure of 1989 fiscal year in Japan(*)

	expenditure (thousand yen)	ratio (%)
1.social security	10,895	18.0
2.education & science	4,937	8.2
3.interest	11,665	19.3
4.veterans	1,856	3.1
5.grant to local government	13,369	22.1
6.national defense	3,920	6.5
7.public works	6,197	10.3
8.ODA	728	1.2
9.small businesses	194	0.3
10.energy	527	0.9
11.staple food control	418	0.7
12.special account	1,300	2.2
13.miscellaneous	4,058	6.7
14.reservation	350	0.5
total	60,414	100.0

(*) based on Japan Inquiry Committee of Public Finance (1988)

Chapter 4

Distributional Equity and the Theory of Tax Reform(*)

Abstract: This chapter discusses a trade-off between efficiency and equity which results from commodity tax reform. First, we derive a theoretical result which characterizes the efficiency effect and the equity effect brought about by a tax reform. Then, we apply the theoretical result to the Japanese liquor tax structure and calculate the welfare variation caused by a tax reform.

1. Introduction

Welfare effects of commodity tax reform in a single consumer economy has been studied by Hatta(1977), Hatta(1986) and Hatta and Haltiwanger(1986). The main result proposed in these articles is that a reform of the commodity taxes, letting the uniform tax structure be the target, will improve the efficiency of the economy and consequently improve the welfare. Distributional considerations, on the other hand, were incorporated in the theory of optimal public sector pricing by Feldstein(1972a,b). The present chapter integrates these two types of contributions to consider the tax reform problem in a many consumer economy in which redistribution through commodity taxes is often required to improve the social welfare that consists of the utility of many different individuals(*1).

After the exposition of the model in section 2, in section 3 we first derive a theoretical proposition on the welfare effects of tax reform in a many consumer economy in which it is shown that the total welfare effect can be decomposed into an efficiency effect and an equity effect. Then, in section 4, we apply the theoretical result obtained in section 3 to the Japanese liquor tax structure (analyzing "Sake" and whisky separately), and calculate the value of the efficiency effect, the equity effect and the total welfare variation caused by the tax reform. In both cases, the efficiency effect is shown to be crucially dominant, and a reduction in the highest tax rate can be recommended.

In principle, we use superscript to index goods, and subscripts to show derivatives.

2. The Model

<u>Consumer</u>: Consider an economy with infinitely many consumers, who have identical preferences but are endowed with different amounts of a homogeneous production factor, such as labor. We choose the labor as the numeraire, and we denote their income by y. We regard y as a random variable with the density function $f(y)$ dis-

tributed over the range $(0,\infty)$, a subset of the real number. (In the following discussion, we will omit the range of the integral when no confusion is expected.) A consumer's preference over n commodities is represented by the indirect utility function. By making use of this function, his utility level is shown as

$$u = v(q,y) \tag{1}$$

where q denotes the consumer's price vector(*2). By Roy's identity, (1) yields the consumer's demand function,

$$x^j(q,y) = -\frac{\partial v(q,y)/\partial q^j}{\partial v(q,y)/\partial y}. \tag{2}$$

Let $X^j(q)$ denote the aggregate demand function of good j, which is given by

$$X^j(q) = \int x^j(q,y)f(y)dy. \tag{3}$$

<u>Producer</u>: We assume that the production technology is of the constant cost type, and that the producers are competitive. Then the production possibility frontier at an equilibrium is given by

$$p' z + g = Y \tag{4}$$

where p, z, g and Y represent the producer's price vector, the output vector of the private goods, the output of the public good and the factor of production existing in the economy, $Y=\int yf(y)dy$, respectively.

<u>Government</u>: The government imposes the commodity taxes on the private goods and spends the revenue on the public good. Due to the commodity taxes, the consumer's

prices diverge from the producer's prices,

$$q = p + t \tag{5}$$

where t denotes the specific tax vector. The government budget constraint is given by

$$t' z = g. \tag{6}$$

Equilibrium: Balance equations of the goods market are given by

$$X(q) = z. \tag{7}$$

(7), together with (5) and (6), determine 2n+1 variables q, z and one of the n tax rates for the other n-1 tax rates and g chosen by the government[(*3)].

3. Welfare Effects of Tax Reform

It will sometimes be convenient to use the effective tax rate, $\tau^j = t^j/q^j$, instead of the specific tax rate itself[(*4)]. Now we will consider a reform which reduces the highest effective tax rate, τ^n, accompanied by a simultaneous change in the lowest effective tax rate, τ^1, so as to maintain the initial government revenue [(*5)]. (In the following discussion, the term just "tax rate" instead of "effective tax rate" will be used when they are with the words like highest or lowest.) Let a variation in t^1 which offsets a change in the government revenue caused by a change in t^n be represented by the function

$$t^1 = t^1(t^n) \tag{8}$$

in which, by making use of (6) and (7),

$$\frac{dt^1}{dt^n} = -\frac{X^n + \Sigma_i \tau^i q^i X^i_n}{X^1 + \Sigma_i \tau^i q^i X^i_1} \tag{9}$$

where $X^i{}_j = \int (\partial x^i/\partial q^j) f(y)dy$, $j=1,n$.

The government is assumed to intend to improve social welfare

$$V = \int w(v(q,y))f(y)dy \tag{10}$$

subject to its budget balance (6). Substituting (8) for t^1 in (10) and differentiating it with respect to t^n, by making use of (2) and (9), we get

$$dV = X^n \int \mu(y)\left\{\frac{x^1}{X^1}\cdot\frac{1+\Sigma_i \tau^i (q^i/X^n) X^i_n}{1+\Sigma_i \tau^i (q^i/X^1) X^i_1} - \frac{x^n}{X^n}\right\} f(y)dy \cdot dt^n \tag{11}$$

where $\mu(y) = (\partial w/\partial u)(\partial v/\partial y)$ denotes the social marginal utility of income.

Following Feldstein(1972a), we now define the distributional characteristic of a good as follows.

<u>Definition 1</u>: The distributional characteristic of good j, denoted δ^j, is defined by

$$\delta^j = \int \{(x^j/X^j)\mu(y)\} f(y)dy. \tag{12}$$

‖

That is, the distributional characteristic is the weighted average of the social marginal utility of income, where the weight is the ratio of the quantity consumed by a consumer to the total quantity of the good.

Suppose, for instance, that good 1 is preferred to good n by the poor, and oppositely that good n is preferred to good 1 by the rich. Then, it is plausible that the value of δ^1 is larger than that of δ^n so long as the social marginal utility of income declines as income increases. Thus, also following Feldstein(1972a), we define the degree of luxuriousness of a good as follows.

<u>Definition 2</u>: Good n is said to be more luxurious (necessary) than good 1 if $\delta^n < \delta^1$ ($\delta^n > \delta^1$) holds.‖

Now, note that the following relation holds,

$$\Sigma_i \tau^i (q^i/X^j) X^i_j = \Sigma_i \tau^i r^{ij} \varepsilon^{ij} \quad j=1,n \tag{13}$$

where $r^{ij}=q^iX^i/q^jX^j$ and $\varepsilon^{ij}=X^i_j(q^j/X^i)$. Then, by making use of (12) and (13), (11) can be rewritten as

$$dV=X^n\{\frac{\Sigma_i \tau^i r^{in} \varepsilon^{in} - \Sigma_i \tau^i r^{i1} \varepsilon^{i1}}{1+\Sigma_i \tau^i r^{i1} \varepsilon^{i1}} \delta^1 + (\delta^1 - \delta^n)\} dt^n. \tag{14}$$

The following lemma will be used to reduce the first term in the brace on the RHS of (14) to a more understandable expression.

<u>Lemma</u>: The following relation holds;

$$\Sigma_i \tau^i r^{ij} \varepsilon^{ij} = \Sigma_i (\tau^i - \tau^j) r^{ij} \varepsilon^{ij} - \tau^j \quad j=1,n. \tag{15}$$

<u>Proof</u>: Sum up all the consumers' budget constraints to get

$$\Sigma_i q^i X^i = Y. \tag{16}$$

Differentiating (16) with respect to q^j, and multiplying both sides by τ^j/X^j, we get

$$\Sigma_i \tau^j r^{ij} \varepsilon^{ij} + \tau^j = 0. \tag{17}$$

Subtracting (17) from the LHS of (15) yields the lemma. ‖

By making use of (15), (14) can be rewritten as

$$dV = X^n\{-A\delta^1 + (\delta^1 - \delta^n)\}dt^n \tag{18}$$

where

$$A = \frac{\Sigma_i(\tau^n - \tau^i)r^{in}\varepsilon^{in} + \Sigma_i(\tau^i - \tau^1)r^{i1}\varepsilon^{i1} + (\tau^n - \tau^1)}{\Sigma_i(\tau^i - \tau^1)r^{i1}\varepsilon^{i1} + (1 - \tau^1)}.$$

We will call the first term in the brace on the RHS of (18) the <u>efficiency effect</u> of tax reform, since this term reflects the variation in the tax-induced distortion (or the excess burden)(*6). It could be easily verified from the definition of the term A that the wider the difference among tax rates and the larger the cross elasticity among tax rates, the larger is the negative number of the efficiency effect. Hence, in this case, decreasing the highest tax rate while simultaneously increasing the lowest tax rate is desiable from social point of view. Consider an extreme case in which the initial commodity tax structure is uniform; i.e. every commodity is taxed in the same effective tax rate. In this case, this term vanishes, which means that there is initially no tax-induced distortions and therefore welfare cannot be improved by any tax reform through an improvement in efficiency.

On the other hand, we will call the second term, which is given by the difference between the values of the distributional characteristic of the goods in question, the <u>equity effect</u> of tax reform. Since the distributional characteristic of a luxurious good is larger in value than that of a necessary good, this term shows that the tax rate imposed on the luxurious good should be increased with a simultaneous decease in the tax rate on the necessary good from a standpoint of social welfare. In another extreme case in which every consumer earns the same income, the equity effect vanishes and only the efficiency effect remains, which corresponds to a single consumer case.

The following proposition presents a sufficient condition for the sign of each effect to be specified.

<u>Proposition</u>: Consider a tax reform that reduces the highest tax rate accompanied by a simultaneous change in the lowest tax rate so as to maintain the initial government revenue. Then:

(a) (Efficiency Effect:) Suppose that every good which is not imposed the highest tax rate is grossly substitutable for the good on which the highest tax rate is imposed, and that every good which is not imposed the lowest tax rate is grossly substitutable for the good on which the lowest tax rate is imposed. Then the efficiency effect turns out to be negative and pushes up the social welfare.

(b) (Equity Effect:) Suppose that the good on which the highest tax rate is imposed is more luxurious (necessary) than the good on which the lowest tax rate is imposed. Then the equity effect turns out to be positive (negative) and pulls down (pushes up) the social welfare. ‖

Suppose that all the conditions presented in (a) are satisfied. Then, if the good on which the highest tax rate is imposed is more necessary than the good on which the lowest tax rate is imposed, the sign of these effects coincides to be negative. Therefore, it turns out to be that a decrease in the highest tax rate is desirable. It is not plausible, however, since more common is the case in which the luxurious goods are taxed more heavily than the necessary goods. Hence, in general, the efficiency effect and the equity effect take the opposite sign. Therefore, a desirable direction of tax reform depends on the relative magnitude of the two effects.

In the following section, we will take the Japanese liquor tax structure as an example of this problem and consider the effect on welfare of the tax reform.

4. Application to the Japanese Liquor Tax Structure

4.1 Specification

Before coming into the numerical study, we will devote this subsection to reducing (18) to an all operational expression.

Let us first replace the specific tax rate with the effective tax rate. By the definition of τ^n, we get

$$dt^n = \{q^n/(1-\tau^n)\}d\tau^n. \qquad (19)$$

Applying this for (18), we get

$$dV = \{-A\delta^1 + (\delta^1-\delta^n)\}B\cdot d\tau^n \qquad (20)$$

where $B=q^nX^n/(1-\tau^n)$. (20) can be decomposed as follows;

$$dV^e = -A\delta^1B\cdot d\tau^n \qquad (21)$$

$$dV^d = (\delta^1-\delta^n)B\cdot d\tau^n. \qquad (22)$$

That is, (21) and (22) show the portion of the efficiency effect and the equity effect respectively in the total welfare variation.

Note that (20), including the value of A, consists of operational elements except the value of the distributional characteristic. The remaining task of this subsection is to reduce the distributional characteristic to an operational form.

In the following subsection, we will use the LES as the regression model, which is given by

$$q^jx^j = q^j\gamma^j + \beta^j(y-\Sigma_i q^i\gamma^i) \quad j=1,\cdots,n \qquad (23)$$

where γ^j and β^j are commonly interpreted as the basic consumption and the marginal propensity to consume, respectively. We now specify the demand function in relation to income as

$$x^j = a^j+b^jy \quad (*7) \qquad (24)$$

where, in order to be consistent with (23), we have only to let

$$a^j = \gamma^j - (\beta^j/q^j)\Sigma_i q^i\gamma^i \qquad (25)$$

$$b^j = \beta^j/q^j. \tag{26}$$

We also specify the social marginal utility of income in relation to income as

$$\mu = y^{-\eta} \tag{27}$$

where $-\eta$ turns out to be the income elasticity of the social marginal utility of income. Suppose, for instance, that consumer A has income twice as high as consumer B, and that the government chooses a value η . Then the social marginal utility of the income of consumer A turns out to be $1/2^{\eta}$ times the magnitude of that of consumer B. That is, the greater the value of η , the more egalitarian the social welfare function.

Let us now assume that y is lognormally distributed. Then, substituting (24) and (27) for the definition of the distributional characteristic (12), we get

$$\delta^j = M \frac{a^j+b^jY(1+\xi)^{-\eta}}{a^j+b^jY} \tag{28}$$

where M is the expected value of the social marginal utility of income, $M = E[\mu]$, and ξ is the relative variance of the income, $\xi = var[y]/(E[y])^2$ (*8). (For the derivation of (28), see Appendix.)

We now let $d\hat{V}=dV/M$, $d\hat{V}^e=dV^e/M$ and $d\hat{V}^d=dV^d/M$, which turn out to represent the money-metric welfare variation in terms of the average value of the social marginal utility of income.

4.2 Data and Results

The object of our study is the tax structure of sake and whisky. Each item is made up with three grades; the second, the first and the special grade, which will be numbered 1, 2 and 3 respectively as the number for each good.

Tax Rates and Ratios of Expenditure: The effective tax rate on each grade of these items is presented in Table 4-1a,b, which is based on the Monthly Report on Fiscal and Monetary Statistics (Zaisei Kinyu Tokei Geppo), Ministry of Finance, 1986.

The ratios of expenditure between the grades can be calculated from Table 4-2a,b, which is based on the Annual Report on the Family Income and Expenditure Survey (Kakei Chosa Nenpo) , Statistics Bureau, Management and Coordination Agency, 1986, and we obtain the following;

$r^{13}=5.768$ $r^{23}=6.057$ $r^{21}=1.050$ $r^{31}=0.173$ for Sake ,

$r^{13}=0.421$ $r^{23}=0.396$ $r^{21}=0.940$ $r^{31}=2.377$ for whisky .

Elasticities: Aggregate all the consumers' specified demand function (23) to get the following social demand function,

$$q^jX^j = q^j\gamma^j+\beta^j(Y-\Sigma_i q^i\gamma^i) \quad j=1\cdots n \qquad (29)$$

where we regard the total (or, more appropriately, the average) expenditure on Sake or whisky as the income.(*9)

The data of the price, quantity consumed and the total expenditure is based on the annual time series data of all households in the Annual Report on the Family Income and Expenditure Survey covering the period 1963 to 1986 for Sake and 1980 to 1986 for whisky. The estimates of the parameters of (29) are presented in Table 4-3a,b (*10). From these estimates, we can get the elasticities in 1986 as follows; (*11)(*12)

ε^{13}=0.300 ε^{23}=0.149 ε^{21}=3.739 ε^{31}=2.108 for Sake,

ε^{13}=1.959 ε^{23}=2.728 ε^{21}=0.583 ε^{31}=0.729 for whisky.

Relative Variance: The value of the relative variance θ can be calculated by using the all households five-classification cross section data in 1986 in the Annual Report on the Family Income and Expenditure Survey, which is also presented in Table 4-2a,b. Then we get the following;

ξ =0.016 for Sake, ξ =0.059 for whisky.

We are now in a position to calculate the welfare variation by using the values obtained above. Corresponding to several values of η , we can calculate the value of δ^1/M and δ^3/M , and consequently the value of dV^e, dV^d and dV(*13). These are presented in Table 4-4a for Sake and Table 4-4b for whisky.

Let us first look at Table 4-4a for Sake. By decreasing the effective tax rate on the special grade by one percent (i.e., from 41.1% to 39.1%), accompanied by an increase in the tax rate on the 2nd grade so as to maintain the revenue level, we can expect the gain of about 18 yen. It is notable that the equity effect is too small to compare with the efficiency effect. We can point out some reasons why the efficiency effect overwhelms the equity effect: (a) The differences in tax rates are quite large between the grades, and moreover the substitutablility between the grades is of large magnitude, so that the efficiency effect becomes large; (b) There is almost no difference in luxuriousness between the special and the second grade, so that the equity effect becomes small.

We can find from Table 4-4b that the similar discussion holds true for whisky. Though there is a bit wider range in the value of the expected welfare gain than the case of Sake corresponding to several values of η, most part of the welfare variation is owed to the improvement in efficiency.

Anyway, in the total welfare effect of tax reform of these items, the effect through an improvement in efficiency is crucially dominant whatever value of η is chosen. Hence, we can recommend the policy which brings the going tax structure of Sake and whisky toward uniformity.

5.Concluding Remarks

Two comments should be noted. First, in the present analysis we assumed that the labor supply is exogeneously determined. This was in order to avoid complexities and to focus our mind on the distributional aspects. If we develop a similar analysis to the present one letting labor supply be determined endogeneously, we will only obtain the mixture of the result of Hatta(1986), which treats the elastic labor supply case, and that of the present paper.

Secondly, since our analysis is based on the piecemeal approach, the results obtained here is hard to apply when quite a major change in tax rates is carried out.

Appendix: Specification of Distributional Characteristic

By the definition of δ^{j}, we have

$$\delta^{j} = \frac{\int x^{j}\mu \, f(y)dy}{\int x^{j} f(y)dy}. \tag{A.1}$$

Substituting (24) and (27) for (A.1), we get

$$\delta^{j} = M \frac{a^{j} + b^{j}\{\int y^{1-\eta} f(y)dy / \int y^{-\eta} f(y)dy\}}{a^{j} + b^{j}Y}. \tag{A.2}$$

Since we assume that y is distributed according to the lognormal distribution, by letting s=logy and a change of variable from y to s, we get

$$\delta^{j} = M \frac{a^{j} + b^{j}\{m(1-\eta)/m(-\eta)\}}{a^{j} + b^{j}Y} \tag{A.3}$$

where m($\cdot$) denotes the moment generating function of the normal distribution. Then we have

$$m(1-\eta)/m(-\eta) = \exp[E[s]+\{(1/2)-\eta\}var[s]]. \tag{A.4}$$

Note that the following relation holds(*14),

$$E[s] = \log Y-(1/2)\log(1+\xi) \tag{A.5}$$

$$var[s] = \log(1+\xi). \tag{A.6}$$

Substituting (A.5) and (A.6) for (A.4), and applying it to (A.3), we get (28) in the text.

Footnotes

(*) This chapter is based on the report at the Western Meeting of the Japan Association of Economics and Econometrics in 25-26 June 1988 at Hiroshima Univesithy, Hiroshima. The original paper of this chapter is forthcoming in Economic Studies Quarterly. The author would like to thank Professor Toshihiro Ihori , Osaka University, and the anonymous referee of the Journal for valuable comments. Remaining errors are the author's responsibility.

(*1) More precisely, we integrate Hatta(1977) and Feldstein(1972a,b), in which labor is substantially assumed to be inelastically supplied, while Hatta(1986) considers an elastic labor supply case.

(*2) Since the amount of public good is kept constant before and after the tax reform, we can exclude the public good from the indirect utility function with no loss of generality.

(*3) The determination of the variables is as follows. Let t^2 through t^n and g be chosen first by the government. Then, since p is given constant, q^2 through q^n are determined by (5). Substituting (7) for (6), we get $t' x(q) = g$, from which t^1, and consequently q^1 are determined. Finally, z^1 through z^n are determined by (7).

(*4) If all commodities are taxed in the same effective tax rate, Pareto optimum is established. By the definition of τ^i, we have $(1-\tau^i)q^i=p^i$. If $\tau^j=\tau^k$ for any j and k, we get $(q^j/q^k)=(p^j/p^k)$, to the LHS of which the marginal rate of substitution of the consumer is equal, and to the RHS of which the marginal rate of transformation of the producer is equal.

(*5) It can be easily shown by making use of (6), (7) and the definition of τ^j that increasing t^j results in an increase in government revenue if and only if the inequality $X^j+\Sigma_i \tau^j q^i X^i_j > 0$ holds. If this condition is satisfied for both of t^n and t^1, t^1 has to be raised as a result of a reduction in t^n in order to maintain the initial government revenue, which means a tax reform toward uniformity.

(*6) Though A is multiplied by an equity element δ^1, it cannot change the sign of the efficiency effect.

(*7) In the specification of the distributional characteristic, Feldstein used the demand function of the form $x^j=c^j y^{\zeta^j}$ instead of (24) (Feldstein(1972) p.35). His way of specification is consistent with the constant elasticity type demand function like $x^j=c^j\Pi_i(q^i)^{\varepsilon^{ji}}y^{\zeta^j}$ for the regression model.

(*8) Variance can be standardized by dividing by the square of the mean.

(*9) We implicitly assume the weak separability of preference. See Deaton and Muellbauer(1980), chapter 5.

(*10) We used the seemingly unrelated regressions (SUR). For Sake, since a sort of change in preference was likely to happen in about late 1960s, we used a dummy variable which takes on the value 1 during the period 1969 to 1986 and is zero otherwise.

(*11) The cross elasticity is given by $\varepsilon^{ij}=-(\beta^{i}/q^{i}x^{i})\cdot q^{j}\gamma^{j}$.

(*12) Noting that the revenue-increasing condition in the footnote 5 can be rewritten as $\Sigma_{i}(\tau^{i}-\tau^{j})r^{ij}\varepsilon^{ij}+(1-\tau^{j}) > 0$, we find that this condition is satisfied for both Sake and whisky with these elasticities.

(*13) We get the value of A and B as follows;

A=0.979 for Sake, A=0.790 for whisky,

B=1,800 for Sake, B=6,819 for whisky.

(*14) See, for example, Mood-Graybill-Boes(1974), p.117.

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Table 4-1a

consumer prices and tax rate of sake

	price (yen/100ml)	tax rate (%)
1. second grade	72.57	14.1
2. first grade	104.20	26.9
3. special grade	158.44	40.1

Table 4-1b

consumer prices and tax rate of whisky

	price (yen/100ml)	tax rate (%)
1. second grace	113.75	29.0
2. first grade	243.34	43.1
3. special grade	507.25	50.3

Table 4-2a

expenditure on sake (yen)

class (1000yen/year)	1.2nd	2.1st	3.special	total
I (-3,140)	5,275	5,589	683	11,547
II (3,140-4,340)	5,821	6,002	746	12,569
III (4,340-5,950)	6,488	6,311	896	13,695
IV (5,950-7,540)	6,877	6,776	1,111	14,764
V (7,540-)	6,629	7,968	1,953	16,550
average	6,218	6,529	1,078	13,825

Table 4-2b

expenditure on whisky (yen)

class (1000yen/year)	1.2nd	2.1st	3.special	total
I (-3,140)	1,169	847	1,872	3,888
II (3,140-4,340)	1,740	1,280	2,435	5,455
III (4,340-5,590)	1,868	1,318	2,884	6,070
IV (5,590-7,540)	1,416	1,583	4,016	7,015
V (7,540-)	935	1,679	5,739	8,353
average	1,426	1,341	3,389	6,156

Table 4-3a

parameter estimates of LES for sake(*)

parameter	estimate(t-value)	
β^1	0.666(7.845)	} 0.636
d^1	-0.030(2.571)	
β^2	0.305(3.998)	} 0.333
d^2	0.028(2.580)	
β^3	0.030(3.245)	} 0.032
d^3	0.002(1.931)	
γ^1	-1010.263(1.864)	
γ^2	-331.171(5.271)	
γ^3	-18.502(4.558)	

(*) d^i: coefficient of dummy variable; γ^i: 100ml

Table 4-3b

parameter estimates of LES for whisky(*)

parameter	estimate(t-value)
β^1	0.155(4.391)
β^2	0.203(7.274)
β^3	0.642(36.167)
γ^1	-3384.485(3.263)
γ^2	-2204.754(3.867)
γ^3	-3553.040(9.534)

(*) γ^i: 100ml

Table 4-4a

value of distributional characteristic and welfare variation caused by decreasing the effective tax rate on the special grade of sake by one percent(*)

η	δ^1/M	δ^3/M	$d\hat{v}^e$	$d\hat{v}^d$	$d\hat{v}$
0	1.000	1.000	17.6	0.0	17.6
1	1.001	1.001	17.6	0.0	17.6
2	1.002	1.002	17.7	0.0	17.7
3	1.003	1.003	17.7	0.0	17.7
4	1.004	1.005	17.7	0.0	17.7
5	1.005	1.006	17.7	0.0	17.7
6	1.006	1.007	17.7	0.0	17.7
7	1.007	1.007	17.7	0.0	17.7
8	1.008	1.009	17.8	0.0	17.8
9	1.009	1.010	17.8	0.0	17.8
10	1.010	1.010	17.8	0.0	17.8
30	1.025	1.027	18.1	0.0	18.1
100	1.052	1.057	18.5	0.1	18.6

(*) $d\hat{v}^e$, $d\hat{v}^d$, $d\hat{v}$: yen

Table 4-4b

Value of distributional characteristic and welfare variation caused by decreasing the effective tax rate on the special grade of whisky by one percent(*)

η	δ^1/M	δ^3/M	$d\hat{v}^e$	$d\hat{v}^d$	$d\hat{v}$
0	1.000	1.000	53.9	0.0	53.9
1	1.008	1.007	54.3	-0.1	54.2
2	1.014	1.014	54.6	0.0	54.6
3	1.021	1.020	55.0	-0.1	54.9
4	1.027	1.026	55.3	-0.1	55.2
5	1.033	1.031	55.6	-0.1	55.1
6	1.039	1.037	56.0	-0.1	55.9
7	1.044	1.042	56.2	-0.1	56.1
8	1.049	1.046	56.5	-0.2	56.3
9	1.054	1.051	56.8	-0.2	56.6
10	1.058	1.055	57.0	-0.2	56.8
30	1.110	1.104	59.8	-0.3	59.5
100	1.133	1.125	61.0	-0.4	60.6

(*) $d\hat{v}^e$, $d\hat{v}^d$, $d\hat{v}$: yen

Chapter 5

Redistribution via Public Goods Reform(*)

Abstract: This chapter investigates redistribution through a change in the composition of public goods provision. Sufficient conditions for a reallocation of government expenditure from one public good to the other to be welfare improving will be derived.

1. Introduction

In this chapter we investigate redistribution among individuals through a change in composition of various kinds of public goods provision. Especially, we examine the welfare effects of reallocating the government expenditure from one public good to another, keeping the government budget balanced.

Concerning the two public goods, when we consider which public good should be increased at the sacrifice of the other in an economy with many heterogeneous consumers, we should take account of the equity aspects as well as the efficiency aspects of public goods provision. Though efficiency of public goods provision under distortionary taxes has been studied by Pigou (1947), Stiglitz-Dasgupta (1971), Atkinson-Stern(1974), and Wildasin(1979,1984), we do not have many works on the distributional function of public goods provision. (A rare exception is King(1986), which investigated the optimum condition of public goods provision under the optimum taxes in a many consumer economy.)On the other hand, it is very valuable to make reference to the articles on distributional function of commodity taxes by Feldstein(1972a,b), Atkinson-Stiglitz(1976,1980), Boadway(1979) and Mikami(1989), since there exists a somewhat symmetrical relationship between commodity taxes and public goods.

The model of the economy is presented in the next section. In section 3, we consider the effects on welfare of reforming the composition of public goods, maintaining the government budget balance. Total welfare effect can be decomposed into efficiency and equity effect, and sufficient conditions for such policies to be welfare improving will be derived. We find there that the sign of equity effect crucially depends on the shape of a function $\beta(\cdot)$, the social cost of the individual utility. $\beta(\cdot)$ reflects the difference in an attribute of consumers (such as ability to work) and the distributional value judgment of policy-makers. Section 4 then inspects the shape of $\beta(\cdot)$ and shows that there can exist such a paradoxical case that luxurious public goods should be increased at the sacrifice of necessary public good, even though the marginal utility of income is decreasing in ability and the policy-maker takes a value judgment which makes much of the poor rather than the rich. At the end, brief remarks will be given in section 5.

In principle, we use superscripts to index goods, and subscripts for derivatives. For instance, x_i means the derivative of demand vector for private goods x with respect to the ith public good g^i.

2. The Model

<u>Consumer</u>: Consider an economy with a continuum of many heterogeneous consumers. They have identical preferences, but differ in abilities. We represent ability by a scalar parameter α. Imagine, for instance, that they are different in ability to work, and therefore, in wage rate. It is convenient, in this case, to suppose that they face the same wage rate and have different amounts of endowment. We regard α as a random variable which is distributed over $[0,\infty)$ with a density function $f(\cdot)$. (Henceforth the range of the integral will be often omitted.) For convenience, we roughly call the person with high (low) ability the rich (the poor). A consumer's preference over n+1 private goods, including leisure, and m public goods is represented by his indirect utility function $v(\cdot)$. The utility level u of a consumer with ability α is then written as

$$u = v(q,g,y;\alpha) \tag{1}$$

where q and g denote the n+1 dimensional consumer price vector and the m dimensional public goods vector, respectively. To avoid complicating the notation, henceforth α is not written explicitly unless it is especially needed. y shows the lump-sum transfer of income and we assume that y=0 at an equilibrium, so that hereafter we often omit it.

Let the expenditure function corresponding to the indirect utility function be m(q,g,u). Then, the shadow price of the ith public good, denoted by s^i, is defined by

$$s^i(q,g,u) = - m_i(q,g,u). \qquad (2)$$

Now, we have the following two properties:

$$v_i(q,g,y) = \lambda (q,g,y)s^i(q,g,u) \qquad (3)$$

$$x_i(q,g,y) = c_i(q,g,u) + s^i(q,g,u) \cdot x_y(q,g,y) \qquad (4)$$

where λ is the marginal utility of income(*1)(*2).

Producer: We assume that the production possibility frontier is of the constant cost type and that producers are competitive. Then, the production possibility frontier can be written as

$$p' z + r' g = 0 \qquad (5)$$

where p, z and r denote the n+1 dimensional producer price vector of the private goods, the n+1 dimensional output vector of the private goods, and the m dimensional producer price vector of the public goods, respectively. The 0th element of z is the labor, which is represented by a negative quantity.

Government: The government imposes commodity taxes on private goods, and spends the revenue on public goods. Due to commodity taxes, the consumer prices diverge from the producer prices:

$$q = p + t. \tag{6}$$

We assume that commodity tax rates will be held constant before and after the public goods reforms.

<u>Equilibrium</u>: At an equilibrium, demand equals supply in every market. Thus (5) is reduced to

$$p' \int x(q,g;\alpha)dF(\alpha) + r'g = 0 \tag{7}$$

where $x(\cdot)$ denotes the n+1 dimensional uncompensated demand vector and $F(\cdot)$ is the cumulative distribution function. Note in (7) that the 0th private good, the leisure, is negatively measured. An equilibrium of the economy is described by (1), (6) and (7).

3. Welfare Effects of Public Goods Reform

First we define social welfare V by

$$V = \int w(v(q,g;\alpha))dF(\alpha) \tag{8}$$

where $w(\cdot)$ is the weight set on each individual utility. The government intends to improve social welfare subject to the resource constraint (7).

We consider a public goods reform such that one of the public goods is increased with a simultaneous change in another public good so as to maintain the government budget balance, in which commodity tax rates and the quantity of the other public goods will be held constant. We will choose g^N and g^L as representative public goods, each representing necessary and luxurious public good. A formal definition of "necessary" or "luxurious" will be given later.

From (7), using the implicit function theorem, we get a function $g^L(g^N)$ in which total differentials of (7) give

$$\frac{dg^L}{dg^N} = -\frac{R_N}{R_L} \tag{9}$$

where $R_i = p' \int x_i dF + r^i$, $i = N, L$. Though the economy we are considering is a closed economy, it would be helpful to assume a foreign country which offers or deprives resources to or from the domestic economy in a lump-sum manner. Then, R_i shows how much more resources have to be brought from or taken out to a foreign country when one more unit of the ith public good is produced in the domestic economy. Suppose for a moment that R_i is either zero or negative at equilibrium. In this case, by increasing g^i exogeneously, social welfare can be improved, while satisfying the resource constraint (7). Thus, the policy that should be chosen in the case of either zero or negative R_i is obvious and the problem becomes trivial. Therefore, hereafter we will only pay attention to the cases in which R_i is positive. It will be convenient to give formal terminology to this concept:

<u>Definition 1</u>: The ith public good is said to be

resource-increasing(-neutral, -decreasing) if R_i is positive(zero, negative).(*3) ‖

We find from (9) that if both the Nth and the Lth public goods are resource-increasing, raising g^N should be followed by a decrease in g^L.

Substituting $g^L(g^N)$ for (8) and differentiating it with respect to g^N, by making use of (3), we get

$$\frac{dV}{dg^N} = \int \mu \left(s^N - s^L \frac{R_N}{R_L} \right) dF(\alpha) \qquad (10)$$

where μ, the "gross social marginal utility of income (*4)," is defined by $\mu = w_u \cdot \lambda$, the marginal utility on income λ multiplied by the social weight w_u.

(10) can be rewritten as

$$\frac{dV}{dg^N} = k\left(\frac{R_N}{\phi^N} - \frac{R_L}{\phi^L}\right) \qquad (11)$$

where $\phi^i = \int \mu s^i dF$, $i=N,L$, and $k = -\phi^N \phi^L / R_L < 0$. By using (4) and the relation $x_y = c_u \cdot \lambda$ (*5), in which c_u is the derivative of the compensated demand vector with respect to utility, (11) is further rewritten as

$$\frac{dV}{dg^N} = k\left\{\left(\frac{R_N^*}{\phi^N} - \frac{R_L^*}{\phi^L}\right) + \int (b^N - b^L)\beta(\alpha) dF(\alpha)\right\} \qquad (12)$$

where $R_i^* = p' \int c_i dF + r^i$, $b^i = \mu s^i / \phi^i$, $i=N,L$, and $\beta(\alpha) = p' c_u / w_u$. b^i indicates the ratio of a consumer's utility gain, μs^i, to the social welfare gain, ϕ^i, evaluated from a social standpoint, when the ith public good is increased by one unit. Notice that $\int b^i dF = 1$: the average value of b^i over all individuals is unity.

On the RHS of (12), we will call the term $R_N^*/\phi^N - R_L^*/\phi^L$ the <u>efficiency effect</u>, and the term $\int (b^N - b^L)\beta\, dF$ the <u>equity effect</u>. We will now examine these terms in detail consecutively.

<u>Efficiency Effect</u>: Let us first look at R_i^*, $i=N,L$, which consists of $p' \int c_i dF$ and r^i. The former indicates how much resources can be saved in the private sector by producing one more unit of the ith public good, leaving every consumer's utility level unchanged. Hence, generally this term is negative and its absolute value shows the magnitude of benefit. On the other hand, r^i is the cost of production of g^i. Therefore, R_i^* as a whole stands for net cost, cost minus benefit(*6). Next, the denominator ϕ^i expresses the increase in benefit in a social sense from additional provision of g^i. Hence, the term R_i^*/ϕ^i represents social cost-benefit ratio.

Suppose, for instance, that net cost per one unit of benefit through an increase in g^N is less than that through an increase in g^L. In this case, we can say that an increase in g^N is more desirable than an increase in g^L from an efficiency point of view. Thus we have:

<u>Definition 2</u>: The Nth public good is said to be socially more(as, less) scarce than(as, than) the Lth public good, if $R_N^*/\phi^N < (=, >)\ R_L^*/\phi^L$ holds.‖

<u>Equity Effect</u>: Next, consider the equity effect. The

value of the term $b^N - b^L$ differs among individuals with different ability. Suppose, for instance, that the Nth public good is preferred to the Lth one by the poor, whereas the Lth public good is preferred to the Nth one by the rich. Then, it is plausible that $b^N > b^L$ holds for the poor, and oppositely the reverse inequality holds for the rich. Accordingly, it would be reasonable to refer to g^N as more necessary (or less luxurious) public good than g^L if the relative desirability for g^N compared with g^L decreases as ability increases. Formally:

Definition 3: The Nth public good is said to be more necessary (less luxurious) than the Lth public good, if

$$\frac{\partial (b^N - b^L)}{\partial \alpha} < 0 \text{ for any } \alpha \text{ in } [0, \infty). \|$$

Another component of the equity effect is $\beta(\cdot)$. First, the term $p' c_u$ represents the amount of additional resources needed to elevate each consumer's utility level by one unit. Dividing each consumer's $p' c_u$ by his or her weight w_u, equity consideration can be taken into account to some extent: the more the government lays emphasis upon the utility of the poor, the less their $p' c_u$'s are counted from a social point of view. Hence, we will call β the social cost of utility(*7).

The following lemma presents sufficient conditions for the sign of the equity effect to be determined.

Lemma: Suppose that $\beta(\cdot)$ is increasing (decreasing) in α for any α in $[0,\infty)$ (*8). Then the equity effect generated by an increase in the necessary public good, accompanied by a simultaneous change in the luxurious public good so as to maintain the government budget balance, becomes negative (positive).

Proof: (We only prove the case of increasing $\beta(\cdot)$, since the other case can be shown in a similar manner. Figure 5-1 will help the reader follow the proof.) Let $(b^N-b^L)f(\alpha)=h(\alpha)$. Through integration by parts, the equity effect in (12) can be rewritten as

$$\int_0^\infty (b^N-b^L)\beta(\alpha)dF(\alpha) = \int_0^\infty \beta(\alpha)h(\alpha)d\alpha$$

$$= [\beta(\alpha)H(\alpha)]_0^\infty - \int_0^\infty \beta'(\alpha)H(\alpha)d\alpha \qquad (13)$$

where $H(\alpha)$ is a primitive function of $h(\alpha)$,

$$H(\alpha) = \int_0^\alpha h(s)ds. \qquad (14)$$

Since

$$\lim_{\alpha \to \infty} H(\alpha) = H(0) = 0 \qquad (15)$$

the first term on the final expression of (13) vanishes (*6). Concerning the second term, first we have assumed that $\beta'(\alpha) > 0$ for any α in $[0,\infty)$. Next, since (b^N-b^L) is decreasing in α by the assumption and $f(\alpha) > 0$ for any α in $[0,\infty)$, (15) implies that $H(\alpha)$ must be positive for all α in $[0,\infty)$. Thus, the second term, without minus, is positive. Therefore, the equity term becomes negative.‖

We are now in a position to state the main result about welfare improving change in the composition of public goods supply.

<u>Proposition</u>: Consider the case in which the luxurious public good is resource-increasing. Suppose that $\beta(\cdot)$ is increasing (decreasing) in α for any α in $[0, \infty)$. Then an increase (a decrease) in the necessary public good, accompanied by a simultaneous change in the luxurious public good so as to maintain the government budget balance, will improve the welfare of the economy, if the necessary public good is socially more(less) scarce than the luxurious public good.

<u>Proof</u>: In (12), since R_L is positive by the assumption and ϕ^i, i=j,k, is positive by the definition, k is negative. The first term, the efficiency effect, is negative (positive) by the assumption, and the second term, the equity effect, is also negative(positive) by the Lemma. Thus, the RHS of (12) is positive(negative) as a whole.‖

It would be interesting to consider two extreme cases. First, when every consumer has the same ability, there accrues no differences in evaluating the two different public goods in question among individuals. Therefore, the equity effect vanishes and only the efficiency effect remains. This is the result which would be obtained in a single consumer framework. Secondly, when these two public goods are equally lacking in the economy, i.e. when there is no differences in the scarcity between the two public goods in a social sense, the efficiency effect vanishes and only the equity effect remains.

It would be convenient to summarize the latter case in the form of corollary of the previous Proposition, since it enables us to concentrate our attention on the distributional matters.

<u>Corollary</u>: Consider the case in which the luxurious public good is resource-increasing. Suppose that the necessary public good is as socially scarce as the luxurious public good and that $\beta(\cdot)$ is increasing (decreasing) in α for all α in $[0,\infty)$. Then an increase (a decrease) in the necessary public good, accompanied by a simultaneous change in the luxurious public good so as to maintain the government budget balance, will improve the welfare of the economy. ‖

4. The Shape of $\beta(\cdot)$

In the previous section, especially in the Proposition and the Corollary, we saw that whether $\beta(\cdot)$ is upward sloping or downward sloping is crucially important in determining the sign of the equity effect. This section inspects the shape of $\beta(\cdot)$.

First, since an equation

$$p' c_u = \frac{p' x_y}{\lambda} = \frac{1-t' x_y}{\lambda} \tag{16}$$

holds, we have

$$\beta(\alpha) = \frac{1-t' x_y}{\mu}. \tag{17}$$

Differentiating it with respect to α to get

$$\beta'(\alpha) = -\frac{1}{\mu}\{\hat{\mu} \cdot p' x_y + \frac{\partial(t' x_y)}{\partial \alpha}\}. \tag{18}$$

where $\hat{\mu} = \mu_\alpha / \mu$.

Let us initially look at the first term in the brace on the RHS of (18). Under the normality condition $p' x_y >$ 0, if gross social marginal utility of income is decreasing in α, this term becomes negative and works as $\beta(\cdot)$ is upward sloping. Note further that $\hat{\mu}$ can be decomposed as

$$\hat{\mu} = \hat{w}_u + \hat{\lambda} \tag{19}$$

where $\hat{w}_u = w_{u\alpha}/w_u$ and $\hat{\lambda} = \lambda_\alpha / \lambda$. It would be quite natural to consider that $\hat{\lambda}$ is negative. On the other hand, the sign and the magnitude of $\hat{w}_u$ depends on the distributional value judgment of policy-makers. If the government takes Benthamian criteria, $\hat{w}_u = 0$; if it takes Rawlsian criteria, $\hat{w}_u = -\infty$ at the lowest α. If we assume that the government takes a value judgment between Benthamian and Rawlsian criteria, $\hat{w}_u$ takes a negative number. In addition, the strongly the government makes much of the utility of the poor compared with the rich, the larger the negative value of $\hat{w}_u$ becomes.

Suppose for the moment that the marginal utility of income is decreasing in ability. Besides that, the government takes a certain equitable value judgment which is represented by a negative $\hat{w}_u$. Then, the first term in the brace on the RHS of (18) is negative. This, however, is not enough for $\beta(\cdot)$ to be increasing in α, because the second term in the brace may or may not take the same sign as the first term. Since the term $t' x_y$ shows a change in tax revenue from a person when he or she receives one unit of lump-sum income, the second term represents how such changes in tax revenue varies as ability differs. Clearly, it depends on the going tax structure and the marginal propensity to consume of commodities among individuals with different ability. The more heavily the goods of which the marginal propensity to consume of the rich (the poor) are taxed, the more plausible it is that this term becomes positive (negative) and then works as $\beta(\cdot)$ is downward (upward) sloping. Let us say that the tax structure is equalizing (or, more precisely, marginally equalizing) if commodities of which the marginal propensity to consume of the rich are taxed heavily. Formally, we now define the strong eegalitarianism of commodity taxes as follows:

<u>Definition 4</u>:(Strong Egalitarianism) We say that the commodity tax structure is strongly egalitarian, if in (15) the term $\partial(t' x_y)/\partial\alpha$ is so large a positive number

that $\beta(\cdot)$ as a whole becomes downward sloping.‖

Now, in order to focus our mind on distributional aspects, let us consider a situation of the Corollary. If the going tax structure is strongly egalitarian, the case of decreasing $\beta(\cdot)$ holds. Then, it appears somewhat paradoxical that increasing a luxurious public good at the sacrifice of the other necessary public good is desirable even if marginal utility of income is decreasing in ability and in addition the government makes much of the utility of the poor. The key to this mystery exists in the income effect of private goods when public good is changed.

Since we have

$$dg^L = -(R_N/R_L)dg^N \tag{20}$$

from (6), total differentials of demand vector for private goods $x(q,g,y)$ with respect to g^N and g^L gives

$$\begin{aligned} dx &= \{x_N - x_L(R_N/R_L)\}dg^N \\ &= \{(c_N + s^N x_y) - (c_L + s^L x_y)(R_N/R_L)\}dg^N \\ &= \{c_N - c_L(R_N/R_L)\}dg^N + x_y dy \end{aligned} \tag{21}$$

where the second equality follows from (4), and

$$dy = \{s^N - s^L(R_N/R_L)\}dg^N. \tag{22}$$

The first and the second term in the final expression of (21) shows the change in demand due to substitution and income effect, respectively. The term in the brace in (22) can be rewritten as

$$s^N - s^L(R_N/R_L)$$

$$= k\{(\frac{R_N^*}{s^N} - \frac{R_L^*}{s^L}) + (\frac{\bar{s}^N - \int (t' x_y) s^N dF}{s^N} - \frac{\bar{s}^L - \int (t x_y) s^L dF}{s^L})\} \qquad (23)$$

where $\bar{s}^i$ shows the simple mean of s^i, $i=N,L$. First look at the term in the first parenthesis in the brace on the RHS of (23). Since we are now considering the case of the Corollary,

$$R_N^*/\phi^N = R_L^*/\phi^L \qquad (24)$$

holds, and, on the other hand, we have

$$s^N < (>) \phi^N, \; s^L > (<) \phi^L \qquad (25)$$

for the rich (the poor). Then, from (24) and (25), we find that the term in the first parenthesis of (23) is positive(negative) for the rich (the poor). Let us next turn to the term in the second parenthesis of (23). Under the assumption of strong egalitarianism, $t' x_y$ is increasing in α, and then we have

$$\bar{s}^N > \int (t' x_y) s^N dF, \; \bar{s}^L < \int (t' x_y) s^L dF. \qquad (26)$$

Therefore, the term in the second parenthesis of (22) is positive.

After all, $s^N - s^L(R_N/R_L)$ is found to be negative for the rich under the strongly egalitarian commodity tax structure. Then, (22) implies that an increase in g^N, $dg^N > 0$, leads to a reduction in the real income of the rich, $dy < 0$, through the income effect. Reduction in real income attributed to the rich diminishes the government revenue via a decrease in demand of heavily taxed commodities. At the same time, the real income distributed to the poor might be raised followed by an increase in g^N and then the government revenue could be expanded via an increase in the demand of taxed commodities. However, since the former reduction in revenue from the rich will overwhelm the latter raise in revenue from the poor, an increase in g^N results in a net decrease in government revenue, which itself is potentially a loss of social welfare. Therefore, under the case of strong egaritarianism of commodity taxes, luxurious public good should be increased rather than a necessary public good, even though the marginal utility of income decreases as ability increases and the government makes more of the utility of the poor rather than the rich.

5. Concluding Remarks

We have shown that when we have taken enough account of equity through commodity taxes, further considerations of redistribution also on the expenditure side may reduce the social welfare. This fact implies the somewhat surprising matter that equitable commodity taxes may contradict equitable public goods composition from the viewpoint of social welfare. The paradox obtained in this chapter suggests a significant relation between tax and expenditure, which cannot be shown by examining tax side and expenditure side separately .

Footnotes

(*) This chapter is based on the report at the Annual Meeting of the Japan Association of Economics and Econometrics on 14-15 October 1989 at Tsukuba University, Tsukuba, where I got valuable comments from Professor Jun Tsuneki, Seikei University, and Professor Mikio Otsuki, Tohoku University.

(*1) Differentiating the identity $v(q,g,m(q,g,u))=u$ with respect to g^i, we get

$$v_i(q,g,y) + \lambda(q,g,y) \cdot m_i(q,g,u) = 0$$

where we let $m(q,g,u)=y$. Some rearranging of this equation yields (3).

(*2) Differentiating the identity $c(q,g,u)= x(q,g,m(q,g,u))$ with respect to g^i, we get

$$c_i(q,g,u) = x_i(q,g,y) + x_y(q,g,y) \cdot m_i(q,g,u)$$

where we let $m(q,g,u)=y$. Some rearranging of this equation yields (4).

(*3) This is an analogous concept to revenue-increasing (-neutral, -decreasing) with respect to a tax rate in Hatta(1986).

(*4) Atinson-Stiglitz(1976) uses this terminology.

(*5) Differential of the identity $x(q,g,y)=c(q,g,v(q,g,y))$ with respect to y yields this equation.

(*6) Rearrange R_i^* to get

$$R_i^* = r^i - \{ \int s^i dF(\alpha) + t' \int c_i dF(\alpha)\}$$

where r^i represents the cost while the term in the brace the real benefit, the sum of the shadow price added by the distortionary term. We can find from this expression that if a lump-sum tax is available and therefore t=0, we get the traditional Samuelson rule: r^i stands for the marginal rate of substitution (MRT) while $\int s^i dF(\alpha)$ corresponds to the sum of each consumer's marginal rate of substitution (Σ MRS).

(*7) Equity effect can be written as

$$\int (b^N - b^L)\beta \, dF = \gamma^N - \gamma^L$$

where

$$\gamma^i = \int \left(\frac{\mu s^i}{\phi^i}\right)\beta \, dF \quad i=N,L.$$

γ^i can be called the "distributional characteristic of the ith public good" which is the corresponding concept of the "distributional characteristic" of private goods by Feldstein(1972a, b): It is defined by

$$\delta^i = \int \left(\frac{x^i}{X^i}\right)\mu \, dF$$

in our notation, where $X^i = \int x^i dF$. Note that we find a symmetrical relation between γ^i and δ^i in a sense that the ratio of individual demand to social demand in δ^i is replaced by the ratio of the individual shadow price to the social shadow price in γ^i, and the social cost of utility in γ^i takes over the social marginal utility of income in δ^i. In addition, it would also be possible to define the luxuriousness of public goods as: "g^N is more necessary than g^L if $\delta^N < \delta^L$."

(*8) As can be seen in the proof, the positivity of $p' c_u$ is not required here.

(*9) Note that in figure 5-1, $H(\alpha)$ shows the area between $h(\alpha)$ and horizontal axis running horizontally from 0 to α.

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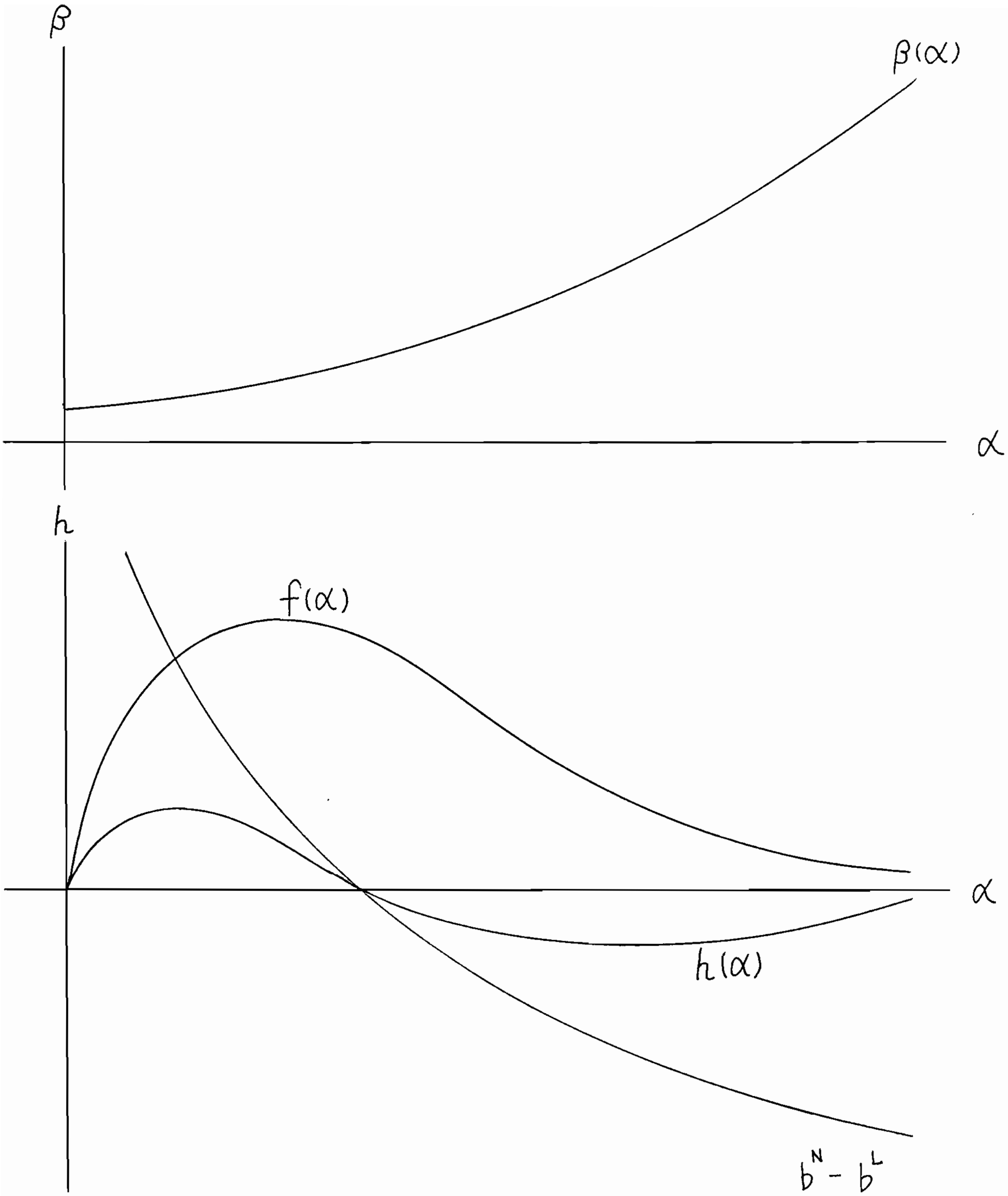

Figure 5-1