Essays on International Trade with Imperfect Competitionand Increasing Returns to Scale

Fujiwara, Kenji

(Degree)
博士（経済学）
(Date of Degree)
2006-03-25
(Date of Publication)
2008-07-17
(Resource Type)
doctoral thesis
(Report Number)
甲3605
(URL)
https://hdl.handle.net/20.500.14094/D1003605

Essays on International Trade with Imperfect Competition and Increasing Returns to Scale

Kenji Fujiwara

Graduate School of Economics, Kobe University

Committee Members

Professor Koji Shimomura (supervisor)
Professor Noritsugu Nakanishi
Associate Professor Toru Kikuchi

Contents

1 Preface **5**
1.1 Introduction 6
1.2 Chapter 2 6
1.3 Chapter 3 8
1.4 Chapter 4 10
1.5 Chapter 5 11

2 Recent Developments on Gains and Losses from Trade: A Selective Survey **15**
2.1 Introduction 16
2.2 Example 1: Shy (1988) 19
2.2.1 An Autarkic Equilibrium 19
2.2.2 A Free Trade Equilibrium 23
2.2.3 Pareto Inferior Trade 26
2.3 Example 2: Willmann (2004) 27
2.3.1 An Autarkic Equilibrium 27
2.3.2 A Free Trade Equilibrium 31
2.3.3 Impossibility of Pareto Gains from Trade 32
2.4 Concluding Remarks 36

3 Gains from Free Trade with International Oligopoly and Increasing Returns **45**
3.1 Introduction 46
3.2 An Autarkic Equilibrium 48
3.3 Losses from Laissez-Faire Free Trade 52
3.4 Impossibility of Trade Gains with Lump-Sum Transfers 56
3.5 Gains from Trade with Non-Lump-Sum Transfers 59
3.6 Multilateral Gains from Trade 64
3.7 Concluding Remarks 67

4 An Oligopolistic Heckscher-Ohlin Theorem with Increasing Returns to Scale **73**
4.1 Introduction 74

4.2 A Model 76
4.3 An Oligopolistic Heckscher-Ohlin Theorem 81
4.4 Concluding Remarks 86

5 Two Heckscher-Ohlin Theorems with Utility Maximizing Oligopolists 93
5.1 Introduction 94
5.2 The Modified Heckscher-Ohlin Theorem 96
5.3 The Heckscher-Ohlin Theorem 104
5.4 Concluding Remarks 109

Chapter 1

Preface

Preface

1.1 Introduction

Since such authors as J. Brander, A. Dixit, E. Helpman and P. Krugman pioneered imperfectly competitive models of international trade, there has been a growing work on trade patterns, gains from trade, and trade policies under imperfect competition and increasing returns to scale. For example, the theory of strategic trade policies and intra-industry trade under monopolistic competition or oligopoly with market segmentation is now standardized in undergraduate and graduate textbooks.

Despite such a development of theory, we must recognize that there are still many unexplored topics. The purpose of this thesis is to pick up a few of them and to give a theoretical answer to them. The brief content of each chapter is as follows.

1.2 Chapter 2

Chapters 2 and 3 concern one of the most classical themes in trade theory: potential gainfulness and harmfulness of trade. It is firmly established that the opening of free trade potentially benefits all the trading countries as long as the Arrow-Debreu economies are assumed. This implies that the gains-from-trade proposition may easily break down in the presence of market distortion. This view has brought numerous papers on gains and losses from trade under various types of market distortions.

Among others, Chapter 2 specializes to surveying the existing literature on losses from trade in incomplete markets and in a dynamic context. As to the former literature, Wong (1995) has already provided a brief survey. However, he developed no formal model by illustrating a simple diagrammatic example. This chapter complements his treatment with the aid of a formal mathematical model.

One notable result with incomplete markets is Pareto inferior trade shown by Newbery and Stiglitz (1984) and later generalized by Shy (1988). Pareto inferior trade refers to the situation in which every agent in every country loses from trade.[1] Their argument is striking and casts a serious doubt on the applicability of the gains-from-trade proposition. However, Kemp and Wong (1995) made a counter-argument by proving that it remains possible to make nobody worse off in free trade than in autarky via lump-sum compensation. This chapter will retrace this strand of research on Pareto inferior trade by generalizing Shy's (1988) model.

Quite recently, another challenge on gains from trade was submitted by Willmann (2004). He considers a two-stage model with endogenous la-

[1]Similar results are known to be obtained in overlapping generations models as well. Wong (1995) gave a formal model and fully explored this case unlike the case of missing markets, whereby we omit surveying it in this thesis.

bor supply. In his setting, the extreme situation like Newbery and Stiglitz (1984) and Shy (1988) does not appear in the sense that the opening up of free trade brings gainers and losers. However, he numerically proves that it is impossible to make everyone better off under free trade relative to autarky by resorting to lump-sum compensation, which crucially depends on one of his artificial assumptions. Nevertheless, his argument is noteworthy and the latter half of the chapter reviews his argument by the help of a generalized model.

1.3 Chapter 3

Since the establishment of the General Agreement of Tariffs and Trade (GATT) and its successor the World Trade Organization (WTO), multilateral trade liberalization has been steadily proceeding. On the other hand, some non-governmental organizations (NGOs) and countries have been persistently against free trade. For what reasons, do they oppose to it? This chapter answers this question by focusing on the role of oligopoly and increasing returns both of which feature an important aspect of modern world trade.

A two-country model of international duopoly and scale economies is constructed. Each country consists of two types of agents named a consumer and a monopolist. Each country's monopolist differs in its marginal cost and fixed cost and plays a Cournot-Nash game in the integrated market under free trade. It will be shown that both the consumer and the monopolist with a higher fixed cost lose from trade.

Furthermore, the above situation can not be remedied via a lump-sum compensation scheme. Someone may criticize that this result is trivial but it is not true because it is in a sharp contract to Kemp and Wong's (1995) gains-from-trade proposition. As mentioned in introducing Chapter 2, they proved the existence of *lump-sum* compensation that can make everyone better off under free trade than under autarky in incomplete markets and overlapping generations even if laissez-faire free trade is Pareto inferior. That is, lump-sum compensation is a nice policy instrument to rescue a country's losers in incomplete markets or overlapping generations, while it is inefficacious under imperfect competition and increasing returns. We believe that our losses-from-trade possibility can answer the question raised on the top of this section.

The above proposition seems to deny the recent progress of trade liberalization. However, the latter half of the chapter proves that there remains possible to make everyone better off under free trade relative to autarky. The key element is *non-lump-sum* redistribution. The redistributive scheme suggested there will take a non-lump-sum form in the sense that it is a function of the oligopolistic firms' strategic variables. In other words, oligopolists' behavior affects the amount of transfers. If this type of scheme is employed, free trade can be Pareto-improving for the country which experiences losses from trade.[2]

Proving the gains-from-trade proposition, we will emphasize that the concept of compensation is an inappropriate interpretation for our scheme. Rather, the scheme can be referred to an incentive scheme because it changes the strategic position of the country's monopolist favorably. That is, the scheme has an aspect of Brander and Spencer's (1985) subsidy in the sense that it shifts the monopolist's reaction curve outward. However, one big difference between Brander and Spencer (1985) and this chapter lies in that we precluded any strategic interaction between the countries' government; no government has a rent-shifting motive in our treatment.

1.4 Chapter 4

The topic treated in Chapters 4 and 5 is that of trade patterns under imperfect competition and increasing returns to scale. Chapter 4, which is based on a joint work with Koji Shimomura, examines the robustness of the Heckscher-Ohlin trade pattern by constructing a two-country, two-good, two-factor model of international duopoly with increasing returns to scale. The model is a straightforward extension of Markusen's (1981)

[2]This gains-from-trade proposition is first established by Kemp and Shimomura (2001).

seminal work.

In Markusen (1981), the difference in relative factor abundance was precluded, whereas this chapter allows for the difference in factor endowment ratios. Even then, it is shown that the Heckscher-Ohlin trade pattern is robust in a rough sense; a capital-abundant country exports the capital-intensive good. We must stress that the interpretation of *relative factor abundance* is different from the standard meaning, thereby our proposition can be called a 'modified' Heckscher-Ohlin theorem.[3]

1.5 Chapter 5

One of the recent hottest topics in trade theory is the implication of non-homothetic preferences for trade patterns and income inequality across countries. According to Trefler (1995), the empirical performance of the Heckscher-Ohlin theorem is poor for at least two reasons. First, the standard Heckscher-Ohlin model assumes identical technologies among countries and second, homothetic preferences are assumed. In general, it is extremely difficult to relax them simultaneously since the analysis becomes too complicated. Then, what can we say about the trade patterns by relaxing each of these two assumptions?

On the other hand, there remains another dissatisfaction in the theory

[3]The details are left to the main text.

of trade patterns. To our knowledge, there is no paper that proves the exact the Heckscher-Ohlin theorem under oligopoly with the number of oligopolists fixed. Although Chapter 4 partially resolves this dissatisfaction, it does not show the exact Heckscher-Ohlin theorem.

This chapter attempts to overcome these two difficulties by employing an argument by Kemp and Shimomura (1995, 2002). Following them, this chapter assumes that each oligopolist maximizes its utility rather than profit. This change in assumption, together with the assumptions of quasi-linear preferences, will make the exact Heckscher-Ohlin valid even under restricted entry oligopoly. This chapter is of some use in establishing a helpfulness of the Kemp-Shimomura (1995, 2002) argument.

Bibliography

[1] Brander, J. A. and B. J. Spencer (1985), "Export Subsidies and International Market Share Rivalry", *Journal of International Economics*, 16, 83-100.

[2] Kemp, M. C. and K. Shimomura (1995), "The Apparently Innocuous Representative Agent", *Japanese Economic Review*, 46, 247-256.

[3] Kemp, M. C. and K. Shimomura (2001), "Gains from Trade in a Cournot-Nash General Equilibrium", *Japanese Economic Review*, 52, 284-302.

[4] Kemp, M. C. and K. Shimomura (2002), "A New Theory of International Trade under Increasing Returns: The Two-Commodities Case", in A. D. Woodland (ed.), *Economic Theory and International Trade: Essays in Honor of Murray C. Kemp*, Aldershot, Hants: Edward Elgar, 3-21.

[5] Kemp, M. C. and K. Wong (1995), 'The Gains from Trade when Markets Are Possibly Incomplete', in M. C. Kemp, *The Gains from Trade*

and the Gains from Aid, Routledge, London, 47-71.

[6] Newbery, D. G. M. and J. E. Stiglitz (1984), "Pareto Inferior Trade", *Review of Economic Studies*, 51, 1-12.

[7] Shy, O. (1988), "A General Equilibrium Model of Pareto Inferior Trade", *Journal of International Economics*, 25, 143-154.

[8] Trefler, D. (1995), "The Case of the Missing Trade and Other Mysteries", *American Economic Review*, 85, 1029-1046.

[9] Willmann, G. (2004), "Pareto Gains from Trade: A Dynamic Counterexample", *Economics Letters*, 83, 199-204.

[10] Wong, K. (1995), *International Trade in Goods and Factor Mobility*, Cambridge, MA: MIT Press.

Chapter 2

Recent Developments on Gains and Losses from Trade: A Selective Survey

Recent Developments on Gains and Losses from Trade: A Selective Survey

2.1 Introduction

Do all agents in a country gain from the opening of free trade? This has been one of the central questions in trade theory as well as economics. The answer depends on what criterion is employed to evaluate the welfare effect of free trade. As long as we are confined to the Pareto criterion, the answer is negative. For example, the Stolper-Samuelson theorem tells us that trade benefits the owners of factors which are abundant in the country while it worsens the owners of scarce factors. Note that this statement is based on the presumption that no income redistribution among factor owners is accompanied. This implies that it may be possible to make nobody worse off in free trade than in autarky when an appropriate compensation scheme is allowed for. Such an idea is called the compensation principle, which enables us to say that free trade *can* benefit all agents. The first proof of this statement was made by Samuelson (1939) and Kemp (1962) and much more rigorous and general proofs were provided by Kemp and Wan (1972) and Grandmont and McFadden (1972).

One common assumption shared by these papers was the Arrow-Debreu economy: perfect competition, constant returns to scale, complete markets, finite agents and goods, and so on. Needless to say, it is very severe and hard to accept as a description of the real world. Shortly after the publication of the above papers, some authors attempted to examine the robustness of the gains-from-trade proposition in non-Arrow-Debreu economies. This branch of literature revealed that losses from trade can occur in uncertainty (e.g., Batra and Russel, 1976), increasing returns to scale (e.g., Ethier, 1982 and Markusen and Melvin, 1981) and imperfect competition (e.g., Markusen, 1981). However, most papers adopted the assumption of social utility functions and ignored the income distribution issue among agents.

Therefore, it remained an open question whether all heterogeneous agents in a country or every country lose from trade.[1] It is Newbery and Stiglitz (1984) that first pointed out this possibility in a model with market incompleteness. Their result was based on a partial equilibrium model and was shown by computer simulations. Later, constructing a general equilibrium model, Shy (1988) analytically proved that free trade worsens all agents' utility in all countries, which is sometimes called Pareto inferior trade. To counter this pessimistic result, Kemp and Wong (1995) showed

[1] Kemp and Shimomura (2002) report the literature on losses from trade under the Arrow-Debreu assumption. However, according to Kemp and Shimomura (2002), the arguments of such literature rest on a specific assumption which is not imposed in the standard gains-from-trade proposition and can be reversed by relaxing it. Indeed, although Tompkinson (1999) once showed the impossibility of potential gains from trade when agents have preferences over occupations, Kemp and Shimomura (2000) proved the potential gainfulness of trade when Tompkinson's specific assumption is rejected.

the existence of a compensation scheme which makes nobody worse off.[2]

On the other hand, Willmann (2002, 2004) offered another example in which all agents in a country lose from trade. His model is a two-period general equilibrium model where each consumer determines to become skilled or unskilled labor in the first period and in the second period, it obtains a higher or lower income depending on the predetermined choice. In this example, it is shown that it is impossible to make nobody worse off in free trade than in autarky when the scheme is introduced *only* in the second period. However, if the scheme is implemented in both periods, all agents can gain from trade. Willmann's (2002, 2004) example is of interest since it is the first to clarify the importance of dynamic contexts in discussing gains from trade.

This paper reviews these two strands of literature on losses from trade. Since our interest is confined to the potential gainfulness of trade, the literature with social utility functions or representative agents is precluded from consideration. Section 2 considers the model with incomplete markets by Shy (1988) and Section 3 treats the example by Willmann (2002, 2004).

[2]A similar result is known to be valid in overlapping generations economies in which the finiteness of agents is violated. This paper does not deal with this branch of literature but the interested reader is referred to Kemp (1995) and Wong (1995).

2.2 Example 1: Shy (1988)

The first example of losses from trade is Shy's (1988) model. He incorporated uncertainty into a two-country Ricardian model to derive the result of Pareto inferior trade, i.e., all agents in all countries become worse off moving from autarky to free trade.

2.2.1 An Autarkic Equilibrium

Suppose a single country producing and consuming two goods (goods X and Y). Both goods are produced from labor which is the only primary factor. Good Y is taken as a numeraire and is produced from one unit of labor. The production of good X is subject to productivity uncertainty such that one unit of labor produces θ_H units of good X with probability of $\pi \in [0,1]$ and θ_L units with probability of $1-\pi$. Without loss of generality, $\theta_H > \theta_L$ is assumed, i.e., θ_H represents a high productivity. Assuming perfect competition in all markets and letting w_Y and w_{XH} (resp. w_{XL}) denote the wage rate in sector Y and sector X with the productivity of θ_H (resp. θ_L), the conditions for the interior profit-maximizing solutions are obtained as

$$\begin{aligned} w_Y &= 1 \\ w_{XH} &= \theta_H p_H \quad \text{with probability of } \pi \end{aligned} \tag{2.1}$$

$$w_{XL} \quad = \quad \theta_L p_L \quad \text{with probability of } \ 1-\pi, \tag{2.2}$$

where p_H (resp. p_L) is the price of good X with the productivity θ_H (resp. θ_L). Labor endowment is normalized to one and $\alpha \in (0,1)$ is employed in sector X and $1-\alpha$ is employed in sector Y. α is endogenously determined and how it is determined is explained later. Since α is employed in sector X and the rest of labor goes to sector Y, the national income, which is denoted by I, is given by

$$I = \begin{cases} \alpha w_{XH} + 1 - \alpha & \text{with probability of } \ \pi \\ \alpha w_{XL} + 1 - \alpha & \text{with probability of } \ 1-\pi \end{cases}. \tag{2.3}$$

Regarding the preference, all the agents have an identically homothetic utility function:[3]

$$U = \frac{1}{1-\rho}[u(C_X, C_Y)]^{1-\rho}, \quad \rho > 1$$

where C_X and C_Y are the consumption of each good. From this specification and making use of (2.3), the market-clearing condition for good X under autarky becomes

$$\frac{e'(p_H)}{e(p_H)}(\alpha w_{XH} + 1 - \alpha) \quad = \quad X \quad \text{with probability of } \ \pi \tag{2.4}$$

$$\frac{e'(p_L)}{e(p_L)}(\alpha w_{XL} + 1 - \alpha) \quad = \quad X \quad \text{with probability of } \ 1-\pi, \tag{2.5}$$

[3]In the original paper of Shy (1988), a Cobb-Douglas utility function is employed for simplicity. As he pointed out, it is an over-specification for deriving the main result. However, it facilitates and simplifies the analysis so drastically that we will sometimes use it in later arguments.

where X is the supply of good X and the function $e(p)$ is defined as

$$e(p) \equiv \min_{C_X, C_Y} \left\{ pC_X + C_Y \mid u(C_X, C_Y) \geq 1 \right\}.$$

The autarkic equilibrium is fully described by introducing the labor market-clearing condition:

$$\begin{aligned} \frac{X}{\theta_H} + Y &= 1 \quad \text{with probability of } \pi \\ \frac{X}{\theta_L} + Y &= 1 \quad \text{with probability of } 1 - \pi. \end{aligned}$$

Let us solve the model following Shy's (1988) method. Substituting the production function $X = \alpha\theta_H$ and (2.1) into (2.4), we have

$$\frac{e'(p_H)}{e(p_H)}(\alpha\theta_H p_H + 1 - \alpha) = \alpha\theta_H.$$

This expression is rearranged to

$$\frac{e(p_H) - e'(p_H)p_H}{e'(p_H)} = \frac{1-\alpha}{\alpha\theta_H},$$

and solving for p_H yields $p_H = P((1-\alpha)/(\alpha\theta_H))$ with $P'(\cdot) > 0$. Now, suppose that the utility function takes a Cobb-Douglas form:[4]

$$U = \frac{1}{1-\rho}\left(a^{-a}(1-a)^{a-1}C_X^a C_Y^{1-a}\right)^{1-\rho}, \quad a \in (0,1),$$

then, p_H is determined by

$$p_H = \frac{a(1-\alpha)}{(1-a)\alpha\theta_H}.$$

[4]Note that the corresponding indirect utility function is given by $p^{-a}w$.

Thus, the equilibrium wage rate becomes

$$w_H = \theta_H p_H = \frac{a(1-\alpha)}{(1-a)\alpha}. \tag{2.6}$$

Through the same procedure, the equilibrium wage rate corresponding to the productivity θ_L is

$$w_L = \frac{a(1-\alpha)}{(1-a)\alpha}. \tag{2.7}$$

In the arguments so far, it has been assumed that α is given. Following Shy (1988), the equilibrium value of α is determined when the expected utility of the workers in sector X and that in sector Y are equalized. Making use of the function $e(p)$, the corresponding indirect utility function is given by

$$\frac{1}{1-\rho}\left[\frac{w}{e(p)}\right]^{1-\rho}.$$

Substituting the equilibrium values of wage and commodity price and denoting the expected utility of the workers in sector X by EU_X, it is obtained as

$$\begin{aligned} EU_X &= \pi\frac{1}{1-\rho}\left[\frac{w_H}{e(p_H)}\right]^{1-\rho} + (1-\pi)\frac{1}{1-\rho}\left[\frac{w_L}{e(p_L)}\right]^{1-\rho} \\ &= \frac{1}{1-\rho}\left\{\pi\left[\frac{a(1-\alpha)}{(1-a)\alpha e(p_H)}\right]^{1-\rho} + (1-\pi)\left[\frac{a(1-\alpha)}{(1-a)\alpha e(p_L)}\right]^{1-\rho}\right\}. \end{aligned}$$

Since those who are employed in sector Y earn 1 unit of wage, their expected utility denoted by EU_Y becomes

$$EU_Y = \frac{1}{1-\rho}\left\{\pi\left[\frac{1}{e(p_H)}\right]^{1-\rho} + (1-\pi)\left[\frac{1}{e(p_L)}\right]^{1-\rho}\right\}.$$

Equating EU_X with EU_Y, we have

$$\pi\left[\frac{a(1-\alpha)}{(1-a)\alpha e(p_H)}\right]^{1-\rho}+(1-\pi)\left[\frac{a(1-\alpha)}{(1-a)\alpha e(p_L)}\right]^{1-\rho}=\pi\left[\frac{1}{e(p_H)}\right]^{1-\rho}+(1-\pi)\left[\frac{1}{e(p_L)}\right]^{1-\rho},$$

from which α is determined as

$$\frac{a(1-\alpha)}{(1-a)\alpha}=1,$$

that is, $\alpha = a$. Substituting back this into (2.6) and (2.7) yields

$$w_H = w_L = 1,$$

and it follows that $p_H = 1/\theta_L$ and $p_L = 1/\theta_L$. Further substitution of these equilibrium values into EU_X or EU_Y, the expected utility in an autarkic equilibrium is

$$EU^A=\frac{1}{1-\rho}\left\{\pi\left[\frac{1}{e\left(\frac{1}{\theta_H}\right)}\right]^{1-\rho}+(1-\pi)\left[\frac{1}{e\left(\frac{1}{\theta_L}\right)}\right]^{1-\rho}\right\}, \tag{2.8}$$

where the superscript A stands for the autarkic equilibrium.

2.2.2 A Free Trade Equilibrium

Let us turn to the analysis of free trade. Shy (1988) assumes that two countries are distinguished according to the realization of θ_H and θ_L. Thus, each country's national income becomes

$$\begin{aligned} I_H &= \alpha w_H + 1 - \alpha \\ I_L &= \alpha w_L + 1 - \alpha, \end{aligned}$$

where I_H (resp. I_L) is the national income in the country of the productivity θ_H (res. θ_L). Therefore, the aggregate demand for good X under free trade is

$$\frac{e'(p)}{e(p)}(I_H + I_L) = \frac{e'(p)}{e(p)}[\alpha(\theta_H + \theta_L)p + 2(1-\alpha)].$$

Since the production of each country is respectively given by $\alpha\theta_H$ and $\alpha\theta_L$, the world market-clearing condition is described by

$$\frac{e'(p)}{e(p)}[\alpha(\theta_H + \theta_L)p + 2(1-\alpha)] = \alpha(\theta_H + \theta_L).$$

Solving for p, it satisfies

$$\frac{e(p) - e'(p)p}{e'(p)} = \frac{2(1-\alpha)}{\alpha(\theta_H + \theta_L)},$$

which gives the equilibrium price as

$$p = P\left(\frac{2(1-\alpha)}{\alpha(\theta_H + \theta_L)}\right).$$

The functional form of $P(\cdot)$ depends on the specification of the utility function. When it is given by a Cobb-Douglas form, p is simplified to

$$p = \frac{2(1-\alpha)}{(1-a)\alpha(\theta_H + \theta_L)}, \tag{2.9}$$

and the wage rate in each country becomes

$$\begin{aligned} w_H &= \theta_H p = \frac{2(1-\alpha)\theta_H}{(1-a)\alpha(\theta_H + \theta_L)} && (2.10)\\ w_L &= \theta_l p = \frac{2(1-\alpha)\theta_L}{(1-a)\alpha(\theta_H + \theta_L)}. && (2.11) \end{aligned}$$

In order to determine the equilibrium value of α, we derive the expected utility of each worker. The expected utility of the workers in sectors X and Y is given by

$$\begin{aligned} EU_X &= \frac{1-\rho}{1}\pi\left[\frac{w_H}{e(p)}\right]^{1-\rho} + \frac{1}{1-\rho}(1-\pi)\left[\frac{w_L}{e(p)}\right]^{1-\rho} \\ EU_Y &= \frac{1}{1-\rho}\left[\frac{1}{e(p)}\right]^{1-\rho}. \end{aligned}$$

α is determined by equating EU_X with EU_Y:

$$\frac{1}{1-\rho}\pi\left[\frac{w_H}{e(p)}\right]^{1-\rho} + \frac{1}{1-\rho}(1-\pi)\left[\frac{w_L}{e(p)}\right]^{1-\rho} = \frac{1}{1-\rho}\left[\frac{1}{e(p)}\right]^{1-\rho},$$

or equivalently,

$$\pi w_H^{1-\rho} + (1-\pi)w_L^{1-\rho} = 1.$$

Substituting (2.10) and (2.11) into this equation, we have

$$\pi\left[\frac{2(1-\alpha)\theta_H}{(1-a)\alpha(\theta_H+\theta_L)}\right]^{1-\rho} + (1-\pi)\left[\frac{2(1-\alpha)\theta_L}{(1-a)\alpha(\theta_H+\theta_L)}\right]^{1-\rho} = 1.$$

Solving for α, its equilibrium value is determined by

$$\alpha = \left\{\frac{(1-a)(\theta_H+\theta_L)}{2}\left[\pi\theta_H^{1-\rho} + (1-\pi)\theta_L^{1-\rho}\right]^{\frac{1}{\rho-1}} + 1\right\}^{-1}.$$

Substituting back this into (2.9) and some messy rearrangements yield

$$p = \left[\pi\theta_H^{1-\rho} + (1-\pi)\theta_L^{1-\rho}\right]^{\frac{1}{\rho-1}}. \tag{2.12}$$

Further substitution of (2.12) into EU_X or EU_Y yields the equilibrium expected utility under free trade as

$$EU^T = \frac{1}{1-\rho}\left[\frac{1}{e\left(\left[\pi\theta_H^{1-\rho} + (1-\pi)\theta_L^{1-\rho}\right]^{\frac{1}{\rho-1}}\right)}\right]^{1-\rho}. \tag{2.13}$$

where the superscript T stands for the free trade equilibrium.

2.2.3 Pareto Inferior Trade

Based on the preliminaries above, let us derive the main result. It is summarized in:

Proposition 1 (Shy, 1988). *All agents in all countries become worse off in free trade than in autarky, i.e., free trade is Pareto inferior to autarky.*

Proof. All we have to do is to show $EU^T < EU^A$. From (2.8) and (2.13), this inequality is equivalent to

$$\left[\frac{1}{e\left(\left[\pi\theta_H^{1-\rho}+(1-\pi)\theta_L^{1-\rho}\right]^{\frac{1}{\rho-1}}\right)}\right]^{1-\rho} > \pi\left[\frac{1}{e\left(\frac{1}{\theta_H}\right)}\right]^{1-\rho} + (1-\pi)\left[\frac{1}{e\left(\frac{1}{\theta_L}\right)}\right]^{1-\rho}. \tag{2.14}$$

To show this inequality, define the function $f(x) \equiv [1/e(x)]^{1-\rho}$. Then, we see that $f'(x) > 0,\ f''(x) < 0$ due to $\rho > 1$. And we can also see that

$$\frac{1}{\theta_H} < \left[\pi\theta_H^{1-\rho}+(1-\pi)\theta_L^{1-\rho}\right]^{\frac{1}{\rho-1}} < \frac{1}{\theta_L}.$$

The concavity of $f(x)$ and this inequality implies the inequality (2.14).

Proposition 1 is striking since it asserts that everyone in all countries becomes a loser from trade. It goes without saying that this proposition casts a serious doubt on the desirability of trade liberalization. However, Kemp and Wong (1995) provided a counter-argument to Shy (1988) by demonstrating the existence of lump-sum compensation such that nobody is worsened by laissez-faire free trade.

2.3 Example 2: Willmann (2004)

The second example of losses from trade is proposed by Willmann (2002, 2004) that shares similar aspects to Shy's (1988) model considered in the previous section. Therefore, we shall use the same notations and functions as before.

2.3.1 An Autarkic Equilibrium

Let us begin with describing a single country's autarkic equilibrium. Suppose a country which produces and consumes two goods both of which are produced by only one factor, labor.[5] Labor is divided into two types: skilled labor denoted by L_X and unskilled labor denoted by L_Y. Good Y, which is taken as a numeraire, is produced by one unit of unskilled labor, which implies that the wage rate of unskilled labor is one. On the other hand, the production function of good X is given by

$$X = \frac{L_X}{c}, \quad c > 0.$$

[5]The model presented here is not exactly the same as Willmann's (2002, 2004) but the essence of his argument is not lost and our modification is only for explanatory convenience.

Letting p denote the price of good X in terms of good Y, the profit maximization condition under perfect competition becomes

$$w = \frac{p}{c},$$

where w is the wage rate of skilled labor.

The consumer side is formulated as follows. As in Shy (1988), let there be a continuum of consumers in the closed interval, $[0, 1]$. Agent $b \in [0, 1]$ is endowed with one unit of consumption good. Occupational choice is endogenized but its determination is different from Shy (1988). Consider a two-stage setting where each agent chooses to be skilled labor or unskilled labor in the stage 1 while it consumes both goods to maximize utility in stage 2.

Let us set up stage 1 in which occupational choice is made. In order to become skilled labor, agent b must sacrifice $1 - b$ amount of its endowment. Therefore, the agent labeled 1 need not sacrifice any amount of its endowment and agent 0 must invest all of its endowment to be skilled. Thus, it is fair to say that agent 1 is the most talented one whereas agent 0 is the least talented. Following Willmann (2002, 2004), agent b's utility in stage 1 equals its consumption; if agent b invests, its utility is b while if it does not invest, its utility is unity.

In stage 2, each agent is endowed with one unit of skilled labor if it invests in stage 1. Similarly, it is endowed with one unit of unskilled labor if it sacrifices nothing. From this assumption and supposing that all agents have an identically homothetic utility function, the indirect utility function for skilled and unskilled labor is[6]

$$\frac{w}{e(p)} \qquad \text{if it chooses to be skilled} \tag{2.15}$$

$$\frac{1}{e(p)} \qquad \text{if it chooses to be unskilled.} \tag{2.16}$$

Intertemporal utility is assumed to be defined as a product of consumption in stage 1 and indirect utility in stage 2, which implies that each agent's intertemporal utility is equal to

$$1 \cdot \frac{1}{e(p)} \qquad \text{if it chooses to be unskilled} \tag{2.17}$$

$$b \cdot \frac{w}{e(p)} \qquad \text{if it chooses to be skilled.} \tag{2.18}$$

The critical value of b, investment level, is determined when the intertemporal utility obtained from choosing to be skilled and that from choosing to be unskilled are equalized:

$$\frac{1}{e(p)} = \frac{bw}{e(p)} \quad \Longrightarrow \quad b = \frac{1}{w}. \tag{2.19}$$

Due to the fact of $b \in [0, 1]$, $w > 1$ follows, i.e., the wage rate for skilled labor is always greater than that of unskilled labor.

[6]Note again that the indirect utility function associated with a homothetic utility function is given by [income]/$e(p)$, where $e(p)$ is the unit expenditure function.

The model of autarky is closed by introducing the market-clearing condition for good X. In stage 2, each skilled labor's demand for good X is given by $e'(p)w/e(p)$ and each unskilled labor's counterpart is given by $e'(p)/e(p)$. The population of skilled labor is $1-b$ and that of unskilled is b. Therefore, the aggregate demand for good X becomes

$$\begin{aligned} & (1-b)\frac{e'(p)}{e(p)}w + b\frac{e'(p)}{e(p)} \\ = \;& \frac{w-1}{w}\frac{e'(p)}{e(p)}w + \frac{1}{w}\frac{e'(p)}{e(p)} \\ = \;& \frac{e'(p)}{e(p)}\frac{w^2-w+1}{w}, \end{aligned}$$

where the two equalities are come from (2.19). On the other hand, the supply of good X is determined through the marker-clearing condition for skilled labor:

$$cX = L_X = 1-b = \frac{w-1}{w},$$

which gives

$$X = \frac{w-1}{wc}.$$

Equating the aggregate demand and supply and some arrangements yield

$$\frac{ce'(p)}{e(p)} = \frac{w-1}{w^2-w+1}. \tag{2.20}$$

Another equilibrium condition is the profit maximization one:

$$p = wc. \tag{2.21}$$

In the autarkic equilibrium, (2.20) and (2.21) determine p and w. Such an equilibrium variable under autarky is attached an upper-bar in what follows. This completes the description of autarky.

2.3.2 A Free Trade Equilibrium

Let this country freely trade with the rest of the world. Following Willmann's (2002, 2004) original presumption, it is assumed that this country, which is a small open country, faces a free trade price denoted by $p^* > \overline{p}$. Due to this change, the wage rate for skilled labor proportionately increases from $\overline{w} = \overline{p}/c$ to $w^* = p^*/c$. Obviously, we can observe that any worker in sector X gains while that in sector Y loses since each worker's indirect utility in free trade is

$$\begin{aligned} b \cdot \frac{w^*}{e(p^*)} = b \cdot \frac{\frac{p^*}{c}}{e(p^*)} > b \cdot \frac{\frac{\overline{p}}{c}}{\overline{p}} = b \cdot \frac{\overline{w}}{e(\overline{p})} \qquad & \text{for skilled labor} \\ \frac{1}{e(p^*)} < \frac{1}{e(\overline{p})} \qquad & \text{for unskilled labor,} \end{aligned}$$

where $w^*/e(p^*) > \overline{w}/e(\overline{p})$ follows from that $p/e(p)$ is increasing in p. Therefore, the opening of free trade brings gainers and losers without any redistributive scheme. And this regime change makes the critical value of b lower than the autarkic level.

These changes in welfare are depicted in Figures 1 and 2. Figure 1 draws the relationship between b and the utility in stage 2 while Figure 2 captures the relationship between b and the intertemporal utility. As mentioned, the change from autarky to free trade lowers b at which the occupational change occurs from $\bar{b}$ to b^*. Those who are below b^* are definite losers since their intertemporal utility in free trade is short of that in autarky. The opposite holds for those who are above $\bar{b}$. On the other hand, the distributional effect on the middle party is ambiguous. That is, depending on their ability to become skilled labor, some gain from trade and the others may lose. In the next subsection, we introduce the government which implements a redistributive policy to seek a Pareto improvement and examines the possibility and impossibility of such an improvement.

2.3.3 Impossibility of Pareto Gains from Trade

It goes without saying that the biased distributional effect of free trade is attributed to the lack of any redistributive scheme and it is a natural phenomenon. According to the standard gains-from-trade proposition, even if the above situation is expected, we can find a proper redistributive scheme which makes nobody worse off in free trade than in autarky. This subsection reexamines the validity of this statement in the present framework.

First, note that we can divide all agents into three categories. The first one is those who choose to be skilled both in both autarky and free trade. The second is those who choose to be unskilled in autarky and to be skilled in free trade. The third is those who choose to be skilled in both regimes.

The compensation scheme is determined to make the intertemporal utility of each agent under post-compensation free trade equal that under free trade. That is, the amount of income transfer is calculated in such a way to equalize each agent's utility under post-scheme free trade with that under autarky, from which everyone enjoys exactly the same utility in both regimes.

Now, we impose one crucial assumption. How it is crucial will be mentioned after deriving the main result.

Assumption. *The government can implement the compensation scheme only in stage 2. In other words, the government observes a laissez-faire policy in stage 1.*

Based on this assumption, let us calculate the amount of income transfers. Letting T_1 be the amount of income compensated for the workers in the first category. Equating their autarkic utility with their post-compensation free trade utility yields

$$\frac{1}{e(\overline{p})} = \frac{1+T_1}{e(p^*)} \quad \Longrightarrow \quad T_1 = \frac{e(p^*)}{e(\overline{p})} - 1,$$

which immediately turns out to be positive, which implies that these people receive positive compensation from the government.

In a similar manner, the amount of compensation for the workers in the second category, which is represented by T_2, is obtained as

$$\frac{1}{e(\overline{p})} = \frac{b(w^* + T_2)}{e(p^*)} \quad \Longrightarrow \quad T_2 = \frac{e(p^*)}{be(\overline{p})} - w^*,$$

whose sign is ambiguous. And finally, compensation for the workers in the third category, denoted by T_3, is derived as

$$\frac{b\overline{w}}{e(\overline{p})} = \frac{b(w^* + T_3)}{e(p^*)} \quad \Longrightarrow \quad T_3 = \frac{e(p^*)\overline{w}}{e(\overline{p})} - w^*,$$

which proves to be negative, i.e., these people pay a lump-sum tax to the government by T_3.

On the basis of these preliminary analysis, we address Willmann's (2002, 2004) main result stated in:

Proposition 2 (Willmann, 2002, 2004). *It can be impossible to achieve a Pareto improvement with this compensation scheme. That is, the scheme proposed above can not make nobody worse off in free trade than autarky.*

Proof. The proof is rather simple. First of all, note that adding up T_1, T_2 and T_3 yields the government net expenditure. Hence, it is impossible to make everybody better off if and only of $T_1 + T_2 + T_3 > 0$ since this inequality violates the government budget constraint such that $T_1 + T_2 + T_3 \leq 0$.

To show the proposition, summing all the compensated income, we have

$$\begin{aligned} T_1 + T_2 + T_3 &= \int_0^{b^*} \left[\frac{e(p^*)}{e(\overline{p})} - 1 \right] db + \int_{b^*}^{\bar{b}} \left[\frac{e(p^*)}{be(\overline{p})} - w^* \right] db + \int_{\bar{b}}^{1} \left[\frac{e(p^*)\overline{w}}{e(\overline{p})} - w^* \right] db \\ &= b^* \left[\frac{e(p^*)}{e(\overline{p})} - 1 \right] + \left[\frac{e(p^*)}{e(\overline{p})} \ln b - w^* b \right]_{b^*}^{\bar{b}} + (1 - \bar{b}) \left[\frac{e(p^*)\overline{w}}{e(\overline{p})} - w^* \right]. \end{aligned}$$

Unfortunately, it is ambiguous whether this summation becomes positive or not. However, Willmann (2002, 2004) showed that it becomes positive for some parameter sets calculated numerically by assuming a Cobb-Douglas utility function. Accordingly, under certain conditions regarding the parameter on the expenditure share on good X, $T_1 + T_2 + T_3 > 0$ is established and hence a Pareto improvement is not accomplished.

Remark. We would like to emphasize here that Assumption imposed plays a crucial role in Proposition 2. Indeed, if we relax it, the proposition can easily break down as pointed out by Willmann (2002, 2004). In particular, instead of the assumption imposed above, let us assume that the government can implement the compensation scheme in stage 1. That is, the government transfers the initial endowment of each agent such that all agent's intertemporal utility is equalized between autarky and free trade. Willmann (2002, 2004) shows that the government net expenditure can be non-positive, i.e., the government has a surplus large enough to achieve a Pareto improvement with this scheme. Therefore, in this alternative scheme, the traditional gains-from-trade proposition revives. However, as Willmann (2002, 2004) commented, the government has to know each agent's ability, b, in order to determine the amount of income compensated for each agent, which is impossible in reality.

2.4 Concluding Remarks

This chapter has provided a brief survey on two papers both of which point out the theoretical possibility of losses from trade: Shy (1988) and Willmann (2002, 2004). Of course, one can find other works which study losses from trade. However, we chose only two works since our predecessors already gave a much better survey on the same topic although they are not up-to-date. For example, Pareto inferior trade in an overlapping generations economy is fully covered in Wong (1995) whereas Tompkinson's (1999) losses-from-trade proposition is intensively reviewed in Kemp and Shimomura (2000, 2002). Therefore, we believe that it is innocuous to select only two works both of which are not formally treated in the existing textbook and survey paper.

Before closing the chapter, we make a remark. Both Shy (1988) and Willmann (2002, 2004) consider an economy in which labor supply is endogenized. In Shy (1988), each agent chooses to work in the industry which yields a larger expected utility, while Willmann (2002, 2004) assumed that each agent chooses an occupation which yields a larger intertemporal utility. And, in Tompkinson (1999), each agent has a preference over occupations and its occupational choice is supposed to be endogenously determined. In this respect, we can say that these three authors share a similar idea. Moreover, all of these authors employed a standard two-good Ricardian model. However, the reason for losses from trade is different among these papers. Therefore, it is an interesting extension to construct a general model which comprises them as a special case and clarify what elements are needed to bring the losses-from-trade result. And it is a further task to prove the existence of an appropriate scheme of redistribution which makes nobody worse off under free trade than under autarky.

Bibliography

[1] Batra, R. N. and W. R. Russel (1974), "Gains from Trade under Uncertainty", *American Economic Review*, 64, 1040-1048.

[2] Ethier, W. J. (1982), "Decreasing Costs in International Trade and Frank Graham's Argument for Protection", *Econometrica*, 50, 1243-1268.

[3] Grandmont, J. M. and D. McFadden (1972), "A Technical Note on Classical Gains from Trade", *Journal of International Economics*, 2, 109-126.

[4] Markusen, J. R. (1981), "Trade and the Gains from Trade with Imperfect Competition", *Journal of International Economics*, 11, 531-551.

[5] Markusen, J. R. and J. R. Melvin (1981), "Trade, Factor Prices and the Gains from Trade with Increasing Returns to Scale", *Canadian Journal of Economics*, 14, 450-169.

[6] Newbery, D. G. M. and J. E. Stiglitz (1984), "Pareto Inferior Trade", *Review of Economic Studies*, 51, 1-12.

[7] Kemp, M. C. (1962), "The Gain from International Trade", *Economic Journal*, 72, 803-819.

[8] Kemp, M. C. (1995), *The Gains from Trade and the Gains from Aid*, London: Routledge.

[9] Kemp, M. C. and K. Shimomura (2000), "A Note on Tompkinson's Losses-from-Trade Proposition (in Japanese)", *Annals of Economics and Business Administration*, 50, 77-87.

[10] Kemp, M. C. and K. Shimomura (2002), "Recent Challenges to the Classical Gains-from-Trade Proposition", *German Economic Review*, 3, 485-489.

[11] Kemp, M. C. and H. Y. Wan, Jr. (1972), "The Gains from Free Trade", *International Economic Review*, 13, 509-522.

[12] Kemp, M. C. and K. Wong (1995), "Gains from Trade when Markets Are Possibly Incomplete", in Kemp (1995).

[13] Samuelson, P. A. (1939), "The Gains from International Trade", *Canadian Journal of Economics and Political Science*, 5, 195-205.

[14] Shy, O. (1988), "A General Equilibrium Model of Pareto Inferior Trade", *Journal of International Economics*, 25, 143-154.

[15] Tompkinson, P. (1999), "The Gains from Trade in a Ricardian Model when Workers Have Preferences among Occupations", *Journal of Post Keynesian Economics*, 21, 611-620.

[16] Willmann, G. (2002), "Redistribution in the Wake of Trade Liberalization- Help or Hindrance?", Mimeo., University of Kiel.

[17] Willmann, G. (2004), "Pareto Gains from Trade: A Dynamic Counterexample", *Economics Letters*, 83, 199-204.

[18] Wong, K. (1995), *International Trade in Goods and Factor Mobility*, Cambridge, MA: MIT Press.

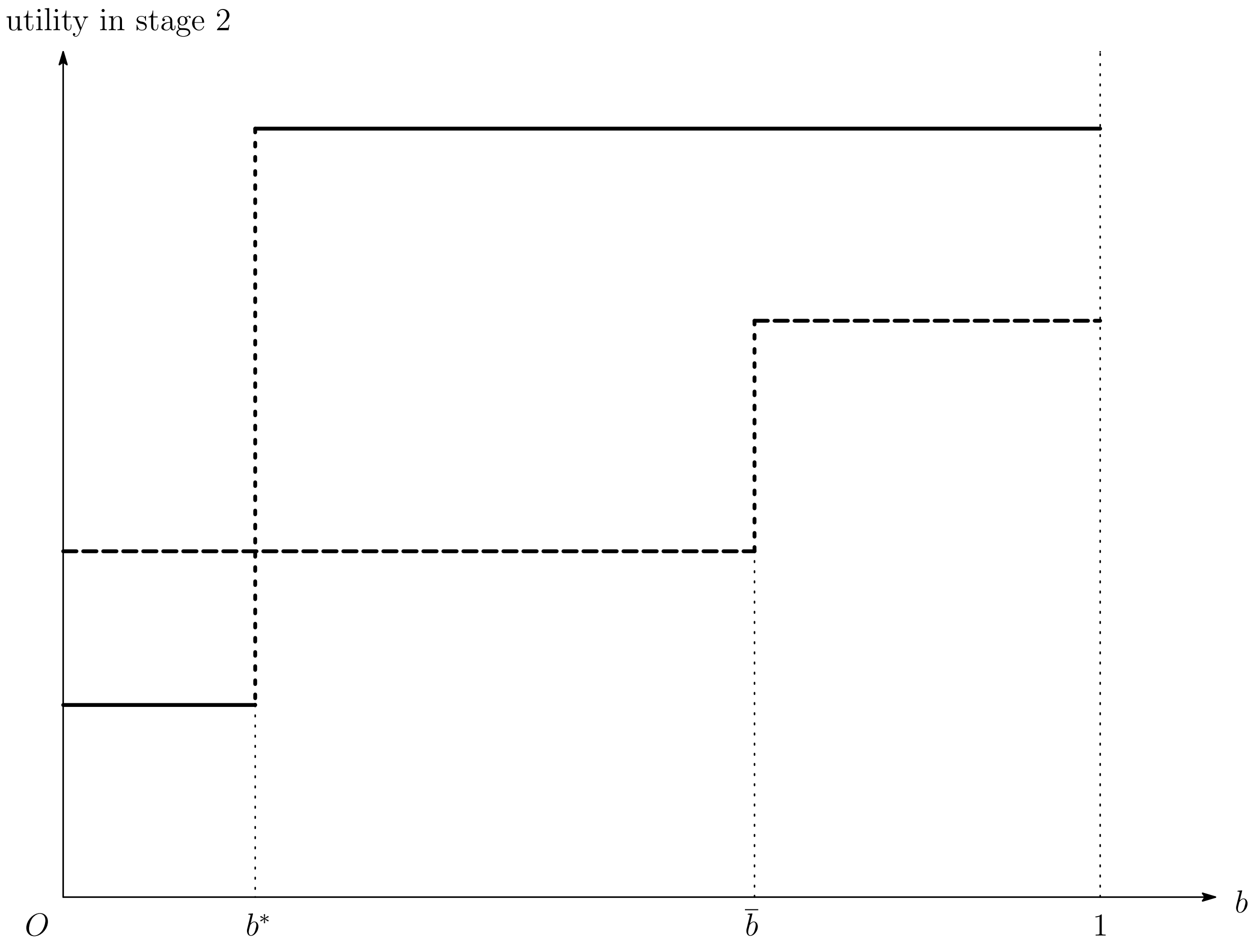

Figure 2.1:

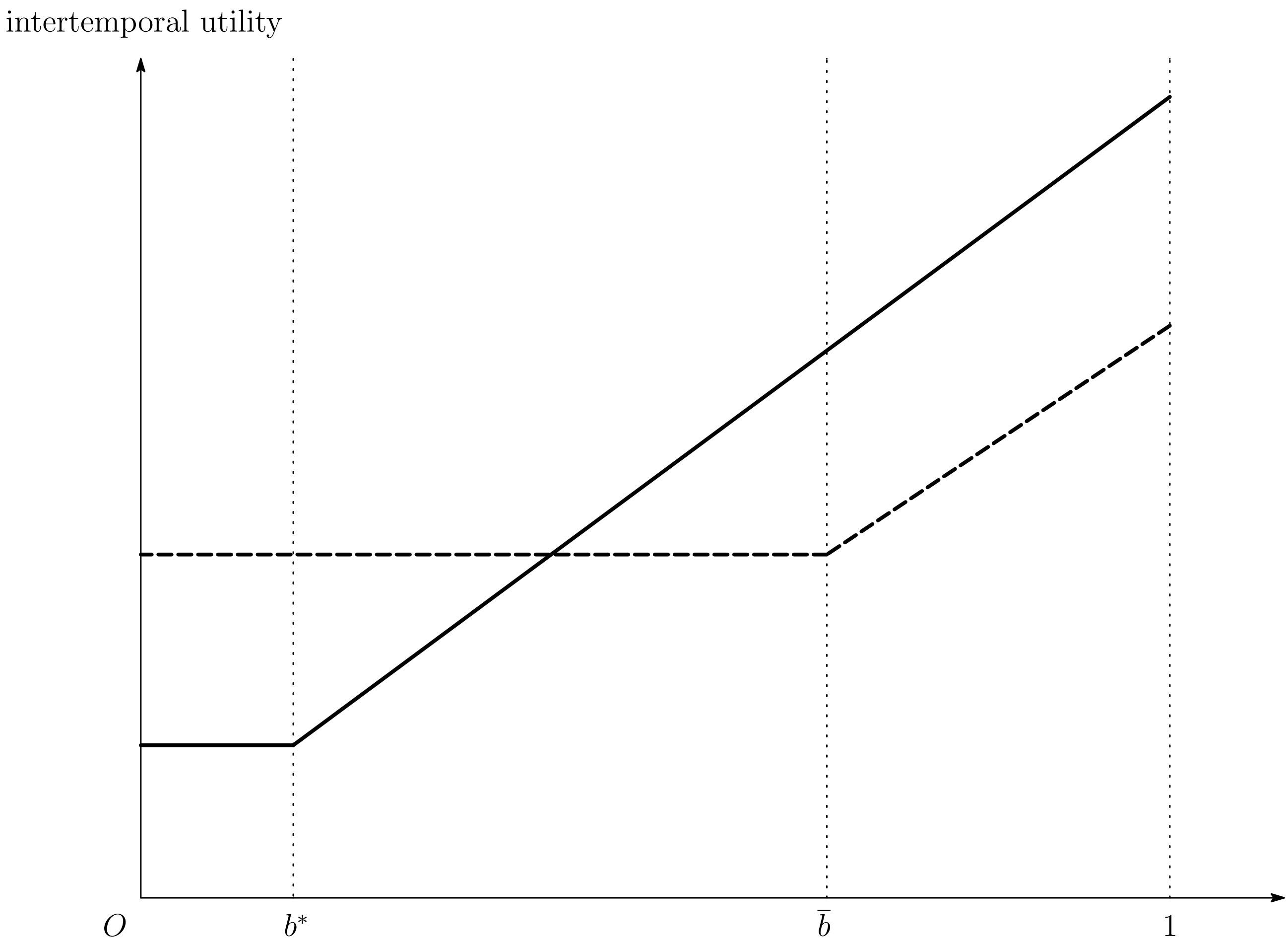

Figure 2.2:

Chapter 3

Gains from Free Trade with International Oligopoly and Increasing Returns

[1]

[1]This chapter is based on Fujiwara (2005). I thank David R. Collie, Wilfred J. Ethier, Fumio Dei, Yunfang Hu, Toru Kikuchi, Ngo Van Long, Kazuo Mino, Noritsugu Nakanishi, Masao Oda, Masayuki Okawa, Takashi Shibata, Kar-yiu Wong and Laixun Zhao for helpful comments. Among others, I express special thanks to Elhanan Helpman, Murray C. Kemp and Koji Shimomura whose help substantially improved the paper above. Furthermore, valuable suggestions from the editor of *Economic Record*, Harry Bloch, and its two referees are gratefully acknowledged. This research is financially supported by the Ministry of Education, Culture, Sports, Science and Technology of Japan (Grant-in-Aid for 21st century COE Program 'Research and Education Center of New Japanese Economic Paradigm') is also acknowledged.

Gains from Free Trade with International Oligopoly and Increasing Returns

3.1 Introduction

Propagation of trade liberalization is one of the most salient economic events after World War II. The General Agreement of Tariffs and Trade (GATT) and its successor, the World Trade Organization (WTO), have promoted multilateral trade liberalization and their participants have steadily increasing. However, it is ambiguous for all countries to gain from freer trade. Indeed, some countries seemingly experience welfare losses by the opening up of trade, which makes them oppose to trade liberalization. The big protests during the 1999 WTO round talks at Seattle are a symbol of such anti-globalization.

In discussing the resistance to globalization or trade liberalization, the role of multinational corporations is sometimes emphasized. Multinational corporations exploit the developing countries' market and people by taking advantage of a big market power and scale economies in most cases. Then, is it correct to judge that the developing countries should not open their markets to the world? The purpose of this paper is to address this question by constructing a simple two-country, two-agent model which comprises imperfect competition and increasing returns.

In view of the literature on gains and losses from trade under oligopoly and/or increasing returns, most works have concentrated on deriving the sufficient condition for positive trade gains or showing potentially gainful free trade. Markusen (1981), Markusen and Melvin (1984) and Schweinberger (1996) are included in the first category, whereas Kemp and Shimomura (2001) in the second. There is no doubt that these studies made a big contribution, but none of them addressed the negative aspects of trade liberalization stated above.

On the other hand, Fujiwara (2005) shows that (i) all agents in a country lose from trade and that (ii) it is impossible to overcome losses from trade through a lump-sum compensation scheme but that (iii) even such a country can still find a *non-lump-sum* redistributive scheme such that everyone gains from trade. That is, although a country loses from trade without any redistribution, even such a country can potentially gain from trade with an appropriate scheme of redistribution. In the arguments, the role of increasing returns due to fixed costs turns out to be crucial.

This paper reviews Fujiwara's (2005) arguments by developing a generalized model of Fujiwara's (2005). While Fujiwara (2005) presumes linear demand and cost functions, we employ a linearly homogeneous preference. We show that Fujiwara's (2005) result basically survives this generalization.

The paper proceeds as follows. Section 2 sets up a basic model and characterizes an autarkic equilibrium of a country. Section 3 extends it to a two-country world and shows the losses-from-trade proposition. Section 4 demonstrates that no lump-sum compensation scheme can overcome such losses from trade. However, Section 5 proves the existence of a proper scheme of *non-lump-sum* redistribution such that everyone in the country gains from trade. Section 7 allows for the implementation of both countries and reestablishes the gains-from-trade proposition. Section 7 sums up the conclusions.

3.2 An Autarkic Equilibrium

A two-country (home and foreign), two-good (goods X and Y), and two-agent (consumer and monopolist) model is developed. We pay attention to an autarkic equilibrium of a country, say, the home country. Parallel arguments straightforwardly apply to the foreign country.

Two goods are produced from only one factor of production called labor. Without loss of generality, one unit of labor produce the same unit of good 2 under conditions of perfect competition, which serves as a numeraire. Throughout the paper, it is assumed that good 2 is positively produced under autarky and free trade, which makes the wage rate fixed to unity. Good 1, on the other hand, is monopolized by a monopolistic firm and its production function is specified by

$$X = \begin{cases} \frac{L_X - f}{c} & \text{if} \quad L_X > f \\ 0 & \text{if} \quad L_X \leq f, \quad c > 0, \quad f > 0, \end{cases},$$

where X is the output of good 1 and L_X the labor input. Thus, when the labor input is so large that $L_X > f$, the cost function associated with this production function is given by $cX + f$. In what follows, we call c a marginal cost and f a fixed cost.

Let us now turn to the demand side. Unlike most of the existing literature, we deviate from the assumption of representative consumers. There are two types of agents in a country. A monopolist has no factor of production but it possesses 100% ownership of the firm and consumes only the numeraire good. It maximizes utility by maximizing profits.[2] The other agent, a consumer, owns L amount of labor and earns its labor income by supplying it inelastically in the labor market. Using the labor income, the consumer maximizes the utility given by

$$U = u(D_X, D_Y), \tag{3.1}$$

where U is the consumer's utility and D_i, $i = X, Y$ the consumption of each good. $u(\cdot)$ is an increasing, strictly quasi-concave, and linearly homo-

[2]It is sometimes problematic to presume profit-maximizing firms in a context of imperfect competition since profit-maximizing solutions become indeterminate depending on the choice of numeraire. In particular, this is true when a non-competitive firm consumes all commodities (goods X and Y in our model). Kemp and Shimomura (1995) and Shimomura (1995) discuss the details of this problem.

geneous. Solving the utility maximization problem, the demand function of good X is obtained as[3]

$$D_X = \frac{e'(p)}{e(p)} L, \tag{3.2}$$

where the function $e(\cdot)$ is a unit expenditure function defined by

$$e(p) \equiv \min_{D_X, D_Y} \{pD_X + D_Y | u(D_X, D_Y) = 1\}.$$

Since the monopolist does not consume its own product, the market-clearing condition under autarky is

$$\frac{e'(p)}{e(p)} L = X,$$

and solving for p yield the following inverse demand function:

$$p = p\left(\frac{X}{L}\right), \quad p'(\cdot) < 0.$$

Then, the monopolist's profit is defined as

$$\left[p\left(\frac{X}{L}\right) - c\right] X - f,$$

and its first-order condition for profit maximization corresponding to the interior solution becomes

$$p'\left(\frac{X}{L}\right)\frac{X}{L} + p\left(\frac{X}{L}\right) - c = 0. \tag{3.3}$$

[3]Note that the labor income is given by $wL = L$ due to the unitary wage rate.

As is immediately seen, the first term of the left-hand side in (3.3) is a function of only X/L. Hence, the solution of X/L to (3.3) is a function of c and expressed by[4]

$$\frac{X}{L} = x^A(c), \quad x^{A'}(c) < 0. \tag{3.4}$$

This is the autarkic equilibrium output of good X. Substituting it into the definition of the profit yields

$$\pi^A \equiv \left[p\left(x^A(c)\right) - c\right] x^A(c) - f, \tag{3.5}$$

and the parameter set satisfies the positivity of (3.5) for guaranteeing the interior maximum.

The consumer's welfare is grasped by a standard indirect utility function:

$$\frac{L}{e(p)}.$$

Substituting (3.4) into this immediately gives the consumer's utility in an autarkic equilibrium as follows.

$$V^A \equiv \frac{L}{e(p(x^A(c)))} \tag{3.6}$$

This completes the analysis of a country's autarkic equilibrium. The rest of the paper considers the welfare consequences of free trade.

[4]Note that we must impose that the first term of the left-hand side in (3.3) is decreasing in X/L to guarantee the interior solution.

3.3 Losses from Laissez-Faire Free Trade

The model developed in the previous section is now extended to a two-country world. All of the foreign variables are distinguished from the home ones by attaching an asterisk (*) to them. In the world market, the home monopolist competes the foreign monopolist in a Cournot-Nash fashion. The Cournot-Nash equilibrium is employed as an equilibrium concept. Following the traditional literature on trade gains, we assume that the two countries' markets are completely integrated rather than segmented. Then, the market-clearing condition under free trade is

$$\frac{e'(p)}{e(p)}(L + L^*) = X + X^*,$$

whose solution constitutes the inverse demand function:

$$p = p\left(\frac{X + X^*}{L + L^*}\right).$$

Using this inverse demand function, each country's monopolist has the following profit.

$$\left[p\left(\frac{X + X^*}{L + L^*}\right) - c\right] X - f$$
$$\left[p\left(\frac{X + X^*}{L + L^*}\right) - c^*\right] X^* - f^*$$

If the parameter sets guarantee the interior solutions of the Cournot-Nash equilibrium, they satisfy the system of the first-order conditions:

$$p'\left(\frac{X + X^*}{L + L^*}\right)\frac{X}{L + L^*} + p\left(\frac{X + X^*}{L + L^*}\right) - c = 0 \qquad (3.7)$$

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X^*}{L+L^*}+p\left(\frac{X+X^*}{L+L^*}\right)-c^* \quad = \quad 0. \tag{3.8}$$

However, the interior solutions are not always observed in the present model due to the presence of increasing returns. Roughly speaking, zero output can be the equilibrium from the profit-maximizing monopolist's viewpoint if its fixed cost is sufficiently large. In other words, a monopolist's reaction curve is discontinuous and contains a vertical or horizontal segment, which tends to make the Nash equilibrium characterized by a corner solution. Therefore, we need to capture this aspect of increasing returns precisely. To do so, the home monopolist's output which satisfies (3.7) is now rewritten by

$$X = R(X^*, c),$$

which is its best response function associated with the interior maximum. Substituting this into the definition of its profit, we have

$$\Pi(X^*, c, f) \equiv \left[p\left(\frac{R(X^*, c)+X^*}{L+L^*}\right)-c\right]R(X^*, c)-f,$$

as a maximized profit of the home monopolist. The interior best response $R(X^*, c)$ is justified as long as $\Pi(X^*, c, f) \geq 0$. Accordingly, the home monopolist's optimal behavior is summarized in:

$$X = \begin{cases} R(X^*, c) & \text{if} \quad \Pi(X^*, c, f) \geq 0 \\ 0 & \text{if} \quad \Pi(X^*, c, f) < 0. \end{cases}$$

Exactly the same is true of the foreign monopolist. Its reaction function is now obtained as

$$X^* = \begin{cases} R(X, c^*) & \text{if} \quad \Pi(X, c^*, f^*) \geq 0 \\ 0 & \text{if} \quad \Pi(X, c^*, f^*) < 0. \end{cases}$$

Based on these preliminaries, we offer the first main result:

Proposition 1. *Both the consumer and the monopolist loses from trade if the sets of parameters satisfy* $c < c^*$ *and*

$$\Pi(R(0, c^*), c, f) < 0, \quad \Pi(0, c^*, f^*) \geq 0. \tag{3.9}$$

Proof. From the above computation of each monopolist's reaction function, condition (3.9) yields

$$X^N = 0, \quad X^{*N} = R(0, c^*) = (L + L^*)x^A(c^*), \tag{3.10}$$

where the superscript N indicates the Nash equilibrium. That is, the world market is now monopolized by the foreign monopolist. Note that the function $x^A(\cdot)$ in (3.4) is used since the foreign monopolist's output is given by the condition of:

$$p'\left(\frac{X^*}{L + L^*}\right)\frac{X^*}{L + L^*} + p\left(\frac{X^*}{L + L^*}\right) - c^* = 0.$$

The resulting price and profits are determined by

$$p^N = p\left(x^A(c^*)\right) \tag{3.11}$$

$$\pi^N = 0 \tag{3.12}$$

$$\pi^{*N} = (L + L^*) \left[p\left(x^A(c^*)\right) - c^*\right] x^A(c^*) - f^*, \tag{3.13}$$

From the other assumption of $c < c^*$, the world price is larger than the home autarkic level and the home monopolist's profit is driven to zero from a positive level, while the foreign monopolist's profit increases since it confronts an enlarged market and there is no rival for it.

The home consumer's welfare is now

$$V\left(p\left(x^A(c^*)\right), L\right) \equiv \frac{L}{e\left(x^A(c^*)\right)} < V^A,$$

which implies that the home consumer is worse off under free trade than under autarky. Thus, both agents in the home country lose from the opening up of free trade under the conditions of $c < c^*$ and (3.9).

Proposition 1 is closely related to a seminal work by Markusen (1981). He shows the now-classical gains-from-trade proposition that a country gains from trade if the country's output of the non-competitive good increases after starting trade.[5] When we take a converse of Markusen's (1981) proposition, a country loses from trade *only if* its output of the non-competitive good decreases by the opening of trade. Indeed, this condition holds in our two-agent model as well since the home country's out-

[5]This production expansion condition is later generalized and sophisticated by Markusen and Melvin (1984) and Schweinberger (1996).

put of good 1 is driven to zero from a positive level. However, note that Markusen's (1981) condition is imposed on endogenous variables, whereas ours on exogenous parameters. In this sense, our condition is one simple example to guarantee the Markusen condition.

3.4 Impossibility of Trade Gains with Lump-Sum Transfers

Proposition 1 states that a country has a possibility of losses from trade if its monopolist's fixed cost is far larger than that of the other country's monopolist. Then, what if the home country implements a lump-sum compensation scheme? In the other market distortions such as market incompleteness and infinite agents characterized by overlapping generations, properly compensated free trade is beneficial to any country even if it loses from trade without any compensation.[6] This section addresses the validity of their assertion in an imperfectly competitive context.

The lump-sum compensation scheme is defined as follows. Letting T_C and T_M be the income transfer from the government to the consumer and the monopolist, respectively. The level of T_C is assumed to be determined through

$$\frac{L}{e\left(p\left(x^A(c)\right)\right)} = \frac{L + T_C}{e\left(p\left(x^A(c^*)\right)\right)},$$

[6]See Kemp and Wong (1995a, 1995b) and Wong (1995) for the formal proofs and arguments.

where the left-hand side represents the autarkic utility and the right-hand side that under free trade with a compensatory transfer. This equality means that the transfer level is calculated such that the consumer enjoys exactly the same utility under autarky and compensated free trade. Then, T_C is explicitly obtained as

$$T_C = \left[\frac{e\left(p\left(x^A(c^*)\right)\right)}{e\left(p\left(x^A(c)\right)\right)} - 1 \right] L > 0,$$

where the above positivity follows from that $x^A(\cdot)$ is decreasing, $p(\cdot)$ is decreasing, and the assumption of $c < c^*$ is imposed. From $T_C > 0$, the home government must transfer a positive amount of income to the consumer to maintain its utility to the autarkic level.[7]

Analogously, the transfer to the monopolist is computed through the formula:

$$\left[p\left(x^A(c)\right) - c\right] Lx^A(c) - f = 0 + T_M,$$

where the left-hand side is the autarkic profit, whereas the right-hand side is the free trade profit after compensation. Solving for T_M, we have

$$T_M = \left[p\left(x^A(c)\right) - c\right] Lx^A(c) - f > 0,$$

which says that the government has to subsidize the monopolist in order to keep its profit to the autarkic level.

[7] T_C is called a compensating variation in microeconomic theory.

In sum, both the consumer and the monopolist must be subsidized in order not to lose from trade. However, the above scheme of compensation is impossible since it violates the government's budget constraint such that $T_C + T_M \leq 0$. Therefore, we arrive at:

Proposition 2. *With the suggested scheme of lump-sum compensation, T_C and T_M, it is impossible to overcome the home country's losses associated with the opening up of free trade.*

Proposition 2 is in sharp contrast with the gains-from-trade propositions by Kemp and Wong (1995a, 1995b). Kemp and Wong (1995a) prove the existence of a *lump-sum* compensation scheme which can achieve a Pareto-improvement in market incompleteness although everyone in every country loses from trade without such a scheme. Kemp and Wong (1995b) do the same task in a context of overlapping generations, i.e., the violation of finite agents. In view of their results, someone may wonder whether a similar conclusion can follow under imperfect competition and increasing returns as well since gains from trade via lump-sum compensation can be assured in incomplete markets and overlapping generations. However, Proposition 2 betrays such an expectation. According to it, we can find no *lump-sum* compensation scheme that achieves a Pareto-improvement, which makes us to explore the possibility of a *non-lump-sum* scheme of redistribution. This is a main task in the next section.

3.5 Gains from Trade with Non-Lump-Sum Transfers

The losses-from-trade propositions in Propositions 1 and 2 seem to cast a serious doubt on globalization which is supported by the WTO. However, this section shows that we can find an appropriate scheme of *non-lump-sum* redistribution. We begin with defining the scheme. Letting Γ be the transfer level to the consumer, it is assumed to be determined through

$$\frac{L}{e\left(p\left(x^{A}(c)\right)\right)} = \frac{L+\Gamma}{e\left(p\left(\frac{X+X^{*}}{L+L^{*}}\right)\right)},$$

where the left-hand side is the consumer's autarkic utility and the right-hand side the consumer's utility with the scheme of Γ. Note that this scheme is basically the same as T_C suggested in the previous section. Solving for Γ, the solution becomes a function of each monopolist's output:

$$\Gamma\left(\frac{X+X^{*}}{L+L^{*}}\right) \equiv \left[\frac{e\left(p\left(\frac{X+X^{*}}{L+L^{*}}\right)\right)}{e\left(p\left(x^{A}(c)\right)\right)} - 1\right] L. \tag{3.14}$$

As mentioned, any redistributive scheme must satisfy the government's budget constraint. That is, Γ has to be financed from taxation on the monopolist. Thus, the monopolist's post-taxation profit is now defined by

$$\left[p\left(\frac{X+X^{*}}{L+L^{*}}\right) - c\right] X - f - \Gamma\left(\frac{X+X^{*}}{L+L^{*}}\right)$$

$$= \left[p\left(\frac{X+X^*}{L+L^*}\right)-c\right]X - f - \left[\frac{e\left(p\left(\frac{X+X^*}{L+L^*}\right)\right)}{e\left(p\left(x^A(c)\right)\right)} - 1\right]L.$$

Based on this newly defined objective function, the home monopolist seeks to maximize it with respect to X. When the interior solutions are assured, the system of the first-order conditions is derived as

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X}{L+L^*} + p\left(\frac{X+X^*}{L+L^*}\right) - \frac{\Gamma'\left(\frac{X+X^*}{L+L^*}\right)}{L+L^*} = c \qquad (3.15)$$

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X^*}{L+L^*} + p\left(\frac{X+X^*}{L+L^*}\right) = c^*. \qquad (3.16)$$

The procedure to solve the Cournot-Nash equilibrium follows that in Bergstrom and Varian (1985). Summing up the two first-order conditions yields

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X+X^*}{L+L^*} + 2p\left(\frac{X+X^*}{L+L^*}\right) - \frac{\Gamma'\left(\frac{X+X^*}{L+L^*}\right)}{L+L^*} = c + c^*,$$

whose solution of $(X+X^*)/(L+L^*)$ is given by

$$\frac{X+X^*}{L+L^*} = x^T(c+c^*), \quad x^{T\prime}(c+c^*) < 0,$$

where the superscript T denotes the free trade equilibrium with the transfer $\Gamma(\cdot)$. Substituting this into each first-order condition and solving for X and X^*, the new Nash equilibrium output of the home monopolist takes the form of

$$X^T = \frac{(L+L^*)\left[p\left(x^T(c+c^*)\right)-c\right] - \Gamma'\left(x^T(c+c^*)\right)}{-p'\left(x^T(c+c^*)\right)} \equiv \phi(c,c^*). \qquad (3.17)$$

And further substitution of X^T into the definition of the post-scheme profit, we have

$$\pi^T \equiv \left[p\left(x^T(c+c^*)\right) - c\right]\phi(c,c^*) - f - \Gamma\left(x^T(c+c^*)\right). \tag{3.18}$$

Note that the interior solutions are guaranteed if and only if (3.18) is non-negative.

Based on the analysis up to now, we shall prove the main proposition. While Fujiwara (2005) proves potential gainfulness of trade by solving for the Nash equilibrium outputs explicitly, it is impossible in the present model. Hence, we employ the way of proof suggested by Kemp and Shimomura (2001). It consists of two steps one of which is to show:

$$\left[p\left(\frac{X^A+X^{*T}}{L+L^*}\right)-c\right]X^A - f - \Gamma\left(\frac{X^A+X^{*T}}{L+L^*}\right) > \left[p\left(\frac{X^A}{L}\right)-c\right]X^A - f.$$

The left-hand side is the post-scheme profit when the home monopolist *hypothetically* chooses X^A while the foreign monopolist chooses the Nash equilibrium output. The right-hand side is the autarkic profit. This condition reduces to

$$\left[p\left(\frac{X^A+X^{*T}}{L+L^*}\right)-p\left(\frac{X^A}{L}\right)\right]X^A - \Gamma\left(\frac{X^A+X^{*T}}{L+L^*}\right) > 0.$$

This condition can be rewritten as a condition concerning the exogenous variables. To show it, we must note that the foreign monopolist's Nash equilibrium output becomes

$$X^{*T} = \frac{(L+L^*)\left[p\left(x^T(c+c^*)\right) - c^*\right]}{-p'\left(x^T(c+c^*)\right)}.$$

Then, we see

$$\frac{X^A + X^{*T}}{L+L^*} = \frac{L}{L+L^*}x^A(c) + \frac{p\left(x^T(c+c^*)\right) - c^*}{-p'\left(x^T(c+c^*)\right)} \equiv \Phi(c, c^*).$$

Accordingly, substituting $X^A = Lx^A(\cdot)$ and $(X^A + X^{*T})/(L+L^*) = \psi(c, c^*)$ into the above inequality, we have

$$\left[p\left(\Phi(c,c^*)\right) - p\left(x^A(c)\right)\right] Lx^A(c) - \Gamma\left(\Phi(c,c^*)\right) > 0, \tag{3.19}$$

and we can state and prove:

Proposition 3. *Nobody in the home country is worse off under free trade with the redistributive scheme* $\Gamma(\cdot)$ *if the pair of marginal costs satisfy (3.19).*

Proof. The proof owes to Kemp and Shimomura's (2001) one. Under (3.19), the following inequalities can hold.

$$\begin{aligned}
\pi^T &= \left[p\left(\frac{X^T + X^{*T}}{L+L^*}\right) - c\right] X^T - f - \Gamma\left(\frac{X^T + X^{*T}}{L+L^*}\right) \\
&\geq \left[p\left(\frac{X^A + X^{*T}}{L+L^*}\right) - c\right] X^A - f - \Gamma\left(\frac{X^A + X^{*T}}{L+L^*}\right) \\
&> \left[p\left(\frac{X^A}{L}\right) - c\right] X^A - f \\
&\equiv \pi^A,
\end{aligned}$$

where the first inequality follows from the definition of Nash equilibrium, while the second follows from condition (3.19). As a result, the home monopolist's post-scheme profit exceeds the autarkic one from which the monopolist gains from trade. On the other hand, the scheme $\Gamma(\cdot)$ is designed so that the consumer's utility under free trade is just equal to the autarkic level, its post-scheme utility is exactly the same as the autarkic one. Hence, a (weak) Pareto-improvement is achieved via the redistributive scheme $\Gamma(\cdot)$.

In what follows, we interpret Proposition 3 intuitively by considering what role the scheme $\Gamma(\cdot)$ plays. From the construction of this scheme, it changes the home monopolist's objective function. Then, the home monopolist re-optimizes based on such a renewed objective function. This changes the home monopolist's reaction curve and makes it shift outward as a *virtual* production subsidy. Thus, a well-known procompetitive effect occurs at the new Nash equilibrium and the world price tends to decline relative to the autarkic level, which is beneficial to the consumer. This implies that the consumer must pay a lump-sum tax to the government, which is transferred to the monopolist as a production subsidy. Therefore, the home monopolist's profit including the subsidy can be larger than the autarkic profit, which results in a Pareto-improvement.

3.6 Multilateral Gains from Trade

The last section allowed the home country to employ the scheme given in (3.14) and showed that the home country benefits from trade. This section further allows for the implementation of the scheme by both countries and shows multilateral gains from trade. This result suggests that multilateral trade liberalization or globalization can potentially benefits all the trading countries like the Kemp-Wan (1972) gains-from-trade proposition in the Arrow-Debreu economies.

Suppose that both countries employ the scheme in (3.14), which makes each oligopolist's objective function take the form of:

$$\left[p\left(\frac{X+X^*}{L+L^*}\right)-c\right]X-f-\Gamma\left(\frac{X+X^*}{L+L^*}\right)$$
$$\left[p\left(\frac{X+X^*}{L+L^*}\right)-c^*\right]X^*-f^*-\Gamma^*\left(\frac{X+X^*}{L+L^*}\right).$$

Then, the new Cournot-Nash equilibrium involves the system of the first-order conditions given by

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X}{L+L^*}+p\left(\frac{X+X^*}{L+L^*}\right)-c-\Gamma'\left(\frac{X+X^*}{L+L^*}\right) = 0 \quad (3.20)$$
$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X^*}{L+L^*}+p\left(\frac{X+X^*}{L+L^*}\right)-c^*-\Gamma^{*\prime}\left(\frac{X+X^*}{L+L^*}\right) = 0 \quad (3.21)$$

This system is solved in a similar way to that in the previous section. First of all, sum up the two equations to get

$$p'\left(\frac{X+X^*}{L+L^*}\right)\frac{X+X^*}{L+L^*}+2p\left(\frac{X+X^*}{L+L^*}\right)-\Gamma'\left(\frac{X+X^*}{L+L^*}\right)-\Gamma^{*\prime}\left(\frac{X+X^*}{L+L^*}\right)=c+c^*,$$

whose solution of $(X+X^*)/(L+L^*)$ is obtained as

$$\frac{X+X^*}{L+L^*}=x^M(c+c^*), \quad x^{M\prime}(c+c^*)<0,$$

where the superscript M indicates the new Cournot-Nash equilibrium. Substituting $x^M(\cdot)$ into each monopolist's first-order condition and solving for X and X^* yield

$$X^M = \frac{(L+L^*)\left[p\left(x^M(c+c^*)\right)-c\right]-\Gamma'\left(x^M(c+c^*)\right)}{-p'\left(x^M(c+c^*)\right)} \equiv \psi(c,c^*) \quad (3.22)$$

$$X^{*M} = \frac{(L+L^*)\left[p\left(x^M(c+c^*)\right)-c^*\right]-\Gamma*'\left(x^M(c+c^*)\right)}{-p'\left(x^M(c+c^*)\right)} \equiv \psi^*(c,c^*) \quad (3.23)$$

Further substitution of $x^M(c+c^*)$ and $\psi(c,c^*)$ into the definition of the home monopolist's profit, its maximized profit is derived as

$$\pi^M \equiv \left[p\left(x^M(c+c^*)\right)-c\right]\psi(c,c^*)-f-\Gamma\left(x^M(c+c^*)\right) \quad (3.24)$$

$$\pi^{*M} \equiv \left[p\left(x^M(c+c^*)\right)-c^*\right]\psi^*(c,c^*)-f^*-\Gamma^*\left(x^M(c+c^*)\right) \quad (3.25)$$

both of which are supposed to be non-negative to assure the interior Nash equilibrium.

Now, we are ready to state and prove the gains-from-trade proposition such that all countries benefit from trade with the bilateral implementation of the scheme. Suppose *hypothetically* the following inequality holds.

$$\begin{aligned}&\left[p\left(\frac{X^A+X^{*M}}{L+L^*}\right)-p\left(\frac{X^A}{L}\right)\right]X^A-\Gamma\left(\frac{X^A+X^{*M}}{L+L^*}\right)\\ =\ &\left[p\left(\Psi(c,c^*)\right)-p\left(x^A(c)\right)\right]Lx^A(c)-\Gamma\left(\Psi(c,c^*)\right)>0,\end{aligned} \quad (3.26)$$

where

$$\Psi(c, c^*) \equiv \frac{Lx^A(c) + \psi^*(c, c^*)}{L + L^*}.$$

Then, we safely have the following inequalities:

$$\begin{aligned}
\pi^M &= \left[p\left(\frac{X^M + X^{*M}}{L + L^*}\right) - c\right] X^M - f - \Gamma\left(\frac{X^M + X^{*M}}{L + L^*}\right) \\
&\geq \left[p\left(\frac{X^A + X^{*M}}{L + L^*}\right) - c\right] X^A - f - \Gamma\left(\frac{X^A + X^{*M}}{L + L^*}\right) \\
&> \left[p\left(\frac{X^A}{L}\right) - c\right] X^A - f \\
&= \pi^A.
\end{aligned}$$

Accordingly, we have reached another main result:

Proposition 4. *It is possible to make nobody worse off under free trade than under autarky by multilateral implementation of the above non-lump-sum scheme of redistribution if the condition (3.26) and its foreign counterpart are satisfied.*

Proposition 4 is a confirmation of Kemp and Shimomura's (2001) gains-from-trade proposition. In this sense, it is not a new contribution. However, the Kemp-Shimomura proposition survives the extreme situation in which everyone becomes a loser from trade, which is a small contribution behind Proposition 4.

3.7 Concluding Remarks

This paper has analyzed the issue of gains and losses from trade in a two-agent model of international oligopoly and increasing returns.

It is no wonder that losses from trade can occur in the non-Arrow-Debreu environments in view of the losses-from-trade propositions such as Newbery and Stiglitz (1984), Shy (1988), Kemp and Long (1979) and Bihn (1991). All of these papers made use of a highly specified model, which allow us to construct a restricted model in this paper as well. As mentioned in Introduction, it is of great interest and importance to explore the theoretical possibility of losses from trade since such a task sheds light on why there are many resisting parties to multilateral trade liberalization promoted by the WTO. Proposition 1 in our paper attributes the cause of losses from trade to the interaction between imperfect competition and economies of scale. Besides, this difficulty can not be overcome through lump-sum compensation according to Proposition 2.

However, if a *non-lump-sum* redistributive scheme is adopted, a country can make free trade potentially Pareto-improving as Propositions 3 and 4 clarify. Thus, it is because the country does not implement the non-lump-sum scheme of redistribution that a country can not escape from losses from trade. The role of the scheme has been discussed. In our context, the scheme plays a role of production subsidy and promotes international competition. This enhances the consumer surplus and hence the consumer is willing to pay a lump-sum tax to finance the subsidy. In addition, in spite of the fierce competition, the monopolist's post-scheme profit can exceed the autarkic one thanks to the subsidy, which results in a Pareto-improvement. In this sense, our scheme has a role beyond a mere compensation. This is a small but another contribution of our paper.

Bibliography

[1] Bihn, T. (1985), "A Neo-Ricardian Model with Overlapping Generations", *Economic Record*, 61, 707-718.

[2] Fujiwara, K. (2004), "The Banana Republic and Losses from Trade", Mimeo., Graduate School of Economics, Kobe University.

[3] Fujiwara, K. (2005), "Unilateral and Multilateral Gains from Trade in International Oligopoly", *Economic Record*, 81, 404-413.

[4] Kemp, M. C. and N. V. Long (1979), "The Under-Exploitation of Natural Resources: A Model with Overlapping Generations", *Economic Record*, 55, 214-221.

[5] Kemp, M. C. and K. Shimomura (1995), "The Apparently Innocuous Representative Agent", *Japanese Economic Review*, 46, 247-256.

[6] Kemp, M. C. and K. Shimomura (2001), "Gains from Trade in a Cournot-Nash General Equilibrium", *Japanese Economic Review*, 52, 284-302.

[7] Kemp, M. C. and K. Wong (1995a), "The Gains from Trade when Markets Are Possibly Incomplete", in M. C. Kemp, *The Gains from Trade and the Gains from Aid*, London, Routledge: 47-71.

[8] Kemp, M. C. and K. Wong (1995b), "Gains from Trade with Overlapping Generations", in M. C. Kemp, *The Gains from Trade and the Gains from Aid*, London, Routledge: 105-128.

[9] Markusen, J. R. (1981), "Trade and the Gains from Trade with Imperfect Competition", *Journal of International Economics*, 11, 531-551.

[10] Markusen, J. R. and J. R. Melvin (1984), "The Gains-from-Trade Proposition with Increasing Returns to Scale", in H. Kierzkowski (ed.), *Monopolistic Competition and International Trade*, Oxford: Clarendon Press, 10-33.

[11] Newbery, D. M. G. and J. E. Stiglitz (1984), "Pareto Inferior Trade", *Review of Economic Studies*, 51, 1-12.

[12] Schweinberger, A. G. (1996), "Procompetitive Gains from Trade and Comparative Advantage", *International Economic Review*, 37, 361-375.

[13] Shimomura, K. (1995), "Some Implications of Imperfect Competition for Recent Trade Theory", *Review of International Economics*, 3, 244-247.

[14] Shy, O. (1988), "A General Equilibrium Model of Pareto Inferior Trade", *Journal of International Economics*, 25, 143-154.

[15] Wong, K. (1995), *International Trade in Goods and Factor Mobility*, Cambridge, MA: MIT Press.

Chapter 4

An Oligopolistic Heckscher-Ohlin Theorem with Increasing Returns to Scale

[1]

[1]This chapter is based on Fujiwara and Shimomura (2005). We thank Taiji Furusawa, Chiaki Hara, Jota Ishikawa, Murray C. Kemp, Ngo Van Long, Michihiro Ohyama, Masayuki Okawa, Yoshimasa Shirai, Makoto Tawada, Makoto Yano, and the participants in the seminars at Keio University and Hitotsubashi University for valuable comments. Constructive comments from the two anonymous referees in *Canadian Journal of Economics* are gratefully acknowledged. This research is supported by the Ministry of Education, Culture, Sports, Science and Technology, Grant-in-Aid for 21st Century COE Program, 'Research and Education Center of New Japanese Paradigm.'

An Oligopolistic Heckscher-Ohlin Theorem with Increasing Returns to Scale

4.1 Introduction

The Heckscher-Ohlin theorem, which attributes the determinant of trade patterns to the cross-country difference in factor endowments, is one of the most fundamental theorems in trade theory. It has been challenged and extended to various frameworks including imperfectly competitive ones. For example, Dixit and Norman (1980) and Helpman and Krugman (1985) prove the theorem under monopolistic competition and increasing returns. Lahiri and Ono (1995) and Shimomura (1998) also show the validity of the theorem under free entry oligopoly. In these models, most of the neoclassical properties such as factor price equalization survive, which makes the Heckscher-Ohlin theorem easily robust.

Then, what conclusion concerning the validity of the Heckscher-Ohlin theorem follows under increasing returns and oligopoly with restricted entry? This is the main question in this paper. In analysis, we employ a well-known two-country, two-good, two-factor model of international duopoly pioneered by Markusen (1981). Incorporating oligopoly aspects into a standard Heckscher-Ohlin model in which the two countries differ only in their size, he addresses factor price equalization, trade patterns, and gains from trade. According to it, the small country exports the monopolized good when both countries' factor endowment ratio is equal. Markusen's (1981) result naturally induces at least two questions. First, what patterns of trade are obtained if we allow for an arbitrary difference in factor endowment ratios between the countries? Second, does the introduction of increasing returns affect the above trade pattern proposition?

Fujiwara and Shimomura (2005) try to answer these questions by constructing a two-country, two-good, two-factor model of international duopoly and increasing returns. They show the rough validity of the Heckscher-Ohlin theorem such that the capital-abundant country exports the capital-intensive good. This paper aims to review their result by providing an alternative model. The difference in their model and that in the present model lies in the number of agents. Following the most of the existing literature including Markusen (1981), Fujiwara and Shimomura (2005) presume the existence of a representative consumer so that the monopolistic firm's profit is transferred to it in a lump-sum fashion. On the other hand, we assume two agents named a factor owner and a monopolist. In this sense, we employ the same assumption as the previous section. However, we will show this difference in assumption does not affect the final result, i.e., the Heckscher-Ohlin theorem can hold roughly in our framework as well.

The paper proceeds as follows. Section 2 sets up the basic model and

Section 3 proves the main result. Section 4 gives concluding remarks.

4.2 A Model

A two-country (home and foreign), two-good (goods 1 and 2), two-factor (capital and labor), two-agent (factor owner and monopolist) model is developed. The factor owner whose number is normalized to one supplies its endowment of capital and labor inelastically and consumes both goods by spending the factor income. On the other hand, each country has a monopolistic firm which has no factor. It has 100% ownership of the firm and employs capital and labor in the factor markets. Following the existing literature, the monopolist is assumed to take the factor prices and factor income as given in maximizing its objective. The monopolist consumes only the numeraire good, which makes utility maximization equivalent to profit maximization, while the factor owner consumes both goods.[2] Both factors are internationally immobile, whereas nationally mobile between the two sectors.

Let us introduce some basic settings. For the time being, we concentrate on the economic structure in the home country only. Both goods are produced by both factors. Good 2 serves as a numeraire and is produced

[2]See, e.g., Kemp and Shimomura (1995, 2002) and Shimomura (1995).

under constant returns to scale:

$$Y_2 = f_2(K_2, L_2),$$

where Y_2 is the output of good 2, and K_2 and L_2 the capital and labor inputs in sector 2. The function $f_2(\cdot)$ is increasing, strictly quasi-concave, and linearly homogeneous.

On the other hand, the technology of good 1 is characterized by increasing returns to scale:

$$Y_1 = F(f_1(K_1, L_1)), \quad F'(\cdot) > 0, \quad F''(\cdot) > 0, \tag{4.1}$$

where Y_1 denotes the output of good 1, and K_1 and L_1 the inputs of capital and labor. $f_1(\cdot)$ satisfies the same properties as $f_2(\cdot)$. This homothetic production function gives the following cost function.

$$c_1(r, w)F^{-1}(Y_1) \equiv c_1(r, w)\phi(Y_1), \quad \phi'(\cdot) > 0, \quad \phi''(\cdot) < 0,$$

where r and w are the capital rental rate and the wage rate, respectively.

The demand side is now introduced. The factor owner's utility function is given by a quasi-linear one:

$$U = u(C_1) + C_2, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0,$$

where U stands for the utility level, and $C_i,\ i = 1, 2$ is the consumption of each good. Due to the quasi-linearity, the demand function of good 1 is a function of price only: $C_1 = D(p),\ D'(\cdot) < 0$ with p denoting the price of good 1 measured by good 2.

The foreign country has the same structure as that specified above. The two countries are different only in their distribution of factor endowments. The two-country world is now described. The two countries' market is fully integrated from which the market-clearing condition of good 1 under free trade is

$$D(p) + D(p) = Y_1 + Y_1,$$

whose solution gives the world inverse demand function:

$$p = p\left(\frac{Y_1 + Y_1^*}{2}\right), \quad p'(\cdot) < 0.$$

Thus, the profit of each country's monopolist is defined as

$$p\left(\frac{Y_1 + Y_1^*}{2}\right) Y_1 - c_1(r, w)\phi(Y_1)$$
$$p\left(\frac{Y_1 + Y_1^*}{2}\right) Y_1^* - c_1(r^*, w^*)\phi(Y_1^*).$$

Two monopolists are supposed to engage in Cournot-Nash duopoly in the integrated market. Then, the free trade Cournot-Nash equilibrium must satisfy the system of the first-order conditions for profit maximization:

$$p'\left(\frac{Y_1 + Y_1^*}{2}\right) \frac{Y_1}{2} + p\left(\frac{Y_1 + Y_1^*}{2}\right) - c_1(r, w)\phi'(Y_1) = 0 \quad (4.2)$$
$$p'\left(\frac{Y_1 + Y_1^*}{2}\right) \frac{Y_1^*}{2} + p\left(\frac{Y_1 + Y_1^*}{2}\right) - c_1(r^*, w^*)\phi'(Y_1^*) = 0. \quad (4.3)$$

As mentioned, each monopolist maximizes profit taking the factor prices as given. However, they become a function of the output in the whole system of general equilibrium. Henceforth, how they depend on the output is explained. To do so, introducing a new variable, q, suppose the following system.

$$c_1(r, w) = q \tag{4.4}$$

$$c_2(r, w) = 1. \tag{4.5}$$

(4.5) is the zero profit condition in sector 2. As in the standard Heckscher-Ohlin model, the two factor prices become the function of q:

$$r(q) \quad \text{and} \quad w(q),$$

and $r(\cdot)$ and $w(\cdot)$ possess the familiar Stolper-Samuelson relationship, i.e., $r'(\cdot) > 0$ and $w'(\cdot) < 0$ if and only if good 1 is capital-intensive and vice versa. Furthermore, q in (4.4) and (4.5) is given by the following equality:[3]

$$\phi(Y_1) = r'(q)K + w'(q)L.$$

The solution to the above equality becomes $q(\phi(Y_1), K, L)$. Therefore, applying $q(\cdot)$ to (4.2) and (4.3), they are rewritten as

$$p'\left(\frac{Y_1 + Y_1^*}{2}\right)\frac{Y_1}{2} + p\left(\frac{Y_1 + Y_1^*}{2}\right) - q(\phi(Y_1), K, L)\phi'(Y_1) = 0 \tag{4.6}$$

$$p'\left(\frac{Y_1 + Y_1^*}{2}\right)\frac{Y_1^*}{2} + p\left(\frac{Y_1 + Y_1^*}{2}\right) - q(\phi(Y_1^*), K^*, L^*)\phi'(Y_1^*) = 0. \tag{4.7}$$

[3]In the neoclassical model, the following equality regarding the GDP function is known to hold: $Y_1 = r'(p)K + w'(p)L$. From this analogy, it is fair to say that $\phi(Y_1)$ and q are called a *virtual* output and price. We owe the term *virtual* to Wong (1995).

Concerning the above system, we make a few assumptions:

Assumption 1. There exists a solution (Y_1^N, Y_1^{*N}) to the system such that both countries incompletely specialize, i.e., (4.6) and (4.7) hold with equality.

Assumption 2. The second-order condition is satisfied:[4]

$$\begin{aligned}
\frac{\partial}{\partial Y_1}\left\{p'\left(\frac{Y_1+Y_1^*}{2}\right)\frac{Y_1}{2}+p\left(\frac{Y_1+Y_1^*}{2}\right)-q(\phi(Y_1),K,L)\phi'(Y_1)\right\} &< 0\\
\frac{\partial}{\partial Y_1^*}\left\{p'\left(\frac{Y_1+Y_1^*}{2}\right)\frac{Y_1^*}{2}+p\left(\frac{Y_1+Y_1^*}{2}\right)-q(\phi(Y_1^*),K^*,L^*)\phi'(Y_1^*)\right\} &< 0.
\end{aligned}$$

Assumption 3. The free trade Cournot-Nash equilibrium (Y_1^N, Y_1^{*N}) satisfies the standard stability condition:

$$\left|\frac{dY_1^*}{dY_1}\right|^H_{Y_1^N,Y_1^{*N}} > \left|\frac{dY_1^*}{dY_1}\right|^F_{Y_1^N,Y_1^{*N}},$$

where the left-hand side stands for the absolute value of the slope of the home monopolist's reaction curve evaluated at the Cournot-Nash equilibrium and the right-hand side is that of the foreign monopolist's reaction curve.[5]

[4] Note that this is different from the second-order condition from a private viewpoint.

[5] Note that the horizontal axis measures Y_1 and the vertical axis Y_1^* in the $Y_1 - Y_1^*$ plane.

We have now prepared for the proof of the main result. In what follows, we show the validity of the factor endowment theory of trade in the present model.

4.3 An Oligopolistic Heckscher-Ohlin Theorem

Based on the preliminaries given so far, we shall show a modified version of the Heckscher-Ohlin theorem. We begin with showing three auxiliary lemmas and then proving the main result: an oligopolistic Heckscher-Ohlin theorem. The first lemma is stated in:

Lemma 1. *The home country exports good 1 if and only if* $Y_1 > Y_1^*$.

Proof. Invoking the world market-clearing condition:

$$D(p) + D(p) = Y_1 + Y_1^*,$$

which is rewritten as

$$Y_1^* = 2D(p) - Y_1.$$

Hence, we have

$$\begin{aligned} Y_1 - Y_1^* &= Y_1 - [2D(p) - Y_1] \\ &= 2[Y_1 - D(p)], \end{aligned}$$

and we can say that the home country exports good 1 if and only if $Y_1 - Y_1^* > 0$.

Lemma 1 considerably facilitates the analysis since all we have to do is to relate the distribution of a country's factor endowment to its output of good 1. The preliminary result which comes from this task is summarized in:

Lemma 2. *Regarding the difference in the Cournot-Nash equilibrium outputs, we have* $Y_1^N < Y_1^{*N}$ *if* $q\left(\phi\left(\overline{Y}_1\right), K, L\right) > q\left(\phi\left(\overline{Y}_1\right), K^*, L^*\right)$ *for any* $\overline{Y}_1$.

Proof. Suppose that $(K^*, L^*) = (K, L)$ holds. Then, it is trivial that $Y_1 = Y_1^* = \overline{Y}_1$, which is the solution to

$$p'\left(\overline{Y}_1\right)\frac{\overline{Y}_1}{2} + p\left(\overline{Y}_1\right) - q\left(\phi\left(\overline{Y}_1\right), K, L\right)\phi'\left(\overline{Y}_1\right) = 0.$$

Therefore, if $q(\phi(\overline{Y}_1), K, L) > q(\phi(\overline{Y}_1), K^*, L^*)$, the following inequality is obtained.

$$\begin{aligned} 0 &= p'\left(\overline{Y}_1\right)\frac{\overline{Y}_1}{2} + p\left(\overline{Y}_1\right) - q\left(\phi\left(\overline{Y}_1\right), K, L\right)\phi'\left(\overline{Y}_1\right) \\ &< p'\left(\overline{Y}_1\right)\frac{\overline{Y}_1}{2} + p\left(\overline{Y}_1\right) - q\left(\phi\left(\overline{Y}_1\right), K^*, L^*\right)\phi'\left(\overline{Y}_1\right). \end{aligned}$$

This inequality implies that $\overline{Y}_1$ is not the best response to $\overline{Y}_1$ from the foreign monopolist's viewpoint. That is, in view of the second-order condition, the foreign monopolist's best response to $\overline{Y}_1$ must be larger than $\overline{Y}_1$. This holds for any value of the home monopolist's output $\overline{Y}_1$ which results in $Y_1^N < Y_1^{*N}$.

Lemmas 1 and 2 assert that trade patterns are solely attributed to the difference in $q(\cdot)$ evaluated at $\overline{Y}_1$. Concerning such a difference in $q(\cdot)$, we can obtain:

Lemma 3. *See Figure 1 and suppose that good 1 is capital-intensive. Then, if the foreign factor endowment pair is above $A'M'$, $q\left(\phi(\overline{Y}_1), K, L\right) > q\left(\phi(\overline{Y}_1), K^*, L^*\right)$.*

Proof. The difference in $q(\cdot)$ can be approximately decomposed into

$$\begin{aligned} & q\left(\phi\left(\overline{Y}_1\right), K^*, L^*\right) - q\left(\phi\left(\overline{Y}_1\right), K, L\right) \\ \approx\ & q_K\left(\phi\left(\overline{Y}_1\right), K, L\right)(K^* - K) + q_L\left(\phi\left(\overline{Y}_1\right), K, L\right)(L^* - L) \\ =\ & q_K\left(\phi\left(\overline{Y}_1\right), K, L\right)(L^* - L)\left[\frac{K^* - K}{L^* - L} + \frac{q_L\left(\phi\left(\overline{Y}_1\right), K, L\right)}{q_K\left(\phi\left(\overline{Y}_1\right), K, L\right)}\right]. \end{aligned}$$

Since $q(\cdot)$ is defined as a solution to $\phi(Y_1) = r'(q)K + w'(q)L$, $q_K(\cdot)$ and $q_L(\cdot)$ become

$$q_K\left(\phi(Y_1), K, L\right) = \frac{-r'(q)}{r''(q)K + w''(q)L} < 0$$

$$q_L\left(\phi(Y_1), K, L\right) = \frac{-w'(q)}{r''(q)K + w''(q)L} > 0,$$

if and only if good 1 is capital-intensive.[6] Therefore, we further have

$$\begin{aligned}
& q_K\left(\phi\left(\overline{Y}_1\right), K, L\right)(L^* - L)\left[\frac{K^* - K}{L^* - L} + \frac{q_L\left(\phi\left(\overline{Y}_1\right), K, L\right)}{q_K\left(\phi\left(\overline{Y}_1\right), K, L\right)}\right] \\
= \ & \frac{-r'(q)}{r''(q)K + w''(q)L}(L^* - L)\left[\frac{K^* - K}{L^* - L} + \frac{w'(q)}{r'(q)}\right] \\
= \ & \frac{-r'(q)}{r''(q)K + w''(q)L}(L^* - L)\left[\frac{K^* - K}{L^* - L} - k_2(r(q), w(q))\right],
\end{aligned}$$

where $k_2(\cdot) \equiv K_2(r(q), w(q))/L_2(r(q), w(q))$ and the last equality follows from the familiar calculation of the Stolper-Samuelson theorem. Without loss of generality, suppose that the foreign factor endowment is above $A'M'$ whose slope equals k_2 and $L^* > L$. Then, $q\left(\phi\left(\overline{Y}_1\right), K^*, L^*\right) - q\left(\phi\left(\overline{Y}_1\right), K, L\right) < 0$. Parallel arguments apply to cover other cases.

Based on Lemmas 1-3, we can finally state and prove:

Proposition. *Suppose that good 1 is capital-intensive. Then, the foreign country exports good 1 if its factor endowment distribution is above $A'M'$.*

Proof. $q\left(\phi\left(\overline{Y}_1\right), K^*, L^*\right) - q\left(\phi\left(\overline{Y}_1\right), K, L\right) < 0$ if good 1 is capital-intensive and the foreign factor endowment pair is above $A'M'$. Then,

[6]Note that $r''(q)K + w''(q)L > 0$ regardless of the factor intensity ranking.

$Y_1^N < Y_1^{*N}$ follows from Lemma 2, which leads to that the foreign country becomes the exporter of good 1 from Lemma 1.

The above proposition confirms the rough validity of the Heckscher-Ohlin theorem in a model of international duopoly and increasing returns. One distinguishable point is that the borderline which determines the exporter of good depends on the factor intensity ranking. As shown in Figure 1, it becomes $A'M'$ when good 1 is capital-intensive, whereas it becomes $A''M''$ when good 1 is labor-intensive. In this sense, it is fair to say that our trade pattern proposition is called a 'modified' Heckscher-Ohlin theorem.

Remark 1. Throughout this paper, our attention has been focused on the model of international duopoly. However, Doi *et al.* (2004) have proved the robustness of the proposition in various market structures that are frequently employed in trade theory.[7]

Remark 2. As mentioned in Introduction, Markusen (1981) is one of the most seminal works in oligopolistic trade theory. According to him, a large country imports good 1 as long as the assumption of constant returns

[7]Perfect competition and constant returns, monopolistic competition, free entry oligopoly, national economies of scale, and international economies of scale are dealt with in Doi *et al.* (2004).

is maintained. However, our proposition can reverse his result. To see this, suppose that good 1 is capital-intensive. Then, the large country exports good 1 from Figure 1 since the foreign endowment point is above $A'M'$. On the other hand, the large country imports good 1 if it is labor-intensive. Accordingly, Markusen's (1981) trade pattern is reconciled in our framework in the case that good 1 is labor-intensive, while it is not in the case that good 1 is capital-intensive.

4.4 Concluding Remarks

This paper has established a Heckscher-Ohlin trade pattern in a two-country model of international duopoly and increasing returns. We have shown that the Heckscher-Ohlin theorem roughly holds in our setting as well. Our model is different from a standard Heckscher-Ohlin model in three respects. First, we have assumed a quasi-linear utility function, which deviates from a traditional Heckscher-Ohlin model that usually assumes a homothetic preference. Second, our model comprises two distinct agents: factor owners and monopolists rather than a representative consumer as in Markusen (1981). Third, factor price equalization does not continue to hold in our model. We can give a couple of remarks on these points.

Regarding the preference specification, a comment can be noted. In a path-breaking paper, Trefler (1995) found out that the empirical performance of the Heckscher-Ohlin theorem is poor enough not to be supported by data. To explain this, Trefler (1995) suggests that it stems from two of the crucial assumptions in the original Heckscher-Ohlin model: homothetic preferences and internationally identical technologies. Between these factors, much attention was paid to the latter in Trefler (1995). On the other hand, this paper shows that the Heckscher-Ohlin theorem roughly survives quasi-linear preferences, which is a by-product of the paper.

Considering two heterogeneous agents is of meaning in addressing gains from trade like the previous chapter. As shown there, an importer of the non-competitive good tends to lose from trade in the sense that both the consumer and the monopolist become worse off. Connecting this view with our 'modified' Heckscher-Ohlin theorem, a labor-abundant country possibly loses from trade if the non-competitive good is capital-intensive. Such a direction of research is left as our future work.

Bibliography

[1] Dixit, A. K. and V. Norman (1980), *Theory of International Trade*, Cambridge: Cambridge University Press.

[2] Doi, J., K. Fujiwara, T. Kikuchi and K. Shimomura (2004), "A Modified Heckscher-Ohlin Theorem under Quasi-Linear Utility Functions", Mimeo., Kobe University.

[3] Fujiwara, K. and K. Shimomura (2005), "A Factor Endowment Theory of International Trade under Imperfect Competition and Increasing Returns", *Canadian Journal of Economics*, 38, 273-289.

[4] Helpman, E. and P. R. Krugman (1985), *Market Structure and Foreign Trade*, Cambridge, MA: MIT Press.

[5] Kemp, M. C. and K. Shimomura (1995), "The Apparently Innocuous Representative Agent", *Japanese Economic Review*, 46, 247-256.

[6] Lahiri, S. and Y. Ono (1995), "The Role of Free Entry in an Oligopolistic Heckscher-Ohlin Model", *International Economic Review*, 36, 609-

624.

[7] Markusen, J. R. (1981), "Trade and the Gains from Trade with Imperfect Competition", *Journal of International Economics*, 11, 531-551.

[8] Shimomura, K. (1995), "Some Implications of Imperfect Competition for Recent Trade Theory", *Review of International Economics*, 3, 244-247.

[9] Shimomura, K. (1998), "Factor Income Function and an Oligopolistic Heckscher-Ohlin Model of International Trade", *Economics Letters*, 61, 91-100.

[10] Trefler, D. (1995), "The Case of the Missing Trade and Other Mysteries", *American Economic Review*, 85, 1029-46.

[11] Wong, K. (1995), *International Trade in Goods and Factor Mobility*, Cambridge, MA: MIT Press.

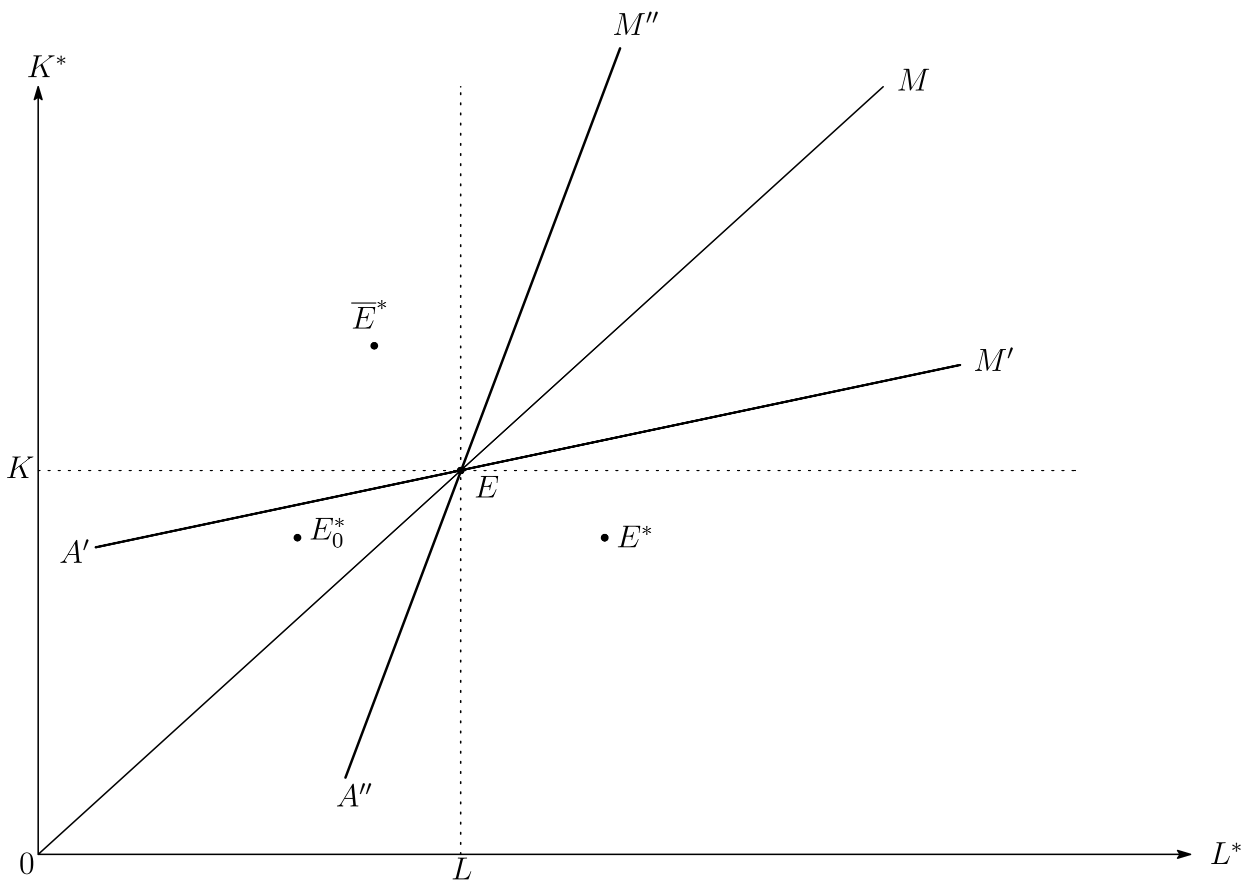

Figure 4.1: The borderline of relative factor abundance

Chapter 5

Two Heckscher-Ohlin Theorems with Utility Maximizing Oligopolists

[1]

[1] I thank Toru Kikuchi and Noritsugu Nakanishi for helpful comments and suggestions.

Two Heckscher-Ohlin Theorems with Utility Maximizing Oligopolists

5.1 Introduction

In the last quarter century, many trade theorists have tried to examine the implications of imperfect competition and increasing returns to scale for the traditional results on international trade. Among others, the Heckscher-Ohlin (henceforth, HO) theorem has been reexamined in imperfectly competitive settings. Dixit and Norman (1980) and Helpman and Krugman (1985) prove the theorem under monopolistic competition and Lahiri and Ono (1995) and Shimomura (1998) reach the same result in oligopolistic models with free entry and exit. In this literature, all these authors shared the assumption of homothetic preferences following the predecessors.

However, recent studies have suggested that homothetic tastes might be problematic both theoretically and empirically. In an influential paper, Trefler (1995) empirically tested the factor content version of the HO theorem, which is called the Heckscher-Ohlin-Vanek theorem, and concluded that its empirical performance is quite poor. Then, he proposed two reasons for it: identically cross-country technologies and homothetic preferences. Thus, a theory of international trade that incorporates non-homothetic preferences is needed to counter his argument.

Moreover, there is another missing link on the HO theorem. To our knowledge, no paper has proved the validity of the theorem under international oligopoly with fixed number of oligopolists. Recently, Fujiwara and Shimomura (2005) proved that the trade pattern roughly follows the HO manner even in an oligopolistic model. However, their theorem is slightly different from the standard HO theorem.

This paper proposes an alternative factor endowment theory of trade under international oligopoly without homothetic preferences. The key element in our theory is that all oligopolists maximize indirect utility rather than profit. Such a treatment dates back to Kemp and Shimomura (1995) that analyzed some implications of utility maximizing oligopolists for the existing results. This paper makes use of their idea for explaining trade patterns.

The paper is organized as follows. Section 2 presents a two-country model with quasi-linear preferences and utility maximizing oligopolists. In this model, the population of the representative consumer is given by unity. Then, we derive the modified version of the HO theorem proved first in Fujiwara and Shimomura (2005). In Section 3, the assumption on unitary population of consumers is replaced by the one with different population sizes. By this change, the exact HO theorem will turn out to be valid. Section 4 sums up the concluding remark.

5.2 The Modified Heckscher-Ohlin Theorem

This section builds up the first model and derives the modified HO theorem. The model employed is basically the same as that in Kemp and Shimomura (2002). Consider a two-country (home and foreign), two-good (goods X and Y), two-factor (capital and labor) world. Good Y is taken as a numeraire and p denotes the price of good X measured by good Y. Both goods are produced from both factors, which are supplied inelastically and in full employment. One deviation from Kemp and Shimomura (2002) is that a country's representative consumer has a quasi-linear utility function:

$$U = u(D_X) + D_Y, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0, \tag{5.1}$$

where U denotes the utility level and $D_i,\ i = X, Y$ the consumption of each good. Utility maximization yields the demand function of good X as a function of price only, $D(p)$, where $D(\cdot) \equiv u^{'-1}(\cdot)$. Both countries share the same preference, which leads to the following market-clearing condition under free trade.

$$D(p) + D(p) = X + X^*, \tag{5.2}$$

where X and X^* are the output of the home and foreign oligopolists. Solving for p, the inverse demand function takes the form of

$$p = p\left(\frac{X + X^*}{2}\right), \quad p'(\cdot) < 0,$$

where $p(\cdot) \equiv D^{-1}(\cdot)$. While most of the existing literature has assumed that each oligopolist maximizes profit in terms of the numeraire, we assume that its objective is to maximize indirect utility. This is justified when a country's consumer has the whole ownership of the oligopolistic firm. The indirect utility function associated with (5.1) is given by

$$u(D(p)) - pD(p) + I,$$

where I is the home national income. To see how I is determined, let us introduce the production side. The production of good X is assumed to be subject to increasing returns to scale formulated as

$$X = F(f_X(K_X, L_X)), \quad F'(\cdot) > 0, \quad F''(\cdot) > 0,$$

where K_X and L_X are the capital and labor inputs and the function $f_X(\cdot)$ is increasing, strictly quasi-concave and linearly homogeneous. On the other hand, the technology of good Y follows a neoclassical production function:

$$Y = f_Y(K_Y, L_Y),$$

where K_Y and L_Y are the capital and labor employment in sector Y. Then, the national income is determined as $I = pX + G(\phi(X), K, L)$, where $G(\phi(X), K, L)$ is the production possibility frontier defined as

$$G(\phi(X), K, L) \equiv \max_{K_i, L_i, i=X,Y} \quad f_Y(K_Y, L_Y)$$

$$\text{subject to} \qquad f_X(K_X, L_X) \geq \phi(X) \equiv F^{-1}(X)$$

$$K_X + K_Y \leq K$$

$$L_X + L_Y \leq L.$$

Note that the function $G(\cdot)$ possesses all the properties of the neoclassical production possibility frontier. Substituting the inverse demand function and the national income defined above into the indirect utility function, the home monopolist's objective function becomes

$$u\left(\frac{X+X^*}{2}\right) - p\left(\frac{X+X^*}{2}\right)\frac{X+X^*}{2} + p\left(\frac{X+X^*}{2}\right)X + G(\phi(X), K, L). \tag{5.3}$$

This objective function is so familiar when we invoke a partial equilibrium model. The first two terms represent a consumer surplus, while the rest is the national income or producer surplus.

A similar argument applies to the foreign monopolist. The home monopolist seeks to maximize (5.3) with respect to X taking X^* as given, which yields the system of the first-order conditions:

$$p'\left(\frac{X+X^*}{2}\right)\frac{X-X^*}{2} + p\left(\frac{X+X^*}{2}\right) + G_\phi(\phi(X), K, L)\phi'(X) = 0 \tag{5.4}$$

$$p'\left(\frac{X+X^*}{2}\right)\frac{X^*-X}{2} + p\left(\frac{X+X^*}{2}\right) + G_\phi(\phi(X^*), K^*, L^*)\phi'(X^*) = 0, \tag{5.5}$$

where $G_\phi(\cdot) \equiv \partial G(\cdot)/\partial\phi$. In this system, X and X^* are determined given each country's factor endowment. Now, we impose:

Assumption 1. There exists at least one Cournot-Nash equilibrium which satisfies the system of (5.4) and (5.5). And any equilibrium involves incomplete specialization in both countries.

Remark 1. Kemp and Shimomura (2002) derived the sufficient conditions for the existence of the Nash equilibrium based on homothetic tastes. Although a different utility function is assumed, a parallel argument is possible.

Following the procedure in Doi *et al.* (2004), we first derive the locus in the $L^* - K^*$ space such that both countries have no opportunity to trade. In what follows, we shall call it the no-trade locus. From the market-clearing condition (5.1), the country whose output of good X exceeds the other country's will export good X, i.e., there is no international trade if and only if we have $X = X^* = \overline{X}$ in the equilibrium.[2] Let us now obtain a few properties of the no-trade locus which causes such a symmetry. Substituting $\overline{X}$ into (5.4) and (5.5) yields

$$\begin{aligned} p\left(\overline{X}\right) + G_\phi\left(\phi\left(\overline{X}\right), K, L\right)\phi'\left(\overline{X}\right) &= 0 \\ p\left(\overline{X}\right) + G_\phi\left(\phi\left(\overline{X}\right), K^*, L^*\right)\phi'\left(\overline{X}\right) &= 0. \end{aligned}$$

[2] See also the previous chapter.

Subtracting the latter equation from the former, we have

$$-G_\phi\left(\phi\left(\overline{X}\right), K, L\right) = -G_\phi\left(\phi\left(\overline{X}\right), K^*, L^*\right). \tag{5.6}$$

Noting that the function $G_\phi(\cdot)$ is a homogeneous of degree zero, (5.6) is equivalent to

$$-G_\phi\left(\frac{\phi\left(\overline{X}\right)}{L}, \frac{K}{L}, 1\right) = -G_\phi\left(\frac{\phi\left(\overline{X}\right)}{L^*}, \frac{K^*}{L^*}, 1\right). \tag{5.7}$$

The relationship between each country's factor endowment which establishes (5.7) is now examined. Suppose $L^* > L$ and $K/L = K^*/L^*$. In view of that $-G_\phi(\cdot)$ is increasing in the first argument, this inequality implies

$$-G_\phi\left(\frac{\phi\left(\overline{X}\right)}{L}, \frac{K}{L}, 1\right) > -G_\phi\left(\frac{\phi\left(\overline{X}\right)}{L^*}, \frac{K^*}{L^*}, 1\right).$$

If good X is a capital-intensive good, $-G_\phi(\cdot)$ becomes decreasing in the second argument. Therefore, in order to restore (5.7), we must have $K/L > K^*/L^*$. Similar arguments can apply to the case in which $L^* < L$ and good X is a labor-intensive good.

The above argument is concerned with the *global* relationship between the factor endowment ration and trade pattern. In what follows, we explore a *local* property of the no-trade locus. To this end, recall the no-trade locus is defined by the pair of K^* and L^* which satisfies (5.6). Let us analyze it as follows. Suppose that $K^* = K$ and $L^* = L$ hold initially, which gives $X = X^* = \overline{X}$. Then, consider a slight change in K^* and L^* from K and L.

Because $\overline{X}$ must be kept to a fixed level from the definition of the no-trade locus, such a deviation must satisfy

$$-G_{\phi K}\left(\phi\left(\overline{X}\right),K,L\right)dK^{*}-G_{\phi L}\left(\phi\left(\overline{X}\right),K,L\right)dL^{*}=0.$$

Note that K^* and L^* are evaluated at K and L since their initial value is given by K and L, respectively. Hence, on the no-trade locus, we have

$$\left.\frac{dK^{*}}{dL^{*}}\right|_{K^{*}=K,L^{*}=L}=-\frac{G_{\phi L}\left(\phi\left(\overline{X}\right),K,L\right)}{G_{\phi K}\left(\phi\left(\overline{X}\right),K,L\right)}=k_{Y},$$

where k_Y is the factor intensity of good Y.[3] This implies that the slope of the no-trade locus is given by the factor intensity of good Y in the neighborhood of (K, L). Together with the *global* result proved beforehand, we have reached an auxiliary result about the no-trade locus.

Lemma 1. *Without loss of generality, suppose that good X is a capital-intensive good. Then, the no-trade locus has the following properties. See Figure 1. (i) It is located above OA in the area of $L > L^*$ and below OA in the area of $L < L^*$. (ii) Its slope in the neighborhood of E $(= (L, K))$ is given by the equilibrium factor intensity of good Y. A similar conclusion follows when good X is labor intensive as well.*

[3]The last equality follows from the property of the production possibility frontier. For more details, see Long (1982).

So far, our focus has been confined to the characterization of the no-trade locus. Based on this preliminary analysis, we move on to the derivation of the trade pattern proposition. The starting point is the system of (5.4) and (5.5) which is abbreviated to

$$\begin{aligned} MR(X, X^*) + G_\phi(\phi(X), K, L)\phi'(X) &= 0 \\ MR(X^*, X) + G_\phi(\phi(X^*), K^*, L^*)\phi'(X^*) &= 0, \end{aligned}$$

where

$$MR(X, X^*) \equiv p'\left(\frac{X + X^*}{2}\right)\frac{X - X^*}{2} + p\left(\frac{X + X^*}{2}\right).$$

Hypothetically assume that the foreign factor endowment is initially given by (K, L), which ensures $X = X^* = \overline{X}$. Now, let only the foreign capital increase by dK^* as Figure 2 depicts. The effect of this change on the system is summarized in the matrix form:

$$\begin{bmatrix} MR_1 + G_{\phi\phi}(\phi')^2 + G_\phi\phi'' & MR_2 \\ MR_2 & MR_1 + G_{\phi\phi}(\phi')^2 + G_\phi\phi'' \end{bmatrix}\begin{bmatrix} dX \\ dX^* \end{bmatrix} = \begin{bmatrix} 0 \\ -G_{\phi K}\phi' \end{bmatrix} dK^*.$$

Since only the capital endowment increases with the labor endowment kept to L in the foreign country, this change makes the foreign country the capital-abundant country. Thus, if good X is a capital-intensive good, we must have $dX^*/dK^* - dX/dK^* > 0$ in order to reach the HO trade pattern. From the above system, $dX^*/dK^* - dX/dK^*$ is obtained as

$$\frac{dX^*}{dK^*} - \frac{dX}{dK^*} = \frac{-G_{\phi K}\phi'}{MR_1 - MR_2 + G_{\phi\phi}(\phi')^2 + G_\phi\phi''}.$$

The sign of the numerator and denominator is determined as follows. From the property of the production possibility frontier, $-G_{\phi K} < 0$ if and only if good X is capital-intensive. To determine the sign of the denominator, let us resort to the stability condition of the Cournot-Nash equilibrium. To this end, let us introduce the following adjustment process.

$$\begin{aligned}\dot{X} &= MR(X, X^*) + G_\phi(\phi(X), K, L)\phi'(X) \\ \dot{X}^* &= MR(X^*, X) + G_\phi(\phi(X^*), K^*, L^*)\phi'(X^*).\end{aligned}$$

The stability of the Nash equilibrium requires that the trace of the coefficient matrix in the above differentiated system be negative and that its determinant be positive. The former is satisfied from the second-order condition for utility maximization. On the other hand, the second requirement evaluated at $(K^*, L^*) = (K, L)$ is given by

$$\left[MR_1 + MR_2 + G_{\phi\phi}(\phi')^2 + G_\phi\phi''\right]\left[MR_1 - MR_2 + G_{\phi\phi}(\phi')^2 + G_\phi\phi''\right] > 0.$$

We consider both the case of $MR_2 > 0$ and that of $MR_2 <$ separately since both cases are theoretically possible. In the former case, the second-order condition ensures that $MR_1 - MR_2 + G_{\phi\phi}(\phi')^2 + G_\phi\phi'' < 0$ and hence the denominator in $dX^*/dK^* - dX/dK^*$ is negative. In the latter case, the terms in the first brackets is negative, which in turn requires that $MR_1 - MR_2 + G_{\phi\phi}(\phi')^2 + G_\phi\phi' < 0$ for the stability. Therefore, as long as the stability is guaranteed, the denominator in the formula of

$dX^*/dK^* - dX/dK^*$ is negative regardless of the sign of MR_2. As a result, if good X is capital-intensive, we can safely say that

$$\frac{dX^*}{dK^*} - \frac{dX}{dK^*} > 0,$$

from which we have derived the first main proposition:

Proposition 1. *See Figure 1. Define the foreign country a capital-abundant (resp. labor-abundant) country if its factor endowment point is above (resp. below) the no-trade locus given by BB (resp. $B'B'$) depending on whether good X is capital-intensive (resp. labor-intensive). Then, the foreign country exports the capital-intensive (resp. labor-intensive) good. In other words, the trade pattern is determined in an HO manner while the borderline which determines it is modified to BB or $B'B'$ from OA.*

5.3 The Heckscher-Ohlin Theorem

This section proposes an alternative model to show the exact HO theorem. Let us assume that there are L ($resp.$ L^*) identical monopolists in the home (resp. foreign) country each of whom supplies one unit of labor. Then, the utility function (5.1) stands for the per-capita one. This change in the assumption makes the market-clearing condition take the form of

$$Ld(p) + L^*d(p) = Lx + L^*x^*, \tag{5.8}$$

where $d(\cdot)$ is the per-capita demand function derived from the quasi-linear utility function while x and x^* denote the per-capita output. The inverse demand function is given by

$$p = p\left(\frac{Lx + L^*x^*}{L + L^*}\right).$$

On the other hand, the per-capita income is determined by

$$px + G\left(\phi(x), k, 1\right),$$

where use is made use of the following definition of the per-capita production possibility frontier:

$$\begin{aligned} G\left(\phi(x), k, 1\right) \equiv \max_{K_i, L_i, i=X,Y} \quad & f_Y(K_Y, L_Y) \\ \text{subject to} \quad & K_X + K_Y = k \\ & L_X + L_Y = 1 \\ & f_X(K_X, L_X) = \phi(x). \end{aligned}$$

Substituting this definition of the per-capita income and inverse demand function into the indirect utility function yields the following objective function:

$$\begin{aligned} & u\left(\frac{Lx + L^*x^*}{L + L^*}\right) - p\left(\frac{Lx + L^*x^*}{L + L^*}\right)\frac{Lx + L^*x^*}{L + L^*} \\ & + p\left(\frac{Lx + L^*x^*}{L + L^*}\right)x + G\left(\phi(x), k, 1\right). \end{aligned} \tag{5.9}$$

Then, the system of the first-order conditions for utility maximization is described by

$$p'\left(\frac{Lx+L^*x^*}{L+L^*}\right)\frac{L^*(x-x^*)}{L+L^*}+p\left(\frac{Lx+L^*x^*}{L+L^*}\right)+G_\phi\left(\phi(x),k,1\right)\phi'(x) \quad (5.10)$$

$$p'\left(\frac{Lx+L^*x^*}{L+L^*}\right)\frac{L(x^*-x)}{L+L^*}+p\left(\frac{Lx+L^*x^*}{L+L^*}\right)+G_\phi\left(\phi(x^*),k^*,1\right)\phi'(x^*) \quad (5.11)$$

Now, let us derive the property of the no-trade locus. In the present context, $x = x^* = \overline{x}$ holds on the no-trade locus. Substituting this into the above system, we have

$$\begin{aligned} p\left(\overline{x}\right)+G_\phi\left(\phi\left(\overline{x}\right),k,1\right)\phi'\left(\overline{x}\right) &= 0 \\ p\left(\overline{x}\right)+G_\phi\left(\phi\left(\overline{x}\right),k^*,1\right)\phi'\left(\overline{x}\right) &= 0. \end{aligned}$$

Subtracting the latter equation from the former yields

$$-G_\phi\left(\phi\left(\overline{x}\right),k,1\right) = -G_\phi\left(\phi\left(\overline{x}\right),k^*,1\right), \quad (5.12)$$

which gives another lemma on the no-trade locus in the present setting:

Lemma 2. *Suppose that good X is a capital-intensive good. Then the no-trade locus is given by OA, that is, it has the following properties. (i) It goes through the origin and the home factor endowment point. (ii) Its slope is equal to the home factor endowment ratio. In other words, the no-trade locus coincides with that of the standard HO theorem.*

Together with Lemma 2, the foregoing argument enables us to state and prove the second main theorem:

Proposition 2. *The standard HO theorem survives international oligopoly and increasing returns to scale with utility maximizing behavior of oligopolists.*

Proof. From Lemma 2, if the foreign factor endowment ratio is initially given by k, the per-capita output in both countries is equalized such that $x = x^* = \overline{x}$ and the equilibrium involves

$$\begin{aligned} p\left(\overline{x}\right) + G_{\phi}\left(\phi\left(\overline{x}\right), k, 1\right) \phi'\left(\overline{x}\right) &= 0 \\ p\left(\overline{x}\right) + G_{\phi}\left(\phi\left(\overline{x}\right), k, 1\right) \phi'\left(\overline{x}\right) &= 0. \end{aligned}$$

Now, suppose that the foreign factor endowment ratio increases from k and that good x is capital-intensive:

$$-\frac{\partial G_{\phi}}{\partial k} \equiv -G_{\phi K}\left(\phi\left(\overline{x}\right), k, 1\right) < 0.$$

Then, this increase in k^* leads to the following inequality.

$$\begin{aligned} 0 &= p\left(\overline{x}\right) + G_{\phi}\left(\phi\left(\overline{x}\right), k, 1\right) \phi'\left(\overline{x}\right) \\ &< p\left(\overline{x}\right) + G_{\phi}\left(\phi\left(\overline{x}\right), k^*, 1\right) \phi'\left(\overline{x}\right). \end{aligned}$$

Therefore, in order for the last right-hand side be equal to zero, the foreign output must be larger than $\overline{x}$ from the second-order condition, i.e.,

$$\frac{\partial}{\partial x^*}\left\{p'\left(\frac{Lx+L^*x^*}{L+L^*}\right)\frac{L(x^*-x)}{L+L^*}+p\left(\frac{Lx+L^*x^*}{L+L^*}\right)+G_\phi\left(\phi(x^*),k^*,1\right)\phi'(x^*)\right\}<0.$$

Accordingly, $x<x^*$ is established in the equilibrium when good x is capital-intensive and the foreign country is capital-abundant. A similar result is concluded in the case where good X is labor-intensive. Thus, we have proved the proposition.

Two remarks concerning Proposition 2 are now offered.

Remark 2. What should be emphasized is that Proposition 2 is a *global* one about the determination of trade patterns. To our knowledge, there is no *global* proposition on trade patterns between the countries whose factor endowment is arbitrarily different in a context of international duopoly and increasing returns. Moreover, the exact HO theorem, not the modified version, survives them, which is another virtue of Proposition 2.

Remark 3. One implication of Proposition 2 is that there is no international trade if and only if both countries' factor endowment ratio is identical. This is a surprising result since the large country is seemingly likely to export the monopolized good invoking the existing studies such as Kemp and Shimomura (2002) that are based on homothetic preferences. In our proposition, however, such a conjecture is not the case.

5.4 Concluding Remarks

We have explored the implications of utility maximizing oligopolists for the determination of trade patterns between the two economies whose factor endowment differs arbitrarily. In the analysis, we have presupposed that a country's monopolist seeks to maximize its utility taking account of the effect of its decision on factor prices and national income. This differs from the existing literature which assumes that a monopolist takes factor prices and national income as given. Relaxing this assumption, Tawada and Okawa (1995) showed that the equilibrium output can deviate when the income effect is taken into account by the monopolist. Their analysis suggests that the income effect and factor price effect may give a serious influence on the existing results on trade. However, even in the presence of such effects, the trade pattern proves to be determined reflecting a country's factor endowment abundance a la HO. This implies that the factor endowment theory of trade has a great validity together with the results in Fujiwara and Shimomura (2005).

Another remark to be mentioned is that we can obtain a *global* trade pattern proposition in a model with restricted entry oligopoly and increasing returns. For example, making use of a model of international duopoly and increasing returns, Fujiwara and Shimomura (2005) showed a validity of the factor endowment theory of trade based on the profit maximizing oligopolists. Nevertheless, their results were confined to the *local* one since they hold only when the two countries' factor endowment is sufficiently close. On the contrary, this paper's results are true even if each country's factor endowment is arbitrarily different. Therefore, the assumption of utility maximizing behavior by oligopolists turns out to be of great use to facilitate analysis and has a number of applications other than the determination of trade patterns.

Bibliography

[1] Dixit, A. K. and V. Norman (1980), *Theory of International Trade*, Cambridge: Cambridge University Press.

[2] Doi, J., K. Fujiwara, T. Kikuchi and K. Shimomura (2004), "The Modified Heckscher-Ohlin Theorem under Quasi-Linear Utility Functions", Mimeo., Kobe University.

[3] Fujiwara, K. and K. Shimomura (2005), "A Factor Endowment Theory of International Trade under Imperfect Competition and Increasing Returns", *Canadian Journal of Economics*, 38, 273-289.

[4] Helpman, E. and P. R. Krugman (1985), *Market Structure and Foreign Trade*, Cambridge, MA: MIT Press.

[5] Kemp, M. C. and K. Shimomura (1995), "The Apparently Innocuous Representative Agent", *Japanese Economic Review*, 46, 247-256.

[6] Kemp, M. C. and K. Shimomura (2002), "A New Theory of International Trade under Increasing Returns: The Two-Commodities Case",

in A. D. Woodland (ed.), *Economic Theory and International Trade: Essays in Honor of Murray C. Kemp*, Aldershot, Hants: Edward Elgar, 3-21.

[7] Lahiri, S and Y. Ono (1995), "The Role of Free Entry in an Oligopolistic Heckscher-Ohlin Model", *International Economic Review*, 36, 609-624.

[8] Long, N. V. (1982), "Some Properties of the Per Capita Production Set in the Two-Sector Model of Economic Growth", in M. C. Kemp (ed.), *Production Sets*, New York: Academic Press, 145-158.

[9] Shimomura, K. (1998), "Factor Income Function and an Oligopolistic Heckscher-Ohlin Model of International Trade", *Economics Letters*, 61, 91-100.

[10] Tawada, M. and M. Okawa (1995), "On the Behavior of Monopoly in the General Equilibrium Trade Models", in W. W. Chang and S. Takayama (eds.), *Imperfect Competition in International Trade*, New York: Kluwer, 63-78.

[11] Trefler, D. (1995), "The Case of the Missing Trade and Other Mysteries", *American Economic Review*, 85, 1029-1046.

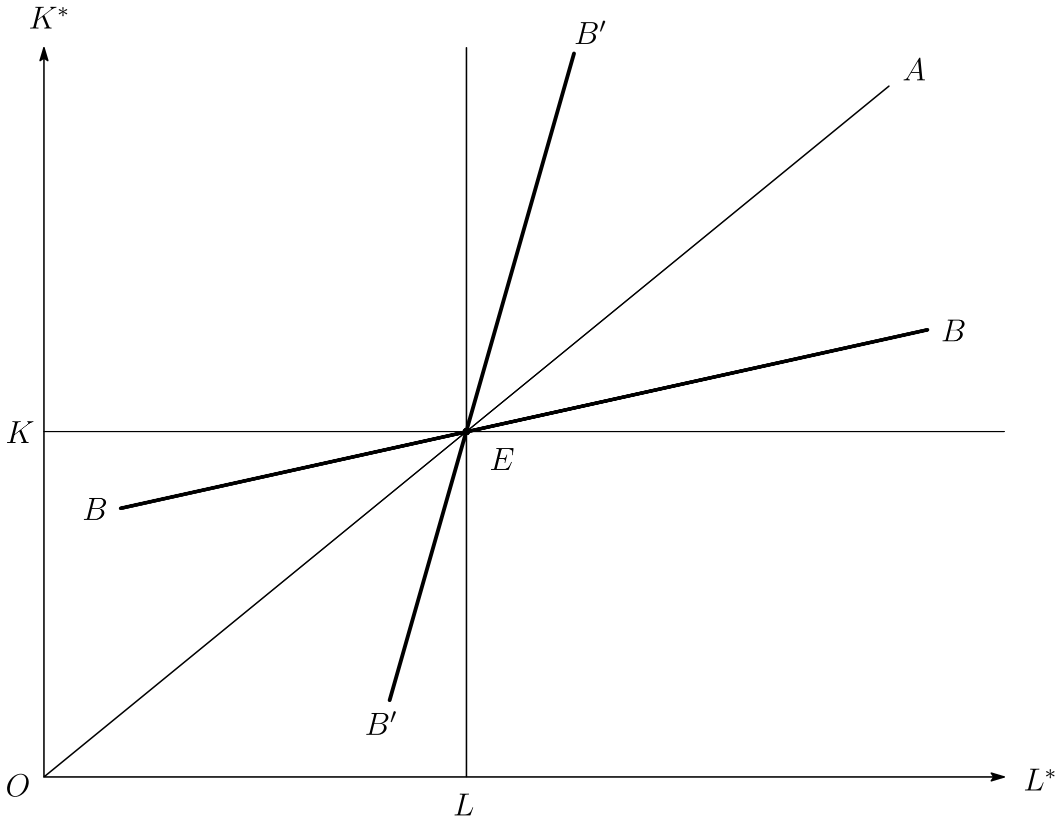

Figure 5.1:

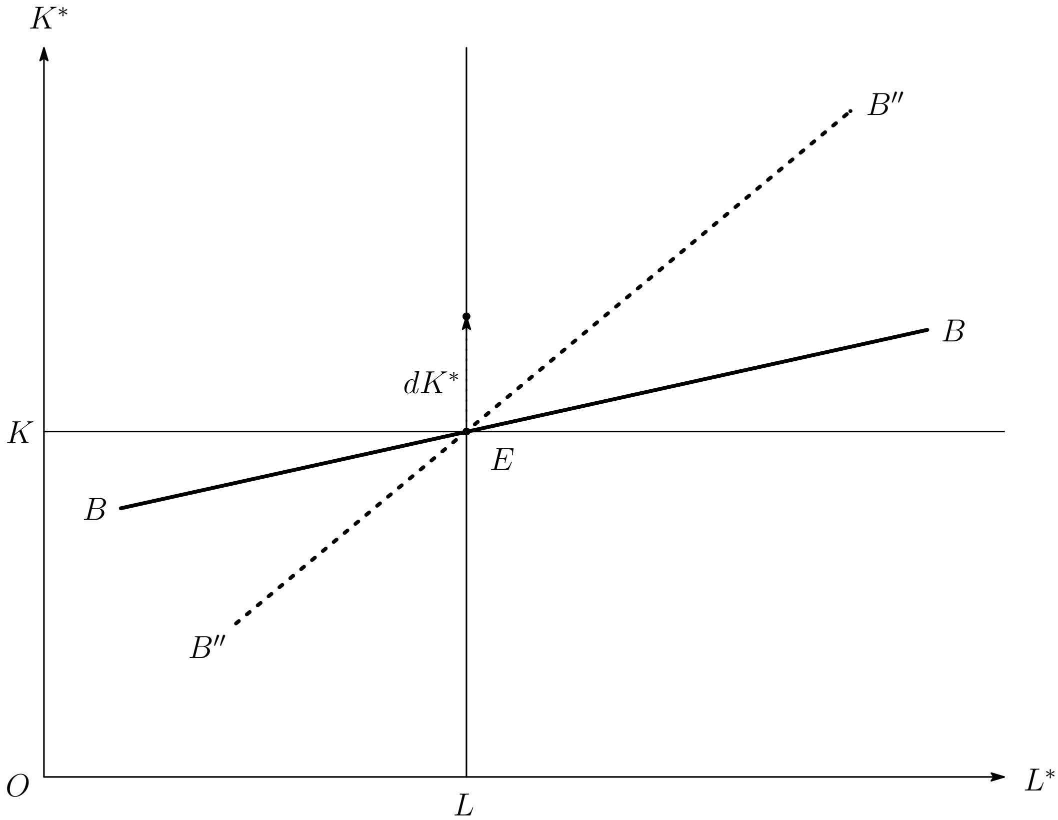

Figure 5.2: