A demonstration of paradoxical aspects derived from the measurement problem under the finite velocity of observation propagation

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(Degree)
博士（学術）
(Date of Degree)
1993-03-31
(Date of Publication)
2008-05-16
(Resource Type)
doctoral thesis
(Report Number)
甲1170
(JaLCDOI)
https://doi.org/10.11501/3070618
(URL)
https://hdl.handle.net/20.500.14094/D1001170

A demonstration of paradoxical aspects derived from the measurement problem under the finite velocity of observation propagation

(邦題:有限観測伝播速度下における観測問題のもたらすパラドキシカルな様相とその例示)

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SUMMARY		······ 3
Chapter 1	Introduction	······ 4
Chapter 2	Measurement Problem in Biology : A Fixed Point Derived from Finite Velocity of Observation Propagation	······ 7
Chapter 3	Algebraic Property of Flow-diagram in Time-reverse Cellular-Automaton	······ 36
Chapter 4	The Extraction of Biological Feature of Learning Process in Man-to-man Game	······ 67
Chapter 5	Conclusion	······ 95
References		······ 97

0. Summary

We consider the measurement problem under the finite velocity of observation propagation and discuss the induced paradox. To incorporate the paradox into formulation, AEB(Autonomously Emerging Boundary) model was proposed[5]. Time-reverse rules for AEB can be decided with the primitive FD(flow diagram). A theorem is presented to the primitive FD. Utilizing the theorem, each box of the time-reverse rule can be classified and the primitive FD can be proved to be closed. Examples of adopting the time-reverse rule are shown referred to the primitive FD. Almost all of the rules, except a few, can be utilized including contradiction.

To demonstrate the paradox from actual data, we adopt a man-to-man game called *Renju*, and simulate the aspect of uncertainity derived from the finite velocity of observation propagation. It is shown that players somehow learn through repeated games, but their moves in further repeated games show new discovers by the players'. It is difficult to describe *a priori* the learning process or the transformation system. The move in a replay game shows a correlation with the move in the first game(SG). Then, it can be concluded that the information generated in SG was certainly utilized in the replay. When we, external observers, confronted paradoxical aspects, then we consider that the player 'considers'. In other words, we find 'subject' in him.

Chapter 1. Introduction

Studies has focused on the measurement problem under the finite velocity of observation propagation and on the information generation accompanied by the process have been intensively made by several reseachers. The problem was proposed by Matsuno[1], who firstly detected the finite velocity of observation in biological object experimentally[2]. If the problem is admitted, we cannot help the discrepancy between the microscopic observation and the macroscopic observation. It means that we can definitely describe the progress only *a posteriori*. Matsuno[1] emphasized the irreversibility of the determination *a posteriori* and the non-determination *a priori*, and proposed 'one-to-many mapping' as the physical basic concept. Gunji, Ito, Kon-no and the author have accepted the problem and discussed procedures to incorporate the concept into description[3-10]. One plausible way is to introduce the time-reverse rule to elementary cellular automata(ECA). The discussion was started from the possibility of the use of time-reverse rules, and investigates the relation between introduction of time-reverse rule and the concept of autopoiesis, the philosophical significance of time-reverse, the construction of the Autonomous Emerging Boundary model(AEB), the form of life, and the construction of the self-referential system. Now some investigators and the author are studying about the biological feature of learning process on this aspect. The author uses a man-to-man game called *Renju*.

This thesis consists of two papers. They are parted in Chapters 3 and 4.

In Chapter 3, discussion is focused on the procedure of construction of the primitive flow-diagram(FD), utilized in AEB. AEB is a basic and probably intrinsic model for living things which are irreversible systems. It is formulated using elementary cellular automata and its fundamental feature lies in the operation of time-reverse rule. The reverse rule could contradict with the ordinarily temporal rule and the discrepancy is expressed between the macroscopic rule and the microscopic rule. In this paper we show a short procedure of AEB and a new recipe for the flow-diagram (primitive FD) which is a reference for the time-reverse rule. We construct the class of Box in the flow-diagram and prove that the flow-diagram is closed. Especially we show the feature of primitive FD of symmetric rules.

In Chapter 4, we discuss the biological feature of learning process,using a man-to-man game called *Renju* as example. To extract the biological feature of learning and to simulate the biological situation, we take the man-to-man game. *Renju* is a finite game and the number of elementary events can be countable in principle. The number, however,is too enormous and the number of local solutions is also enormous. A Player takes a move at a wild guess. The finite velocity of observation propagation induces the uncertainity in the motion of two particles. Such uncertainity is replaced in the present case of *Renju* game by the wild guess of Players. In the first place, two Players play a game and the orbit(the sequence of moves) is called SG(sample game). The same two replay the game starting from the i-th step. SG and Replay show correlation, because players learn by experience. The orbit of Replay does not converge, however, i is a few or less steps earlier than the ending step of SG in spite of that the number of solutions is finite. The learning system is difficult to describe. We can see somehow biological feature in the variance of Z, which is the summation of the value of the Replay orbit.

Both two papers are concerned with the measurement problem. The first paper explains the mathematical procedure of AEB to demonstrate biological aspects. If we accept the measurement problem, we cannot help accepting that the dynamics cannot be *a priori* given. Then, the acceptance of it is represented by the construction of contradiction for themselves in description. One way to construct the contradiction is to introduce the time-reverse rule in ECA, in which the ordinally temporal rule is 'many-to-one' mapping and invertible. In the time-reverse rule, the invertibility induces contradiction. We intend to demonstrate the biological process with incorporating the contradiction into description. The second paper reports the analysis of an experiment which simulates the biological aspect under the finite velocity of observation propagation. The uncertainity of dynamics *a priori* is replaced by the wild guess of Players. External observers, who can measure the Players' system for prediction under infinite velocity of observation propagation, can also construct the transformation system or learning process, but if they intend to apply the past transformation system to the future, the obtained data will betray them. Then we will find that the Players are subject, even if at first we take the standpoint of external observers and suppose the Players are object. This is caused by the Players' wild guess and probably the finite velocity of observation propagation.

The introductory lines will be shown with respect to the relation between the syntax(adoption of dynamics) and the semantics(data), the object(the external observers) and the subject (the internal observers), the measurement and the paradox.

Chapter 2.

Measurement Problem in Biology: A Fixed Point Derived from Finite Velocity of Observation Propagation

2-1. The questions about life

In biological and/or brain sciences, we often ask ourselves, "what is life?", "Is it possible to construct an artificail intelligence?", "Does an animal have a mind?", or "Is this behavior programmed by gene or acquired in ontogeny?", and those problems are regarded as one of the lethal questions in science. Those are just various expressions for a unique question in the context of language game. In naive realism we assume subject/object dualism, and we erase the token of a subject or an observer from any description. As a result the descriptions are regarded as objective ones. A subject is different from an object with respect to logical status.

However, all questions mentioned above involves something beyond subject/object dualism. The terms, life, intelligence and mind have something to do with a subject. Hence the questions can involve the mixture of terms at different logical status. Therefore, those questions cannot be well-defined because each one is self-referential form as well as Goedel's theorem of incompleteness. In the previous researches the biological features relevant to self-referential forms were estimated in the comparison with classical machines (e.g. Varela[21]; Hofstadter [9]; Gunji & Nakamura[8]), and there are many discourses how we can talk about a paradox. On the one hand, Varela and Hofstadter argued that organisms prove a paradox and it leads to the transformation of the structure as the process of evolution, and that a paradox accidentally originated causes the evolution. On the other hand, we claimed that a paradox is not only the cause of evolution but also the result of evolution(Gunji[4-6]), and that there is no causal relation between evolution and a paradox. They are different with respect to the stance relevant to "time" in self-referential system or autopoietic system.

Autopoiesis(Maturana & Varela[15]; Varela[21]; Fleishaker [3]) is defined by self-organizing system of which the system's own boundary is organized by itself and it entails to a logical paradox. When we call objective descriptions the inside and call the stance of subject which cannot explicitly appear in the description the outside, a paradox is originated from the mixture of the inside and the outside (Maturana & Varela[15]; Gunji & Nakamura[8]). Hence, an autopoietic system has the specific topology in which there is no distinction between the inside and the outside. However, it is too speculative to estimate the relation of a paradox and life. Now, we have to examine whether an organism live against a paradox, live in a paradox or live as a paradox. Because autopoiesis has ambiguity on this point, we had some controversial discourses in system theory.

The most important thing is to clarify whether a paradox which appears in describing an organism is real existence or not. If it is real, it implies that our described paradox must completely coincide with an organism's paradox. How to improve a paradox must be real in biological processes. Otherwise, finding a paradox just implies the aspect that we cannot formally describe life in principle. Of course, we can constitute an alternative formal logic by embeding a paradox in the logic. However, we can no longer regard a paradox as a real existence.

Hence, how to use a paradox, the performativeness of a paradox (Gunji[4]) has to be examined. We think that talking about a paradox in the context of "time" and/or evolution can be performative in science. Therefore we prove how a paradox appear in the context of time/space geometry in detail. It is discussed with respect to velocity of observation propagation, VOP (Matsuno [13]).

In the present paper, first we criticize the naive realism believed by most scientists, and claim the disagreement with the concept of reality. As a result science is proved to be a specific language game in which logical consistency is established without any foundations, in the sense of Wittgenstein [22] and Kripke[10]. Second, we prove that we implicitly assume infinite VOP whenever we describe interactions consistently, and prove that a paradox in the form of a fixed point appear in describing the interaction under finite VOP in the logic with the assumption of infinite VOP. Finally we discuss that various non-well-defined questions originated from the same reason that the question has a fixed point in principle, and refer to the significance of embeding a paradox in dynamical space.

2-2. Naive Realism and Paradox in Rule Following

We always construct the concept or the structure of a system, not from a "raw" system itself, but from the symbolic sequences which is regarded as representation for a "raw" system. According to Rosen[19], all representations leads to the symbolic systems called *formal system* and "raw" materials are called *natural system*. That dualism is clearly derived from naive realism, and Rosen tried to describe the perspective of naive realism in the context of relativism. However, it can be failed because whole his perspective is also in the domain of naive realism. First, we have to be liberated from naive realism that most scientists implicitly believe.

Imagine the case of the Fibonacci series, which is one of the structure in biological science. We believe that Fibonacci series represents the process of cell proliferation which belongs to what we call natural system. However, though it can refer (it looks as if it referred) to the specific natural phenomenon, they can by no means indicate it(Gunji[4]). In the context of Fibonacci series, all what we can find is the correspondence between the transition rule

$$s^{t+1} = s^t + s^{t-1} \qquad (1)$$

where s^t represents the number of cells at the t-th step, and the collection of symbols(i.e. the number of cells with time) such as

$$\{(1,\ 2,\ 3,\ 5..),\ (12,\ 15,\ 27,\ 42..),\ ..\}. \qquad (2)$$

The former is called syntax or axiomatic system and the latter is called semantics. In other words, the latter form (2) implies the existence of a model for the former one (1). Therefore any descriptions leads to the consequences of formal language in principle. The problem that is originated in describing natural system is formally examined and/or discussed.

Of course, in formal language we cannot ignore Goedel's theorem of incompleteness, in which if we assume the soundness of a system it is proved incomplete. In other words, we can at most choose a model(Set-(2)) for an axiomatic system(Eq-(1)), but cannot prove a unique relation. We cannot prove that there is a unique rule, (1), by which the number of series, Set-(2) follow. Though it sounds trivial, we must not ignore that it is hidden assumption in formal systems, because it is, in fact, the essential point in the question on the artificial intelligence, mind of animals or what is life. We here suggest the following working hypothesis; (i) there is no explicit relation between natural ("raw" phenomena) and formal system in science, and the relationship regarded as that of natural and formal ones, itself, is embeded in a formal system as the relation of semantics and syntax. (ii) As well as that a formal system cannot refer to a natural system, an explanation which is the correspondent relation between syntax and semantics is not uniquely determined.

The first working hypothesis can also be exposed by the questions from skeptic naive realists, such that "We agree that the rule-(1) is related not to cell but to the set-(2), but the set is just one of the representations of "raw" cell proliferation. It implies that the set-(2) indicates the referent of a cell. That is why we can regard Fibonacci series as the structure of cell proliferation, isn't it?" Needless to say, naive realism is defined by the dogma such that ***if X is described then X exists***. Therefore any words and/or descriptions represent specific referents, and it supplies the foundation for how to use a word. However, we do not have to answer this question directly. Naive realists' refutation is well-defined only if we can prove the certain correspondence between the rule and the series of numbers. In other words, naive realists believe that it is certain that the rule refers to the series of numbers and back, and that the series of number refers to natural referent as well as the relationship in the formal system.

Now, the proof of the hypothesis (ii) makes it clear. The answer can be found in the discourses of Kripke[10] and Wittgenstein[22], and it is another and general expression of Goedel's theorem of incopleteness. We cannot prove that a series (1, 2, 3, 5) follows the rule $s^{t+1}=s^{t}+s^{t-1}$, because another rule $s^{t+1}=2s^{t}-1$ is also possible and we cannot deny the existence of other possible rules. Let a set of a series of numbers A and a set of meaning Y, where for example $Y=2=\{0,1\}$ and the meaning of $a \in A$ is 1 if we can regard a as Fibonacci series, and it is 0 otherwise. We can regard the series a as Fibonacci series whenever we find that the series a follows $s^{t+1}=s^{t}+s^{t-1}$. Hence, in order to prove that a series a follows the rule $s^{t+1}=s^{t}+s^{t-1}$, we have to prove that there exists a unique rule $s^{t+1}=s^{t}+s^{t-1}$ which a series a follows. It must fail because the number of rule which satisfies Fibonacci series is always larger than that of a series. It can be expressed by

$$\mathrm{Card}(Y^{A})>\mathrm{Card}(A), \tag{3}$$

where Card(S) represents the cardinality of a set S and Y^{A} $=\mathrm{Hom}\ (A,\ Y)=\{f:A\to Y\mid {}^{\forall}a\in A,\ {}^{\exists}y\in Y,\ y=f(a)\}$. Y^{A} is a set of rules. In other words, the form (3) implies that $\Phi:A\to Y^{A}$ must not be surjective while we have to assume that $\Phi:A\to Y^{A}$ is surjective because we have to prove a unique correspondence between $a\in A$ and $f\in Y^{A}$. The proof of that unique correspondence or conviction of a choice of f, which a series a follows, must be founded by the proof that there is no other possible rule. That is why we assume $\Phi:A\to Y^{A}$ is surjective. Finally the assumption entails to contradiction, which is expressed by the a fixed point; there exists $b \in A$ for any $h: Y \to Y$, such that $\phi(b)(b) = h(\phi(b)(b))$. That procedure proposed by Lawvere to prove a fixed point is as same as Cantor's diagonal arguments, Goedel/Tarski's or Russel's paradox(Lawvere[11]).

You may disagree with our proof which leads to contradiction, because we at first set an infinite series like (1, 2, 3, 5, ...) but we mention a finite set (1, 2, 3, 5) in the proof. However, now, any symbols like <...> cannot indicate infinite sequence which implies a specific sequence. If we declare that <...> implies a specific sequence obtained from the same operation as (1, 2, 3, 5), then we must indicate the rule which (1, 2, 3, 5) follows. Because we cannot prove that a specific rule is a referent of the sequence (1, 2, 3, 5), we cannot prove that the symbol <...> implies a specific sequence. Hence the proof mentioned above is sufficient.

That is why we cannot prove any relationship between a rule and a series. It suggests that the relation is generally replaced also by those of a referent and a word, function and structure, or the usage of a machine and a machine. Therefore, we must say that we can use a language without the foundation of the usage or the conviction of a referent, that the function cannot be decided dependent on the structure, and that we can use a machine independent of the conviction of how to use a machine. We can repeat why we can use a language without the correspondence-table between a word and its referent, but it must fail again. Wittgenstein[22] showed that the question is not even a problem. Strictly speaking, he reversed the direction of the usage of a language and the rule of a language. In the context of naive realism we believe that if there exists the rule of a language then we can use a language. However, Wittgenstein said that, first, a language game proceeds and that the ad hoc rule is invented a posteriori. Acceptance of a language game is Wittgenstein's (and/or Kripke's) philosophy. That is why we do not have to think about the reason or foundation(Gunji[5]).

In a language game we use even a formal language. Now usage of a language implicates the usage without any foundations. We just use. The correspondence between syntax and semantics is invented a posteriori in a language game, and the illusion that we can use a formal language owing to the real correspondence-table, does not hold. We use a formal language, formal system or scientific language not by naive realism but in a language game. Science is the specific language game to find a rule but finding a rule, itself, is not founded, then it is independent of naive realism whether scientists believe naive realism or not.

Therefore, the usage of a formal language, itself, must conflict the concept of a rule, while that conflict must not appear in sofar as we just use the language and do not ask the rule of the usage of the language. Asking the rule of how to use the language in a formal language leads to Goedel's theorem of incompleteness or a paradox. If we ignore the foundation of the usage of a language then we use a language as if we could use a language founded by the correspondence-table between a word and its referent. Otherwise, we find that there exists no correspondence-table or rule. In the former case it looks as if a universe was based on naive realism, and in the latter case a language game appears in the form of Goedel's theorem of incompleteness. That aspect is found in biological system and discourses on life. Goedelness yields a key for the problem, what is life.

2-3. To Observe, To be Observed and Goedelness

On the one hand, the estimation on the animal mind or intelligence is to find something like us, an observer. It leads to finding something like a subject in an object. On the other hand, observation or description itself is something to discard the token of a subject. Therefore the problem, what is animal mind, is suggested not to be well-defined problem. Also, the problem of an artificial intelligence and what is life are discussed as well as that problem.

In that context, we distinguish ***life*** from ***organisms***. We generally use the term of, not the origin of organisms but the origin of life. On other hand, we use the evolution of organisms instead of the evolution of life(Nagano[16]). It suggests that the term life involves something like injective limit and that life is regarded as a limit in the retrogression of the evolution, because life involves something changeable which is intrinsic change from a material to an organism. While we account for organisms that they are originated from the appearance of life, described organisms do not involve something changeable without the effect of the accident boundary condition(Conrad & Matsuno[2]). In the context of organisms we accept the accident external cause of change and simulataneously ignore it, while in the context of life we have to be faced with the cause of change itself because the question with respect to life focuses just on that point.

Modern synthesis concerning the evolution of organisms shelved something changeable by the separation of organisms and environments based on Weisemann hypothesis (Matsuno[14]). That separation is believed to have been judged by their empirical accuracy, hence it is called the fact. However, any scientific discourses are judged also by how close they come to Platonic ideal, that implicates self-consistency of the logic and that by no means involve something changeable. In principle, entity of changeable conflicts consistent logic. In other words, whenever we ignore the logic's own consistency or the conviction of the correspondence between syntax and semantics, the separation between syntax and semantics is regarded as the trivial and leap in the darkness in using a logic(Kripke[10]) is hidden. The leap in the darkness - you *comprehend* a pen when I say "a pen" without a referent of a word, pen - implicates something changeable. Therefore, something changeable is concealed out of a logic when we use a formal logic, and it appears in the form of paradox, Goedelness or leap in the darkness when we overlook the aspect of the usage of a language. Note that in the former case it looks as if a syntax referred to a specific semantics. That is why the separation can be judged by empirical accumulation (empirical data are also in the logic).

As mentioned above, science is a specific language game independent of naive realism. However, if one believes that it is founded by naive realism, referents (noted here by Y) of the described sentence, system, data, and discourses (noted here by X) must exists in the sense of $X \simeq Y$(X and Y are isomorphic). At the same time X constitutes objectivism because X does not involve something changeable and/or ambiguity. In biology X is called an organism compared to life. Regarding X as non-changeable objective existence prohibits from talking about the usage of X or from considering a subject who uses X. Hence, life such that is as same as a subject leads to the de-construction of X. Logical paradox cannot be improved in principle, because on the one hand a paradox inevitably appears in finding a rule and on the other hand the improvement of a paradox has to be realized by a rule proving a paradox. Only by Wittgenstein's skeptic proof a paradox is proved without finding a rule of the proof(Kripke, [10]). Then, the gap between an organism as an object and life as a subject must not disappear. It entails to life as a limit of organisms. We summarize the aspect as shown in Table 2-1.

In the aspect of description, it is assumed that there exists subject/object dualism. An object is regarded as an object only because it is observed or measured by a subject, and a subject is distinguished from an object only because he observes an object. Needless to say, even a subject does not exist and it is just one singular point in the network of a language game. A subject is also a result of the ad hoc invention derived from a language game. Subject/object dualism is also established without foundation. We have to find the formal aspect in observation or measurement to describe the token of a subject(i.e. to describe life), which can be replaced by the token of the usage of a language, and it leads to Goedel's theorem of incompleteness

Those who just use a language cannot be faced with a paradox whether he believes the objectivism or not, and the specific language game proceeds as if there existed objective ideal. With respect to observation an observer do not pay attention to an observation itself, and it leads to an objective observation. We call that observer an exo-observer, and call those who observe the observation itself an endo-observer. However, an endo-and exo-observer cannot be examined as the same logical status. Whenever we describe or observe something, even an observation itself, we cannot look out over our own observation or description. Hence, we always take the stance of an exo-observer, and an endo-observer is constructed in the form of a paradox by the stance of an exo-observer.

Now, the answer to the problem, what is life, is to construct an endo-observer, but constructing cannot be here distinguished from finding because there does not exist a reality of referent to be find. Determinant of construction and discovery is originated from naive realism. We formalize an endo-observer as a paradox or a fixed point with respect to measurement, especially to velocity of observation propagation.

2-4. "Measurement" in Measurement and Fixed Point

Now, we can distinguish an object from a subject(i.e. distinguish X by an exo-observer from X' as an endo-observer) with respect to the measurement. As far as we understand the objective description in the context of causality, it must leads to the formal description such that for any cause there uniquely exists a result. If we take measurement in space into consideration, a cause is articulated into an internal observer (i.e. internal state) and an input. When the space is defined by the lattice of products, such measurement can be expressed by $f: A\times B\to B$, that for $\forall(a, b)\in A\times B$, there exists $b'\in B$ such that $b'=f(a, b)$, where we call A the set of inputs, B the set of internal states of an internal observer. For example, when we describe that a frog(an internal observer), whose state at the t-th time step is b^t, observes a fry, whose state at the t-th step is a^t and it leads to an action $b^{t+\Delta t}$ at the $(t+\Delta t)$-th step, it is expressed by

$$b^{t+\Delta t}=f(a^t,\ b^t). \qquad (4)$$

Eq-(4) is a result of an external measurement, and i.e., we, an external observer observes the computation of an internal observer, a frog. That expression can be articulated into internal measurement and computation. The former is identification process of an input and is expressed by the disjunction, $(a^t,\ b^t)$. The latter is realization of computation or the operation of a function, f. All programmable computation processes are realized in this way, namely after receiving an input (internal measurement) an output is computed. In this sense we find measurement in any formal descriptions. In other words, this scheme implies that the result of an internal measurement coincides with that of an external measurement, by the commutable diagram, computation(internal measurement)=external measurement. As discussed later, it is a character of an exo-observer.

Such "formal" observation or measurement is different from our (subject's) measurement or measurement by measurement. We distinguish them by naming the latter "measurement". On the one hand, "measurement" involves finite time to measure and simultaneous change of a state. Therefore "measurement" conflicts the formal articulation of an internal measurement and computation. The "measurement" is defined by impossible articulation, and the form of impossibility can be regarded as "measurement". Now we have to formalize "measurement" in measurement. Now, measurement is realized by an exo-observer, and "measurement" is realized by an endo-observer.

First, we emphasize that in measurement we assume finite velocity of observation propagation. When we describe $f:A\times B\rightarrow B$, category theory is very convenient. In that theory we define category $\boldsymbol{C}$ which has objects denoted such as A, B, C, and morphisms denoted such as $f:A\to B$, g,..., and define a specific axiomatic system such that $\boldsymbol{C}$ satisfies the axiom of category theory. If we adopt a set as an object, a function corresponds to a morphism and we can construct a set theory. It is the most important that we define the power and the limit in the cartesian closed category in which we can construct a product $A\times B$ and can articulate any $f:A\times B\to C$ into computation and an internal measurement by the commutable diagram,

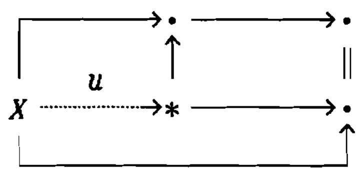
$$
\begin{array}{ccc}
A\times C^B & \xrightarrow{\ \mathrm{ev}\ } & C \\
{\scriptstyle \mathrm{id}_A\times\Phi}\uparrow & & \| \\
A\times B & \xrightarrow{\ f\ } & C
\end{array}
\quad , \tag{5}
$$

where C^B represents the power. Here, an internal measurement (resp. computation) can be replaced by the transpose of f, Φ(resp. ev)(Gunji[5]).

The definition of power leads to the extension of the form of limit, and limit in category implies the existence of an observer who observes taking no time. If it takes no time to measure or observe something, we define that observation is realized under infinite velocity of observation(Matsuno[13]). Now, measurement can be comprehended by another name of morphism in a commutable diagram. The definition that an external measurement is articulated into an internal measurement and a computation is described by

$$
\begin{array}{ccc}
B & \xrightarrow{\ f\ } & C \\
{\scriptstyle g}\uparrow & & \| \\
A & \xrightarrow{\ h\ } & C
\end{array}
\quad , \tag{6}
$$

where we call g and h an internal and external measurement respectively. Because any object X in a category has identity $id_X: X \to X$, we can always find a commutable diagram such as (6). If $B=C$, we obtain a commutable diagram $h = id_C \circ h$ for $h: A \to C$. We can say that an observer A observes a morphism f by the means of measurement g and h (e.g. Given $h = id_C \circ h$, an observer A observes id_C or C by the measurement $h: A \to C$). Therefore, if there uniquely exists a morphism u which commutes



(7)

for any objects X, the diagram (7) implies the existence of objective observation or measurement, because the results of any measurement or observation (i.e. by any observers) can be uniquely transformed by that of *. Eternal measurement can here be constructed by the consensus of various measurements by the diagram (7), and * is called a limit. It is clear that eternal measurement can be constructed if and only if velocity of observation propagation is infinite. That is why we call a limit an exo-observer. Limit leads to product, and then we can define space by the extension of products like $A \times A \times A \times .. \times A$, hence it leads to eternal measurement of the space, which implies infinite velocity of observation propagation(VOP). Hence, the definition of limit implies the assumption of infinite velocity of observation propagation. Now, "measurement" in measurement can be replaced by the measurement under finite velocity of observation propagation in cartesian closed category.

In the form (4), we can find an internal measurement as (a^t, b^t), however, it takes no time in measurement. In order to describe "measurement" we can rewrite (4) by,

$$\begin{cases} b^{t+\Delta t_1} = f_0(a^{t+\Delta t_2},\ b^t) \\ b^{t+\Delta t_2} = f_1(a^t,\ b^t), \end{cases} \tag{8}$$

where $\Delta t_1 >> \Delta t_2$. However, even in this form we find measurement as (a^t, b^t) and then we rewrite again. That procedure must fall into infinite regression, and finally we obtain infinite series of equations as

$$\begin{cases} b^{t+\Delta t_1} = f_0(a^{t+\Delta t_2},\ b^t) \\ b^{t+\Delta t_2} = f_1(a^{t+\Delta t_3},\ b^t) \\ b^{t+\Delta t_3} = f_2(a^{t+\Delta t_4},\ b^t) \\ \quad\vdots \\ b^{t+\Delta t_i} = f_{i-1}(a^{t+\Delta t_{i+1}},\ b^t) \\ \quad\vdots \end{cases} \tag{9}$$

where $\Delta t_1 >> \Delta t_2 >> \Delta t_3 >> .. >> \Delta t_i >> \Delta t_{i+1} >> ...$ As far as we start from cartesian closed category or measurement, "measurement" entails to infinite form. On the one hand, if we assume infinite VOP we can measure with finite (coarse) degree of accuracy. On the other hand, if we assume finite velocity of observation propagation, we have to cut finite sequence from infinite one and it requires infinite degree of accuracy. In spite of the infinite form (9), we, those who observe under finite velocity of observation, can "measure". Hence, when we describe "measurement" in measurement, we also have to describe "measurement" for the infinite form (9) in the term of measurement. It implies that we can "measure" by the correspondence table between the meaning (i.e. referent) and symbol sequence (9). The infinite symbol sequences are expressed by

$$
\begin{cases}
A_0 = (f_{00},\ f_{01},\ f_{02},\ f_{03},\ \ldots) \\
A_1 = (f_{10},\ f_{11},\ f_{12},\ f_{13},\ \ldots) \\
\quad \vdots \\
A_i = (f_{i0},\ f_{i1},\ f_{i2},\ f_{i3},\ \ldots) \\
\quad \vdots
\end{cases}
\qquad (10)
$$

where i in the infinite sequence A_i represents the kind of the sequence, and f_{ij} represents that f_{ij} belongs to A_i. Let the meaning of A_i, $g_i(A_i)$, where g_i denotes which f_{ij} indicates the meaning of A_i (e.g. $g_i(A_i)=f_{i3}$). Therefore, we define $g_i: S \to F$, for $^\forall A_i \in S = F^\infty$, $^\exists g_i(A_i) \in F$. Also, we define $\Phi: S \to F^S$, where F^S represents a set of functions from S to F, and i.e. F^S=Home(S, F)=$\{g_i: S \to F\}$. What we can "measure" in terms of measurement can be replaced by a unique existence of the choice morphism Φ.

Is it possible to formally describe "measurement" in measurement? It can be replaced by the problem, whether we can observe under finite velocity of observation propagation by the way of observation under infinite velocity of observation propagation. If can, it implies that we have the correspondence-table between infinite sequences of measurements, A_i, and their meanings, $g_i(A_i)$, and can choose one of correspondences or one of A_i. It implicates that for any $g \in F^S$, we can decide B (i.e. there exists $B \in S$) such that for any $A_i \in S$,

$$g(A_i) = \Phi(B)(A_i). \qquad (11)$$

In spite of that assumption which we can describe "measurement" in measurement, using diagonal arguments, we can obtain $g^*: S \to F$ such that for $^\forall A_i \in S$,

$$g^*(A_i)=h(\Phi(A_i)(A_i)) \quad (12)$$

for any functions $h:F\to F$. It implies that we can construct unknown meaning in the form of g^*, and then it leads to the contradiction, because, on the one hand we know all meaning and/or all infinite sequences of measurement, and on the other hand we can construct unknown meaning or unknown infinite sequence. That paradox is formally expressed by the following. Eq-(11) holds for any g and A_i, we obtain $g^*(B)=\Phi(B)(B)$. At the same time from Eq-(12), we obtain $g^*(B)=h(\Phi(B)(B))$. Hence, we obtain, for any $h:F\to F$,

$$\Phi(B)(B)=h(\Phi(B)(B)). \quad (13)$$

There must exists h that changes the state of $\Phi(B)(B)$ in spite of Eq-(13). It is a paradox.

Finally when we describe the "measurement" which involves the interaction under finite VOP in the terms of measurement which involves infinite VOP, "measurement" can be replaced by a fixed point. We constitute an endo-observer as a fixed point as well as Maxwel's demon in endo-physics(Roessler[17]).

2-5. Not-well-defined problem in Hierarchical System

A fixed point $\Phi(B)(B)$ in Eq-(13) implies that the meaning for any measurement, $A_i=(f_{i0}, f_{i1}, f_{i2}, f_{i3}, \ldots)$, cannot be uniquely determined, and that we cannot ignore other possibilities of f-sequences. Redefine that $g:S\to N$, N represents a set of natural numbers, such that $g(A_i)=j$ which indicates the

j-th symbol in f-sequences of A_i. Also define that we can express by the finite form or can obtain the approximated form,

$$\begin{cases} b^{t+\Delta t_1}=f_0(a^{t+\Delta t_2},\ b^t) \\ b^{t+\Delta t_2}=f_1(a^{t+\Delta t_3},\ b^t) \\ \vdots \\ b^{t+\Delta t_j}=f_{j-1}(a^t,\ b^t), \end{cases} \tag{14}$$

if $g(A_i)=j$. Therefore, if $g(A_i)=0$, we can express $b^{t+\Delta t}=f_0(a^t,\ b^t)$. However, a fixed point implies that we cannot uniquely determined a finite number j like $g(A_i)=0$. It leads to the aspect that $b^{t+\Delta t}=f_0(a^t,\ b^t)$ is possible but we cannot deny other possibilities, such as

$$\begin{cases} b^{t+\Delta t_1}=f_0(a^{t+\Delta t_2},\ b^t) \\ b^{t+\Delta t_2}=f_1(a^t,\ b^t). \end{cases} \tag{15}$$

Hence, we must take another possible form, $b^{t+\Delta t}=f_0{}'(a^t,\ b^t)$, derived from Eq-(15) into consideration. It implicates the existence of $\Phi:B\rightarrow\mathrm{Hom}(A\times B,\ B)$ defined that, for $^\forall b\in B$, $^\exists f\in\mathrm{Hom}(A\times B,\ B)$ such that

$$f=\Phi(b). \tag{16}$$

Now, we describe the interaction under finite VOP assuming infinite VOP and then arbitrarily cut the f-sequence by $g(A_i)=0$. As a result, we are faced with the aspect that the arbitrary form $b^{t+\Delta t}=f_0(a^t,\ b^t)$ is instable in principle. However, such an instability does not imply indefinite possibilities.

Formal description in dynamical theory derived from cartesian closed category always imply state space in which the trajectory can be defined. If we formally describe (formal description itself involves the assumption of infinite VOP) the interaction under finite VOP in spite of a fixed point, we have to embed a fixed point in a state space (Gunji[6]). It implies that any trajectories cannot be defined and that dynamics must be one-to-many type mapping. Simultaneously, a fixed point is embeded and constructed in order to minimize logical contradiction using forward and backward dynamics (Gunji[4-6]). Therefore, the system in which a fixed point is embeded behaves as if it showed structural stability. If the distance like $f_\theta(a^t, b^t) \sim f_\theta'(a^t, b^t)$, which suggests the degree of logical contradiction is small, then the system can recover to $f_\theta(a^{t+1}, b^{t+1})$. It suggests a self-repairing system, however a self-repairing system does not exist a priori and it is invented a posteriori. Also, if the distance is fairly large, the system can move as if it followed $f_\theta''(a^{t+1}, b^{t+1})$. After the transformation from f_θ to f_θ'', the system can show the structural stability around the emergent rule f_θ''. It looks as if the system was attracted into another stable structure or as if the system learned something.

A fixed point is derived from the assumption in formal description, infinite VOP. Therefore, to formally describe by embeding a fixed point implies both the generation of a paradox and removing a paradox. As a result the defined system has critical logical status and then the system demonstrates both the generation of information and the learning (i.e. removing information). It suggests that the interaction under finite VOP is described as if it was constructed by

$$A \times B \xrightarrow{f} B \xrightarrow{\Phi} \mathrm{Hom}(A \times B,\ B). \qquad (17)$$

As far as we describe in cartesian closed category we assume infinite VOP. Hence the form (17) can be rewritten by the sequence of metabolic repair(Rosen[18];Casti[1]) system as

$$D \xrightarrow{f} D \xrightarrow{\Phi} \mathrm{Hom}(D,\ D) \xrightarrow{\Phi} \mathrm{Hom}(D,\ \mathrm{Hom}(D,\ D)) \xrightarrow{\Gamma} \cdots, \qquad (18)$$

where Φ represents the choice or estimation of f. Note that the metabolic rule $f:D\rightarrow D$ under finite VOP is regarded as instable rule as far as we describe assuming infinite VOP, and that the sequence (18) can be infinitely continued in principle. Hence, the feature of instability and the critical logic leads to the illusion which the hierarchical structure like the form (18) a priori exists. It causes various non-well defined problems.

Ethologists and/or ecologists often estimate whether a given behavior is programmed by gene or acquired (learned) in ontogeny. Tinbergen[20] claimed that the problem was nonsense because any behavior consists of both something instinct and something learned. Against the claim Lorentz[12] said that learning mechanism was released by the trigger of the key stimuli and then learning was attained, and that the question was not nonsense because the hierarchical structure (either $D\rightarrow D$ or $D\rightarrow D\rightarrow \mathrm{Hom}(D,\ D)$) exists. Both stances are founded by the assumption that a hierarchical structure exists as a result of adaptation and that any behavior including learning process can be programmed without ambiguity. Modern social biologists succeed to that perspectives (e.g. Maynard-Smith) and the controversy between instinct and learning is believed to have been proved because the learning mechanism itself resulted from adaptation. However, we argue that a hierarchical structure is ad hoc invented and that any behavior involves ambiguity or behavioral plasticity due to the paradox originated from the finite VOP. Our argument sounds like Tinbergen, but is different from his.

As we recognize behavioral plasticity that cannot formally be programmed, any behavior is articulated into the part which can be programmed and the other part that conflicts the programmed part. That articulation is dependent on an observer, is arbitrary, and is continuous. On the one hand, the rate among them is expressed by p, $|p|\leq 1$, and has the cardinality of real number(R). On the other hand, once we determine whether the behavior is instinct or learning, we describe the behavior either by $D\to D$ or by $D\to D\to \mathrm{Hom}(D, D)$, called the mechanism. The mechanism is always hierarchical structure whose number of ranks is one, two, or more (as expressed as the form (18)), hence it has the cardinality of natural number(N). Therefore, as far as we recognize that any behavior cannot be completely described in formal language and that behavioral plasticity is always observed more or less, we are faced with

$$\mathrm{Card}(R)>\mathrm{Card}(N), \qquad (19)$$

in determining whether an observed behavior including behavioral plasticity is instinct or is learned.

It is one example of non-well-defined problem in modern science. We argue the problem of artificial intelligence in the same context. When we ask whether the machine has the intelligence or not, we assume that the machine exits independent of us, the users of the machine. However, the architecture of the machine is constructed dependent on the users. In any computers in which we can use programs, the access to the computational element which is in the midst of computation is completely prohibited in order to prevent from the dead-lock. Therefore, we have to determine the time interval of which it takes for a computational element to compute, or the time duration that it takes till a computational element recover its original state. It is another expression of finite VOP. If we can definitely determine that time interval, then we can separate the machine from th users. However, the determinant of that time interval involves the measurement problem as well as the argument discussed above section. That determinant is impossible in principle. The reason why we can use a computer in spite of the difficulty that we cannot determine that time interval is that we use a computer. We roughly define the time interval depending on whether we can understand the output of the computer. In other words, the self-referential form which consists of the machine and the user(Fig. 2-2) covered the mystery that we can use a machine and leads the illusion that a machine exists independently separated.

As well as discussed above various problems in modern science is originated from misunderstanding originated from naive realism. They can be proved by point out that the problem is non-well-defined problem. It is skeptic proof according to Kripke [10]. It is shown by the construction of a paradox, and the paradox cannot be improved. If we construct the new logic embeding a paradox or a fixed point, it looks as if we went back the naive realism again. However, we have already understand that science is just specific language game but is not founded by naive realism. In that context an embeded paradox is not real existence even if a paradox is embeded in the new logic. Therefore, we can demonstrate what is a paradox by exhibiting a specific language game in which a fixed point is embeded in a state space. The problem whether the specific language game is good or bad is just whether it is performative or not.

In formalizing a paradox in the context of evolution, we cannot separate the generation of a contradiction from the removing a contradiction, while in modern synthesis the generation of variants (contradiction) is separated from selection process (removing contradiction). Therefore, in our perspectives, a paradox which is a kind of the uncertainty principle leads to the generation of variants(Matsuno[14]).

We claim against Varela[21] because he regarded a paradox as the real existence in spite of his will against naive realism. He argued that evolution is the process in which a paradox is improved. The generation of a paradox is independently separated from the process improving a paradox, illustrating a structure coupling like symbiosis (Symbiotic element accident encounter). However, in our stance, the generation of a paradox is originated from the effort of improving a paradox, and it is perpetually maintained because a paradox cannot be proved in principle. The difference looks tiny but it leads to the difference between naive realism and a language game. The reason why we embed a fixed point as a dynamical rule itself or the structure of the "time"(Gunji[4-6]; Gunji & Nakamura[8]; Gunji & Konno[7]) is shown as that difference. We can no longer simulate a system but can demonstrate it.

2-6. Discussion

It has been suggested that a biological system is relevant to a self-referential form, a paradox and a fixed point. We have to understand that in describing a biological system we are inevitably faced with a paradox and the accept that a biological system cannot be predictable in principle. As far as we do not hope to overcome a paradox but accept that stance, we regard a biological system as a life. It is as same as a skeptic proof against the pragmatic proof of the question, what meaning is (Kripke[10]). Therefore, we cannot regard a paradox itself as the real existence, and then constitute a logic embeding a paradox in order to make emergent unified theory of life.

In using a paradox or a fixed point, we constitute a specific language game according to Wittgenstein and cannot regard a new logic as the new better logic in order to describe life. However, when we demonstrate a new specific language game, we can simultaneously demonstrate a life. In demonstrating a life, we focus on the feature of evolution. Hence we prove a fixed point relevant to the time/space geometry, with respect to finite velocity of observation propagation.

We proved a fixed point in a cartesian closed category in which we assume infinite velocity of observation propagation in the form of a limit when we describe the interaction under finite velocity of observation propagation. It is originated from the effort that we wish to formally describe as consistent as possible. It is not invented ad hoc. Therefore we find formal (and/or natural) description has two faces; the one is removing a paradox and the other is generating a paradox. The measurement under finite velocity of observation propagation called "measurement" cannot be articulated into measurement and computation. It implies that we cannot compute an action after identifying an input. As well as "measurement", we cannot distinguish the engine of a paradox from an agent removing a paradox(Matsuno[14]). We find life in that aspect that is nothing but motion or evolution.

A fixed point with respect to finite velocity of observation propagation implies that a dynamical rule by which we predict the time evolution of the system cannot be uniquely determined, and that described dynamical rules are perpetually degenerated into a unique rule approximately but must not be unique in principle. It is resulted from both sides of a paradox. Therefore, we can find evolution in that dynamical process, and can constitute a new theory of evolution only by focusing finite velocity of observation propagation. It entails to the proof that the dualism between operand(structure, gene) and operator(function, selection) is not well-defined. Evolution is not a result of a real existence of the replication rule, mutation and selection. We can constitute the generative process of both replication and mutation in that perspective.

	Implication		Demonstration
Life	subject	performativeness of a langauge	Metalogic/Logic →Goedelness
Organisms	object	inside of a language	Logic

Table 2-1. Comparison Life with Organisms. On the one hand, whenever we use the term ***organisms***, it implicates objectivism and we always ignore the usage of the language by which we describe organisms. We do not look out over the aspect in which a language is used. On the other hand, the term ***life*** implicates something sounds like an observer. We use the term life when we find that an organism behaves as well as us. Hence, it looks as if it referred to a subject or to the aspect in which a language is used and performative. The performativeness itself cannot be pragmatically described but it is demonstrated by the proof of a paradox which appears in describing the foundation of the usage of a language.

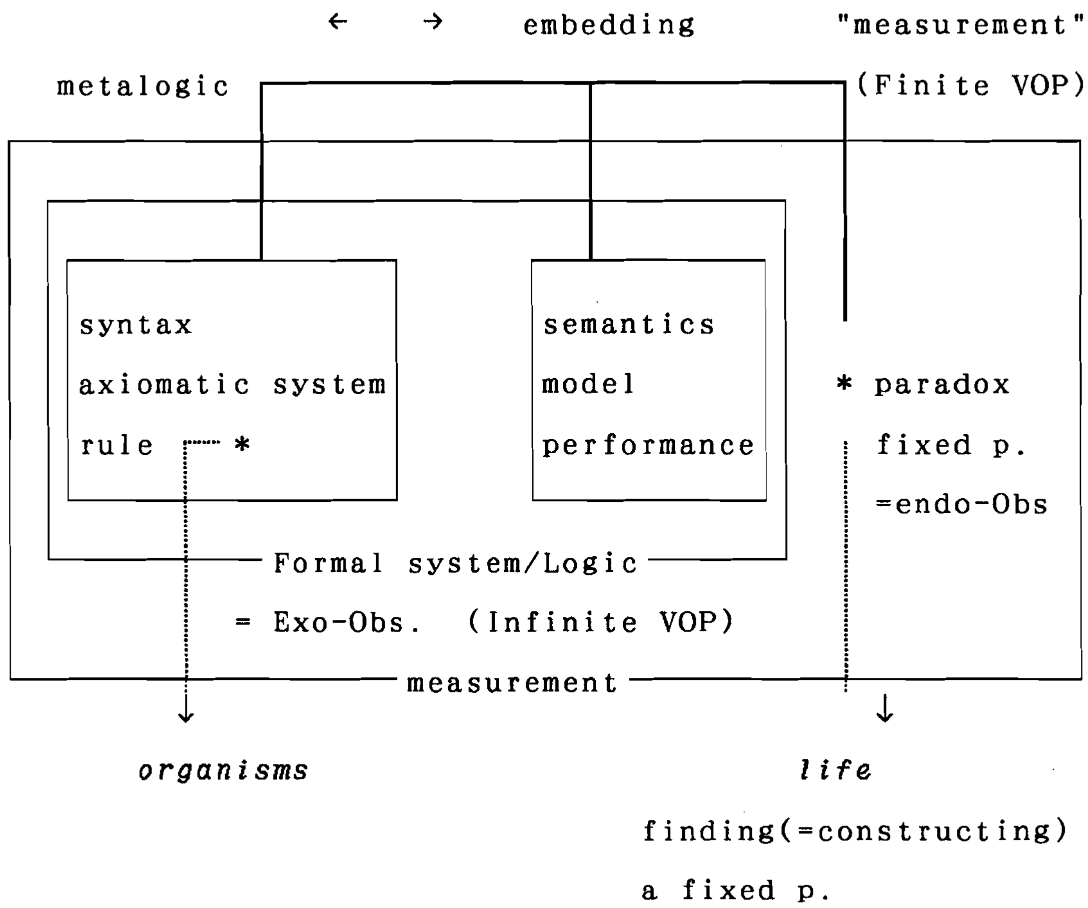


Fig. 2-1. Schematic diagram of the comparison of an organism and life with respect to formal logic. A category (formal system) is complete and objective if a limit is defined in it. The definition of a limit also implies the existence of an exo-observer under infinite velocity of observation propagation and measurement. As a statement in a complete language we find the term organisms. The term life which implies the entity moving under finite velocity of observation propagation can be demonstrated only by proof of a fixed point(paradox). "Measurement" or an endo-observer always appears as a fixed point.

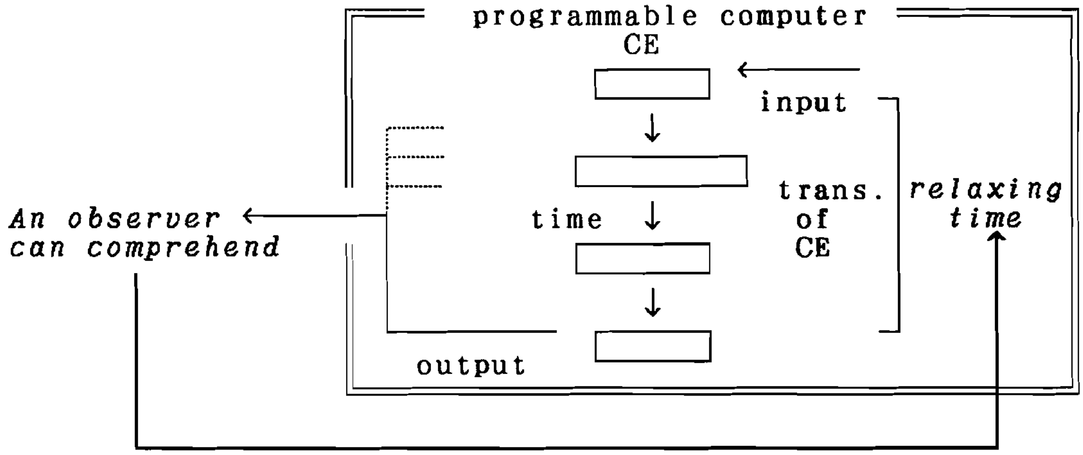


Fig. 2-2. Schematic diagram of the aspect in which we use the programmable computer. The problem, "is possible to construct an artificial intelligence?", is well-defined if a machine can be separated from the user of a machine. However, in any programmable computers the relaxing time of a computational element (CE) which prohibits from another input not to fall into dead-lock, is given, and we cannot pragmatically define the relaxing time. We actually use the computer without the definition of the relaxing time. It is mystery. The mystery is hidden because we comprehend the output of the computer without any definition or foundation of the output. It implies the usage of a computer and implies that a computer is performative. See text for detailed discussions.

Chapter 3.
Algebraic property of Flow-diagram in Time-reverse Cellular-Automaton

3-1. Irreversible System with many-to-one mapping

In the exsiting models of irreversible systems presented in the past, the systems are defined, as fixed and not changeable[1]. Such fixed systems are formulated with rules of many-to-one mapping. Many-to-one mapping is not invertible and this is why the model is called to be the description for irreversible systems. In such models it is important that initial conditions, boundary conditions, nonlinear deterministic rules, parameters, and terms for perturbation are given independently. Especially if the boundary condition and the term for perturbation, both of which represent the external environments for the system, are *a priori* given, the deterministic rule will never cause the external environments to change and will never be forced to be changed by the external environments. Even if a rule dynamics is introduced to change the deterministic rule itself depending on time step or certain values of initial condition, the case is not essentially different. The rule dynamics and the external environments are independent each other, so long as the rule dynamics is *a priori* given[3,4,5,6,7,8,14].

It is sure that such a model is good enough to implicate some phenomena, such as earthquakes. Self-organized criticality[2] explained the distribution of released energy well, which is found to obey the Gutenberg-Richter law. It is important that earthquakes as phenomena are caused by the tectonic plate motion, which can be considered to be uniform motion. In this model the term for perturbation is given in the form of probability which is uniform in time and space. The perturbation is just accidental in the sense that we cannot know where or when it is taken.If the probabilistic perturbation is taken through enough long times, the perturbation itself can be approximated to be totally averaged in time and space. This corresponds to the uniform motion of the tectonic plate. The boundary condition is *a priori* given at which macroscopic forces are released. If the stick-slip mechanism forced by increasing energy has been stabilized through long time enough, we can describe such a model as implication. Because the very mechanism is stabilized against perturbation, it can be discussed in the context of structural stability based on catastorophe theory.

The most remarkable difference of AEB(Autonomously Emerging Boundary model,[3,4,5,6,7,8,15]) from what is called the model for irreversible system involving self-organized criticality is that the boundary condition is given *a posteriori* and the deterministic rule can be changed dependent on the boundary condition. The configuration at the t-th step is *a posteriori* obtained after calculating the configuration at the ($t+1$)-th step. Therefore, we can find one-to-many mapping in this process, as a result. It means that we cannot define a phase space of a given dynamics(deterministic rule) *a priori*. Because we can no longer define homeomorphism between structure, we cannot use, what is called, structural stability theory in order to express one-to-many type mapping. We will show the aim and the mathematical procedure of AEB.

3-2. Automously Emerging Boundary model(AEB)

The meaning of boundary condition in AEB is also different. What is the external environment for an object[8]? If we recognize that the object does not interact with the external environment, we need not consider the external environment. We have to consider the effect of external environment only when we recognize that the object is clearly discriminated from the external environment. In other words, the object should be described in a different logic from one for the external environment, it would change or be changed by the external environment, and furthermore the possibility of the change is intrinsic for the object. It is important that we external observers can know the change and the effect of external environment only after the change occurs. Then, the external environment for the object should be given *a posteriori* and the external environment might have the force to change the system which we have used. If we see the object which is discriminated from the external environment and interacts with the external with possibility of change, the object should be described with the external environment, which can positively force to change the system and can be changed.

Hence, the aim of AEB is the formulation to describe how we see an object discriminated from the external and interacts with the external environment, which may force to change the object and be forced to be changed by the object. It should be remarked that the logical space is constructed for the object and not necessarily for the external environment. The situation in which we should use AEB is that the object and the external environment interact each other and that we observers recognize both two should be described with different formal logics. Furthermore, it is important that we observers think that the interaction and the difference of logics is intrinsic for the object.

First, we show the basic concept of AEB, and prepare necessary technical terms[3,4,5,6,7,8,15]. Let us take units to be counted in space, and suppose that n pieces of units are placed in a directed row along which we can count them. The i-th unit is expressed as $a_i \in \{0,1\}$, $1 \leq i \leq n$, and the local and ordinarily temporal rule $f \in R \equiv \{f : \{0,1\}^3 \rightarrow \{0,1\}\}$ is defined as $a_i^{t+1} = f(a_{i-1}^t, a_i^t, a_{i+1}^t)$. f represents the spatial interaction between units, and a_0, $a_{n+1} \in \{0,1\}$ are the boundary condition(BC) which means the external environment of system. The family of finite units of space, local and temporal rule and boundary condition is called elementary cellular automaton(ECA)[9]. Elementary cellular automaton is probably one of the most simple formulation in order to consider a spatial pattern transferred through time evolution and the relation between time and space. Especially, the finiteness of space is very important, because objects like living things are spacially finite and the propagation velocity of interaction between the objects is much lower than that of light.

The definition of AEB consists of the followings,

1. Local and ordinarily temporal rule : f ;
2. Non-local and time-reverse rule : g ;
3. System : The subset of R : R_s ;
4. The relation of 1 and 2 : $g(\overline{f}(\eta \cup \xi) \cup \xi')$ can be approximated to be an identity map ;
5. BC, local rule f : these are decided to satisfy the relation in 4 ;

where, $\overline{f}$ represents a non-local and ordinarily temporal rule and is applying the local rule f to whole space including BC, where $\xi, \xi' \in \{0,1\}^2$ represents BC at the t-th step, and at the $(t+1)$-th step, respectively.

We will show the detail of 2 later. g does not generally satisfy the identity map in 4. To be close to the identity map, operation 5 is provided in which BC is decided and then rule f at the $(t+1)$-th step is decided. The operation to decide BC is called Macroscopic Perpetual Disequilibration(Macro PD) and that to decide f at the next step is Micro PD. Macro PD is formulated as,

$$\min_{1 \leq k \leq m} \min_{\xi, \xi'} d_H(g_f^{(k)}(\bar{f}(\eta \cup \xi) \cup \xi'), \eta \cup \xi). \tag{1}$$

where d_H represents haming distance, $g_f^{(k)}$ an inverse rule of f, $\eta \in \{0,1\}^n$ the sequence of spatial units at t-th step, $\xi \in \{0,1\}^2$ the BC at time t , $\xi' \in \{0,1\}^2$ the BC at time $t+1$, and m is the number of elements of the set of inverse rules of f defined in 2. In Eq-(1), though one inverse rule as well as BC are selected, only BC is important for the next procedure. Micro PD is formulated such that,

$$\min_{g_{f'}^{(k')}} d_H(g_{f'}^{(k')}(\bar{f}(\eta \cup \xi^*) \cup \xi'^*), \eta \cup \xi^*). \tag{2}$$

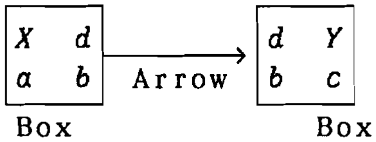
where $g_{f'}$ represents an inverse rule of f' which is any element of R_s defined in 3. Boundary states ξ^* and ξ'^* are BC which is selected in Macro PD. That is, Micro PD is the operation to select one inverse rule from the disjoint union of the sets of the inverse rule of all elements of R_s. Rule $f' \in R_s$ whose inverse rule $g_{f'}^{(k')}$ is selected in (2) is given as the ordinal temporal rule at time $t+1$. Fig.3-1 shows one example of AEB.

It can be also said that AEB is formulation for the concept 'unprogrammability'[5]. Unprogrammability[11][12] suggests that the logic for macroscopic phenomenon and the logic for microscopic phenomenon are neither dependent on nor independent of each other when we observe living things or especially the brain. Macroscopic objects and microscopic ones always interact and both are causal of each other, but each of them is not dependent on only the other. That is, each logic can neither be *a priori* constructed independently of the other nor be *a priori* deduced from the other. This is probably because the velocity of the observation propagation is much smaller than that of light. Matsuno[13][14] suggests that the small value of this is intrinsic for living things and proposed the significance of one-to-many mapping is living things. On the other hand, we must *a priori* prepare operand and operator in mathematical model. Then we cannot express the logical discrepancy between macroscopic rule and microscopic one, if we continue to adopt the former rule which can deduced from the latter or *vice versa*. Using AEB, we can deduce the discrepancy between macro- and micro- because reverse rule g_f can involve contradiction with ordinal rule f. The contradiction depends not only on the rule but on the sequential values. The contradiction can be introduced directly from *DMB* in primitive *FD* and indirectly form *PMB* or *HMB*, and the number of Daughters in primitive FD can be deduced by Theorem 1. That will be shown in following.

3-3. Primitive flow diagram

3-3-1.Basic definition

We can construct the spatial transition rule g represented in flow-diagram(FD), following the procedure([3,4,5,6,7,8,15]). Local spatial transition can be expressed with two nodes(Box) and one directional edge(Arrow).



, (3)

where d, X, Y denotes the site at time t and a, b, c does the site at time $t+1$. In the case that $\overline{f}$ is many-to-one mapping, a sequence η might be different from $g(\overline{f}(\eta \cup \xi) \cup \xi')$, and g cannot be defined uniquely. And g can deduce the sequence only if each Box is linked to two Box and then g could contradicts to f. In this paper we will show the algorithmic recipe for the flow-diagram which does not contradict to f and which would give the basis of the definition of g, and will show some property of it, particularly, in the case that f is symmetric rule. Now we will call this flow-diagram primitive FD hereafter.

In the local transition, we call the initial node Mother and the terminal node Child. Especially, we call Daughter to that Daughter which can play the role of Mother in the next spatial step. We must remember that the values of the second column of Mother coincides with the values of the first column of Child. $Y, c \in \{0, 1\}$ and this is why for arbitrary Mother there are four candidates of Child. If we take the rule f into consideration, the number of candidates would be selected and decreased. When the equation $d=f(a, b, c)$ is satisfied in (1), then we say that 'Mother has Child'.

Definition 1. Box-vector$(\alpha, \beta, \gamma, \delta)$ is defined for the box $\begin{array}{|cc|}\hline \alpha & \beta \\ \gamma & \delta \\ \hline \end{array}$, where $\alpha, \beta, \gamma, \delta \in \{0, 1\}$.

Definition 2. 44M is matrix of all possible Box like that,

$$44M = \begin{bmatrix} \boxed{\begin{smallmatrix}00\\00\end{smallmatrix}} & \boxed{\begin{smallmatrix}10\\00\end{smallmatrix}} & \boxed{\begin{smallmatrix}00\\10\end{smallmatrix}} & \boxed{\begin{smallmatrix}10\\10\end{smallmatrix}} \\ \boxed{\begin{smallmatrix}01\\00\end{smallmatrix}} & \boxed{\begin{smallmatrix}11\\00\end{smallmatrix}} & \boxed{\begin{smallmatrix}01\\10\end{smallmatrix}} & \boxed{\begin{smallmatrix}11\\10\end{smallmatrix}} \\ \boxed{\begin{smallmatrix}00\\01\end{smallmatrix}} & \boxed{\begin{smallmatrix}10\\01\end{smallmatrix}} & \boxed{\begin{smallmatrix}00\\11\end{smallmatrix}} & \boxed{\begin{smallmatrix}10\\11\end{smallmatrix}} \\ \boxed{\begin{smallmatrix}01\\01\end{smallmatrix}} & \boxed{\begin{smallmatrix}11\\01\end{smallmatrix}} & \boxed{\begin{smallmatrix}01\\11\end{smallmatrix}} & \boxed{\begin{smallmatrix}11\\11\end{smallmatrix}} \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \overset{\cdot}{B_{i,j}} & \vdots \end{bmatrix}.$$

where $i,j \in \{0,1,2,3\}$. Each second row of the Box is similar in each row in 44M and the first row of the Box is similar in each column in 44M. Each component in i-th row and in j-th column is represented as $B_{i,j}$ and Box-vector for it is $(j\%2, i\%2, j¥2, i¥2)$, where $(j\%2)$ presents the modulo of the operation j is divided 2 and $(j¥2)$ is the quotient of the same operation. The low suffix i,j for $B_{i,j}$ is called Box-number.

Remark : Children of Mother-Box $B_{i,j}$ are necessarily Boxes in i-th Column.

Remark : Using Box-vector$(\alpha,\beta,\gamma,\delta)$, we can obtain the suffix of B in 44M as $B_{2\delta+\beta,2\gamma+\alpha}$.

Definition 3. RT(Right Triangle) of the Box is a vector whose three components are respectively 2nd, 3rd and 4th component of the Box-vector. Similarly LT(Left Triangle) is constituted of its 1st, 3rd and 4th component.

RT of $(\alpha,\beta,\gamma,\delta)$ is (β,γ,δ) and LT is (α,γ,δ),

i.e. $\boxed{\begin{array}{cc}\cdot & \beta\\ \gamma & \delta\end{array}}$ is defined as RT of $\boxed{\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}}$, $\boxed{\begin{array}{cc}\alpha & \cdot\\ \gamma & \delta\end{array}}$ is LT of $\boxed{\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}}$.

In 44M the RT of $B_{i,0}$ is same as $B_{i,1}$'s and $B_{i,2}$'s is as $B_{i,3}$'s for each $i \in \{0,1,2,3\}$. Consequently, $B_{i,2n}$ and $B_{i,2n+1}$ as Mother(for $n \in \{0,1\}$) has the same Child because only RT of them plays the part role of determination of the transition. In nearly sense, a Box $B_{i,j}$ has both $B_{2n,i}$ and $B_{2n+1,i}$ as Child for $n \in \{0,1\}$, because necessary condition of Child is determined by LT of them. So, let define useful set in which two elements are Boxes whose RT or LT are equivalent. Then, as a useful tool, we will define the sets whose two elements' RT or LT are co-incident. Each set can be obtained by dividing 44M to eight parts.

Definition 4. Set H_R and H_L

$H_{Rm}{}^{n} = \{B_{m,2n}\ ,\ B_{m,2n+1}\}$ where m=0,1,2,3 n=0,1

$H_{Lm}{}^{n} = \{B_{2n,m}\ ,\ B_{2n+1,m}\}$

i.e. $H_{Rm}{}^{n} = \{ \begin{array}{|cc|} \# & (m\%2) \\ n & (m¥2) \end{array} \ ;\ \#=0,1 \}$,

$H_{Lm}{}^{n} = \{ \begin{array}{|cc|} (m\%2) & \# \\ (m¥2) & n \end{array} \ ;\ \#=0,1 \}$.

The correspondence between the sets and the parts of 44M is illustrated in Fig. 3-2. It can be deduced that $B_{i,j}$ is an element of $H_{Ri}{}^{j¥2}$ or of $H_{Lj}{}^{i¥2}$. Given an arbitrary Box whose Box-vector=($\alpha,\beta,\gamma,\delta$), it is an element of $H_{R2\delta+\beta}{}^{\gamma}$ or of $H_{L2\gamma+\alpha}{}^{\delta}$.

Notation. Let # be a wild card symbol denoting both of the values {0,1}.

Lemma 1. H_{Rm}^{n} always has a intersection with $H_{L2n}^{m¥2}$ or $H_{L2n+1}^{m¥2}$, and H_{Lm}^{n} always has one with $H_{R2n}^{m¥2}$ or $H_{R2n+1}^{m¥2}$.

Proof. The elements of $H_{L2n}^{m¥2}$ are $((2n)\%2,\ \#,\ (2n)¥2,\ m¥2)=(0,\ \#,\ n,\ m¥2)$ as Box-vector. Similarly, $H_{L2n+1}^{m¥2}$'s are $(1,\ \#,\ n,\ m¥2)$. And the elements of H_{Rm}^{n} is $(\#,\ m\%2,\ n,\ m¥2)$. Consequently, $(0,\ m\%2,\ n,\ m¥2)$ is element both of H_{Rm}^{n} and of $H_{L2n}^{m¥2}$ and $(1,\ m\%2,\ n,\ m¥2)$ is element both of H_{Rm}^{n} and of $H_{L2n+1}^{m¥2}$. The intersection of H_{Lm}^{n} and $H_{R2n}^{m¥2}$ or one of H_{Lm}^{n} and $H_{R2n+1}^{m¥2}$ can be proved in the similar manner.

3-3-2. The relation between Mother and Daughter

Local rule f can be specified such that

$$\begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ d_0 & d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 \end{array} .$$

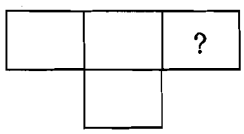
Since $d_i \in \{0,1\},\ i=0,1,\cdots,7,$ there is $2^8=256$ possible rules. Wolfram[9] has defined a labeling scheme according to rule f as

$$\text{rule number} = \sum_{i=0}^{7} d_i * 2^i .$$

Jen[10] analised elementary cellular automaton and labeled to some classes of rules to exhibit the deterministic structure. In the case that $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$, $d_6 \neq d_7$, such rule should be called to hold the particular deterministic structure as,

t

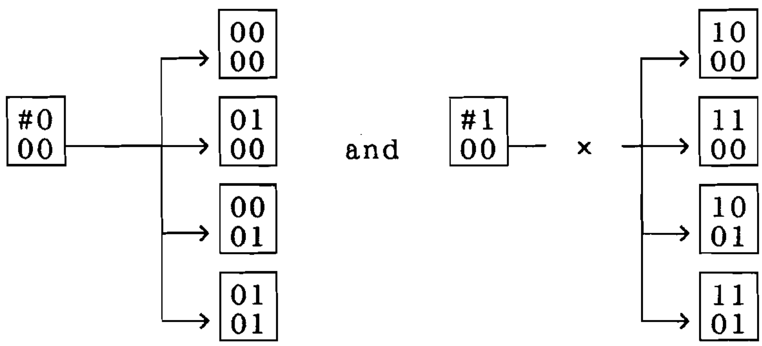
$t+1$



i.e. $a_s^{t+1}, a_{s-1}^{t}, a_s^{t}$
$\rightarrow a_{s+1}^{t}$.

This structure could be reinterpreted in terms of local spatial transition.

Since the ordiarily temporal rule f determines the local spatial transition between Mother and Child in primitive FD, it is determined by RT of each Box whether it could play the role of Mother and be linked to some Boxes. Similarly, LT of each Box determines the possibility of the role of Child. For example, when rule f satisfies $f(0,0,0)=f(0,0,1)=0$, i.e. $d_0=d_1=0$, both elements of H_{R0}^0 whose RT is (0,0,0) can be linked to all of the Boxes in the 0th column in 44M, whereas both elements H_{R1}^0 cannot be linked to any Boxes. Such relation can be illustrated as,



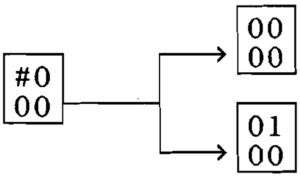
where × denotes no linkage.

i.e. the elements of $H_{R0}^0 \longrightarrow$ the elements of $\bigcup_n H_{L0}^n$, $n=\{0,1\}$.

the elements of $H_{R1}^0 \longrightarrow \emptyset$. (4)

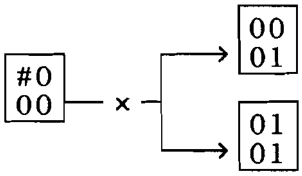
If $d_0=d_1=1$, both elements of H_{R1}^0 can be linked to four Boxes and H_{R0}^0's can be linked to no Box. Of course the value of $d_2, d_3, \cdots, d_7$ is independent of the determination of Child according to the elements of H_{R0}^0 and H_{R1}^0, so it is convinced that only the value of d_0 and d_1 decides the condition whether the elements of H_{R0}^0 or H_{R1}^0 could play the role of Mother.

Now consider the case $d_0=1-d_1=0$. The both elements of H_{R0}^0 could be linked to at most four Boxes in the 0th column in 44M i.e. the elements of H_{L0}^0 and H_{L1}^0. Following Definition 3, RT of H_{R0}^0's is (0,0,0) and LT of H_{L0}^0 is (0,0,0) and then $d_0=f(0, 0, 0)=0$ can be satisfied. Hence, the element of H_{L0}^0 could be linked with H_{R0}^{0}'s as Child.



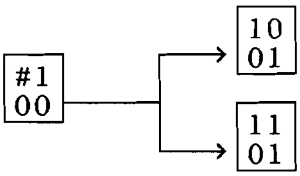
. i.e. H_{R0}^{0}'s $\longrightarrow$ H_{L0}^{0}'s .

On the contrary, LT of H_{L1}^{0}'s is (0,0,1) and when we assume H_{R0}^{0}'s would be linked to H_{L1}^{0}'s, then $0=f(0,0,1)$ should be satisfied and this will contradicts to rule f such that $d_0=1-d_1=0$.

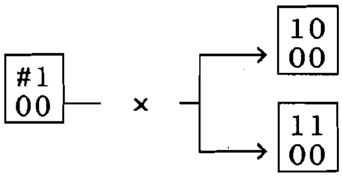


. since $\overset{0}{\uparrow}$ 000 and $\overset{1}{\uparrow}$ 001 .

where × denotes no linkage. Therefore, for such f, the elements of H_{R0}^{0} must be linked to only H_{L0}^{0}'s and similarly, H_{R1}^{0}'s must be to only H_{L1}^{1}'s.



and



.

i.e. H_{R1}^{0}'s $\longrightarrow$ H_{L1}^{1}'s .

And if $d_0=1-d_1=1$, H_{R0}^{0}'s be to H_{L1}^{0}'s and H_{R1}^{0}'s be to H_{L0}^{0}'s.

As suggested above, in the case that $d_0=d_1=\alpha$, the elements of $H_{R\alpha}^{0}$ will be linked to both $H_{L\alpha}^{0}$'s and $H_{L\alpha}^{1}$'s i.e. totally four Boxes(say "each $H_{R\alpha}^{0}$'s has four Children.") and $H_{R1-\alpha}^{0}$'s will be to no Boxes(say "each $H_{R1-\alpha}^{0}$'s has no Child."), and in the other case that $d_0=1-d_1=\beta$, H_{R0}^{0}'s will be linked to H_{L0}^{β}'s and H_{R1}^{0}'s will be to $H_{L1}^{(1+\beta)\%2}$'s(say "each H_{R0}^{0}'s or H_{R1}^{0}'s has two Children, H_{L0}^{β}'s or $H_{L1}^{(1+\beta)\%2}$'s respectively."). Since the values of the second row of H_{R0}^{0}'s and H_{R1}^{0}'s are (00), the values of $d_2, d_3, \cdots, d_7$ are independent of the condition for them as Mother and the link of them to Children. Similarly, only the values of d_2 and d_3 will decide the condition for H_{R2}^{0}'s and H_{R3}^{0}'s and the link of them, and H_{R0}^{1}'s and H_{R1}^{1}'s depend on only d_4's and d_5's, and H_{R2}^{1}'s and H_{R3}^{1}'s do on d_6's and d_7's. For general and algorithmic description, we will extend the relation between the value of d_{2x} and d_{2x+1} (x=0,1,2,3) and the link of $H_{R2(x\%2)+r}{}^{x¥2}$'s to H_L's(r=0,1). Concretely, it is sufficient to specify the suffix of H_L for any given values of d_{2x} and d_{2x+1}.

Definition 5. Function D

$$D: H_{Rm}{}^{n} \rightarrow \bigcup_k H_{Lm}{}^{k} . \qquad \text{for } k=0,1$$

This function D represents the link of any given Box which is an element of $H_{Rm}{}^{n}$ as Mother to the union of the sets whose two elements are Child of the Box.

Theorem 1. for x=0,1,2,3

1) if $d_{2x}=d_{2x+1}=\alpha$, then

$$D(H_{R2(x\%2)+\alpha}{}^{x¥2}) = H_{L2(x\%2)+\alpha}{}^{0} \cup H_{L2(x\%2)+\alpha}{}^{1} .$$

$$D(H_{R2(x\%2)+(1-\alpha)}{}^{x¥2}) = \emptyset .$$

2) if $d_{2x}=1-d_{2x+1}=\beta$, then

$$D(H_{R2(x\%2)+r}{}^{x¥2}) = H_{L2(x\%2)+r}{}^{(\beta+r)\%2} ,$$

for r=0,1.

proof. In general, for x=0,1,2,3, the second row of the each element of $H_{R(x\%2)*2}{}^{x¥2}$ and $H_{R2(x\%2)+1}{}^{x¥2}$ is (x¥2 x%2). If $d_{2x}=f(x¥2, x\%2, 0)=\theta$, then $D(H_{R2(x\%2)+\theta}{}^{x¥2})$ must be within the two

H_L of which the second row of the each element is (x%2 0), i.e. $H_{L2(x\%2)}^{0}$ and $H_{L2(x\%2)+1}^{0}$. Either of the two sets cannot be linked with $H_{R2(x\%2)+\theta}^{x¥2}$ since the low suffix must be equivalent according to the link between H_R and H_L. Consequently $H_{L2(x\%2)+\theta}^{0}$ can be linked with $H_{R2(x\%2)+\theta}^{x¥2}$. And If $d_{2x+1} = f(x¥2, x\%2, 1) = \sigma$, similarly $D(H_{R2(x\%2)+\sigma}^{x¥2})$ must be within the two sets $H_{L2(x\%2)}^{1}$ and $H_{L2(x\%2)+1}^{1}$ and only one of theirs, $H_{L2(x\%2)+\sigma}^{1}$ can be linked. In the case that $\theta=\sigma$, then the element of $H_{R2(x\%2)+1-\theta}^{x¥2}$ whose RT=$(1-\theta, x¥2, x\%2)$ cannot be linked to H_L since $f(x¥2, x\%2, 0) = f(x¥2, x\%2, 1) = \theta$, whereas both $H_{L2(x\%2)+\theta}^{0}$ and $H_{L2(x\%2)+\sigma}^{1}$ will be linked to $H_{R2(x\%2)+\theta}^{x¥2}$, i.e. $D(H_{R2(x\%2)+\theta}^{x¥2}) = \bigcup_{n=0,1} H_{L2(x\%2)+\theta}^{n}$ since $\theta=\sigma$. In the other case that $\theta\neq\sigma$, then $2(x\%2)+\theta \neq 2(x\%2)+\sigma$, and both $H_{R2(x\%2)+\theta}^{x¥2}$ and $H_{R2(x\%2)+\sigma}^{x¥2}$ can be linked to only one set. Hence, $D(H_{R2(x\%2)+\theta}^{x¥2}) = H_{L2(x\%2)+\theta}^{0}$ and $D(H_{R2(x\%2)+1-\theta}^{x¥2}) = H_{L2(x\%2)+1-\theta}^{1}$, and in the unified form, we obtain $D(H_{R2(x\%2)+r}^{x¥2}) = H_{L2(x\%2)+r}^{(\theta+r)\%2}$, for $r=0,1$.

In the similar way to the decision of the condition of Mother, LT of an arbitrary Box will also decide the condition whether the Box could play the role of Child. In general, when $d_y = d_{y+4} = \alpha$ for $y=0,1,2,3$, the Box whose LT is $(1-\alpha, y¥2, y\%2)$ cannot play the role of Child, whereas the Box whose LT is $(\alpha, y¥2, y\%2)$ can be linked with four Boxes. Of course in the case that $d_y = 1 - d_{y+4} = \alpha$, the Box whose LT is $(\alpha, y¥2, y\%2)$ or $(1-\alpha, y¥2, y\%2)$ can be to two Boxes.

3-4. The property of Primitive FD

3-4-1. Class of Box

We can construct primitive FD by sequential linkage of Boxes according to the local spatial linkage between Mother and Child. Remark primitive FD is not R_g^{-1} which is the global inverse rule. In this FD, some Boxes may have different number of Child(or Daughter) than others, depended on rule f, i.e. the value of d_i (i=0,1,•••,7). Gunji & Kon-no[5] and Kon-no & Gunji[6] classify Boxes to five types according to the number of outdegrees of them which is the number of Daughter. The number can be 0,1,2,3 or 4 and each class of Box is called ***SMB***, ***DMB***, ***NMB***, ***PHB*** or ***HMB***, respectively. Theorem 1 can support and generalize the classification. If $d_{2x}=d_{2x+1}=\alpha$ (x=0,1,2,3), then the Box which is each element of $\mathrm{H}_{R2(x\%2)+\alpha}^{x\yen 2}$ has four Children, but does not necessarily have four Daughters. That is, the Children are elements of $\mathrm{H}_{L2(x\%2)+\alpha}^{0}$ and $\mathrm{H}_{L2(x\%2)+\alpha}^{1}$ and some of them could not be Daughters. From Lemma 1 $\mathrm{H}_{L2(x\%2)+\alpha}^{0}$ has a intersection with $\mathrm{H}_{R0}^{(2(x\%2)+\alpha)\yen 2}=\mathrm{H}_{R0}^{x\%2}$ and $\mathrm{H}_{R1}^{x\%2}$, and from Theorem 1-1 the elements of either of them have no Child(nor Daughter), i.e. either $D(\mathrm{H}_{R0}^{x\%2})$ or $D(\mathrm{H}_{R1}^{x\%2})$ is $\emptyset$ in the case of $d_{4(x\%2)}=d_{4(x\%2)+1}$. Whether each element of $\mathrm{H}_{L2(x\%2)+\alpha}^{0}$ could be Child of $\mathrm{H}_{R2(x\%2)+\alpha}^{x\yen 2}$'s depends on the value of $|d_{4(x\%2)}-d_{4(x\%2)+1}|$. Similarly, $\mathrm{H}_{L2(x\%2)+\alpha}^{0}$'s does on the value $|d_{4(x\%2)+2}-d_{4(x\%2)+3}|$. When both of them are equal to 1, $\mathrm{H}_{R2(x\%2)+\alpha}^{x\yen 2}$'s will have four Daughters or be called ***HMB***, and when either of them is equal to 1 it will be called ***PMB***, and when neither of them is 1 it will be ***NMB***. ***NMB*** is founded in the case of $d_{2x}=1-d_{2x+1}=\beta$, or rather, the origin of the name ***NMB*** is this case.

From Theorem 1-2 the elements of $\mathrm{H}_{R2(x\%2)+r}^{x\yen 2}$, r=0,1 will be linked to the elements of $\mathrm{H}_{L2(x\%2)+r}^{(\beta+r)\%2}$. When $|d_{4(x\%2)+2((\beta+r)\%2)}-d_{4(x\%2)+2((\beta+r)\%2)}|$=0, either of $\mathrm{H}_{L2(x\%2)+r}^{(\beta+r)\%2}$'s has no Child(nor Daughter). The other will necessarily have four

Children and at least two Daughters, and it is important that $H_{R2(x\%2)+r}^{x¥2}$'s will have at least one Daughter in $d_{2x}=1-d_{2x+1}=\beta, r=0,1$. Consequently, each element of $H_{R2(x\%2)+r}^{x¥2}$, $r=0,1$ will be called *NMB* when $d_{2x}=1-d_{2x+1}=\beta$ and $|d_{4(x\%2)+2((\beta+r)\%2)}-d_{4(x\%2)+2((\beta+r)\%2)}|=1$, and it will be *DMB* when $d_{2x}=1-d_{2x+1}=\beta$ and $|d_{4(x\%2)+2((\beta+r)\%2)}-d_{4(x\%2)+2((\beta+r)\%2)}|=0$. It is clear that in the case of $d_{2x}=d_{2x+1}=\alpha (x=0,1,2,3)$ each element of $H_{R2(x\%2)+(1-\alpha)}^{x¥2}$ is *SMB*.

In the discussion above, we dealt with the two elements of $H_{R2(x\%2)+r}^{x¥2}$ in the same way, but we concentrated only Theorem 1 which is introduced by the condition of Mother and Child. That is, Theorem 1 does not take it into consideration whether the Box which satisfies the condition of Mother can play the role of Child for other Boxes. Gunji and Kon-no[5][6] defined that the Box called *SMB* does not have both Mother and Child. Then for effective usage of Theorem 1, we would newly divide *SMB* into three classes, the ***Orphan Mother Box(OMB)***, the ***Lonely Mother Box(LMB)***, and the ***Lonely Orphan Mother Box(LOMB)***.

Definition 6. *OMB*, *LMB* and *LOMB*

for any given rule f or $d_0, d_1, \cdots$, and d_7,

1) Iff $d_{2x}=d_{2x+1}=\alpha$ $(x=0,1,2,3)$, the Box whose RT=$(1-\alpha, x¥2, x\%2)$, or the element of $H_{R2(x\%2)+(1-\alpha)}^{x¥2}$ is called *LMB*.

2) Iff $d_y=d_{y+4}=\alpha$ $(y=0,1,2,3)$, the Box whose LT=$(1-\alpha, y¥2, y\%2)$, or the element of $H_{L2(y¥2)+(1-\alpha)}^{y\%2}$ is called *OMB*.

3) *LOMB* is the Box which is both *LMB* and *OMB*.

The Box which is neither *OMB* nor *LMB*, is necessarily either *DMB*, *NMB*, *PMB* or *HMB*, and can play the role of Daughter in primitive FD. For later discussion, it is convinced that the Box which is not *OMB*(of course nor *LOMB*) must have at least two Children or one Daughter.

Lemma 2. If one element of H_{Lm}^{n} is not *OMB*, it has at least one Mother.

proof:It is natural, following the spatial reverse procedure of the proof of Theorem 1.

Lemma 3. If one element of H_{Rm}^{n} or H_{Lm}^{n} is *LOMB*, the other element is not *LOMB*.

proof: If one element of H_{Rm}^{n} is *LOMB*, both elements are necessarily *LMB*. From Lemma 1, each of them is also an element of $H_{L2n}^{m¥2}$ or $H_{L2n+1}^{m¥2}$. If $d_{2n+m¥2}-d_{2n+m¥2+4}=\alpha$, from Definition 5, the Box whose LT=($1-\alpha, n, m¥2$), is *OMB* and it is an element of $H_{L2n+(1-\alpha)}^{m¥2}$. In this case $H_{L2n+\alpha}^{m¥2}$'s must not be *OMB* and consequently, either $H_{L2n}^{m¥2}$'s or $H_{L2n+1}^{m¥2}$'s is *OMB* and the other is not *OMB*. So, when one element of H_{Rm}^{n} is *OMB*(*LOMB*), the other is not. The case of H_{Lm}^{n} can be followed in the similar way.

3-4-2. Primitive FD is closed

Primitive FD can be constructed by the linkage of local spatial transition. Depended on Box-number or rule *f*, it is varied how many Daughter or Mother the Box has and which Boxes it is linked with. In general, digraph should not be always closed and might extend infinitely, or it might have the two or more paths either of which is selected with the initial Box. Now, we will show that the primitive FD is closed and it does not have initial Box dependence, or concretely,

Proposition 1. an arbitrary Box except for *LMB* is linked to an arbitrary Box except for *OMB* in 44M with finite(at most 4) steps of spatial transition.

proof:It is shown in 4 steps. The 1st, 2nd or 3rd step follows the similar procedure.

1)Let take an element of $H_{Rm}{}^{n}$,m=0,1,2,3 and n=0,1. If it is not *LMB*, it has always two or four Children from Theorem 1.

1-1)When $d_{2(2n+m¥2)}=d_{2(2n+m¥2)+1}=\alpha$, then

$$D(H_{Rm}{}^{n})=\bigcup_{k=0,1} H_{Lm}{}^{k}=H_{Lm}{}^{0} \cup H_{Lm}{}^{1}.$$

From Lemma 1, $H_{Lm}{}^{0}$ has an intersection with $H_{R0}{}^{m¥2}$ or with $H_{R1}{}^{m¥2}$ and $H_{Lm}{}^{1}$ does with $H_{R2}{}^{m¥2}$ or with $H_{R3}{}^{m¥2}$. That is, each of the four Children is an element of $H_{R0}{}^{m¥2}$, $H_{R1}{}^{m¥2}$, $H_{R2}{}^{m¥2}$ or $H_{R3}{}^{m¥2}$, respectively.

1-2)When $d_{2(2n+m¥2)}=1-d_{2(2n+m¥2)+1}=\beta$, then

$$D(H_{Rm}{}^{n}) = H_{Lm}{}^{(\beta+m\%2)\%2} = H_{Lm}{}^{(\beta+m)\%2}.$$

from Lemma 1, $H_{Lm}{}^{(\beta+m)\%2}$ has an intersection with $H_{R2((\beta+m)\%2)}{}^{m¥2}$ or with $H_{R2((\beta+m)\%2)+1}{}^{m¥2}$.

2)From 1-2), we will take an element of $H_{R2((\beta+m)\%2)}{}^{m¥2}$ and $H_{R2((\beta+m)\%2)+1}{}^{m¥2}$.

2-1)When $d_{2(2(m¥2)+(\beta+m)\%2)}=d_{2(2(m¥2)+(\beta+m)\%2)+1}$, then either of them will be *LMB* and the other have four Child. In the similar way of 1-1), each of the four Child is an element of $H_{R0}{}^{(\beta+m)\%2}$, $H_{R1}{}^{(\beta+m)\%2}$, $H_{R2}{}^{(\beta+m)\%2}$ or $H_{R3}{}^{(\beta+m)\%2}$, respectively.

2-2)When $d_{2(2(m¥2)+(\beta+m)\%2)}=1-d_{2(2(m¥2)+(\beta+m)\%2)+1}=\gamma$,

$$D(H_{R2((\beta+m)\%2)}{}^{m¥2}) = H_{L2((\beta+m)\%2)}{}^{\gamma\%2}.$$

$D(H_{R2((\beta+m)\%2)+1}{}^{m¥2}) = H_{L2((\beta+m)\%2)+1}{}^{(\gamma+1)\%2}$.

From Lemma 1, $H_{L2((\beta+m)\%2)}{}^{\gamma\%2}$ has an intersection with $H_{R2(\gamma\%2)}{}^{(\beta+m)\%2}$ or with $H_{R2(\gamma\%2)+1}{}^{(\beta+m)\%2}$ and $H_{L2((\beta+m)\%2)+1}{}^{(\gamma+1)\%2}$ has an intersection with $H_{R2((\gamma+1)\%2)}{}^{(\beta+m)\%2}$ or with $H_{R2((\gamma+1)\%2)+1}{}^{(\beta+m)\%2}$. Whether γ=0 or 1, each of the Child of the two elements of $H_{Lm}{}^{(\beta+m)\%2}$ is an element of $H_{R0}{}^{(\beta+m)\%2}$, $H_{R1}{}^{(\beta+m)\%2}$, $H_{R2}{}^{(\beta+m)\%2}$ or $H_{R3}{}^{(\beta+m)\%2}$, respectively.

3)From 1-1) and 2), we will take an element of $H_{R0}{}^{p}$, $H_{R1}{}^{p}$, $H_{R2}{}^{p}$ and $H_{R3}{}^{p}$, p=m¥2 or $(\beta+m)\%2$.

In the similar way to 2), each of the Child of $H_{R0}{}^{p}$'s and $H_{R1}{}^{p}$'s, even if either of them is *LMB*, $H_{R0}{}^{0}$, $H_{R1}{}^{0}$, $H_{R2}{}^{0}$ or $H_{R3}{}^{0}$ respectively and also each of the Child of $H_{R2}{}^{p}$'s and $H_{R3}{}^{p}$'s, even if either of them is *LMB*, $H_{R0}{}^{1}$, $H_{R1}{}^{1}$, $H_{R2}{}^{1}$ or $H_{R3}{}^{1}$, respectively.

Consequently, an element of $H_{Rm}{}^{n}$, which is not *LMB*, is linked to one element of $H_{Rq}{}^{r}$, for arbitrary $q \in \{0,1,2,3\}$, $r \in \{0,1\}$, with two or three steps.

4)If a Box is not OMB, form lemma 2 it is linked with at least one Mother, i.e. it is necessarily Child of $H_{Rm}{}^{n}$, in which m=0,1,2,3 and n=0,1. Then, from 3), an arbitrarily Box, which is not *LMB* is linked to any Box in 44M which is not *OMB* with at most 4 spatial steps.

3-4-3. Symmetric rule

Since we fundamentally concentrate symmetric rule and should imply to asymmetric one, we will show some theorems as useful tools according to symmetric rule which satisfies $d_1 = d_4$ and $d_3 = d_6$.

Lemma 4-1. On the condition that f is symmetric($d_1=d_4$,$d_3=d_6$),when $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6 \neq d_7$, in 44M there are no *SMB*.

proof:From Theorem 1, each element of $H_{Rm}{}^{n}$ for arbitrary $m \in \{0,1,2,3\}$ or $n \in \{0,1\}$ has two Child and there is no *LMB*. For symmetry($d_1=d_4$), $d_0 \neq d_1$ follows $d_0 \neq d_4$ and $d_4 \neq d_5$ does $d_1 \neq d_5$, and similarly $d_2 \neq d_6$ and $d_3 \neq d_7$ are concluded. Then there is no *OMB* and consequently there is no *SMB*.

Lemma 4-2. On the condition that f is symmetric($d_1=d_4$,$d_3=d_6$),when $d_0=d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6 \neq d_7$, in 44M there are three *SMB*es or particularly one *LOMB*. The case $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6=d_7$ is so on.

proof:If $d_0=d_1=\alpha$, $d_4=1-d_5=\alpha$ for symmetric rule $d_1=d_4$, and then each element of $H_{R2(0\%2)+(1-\alpha)}{}^{0*2}$ (=$H_{R1-\alpha}{}^{0}$) is *LMB* and $H_{L2(0*2)+(1-\alpha)}{}^{0\%2}$'s (=$H_{L1-\alpha}{}^{0}$'s) is *OMB* from Definition 5 and there is no other *SMB* in 44M. And $H_{R1-\alpha}{}^{0}$ has an intersection with $H_{L1-\alpha}{}^{0}$ from Lemma 1. So, one element of $H_{R1-\alpha}{}^{0}$ is equivalent to one element of $H_{L1-\alpha}{}^{0}$ and consequently it is *LOMB* and totally there is three *SMB*es. In the case $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6=d_7$, $H_{R3-\alpha}{}^{0}$'s is *LMB*, $H_{L3-\alpha}{}^{0}$'s is *OMB*, and the intersection $B_{3-\alpha,3-\alpha}$ is *LOMB*.

Lemma 4-3. On the condition that f is symmetric($d_1=d_4$, $d_3=d_6$), when $d_0 \neq d_1$, $d_2=d_3$, $d_4 \neq d_5$ and $d_6 \neq d_7$, in 44M there are four SMBes, or particularly two *LMB*es and two *OMB*es and no *LOMB*. In the case $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4=d_5$ and $d_6 \neq d_7$ is, Boxes can be decided as well as the former case.

proof: If $d_2=d_3=\alpha$, $d_6=1-d_7=\alpha$ for symmetric rule $d_3=d_6$, and then each element of $H_{R2(1*2)+(1-\alpha)}{}^{1*2}$ ($=H_{R3-\alpha}{}^{0}$) is *LMB* and $H_{L2(2*2)+(1-\alpha)}{}^{2*2}$'s ($=H_{L3-\alpha}{}^{0}$'s) is OMB from Definition 5 and there is no other SMB in 44M. From Lemma 1 there is no intersection between $H_{R3-\alpha}{}^{0}$ and $H_{L3-\alpha}{}^{0}$ and so there is no *LOMB*. In the case $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4=d_5$ and $d_6 \neq d_7$, $H_{R1-\alpha}{}^{1}$'s is *LMB* and $H_{L1-\alpha}{}^{1}$'s is *OMB*, and there is no intersection between them. Consequently there are totally four *SMB*es.

Lemma 4-4. On the condition that f is is symmetric($d_1=d_4$, $d_3=d_6$), when $d_0=d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6=d_7$ or when $d_0 \neq d_1$, $d_2=d_3$, $d_4=d_5$ and $d_6 \neq d_7$, there are six SMBes or particularly there are two *LOMB*es.

proof: If $d_0=d_1=\alpha$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6=d_7=\beta$, from lemma 4-1, $H_{R1-\alpha}{}^{0}$'s or $H_{R3-\beta}{}^{0}$'s is *LMB*, and $H_{L1-\alpha}{}^{0}$'s or $H_{L3-\beta}{}^{0}$'s is *OMB*, and $B_{1-\alpha,1-\alpha}$ or $B_{3-\beta,3-\beta}$ is the intersection. Then there are two *LOMB*es and totally there are six *SMB*es. If $d_0 \neq d_1$, $d_2=d_3=\alpha$, $d_4=d_5=\beta$ and $d_6 \neq d_7$, $H_{R1-\alpha}{}^{1}$'s or $H_{R3-\alpha}{}^{0}$'s is *LMB* and $H_{L1-\alpha}{}^{1}$'s or $H_{L3-\alpha}{}^{0}$'s are *OMB*. From lemma 1, there is one intersection between $H_{R1-\alpha}{}^{1}$ and $H_{L3-\alpha}{}^{0}$ or between $H_{R3-\alpha}{}^{0}$ and $H_{L1-\alpha}{}^{1}$. Then there are two *LOMB* and totally there are six *SMB*es.

Theorem 2 can be obtained from Lemma 4.

Theorem 2. Let f be symmetric rule ($d_1=d_4$, $d_3=d_6$). The elements of $H_{Rm}{}^{n}$ are *OMB*, iff $H_{Lm}{}^{n}$'s are *LMB*.

It was proved in Lemma 4 that for each $k \in \{0,1\}$ $d_{2k}=d_{2k+1}=\alpha$ or $d_{2k+4}=d_{2k+5}=\beta$ which are the condition that $H_{R2k+(1-\alpha)}{}^{0}$'s or

$H_{R2k+(1-\beta)}^{1}$'s are respectively *LMB*, always result in $d_{2k}=d_{2k+4}=\alpha$ or $d_{2k+1}=d_{2k+5}=\beta$ which are the condition that $H_{L2k+(1-\alpha)}^{0}$'s or $H_{L2k+(1-\beta)}^{1}$'s are respectively *OMB*. On the contrary $d_{2k}=d_{2k+4}=\gamma$ or $d_{2k+1}=d_{2k+5}=\delta$ conclude $d_{2k}=d_{2k+1}=\gamma$ or $d_{2k+4}=d_{2k+5}=\delta$ since f is symmetric. Then in symmetric rule we can know which Box is *OMB* in 44M only if we obtain the value of $|d_{2x}-d_{2x+1}|$, $x=0,1,2,3$.

Theorem 3. Suppose f is symmetric ($d_1=d_4, d_3=d_6$) and satisfies $d_{2k}\neq d_{2k+1}$ and $d_{2k+4}\neq d_{2k+5}$ such that $k=0$ or 1. Then $D(H_{R2k}^{0})\neq D(H_{R2k}^{1})\neq D(H_{R2k+1}^{0})\neq D(H_{R2k+1}^{1})$.

proof: If $d_{2k}=1-d_{2k+1}=\alpha$, then $1-d_{2k+4}=d_{2k+5}=\alpha$ since f is symmetric. From Theorem 1, none of H_{R2k}^{0}'s, H_{R2k}^{1}'s, H_{R2k+1}^{0} and H_{R2k+1}^{1} is *LMB* and from Theorem 2, none of H_{L2k}^{0}'s, H_{L2k}^{1}'s, H_{L2k+1}^{0} and H_{L2k+1}^{1} is *OMB*. And From Theorem 1, $D(H_{R2k}^{0})=H_{L2k}^{\alpha\%2}$, $D(H_{R2k}^{1})=H_{L2k}^{(1-\alpha)\%2}$, $D(H_{R2k+1}^{0})=H_{L2k+1}^{((1-\alpha)+1)\%2}=H_{L2k+1}^{\alpha\%2}$ and $D(H_{R2k+1}^{0})=H_{L2k+1}^{(\alpha+1)\%2}=H_{L2k+1}^{(1-\alpha)\%2}$. According to the lower suffix $2k\neq 2k+1$ and to the upper suffix $(\alpha\%2)\neq((1-\alpha)\%2)$, then each of the four H_L is different from the others.

Corollary 1. In symmetric and legal($d_0=0$) rule, there are always *DMB*es without rule 0,90,150,204.

Corollary 2. If there is at least one *DMB* in rule f, there is at least a *PMB* or *HMB* in symmetric and legal rule.

Corollary 3. If there is at least one *PMB* in rule f, there is no *HMB* in symmetric and legal rule, and *vice versa*.

3-5. Discussion

For example, let us construct the primitive FD for rule 22. In rule 22, $d_0=1-d_1=0$, $d_2=1-d_3=1$, $d_4=1-d_5=1$ and $d_6=d_7=0$. For definition 6-1, $d_6=d_7=0$ is resulted in that the elements of $H_{R3}{}^1$ is *LMB* and for definition 6-2 or Theorem 2, the elements of $H_{L3}{}^1$ is *OMB*. From Lemma 1, $H_{R3}{}^1$ has an intersection $B_{3,3}$ with $H_{L3}{}^1$ and for definition 6, $B_{3,3}$ is *LOMB*. This case corresponds to lemma 4-2, then only three Boxes $B_{2,3}$, $B_{3,2}$, $B_{3,3}$ are *SMB*(*LMB* or *OMB*) and each of them is an element of $H_{R2}{}^1$ or $H_{R3}{}^1$ and of $H_{L2}{}^1$ or $H_{L3}{}^1$. From Theorem 1, we can get

$$D(H_{R0}{}^0)=H_{L0}{}^0 , \qquad D(H_{R0}{}^1)=H_{L0}{}^1 ,$$
$$D(H_{R1}{}^0)=H_{L1}{}^1 , \qquad D(H_{R1}{}^1)=H_{L1}{}^0 ,$$
$$D(H_{R2}{}^0)=H_{L2}{}^1 , \qquad D(H_{R2}{}^1)=H_{L2}{}^0 \cup H_{L2}{}^1 ,$$
$$D(H_{R3}{}^0)=H_{L3}{}^0 .$$

An element of $H_{R2}{}^1$ is *OMB* and only the other element organizes primitive FD. One element of $H_{L2}{}^1$ is *SMB*, then two elements of $H_{R2}{}^0$ are *DMB*es and $B_{2,2}$ or one element of $H_{R2}{}^1$ is *PMB*. The other ten Boxes are *NMB*es and then all of Boxes' Daughters except for *SMB* are decided(Fig.3-3).

Rule 90 and rule 150 satisfys $d_0 \neq d_1$, $d_2 \neq d_3$, $d_4 \neq d_5$ and $d_6 \neq d_7$. In this case there is no SMB in primitive FD. It is shown in Fig. 3-4.

Now we will show one example for construction of time-reverse rule g for rule 22 utilizing primitive FD. *NMB* has two Daughters, and then we can describe in functional form. In order to describe as a function, *PMB* must be also linked to only two Boxes. If two Boxes are prepared independent of spatial position, we must choose two Boxes from the three Boxes. The second component of selected two Box-vector must be different, we will choose one Box from $B_{0,2}$ and $B_{2,2}$, then there are two ways that $B_{2,2}$ is linked in spatial direction.

In the case of *DMB* it is more difficult. The selection for *PMB* can be performed within local rule f, but in order to describe *DMB*'s linkage in the form of function, we can no longer follow f. If we follow only Remark in Section 3-3-1," Children of Mother-Box $B_{i,j}$ are necessarily Boxes in i-th Column.", the candidate for $B_{2,0}$ and $B_{2,1}$ is $B_{0,2}$, $B_{1,2}$ and $B_{2,2}$. Only $B_{2,2}$ is Daughter of $B_{2,0}$ or $B_{2,1}$, then $B_{2,2}$ will be selected. If $B_{2,2}$ is selected, $B_{2,1}$ must be chosen since the second components of the two should be different. In this case if Box following f is selected, the other is autonomously selected in order to construct functional form, so long as we follow the Remark in Section 3-3-1.

Consequently there are two reverse rules g for rule 22. Fig. 3-5 shows the two patterns.

But there are some rules that cannot construct g in the procedure above. They are two rules 54, 250 in legal symmetric rules. In case of rule 54, $d_0=1-d_1=0$, $d_2=1-d_3=1$, $d_4=d_5=1$, and $d_6=d_7=0$. Then it is deduced by Theorem 1 and 2 that each element of H_{R0}^1 and of H_{R3}^1 is *LMB* and H_{L0}^1's and H_{L3}^1's is *OMB*(i.e. $B_{3,3}$ is *LOMB*). Consequently the three Boxes $B_{0,3}$, $B_{2,3}$, $B_{3,3}$ which are in the 3rd column in 44M are *SMB*es. Then in the 3rd column in 44M there is only one Box which is not *SMB*, that is, which can play the role of Daughter in primitive FD(Fig. 3-6). Therefore, we cannot link $B_{3,1}$ to two Boxes if we follow the remark 3-1 and seek the successor from the Boxes in 3rd column in 44M. Then, we have to select a way from the following three ways in order to construct time reverse rule g for rules 54 and 250.

1) $B_{3,1}$'s successor can be *OMB* but not *LOMB* in 3rd column.

2) $B_{3,1}$'s successor can be *LMB* included *LOMB* in 3rd column.

3) $B_{3,1}$'s successor can be the Box in the other column than 3rd one.

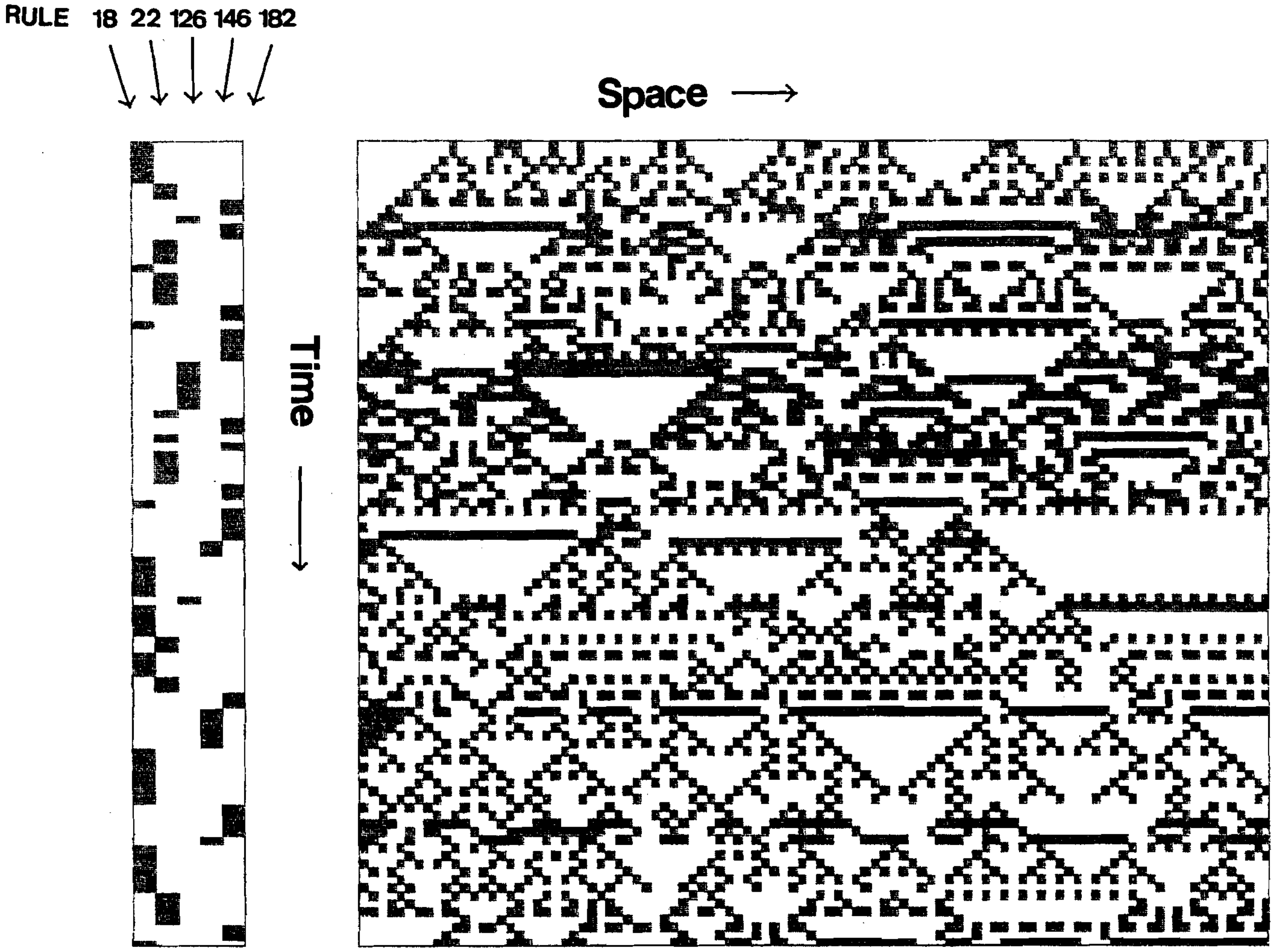


Fig. 3-1. An example of AEB, in which $R_s = \{f_{18}, f_{22}, f_{126}, f_{146}, f_{182}\}$. The low suffix shows the rule numberof elementary cellular automaton. As for the rule number, see section 3-3-2. The right figure shows the temporal change of the spatial patterns and the left one shows the adopted rule in each step determined with Micro-PD.

for H_R

00 00	10 00	00 10	10 10
01 00	11 00	01 10	11 10
00 01	10 01	00 11	10 11
01 01	11 01	01 11	11 11

→

H_{R0}^{0}	H_{R0}^{1}
H_{R1}^{0}	H_{R1}^{1}
H_{R2}^{0}	H_{R2}^{1}
H_{R3}^{0}	H_{R3}^{1}

for H_L

00 00	10 00	00 10	10 10
01 00	11 00	01 10	11 10
00 01	10 01	00 11	10 11
01 01	11 01	01 11	11 11

→

H_{L0}^{0}	H_{L1}^{0}	H_{L2}^{0}	H_{L3}^{0}
H_{L0}^{1}	H_{L1}^{1}	H_{L2}^{1}	H_{L3}^{1}

Fig. 3-2. The correspondence of H_R and H_L to components in 44M.

Rule 22

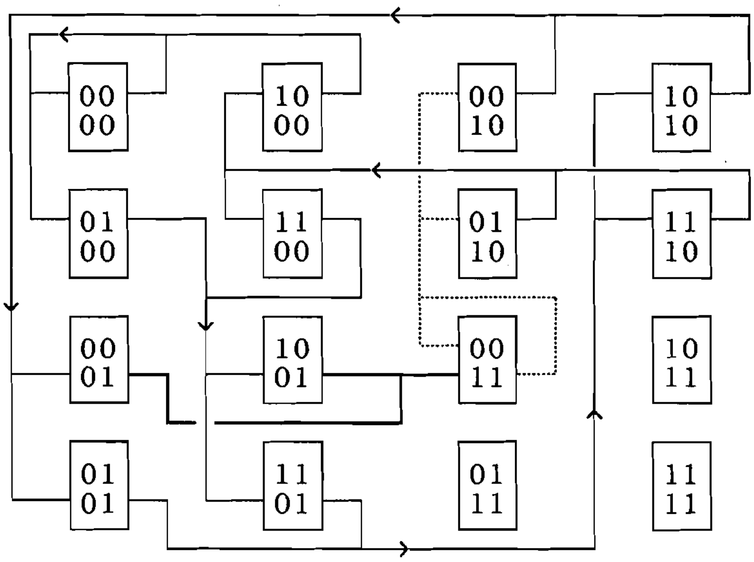
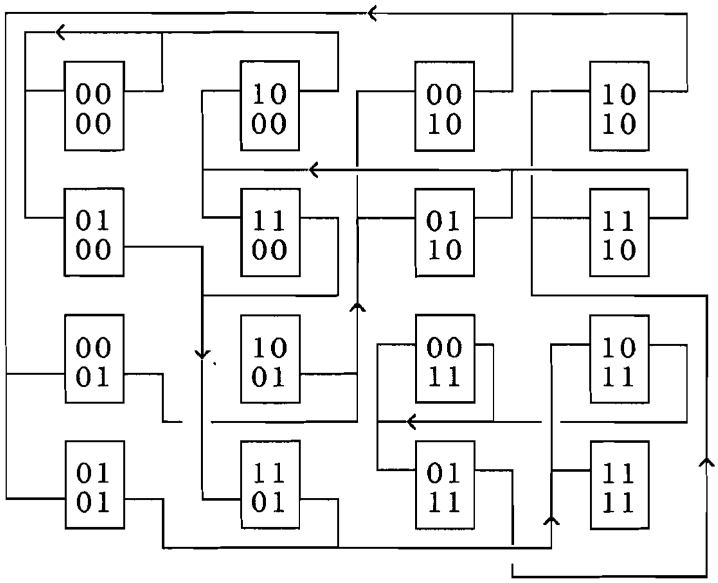


Fig. 3-3. Primitive FD of rule 22. Slender lines show that the initial node is ***NMB***. Similarly bold lines are for ***DMB*** and dotted ones are for ***PMB***. $B_{2,3}$, $B_{3,2}$ and $B_{3,3}$ are ***SMB***es and they are not in primitive FD.

Rule 90



Rule 150

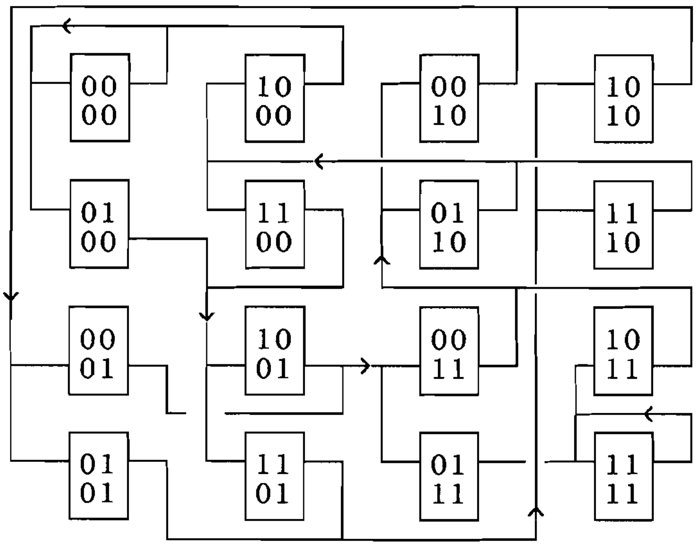
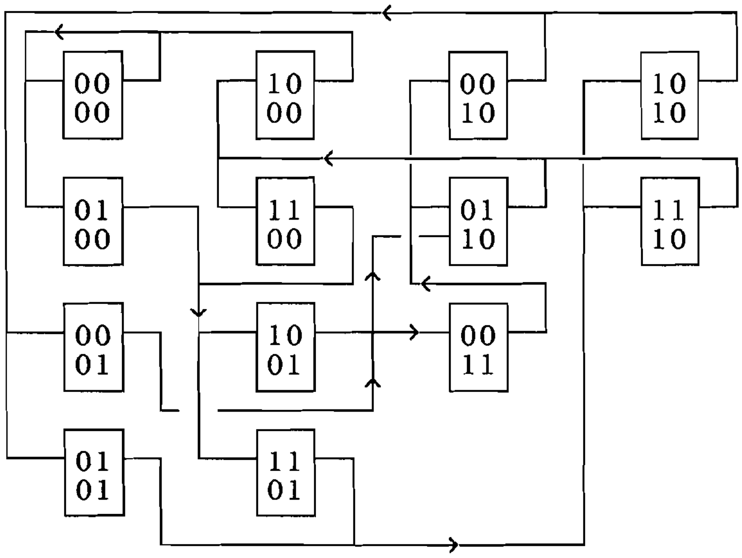


Fig. 3-4. Primitive FD of rule 90 and rule 150. There is no *SMB* and each Box is *NMB*. See text.

[1]



[2]

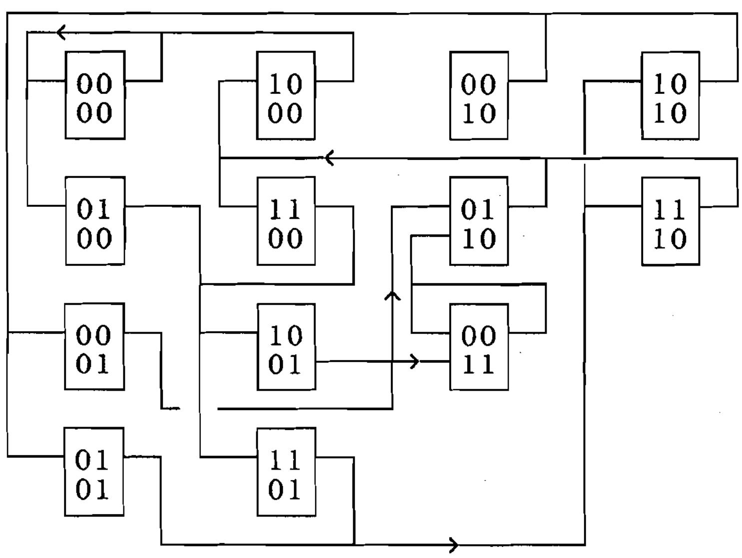


Fig. 3-5. Example of time-reverse rules for rule 22. The difference between the two is the successor of $B_{2,2}$(Box-vector =(0,0,1,1)).

Rule 54

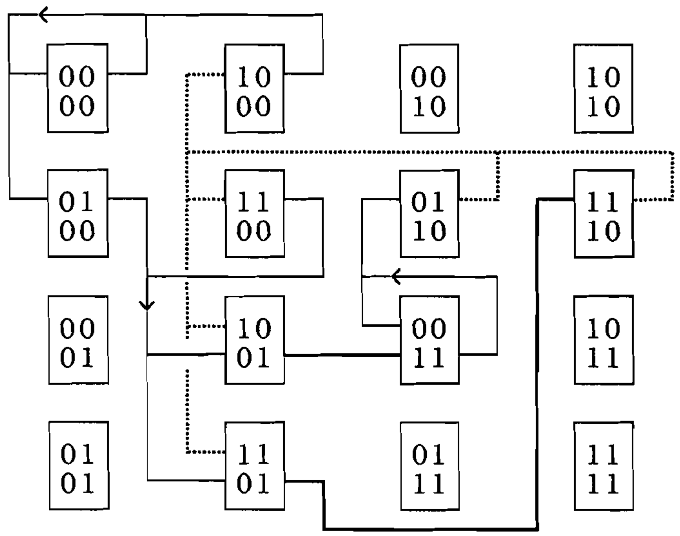


Fig. 3-6. Primitive FD of rule 54. Slender line is for *NMB*, bold one is for *DMB* and dotted one is for *HMB*. There is three *SMB*es in the 3rd column.

Chapter 4.
The Extraction of Biological Feature of Learning Process in Man-to-man Game

4-1. The learning process of a machine

This chapter focuses on the biological feature of learning to think about evolutionionary aspects. Like the theory of back-propagation[1], learning is generally so defined that the desired output is settled and the function to estimate is readied and any initial sequence is approached to the desired output. It is technically useful, for example, in recognition of handwriting letters. Our question is whether the learning system matches to the biological learning as proposed in computational nervous science[2].

Observation is generally articulatedd into receiveing an object $\boldsymbol{a} \in \boldsymbol{A}^n$ $(A \times A \times \cdots \times A)$ at the t-th time and computing an output $\boldsymbol{b} \in \boldsymbol{B}^n$ $(B \times B \times \cdots \times B)$ at the $t+\Delta t$-th time . Then, even if we interpret that learning process is resulted from observation, we regard a learning system as a black box which transforms $f: \boldsymbol{A}^n \to \boldsymbol{B}^n$ dependent on the condition. In this, we need the system to estimate the output. Rather, we had better say that the time is proceeding with the object if the estimated values are different between the two output. And we may say that the system has learned or 'evolved' if we are able to find the estimation map in that system.

We may be somehow able to find that the system to estimate the symbols to explain the variance of object. It is true we can explain learning or 'evolution' with the system, but we cannot necessarily define that the referent for the system exists. The aim of this work is to show that it is possible to describe the learning system of man-to-man game but not necessarily[3-5]. That will be indirectly shown.

4-2. *Renju*

Renju(*Go-bang*) is a finite zero-sum game by two players in terms of the game theory[6]. It follows by the rule such that;

1 : Each player alternately takes a move, or puts one '*ishi*', in his color, black or white, on the site on the board(two-dimensional lattice, 15×15 or 17×17).

2 : When *n* pieces of *ishi* in either player's color are arranged in a horizontal, perpendicular or diagonal line without open site on the board, we call the sequence '*n-ren*'. If '*5-ren*' is arranged, the game is over and the player wins(see Fig.4-1).

3 : When 3(resp. 4) pieces of *ishi* in a color are arranged in a line with one open site, we call the sequence '*tobi-3*'(resp. '*tobi-4*'). Player-1, who takes his move in the odd numbered step with black *ishi*, is prohibited to arrange two '*3-ren*' or '*tobi-3*' in one move. This prohibited move is called '*3-3*' and he will lose the game if he takes it. He is also prohibited to take '*6-ren*' or '*4-4*'(see Fig.4-2). Player-2 who takes in the even numbered step is not prohibited. The unbalance between the two players is not important in this work.

4 : The most probable strategy for Player-1 is to form '*4-3*'(see Fig.4-2). For Player-2 that is to form '*4-3*', '*3-3*', '*4-4*' or to force Player-1 to take a prohibited move.

5 : '*3-ren*', '*tobi-3*', '*4-ren*', and '*tobi-4*' are called '*oi-te*' or the offensive moves. '*Misete*' and '*Ryougati*' are also offensive moves. That is the preparatory move to form '*4-3*(or *4-4*)' in

the next turn, and this is the double purposeful '*Misete*' for two different '*4-3*(or *4-4*)' in the next turn. If one Player continues to take '*4-ren*' or '*tobi-4*' in his turn and at last he takes '*4-3*' or '*4-4*', the way to win is called '*4-oigati*'[7].

The most important feature is that players can take one move in their turns and that both players and external observers can observe all of the moves taken till the present time step. Chess or Othello-game has the same feature, not like the card game nor mahjong, in which each player has a different hand respectively[8]. So, both of them can in principle count up all of the elementary events to happen. But it is too enormous. In each time step the most effective move for the situation may be somehow decided, but in fact each player cannot help taking moves at a wild guess. This is why the progress of the game is surprising and interesting, though the probabilistic uncertainty is not induced from the rule. Why we did not adopt chess or Othello-game in our experiments will be told after.

Most of us have experienced *Renju* or Chess. For convenience, we will discuss in the stance of players in a while. The discrepancy of the wild guess between the two is exposed with the proceeding of the game. We may in our own experience that both players and observers cannot become aware of the discrepancy with respect to the meaning of a configuration and/or a move, till the move is taken in each time step. Also, we say that the player can *a posteriori* know the mistake of the wild guess in the past step. If the playmate allowed to turn back to the past step, the player would take another move referring to the mistake. It should be remarked here that he would do still at another wild guess because he knew the mistake about only one move and he did not necessarily know more effective move, far from the optimal move('optimal move' means the most effctive move which is decided by god or who knows all of the elementary events and is possible to estimate them.) Then, even if he takes an another move from the past move, it is not until the playmate takes the next move that he knows whether his own move is more effective or not.

In various situations of the game, players must 'consider' because they do not necessarily know the indication for the more effective move. Therefore, we always ready the progress of the state with uncertainty. The uncertain aspect can be replaced by the appearance of another move, if the same players replay the game from the same situation. The expression above was from the stance of players and we might say that a player 'consider' and take moves at a wild guess from our own experience. On the other hand, in the experiment we cannot help taking the stance of the external observers, because we do not participate in the game.

4-3. Method

We asked some pairs of persons to take the following procedure that

1:They play ***Renju*** once till the end. We call this game 'Sample Game(SG)'. It ends in the N-th step.

2:The same men replay the game K times from the i-th situation that is the same one in SG($K \geq 1$, $1 \leq i \leq (N-1)$).

See Fig.4-3. If an player is interpreted as an black box or the mechanism that receives inputs and shows an output, we can say that he receives the same input in SG and each replay. We call 'the i-th situation' to what to be formed with totally i pieces of '***ishi***' after the move is taken in the t-th step. In the (i+1)-th step, the Player in his turn takes a move refering the

i-th situation. We call the replay from the i-th situation of SG 'i-Replay' and we call i-Replay in the k-th times repeated 'k-time-i-Replay'. Of course in Replay they can take another move from in SG if they wish. As mentioned above, the known orbits cannot always be the indication for the more effective move so long as he still takes a move at a wild guess in Replay.

For each Replay, we will define four measures for the orbit of Replay. Partly because we experimenters do not know the optimal path for any situation and partly because there exists numberless local solutions, we do not measure the distance of the move from the optimal path nor local solutions. We attend to offensive moves. We suppose that Players wish to win. Offensive moves do not always approach them to win but they can induce Players more advantageous situation compared to other moves and Players cannot win without offensive moves. So offensive move is interpreted as meaningful. In Replay, the same move as in SG is interpreted as meaningless since the uncertain aspect from wild guess in SG is replaced by another move and we regard that he does not 'learn', if he takes the same move. In i-Replay, if the orbit reflects the uncertainity with the i-th situation, the earilier move is more meaningful. Then the move in the $(i+j)$-th step in i-Replay should be estimated in inverse proportion to j.

Definition 1. Advantage

$$A(i,k) = \sum_{j=1}^{J} \{ E(j)C(j)\delta(i,j)/m - E(j)C(j)\varepsilon(i,j)/m \} ,$$

where $E(j)$ = 1, when the move is offensive,

= 0, otherwise,

$C(j)$ = 1, when the move is different from the move in SG,

= 0, when the move is the same as in SG,

$\delta(i,j)=(i+j+1) \bmod 2$,

$\varepsilon(i,j)=(i+j) \bmod 2$,

$m \quad =(j-1)\%2+1$,

in the $(i+j)$-th step in the k-time-i-Replay, in which '%' is the operation for the quotient. J shows the length of the obrit in k-time-i-Replay estimated. Then $A(i,k)>0$ shows that Player-1 is more offensive or gains an advantage of Player-2 in the some steps from the beginning in the k-time-i-Replay, and $A(i,k)<0$ shows that Player-2 gains.

Definition 2. Fraction of information

$$F(i,k) = \sum_{j=1}^{J} E(j)C'(j)/j \ ,$$

where $C'(j) = 1$, when the move is different from the move in SG and from the move in p-time-i-Replay($1 \leq p<k$) ,

$C'(j) = 0$, when the move is the same in SG or in p-time-i-Replay($1 \leq p<k$) ,

in the $(i+j)$-th step in the k-time-i-Replay. E and J is the same as in Def. 1.

Definition 3. Fraction of information for players

$$F_1(i,k) = \sum_{j=1}^{J} \delta(i,j)E(j)C'(j)/j \ ,$$

$$F_2(i,k) = \sum_{j=1}^{J} \varepsilon(i,j)E(j)C'(j)/j \ ,$$

where $\delta(i,j)$, $\varepsilon(i,j)$ is the same as in Def. 1, and $E(j)$, $C'(j)$ is the same as in Def. 2. F_1(resp. F_2) shows how offensive Player-1(resp. Player-2) is and how different the move is in k-time-i-Replay.

Definition 4. The amount of information for players

$$I_x(i) = \sum_{k=1}^{K} F_x(i,k),$$

where x=1,2. K shows the number of times the Replay were repeated.

Tp shows the step in which I_x is peaked and T_0 shows the step in which I_x=0(x=1,2). $Z_x(i)$ shows the summation of I_x from the step i to the nearest T_0. $St(T_p)$ shows the number of offensive moves from the step T_p to the nearest T_0 in SG.

We asked our friends or high school students, totally more than 200 persons to follow the mentioned procedure. Each player was familiar with Renju almost as well as the playmate. Some did not play the game and some did not understand the procedure.

We can see the obvious correlation between the sequence of the move in SG and the advantage A in Replay at k=1. For example, the progress of SG of the example in Fig.4-3 can be represented as 'nnnnnnnn1nn21n1n12n21n1', where 1 shows that Player-1 took an offensive move in the step, 2 shows Player-2 did and n shows the move is not offensive, and N=23, at which Player-1 took '*4-3*'. In the 9-th step Player-1 took the initiative firstly and in the 13-th, and 21-th step he turned the tables, and in the 12-th and 18-th step Player-2 turned the tables. The sign of $A(i,1)$ is represented as '000----+-+-+++++--++00' through $i(1 \leq i \leq 22)$ with J=6. This is shown in Fig. 4-3-I with interrupted line. It should be remarked that $A(i,1)<0$ when i=5,9 and 17 and that $A(i,1)>0$ when i=8 and 14. In this example, we can say that if in the t-th step either player took the initiative firstly or turned the tables in SG, in 1-time-(t-*4*)-Replay the other player took the advantage. We can see the same correlation in Fig. 4-3-II~V.

We could obtain totally 90 examples for the first initiative or the turning of the tables in SG. We introduce the notation t for the steps of the taking of first initiative of the turn of the tables, and call 'τ-rollback' to k-time-($t-\tau$)-Replay if the t-th move in SG is represented as '+' and $A(t-\tau, k)\leq 0$ is satisfied(resp. it is '-' and $A(t-\tau, k)\geq 0$). As for 91.1% of the data, τ-rollback was successful when k=1 and τ=4[9]. The success explicitly shows that Players somehow 'learn' from the progress of SG. The reason why it is more successful in τ=4 will be explained following section.

One example of the change of the fraction of information F for k is shown in Fig.4-5. The obvious feature of the variation is as follows

1: For k, F does not seem to change remarkably, when i=1 or 2.

2: F is rapidly saturated to become zero for several steps i before N-1.

3: When i is further earlier, F seems to be once saturated, rise again, and be saturated again.

4: As SG proceeds(i.e. i increases), F tends to decrease with k more rapidly.

A remarkable feature is obvious in the third figure, in which K=20 and N=20. When i=14, F keeps to fluctuate, and peaks are emerged several times. Especially at k=19, F attains the maximum value in the 14-Replay. F is so defined as to be zero if the orbit of k-time-i-Replay is the same as the SG orbit or that of p-time-i-Replay($1\leq p<k$). Then the maxium value shows that the orbit of 19-time-14-Replay is different from them.

Fig. 4-6 shows an example of the variation of F_1 and F_2 through i for one SG with K=1. The upper figure in Fig.4-7 shows the correlation between $Zx(T_p)$ and $|T_p - T_o|$ and the lower one shows the correlation between $Zx(T_p)$ and $St(T_p)$ with both of Players(x=1,2).

4-4. Results and analysis

4-4-1. The success of τ-rollback

The success of τ-rollback(k=1 and τ=4) suggests that from a situation the players can know at least the one orbit whose length is more than 4, if it has been realized. Needless to say, they cannot always search all of the orbits whose length is 4. We will discuss about how the knowledge newly gained is 'effective' to the progress of Replay according to the character of ***Renju***.

Suppose that the player-1 takes the first initiative or turns the tables in the t-th step in SG.

1 : In 1-time-(t-1)-Replay, Player-1 takes the first move in the (t-1+1)-th step. He took the offensive move in the t-th step in SG on the above supposition, and he is expected to take the same offensive move or another offensive one in Replay. Therefore Player-2 must defend it and he may select the other way to defend from the one in SG. In most cases, Player-1 will take an offensive move in the (t-1+3)-th move and Player-2 has little chance to success 1-rollback. Player-2 may success only if Player-1 forget the orbit of SG or take the offensive move whose defensive move is offensive for Player-2.

2 : In 1-time-(t-2)-Replay, Player-2 takes the first move in the (t-2+1)-th step. He can take the move to interrupt directly the orbit realized in SG or take on the site where Player-1 put 'ishi'in the (t-2+2)-th step in SG. Player-2 can always interrupt

the orbit except for in the case that Player-1's move is offensive in the (t-2)-th step in SG. The character is induced from the rule of Renju which allow the players to put one of '*ishi*' on any open sites and this can be found in neither Chess nor Othello. But even if he interrupt, the offensive move of Player-1's in the (t-2+2)-th step would be inevitable. Because in most cases more than two ways to offend were ready with the Player-1's move(the move is called '*co-shu*' in terms of *Renju*-rule) in the (t-2)-th step in SG and Player-1 can take another offensive move in the (t-2+2)-th step in (t-2)-Replay. Rather, in most cases, since there were the plural ways to offend in the (t-2)-th step, Player-2 could not prevent all of them with one move and Player-1 could take the offensive move in the t-th step in SG. Of course we cannot say that Player-2 can never success 2-rollback, but that will hardly happen as well as 1-rollback.

3 : In 1-time-(t-3)-Replay, Player-1 takes the first move in the (t-3+1)-th move. He can take the same move in SG or make preparation for the offensive move in the (t-3+3)-th step. If he does, Player-2 has little chance for 3-rollback as well as the case of 2-rollback. Player-1 would takes another move from the one in SG which could cause the favorable situation, only when he estimates that it is more effective.

4 : In 1-time-(t-4)-Replay, Player-1 can firstly take an offensive move or a defensive one from Player-1's *co-shu* in the next step. In some cases there may exists other ways to make more advantageous situation for Player-2, we can search for all possible way in principle, but we cannot practically, because the search to take a move is terminated before all of the elementary events are analyzed, and we takes an *ad hoc* move. Therefore we find the observation or search under finite velocity of obser-

vation propagation in the progression of the game, *Renju*. But it is not so simple. Player-2 has to take the move in the (t-4+1)-th step to kill the effect of Player-1's move on the orbit in SG, and so he will take another move from the one in SG. Then the effect of the move in the (t-3)-th step in SG can be lost and it might make even more dangerous situation because the (t-4)-th situation is so advantageous for Player-1 that he could reach the turning point through the orbit in SG. Though it would not be easy, we can say that Player-2 has more chance to success τ-rollback($\tau \geq 4$) while he has little chance in the case of τ'-rollback(τ'=1,2 or 3).

5 : In 1-time-(t-5)-Replay, Player-1 takes the first move in the (t-5+1)-th step. He can take another move and then the situation is changed. It may be more effective than in SG and anyway Player-2 must 'consider' over again in the next step. So, 5-rollback is less successive than 4-rollback. And the larger τ, the larger the times of the turning tables. Then, memory for the system must be larger if the learning system, the player, has the momory store. More or less, we may say with experience that we are impossible to remember all of them. And our players in this experiment are not experts of such games. So we may say that τ-rollback($\tau \geq 5$) is less successive than 4-rollback.

As mentioned in the previous chapter, we can see 92% successful τ-rollback with k=1 and τ=4, and we are certain that τ-rollback(τ=4) could happen though it is difficult. If we take the standpoint of the external observer and suppose that we can describe the system for prediction of Player's definitely, what is introduced with the success of τ-rollback? (Here the system for prediction means what estimates all the candidates of the next move and decides the order of priority in each time step. In

Renju the number of the candidates is countable and/or finite.)

1 : If the system for prediction in Replay is the same in the (t-3)-th step and another element is selected out of the same set of the maximum priority as in SG, then it will be induced that in SG Players continued to choose the move which resulted in mistake and in Replay they keep up to select the successful moves from among the same set at least in the (t-3)-th step(t shows the turning step in SG). So this is hardly prospective.

2 : Suppose that the system for prediction in Replay is almost the same in the (t-3)-th step in SG. The difference is that it estimates the orbit of SG is the most 'dangerous' and gives the path the maximum order of priority if the t-th step was the turning point in SG. Now we must remember that 4-rollback is can be successful but it is not easy. this is because Player has to take a different move from SG and it is likely that the move's effect in SG, which he had thought, will be lost and then the playmate might be more offensive before rollback. If he had taken to prevent the playmate's offend beforehand in SG, in Replay he would be wide open and give the playmate a chance. To succeed rollback, Player must find the move not to lose the effect in SG or the more effective one. In the (t-4+1)-th step in (t-4)-Replay, if one player takes on the same site where the playmate took in the (t-2)-th step in SG, he can defend the move which often prepared more than two ways to offend for the playmate. But in most cases he only defends at most two or three ways though there may exists many other ways, and in most cases he loses the effect of the move in SG. So, such easy idea is not prospective. Anyway, τ-rollback(τ=4) is so difficult that it would not success if the realized orbit in SG was interpreted as data or mere 'knowledge'.

Then we can say that we external observers cannot ignore the proceeding of SG and the system for prediction must be somehow transformed between in SG and in Replay. This transformation implies the existence of the meta-system estimalting the prediction system or 'learning'.

4-4-2. The variation of the information through *k*

If we go on the assumption that the system for prediction is definitive, then it is induced that the learning process can be described definitely. If Player's system for prediction is transformed through some learning process with the increase of *k*, in *k*-time-*i*-Replay Player would take the more 'effective' move than in SG or in the *i'*-Replay(*i'*<*i*). A Player takes the move on the open site in the board, and the number of the candidate for the next move is always finite in *Renju*-rule. Player cannot continue to take the different move through *k*. So, in any learning process the orbit of *k*-time-*i*-Replay must be saturated with arbitrary *i*, if *k* is sufficiently large.

When $k \geq 2$, the rate of τ-rollback(τ=4) decreases as *k* increases. Because Players repeatedly take moves in the same situation and they know the most offensive orbit for themselves after SG and some trials of Replay. Of course both of them know the orbit at wild guess and sometimes they can know that they were mistaken. They can find the 'more' offensive one in *k*-time-*i*-Replay as *k* is increased, yet it may not be the most offensive one, since they always take moves at wild guess. If we take the standpoint of Players, such 'discover' is very natural and rather familiar, because we have experienced in many times that our guess is groundless. But if we take the standpoint of external observers and plan to describe the learning process consistently, the 'discover' makes external observers embarrassed

Fig. 4-5 shows some examples of *k*-times-*i*-Replay(the horizontal axis is for k). When *i*=1 or less than a few, *F* is hardly saturated for *k*. The behavior of F in *i*-Replay is almost the same an utterly different SG, though the number of elementary events is countless. Taking the standpoint of external observers, we can interpret that Player can select a move from many candidates for the next step. So it is a matter of course for external observers that *F* is rarely saturated. Otherwise, some effective orbits were ***a priori*** readied and Renju was neither interesting nor surprising.

When *i*=5 or a few more, *F* is gradually saturated for *k*. *i*=5 is the first step in which Player-1 put just the third '***ishi***' in his color. In one case he can take an offensive move with least pieces of '***ishi***' and in one case he can take 'co-shu' *i*=6 is as well for Player-2. Then, if Player does not take an offensive move in his turn, in many cases the playmate offends at the next step. If Player is offended, in the next step the playmate must take a defensive move which is selected among from a few candidates(in the case of '***3-ren***', the two sites neighbor to '***3-ren***' in the direction are defensive sites. See Fig. 4-2). So, if Player can take an offensive move in his turn, he tends to take it, or he would be offended in the next step.(It should be remarked that an offensive move is not always the most effective move for the situation.) In these steps he has at most two or a little more offensive ways. So, in the beginning of *i*-Replay Players would select the same move as in SG or in *i'*-Replay (*i'*<*i*). A few ways to defend for Player exist and after that a few ways to offend for the other may exist. Dependent on a situation, offensive ways may be exhausted or some ways to defend is also offending moves. In any case path is diverged into a few or more branches. So, it is not strange that F is gradually saturated with k.

When i is further more, the saturation is more rapid. Because the more effective move for offence and/or for defence more clearly appear.(Of course it is Player's interpretation and there may exist the more effective moves.) In these steps, the two defending sites for '*3-ren*' may be utterly different since the situation is already non-symmetric. They may be differently associated with the neighbor sites in the other direction from '*3-ren*'. The path is still diverged, but Player would estimate which path is more hopeful to maintain the initiative among from ways to offend in his tables. In other cases he would estimate which path is more effective for interrupting the playmate's strategy among from ways to defend. So, taking the standpoint of external observers, we can say that the cardinal number (of the most highly estimated moves extracted from Player's system for prediction in i-Replay) is smaller than in $\hat{i}$-Replay, where $\hat{i}$=5 or a few more. Then the saturation tends to happen and is more rapid.

When $i=N-1$ or a few earlier, F is much more rapidly saturated. In these steps the winner of SG knows the best strategy for his win and in i-Replay he has only to follow it however the playmate change the way to defend. Then the loser hardly takes the defensive move and the number of defensive moves is at most one or two. Unless the winner forget the strategy, the same orbits will be traced after a few times of i-Replay. Here the loser does not utterly have ways to success rollback or to tide over the unfavorable situation. But even though the way exists, at least the loser could not find the way to recover in SG and also in i-Replay he could be still unconscious of it.

Call 'the convicted situation' to the situation from which one Player can continue to offend and always win, however the other tries to defend. Unless all of the elementary events are searched, we cannot judge whether a situation is the convicted one or not. Rather, we can say that the progress of the game depends on the judgement of Players'. At least the $(N-1)$-th situation in SG is the convicted one. If SG ends with '***4-oigati***', the situation in the step is convicted. And that situation is not in which the winner of SG did not offend. If in SG both Players knows that the situation in the $\tilde{\imath}$-th step is the convicted situation, then $\tilde{\imath}$-Replay will be soon saturated with $\boldsymbol{k}$. As external observers, we sometimes find that a situation is the convicted one but it is often overlooked and Players make it invalid. For example, though one Player took '***Misete***' in his turn, not only the playmate but also the Player himself took moves on the different sites after that. Players take one of the most 'effective' moves in SG. It is not until the situation was the convicted one that the loser was conscious of it, otherwise he would not lose. If we take the loser's standpoint, we would 'consider' another move in i-Replay, where i is a little earlier than $N-1$. If he finds that the situation is the convicted one, F will be saturated with k as i-Replay when $i=N-1$ or a few earlier. But we, Player, cannot decide that a situation is convicted except for the $(N-1)$-th situation in SG or the situation in the step of '***4-oigati***'. In fact, though both of Players judged the situation was convicted, later they often find in Replay that the more effective move exists and it is not convicted. The winner of SG also knows it is not, though he might judge it.(Of course we, as external obsevers, cannot say it is not truly convicted. It is so judged in many cases only since the number of the paths is increased and Both Players become unable to follow the progress.) The mistake is caused that Players cannot help judging it at wild guess. And the mistake is never exposed till the move is taken.

When i is several times before N, sometimes F is once saturated and rise again repeated i-Replay in Fig. 4-5. Because Players may find that the situation is not convicted after taking moves, and then they will come to be faced with the 'discovered' situation. They will take moves still at wild guess. If they will take the most 'effective' move, and then F can be gradually or rapidly saturated with k. But the possibility for Players to discover the more effective path always remains low, so long as Players take moves at wild guess. For example, the loser in SG may find a new path which induce him to advantageous situation in k-time-i-Replay with k=1. After the Replay the winner of SG may find the move which make the new move invalid and after that the loser may discover the move which make the lost move valid again or the more effective move. Taking the standpoint of Players, we take it a matter of course that F can always rise however many times we repeated from the same situation.

If we take the standpoint of external observers and intend to describe the learning process *a priori*, the above-mentioned situation leads to difficulty in principle, because the rise of F or the discovery of a new path can happen in any i-Replay except for i=N-1 and in the step of '*4-oigati*'. It tends to happen when i is a few or further earlier than N-1, though the number of the elementary events is much smaller than in $\tilde{\tau}$-Replay with $\tilde{\tau}$=1 or a few more. As described above, if we define the learning process in the context of an external observer, the value of all of the candidates for the next move must be estimated and the value of the output in k-time-i-Replay must continue to be increased(or decreased) through k. The number of the candidates is always finite in ***Renju***, then the most effective orbit exists for any situation and F will be saturated with k. But the obtained data shows that the new path could be taken and F can rise in spite of the decrease of the elementary events through i. So, we are hardly able to decide how many times i-Replay should be repeated. This is independent of whether we, external observers, know the best strategy, or the optimal path for a situation or not in theory.

4-4-3. The correlation between $Zx(T_p)$, $|T_p - T_o|$, and $St(T_p)$

When $I_1(i)=0$, in i-Replay Player-1 cannot take another move from in SG and/or cannot take offensive moves. ***Renju*** is originally the game in which Player intend to force the playmate to stalemate. Player-1 is in the stalemated situation in i-Replay if $I_1(i)=0$. If we take the standpoint of Players and they take the progress of SG into consideration, the correlation between $Zx(T_p)$ and $|T_p - T_o|$ will show that clearly they are conscious of the stalemated situation and they 'struggle' or consider the more effective ways somehow or other till they are stalemated. And the correlation between $Zx(T_p)$ and $St(T_p)$ will show that the moves in i-Replay are somehow dependent on those of SG. The systems for prediction are also dependent somehow. As external observers, the correlations suggests that the information generated in i-Replay, which is always generated after SG, can be somehow utilzed in the SG itself. It implies that the decision change for a move *a* ***posteriori*** perpetually proceeds in SG, and that the system for prediction is perpetually transformed without the estimation function for that system.

The learning process could be said to be autonomous because it is remarkable with biological object. Fig.4-8 shows a characteristic feature with biological process. $Z_x(i)$, which is summation of I_x from the step i to the nearest T_0, rapidly rise, and gradually decrease till the step T_0. The similar tendency is the time variance of the amount of DNA in a cell[10] or the punctuated equilibration in evolutionary process.[11].

4-5. Discussion

One reason why we adopt *Renju* is that the probablistic progress is omitted. If it is included like card game or mahjong, then we cannot 'hold' the input or the initial condition through repeated games and the change of F may be explained merely with the change of input. The rule of chess game and Othello game also omit the probablistic progress but they are more complex. It should be remarked that Player can put '*ishi*' on any open site in the board and the slight change of the move will cause remakable change of the progress in *Renju*. On the other hand, in chess game he can move a chessman to some sites following the rule and the turning of the tables is not frequent. Then Player's 'learning' is more obvious in *Renju* with τ-rollback(τ=4). And the knowledge of formulas will influence the progress. In othello game the number of the candidates for the next move is always small and the formulas is known. Most people do not know the formulas of *Renju* and the number of the candidates for the next move is always many and Player often fail to notice the more effective move. It may be found in Replay and shown as τ-rollback or the change of *F*, or it may continue to be unknown. Then 'discover' is more possible in *Renju*.

One way to associate with an object is to understand that it is a black box which necessarily shows the fixed output for a certain input. If any input can be linked to the corresponded output, the description as the black box is completed and we could say the object is programmable or controllable[12]. If the output is changed, the 'history' can be taken into consideration and the learning (or evolution) system can be described. Here the system is also programmable or controllable.

The black box and/or the system may be described but it is not necessary that the black box and/or the system is the object itself and that the object will be transformed through the learning (or evolution) system hereafter. The change of *F* suggests that. The success of τ-rollback(τ=4) point out that Players certainly 'learn' somehow. The candidate for the next move is always finite in Renju and the learning system would be described as for the past transformation, but it is difficult to apply the system to the future transformation, even if Players' system for prediction can be described completely and external observers know the most effective moves for any situation. Then we may say Players 'consider'. The correlation between $Zx(T_p)$ and $|T_p - T_o|$ or $St(T_p)$ also tell us that Players 'learn' from the progress of SG. The variance of $Zx(i)$ seems alike to the feature of biological evolution.

We may say that the biological object is creative or evolutional since it can always betray our intention to apply the transformation system of past to the future. The feature of the learning in the man-to-man game suggests that Players 'consider' or they may be creative.

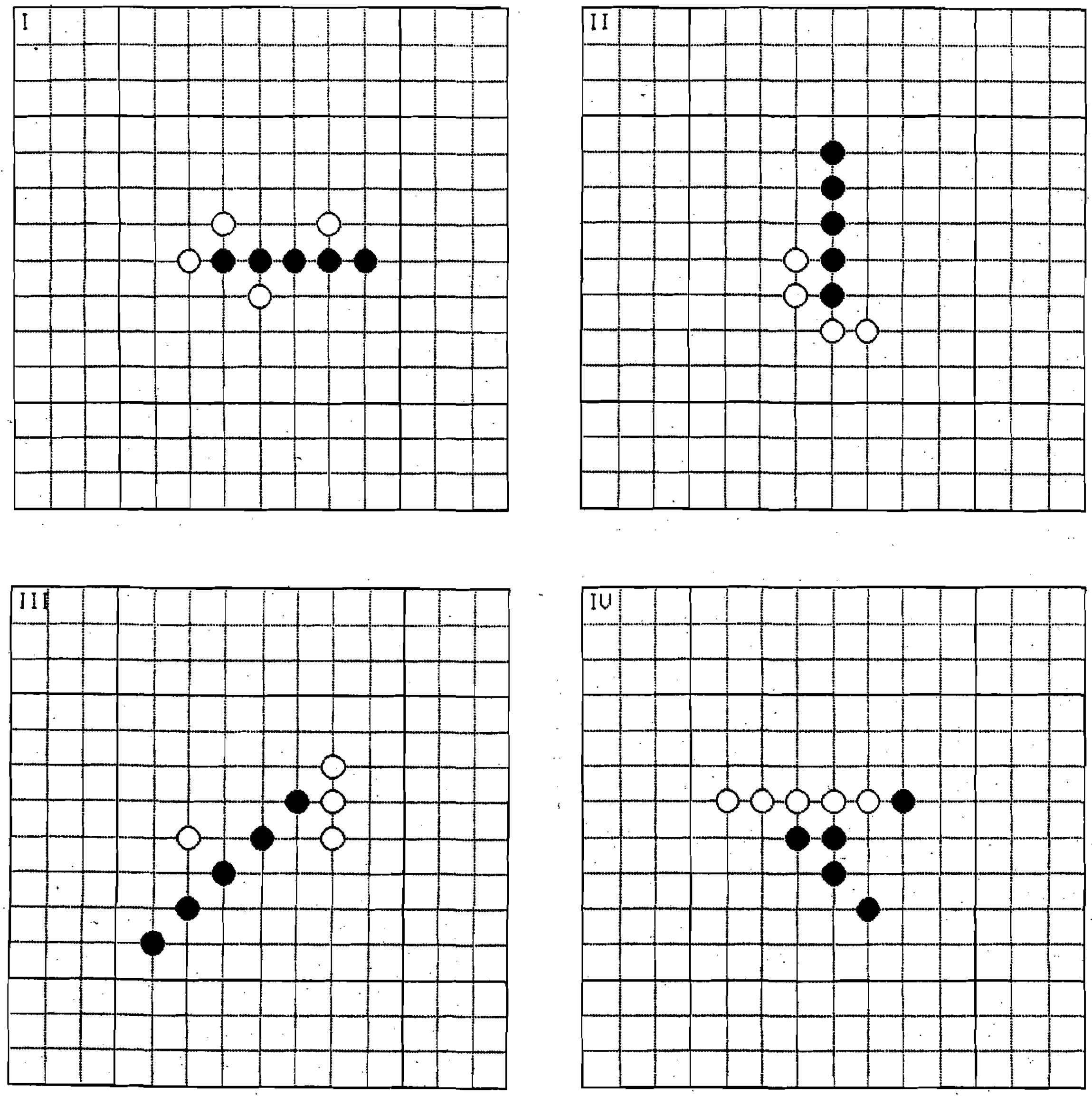


Fig.4-1 : I,II and III show '5-ren' for Player-1 respectively in the horizontal, perpendicular and diagonal direction. Player-1 wins in these game. In IV Player-2 arranges '*5-ren*' in the horizontal direction.

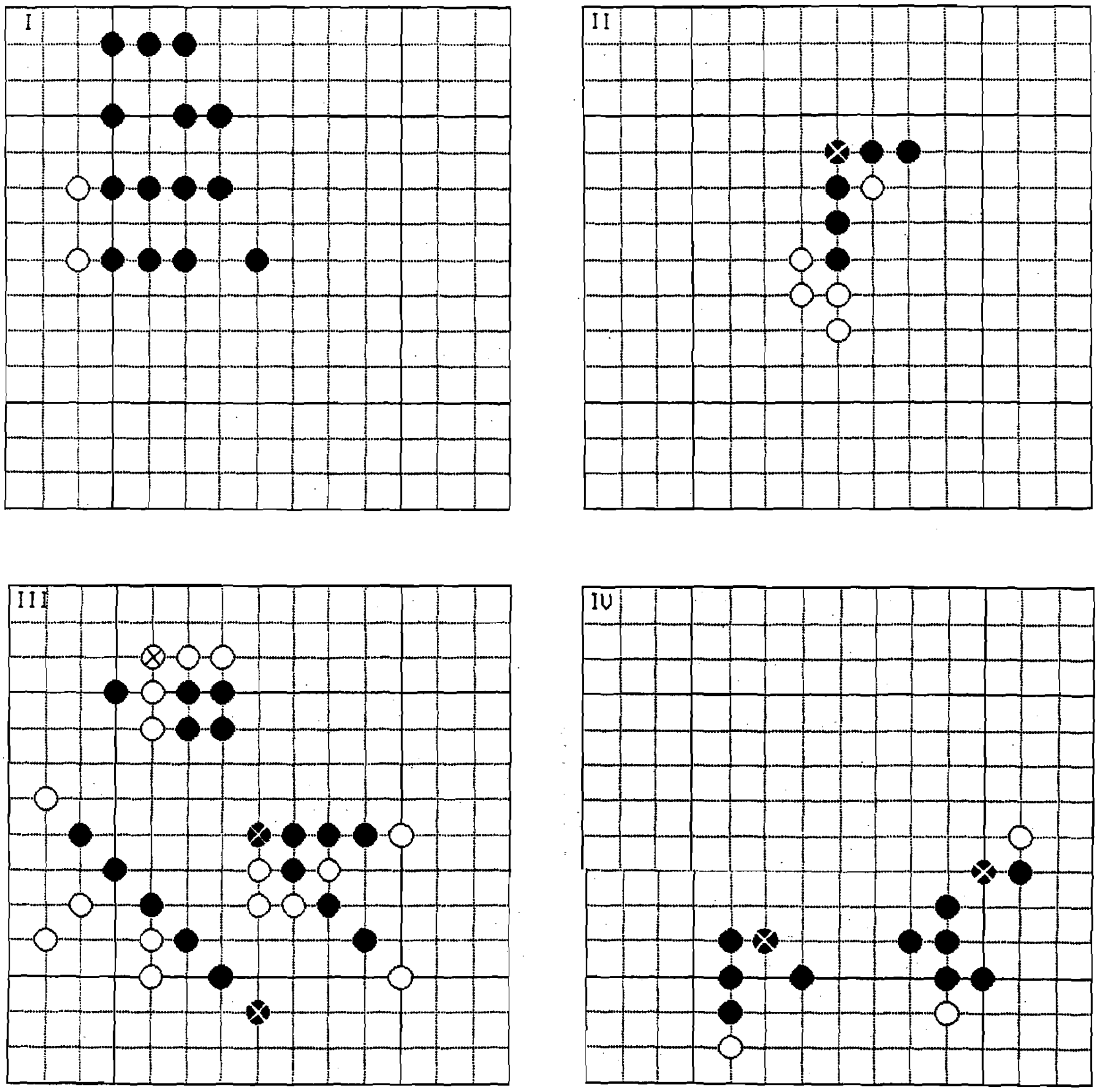


Fig.4-2 : I shows respectively '*3-ren*', '*tobi-3*', '*4-ren*' and '*tobi-4*' for Player-1. II shows '*4-3*' for Player-1. The marked '*ishi*' shows the current move and it forms '*4-ren*' and '*3-ren*' at once. In III, the above situation shows '*3-3*' for Player-2 is formed. The low situations show respectively '*6-ren*' and '*4-4*' for Player-1 are arranged. If Player-1 takes the marked move in the left figure in IV, it is called '*Misete*'. In the right figure, it is called '*Ryougati*'.

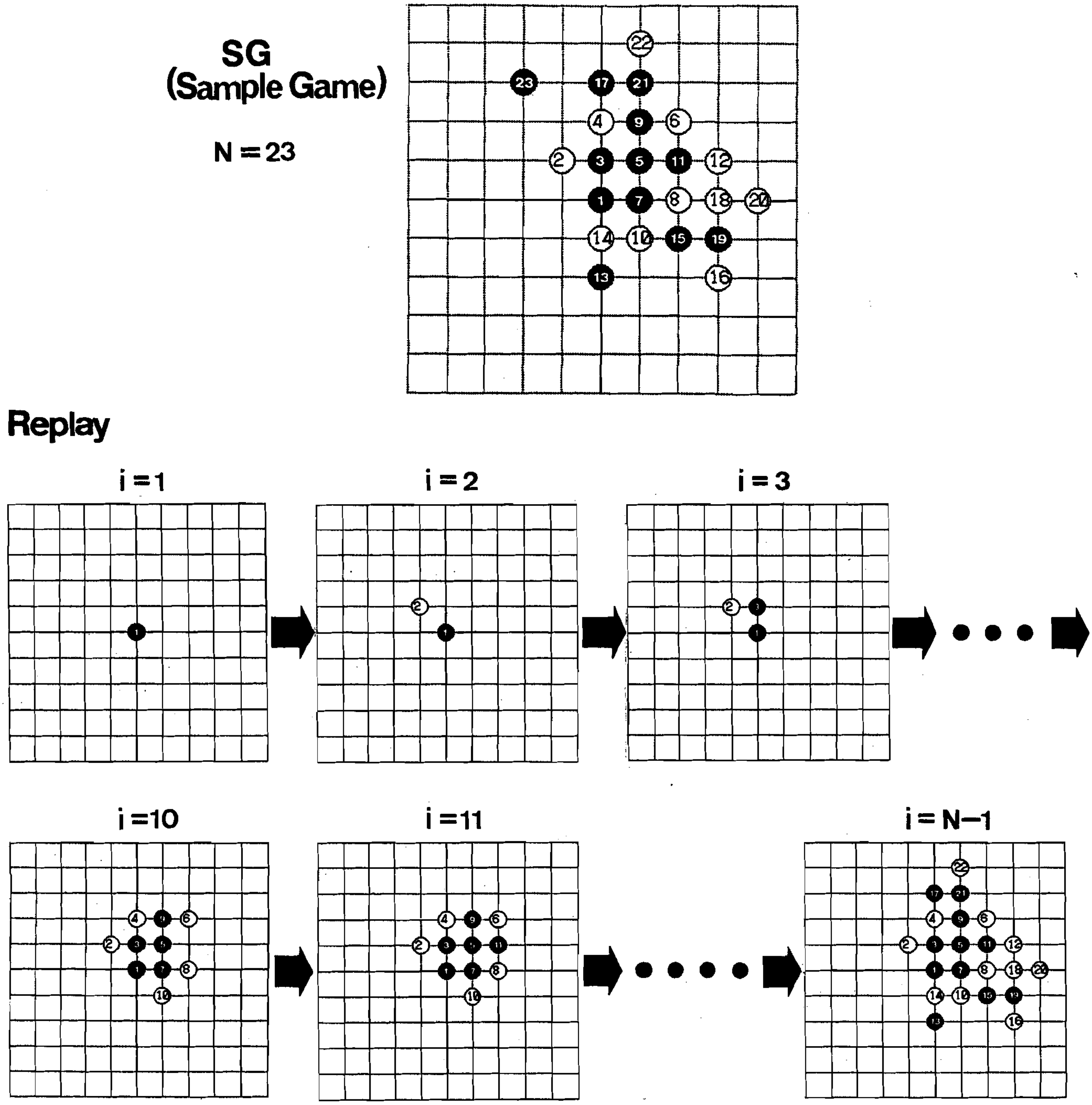


Fig.4-3 : One example of SG and the Replay(Player-1 took '*4-3*' in the 23-th step). The white letter against black circle shows Player-1's move. The number is the order of step. The black letter is Player-2's. They start i-Replay from the *i*-th situation of SG($1 \leq i \leq (N-1)$).

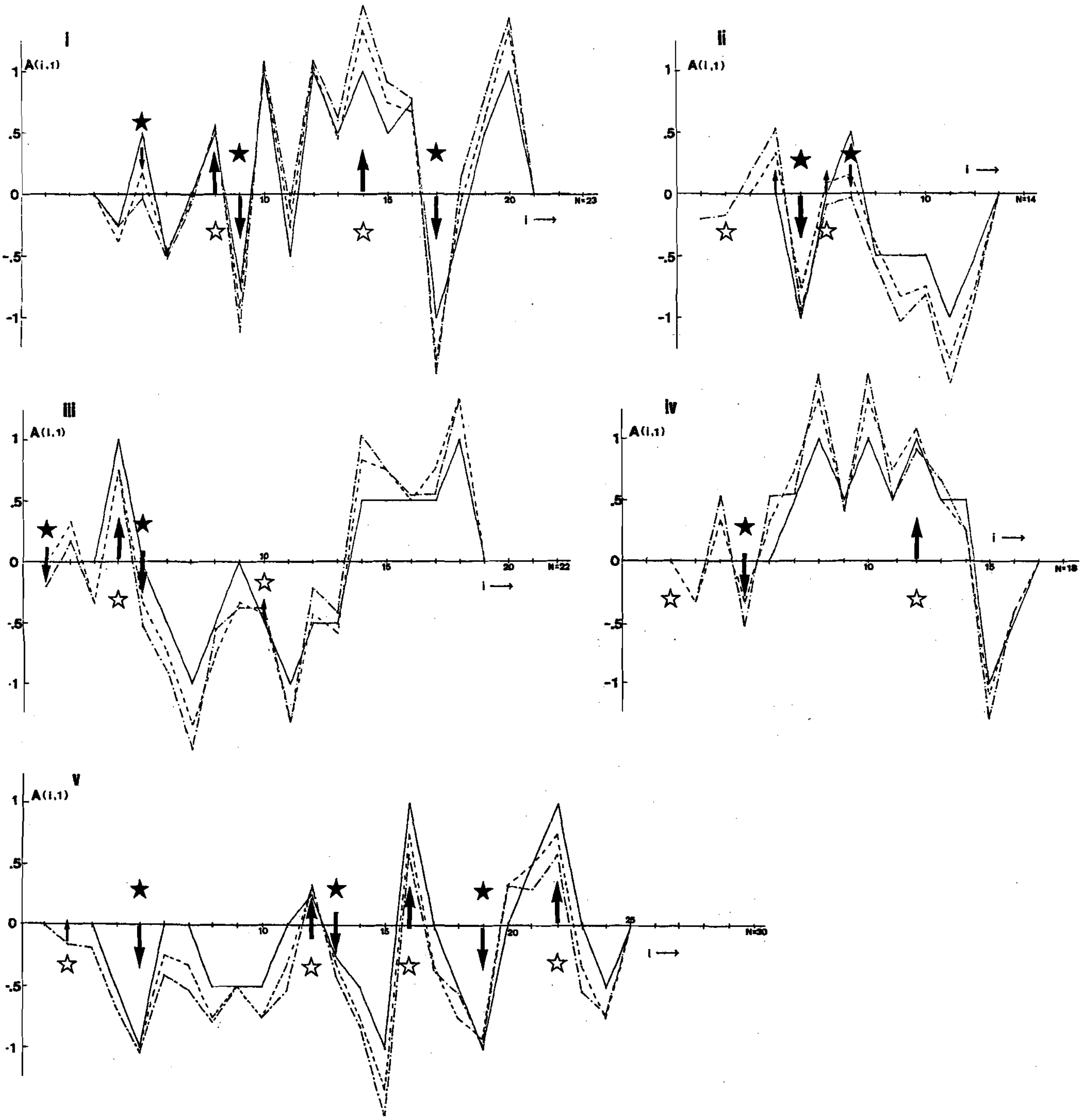


Fig.4-4 : Five examples for the change of A(i,1) through i for one SG and the Replays. The real line, broken line and interrupted line show respectively the values for J=2, J=4 and J=6. Suppose that Player-1 took the initiative or turned the table in the t_1-th step and Player-2 did in the t_2-th step. The black stars are plotted at $i=t_1-4$. The arrow shows that $A(t_1-4,1)<0$. The white stars are plotted at $i=t_2-4$ and $A(t_2-4,1)>0$.

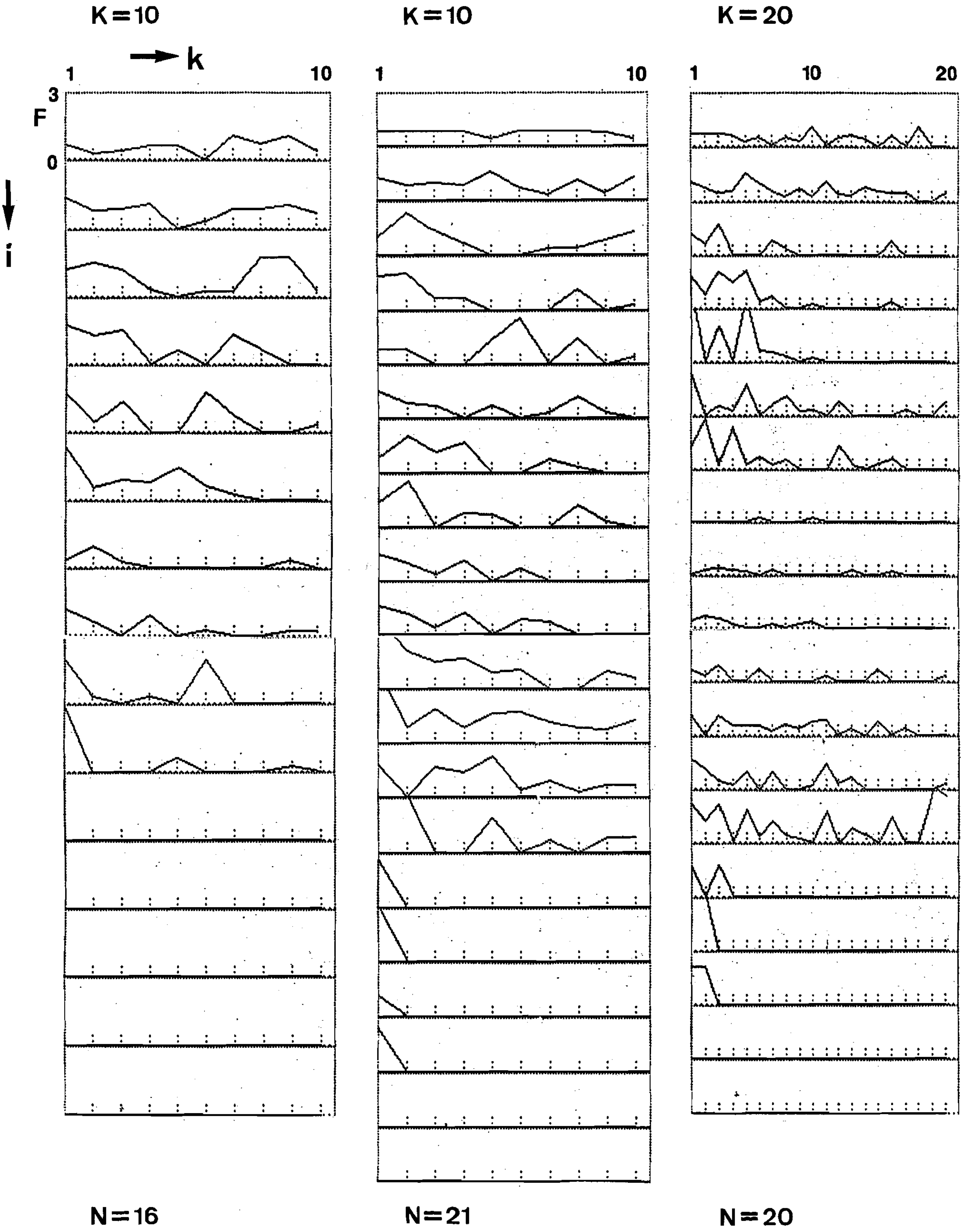


Fig.4-5 : Three examples for the change of $F(i,k)$ through k for each i-Replay.

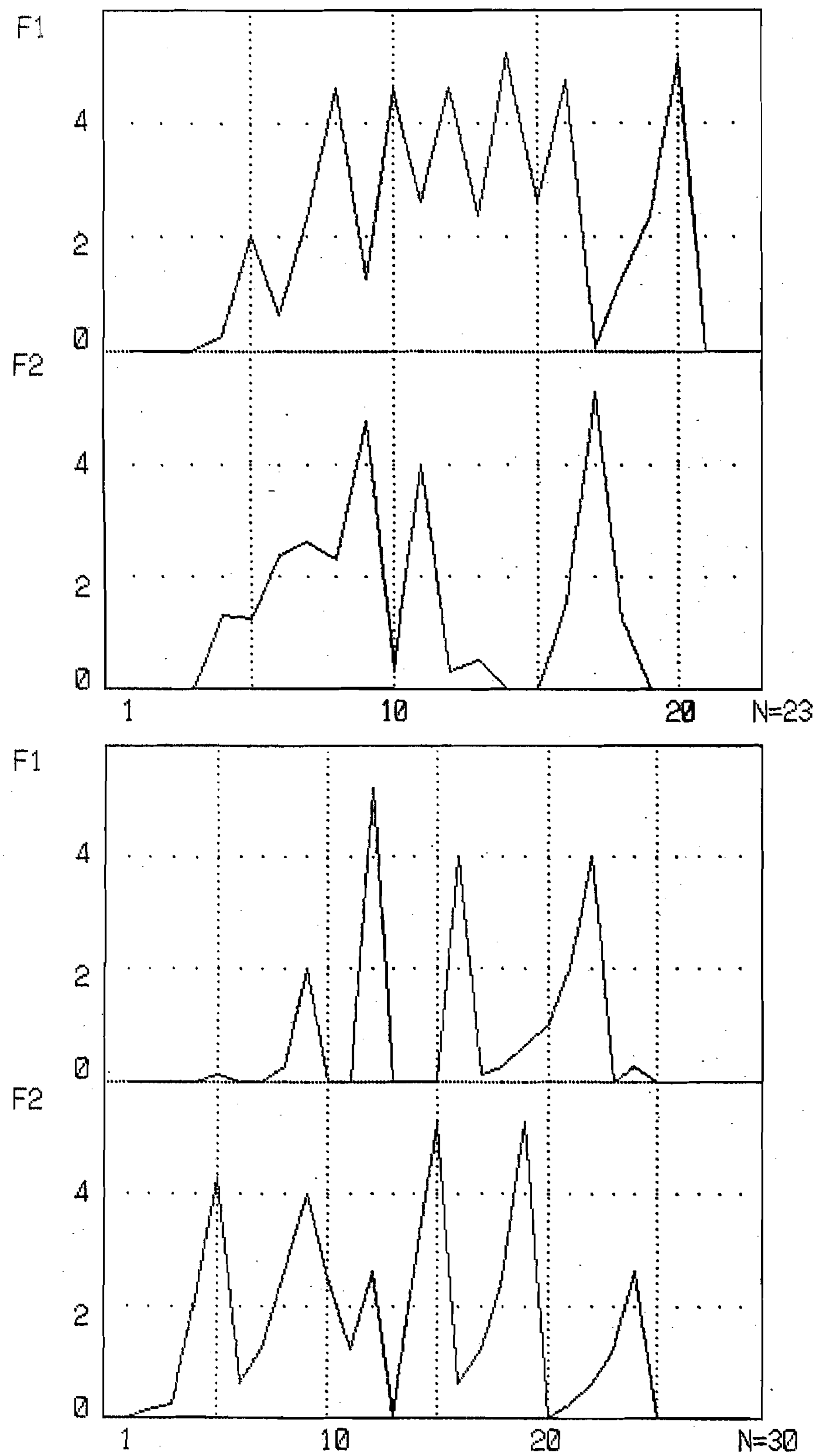
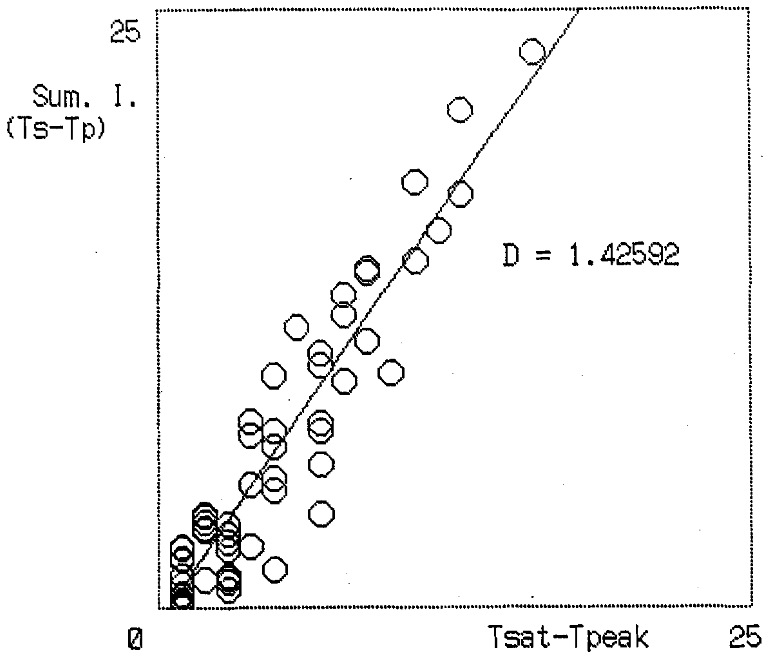


Fig.4-6 : Two examples of the change of $F_1(i,1)$ and $F_2(i,1)$ through i(The upper figure is in the same SG and Replay as in Fig.4-3 or Fig.4-4-i. The lower is in the same as in Fig.4-4-v). T_p shows the peaked step with F_1 or F_2. T_p for F_1 are 5,8,10,12,14,16 and 20. T_o for F_1 are 17 and 21.



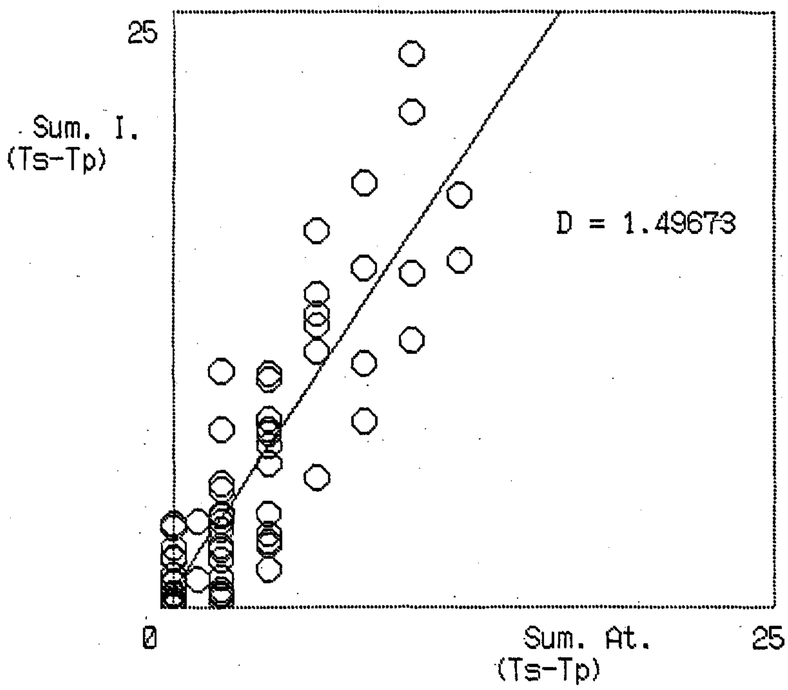


Fig.4-7 : The upper figure shows the correlation between Zx and $|T_p - T_o|$. The lower shows the correlation between Zx and $St(T_p)$. More than 120 point is plotted.

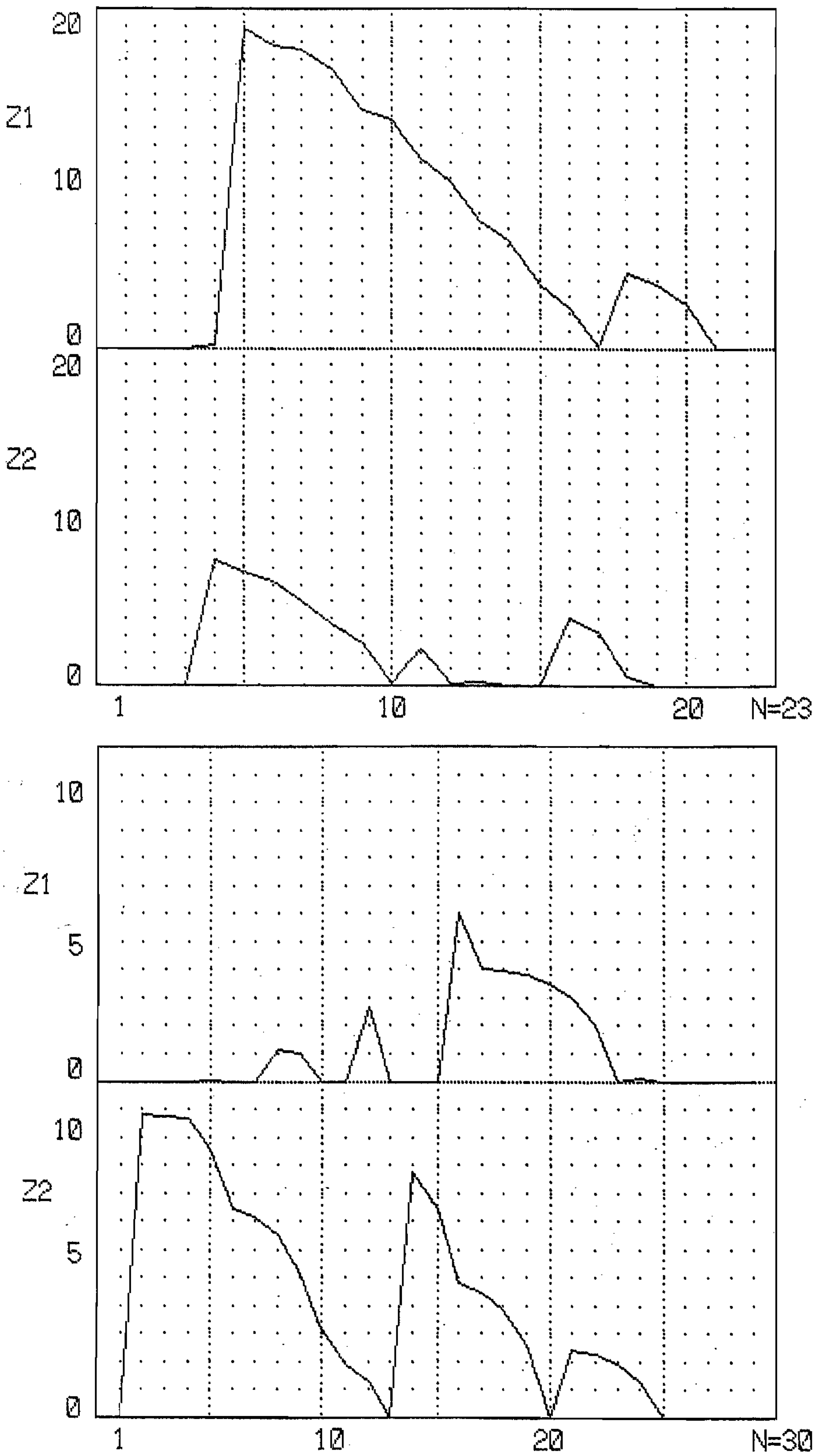


Fig.4-8 : Two examples of the change of $Z_1(i,1)$ and $Z_2(i,1)$ through i for the same two games as in Fig.4-6.

Chapter 5. Conclusion

The finite velocity of the observation propagation in biological aspect were investigated emprically. If we accept the finiteness, we cannot help confronting the measurement problem. In the description the problem is represnted as the paradox. So we have to accept the paradox to approach to biological aspect and to take part in the scientific language game.

Our intention is to accept the measurement problem and to incorpolate the paradox into description. In AEB, the time-reverse rule represented with FD contain the contradiction with the ordinal rule and the contradictive effect directly depends on the Child for *DMB*, *PMB* and *HMB*. As for Theorem 1, primitive FD is deduced and each box is classified. Then dependent on the selection way of Child for *DMB*, *PMB* and *HMB*, the number of time-reverse rule is identified. Espesially for Rule 54 and 250, the contradiction can be dually occured. These rule may be difficult to take in AEB.

Renju is utilized to make the paradox obvious though indirectly. We take the assumption that external observers can measure the players' system for prediction at a moment and the players wish to take as an effective move as possible to win. Since the number of the candidates for the next step is always finite in *Renju*, all of the open sites have to be estimated and the orbit of Replay have to become fixed, for any learning process or the transformation system. So, the fraction of information F should be saturated when the repeated times k is sufficiently large. The obtained data suggests that F would not. Then we show the paradoxical aspect and we may find that the players must 'consider' or they are 'subject'.

We continue to associate with the paradox both in formulation and in experiment. The way may approach to the understanding of life.

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