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博士論文

**Probing  $CP$  Violation via  
Top Quark Anomalous Interaction  
at High Energy Collider**

(高エネルギー加速器実験におけるトップクォークの異常な相互作用を  
通じて生じる  $CP$  の破れについて)

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March 2001

DOCTORAL THESIS

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# ABSTRACT

The standard model has successfully described all existing experimental data related to the strong and electroweak interactions. However, since there are still several unsolved problems in the standard model, people believe that the new physics beyond the standard model is necessary to solve them. To find the evidence for new physics is very important for further development of particle physics.

In this work, we try to find a signal of new physics by studying the  $CP$ -violation effect via top quark anomalous interaction which is induced by new physics. The following two processes been studied. First, expecting the forthcoming experiment with future high energy electron-positron linear collider, we calculated the  $CP$ -violating polarization asymmetry of  $t\bar{t}$ ,  $A_{CP}^{e\bar{e}} \equiv [\sigma(e\bar{e} \rightarrow t(-)\bar{t}(-)) - \sigma(e\bar{e} \rightarrow t(+)\bar{t}(+))] / \sigma(e\bar{e} \rightarrow t\bar{t})$ , by taking the most general  $t\bar{t}\gamma/Z$  couplings without neglecting any non-standard contribution. We found that a contribution of the term which was usually neglected in many other works done so far, increases with the square of the center-of-mass energy and eventually becomes non-negligible at very high energy. Next, expecting the forthcoming experiment with the upgraded Fermilab Tevatron, we also calculated  $CP$ -violating polarization asymmetry of  $t\bar{t}$ ,  $A_{CP}^{p\bar{p}} \equiv [\sigma(p\bar{p} \rightarrow t(-)\bar{t}(-) + X) - \sigma(p\bar{p} \rightarrow t(+)\bar{t}(+) + X)] / \sigma(p\bar{p} \rightarrow t\bar{t}X)$ , by assuming anomalous chromomagnetic ( $\kappa$ ) and chromoelectric ( $\tilde{\kappa}$ ) couplings of the gluon to the top quark. It was seen that the magnitude of  $A_{CP}^{p\bar{p}}$  depends only on  $\text{Im}(\tilde{\kappa})$  and  $\text{Im}(\kappa^*\tilde{\kappa})$ . In this calculation, we found that when the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  are larger than about  $|0.5|$ , one can possibly detect the  $CP$ -violation effect as a signal of new physics even if the magnitude of the  $\text{Im}(\tilde{\kappa})$  is zero.





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# CHAPTER 1

## INTRODUCTION

The unified gauge theory of weak and electromagnetic interactions with  $SU(2)_L \times U(1)_Y$  gauge symmetry and Higgs mechanism [1] with spontaneously symmetry breaking which was beautifully formulated by Glashow [2], Weinberg [3] and Salam [4] in 1960's, has been one of the most significant advances of the last thirty years in the development of particle physics. Furthermore, the underlying field theory of quarks and gluons with  $SU(3)_C$  color gauge symmetry called quantum chromodynamics (QCD), proposed by Fritzsche, Gell-Mann and Leutwyler in 1974 [5], has been very successful in explaining strong interaction phenomena. These two theories were combined into a unified gauge theory called *Standard Model* (SM) with  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge symmetry. Since this model is renormalizable, we can precisely predict the physical observables because we can, in principle, calculate the higher order correction of these physical values without ambiguity. The SM is excellently in good agreement with all experimental data [6], though the Higgs boson has not yet been discovered. However, despite its success in describing physics of leptons, quarks and their interactions, the SM is currently not accepted as the final theory of Nature because this model can neither predict the fundamental parameters such as masses and couplings, nor explain gravitational interactions. Recent atmospheric neutrino observation at Super-Kamiokande [7] shows the strong evidence of massive neutrino. This is the first signal of new physics beyond the SM because neutrinos are treated as massless particles in the SM. However, so far we have no signal of new physics at the collider experiment. Therefore, it is very interesting to search for the signal of new physics at the high energy colliders.

In 1995, the CDF [8] and D0 [9] collaborations discovered the top quark at the Fermilab Tevatron with center-of-mass energy  $\sqrt{s} = 1.8$  TeV. It was discovered as a very heavy particle with the mass,  $m_t \simeq 180$  GeV. Since the mass of the top quark is much larger than the masses of the other quarks and leptons (even of the electroweak gauge bosons), studies on the role of this particle in Nature is expected to lead us to the physics beyond the SM. For example, the discovery of the top quark might be helpful for finding the evidence of the  $CP$ -violation, as predicted in the Cabbibo–Kabayashi–Maskawa (CKM) mechanism [10, 11]. This is because the sixth quark, i.e. the top quark, must exist in order

to cause the  $CP$ -violation. However, it is well-known that the  $CP$ -violation in the top quark decay and also in the top quark pair production is estimated to be extremely small in the SM [12]. Therefore, it is interesting to search for other possibilities of  $CP$ -violation in the top quark sector which originates from new physics. Furthermore, it is remarkable that due to its huge mass, the top quark decays before it hadronizes [13]. This means that we can easily get information about the produced top quark from the decay distribution of the secondary leptons and hadrons [14]. This property is very advantageous for searching the  $CP$ -violation in top quark sector as a signal of new physics.

Because of the above reasons, top quark physics is one of the most interesting subjects in the forthcoming collider experiments. The Tevatron at Fermilab which is a proton-antiproton collider with center-of-mass energy  $\sqrt{s} = 2.0$  TeV, and Large Hadron Collider (LHC) at CERN which is a proton-proton collider with center-of-mass energy  $\sqrt{s} = 14.0$  TeV, will not only search for new particles such as Higgs boson, supersymmetric particles, etc, and their new phenomena, but also study more precisely the properties of the top quark itself. In addition, there are also some plans for the future high energy electron-positron linear collider whose center-of-mass energy  $\sqrt{s} = 0.5 \sim 5.0$  TeV. In the future electron-positron linear collider experiments, it is expected that we can get the clean signals of new physics beyond the SM because noises originating from initial hadron fragmentation will be absent.

The main purpose of this thesis is to discuss the  $CP$ -violation in the top quark pair production process in order to search for a signal of new physics. In this work, we focus on the following two kinds of the top quark pair production processes:

- (a) The electron-positron annihilation process [15];

$$e^+ + e^- \rightarrow t + \bar{t}.$$

This process can be studied in the future high energy electron-positron linear collider. In analyzing this process, we assume the most general couplings of the top quark to gauge boson ( $\gamma/Z$ ). Here, we estimate the contribution from the cross term of non-standard couplings which is usually neglected in many calculations done so far and discuss their effect on the  $CP$ -violation.

- (b) The proton-antiproton scattering process [16];

$$p + \bar{p} \rightarrow t + \bar{t} + X.$$

This process will be studied soon at the upgraded Fermilab Tevatron. For this process, we assume the anomalous chromomagnetic and chromoelectric couplings of the gluon to the top quark and discuss the possibility of the discovery of new physics.



This thesis is organized as follows. In Chapter 2, the standard model and the  $CP$  transformation properties are summarized. After briefly reviewing the SM which is composed of electroweak theory and QCD, we discuss the properties of the parity transformation ( $P$ ), charge transformation ( $C$ ) and  $CP$  transformation. In Chapter 3, we briefly describe the future plans of the high energy electron-positron linear collider and the Fermilab Tevatron. In Chapter 4, we discuss the effect of the possible  $CP$ -violation in the top quark pair production via the anomalous interaction which could be induced by new physics. In the first section of this Chapter, we introduce the  $CP$ -violating observable for the top quark pair production. In the following sections, we calculate the effect of the  $CP$ -violation for  $e^+e^- \rightarrow t\bar{t}$  and  $p\bar{p} \rightarrow t\bar{t}X$  processes and discuss the results. Chapter 5 is devoted to the summary of our study and the future outlook.



# CHAPTER 2

## THE STANDARD MODEL AND *CP* TRANSFORMATION

The purpose of this chapter is to review the standard model for the elementary particle physics and *CP* transformation which will be widely used in the following chapters. The standard model is consist of the electroweak theory and the quantum chromodynamics. Since *CP* transformation is an important concept in order to understand *CP*-violation, hence Parity (*P*), Charge (*C*) and *CP* transformations are also explained.

### 2.1 The Standard Model

The present theory of the electroweak and strong interaction known as “*Standard Model*”, is composed of two theories; One is the electroweak theory which describes the electromagnetic and weak interaction for quarks and leptons. The other is Quantum Chromodynamics (QCD) which describes the strong interaction.

#### 2.1.1 The Electroweak theory

The electroweak theory which is the so-called “Glashow–Weinberg–Salam theory” is based on the non-Abelian gauge group  $SU(2)_L \times U(1)_Y$  [2, 3, 4]. In addition, in this model, the masses of gauge bosons and fermions are generated by Higgs mechanism [1] induced by a massive scalar particle via spontaneously symmetry breaking.

#### The gauge and Higgs sector

The generators of the corresponding Lie algebra to  $SU(2)_L \times U(1)_Y$  gauge group are the three weak-isospin operators  $I_1, I_2, I_3$  and the hypercharge  $Y$ . Since each of these operators is associated with a vector field (gauge field), the electroweak theory includes the isotriplet  $W_\mu^a$  ( $a = 1, 2, 3$ ) and the isosinglet  $B_\mu$ . The corresponding field strengths are defined as

$$W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc}W_\mu^b W_\nu^c, \quad (2.1)$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (2.2)$$

where  $\epsilon^{abc}$  is the totally antisymmetric Levi-Civita tensor. The Lagrangian for the pure gauge fields in electroweak theory can be written as

$$\mathcal{L}_G^{EW} = -\frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \quad (2.3)$$

In addition to these vector particles, this model also includes an  $SU(2)_L$  doublet complex scalar fields  $\Phi$  with hypercharge  $Y = 1$  in order to generate the masses of gauge bosons and fermions. This is known as *Higgs* field and the corresponding Lagrangian is written as

$$\mathcal{L}_H = (D_\mu\Phi)^\dagger (D_\mu\Phi) - V(\Phi^\dagger\Phi), \quad (2.4)$$

$$V(\Phi^\dagger\Phi) = \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 - \mu^2\Phi^\dagger\Phi \quad (2.5)$$

with the covariant derivative

$$D_\mu\Phi = \left( \partial_\mu + igW_\mu^a I_a + \frac{1}{2}ig'B_\mu Y \right) \Phi, \quad (2.6)$$

where  $\lambda$ ,  $\mu$ ,  $g$  and  $g'$  are arbitrary real parameters. The whole Lagrangian  $\mathcal{L}_G^{EW} + \mathcal{L}_H$  is invariant under  $SU(2)_L \times U(1)_Y$  gauge transformations<sup>#1</sup>:

$$B_\mu \rightarrow B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu\alpha, \quad (2.7)$$

$$W_\mu \equiv W_\mu^a I_a \rightarrow W'_\mu = e^{i\beta_a I_a} \left( W_\mu - \frac{i}{g}\partial_\mu \right) e^{-i\beta_a I_a}, \quad (2.8)$$

$$\Phi \rightarrow \Phi' = e^{i\alpha Y/2 + i\beta_a I_a} \Phi, \quad (2.9)$$

where parameters  $\alpha$  and  $\beta_a$  depend on the space-time coordinates:  $\alpha \equiv \alpha(x)$ ,  $\beta_a \equiv \beta_a(x)$ .

Now, in order to understand the mass generation for gauge bosons in the SM, we focus on the Eqs. (2.4–2.5). Because of the negative sign of the quadratic term in Eq. (2.5), the symmetric solution  $\langle 0|\Phi|0\rangle=0$  becomes unstable(Fig. 2.1), and when  $\lambda > 0$ , the favored solution has a non-zero vacuum expectation value which may be written in the form:

$$\langle 0|\Phi|0\rangle = \langle 0|\Phi^\dagger|0\rangle = v \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad : \quad v^2 \equiv \frac{\mu^2}{2\lambda}. \quad (2.10)$$

This corresponds to *Spontaneous breakdown of the electroweak gauge symmetry:  $SU(2) \times U(1)_Y \rightarrow U(1)_{em}$* . In order to expand  $\Phi$  into around the vacuum, it is convenient to

<sup>#1</sup>In general, the gauge field  $V_\mu$  obeys the transformation law:  $V_\mu \rightarrow V'_\mu = UV_\mu U^{-1} + (i/g)(\partial_\mu U)U^{-1}$ , where  $U$  depending on the parameters,  $\alpha^A(x)$ , is defined as a  $D$  dimensional continuous group of transformations:  $\phi' = U(\alpha^A(x))\phi = \exp[ig\alpha_A(x)T^A]\phi$  with the generators  $T^A$  ( $A=1, 2, 3, \dots, D$ ) and parameter,  $g$ . This transformation of  $V_\mu$  keeps Lagrangian,  $\mathcal{L}[\phi, \partial_\mu\phi]$ , invariant under gauge transformation with the covariant derivative;  $D_\mu = \partial_\mu + igT^A V_{A\mu}$ .

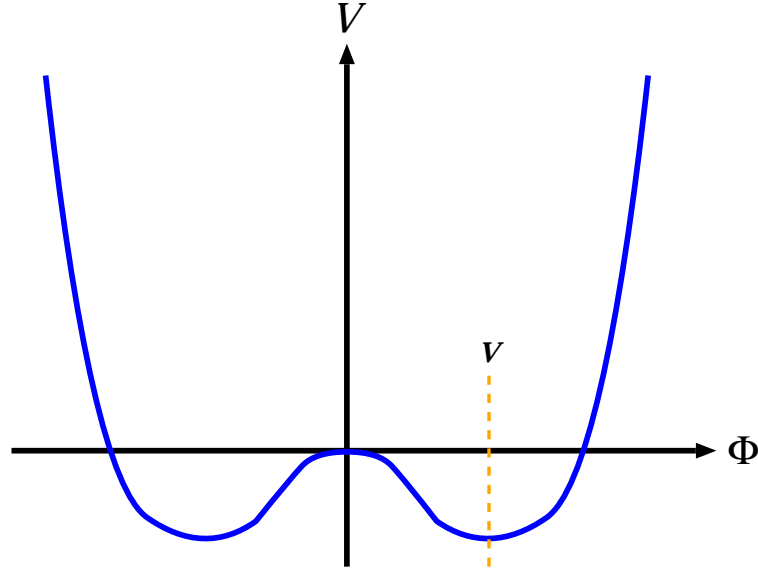


FIGURE 2.1: Higgs potential in SM.

rewrite  $\Phi$  in the following way;

$$\Phi = \frac{1}{\sqrt{2}} e^{-i\theta_a I_a} \begin{pmatrix} 0 \\ v + H \end{pmatrix}, \quad (2.11)$$

where  $\theta_a$  and  $H$  are real scalar fields which depend on the space-time coordinate:  $\theta_a \equiv \theta_a(x)$ ,  $H \equiv H(x)$ . To understand how it happens, let us substitute Eq. (2.11) into the Lagrangian  $\mathcal{L}_H$  of Eq. (2.4). It is straightforward to rewrite  $\mathcal{L}_H$  in terms of the new fields  $\theta_a$  and  $H$ . we can easily see that all the  $\theta_a$  can be reabsorbed into  $W_\mu^a$  by a gauge transformation (2.7–2.9) with  $\beta_a = \theta_a$  and  $\alpha = 0$ :

$$B'_\mu = B_\mu, \quad (2.12)$$

$$W'_\mu = e^{i\theta_a I_a} \left( W_\mu - \frac{i}{g} \partial_\mu \right) e^{-i\theta_a I_a}, \quad (2.13)$$

$$\Phi' = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}. \quad (2.14)$$

Since both  $\mathcal{L}_G^{EW}$  and  $\mathcal{L}_H$  are invariant under transformations (2.7–2.9), their new expressions in terms of  $B'_\mu$ ,  $W'^a_\mu$  and  $\Phi'$  are identical to the old ones in terms of  $B_\mu$ ,  $W_\mu^a$  and  $\Phi$ . Therefore, the only visible effect of (2.12–2.14) is that the fields  $\theta_a$  disappear from the Lagrangian. However, to get this result, we have to fix three of the four gauge parameters in a proper way, and as a consequence, the symmetry of the model decreases. The only gauge transformation which is still allowed is the one preserving the form (2.14) of  $\Phi'$ ,

*i.e.* the  $U(1)_{em}$  subgroup of transformations (2.7-2.9) with  $\beta_1 = \beta_2 = 0$  and  $\beta_3 = -\alpha$ . In this way, the original  $SU(2)_L \times U(1)_Y$  symmetry has been broken down to  $U(1)_{em}$ .

For simplicity, let us rewrite the new fields  $B'_\mu$ ,  $W'^a_\mu$  and  $\Phi'$  as  $B_\mu$ ,  $W^a_\mu$  and  $\Phi$  in the following discussions. Now, by using the new expression of  $\Phi$ , the scalar potential  $V$  which is given in Eq. (2.5) can be written as

$$V(H) = \frac{\lambda v^2}{4} H^2 + \frac{\lambda v}{4} H^3 + \frac{\lambda}{16} H^4, \quad (2.15)$$

while for the kinematical term in Eq.(2.4), we have

$$\begin{aligned} (D_\mu \Phi)^\dagger (D_\mu \Phi) &= \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{g^2}{8} (W_\mu^1 W_1^\mu + W_\mu^2 W_2^\mu) (v + H)^2 \\ &\quad + \frac{1}{8} (g W_\mu^3 - g' B_\mu)^2 (v + H)^2. \end{aligned} \quad (2.16)$$

As a consequence of above expansions, we see that the real field  $H$  (called the *physical Higgs field*) denotes a particle with mass  $m_H = v\sqrt{\lambda/2}$ . Also, Eq. (2.16) provides the mass terms for the  $W^a_\mu$  and  $B_\mu$  fields, which therefore are no longer massless fields. The mass term of the Lagrangian is

$$\mathcal{L}_m = \frac{g^2 v^2}{8} (W_\mu^1 W_1^\mu + W_\mu^2 W_2^\mu) + \frac{v^2}{8} (W_\mu^3 \quad B_\mu) \begin{pmatrix} gg & -g'g \\ -g'g & g'g' \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}. \quad (2.17)$$

To diagonalize the mass matrix in the second term of Eq. (2.17), let us introduce an angle  $\theta_W$  which is known as the *weak mixing angle* and define the following fields:

$$W_\mu^\pm \equiv \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}, \quad (2.18)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \quad (2.19)$$

Substituting Eq. (2.19) into Eq. (2.17), we see that  $(A_\mu, Z_\mu)$  are mass eigenstates if  $\theta_W$  is related to  $g$  and  $g'$  by the expression:

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}. \quad (2.20)$$

It is easy to see that the field  $A_\mu$  is massless. Therefore, we can identify  $A_\mu$  with the electromagnetic field. The masses of the  $W^\pm$  and  $Z$  bosons are

$$\left. \begin{aligned} m_W &= \frac{1}{2} g v \\ m_Z &= \frac{1}{2} f v \end{aligned} \right\} \Rightarrow m_W = m_Z \cos \theta_W, \quad (2.21)$$

I	II	III	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q$
$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$	<b>1</b>	<b>2</b>	-1	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_R$	$\mu_R$	$\tau_R$	<b>1</b>	<b>1</b>	-2	-1
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	<b>3</b>	<b>2</b>	1/3	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
$u_R$	$c_R$	$t_R$	<b>3</b>	<b>1</b>	4/3	2/3
$d_R$	$s_R$	$b_R$	<b>3</b>	<b>1</b>	-2/3	-1/3

**TABLE 2.1:** The particle contents of the fermionic sector of the Standard Model. If the mixing among quarks is neglected, the three generations are identical to one another (apart from particle masses) and independent. *Leptons* are  $SU(3)_c$  singlets, and *quarks* are  $SU(3)_c$  triplets. Note that no right-handed neutral lepton is present in the minimal model.

where  $f \equiv \sqrt{g^2 + g'^2}$ .

Interactions among  $W^\pm$ ,  $Z$ ,  $A$  and  $H$  are described by the cubic and quartic terms in the Lagrangian  $\mathcal{L}_G^{EW}$  and  $\mathcal{L}_H$ . In particular, we can see that  $Z$  and  $H$  do not couple to  $A$  (i.e. they are *neutral*), while  $W^\pm$  is *charged* and its coupling constant to the photon field is  $g \sin \theta_W$ .

### The fermion sector

In the Nature, fermions can be classified into the leptons and quarks. This classification is due to the strong interaction. Thus, though leptons participate to the electromagnetic and weak interactions, they do not participate in strong (color) interactions. On the other hand, quarks participate in electroweak interaction as well as strong interaction. Such a difference distinguishes the leptons and the quarks. Furthermore, both of them can be also divided into three generations. The general structure of the fermionic sector of the SM is shown in Table 2.1.

In the following discussion, we will neglect mixing among quarks, since in this thesis, Cabbibo–Kobayashi–Maskawa mechanism of  $CP$ -violation will not be treated <sup>#2</sup>. With this simplifying assumption, all the three families are independent from one another, and the Lagrangian describing interactions of leptons and quarks with  $W^\pm$ ,  $Z$ ,  $A$ ,  $H$  bosons is naturally split into three identical parts. Therefore, to describe fermions in a simple way, we will introduce a generation index  $f = 1, 2, 3$  and denote each lepton and quark in the left  $SU(2)_L$  doublet by  $l_L^f$ ,  $q_L^f$  and each down-lepton, up-quark and down-quark in

<sup>#2</sup>In general, Cabbibo–Kobayashi–Maskawa (CKM) mechanism is very important concept of the  $CP$ -violation. However, since the  $CP$ -violation originated from the CKM mechanism is extremely small in top quark sector which is focused in this thesis, we will neglect mixing among quarks in this thesis.

the right  $SU(2)_L$  singlet by  $E_R^f$ ,  $U_R^f$ ,  $D_R^f$ , respectively. With this notation, the fermionic part of the SM Lagrangian is:

$$\mathcal{L}_l = i \sum_f \left[ l_L^{f\dagger} \bar{\sigma}^\mu D_\mu l_L^f + E_R^{f\dagger} \sigma^\mu D_\mu E_R^f \right] - \sum_f \lambda_E^f \left[ l_L^{f\dagger} \Phi E_R^f + \text{h.c.} \right], \quad (2.22)$$

$$\begin{aligned} \mathcal{L}_q = i \sum_f \left[ q_L^{f\dagger} \bar{\sigma}^\mu D_\mu q_L^f + U_R^{f\dagger} \sigma^\mu D_\mu U_R^f + D_R^{f\dagger} \sigma^\mu D_\mu D_R^f \right] \\ - \sum_f \lambda_U^f \left[ q_L^{f\dagger} \tilde{\Phi} U_R^f + \text{h.c.} \right] - \sum_f \lambda_D^f \left[ q_L^{f\dagger} \Phi D_R^f + \text{h.c.} \right], \end{aligned} \quad (2.23)$$

where  $\sigma^i$  are the Pauli matrices:

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2.24)$$

$$\bar{\sigma}^0 = \sigma^0, \quad \bar{\sigma}^i = -\sigma^i, \quad (2.25)$$

and the covariant derivatives are:

$$D_\mu X_L^f = \left( \partial_\mu + ig W_\mu^a I_a + ig' \frac{Y_X}{2} B_\mu \right) X_L^f \quad X = l, q; \quad (2.26)$$

$$D_\mu X_R^f = \left( \partial_\mu + ig' \frac{Y_X}{2} B_\mu \right) X_R^f \quad X = E, U, D. \quad (2.27)$$

The field  $\tilde{\Phi}$  is used to give mass to the up-quarks, and is defined as

$$\tilde{\Phi} = i\sigma^2 \Phi^*. \quad (2.28)$$

After spontaneous symmetry breaking,  $\Phi$  acquires a nonzero vacuum expectation value and it is convenient to rewrite it as in Eq. (2.11); again, the unphysical fields  $\theta_a$  can be gauged away and we have

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}, \quad \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H \\ 0 \end{pmatrix}. \quad (2.29)$$

Replacing Eq. (2.29) into  $\mathcal{L}_l$  and  $\mathcal{L}_q$ , we obtain both fermion masses and fermion-Higgs interaction terms:

$$\mathcal{L}_H^{ff} = -\frac{v+H}{\sqrt{2}} \sum_f \left[ \lambda_E^f \left( E_L^{f\dagger} E_R^f + \text{h.c.} \right) + \lambda_U^f \left( U_L^{f\dagger} U_R^f + \text{h.c.} \right) + \lambda_D^f \left( D_L^{f\dagger} D_R^f + \text{h.c.} \right) \right]. \quad (2.30)$$

According to this expression, we see that all the lepton and quark masses are proportional to the vacuum expectation value  $v$  of the Higgs field, just as for the gauge bosons  $W$  and  $Z$ :

$$m_X^f = \frac{1}{\sqrt{2}} \lambda_X^f v \quad \Rightarrow \quad \lambda_X^f = \sqrt{2} \frac{m_X^f}{v}, \quad X = E, U, D. \quad (2.31)$$



Comparing Eq. (2.31) with (2.30) it is clear that the coupling of the physical Higgs with fermions is proportional to the fermion mass.

To derive an expression for the interaction between fermions and  $A$ ,  $W^\pm$ ,  $Z$  gauge bosons, we must insert Eqs. (2.18–2.20) and (2.26–2.27) into (2.22) and (2.23). Straight-forward calculations lead us to the interaction Lagrangian which includes 4-components:

$$\mathcal{L}_{W/Z}^{f\bar{f}} = -g_s A^\mu J_\mu^{\text{em}} - g [W^{-\mu} J_\mu^+ + \text{h.c.}] - f Z^\mu J_\mu^0, \quad (2.32)$$

where  $J_\mu^{\text{em}}$  is the *electromagnetic* current and  $J_\mu^+$ ,  $J_\mu^0$  are the *charged* and *neutral* weak currents:

$$J_\mu^{\text{em}} = \sum_f \left[ -\bar{E}^f \gamma_\mu E^f + \frac{2}{3} \bar{U}^f \gamma_\mu U^f - \frac{1}{3} \bar{D}^f \gamma_\mu D^f \right], \quad (2.33)$$

$$J_\mu^+ = \frac{1}{2\sqrt{2}} \sum_f [\bar{E}^f \gamma_\mu (1 - \gamma_5) N^f + \bar{D}^f \gamma_\mu (1 - \gamma_5) U^f], \quad (2.34)$$

$$J_\mu^0 = \frac{1}{2} \sum_f \sum_X \bar{X}^f \gamma_\mu (g_V^X - g_A^X \gamma_5) X \quad \text{for } X = N, E, U, D, \quad (2.35)$$

and the *vector* and *axial* neutral current coupling constant  $g_V$  and  $g_A$  are

$$g_V = I_3 - 2Q \sin^2 \theta_W, \quad (2.36)$$

$$g_A = I_3, \quad (2.37)$$

where  $Q$  denote a charge operator defined as  $Q = I_3 + \frac{Y}{2}$ .

In the low-energy limit, the electroweak sector of the SM must produce as an effective theory of the Fermi model for weak interactions. It is easy to see that this requirement is satisfied only if the following relation between the Fermi coupling constant  $G_F$  and the vacuum expectation value of the Higgs field  $v$  holds:

$$G_F = \frac{1}{\sqrt{2}v^2} \quad \Rightarrow \quad v = \frac{1}{\sqrt{\sqrt{2}G_F}}, \quad (2.38)$$

and from the numerical value  $G_F \approx 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$ , we get  $v \approx 246 \text{ GeV}$ .

### 2.1.2 The Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the non-Abelian gauge theory for strong interaction based on  $SU(3)_C$  symmetry. Here,  $C$  refers to colors and 3 means three color states of the quarks which are assumed to be in the fundamental representation of the group of dimension three. In this theory, the number of gauge boson, so-called *gluon*, associated

with this  $SU(3)_C$  symmetry group, is eight in number and corresponds to the number of  $SU(3)$  generators. Therefore, the quark and gluons are denoted by  $q_i (i = 1, 2, 3)$ ,  $G_A, (A = 1, 2, \dots, 8)$ .

The building of the QCD Lagrangian is done by following the same steps as in the electroweak theory reviewed in the previous subsection. The QCD Lagrangian is written as

$$\mathcal{L}^{QCD} = -\frac{1}{4}G_{\mu\nu}^A G_{\mu\nu}^A + \sum_q \bar{q}(i\not{D} - m_q)q \quad (2.39)$$

with the covariant derivatives:

$$D_\mu q = (\partial_\mu - ig_s T^A G_{\mu A}) q, \quad (2.40)$$

where  $g_s$  and  $T^A$  denote the strong coupling constant and  $SU(3)$  generators. In addition,  $G_{\mu\nu}^A$  is the gluon field strength which is defined as

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f^{ABC} G_{\mu B} G_{\nu C} \quad (2.41)$$

where  $f^{ABC}$  means structure constants corresponding to  $SU(3)$  gauge group.

It is straightforward to see that the QCD Lagrangian is invariant under  $SU(3)_C$  gauge transformations;

$$q \rightarrow q' = e^{i\theta_A T^A} q \quad (2.42)$$

$$D_\mu q \rightarrow D'_\mu q = e^{i\theta_A T^A} D_\mu q \quad (2.43)$$

$$G_{\mu A} T^A \rightarrow G'_\mu = e^{i\theta_A T^A} \left( G_\mu - \frac{i}{g} \partial_\mu \right) e^{-i\theta_A T^A} \quad (2.44)$$

where  $\theta_A (A = 1, 2, \dots, 8)$  are parameters of the transformation depending on  $x$ .

Similarly to the electroweak case, the interaction Lagrangian  $\mathcal{L}_G^{ff}$  for quarks and gluons are derived from substituting Eq. (2.40) into the right side of the second term of Eq. (2.39);

$$\mathcal{L}_G^{ff} = g_s T^A q \gamma^\mu q G_{\mu A} \quad (2.45)$$

In addition, it is remarkable that the gluon kinetic term which is the first term in the right side of Eq. (2.39), contains three gluon and four gluon self-interactions.

## 2.2 Parity Transformation, Charge Conjugation and $CP$ Transformation

Symmetries play very crucial role in particle physics as is known from the previous section. Though in that section, we focused on continuous symmetries,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ ,

but in order to describe the physics of elementary particles, discrete symmetries are also important to understand the general and fundamental relevance of physics in Nature. Among many discrete symmetries, parity transformation ( $P$ ), charge conjugation ( $C$ ) and  $CP$  transformation which is combined transformation of  $C$  and  $P$ , are related even to physics of the excess of baryons over anti-baryons in our universe [17]. Here, we briefly review those symmetry.

### 2.2.1 Parity transformation

The essential feature of the parity transformation, also called the space inversion, is that under this transformation, the space coordinates are reversed;  $x^\mu = (t, \vec{x}) \equiv x \rightarrow x_\mu = (t, -\vec{x}) \equiv \tilde{x}$ . Though parity operation does not affect the spins, but the direction of momenta are all reversed in the momentum space. Furthermore, in the field theory, the fields transform as the following<sup>#3</sup>;

$$\begin{array}{lll}
 \text{Scalar field} & \phi(x) & \rightarrow \phi(\tilde{x}) \\
 \text{Pseudoscalar field} & \mathcal{P}(x) & \rightarrow -\mathcal{P}(\tilde{x}) \\
 \text{Dirac spinor} & \psi(x) & \rightarrow \gamma^0 \psi(\tilde{x}) \\
 & \bar{\psi}(x) & \rightarrow \bar{\psi}(\tilde{x})\gamma^0 \\
 \text{Vector field} & \mathcal{V}_\mu(x) & \rightarrow \mathcal{V}^\mu(\tilde{x}) \\
 \text{Axial vector field} & \mathcal{A}_\mu(x) & \rightarrow -\mathcal{A}^\mu(\tilde{x}),
 \end{array} \tag{2.46}$$

Note that  $\mu = 0, 1, 2, 3$  and we use the Feynman metric, *i.e.*  $X^k = -X_k$  where  $k = 1, 2, 3$  and  $X_0 = X^0$ , for any four-vectors  $X^\mu$ . In addition, current densities which are composed of spinors corresponding to quarks and leptons also transform as

$$\begin{array}{lll}
 \text{Scalar} & \bar{\psi}_1(x)\psi_2(x) & \rightarrow \bar{\psi}_1(\tilde{x})\psi_2(\tilde{x}) \\
 \text{Pseudoscalar} & \bar{\psi}_1(x)\gamma_5\psi_2(x) & \rightarrow -\bar{\psi}_1(\tilde{x})\gamma_5\psi_2(\tilde{x}) \\
 \text{Vector} & \bar{\psi}_1(x)\gamma_\mu\psi_2(x) & \rightarrow \bar{\psi}_1(\tilde{x})\gamma^\mu\psi_2(\tilde{x}) \\
 \text{Axialvector} & \bar{\psi}_1(x)\gamma_\mu\gamma_5\psi_2(x) & \rightarrow -\bar{\psi}_1(\tilde{x})\gamma^\mu\gamma_5\psi_2(\tilde{x}) \\
 \text{Tensor} & \bar{\psi}_1(x)\sigma_{\mu\nu}\psi_2(x) & \rightarrow \bar{\psi}_1(\tilde{x})\sigma^{\mu\nu}\psi_2(\tilde{x}),
 \end{array} \tag{2.47}$$

where  $\psi_1$  and  $\psi_2$  are two Dirac fields, which are not necessarily identical. The proof of the above relations follows very simply by using the transformation formulae for the Dirac spinors in the relation of the field transformations.

### 2.2.2 Charge conjugation

The charge conjugation is the operation which change a particle into its anti-particle. In order to understand the operation of charge conjugation, we introduce the symbolic

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<sup>#3</sup>For simplicity, we will omit the the phase factors,  $\eta$ , though in general we must take account of such factors.

form  $\phi(x) = \sum[(\dots)a + (\dots)b^\dagger]$  where  $a(a^\dagger)$  and  $b(b^\dagger)$  are the annihilation (creation) operators for the *particle* and its *anti-particle*, respectively and  $(\dots)$  is an abbreviation for the plane waves as well as indices, such as the momenta, spins, color, etc. Under the charge conjugation, the role of the operators  $a$  and  $b$  is interchanged and flips the signs of internal charges, such as the electric charge, baryon number, etc. Therefore, under the charge conjugation, the free fields are transformed as follows;

$$\begin{array}{llll}
\text{Scalar field} & \phi(x) & \rightarrow & \phi^\dagger(x) \\
\text{Pseudoscalar field} & \mathcal{P}(x) & \rightarrow & \mathcal{P}^\dagger(x) \\
\text{Dirac spinor} & \psi(x) & \rightarrow & C\bar{\psi}^T(x) \\
& \bar{\psi}(x) & \rightarrow & -\psi^T(x)C^{-1} \\
\text{Vector field} & \mathcal{V}_\mu(x) & \rightarrow & -\mathcal{V}_\mu^\dagger(x) \\
\text{Axial vector field} & \mathcal{A}_\mu(x) & \rightarrow & \mathcal{A}_\mu^\dagger(x),
\end{array} \tag{2.48}$$

where  $C$  is a  $4 \times 4$  unitary matrix which satisfies the conditions  $C^{-1}\gamma_\mu C = -\gamma_\mu^T$  and  $C = -C^\dagger = C^{-1} = -C^T = C^{*\#4}$ . Therefore, we usually take  $C$  as  $i\gamma^2\gamma^0$ . Furthermore, we show densities which are composed of the spinor and gamma matrices, as follows;

$$\begin{array}{llll}
\text{Scalar} & \bar{\psi}_1(x)\psi_2(x) & \rightarrow & \bar{\psi}_2(x)\psi_1(x) \\
\text{Pseudoscalar} & \bar{\psi}_1(x)\gamma_5\psi_2(x) & \rightarrow & \bar{\psi}_2(x)\gamma_5\psi_1(x) \\
\text{Vector} & \bar{\psi}_1(x)\gamma_\mu\psi_2(x) & \rightarrow & -\bar{\psi}_2(x)\gamma_\mu\psi_1(x) \\
\text{Axialvector} & \bar{\psi}_1(x)\gamma_\mu\gamma_5\psi_2(x) & \rightarrow & \bar{\psi}_2(x)\gamma_\mu\gamma_5\psi_1(x) \\
\text{Tensor} & \bar{\psi}_1(x)\sigma_{\mu\nu}\psi_2(x) & \rightarrow & -\bar{\psi}_2(x)\sigma_{\mu\nu}\psi_1(x).
\end{array} \tag{2.49}$$

### 2.2.3 CP transformation

The operations of  $CP$  or  $PC$  transformation are obtained by performing the parity transformation and charge transformation in order. Therefore, in the same way as the parity transformation and charge conjugation, we can easily derive the transformation of the free fields and spinor bilinears as follows;

$$\begin{array}{llll}
\text{Scalar field} & \phi(x) & \rightarrow & \phi^\dagger(\tilde{x}) \\
\text{Pseudoscalar field} & \mathcal{P}(x) & \rightarrow & -\mathcal{P}^\dagger(\tilde{x}) \\
\text{Dirac spinor} & \psi(x) & \rightarrow & \pm i\gamma^2\bar{\psi}^T(\tilde{x}) \\
& \bar{\psi}(x) & \rightarrow & \mp \psi^T(\tilde{x})i\gamma^2 \\
\text{Vector field} & \mathcal{V}_\mu(x) & \rightarrow & -\mathcal{V}_\mu^\dagger(\tilde{x}) \\
\text{Axial vector field} & \mathcal{A}_\mu(x) & \rightarrow & -\mathcal{A}_\mu^\dagger(\tilde{x}),
\end{array} \tag{2.50}$$

and

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<sup>#4</sup>Notice that the symbol  $T$  means *transpose* which should not be confused with a symbol of the time reversal.

$$\begin{array}{llll}
 \text{Scalar} & \bar{\psi}_1(x)\psi_2(x) & \rightarrow & \bar{\psi}_2(\tilde{x})\psi_1(\tilde{x}) \\
 \text{Pseudoscalar} & \bar{\psi}_1(x)\gamma_5\psi_2(x) & \rightarrow & -\bar{\psi}_2(\tilde{x})\gamma_5\psi_1(\tilde{x}) \\
 \text{Vector} & \bar{\psi}_1(x)\gamma_\mu\psi_2(x) & \rightarrow & -\bar{\psi}_2(\tilde{x})\gamma^\mu\psi_1(\tilde{x}) \\
 \text{Axialvector} & \bar{\psi}_1(x)\gamma_\mu\gamma_5\psi_2(x) & \rightarrow & -\bar{\psi}_2(\tilde{x})\gamma^\mu\gamma_5\psi_1(\tilde{x}) \\
 \text{Tensor} & \bar{\psi}_1(x)\sigma_{\mu\nu}\psi_2(x) & \rightarrow & -\bar{\psi}_2(\tilde{x})\sigma^{\mu\nu}\psi_1(\tilde{x}).
 \end{array} \tag{2.51}$$

It is remarkable that the  $CP$  transformation of Dirac spinors generally produces different phases for different orders of operations. However, for spinor bilinear, the phases do not depend on the orders of operation because of  $\eta^*\eta = 1$ .

### 2.2.4 $P$ and $CP$ violation

Though parity and  $CP$  seem to be good symmetries at a glance, we know that these symmetries are not conserved in Nature. Here, in order to understand the property of parity and  $CP$  violation, we consider the parity violation due to weak neutral currents and the  $CP$ -violation due to weak charged currents.

#### Parity violation

In the quantum electromagnetic dynamics, the interaction Lagrangian is simply written as

$$\mathcal{L}_{int}^{QED} = -e\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) \tag{2.52}$$

where  $e$  and  $A^\mu(x)$  denote real coupling constant and photon field. In addition, we regard the electromagnetic current as  $\bar{\psi}(x)\gamma_\mu\psi(x)$ ;  $J_\mu^{QED}(x) \equiv \bar{\psi}(x)\gamma_\mu\psi(x)$ . Then, as is known from the relation (2.47), under the parity transformation, the electromagnetic current is transformed as

$$\bar{\psi}(x)\gamma_\mu\psi(x) \xrightarrow{P} \bar{\psi}(\tilde{x})\gamma^\mu\psi(\tilde{x}). \tag{2.53}$$

Thus, the Lagrangian of Eq. (2.52) is invariant under the parity transformation when we take  $A^\mu(x) \xrightarrow{P} A_\mu(\tilde{x})$ ;

$$\mathcal{L}_{int}^{QED} = -e\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) \xrightarrow{P} -e\bar{\psi}(\tilde{x})\gamma^\mu\psi(\tilde{x})A_\mu(\tilde{x}) = \mathcal{L}_{int}^{QED}. \tag{2.54}$$

On the other hand, in the weak interaction, the Lagrangian of the neutral interactions which are caused by  $Z$  boson exchange, is simply given as

$$\mathcal{L}_{int}^{NC} = J_\mu^{NC}(x)Z^\mu(x) \tag{2.55}$$

with

$$J_\mu^{NC} = \bar{\psi}(x)\gamma_\mu(a + b\gamma_5)\psi(x), \tag{2.56}$$

where  $a$  and  $b$  are real coupling constants. This current does not change the flavor in the standard model<sup>#5</sup>. Then, under parity transformation,  $J_\mu^{NC}$  is transformed into

$$\bar{\psi}(x)\gamma_\mu(a + b\gamma_5)\psi(x) \xrightarrow{P} \bar{\psi}(\tilde{x})\gamma^\mu(a - b\gamma_5)\psi(\tilde{x}). \quad (2.57)$$

Notice that the vector current,  $\bar{\psi}(x)\gamma_\mu\psi(x)$ , and axialvector current,  $\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$ , have a different sign after the parity transformation. Therefore,  $\mathcal{L}_{int}^{NC}$  is not invariant under the parity transformation even if  $Z^\mu(x) \xrightarrow{P} \pm Z^\mu(\tilde{x})$ ;

$$\mathcal{L}_{int}^{NC} = \bar{\psi}(x)\gamma_\mu(a + b\gamma_5)\psi(x)Z^\mu(x) \xrightarrow{P} \pm\bar{\psi}(\tilde{x})\gamma^\mu(a - b\gamma_5)\psi(\tilde{x})Z_\mu(\tilde{x}) \neq \mathcal{L}_{int}^{NC}. \quad (2.58)$$

This means that weak interaction can induce parity violations.

### CP violation

The Lagrangian of charged weak interaction which is caused by  $W^\pm$  boson exchange, is given as

$$\begin{aligned} \mathcal{L}_{int}^{CC} &= gJ_\mu^{CC}(x)W^{+\mu}(x) + g^*J_\mu^{\dagger CC}(x)W^{-\mu}(x) \\ &= g\bar{\psi}_i(x)\gamma_\mu(1 - \gamma_5)\psi_j(x)W^{+\mu}(x) \\ &\quad + g^*\bar{\psi}_j(x)\gamma_\mu(1 - \gamma_5)\psi_i(x)W^{-\mu}(x), \end{aligned} \quad (2.59)$$

where  $g$  is a coupling constant which can be a complex number. Under  $CP$  transformation, this Lagrangian is transformed into

$$gJ_\mu^{CC}(x)W^{+\mu}(x) + g^*J_\mu^{\dagger CC}(x)W^{-\mu}(x) \xrightarrow{CP} gJ^{\dagger CC\mu}(\tilde{x})W_\mu^-(\tilde{x}) + g^*J^{CC\mu}(\tilde{x})W_\mu^+(\tilde{x}), \quad (2.60)$$

if we define the  $CP$  transformation of the  $W^{\pm\mu}(x)$  as  $W^{\pm\mu}(x) \xrightarrow{CP} -W_\mu^\mp(\tilde{x})$  because under  $CP$  transformation, the  $J_\mu^{CC}(x)$  transforms as

$$J_\mu^{CC}(x) = \bar{\psi}_i(x)\gamma_\mu(1 - \gamma_5)\psi_j(x) \xrightarrow{CP} -\bar{\psi}_j(\tilde{x})\gamma^\mu(1 - \gamma_5)\psi_i(\tilde{x}) = -J^{\dagger CC\mu}(\tilde{x}). \quad (2.61)$$

It is remarkable that though  $\mathcal{L}_{int}^{CC}$  is invariant under  $CP$  transformation if the coupling constant is real,  $\mathcal{L}_{int}^{CC}$  with complex coupling is not invariant under this transformation due to transformations of its imaginary parts;

$$\begin{aligned} &i \operatorname{Im}(g) [J_\mu(x)W^{+\mu}(x) - J_\mu^\dagger(x)W^{-\mu}(x)] \\ &\quad \downarrow CP \\ &-i \operatorname{Im}(g) [J_\mu(\tilde{x})W^{+\mu}(x) - J_\mu^\dagger(\tilde{x})W^{-\mu}(\tilde{x})]. \end{aligned} \quad (2.62)$$

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<sup>#5</sup>In the standard model,  $a$  and  $b$  are given from Eq.(2.35-2.37).

Therefore, it is important for  $CP$  violation whether the coupling constant is real or complex.

Since quarks have three generations in our world, some of the couplings can be complex numbers because quark mixing matrix for three generations includes a phase which can not be absorbed into the coupling and make it complex. This is the basic concept of the  $CP$ -violation mechanism which proposed by Kobayashi and Maskawa [11].





# CHAPTER 3

## HIGH ENERGY COLLIDERS

In this Chapter, we briefly introduce the future high energy colliders related to our study. In Section 3.1, the plans of the future high energy electron-positron linear collider are introduced. These colliders are useful to study the  $e^+e^- \rightarrow t\bar{t}$  process which is discussed in Section 4.2. The Tevatron which is a powerful tool for the study of  $p\bar{p} \rightarrow t\bar{t}X$  process discussed in Section 4.3 is introduced in Section 3.2.

### 3.1 High Energy Electron-Positron Liner Collider

The future high energy electron-positron liner collider (FLC) seems to be the essential tool for studying new physics beyond the SM and giving precise measurements. Although the high energy hadron colliders, Tevatron at Fermilab and Large Hadron Collider (LHC) at CERN, might find new physical phenomena beyond the SM, measurements at FLC will confirm it and may help to find even the new gauge boson. Therefore, Tera Electron volt energy Superconducting Linear Accelerator (TESLA) [18], Japan Liner Collider (JLC) [19], Next Liner Collider (NLC) [20] and Compact Liner Collider (CLIC) [21] projects are very attractive proposals. The designed luminosity for TESLA (DESY) at 500 GeV is  $\mathcal{L} = 3.4 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ . In addition, TESLA could be upgraded to about 800 GeV. Similarly, the luminosity for JLC (KEK) and NCL (SLAC, LBNL, LLNL, FNAL) at 500 GeV is expected to be  $2.2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ . Both of them have a plan to upgrade the center-of-mass energy up to 1.5 TeV which is roughly equivalent to the LHC in its ability to detect the new physics.

On the other hand, CLIC project could be a pioneer of multi-TeV physics because the upper limit of center-of-mass energy is aimed around at 5.0 TeV. This energy scale will enable us to directly search for the extra gauge boson such as neutral  $Z'$  gauge boson which is induced by both GUT-inspired  $E_6$  models and Left-Right symmetric model. In addition, there are possibilities to observe the new resonances which are predicted by recent theories of gravity with extra dimensions.

For all proposals, electron beam polarization of 80% is expected. This can be used to enhance signals and suppress background. Furthermore, FLC can also be operated

Project name	TESLA	JLC	NLC	CLIC
Laboratory	DESY	KEK	SLAC <i>etc.</i>	CERN
Total length [km]	33	25	26	38
Energy $\sqrt{s}$ [TeV]	0.5~0.8	0.5~1.5		0.5~5.0
Peak luminosity (at 500 GeV) [ $\text{cm}^{-2}\text{s}^{-1}$ ]	$3.4 \times 10^{34}$	$2.2 \times 10^{34}$		$O(10^{34})$
Electron beam polarization	80%			

**TABLE 3.1:** Parameters of FLC.

for electron-electron collisions with a little less luminosity and it may be possible to create a high luminosity photon-photon and photon-electron collider using Compton back-scattering of laser light. These optional programs are also very interesting to search for new physics and Higgs boson.

Though there remain important Research and Development issues to be resolved, a 500 GeV electron-positron linear collider is expected to be realized as a first stage of the electron-positron physics in next twenty years.

The parameters of TESLA, JLC, NLC and CLIC are shown in Table 3.1.

The FLC will give us the first opportunity to study the properties of the top quark at the electron-positron collider, because top quark is too heavy to be produced at present electron-positron colliders; e.g. Large Electron Positron (LEP) collider whose center-of-mass energy is about 210 GeV. Since at the electron-positron collider, top quarks produced via virtual photon or  $Z$  boson are induced from the electron-positron annihilation, we can precisely measure the top quark couplings to photon and  $Z$  boson. Such measurements are able to constrain the  $t\bar{t}Z/\gamma$  form factors. These form factors are expected to receive large contributions from non-standard Higgs or technifermions which are related to the electroweak symmetry breaking. Therefore, study of top quark physics must be an important purpose of the FCL experiments. At  $\sqrt{s} = 500$  GeV, the lowest-order total cross section for unpolarized beams with  $m_t = 180$  GeV is 0.54 pb. Furthermore, the lowest-order cross section becomes 0.75 pb (0.39 pb) for a fully left-hand (right-hand) polarized electron beam. Thus, 25,000 top pair events are expected to be produced with integrated luminosity  $\mathcal{L} = 50 \text{ fb}^{-1}$  for the unpolarized electron beam [22].

## 3.2 Fermilab Tevatron

The Fermilab Tevatron is a high energy proton-antiproton collider [23]. This collider was the first collider to study the TeV scale physics. It has two major collider detectors; one is the Collider Detector at Fermilab (CDF), the another is the D0.

Run name	Run I	Run II	Run III
Energy $\sqrt{s}$ [TeV]	1.8	2.0	2.0
Run Dates	1992~1995	2001~2003	2004~2007
Peak luminosity [ $\text{cm}^{-2}\text{s}^{-1}$ ]	$2 \times 10^{31}$	$2 \times 10^{31}$	$5 \times 10^{32}$
Integrated Luminosity [ $\text{fb}^{-1}$ ]	0.1	2.0	$30 \geq 15$

**TABLE 3.2:** Tevatron run parameters.

Run I obtained an integrated luminosity of about  $100 \text{ pb}^{-1}$  in each detector at a center-of-mass energy of  $\sqrt{s} = 1.8 \text{ TeV}$ . In this Run, the CDF and D0 collaboration discovered the top quark [8, 9].

Run II will start this year after constructing the Main Injector which will store and pre-accelerate proton and antiproton for injection into the Tevatron ring itself and improve the detectors. In addition, the beam energy will be raised to  $\sqrt{s} = 2.0 \text{ TeV}$ . In this Run, the peak luminosity is expected to rise by  $2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$  and  $2 \text{ fb}^{-1}$  of data will be accumulated in two years operation.

Run III, known as “TeV 33” with the expected instantaneous luminosity of  $\sim 10^{33}$ , will run from 2003 to 2006 with center-of-mass energy  $\sqrt{s} = 2.0 \text{ TeV}$ . In three years operation, the integrated luminosity is expected to be  $30 \text{ fb}^{-1}$ .

The parameters for each Run are summarized in Table 3.2.

Since top quark was discovered at Run I, the properties of top quarks will be studied in detail at next Runs. The dominant top quark production mechanism at Tevatron is pair production through  $q\bar{q}$  annihilation<sup>#1</sup>. For the upgraded Tevatron, the cross section for top ( $m_t=175 \text{ GeV}$ ) pair productions is calculated to be  $7.5 \text{ pb}$ , with an uncertainty estimated by various groups to be about  $10\sim 30\%$  [24]. Thus, a  $2$  ( $30$ )  $\text{pb}^{-1}$  Run II (III) will produce  $15,000$  ( $225,000$ ) events of top pair production [22]. In addition, a single top quark can be produced through electroweak processes such as  $W$ -gluon fusion or the production of a virtual  $W$  that decays to  $t\bar{b}$ . Though the single-top quark production cross section is about  $1/3$  of the top quark pair production, these channels are sensitive to the Cabbibo-Kobayashi-Maskawa parameter  $V_{tb}$  and the decay width of top quark.

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<sup>#1</sup>Though there is also a  $gg$  fusion production in principle, such a contribution is only  $10\%$ . Therefore, we ignore its contribution in the following work.



# CHAPTER 4

## *CP* VIOLATION OF TOP QUARK PAIR PRODUCTION

In this chapter, we discuss the *CP*-violation of the top quark pair production via the anomalous couplings. First, we introduce the *CP*-violating observable for top quark pair production in Section 4.1. In Sections 4.2 and 4.3, we calculate the *CP*-violating observable which is defined in Section 4.1 for the two kinds of processes:  $e^+e^- \rightarrow t\bar{t}$  and  $p\bar{p} \rightarrow t\bar{t}X$ , respectively.

### 4.1 *CP* Violating Observable for Top Quark Pair Productions

Since quark pairs are mainly produced through the vector-boson exchange at the electron-positron linear collider and the proton-antiproton collider, the helicities of  $t\bar{t}$  would be  $(+-)$  or  $(-+)$  due to the helicity conservation which is realized if the quark mass is much smaller than  $\sqrt{s}$ . However, since top quark mass is about 180 GeV, we can also expect to have  $(++)$  and  $(--)$  combinations as a consequence of the breaking of the helicity conservation. We can use these combinations to study *CP* properties of the  $t\bar{t}$  state;  $| - - \rangle$  and  $| + + \rangle$  transform each other under *CP* operation as

$$\hat{C}\hat{P}| \mp \mp \rangle = \hat{C}| \pm \pm \rangle = | \pm \pm \rangle. \quad (4.1)$$

Therefore, the difference between the events  $N(++)$  and  $N(--)$  could be a useful measure of *CP* violation [25];

$$\begin{aligned} A_{cp} &= \frac{N(--)-N(++)}{N(all)} \\ &= \frac{\sigma_{--}-\sigma_{++}}{\sigma_{all}} \equiv \frac{\Delta\sigma}{\sigma_{all}}, \\ N(all) &\equiv N(++)+N(+-)+N(-+)+N(--), \end{aligned} \quad (4.2)$$

where  $\sigma_{++}$  and  $\sigma_{--}$  is the cross section of the  $t\bar{t}$  production with the helicities  $(++)$  and  $(--)$ .

Though this observable can not be directly detected, we can easily reconstruct it through the top quark decay distribution of the secondary leptons [14, 26]. Notice that there are some remarkable properties about the top quark decays [25].

- (1) Since top quark is very heavy ( $m_t \sim 180$  GeV) and its lifetime is much shorter than  $10^{-23}$  s which is smaller than the hadronization time [13], the top would decay without the hadronic effect.
- (2) Since  $m_t > m_W$ , the dominant decay process of the top quark should be  $t \rightarrow W^+ b$ . Because of large top mass, the  $W$  will be predominantly longitudinal while the  $b$  is always left-handed in the SM, if  $m_b/\sqrt{s} \ll 1$  <sup>#1</sup>.
- (3) Because of (2), a  $t(-)$  will decay to an energetic  $b(-)$  <sup>#2</sup>, which must go forward to carry the quark spin, and to a less energetic  $W^+$ ; for  $t(+)$ , the relative energies of  $b$  and  $W$  are roughly reversed.
- (4) Therefore, we can effectively get information about the polarization of the top quark by observing the energy distribution of the  $W$  bosons or their decay leptons.

In addition, it is interesting to know that the *CP*-violation in top quark sector is estimated to be extremely small in the CKM mechanism, because at least three-loop as depicted in Fig. 4.1, is necessary to generate a nonzero *CP*-violation in the SM with the CKM mechanism [12, 27, 28]. Thus, we have a good opportunity for investigating the non-standard origin of *CP*-violation in top quark sector.

In the following sections, we calculate the *CP*-violating observable for two different processes.

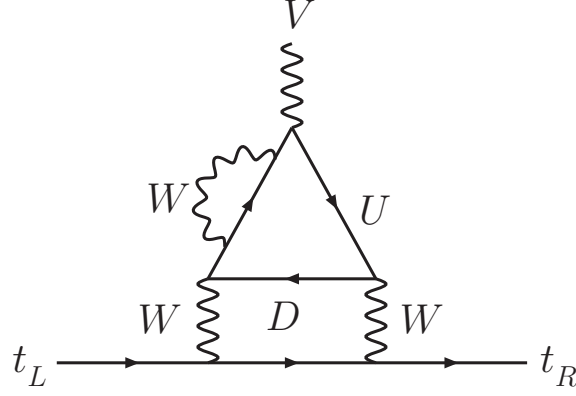
## 4.2 *CP* Violation in $e^+e^- \rightarrow t\bar{t}$

As mentioned in Section 3.1, at the future high energy electron-positron linear collider (FLC), experiments of top quark pair production via electron-positron annihilation are expected to become possible. Indeed, the FLC will be a powerful tool for new physics search beyond the SM and will provide useful data for probing the top quark couplings to the photon and  $Z$  boson.

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<sup>#1</sup>For the collider which we focus on this thesis, it is a good approximation to ignore the  $b$  quark mass. Therefore, since charged currents of the quarks are pure  $V - A$  like lepton currents, where  $V$  and  $A$  mean vector and axial currents, respectively,  $b$  quark are always produced as left-handed one.

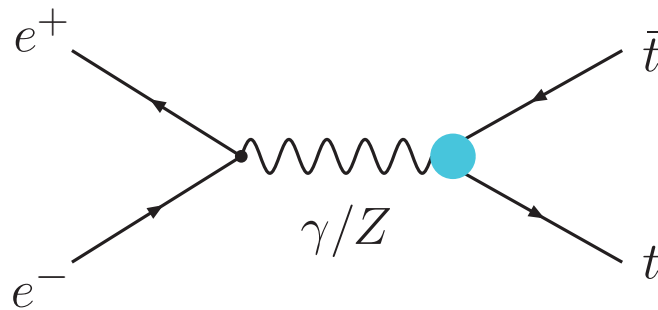
<sup>#2</sup> $t(\dots)$  and  $b(\dots)$  denote a top quark and bottom quark with the indicated helicity.



**FIGURE 4.1:** Typical diagram of *CP*-violation in the standard model. Here,  $D$  ( $U$ ) and  $V$  denote Down ( $UP$ ) type quarks and Vector bosons.

One such study will be a measurement of *CP*-violating observable defined by Eq. (4.2) which can be nonzero through possible anomalous couplings. Many papers have already appeared so far on this theme [25, 29, 30, 31, 32, 33]. In those papers, product terms of non-standard parameters were usually neglected and only interference between the SM and non-SM terms was taken into account under the assumption that non-SM effects are tiny. At much higher energy, however, such neglected terms might come to play an important role. Therefore, in this work, we calculate the cross section of polarized top quark pair productions in  $e^+e^-$  collisions by using the most general form of top quark interaction (Fig. 4.2) and keeping those terms which are usually neglected so far. In our calculation, we assume the following  $t\bar{t}\gamma/Z$  couplings:

$$\Gamma_{vt\bar{t}}^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \{A_v + \delta A_v - (B_v + \delta B_v)\gamma_5\} + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t} (\delta C_v - \delta D_v \gamma_5) \right] v(p_{\bar{t}}), \quad (4.3)$$



**FIGURE 4.2:** Feynman diagram for top quark pair production at the electron-positron linear collider.

where  $g$  denotes the  $SU(2)$  gauge coupling constant,  $v = \gamma, Z$ , and

$$A_\gamma = \frac{4}{3} \sin \theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{v_t}{2 \cos \theta_W}, \quad B_Z = \frac{1}{2 \cos \theta_W}$$

with  $v_t \equiv 1 - (8/3) \sin^2 \theta_W$ .  $\delta A_v, \delta B_v, \delta C_v$  and  $\delta D_v$  are parameters expressing non-standard effects.<sup>#3</sup>

On the other hand, we assume the standard form for the electron-positron couplings:

$$\Gamma_{\gamma e\bar{e}}^\mu = -e \bar{v}(p_{\bar{e}}) \gamma^\mu u(p_e), \quad (4.4)$$

$$\Gamma_{Z e\bar{e}}^\mu = \frac{g}{4 \cos \theta_W} \bar{v}(p_{\bar{e}}) \gamma^\mu (v_e + \gamma_5) u(p_e), \quad (4.5)$$

where  $v_e \equiv -1 + 4 \sin^2 \theta_W$ .

Now, using the above vertices and propagators  $d_\gamma = 1/s$ ,  $d_Z = 1/(s - M_Z^2)$ , we can represent the invariant amplitude for  $e^+e^- \rightarrow t\bar{t}$  as follows:

$$\begin{aligned} M(e\bar{e} \rightarrow t\bar{t}) &= g_{\mu\nu} (d_\gamma \Gamma_{\gamma e\bar{e}}^\mu \Gamma_{\gamma t\bar{t}}^\nu + d_Z \Gamma_{Z e\bar{e}}^\mu \Gamma_{Z t\bar{t}}^\nu) \\ &= C_{VV} [\bar{v}_e \gamma_\mu u_e \cdot \bar{u}_t \gamma^\mu v_{\bar{t}}] + C_{VA} [\bar{v}_e \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 \gamma^\mu v_{\bar{t}}] \\ &+ C_{AV} [\bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma^\mu v_{\bar{t}}] + C_{AA} [\bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 \gamma^\mu v_{\bar{t}}] \\ &+ C_{VS} [\bar{v}_e \not{q} u_e \cdot \bar{u}_t v_{\bar{t}}] + C_{VP} [\bar{v}_e \not{q} u_e \cdot \bar{u}_t \gamma_5 v_{\bar{t}}] \\ &+ C_{AS} [\bar{v}_e \gamma_5 \not{q} u_e \cdot \bar{u}_t v_{\bar{t}}] + C_{AP} [\bar{v}_e \gamma_5 \not{q} u_e \cdot \bar{u}_t \gamma_5 v_{\bar{t}}], \end{aligned} \quad (4.6)$$

where

$$\begin{aligned} C_{VV} &= -\frac{ge}{2s} [(A_\gamma + \delta A_\gamma) - v_e d'(A_Z + \delta A_Z)], \\ C_{VA} &= -\frac{ge}{2s} [\delta B_\gamma - v_e d'(B_Z + \delta B_Z)], \\ C_{AV} &= -\frac{ge}{2s} d'(A_Z + \delta A_Z), \\ C_{AA} &= -\frac{ge}{2s} d'(B_Z + \delta B_Z), \\ C_{VS} &= -\frac{ge}{4m_t s} [\delta C_\gamma - v_e d' \delta C_Z], \\ C_{VP} &= \frac{ge}{4m_t s} [\delta D_\gamma - v_e d' \delta D_Z], \\ C_{AS} &= -\frac{ge}{4m_t s} d' \delta C_Z, \\ C_{AP} &= \frac{ge}{4m_t s} d' \delta D_Z, \end{aligned}$$

<sup>#3</sup>In fact, the most general form contains also a term proportional to  $(p_t + p_{\bar{t}})^\mu$ , but this term gives vanishing contribution in the limit of zero electron mass.



with

$$d' \equiv \frac{s}{4 \sin \theta_W \cos \theta_W} d_Z.$$

A straightforward calculation leads to the spin dependent differential cross section (see Appendix A) and using this result, the asymmetry  $A_{cp}^{e\bar{e}}$  can be calculated as

$$A_{cp}^{e\bar{e}} = \frac{-2\beta \operatorname{Re}[F_1 - \beta^2(s/m_t^2)D_{SP}]}{(3 - \beta^2)D_V + 2\beta^2 D_A - 2\beta^2 \operatorname{Re}(G_1) + \beta^2(s/m_t^2)(\beta^2 D_S + D_P)} \quad (4.7)$$

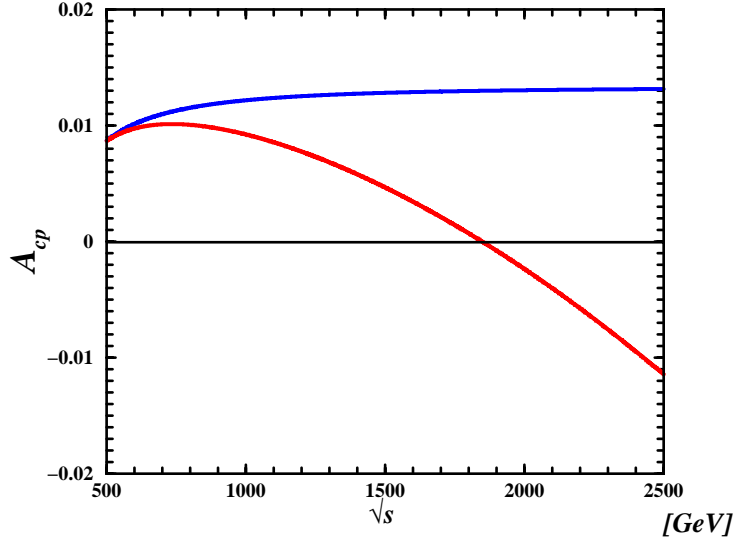
with

$$\begin{aligned} D_V &= (s^2/e^4)(|C_{VV}|^2 + |C_{AV}|^2), & D_A &= (s^2/e^4)(|C_{VA}|^2 + |C_{AA}|^2), \\ D_S &= m_t^2(s^2/e^4)(|C_{VS}|^2 + |C_{AS}|^2), & D_P &= m_t^2(s^2/e^4)(|C_{VP}|^2 + |C_{AP}|^2), \\ D_{SP} &= m_t^2(s^2/e^4)(C_{VS}^* C_{VP} + C_{AS}^* C_{AP}), \\ F_1 &= 2m_t(s^2/e^4)(C_{VV}^* C_{VP} + C_{AV}^* C_{AP}), \\ G_1 &= 2m_t(s^2/e^4)(C_{VV}^* C_{VS} + C_{AV}^* C_{AS}). \end{aligned}$$

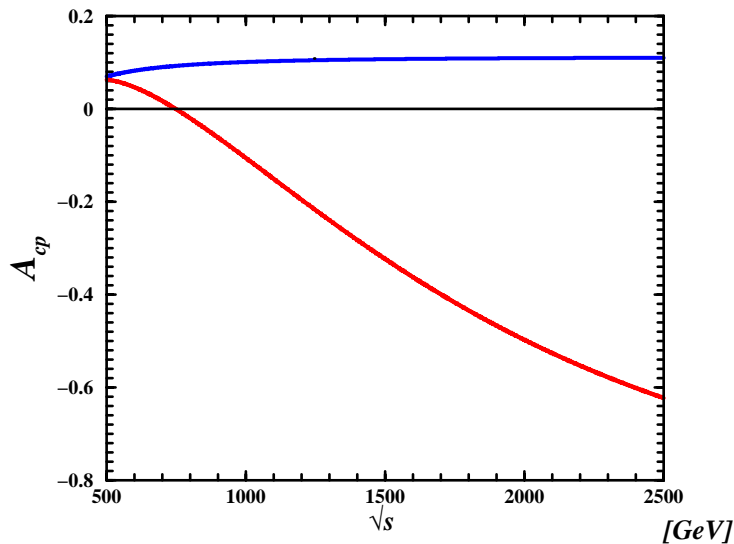
If we keep only  $D_V$ ,  $D_A$  and  $F_1$  terms, this  $A_{cp}^{e\bar{e}}$  agrees with the one calculated in [29, 30, 31]. We find that the term including  $D_{SP}$  in the numerator, which consists of  $\delta C_v$  and  $\delta D_v$ , is proportional to  $s$ . This means there is a possibility that  $D_{SP}$  contributes greatly to *CP*-violation at very high energy, depending on the size of  $\delta C_v$  and  $\delta D_v$ .

Let us visually show the difference between our calculations and other calculations in Fig. 4.3 and Fig. 4.4, where we use, as input data,  $m_t = 173.9$  GeV [37],  $M_Z = 91.1867$  GeV [38] and  $\sin^2 \theta_W = 0.2315$  [38], and assume, as examples, all the real and imaginary parts of  $\delta A_v, \dots, \delta D_v$  in Eq. (4.3) to be 0.01 in Fig. 4.3 and 0.1 in Fig. 4.4. These figures show there is no difference around  $\sqrt{s} = 500$  GeV, but our  $A_{cp}^{e\bar{e}}$  decreases rapidly for higher  $\sqrt{s}$ . Our calculations here are exact as long as we can treat the anomalous couplings as constant parameters. Some comments are necessary, however, on this point before going to summary. If these couplings from some new physics appear at a higher-energy scale  $\Lambda$ , then our results are applicable only for  $\sqrt{s} < \Lambda$ . One plausible way to estimate this  $\Lambda$  will be given by the effective Lagrangian approach [34]. According to it, both of  $\delta C_v$  and  $\delta D_v$  are order of  $O(m_t^2/g\Lambda^2)$  [35], which lead to  $\Lambda \sim m_t/\sqrt{g\delta_v}$  ( $\delta_v = \delta C_v$  or  $\delta D_v$ ) and roughly 2.5 TeV (0.8 TeV) for  $\delta_v = 0.01$  (0.1). Therefore, our results, especially the one in Fig. 4.4, must be seriously considered if this approach describes the Nature correctly, and in that case all the other usual calculations also lose their validity anyway.

To summary of this section, we studied contribution of the product terms of non-standard parameters, which are usually neglected in other calculations done so far, to *CP* violation in  $e^+e^- \rightarrow t\bar{t}$  by assuming the most general  $t\bar{t}\gamma/Z$  couplings. We showed there



**FIGURE 4.3:** *CP* violation asymmetry  $A_{cp}^{e\bar{e}}$  via our calculations (red line) and other calculations (blue line) assuming all the non-SM parameters to be 0.01.



**FIGURE 4.4:** *CP* violation asymmetry  $A_{cp}^{e\bar{e}}$  via our calculations (red line) and other calculations (blue line) assuming all the non-SM parameters to be 0.1.

is a possibility that other calculations may fail to give accurate results at very high  $\sqrt{s}$ . We considered top quark pair productions at FLC in this work, but the same discussion can be applied also for hadron colliders [36] if we replace  $e^+$  and  $e^-$  with the light quarks (and add the gluon-fusion diagram). The following section is devoted to this subject.

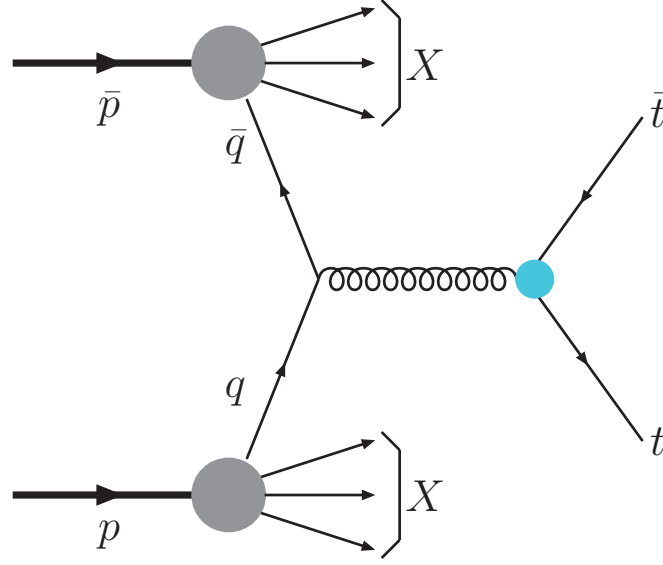


FIGURE 4.5: Feynman diagram for top quark pair production at Tevatron.

### 4.3 *CP* Violation in $p\bar{p} \rightarrow t\bar{t}X$

In this section, we consider the contribution of the anomalous chromomagnetic and chromoelectric dipole moments to the polarization asymmetry  $A_{cp}^{p\bar{p}}$  which might induce the *CP*-violation for the process (Fig. 4.5);

$$p + \bar{p} \rightarrow t + \bar{t} + X. \quad (4.8)$$

This process will be studied with the upgraded Fermilab Tevatron at  $\sqrt{s} = 2.0$  TeV. Though many works have been done so far on the anomalous chromomagnetic and chromoelectric dipole moments [12, 26, 40, 41, 42], those authors have treated them as real parameters because they have focused only on the effective Lagrangian whose dimension is smaller than or equal to five. However, since in general the anomalous chromomagnetic and chromoelectric dipole moments can be originated from some loop corrections, they can also have imaginary parts. Therefore, we will treat them as complex numbers in our calculations.

Here, in order to estimate the effect of *CP*-violation in the process (4.8), we take the following effective Lagrangian for top-gluon interaction:

$$\mathcal{L}_{t\bar{t}g} = g_s T^a \bar{v}_{\bar{t}} \left[ -\gamma^\mu G_\mu^a - \frac{\kappa}{4m_t} \sigma^{\mu\nu} G_{\mu\nu}^a - \frac{i\tilde{\kappa}}{4m_t} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}^a \right] u_t, \quad (4.9)$$

where  $g_s$ ,  $T^a$  and  $m_t$  are the strong coupling constant, SU(3) color matrices and top quark mass, respectively. In addition,  $G_{\mu\nu}^a$  means the gluon field strength and  $\sigma^{\mu\nu} \equiv i/2[\gamma^\mu, \gamma^\nu]$ . The  $\kappa$  and  $\tilde{\kappa}$  are the chromomagnetic and chromoelectric coupling, respectively. From Eq. (4.9), we can derive the effective couplings of  $t\bar{t}g$  interaction:

$$\Gamma_{t\bar{t}g} = -ig_s T^a \bar{v}(p_{\bar{t}}, s_{\bar{t}}) \left[ \gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_t} q_\nu (\kappa + i\tilde{\kappa}\gamma^5) \right] u(p_t, s_t), \quad (4.10)$$

where  $p_t(p_{\bar{t}})$ ,  $s_t(s_{\bar{t}})$  and  $q_\nu$  denote the four-momenta of the top (anti-top) quark, spin vector of the top (anti-top) quark and incoming gluon momentum, respectively. Though a  $ggt\bar{t}$  four-point interaction is induced as a result of gauge invariance, we can neglect such an interaction because top quark pair is dominantly produced through quark–anti-quark annihilation processes at the upgraded Tevatron energy. Furthermore, since we assume that there is no new physics except for the interaction related to the top quark, we use the standard form for quark-gluon couplings:

$$\Gamma_{q\bar{q}g} = -ig_s T^a \bar{v}(p_a) \gamma^\mu u(p_b), \quad (4.11)$$

where  $p_a(p_b)$  are four-momenta of quarks which are emitted from the initial proton (anti-proton).

It is remarkable that our approach does not depend on the specific models because we treat the the anomalous chromomagnetic and chromoelectric couplings as free parameters. As is known, in the SM, these couplings are estimated to be too small to be detected at the Tevatron. Therefore, if we detect the signal of these couplings, it can be a good evidence of new physics beyond the SM.

First, let us focus only on  $\Delta\sigma$  at the numerator of Eq. (4.2) defined by

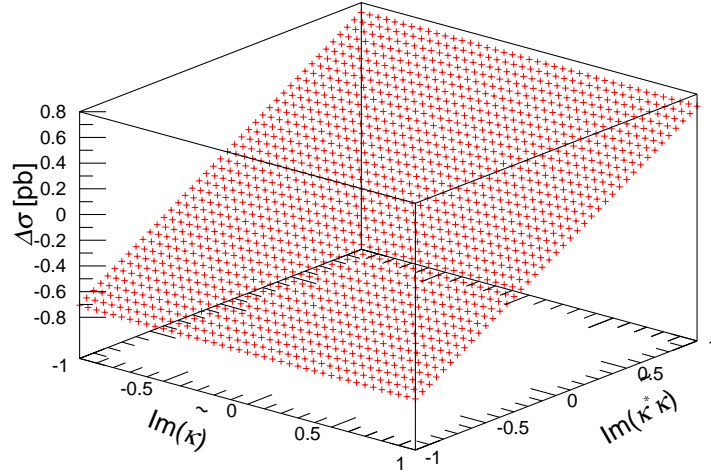
$$\Delta\sigma \equiv [\sigma(p\bar{p} \rightarrow t(-)\bar{t}(-) + X) - \sigma(p\bar{p} \rightarrow t(+)\bar{t}(+) + X)], \quad (4.12)$$

where  $\sigma(p\bar{p} \rightarrow \dots)$  denotes the cross section of top pair production for each helicity state at the Tevatron. Furthermore, in the parton center-of-mass system,  $\Delta\sigma$  described as

$$\begin{aligned} \Delta\sigma &= \sum_q \int_0^1 dx_a \int_0^1 dx_b \int_{-1}^1 d\cos\hat{\theta} f_{q/p}(x_a) f_{\bar{q}/\bar{p}}(x_b) \\ &\quad \times \Theta(x_a x_b s - 4m_t^2) \frac{d\Delta\hat{\sigma}}{d\cos\hat{\theta}} \mathcal{J}, \end{aligned} \quad (4.13)$$

where the sum runs over quark flavors;  $q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}, b, \bar{b}$  <sup>#4</sup>.  $f_{q/p}(x_a)$ ,  $f_{\bar{q}/\bar{p}}(x_b)$  and  $\Theta$  are parton distribution functions and usual step function, respectively. By using

<sup>#4</sup>We assume that there are no top quarks in the proton as partons



**FIGURE 4.6:** Surface plots displaying the dependence of the polarization asymmetry  $\Delta\sigma$  on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  at  $\sqrt{s}=2.0$  TeV.

$\Delta\sigma$ [Pb]		$\text{Im}(\tilde{\kappa})$						
		1.0	0.1	0.01	0	-0.01	-0.1	-1.0
$\text{Im}(\kappa^*\tilde{\kappa})$	1.0	$7.06 \times 10^{-1}$	$7.13 \times 10^{-1}$	$7.14 \times 10^{-1}$	$7.14 \times 10^{-1}$	$7.14 \times 10^{-1}$	$7.15 \times 10^{-1}$	$7.23 \times 10^{-1}$
	0.5	$3.48 \times 10^{-1}$	$3.56 \times 10^{-1}$	$3.57 \times 10^{-1}$	$3.57 \times 10^{-1}$	$3.57 \times 10^{-1}$	$3.58 \times 10^{-1}$	$3.67 \times 10^{-1}$
	0.1	$6.28 \times 10^{-2}$	$7.06 \times 10^{-2}$	$7.13 \times 10^{-2}$	$7.14 \times 10^{-2}$	$7.15 \times 10^{-2}$	$7.23 \times 10^{-2}$	$8.00 \times 10^{-2}$
	0.01	$-1.49 \times 10^{-3}$	$6.28 \times 10^{-3}$	$7.06 \times 10^{-3}$	$7.14 \times 10^{-3}$	$7.23 \times 10^{-3}$	$8.00 \times 10^{-3}$	$1.58 \times 10^{-2}$
	0	$-8.62 \times 10^{-3}$	$-8.62 \times 10^{-4}$	$8.62 \times 10^{-5}$	0	$8.62 \times 10^{-5}$	$8.62 \times 10^{-4}$	$8.62 \times 10^{-3}$
	-0.01	$-1.58 \times 10^{-2}$	$-8.00 \times 10^{-3}$	$-7.23 \times 10^{-3}$	$-7.14 \times 10^{-3}$	$-7.06 \times 10^{-3}$	$6.28 \times 10^{-3}$	$1.49 \times 10^{-3}$
	-0.1	$-8.00 \times 10^{-2}$	$-7.23 \times 10^{-2}$	$-7.15 \times 10^{-2}$	$-7.14 \times 10^{-2}$	$-7.13 \times 10^{-2}$	$-7.06 \times 10^{-2}$	$6.28 \times 10^{-2}$
	-0.5	$-3.67 \times 10^{-1}$	$-3.58 \times 10^{-1}$	$-3.57 \times 10^{-1}$	$-3.57 \times 10^{-1}$	$-3.57 \times 10^{-1}$	$-3.56 \times 10^{-1}$	$-3.48 \times 10^{-1}$
	-1.0	$-7.23 \times 10^{-1}$	$-7.15 \times 10^{-1}$	$-7.14 \times 10^{-1}$	$-7.14 \times 10^{-1}$	$-7.14 \times 10^{-1}$	$-7.13 \times 10^{-1}$	$-7.06 \times 10^{-1}$

**TABLE 4.1:** The dependence of the polarization symmetry,  $\Delta\sigma$  [pb] on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .

Eqs. (4.10-4.11), the subprocess cross section,  $d\Delta\hat{\sigma}/d\cos\hat{\theta}$ , is calculated as

$$\frac{d\Delta\hat{\sigma}}{d\cos\hat{\theta}} = \frac{\pi\alpha_s^2\beta_t}{9\gamma_t m_t \hat{s}^2 \sqrt{\hat{s}}} \left[ \text{Im}\tilde{\kappa} \left\{ (2 - \beta_t^2)\hat{s}\cos^2\hat{\theta} - 4m_t^2 \right\} - \text{Im}(\kappa^*\tilde{\kappa})\hat{s}(1 - 2\cos^2\hat{\theta}) \right], \quad (4.14)$$

where  $\beta_t \equiv \sqrt{1 - 4m_t^2/\hat{s}}$ ,  $\gamma_t \equiv \sqrt{1 - \beta_t^2}$  and  $\hat{\theta}$  denote the emission angle of the top quark in the parton center-of-mass system which is defined in Appendix B. In addition  $\mathcal{J}$  means the Jacobian which transform the variable  $\hat{t}$  to  $\cos\hat{\theta}$ , given as  $\mathcal{J} = |\hat{s}\beta_t/2|$ . Notice that possibilities of the  $CP$ -violation roughly depend only on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  in the  $d\Delta\hat{\sigma}/d\cos\hat{\theta}$ . Therefore, it is possible to measure  $CP$ -violating observable  $\Delta\sigma$  in some combinations of the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ .

We show the dependence of  $\Delta\sigma$  on  $\text{Im}(\kappa^*\tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  at the upgraded Tevatron Energy ( $\sqrt{s}=2.0$  TeV) in Fig.4.6. In addition, the magnitude of  $\Delta\sigma$  for some combinations of the

magnitude of  $\text{Im}(\kappa^* \tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$  are listed in Table 4.1. In our numerical calculation, we used the parton distribution functions of CTEQ5M [43] and as input data  $m_t=174.3$  GeV [39]. In Fig. 4.6, we can easily see that  $\Delta\sigma$  strongly depends on  $\text{Im}(\kappa^* \tilde{\kappa})$  though we cannot individually determine these values. This dependence can be also seen in the same approach to  $CP$ -violation for  $e^+e^- \rightarrow t\bar{t}g$  [44].

Secondly, using  $\Delta\sigma$ , we estimate the  $CP$ -violating observable  $A_{cp}^{p\bar{p}}$  which is defined by Eq. (4.2);  $A_{cp} \equiv \Delta\sigma/\sigma_{all}$ . Since the total cross section  $\sigma_{all}$  calculated using (4.9) has many undetermined parameters such as  $|\kappa|^2$ ,  $|\tilde{\kappa}|^2$  and  $\text{Re}(\kappa)$ , we approximated  $\sigma_{all}$  as  $\sigma_{SM}$  predicted by SM. This approximation is not unreasonable because the cross section given by SM is in good agreement with experimental result. In order to observe  $A_{cp}^{p\bar{p}}$  at the 90% Confidence Level (CL),  $A_{cp}^{p\bar{p}}$  must satisfy the condition:

$$|A_{cp}^{p\bar{p}}| \geq \frac{1.64}{\sqrt{N_{events}}}, \quad (4.15)$$

where  $N_{events}$  means the total number of events which can be experimentally reconstructed.

At the upgraded Tevatron with center-of-mass energy  $\sqrt{s} = 2.0$  TeV, the decay mode which is the most sensitive to the helicities of the produced top and anti-top quark, is the dilepton mode:  $t\bar{t} \rightarrow b\bar{b}W^+(\rightarrow l^+\nu_l)W^-(\rightarrow l^-\bar{\nu}_l)$  because the energies of leptons produced from  $W$  bosons and  $b$  ( $\bar{b}$ ) are sensitive to the helicities of the produced top and anti-top quark as mentioned in Section 4.1. The total number of events of the dilepton mode are expected to be detected about 1200 events for  $m_t = 175\text{GeV}$  [22]<sup>#5</sup>. Therefore, Eq. (4.15) becomes

$$|A_{cp}^{p\bar{p}}| = \left| \frac{\Delta\sigma}{\sigma_{sm}} \right| \geq \frac{1.64}{\sqrt{N_{events}}} = \frac{1.64}{\sqrt{1200}}. \quad (4.16)$$

Then, with  $\sigma_{SM} = 7.5\text{pb}$  [22], the  $\Delta\sigma$  should satisfy the relation:

$$|\Delta\sigma| \geq 0.35[\text{pb}]. \quad (4.17)$$

In the Table 4.2, we present the magnitude of  $A_{cp}^{p\bar{p}}$  at 90 % CL for the typical combinations of the magnitude of  $\text{Im}(\kappa^* \tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . It is interesting that we can observe the 90 % CL  $A_{cp}^{p\bar{p}}$  when the magnitude of the  $\text{Im}(\kappa^* \tilde{\kappa})$  is larger than  $|0.5|$ , even if  $\text{Im}(\tilde{\kappa})=0$ , as shown in the Table 4.2.

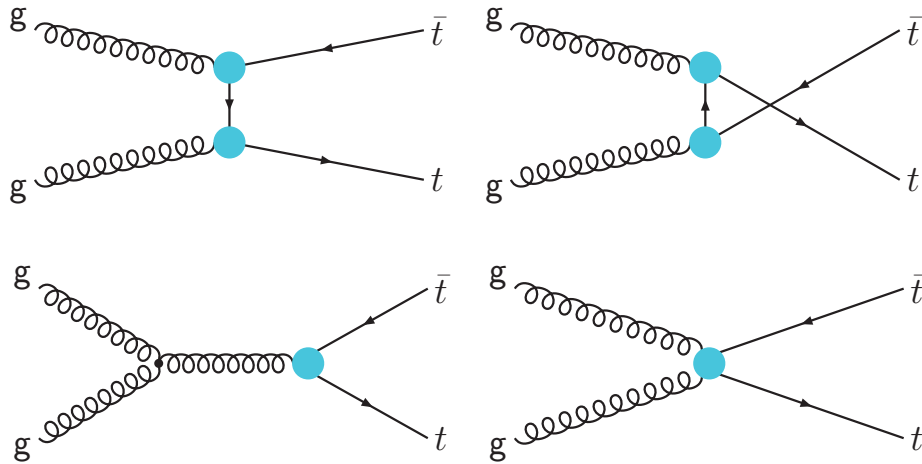
Even if we do not observe signals of  $A_{cp}^{p\bar{p}}$  at the upgraded Tevatron, we can obtain the constraint of the  $\text{Im}(\kappa^* \tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . In addition, it must be useful to search for

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<sup>#5</sup>In this work, we focus only on Run III (TeV 33) due to the statistically advantage. though there is also Run II.

$\Delta A_{cp}^{p\bar{p}}$ [ $\times 10^{-2}$ ]		$\text{Im}(\tilde{\kappa})$						
		1.0	0.1	0.01	0	-0.01	-0.1	-1.0
$\text{Im}(\kappa^* \tilde{\kappa})$	1.0	9.41	9.51	9.52	9.52	9.52	9.52	9.63
	0.7	6.55	6.65	6.66	6.67	6.67	6.68	6.78
	0.5	×	4.75	4.76	4.76	4.76	4.77	4.88
	0.1	×	×	×	×	×	×	×
	0	×	×	×	×	×	×	×
	-0.1	×	×	×	×	×	×	×
	-0.5	-4.68	-4.77	-4.76	-4.76	-4.76	-4.75	×
	-0.7	-6.78	-6.68	-6.67	-6.67	-6.66	-6.65	-6.55
	-1.0	-9.63	-9.52	-9.52	-9.52	-9.52	-9.51	-9.41

**TABLE 4.2:** The expected magnitude of the  $A_{cp}^{p\bar{p}}$  at the Upgraded Tevatron. A symbol “ $\times$ ” means that the magnitude of the  $A_{cp}^{p\bar{p}}$  does not reach by 90% CL.



**FIGURE 4.7:** Dominant subprocess diagrams of top pair productions at LHC.

the signal of  $A_{cp}$  at Large Hadron Collider (LHC) being the forthcoming proton–proton collider. It is remarkable that at the LHC, we must take into account additional diagrams shown in Fig 4.7 because the dominant mechanism of top quark pair productions is two-gluon fusion. Therefore, it is very interesting for LHC physics to apply the same analysis developed in this section because there are possibilities that  $CP$ -violation due to anomalous chromomagnetic and chromoelectric interactions, will be enhanced by such additional diagrams.

To summary this section, expecting the observation of  $p\bar{p} \rightarrow t\bar{t}X$  process at the up-graded Tevatron, we estimated the magnitude of  $CP$ -violation effect by using effective Lagrangian which includes the anomalous chromomagnetic and chromoelectric coupling.

Our analysis does not depend on the models because we taken the anomalous chromomagnetic and chromoelectric coupling as a free parameter. We showed that *CP*-violation depends only on  $\text{Im}(\kappa^* \tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . Especially, dependence of  $\text{Im}(\kappa^* \tilde{\kappa})$  is stronger than that of  $\text{Im}(\tilde{\kappa})$  in this process. Furthermore, we pointed out that *CP*-violating observables,  $A_{cp}^{p\bar{p}}$ , can be measured at 90 % CL in some combinations of the magnitude of  $\text{Im}(\kappa^* \tilde{\kappa})$  and  $\text{Im}(\tilde{\kappa})$ . Finally, we touched on the applicability of this analysis to LHC.



# CHAPTER 5

## SUMMARY AND DISCUSSION

The standard model with  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge symmetry is the extremely successful in describing the physics of leptons, quarks and their interactions. However, since the standard model has serious unsolved problems such as how to calculate many physical parameters, how to treat the gravitational interaction and so on, it is believed that there exists new physics beyond the standard model which can solve such problems. Therefore, it is very challenging and important to search for signals of new physics in the theoretical and experimental studies.

In this thesis, the effect of possible  $CP$ -violation via top quark anomalous interaction related to new physics, is discussed for two kind of processes;

(I) The electron-positron annihilation process:

$$e^+ + e^- \rightarrow t + \bar{t},$$

(II) The proton-antiproton scattering process:

$$p + \bar{p} \rightarrow t + \bar{t} + X.$$

In this analysis, it is remarkable to take account of some properties of the top quark:

- (a) In the top quark production and its subsequent decay,  $CP$ -violation is known to be extremely small in Cabbibo–Kobayashi–Maskawa mechanism. Thus, the top quark sector is quite suitable for searching the effect of another  $CP$ -violation originated from the physics beyond the standard model.
- (b) In the top quark pair production via gauge boson exchange, the allowed helicity states of the produced top quark pair can be not only  $(+-)$  and  $(-+)$  but also  $(++)$  and  $(--)$  because of the breakdown of the helicity conservation caused by large top quark masses. In addition, the helicity states  $|++\rangle$  and  $|--\rangle$  transform into each other under  $CP$  operation as  $\hat{C}\hat{P}| \mp \mp \rangle = | \pm \pm \rangle$ . This means the event asymmetry  $N(--)-N(++)$  can be a signal of the  $CP$ -violation. This asymmetry was calculated in this analysis.

- (c) Since the top quark has huge mass,  $m_t \simeq 180\text{GeV}$ , it can quickly decays without the effect of hadronization. This fact enables us to easily analyze the polarization of top quark.

For the process (I), by using the most general couplings for top quarks and gauge bosons( $\gamma/Z$ ), the magnitude of the  $CP$ -violation was estimated without neglecting products of non-standard couplings which were usually not considered in many calculations done so far. As a result, it was pointed out that

- (1) The terms which has been usually neglected contribute to the  $CP$ -violation with the square of center of mass energy. Therefore, there are possibilities that usual calculations can not correctly estimate the  $CP$ -violation at very high energy.
- (2) In the two cases of all non-standard couplings to be 0.01 and 0.1, the  $CP$ -violation given by usual calculation was numerically compared with ours at the region of center-of-mass energy:  $\sqrt{s} = 0.5 \sim 2.5$  TeV. Though at  $\sqrt{s} = 0.5$  TeV, there are no differences between usual calculation and ours, at the energy larger than  $\sqrt{s} = 1.0$  TeV, the differences become remarkable.

For the process (II), the  $CP$ -violation via the anomalous chromomagnetic ( $\kappa$ ) and chromoelectric ( $\tilde{\kappa}$ ) type couplings for top quark and gluons could be large in new physics models. It should be noted that in the present calculation, the two gluon fusion processes were neglected because its contribution is only 10% at the upgraded Tevatron energy. The results for this analysis are following;

- (1) The  $CP$ -violation effects only depend on  $\text{Im}(\tilde{\kappa})$  and  $\text{Im}(\kappa^*\tilde{\kappa})$ . Especially,  $CP$ -violation strongly depends on the magnitude of  $\text{Im}(\kappa^*\tilde{\kappa})$  in this process.
- (2) At the upgraded Tevatron, sizable  $CP$  asymmetry at 95% CL can be measured for some parameter resins;

As an example, in the case of the magnitude of  $\text{Im}(\tilde{\kappa})$  to be 0,  $\text{Im}(\kappa^*\tilde{\kappa})$  should be at least larger than  $|0.5|$  in order to observe the signal of  $CP$ -violation.

Though the same analysis can be applied for the Large Hadron colliders (LHC) at CERN which will start in 2005, the behavior of the  $CP$ -violation may be different from this analysis because in this case the dominant process of top quark pair production is two-gluon fusion.

Since this analysis was done without specific models, the model could not be specified from this analysis, even if the  $CP$ -violation measured in above processes. However, such measurements give the constraints on parameters in any models.

Here we did not analyze of decays of top quarks in both processes, which needs further investigation. The analysis of  $CP$ -violation at the LHC is also important. These subjects will be studied further in future work.

Finally, I hope that the discovery of signals due to new physics beyond standard model lead us to the new step to more fundamental theory of Nature.



# APPENDIX A

## THE CROSS SECTION FOR $e^+e^- \rightarrow t(s_t)\bar{t}(s_{\bar{t}})$

In this appendix, we write down the complete form of the spin dependent differential cross section which is discussed in Section 4.2.

By using Eqs.(4.3-4.5), the differential cross section in which  $t$  and  $\bar{t}$  have spins  $s_+$  and  $s_-$ , respectively, is calculated as

$$\begin{aligned}
& \frac{d\sigma}{d\Omega}(e^+e^- \rightarrow t(s_+)\bar{t}(s_-)) \\
&= \frac{3\beta\alpha^2}{16s^3} \left[ D_V [ \{4m_t^2s + (lq)^2\}(1 - s_+s_-) + s^2(1 + s_+s_-) \right. \\
&\quad \left. + 2s(ls_+ls_- - Ps_+Ps_-) + 2lq(ls_+Ps_- - ls_-Ps_+) ] \right. \\
&\quad + D_A [ (lq)^2(1 + s_+s_-) - (4m_t^2s - s^2)(1 - s_+s_-) \\
&\quad \left. - 2(s - 4m_t^2)(ls_+ls_- - Ps_+Ps_-) - 2lq(ls_+Ps_- - ls_-Ps_+) ] \right. \\
&\quad - 4 \operatorname{Re}(D_{VA}) m_t [ s(Ps_t - Ps_{\bar{t}}) + lq(ls_t + ls_{\bar{t}}) ] \\
&\quad + 2 \operatorname{Im}(D_{VA}) [ lq \epsilon(s_+, s_-, q, l) + ls_{\bar{t}}\epsilon(s_+, P, q, l) + ls_t\epsilon(s_-, P, q, l) ] \\
&\quad + 4 E_V m_t s(ls_t + ls_{\bar{t}}) + 4 E_A m_t lq(Ps_t - Ps_{\bar{t}}) \\
&\quad + 4 \operatorname{Re}(E_{VA}) [ 2m_t^2(ls_t Ps_{\bar{t}} - ls_{\bar{t}} Ps_t) - lq s ] \\
&\quad + 4 \operatorname{Im}(E_{VA}) m_t [ \epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l) ] \\
&\quad + D_S \frac{1}{m_t^2} [ (lq)^2 + 4m_t^2s - s^2 ] [ (4m_t^2 - s)(1 - s_+s_-) - 2Ps_+Ps_- ] \\
&\quad - D_P \frac{1}{m_t^2} [ (lq)^2 + 4m_t^2s - s^2 ] [ s(1 + s_+s_-) - 2Ps_+Ps_- ] \\
&\quad + 4 \operatorname{Re}(D_{SP}) \frac{1}{m_t} [ (lq)^2 + 4m_t^2s - s^2 ] (Ps_+ + Ps_-) \\
&\quad + 2 \operatorname{Im}(D_{SP}) \frac{1}{m_t^2} [ (lq)^2 + 4m_t^2s - s^2 ] \epsilon(s_+, s_-, P, q) \\
&\quad - \operatorname{Re}(F_1) \frac{1}{m_t} [ lq s(ls_t - ls_{\bar{t}}) - \{(lq)^2 + 4m_t^2s\}(Ps_t + Ps_{\bar{t}}) ] \\
&\quad + 2 \operatorname{Im}(F_1) [ s \epsilon(s_+, s_-, P, q) + lq \epsilon(s_+, s_-, P, l) ] \\
&\quad + 2 \operatorname{Re}(F_2) s(Ps_t ls_{\bar{t}} + Ps_{\bar{t}} ls_t)
\end{aligned}$$

$$\begin{aligned}
& -\text{Im}(F_2) \frac{s}{m_t} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)] \\
& -2 \text{Re}(F_3) lq (Ps_t ls_{\bar{t}} + Ps_{\bar{t}} ls_t) \\
& + \text{Im}(F_3) \frac{lq}{m_t} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)] \\
& - \text{Re}(F_4) \frac{s}{m_t} [lq (Ps_t + Ps_{\bar{t}}) - (s - 4m_t^2)(ls_t - ls_{\bar{t}})] \\
& -2 \text{Im}(F_4) [Ps_t \epsilon(s_-, P, q, l) + Ps_{\bar{t}} \epsilon(s_+, P, q, l)] \\
& +2 \text{Re}(G_1) [\{4m_t^2 s + (lq)^2 - s^2\}(1 - s_t s_{\bar{t}}) - 2s Ps_t Ps_{\bar{t}} \\
& \quad + lq(ls_t Ps_{\bar{t}} - ls_{\bar{t}} Ps_t)] \\
& - \text{Im}(G_1) \frac{lq}{m_t} [\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)] \\
& - \text{Re}(G_2) \frac{s}{m_t} [(s - 4m_t^2)(ls_t + ls_{\bar{t}}) - lq (Ps_t - Ps_{\bar{t}})] \\
& -2 \text{Im}(G_2) [Ps_t \epsilon(s_-, P, q, l) - Ps_{\bar{t}} \epsilon(s_+, P, q, l)] \\
& - \text{Re}(G_3) \frac{lq}{m_t} [lq (Ps_t - Ps_{\bar{t}}) - (s - 4m_t^2)(ls_t + ls_{\bar{t}})] \\
& -2 \text{Im}(G_3) lq \epsilon(s_+, s_-, q, l) \\
& +2 \text{Re}(G_4) [(s - 4m_t^2)(Ps_t ls_{\bar{t}} - Ps_{\bar{t}} ls_t) + 2lq Ps_t Ps_{\bar{t}}] \\
& + \text{Im}(G_4) \frac{1}{m_t} (s - 4m_t^2) [\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)] \Big], \tag{A.1}
\end{aligned}$$

where  $\beta \equiv \sqrt{1 - 4m_t^2/s}$ ,  $P$ ,  $q$  and  $l$  are defined as  $P \equiv p_e + p_{\bar{e}} (= p_t + p_{\bar{t}})$ ,  $l \equiv p_e - p_{\bar{e}}$ ,  $q \equiv p_t - p_{\bar{t}}$ , the symbol  $\epsilon(a, b, c, d)$  means  $\epsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma$  for  $\epsilon_{0123} = +1$ , and

$$\begin{aligned}
D_V &= (s^2/e^4)(|C_{VV}|^2 + |C_{AV}|^2), \quad D_A = (s^2/e^4)(|C_{VA}|^2 + |C_{AA}|^2), \\
D_{VA} &= (s^2/e^4)(C_{VV}^* C_{VA} + C_{AV}^* C_{AA}),
\end{aligned}$$

$$\begin{aligned}
E_V &= 2(s^2/e^4)\text{Re}(C_{AV}^* C_{VV}), \quad E_A = 2(s^2/e^4)\text{Re}(C_{AA}^* C_{VA}), \\
E_{VA} &= (s^2/e^4)(C_{VV}^* C_{AA} + C_{AV}^* C_{VA}),
\end{aligned}$$

$$\begin{aligned}
D_S &= m_t^2 (s^2/e^4)(|C_{VS}|^2 + |C_{AS}|^2), \quad D_P = m_t^2 (s^2/e^4)(|C_{VP}|^2 + |C_{AP}|^2), \\
D_{SP} &= m_t^2 (s^2/e^4)(C_{VS}^* C_{VP} + C_{AS}^* C_{AP}),
\end{aligned}$$

$$\begin{aligned}
F_1 &= 2m_t (s^2/e^4)(C_{VV}^* C_{VP} + C_{AV}^* C_{AP}), \\
F_2 &= 2m_t (s^2/e^4)(C_{VV}^* C_{AP} + C_{AV}^* C_{VP}), \\
F_3 &= 2m_t (s^2/e^4)(C_{VA}^* C_{VP} + C_{AA}^* C_{AP}), \\
F_4 &= 2m_t (s^2/e^4)(C_{VA}^* C_{AP} + C_{AA}^* C_{VP}),
\end{aligned}$$

$$\begin{aligned}G_1 &= 2m_t(s^2/e^4)(C_{VV}^*C_{VS} + C_{AV}^*C_{AS}), \\G_2 &= 2m_t(s^2/e^4)(C_{VV}^*C_{AS} + C_{AV}^*C_{VS}), \\G_3 &= 2m_t(s^2/e^4)(C_{VA}^*C_{VS} + C_{AA}^*C_{AS}), \\G_4 &= 2m_t(s^2/e^4)(C_{VA}^*C_{AS} + C_{AA}^*C_{VS}).\end{aligned}$$

$D_{S,P,SP}$  are coefficients which are usually neglected and other coefficients are defined in the same way as in ref.[32].





# APPENDIX B

## THE SPIN VECTORS AND FOUR-MOMENTA

The definition of the spin vectors and four-momenta in the parton center-of-mass system is given in this appendix.

We define the spin four-vectors,  $s_t, s_{\bar{t}}$ , in the parton center-of-mass system (CMS) in terms of a spin angle  $\xi$  as it is illustrated in Ref[45].

$$s_t = \frac{1}{\gamma_t}(\beta_t \cos \xi; \gamma_t \sin \xi, 0, -\cos \xi) \quad (\text{B.1})$$

$$s_{\bar{t}} = \frac{1}{\gamma_t}(\beta_t \cos \xi; \gamma_t \sin \xi, 0, \cos \xi) \quad (\text{B.2})$$

with  $\gamma_t = \sqrt{1 - \beta_t^2}$ .

Here, we set as  $\xi = \pi$  because we calculated in the helicity basis.

In the parton CMS with the z-axis chosen to be along the top quark direction of motion, the four-momenta read as follows:

$$\begin{aligned} p_t &= \frac{\sqrt{\hat{s}}}{2}(1; 0, 0, \beta_t) \\ p_{\bar{t}} &= \frac{\sqrt{\hat{s}}}{2}(1; 0, 0, -\beta_t) \\ p_a &= \frac{\sqrt{\hat{s}}}{2}(1; \sin \hat{\theta}, 0, \cos \hat{\theta}) \\ p_b &= \frac{\sqrt{\hat{s}}}{2}(1; -\sin \hat{\theta}, 0, -\cos \hat{\theta}). \end{aligned} \quad (\text{B.3})$$



# APPENDIX C

## THE $CP$ TRANSFORMATION OF THE NON-STANDARD COUPLINGS

In this appendix, we perform the  $CP$  transformation of the non-standard couplings discussed in this thesis and discuss the origin of the  $CP$ -violation with these couplings.

By referring to the relations of (2.51), we can perform  $CP$  transformation of the every possible combinations of spinor bilinears. In the following, we perform the  $CP$  transformation of non-standard couplings of top quark.

### (I) The $CP$ transformation of most general couplings of top quark

Though this coupling was discussed in Section 4.2, here we replace some parameters by simple one. Then, under  $CP$ -transformation, this coupling transforms as:

$$\frac{g}{2} \bar{u}(p_t, s_t) \left[ \gamma^\mu \{A_v + B_v \gamma_5\} + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t} (C_v + D_v \gamma_5) \right] v(p_{\bar{t}}, s_{\bar{t}}), \quad (\text{C.1})$$

$\downarrow CP$

$$-\frac{g}{2} \bar{u}(p_t, s_t) \left[ \gamma_\mu \{A_v + B_v \gamma_5\} + \frac{(p_t - p_{\bar{t}})_\mu}{2m_t} (C_v - D_v \gamma_5) \right] v(p_{\bar{t}}, s_{\bar{t}}). \quad (\text{C.2})$$

By comparing Eq. (C.1) with Eq. (C.2), one can see that nonzero  $D_v$  induces the  $CP$ -violation because  $D_v$  term has an opposite sign to other terms.

### (II) The $CP$ transformation of the couplings which include the anomalous chromomagnetic and chromoelectric couplings

This coupling is used in Section 4.3. Using another relation of  $CP$  transformation, *i.e.*  $\bar{\psi}_1(x) \sigma_{\mu\nu} \gamma_5 \psi_2(x) \rightarrow \bar{\psi}_2(\tilde{x}) \sigma^{\mu\nu} \gamma_5 \psi_1(\tilde{x})$ , under  $CP$  transformation, we can transform this coupling as:

$$g_s \bar{u}(p_t, s_t) \left[ \gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_t} q_\nu (\kappa + i\tilde{\kappa} \gamma_5) \right] v(p_{\bar{t}}, s_{\bar{t}}) \quad (\text{C.3})$$

$$\begin{aligned} & \downarrow CP \\ & -g_s \bar{u}(p_t, s_t) \left[ \gamma_\mu + \frac{i\sigma_{\mu\nu} q^\nu}{2m_t} (\kappa - i\tilde{\kappa}\gamma^5) \right] v(p_{\bar{t}}, s_{\bar{t}}) \end{aligned} \quad (\text{C.4})$$

Then,  $\tilde{\kappa}$  can be a source of  $CP$ -violation because of the same reason in (I) above.

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