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博士論文

Heterarchical structure in coupled
Time-State-scale Re-entrant System

結合時間状態スケール相互再参入系におけるヘテラルキー構造

平成20年1月

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Preface

This doctoral dissertation is a conclusion of my study for the Graduate School of Science and Technology, Kobe University. From the final-year of baccalaureate degree program to the doctoral course, I and supervisor have been pursuing the subject of paradoxical expression with respect to autonomy and/or life in the terms of self-reference. As the dynamical system representing such a paradoxical property of autonomy, we have proposed time-state-scale re-entrant system (TSSRS). At the beginning, this attempt simply concentrates into the goal to understand the uncertainty and/or complexity of autonomous systems, that is thought such as the term randomness or chaos. However, recently, the term ”network” increasing importance for the study of biological systems. Thus, we have to extend TSSRS to global study including both of the system and the network, and then we apply TSSRS to the study of self-referential modeling of molecular chemical reaction network. We hope that this abstract but important study provides a rich system theoretic perspective of biological systems to the related study.

Contents

Chapter 1

Introduction

The transition from a system to a network is increasing necessity in the paradigm of system theoretical approach for a wide variety of organism. At many perspectives of scales, subjects, definitions and levels of nature, the indicated and/or described systems have made been thought of the complex networks. However, the many way of understanding the systems with using the lower level structural property of complex network is reduced the system theoretic analysis of systems. The problem of the system theory is that it can not include the discussion about a perspective of observer, and because of that, the system theory can not directly express the notion of autonomy and/or life but indicate them as the complexity and/or uncertainty. A complex network property of a system appears if the elements are seemed to be standing on the same levels, namely the uncertainty of a system is assimilated to the complexity as the result of execution of the network. This paradigm in the description of the natural systems is based on the ideal system that is closed and completely described. To understand the systems in nature, we have to find the way to understand the open systems that extremely connected each other, and then the network property with the definition on the same levels is invalidated. Hence, the way to understand autonomy and/or life is replaced with the construction of the interaction among different levels that is called heterarchy. In this paper, we take the notion of observation into consideration, and define that the system and the network are the question of degree. If this question of degree is possible, the network property can not simply defined on the same levels, and then we can present the model of the network consisting of interacted different levels.

Autopoiesis

The word ”system” have been explicitly defined by Bertalanfy as in the general system theory (Bertalanfy, 1968). He has defined a system as follows;

- A system consists of interrelated components.
- A system can not be reduced to a subclass.
- A system moves toward one purpose.
- A system have some subsystems that have original structure.
- Subsystems make the system with interacting and regulating each other.

The definitions of general systems negated the ordinal reductionism. General system theory has one problem that is about observer because the nonreduced system derives from external observer, and then the autonomous system that does not need a external observation, e.g. life, itself plays role of observer. Such a observer oriented problem of autonomy and/or life reveals the self-referential paradox in logic, as shown later. Because of this difficulty of formation the self-referential property, the system theoretic approach has been mainly developed in cybanetics (von Foerster, 1995) that is about open systems. The feature of developed system theory is open system that interacts with their environment.

Maturana and Valera (Maturana and Varela, 1991) have proposed the developed theory of open systems called autopoiesis. Autopoiesis is defined that both the components and the boundary of a system does not exist from beginning but is yielded when the system executes, and then the system configures united wholeness, after that, the system yields its own components. By this cycle of the boundary formation, realization of the system is possible. However, in spite that the autopoietic system plays a role of internal observer, the cyclic process needs foundation from external observer since the cyclic process itself is closed (Luisi, 2003). Thus, the theory of autopoiesis also reveals self-referential paradox. So, what is meaning of the self-reference to the system theoretic approach to life or complex systems? We, here, take one attitude that self-reference is not a clasp of evolution and/or duration of living organism but a genesis of the asymmetric property of life.

Self-reference and frame-problem

Basically, the self-referential paradox is presented in the field of mathematical logic that is explicitly expressed in a category theoretical form by Lawvere (Lawvere, 1969). He have defined the logical self-reference as the paradox in the cartesian closed category. This types of paradox reveals from the basis of completely indicated wholeness to the consequence of infinite reduction of the elements. Such a consequence of infinite reduction only reveals internal observation but not the external observation. Thus, the self-reference reveals the impossibility of representing the system that plays the role of internal observer. Here, we construct the alternative view of self-reference that is co-exitence of internal and external observer from the perspective of paradoxical formation of the systems. In the field of artificial intelligence, on the other hand, the similar impossibility of description of the system referred, that is called frame-problem (McCarthy and Hayes, 1969). Frame-problem is basically defined as the impossibility of defining wholeness that is based on the premise of indicated elements. The two notions self-reference and frame-problem, here, are complementary since the consequence of paradox invalidate the premise of each other, in other word, the frame-problem reveals the impossibility of indicating the perspective of external observer. If we take both perspective of self-reference and frame-problem, internal and external observer are not completely definable, however, we think that the incomplete view of measurements of the observer paradoxically makes the expression of open system, namely the formalization of the autonomy and/or life.

The more explicit the structural over view of the systems presented by the notion of heterarchy that is based on the network based analysis of the systems in nature. The reason why we refer the notion of heterarchy is that it is easy to be applied any other systems that have a basic property of network structure.

Heterarchy

Heterarchy (McCulloch, 1945; Stark, 1999) is a dynamical hierarchy that have an interplay and/or overlapping multiple scale of network structure. This structural property of network is deeply related to the notion of robustness (Jen, 2003) and evolvability (Kirschner and Gerhart, 1998). Jen, in her paper, has characterized robustness and evolvability of living organism as the possibility of structural change against future perturbation from environment, and the term structure in this discussion is realized only on the basis of the united wholeness of the systems, then it reveals the heterarchical structure. However, the model of heterarchical systems can not be easily expressed since the notion of interplay is basically defined on the same levels of systems, namely among elements or systems. Thus, in order to assume the interplay among hierarchy, it needs to assimilation between the element and the wholeness. Such a assimilation among hierarchy instantly reveals an inconsistency between wholeness and elements, in other word intent and extent. Now, the possibility of modeling heterarchy is replaced with addressing the inconsistency. The mathematical modeling of addressing inconsistency has been presented in some framework (Gunji et al., 2004; Gunji and Kamiura, 2004). This schema is very similar to the discussion about the system theoretic possibility of describing autonomy, and then the purpose of this paper become more clear, that is to consistently discuss the paradoxical construction of the self-referential property of autonomous systems and the heterarchical network structure.

Time-State-scale Re-entrant system

To construct the consistent modeling as mentioned above, we firstly propose the formal model of autonomous systems called time-state-scale re-entarant system (TSSRS) (Gunji and Sasai, (in press; Gunji et al., 2007). TSSRS is based on a logic-based dynamical systems that is presented in order to avoid the self-referential paradox in liar statement. A foundation of avoidance of paradox in liar statement is by introducing the time spent for inference process (Spencer-Brown, 1969; Lefebvre, 2001), that corresponds to the boundary formation process mentioned in the discussion about autopoiesis. Grim et al. have applied this discussion to fuzzy logic and express the self-reference as a chaotic dynamics. We extend their work according to the formation both of self-reference and frame-problem, and two dynamical systems corresponding to them are obtained. By connecting the two dynamical systems, we can construct the paradoxical modeling of autonomous systems, and we call this connecting schema of self-reference and frame-problem as time-state-scale re-entrant form. The introduction and definition of TSSRS and back ground more specifically discussed in chapter 2.

As mentioned above, our aim of this research is to discuss the consistent modeling of the system to the network structure on the formulation of the heterarchy. TSSRS derives from one self-referential liar statement, and further we extend TSSRS to a self-referential representation of a biological network, and show that the time-state-scale re-entarnt form reveals the autonomous robust and evolvable property through the system to the network structure.

Biological network

In the living organism, the network structure is thought as the essential genesis of diverseness. For example, the most primitive network structure is the chemical reaction network in a cell. The diverseness of cells derives from the phenotypic multiplicity at the production of proteins, and the chemical network among multiple protein interactions configure the functional manifestation of the cell as the biological signal processing. The heterarchical property can be also found in this types of network structure such that, in the chemical reaction network have the two types of feedback mechanisms: one is the results of the recurrent process of local interaction among the activation sites, and the other is the influence from the consequent infection to the target proteins (Albert, 1994). The two feedback mechanisms interplay on the development of each node in the chemical reaction network, and it yields the complex structural property of functional manifestation in the living organism. However, it is difficult to construct the model of this interplay and overlapping property of biological network since the two feedback mechanism have extremely different time scale, and only the unilateral interaction affects the individual protein, namely the other influence is ignored. Recently, the experimental results that reveals the co-existence of different time scale is presented, that is about the structural transformation of protein kinase at the phase transition temperature (Henzler-Wildman et al., 2007). Thus, modeling the network property of interplay among two scales of feedbacks is rapidly needed to understand such a property of biological network.

The biological network is studied in the form of computational network analysis such as the boolean network (Kauffman and Glass, 1973). The boolean network consists of the simple definition of the behavior at the nodes of network as an activation and an inactivation of each node. Signal propagation along network is defined by boolean function based on the bivalent valued logic. System theoretical approach to these analytical framework of cell biology has been presented by Tsuda et al. (Tsuda and Tadaki, 1997) They proposed the logical framework directly indicating the feedback mechanism, and that reveals the self-referential property of biological functional manifestation. We apply time-state-scale re-entrant system to Tsuda's model of biological network to provide the expression of heterarchy not only in system theoretic modeling but in the form of network architecture. Actually, the model of the network version of the time-state-scale re-entrant form shows heterarchical property of network structure as the co-existing time scale and fractal switching the dynamical property.

Our purpose of this work is simply to provide the heterarchical property of network structure as direct modeling feedback mechanism in the form of self-reference. The proposed model time-state-scale re-entrant form is useful to construct the robust and evolvable network structure. In the next chapter, we introduce the basic framework of time-state-scale re-entarn form and applied models time-state-scale re-entarant system (TSSRS). The chapter 3 shows the example network version of time-state-scale re-entrant form called coupled TSSRS and its numerical results. The last chapter 4 includes the conclusion of this study and the possibility of application to other framework.

Chapter 2

Time-State-scale Re-entrant System

In this chapter, we introduce the model of time-state-scale Re-entrant System (TSSRS). As mentioned previous chapter, representation of heterarchical structure and invalidation of self-reference are deeply related each other. Here, we take the model of self-reference presented by Grim et al. (Grim et al., 1994), and reconstruct it by the heterarchical meaning. Heterarchicy consists of two levels of perspectives, subsystem level and wholesystem level. As a subsystem level, we define the perspective of frame problem, and as a wholesystem level, we define the perspective of logical self-reference, respectively. The simultaneous realization of two perspectives enables invalidation of self-referential paradox in terms of heterarchical meaning. Heterarchical meaning is represented by connection between two perspectives, and we call the connected two perspectives time-state-scale re-entrant form. Firstly, we introduce the basic model of self-reference. As a second, we define two perspectives, frame problem and logical self-reference. Finally, we construct the connection of two perspectives.

2.1 Chaotic liar

In this section, we briefly introduce the study of Grim et al. (Grim et al., 1994) The most primitive liar statement is represented by 'This statement is false.'. The paradox holds if we assume that the segment 'this statement' and the whole statement 'This statement is false.' are the same implication. In logical expression,

$$X = \neg X, \tag{2.1}$$

where, $X \in \{0, 1\}$ is the truth value of the statement. Grim et al. introduce the time transition from left hand side to right hand side. This time transition primarily presented by Spencer-Brown (Spencer-Brown, 1969). In addition, applying Łukaiewicts logic (Łukasiewicts, 1989) which is a kind of fuzzy logic (Zadeh, 1965), liar statement is interpreted to a simply oscillating dynamics. Definition of Łukasiewicts logic is a infinite valued logic that takes the real truth value on $[0, 1]$. The real truth value indicates the degree or probability of truth. In Łukasiewicts logic logical connectives are interpreted as following operator on $[0, 1]$ intervals, for $x, y \in [0, 1]$,

$$\begin{aligned}
\neg x &= 1 - x, \\
x \wedge y &= \min(x, y), \\
x \vee y &= \max(x, y), \\
x \rightarrow y &= \max(1 - x, y) \\
x \leftrightarrow y &= \min(\max(x, 1 - y), \max(1 - x, y))
\end{aligned} \tag{2.2}$$

Logical connectives $\rightarrow$ and $\leftrightarrow$ do not satisfy classical equality $x \leftrightarrow x = 1$. To avoid this inconsistency, Łukasiewicts logic defines these connectives as follows,

$$x \rightarrow y = \min(1, 1 - x + y), \tag{2.3}$$

$$x \leftrightarrow y = 1 - |x - y|. \tag{2.4}$$

Apllying this logic system to (2.1.1.), dynamical representation of liar statement is arise,

$$x_{t+1} = 1 - x_t. \tag{2.5}$$

Clearly, this dynamical system shows an periodic oscillation between x and $1 - x$. Grim et al. have introduced chaotic liar to represent a logical paradox in liar statement. Chaotic liar is the statement 'This statement is as true as it is estimated to be false.', as a logical equation,

$$X = X \leftrightarrow \neg X. \tag{2.6}$$

Introducing Łukasiewicts logic system and time development to this equation, the dynamical expression of chaotic liar is obtained as follows,

$$x_{t+1} = 1 - |1 - 2x_t|. \tag{2.7}$$

This dynamical expression shows chaotic behavior as shown in Fig. 2.1. By the shape of return map structure, this chaotic map is called tent map.

2.2 Notion of Time and State-scale

We construct the invalidation of self-reference by alternative way to time development. It is connecting logical self-reference and frame problem. In this section, we introduce the two perspectives, logical self-reference and frame problem.

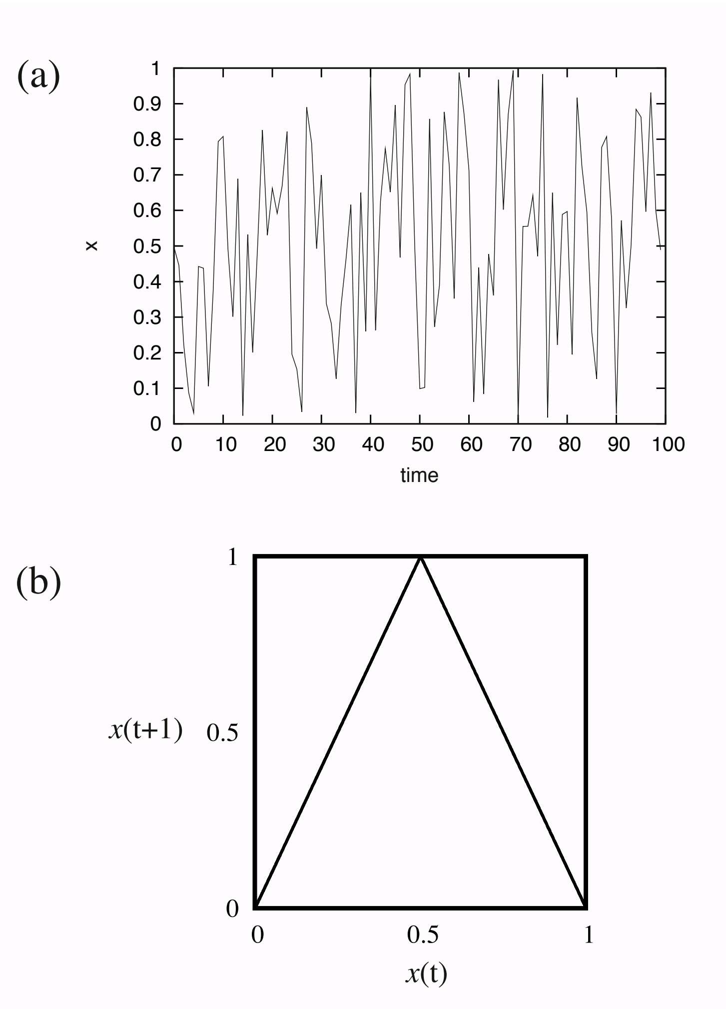

Figure 2.1: Behavior of chaotic liar. (a) chaotic time evolution of truth value. (b) the return map structure of the dynamics.

Let us imagine the following situation (Fig. 2.2); We are looking at a statement 'This statement is false' written on the black board. Then, we estimate the truth value of this statement, and fall into self-referential paradox. Where, we call this self-reference "logical self-reference". In addition, imagine that the word 'not' is written on the corner of black board. Then, we can not indicate the semantics of

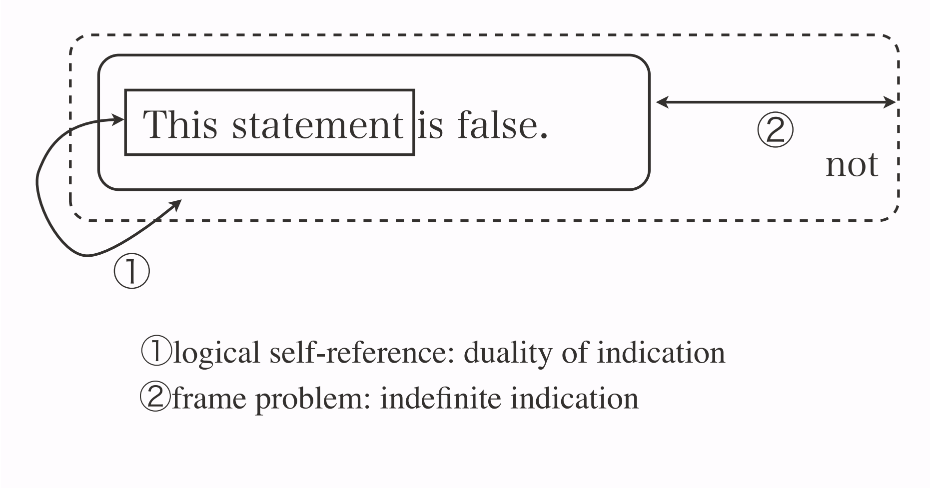

Figure 2.2: The definition of logical self-reference and frame problem in the particular situation.

the word 'this'. We call this indefinite indication "frame problem". These two perspectives have a different logical states. The former is based on the completely external observer, since we can completely indicate the statement. The latter is based on the incomplete observation, since we can not completely indicate the statement. This incomplete observation is deeply related to the study of internal measurement (Matsuno, 1989) or endo-physics (Rössler, 1988). Our heterarchical meaning is established by the crossover of the internal and external observers.

Next, we express the heterarchical meaning of logical self-reference and frame problem. In Fig. 2.3, we show the schematic diagram of heterarchical meaning of logical self-reference and frame problem. Logical self-reference, as mentioned above, needs to indicate the whole statement at first, and leads to the indefinite word as a part of the statement. Thus, logical self-reference can be applied to only wholesystem. On the other hand, frame problem needs to indicate the words at

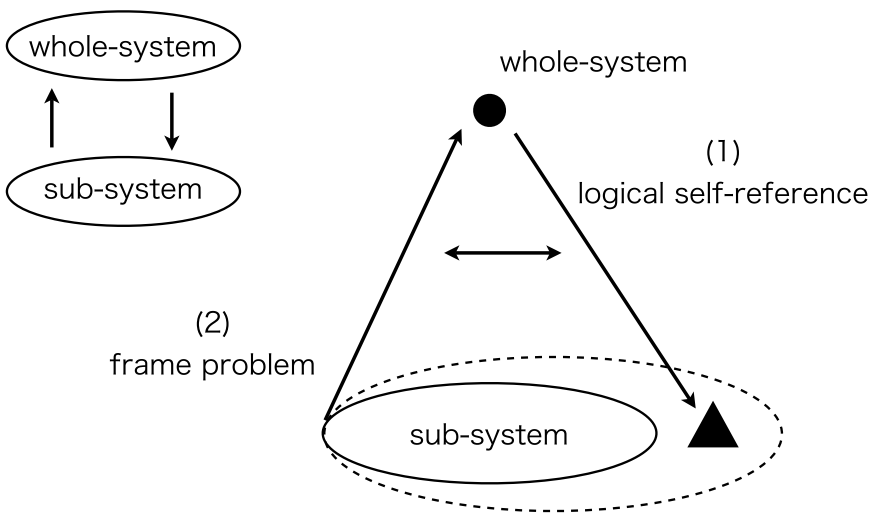

Figure 2.3: Schematic diagram of heterarchical meaning of invalidation of self-reference.

first, and leads to indefinite wholeness as a consequence. Thus, frame problem is applied to only subsystem level. Here, we can following analogy; If logical self-reference happens, then it implies indefinite sub-system and it puts at the extent in the subsystem level description. Then, reconstructed extent of subsystem level description leads to frame problem and indefinite wholeness happens. If these process simultaneously occur, indication in the first condition of two perspectives are made by indefinite ones. Our invalidation of self-reference is defined such a indefinite foundation of perspectives[1].

[1] The idea of invalidation of self-reference is deeply related to Kripke's attitude toward Wittgenstein's paradoxical definition of language(Kripke, 1982). He has introduced another skepticism

Now we have defined logical self-reference as a whole-system level perspective, and frame problem as a sub- ssystem level perspective, respectively. In dynamical system, this whole-system level perspective corresponds to time transition and subsystem level perspective corresponds to indicate the scale of state space. Hence, we define these two perspectives as the dynamics of the time direction and the direction of state-scale, respectively.

We name them the form of time and the form of state-scale. They are expressed by the following maps.

$$x^{t+1} = f(x^t), \tag{2.8}$$

$$x_{s+1} = g(x_s), \tag{2.9}$$

where x is state value and $x \in [0, 1]$. Eqs.(2.8) and (2.9) are expressed as logic-based dynamics derived from logical statements consisting of combination of logical connectives. We define dynamical heterarchical system as an interaction between the two forms, (2.8) and (2.9). In this section, we define the conditions for (2.8) and (2.9).

Here, if it assumes that liar statement forms self-referential system as a heterarchical system, these perspectives may serve as the form of time, and the form of state-scale in the system derived from the liar statement. We redefine a system derived from liar statement by the form including these two perspectives.

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toward classical skepticism. The introduced skepticism invalidate the indication of wholeness to establish the completely defined rule of connecting word. It concludes the invalidated classical skepticism.

2.2.1 The form of Time

In Łukasiewictz logic system, we refer to Cantor's diagonal argument to construct logical self-reference on the $[0, 1]$ interval. Logical self-reference is replaced with the value different from any other values which can be specified on $[0, 1]$ interval in our logic system. It is equivalent to the consequence in Cantor's diagonal argument not being contradictory. The consequence as follows; In Cantor's diagonal argument one arranges the binary sequence corresponding to real number perpendicularly, and assumes that it is enumerated by the natural number[2]. Then, the reversed diagonal sequence differs from any sequences in order. Hence, the sequence added on the table causes contradiction on an intersection.

Consequence in Cantor's diagonal argument constitutes a contradiction resulting from self-reference in a term of intersection. Let $X, \mathrm{NOT}(X)$ be the binary sequences such that for all k, $X = \langle X_1, X_2, \cdots \rangle$,

$$\mathrm{NOT}(X) = \langle \mathrm{NOT}(X_1), \mathrm{NOT}(X_2), \cdots, \mathrm{NOT}(X_k), \cdots \rangle, \tag{2.10}$$

where $\mathrm{NOT}(X_k) = 1 - X_k$. The binary operation, $X \cap Y$, is defined by, for each i, $X_i \cap Y_i$ follows the truth table such that $0 \cap 0 = 0 \cap 1 = 1 \cap 0 = 0$ and $1 \cap 1 = 1$. The contradiction appearing in Cantor's diagonal argument is expressed as,

$$X \cap \mathrm{NOT}(X) = \mathbf{0} \Longleftrightarrow \forall n \in \mathbf{N}\, (X_n \neq \mathrm{NOT}(X_n)), \tag{2.11}$$

where $\mathbf{0}$ represents a all-0's sequence, $\langle 0, 0, 0, \cdots \rangle$.

We redefine AND operation so that the consequence of diagonal argument does not reveal a contradiction. We define logical self-reference in our logic

[2]Even if it puts the real number in order as it is, it is essentially the same.

system. In Kripke's argument, a new value is added and the context is redefined so that the new value invalidates a contradiction. In our statement of diagonal argument, (2.11), a new value Z_n is introduced and added to a pair of terms, X and NOT(X). A new pair, X' and (NOT(X))$'$ are derived from Z_n so that $X' \cap (\mathrm{NOT}(X))' \neq \mathbf{0}$. Formally, for an infinite sequence X and NOT(X), such as $\langle X_1, X_2, \cdots, X_k, \cdots \rangle$ and $\langle \mathrm{NOT}(X_1), \mathrm{NOT}(X_2), \cdots, \mathrm{NOT}(X_k), \cdots \rangle$, a new number $Z_n \in \{0, 1\}$ is inserted at the n-th digit, and it leads that,

$$X' = \langle X_1, X_2, \cdots, Z_n, \cdots \rangle, \quad (\mathrm{NOT}(X))' = \langle \mathrm{NOT}(X_1), \mathrm{NOT}(X_2), \cdots, Z_n, \cdots \rangle. \tag{2.12}$$

As a result, we obtain,

$$X' \cap (\mathrm{NOT}(X))' \neq \mathbf{0} \Longleftrightarrow \exists n \in \mathbf{N}\, (X_n = (\mathrm{NOT}(X))_n = Z_n). \tag{2.13}$$

Let $Z = \langle Z_1, Z_2, \cdots, Z_{n-1}, Z_n, Z_{n+1}, \cdots \rangle = \langle 0, 0, \cdots, 0, Z_n, 0, \cdots \rangle$ (i.e., $\forall i \in \mathbf{N}\, (i \neq n), Z_i = 0$), and decimal expressions of X' and (NOT(X))$'$ are

$$x' = \sum_i 2^{-i}(X_i + Z_i) = x + z, \quad y' = \sum_i 2^{-i}((\mathrm{NOT}(X))_i + Z_i) = y + z. \tag{2.14}$$

The replacement (X, Y) of (X', Y') is operationally re-expressed by X AND NOT(X) $=$ $X' \cap \mathrm{NOT}(X')$. In decimal expression, binary operation AND corresponds to MIN such that,

$$\mathrm{MIN}(x, y) = \min(x', y'). \tag{2.15}$$

Due to the statement (2.13), $\mathrm{MIN}(x, y) \geq \min(x, y)$, and in addition, we define $\mathrm{MIN}(x, y)$ to satisfy the condition of logical equivalent operator $x \leftrightarrow x = 1$. In classical logic system, $x \leftrightarrow y$ can be rewritten with $\wedge$, $\vee$, and $\neg$, such as $x \leftrightarrow y = (x \vee \neg y) \wedge (\neg x \vee y)$. In our logic system, this can be represented with,

$$x \leftrightarrow y = \min(\max(x, 1 - y), \max(1 - x, y)). \tag{2.16}$$

Consequently, we define $\mathrm{MIN}(x, y)$ to satisfy the conditions $\mathrm{MIN}(x, y) \geq min(x, y)$ and $x \leftrightarrow x = 1$, such that

$$\mathrm{MIN}(x, y) := \frac{\min(x, y)}{\max(x, y)}. \tag{2.17}$$

Clearly, this definition (2.17) satisfies $\mathrm{MIN}(x, y) \geq min(x, y)$ and $x \leftrightarrow x = 1$. We consequently obtain the form of time as the logical statement with AND operation. It is clear to see that the form of time reveals logical self-reference and whole-system level statement.

2.2.2 The form of State-scale

The form of state-scale can be also defined based on diagonal argument. Frame-problem assumes that the element is already decided. The process of our understanding gives rise to reconstruction of the boundary of the context. In our logic system, a binary sequence is expressed with finite length such as,

$$x = \sum_{i=1}^{N} 2^{-i} X_i, \tag{2.18}$$

where N is a length of the binary sequence $X = \{X_1, X_2, \cdots, X_N\}$. Since X is finite binary sequence, a diagonal argument consisting of any x on $[0, 1]$ interval has not a contradictory intersection. We call this diagonal argument ”limited diagonal argument”. In general,

$$X \cup \mathrm{NOT}(X) = \mathbf{1}, \tag{2.19}$$

holds, where $\mathbf{1}$ represents a all-1’s sequence, $\langle 1, 1, 1, \cdots \rangle$. In Cantor’s diagonal argument the statement (2.19) allows that a whole set of sequence is invariant even by adding a new sequence derived from diagonal argument and that results in a contradiction. In order to invalidate a contradiction, the idea of ”growing wholeness” is introduced such that $X' \cup (\mathrm{NOT}(X))' \neq \mathbf{1}$. This idea can be easily implemented in the limited diagonal argument.

We redefine OR operation so that the consequence of diagonal argument does not reveal a contradiction. We define frame-problem in our logical system. In Kripke’s argument, a new word is added and the context is redefined so that the new word invalidates a contradiction. In our statement of limited diagonal argument(2.19), since N is a finite number, a new word Z_{N+1} can be introduced and added to a pair of terms, X and $\mathrm{NOT}(X)$. A new pair, X' and $(\mathrm{NOT}(X))'$ are derived from Z_{N+1} so that $X' \cup (\mathrm{NOT}(X))' \neq \mathbf{1}$. Formally, for a finite sequence X and $\mathrm{NOT}(X)$, such as $\langle X_1, X_2, \cdots, X_N \rangle$ and $\langle \mathrm{NOT}(X_1), \mathrm{NOT}(X_2), \cdots, \mathrm{NOT}(X_N) \rangle$, a new number $Z_{N+1} \in \{0, 1\}$ ia added at the $N+1$-th digit, and it leads that,

$$X' = \langle X_1, X_2, \cdots, X_N, Z_{N+1} \rangle, \tag{2.20}$$

$$(\mathrm{NOT}(X))' = \langle \mathrm{NOT}(X_1), \mathrm{NOT}(X_2), \cdots, \mathrm{NOT}(X_N), Z_{N+1} \rangle. \tag{2.21}$$

As a result, we obtain,

$$X' \cup (\mathrm{NOT}(X))' \neq \mathbf{1}. \tag{2.22}$$

Let $Z = \langle Z_1, Z_2, \cdots, Z_{N-1}, Z_N, Z_{N+1} \rangle$ (i.e., $\forall i \in \mathbf{N}\,(i \leq N), Z_i = 0$), and decimal expressions of X' and $(\mathrm{NOT}(X))'$ are

$$x' = \sum_{i=1}^{N} 2^{-i} X_i + \sum_{i=1}^{N+1} 2^{-i} Z_i = x + z, \quad y' = \sum_{i=1}^{N} 2^{-i} (\mathrm{NOT}(X))_i + \sum_{i=1}^{N+1} 2^{-i} Z_i = y + z. \tag{2.23}$$

The replacement (X, Y) of (X', Y') is operationally re-expressed by X OR $\mathrm{NOT}(X) = X' \cup \mathrm{NOT}(X')$. In decimal expression, binary operation OR corresponds to MAX such that,

$$\text{MAX}(x, y) = \max(x', y') \geq x, y. \tag{2.24}$$

We define $\text{MAX}(x, y)$ to satisfy the conditions (2.24) and $x \leftrightarrow x = 1$, as follows;

$$\text{MAX}(x, y) := \text{sgn}(x + y); \quad \text{sgn}(z) = \begin{cases} z & \text{if } z \leq 1, \\ 1 & \text{otherwise.} \end{cases} \tag{2.25}$$

Clearly, the definition (2.25) satisfies (2.24) and $x \leftrightarrow x = 1$. We consequently obtain the form of state-scale as the logical statement containing redefined OR operation. It also reveals frame-problem.

By using definition (2.17) and (2.25), self-referential sentence leads both to the form of time (2.8) and to the form of state-scale (2.9). In the following section, we actually implement the system derived from liar statement.

2.3 Time-State-scale Re-entrant System

time-state-scale Re-entrant System is defined as the interaction between the form of time and that of state-scale. We define the interaction as follows;

$$x^{t+1} = f\left(U\left(g^N\left(V(x^t)\right)\right)\right), \tag{2.26}$$

where, U, V are operational functions defined later. Fig.2.4 shows the schematic diagram for the interaction.

As mentioned above, two functions f and g are defined by a particular statement with (2.17) and (2.25), respectively. To construct interaction between them, we define operational functions that twist binary sequence from a row to a column and vice versa.

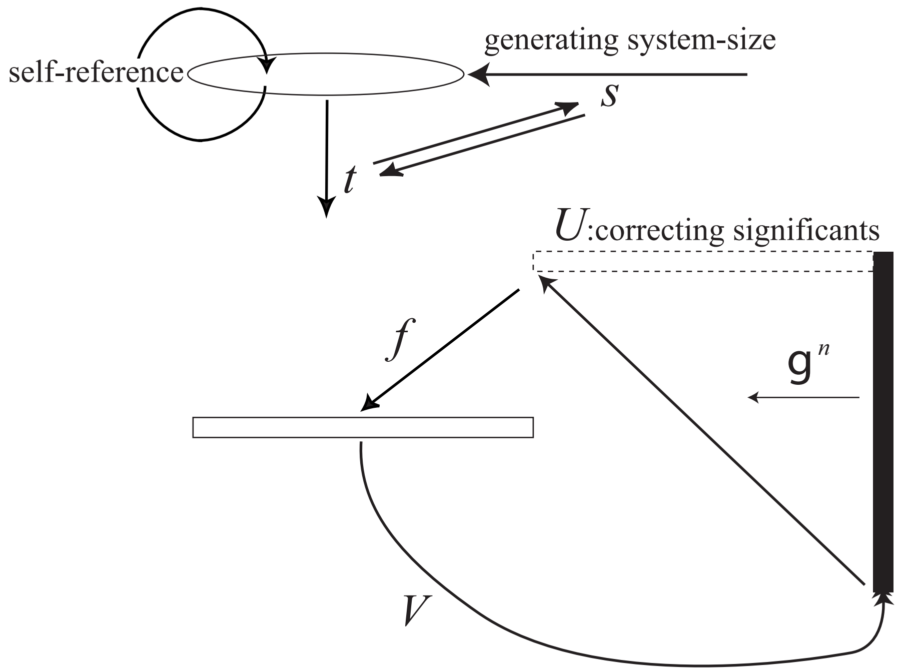

Figure 2.4: schematic diagram of time-state-scale Re-entrant system and its component functions.

2.3.1 Definition of $U(x)$

Let A be a binary sequence expression of state-value $x \in [0, 1]$.

$$A = \begin{pmatrix} a(1) \\ a(2) \\ \vdots \\ a(N) \end{pmatrix}, \qquad x = \sum_{i=1}^{N} 2^{-i} a(i). \tag{2.27}$$

where N is a length of binary sequence, and $a \in \{0, 1\}$.

A binary sequence A_n is obtained by n times applying $g(x)$ to the state value x.

$$A_n = \begin{pmatrix} a_n(1) \\ a_n(2) \\ \vdots \\ a_n(n) \end{pmatrix}., \qquad g^n(x) = \sum_{i=0}^{N} 2^{-i} a_n(i). \tag{2.28}$$

The operational function $U(g^n(x))$ is defined by

$$U(g^N(x)) = \sum_{i=1}^{N} 2^{-(N-i-1)} a_i(1). \tag{2.29}$$

This equation means that U outputs the decimal value expression of the binary sequence consisting of most significant bits in the binary sequences $A_1, A_2, \cdots, A_N$. Note that the order of the binary sequence is inverted by applying function $U(g^n(x))$.

2.3.2 Definition of $V(x)$

Function $V(x)$ plays a role in inversion of the order of the binary sequence A corresponding to state value x.

$$V(x) = \sum_{i=1}^{N} 2^{-i} a(N-i-1) \tag{2.30}$$

From the definition of $V(x)$, the order of binary sequence is preserved in each transition when the direction is inverted. If $f(x)$ and $g(x)$ are given, time-state-scale re-entrant form can be implemented.

2.4 Time-state-scale re-entrant form

To construct the functions $f(x)$ and $g(x)$, we introduce Chaotic liar proposed by Grim et al. Chaotic liar is expressed as the statement *”This statement is as true as* *false.*" that evokes self-referential indication of liar statement, and it is rewritten, in terms of logical operations, as

$$x = x \leftrightarrow 1 - x \tag{2.31}$$

$$= \min(\max(x, x), \max(1 - x, 1 - x). \tag{2.32}$$

In replacing min by MIN, eq.(2.17), a function $f(x)$ corresponding to the form of time is obtained, such that,

$$f(x) = \mathrm{MIN}(\max(x, x), \max(1 - x, 1 - x)) \tag{2.33}$$

$$= \begin{cases} \dfrac{x}{1-x} & \text{if } x \leq 0.5, \\ \dfrac{1-x}{x} & \text{otherwise.} \end{cases} \tag{2.34}$$

On the other hand, in replacing max by MAX, eq.(2.25), a function $g(x)$ corresponding to the form of state-scale is obtained, such that,

$$g(x) = \min(\mathrm{MAX}(x, x), \mathrm{MAX}(1 - x, 1 - x)) \tag{2.35}$$

$$= \begin{cases} 2x & \text{if } x \leq 0.5, \\ 2(1-x) & \text{otherwise.} \end{cases} \tag{2.36}$$

Substituting (2.34), (2.36) into (2.26), we obtained one time-state-scale Re-entrant System (TSSRS).

In Fig. 2.5, shows the numerical results of TSSRS. Upper figure indicates the time series of x^t. The behavior of TSSRS is complex intermittent pulse. This intermittent pulse is basically derived from the form of time implying logical self-reference. The affects from the re-entry via the state-scale form reveal the fractal

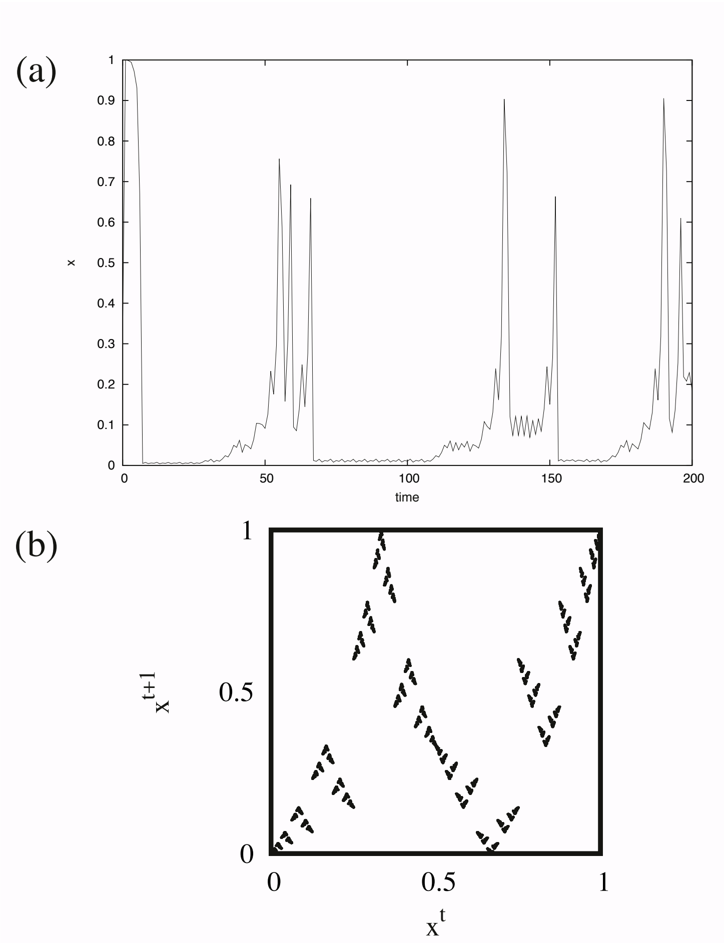

Figure 2.5: Behavior of TSSRS. (a) the intermittent time evolution of truth value. (b) the return map structure of the dynamics.

structure of dynamical property in phase space as shown in lower of Fig. 2.5. A dynamical property of fractal structure in phase space indicates a nowhere differentiable structure corresponding to jump among fractal structures. These fractal jumps in TSSRS implies that the behavior in the laminar phase is not a simple monotonic increase but the fast jumps. (see (Gunji et al., 2007)) Analytically, this fractal structure is yielded by the term $U(g^N(V(x)))$, and it relates to a rule of cellular automata (see Appendix A).

Grim noted that the semantic dynamical property of self-referential statement represents the reliability or the degree of truth with respect to the uncertainty or indetermination in real world. The property of robustness is stability or adaptability for the future transition of the environmental conditions. An itermittent dynamical system basically includes stability and instability of its dynamical phase. Thus, TSSRS represents the feature of robustness by a kind of intermittency.

To confirm the relationship between robustness and heterarchy with a dynamical property, we construct the control experiment that the interaction between time and state-scale is simple map composition. Concretely, the time development is expressed as following,

$$x^{t+1} = f(g(x^t)). \tag{2.37}$$

Since the simple map composition does not need two direction, the both forms of time and state-scale are applied same direction, namely time direction. In Fig. 2.6, we show the behavior of the control experiment (2.37) and its return map. The return map shows linear curve, and the behavior is chaotic oscillation. There is not a robust intermittent behavior. Thus, the comparison between TSSRS and control experiment tells us the importance of the introduction of twisted orthogonality corresponding to the heterarchical meaning.

To address the difficulty of constructing heterarchical system, we introduce the two forms that represent the logical self-reference and frame problem, respectively. The two forms indicate the inconsistency between the whole and subsystem revels. As a dynamical system with connected two forms we define the time-state-scale re-entarnt system (TSSRS). The connection between two forms

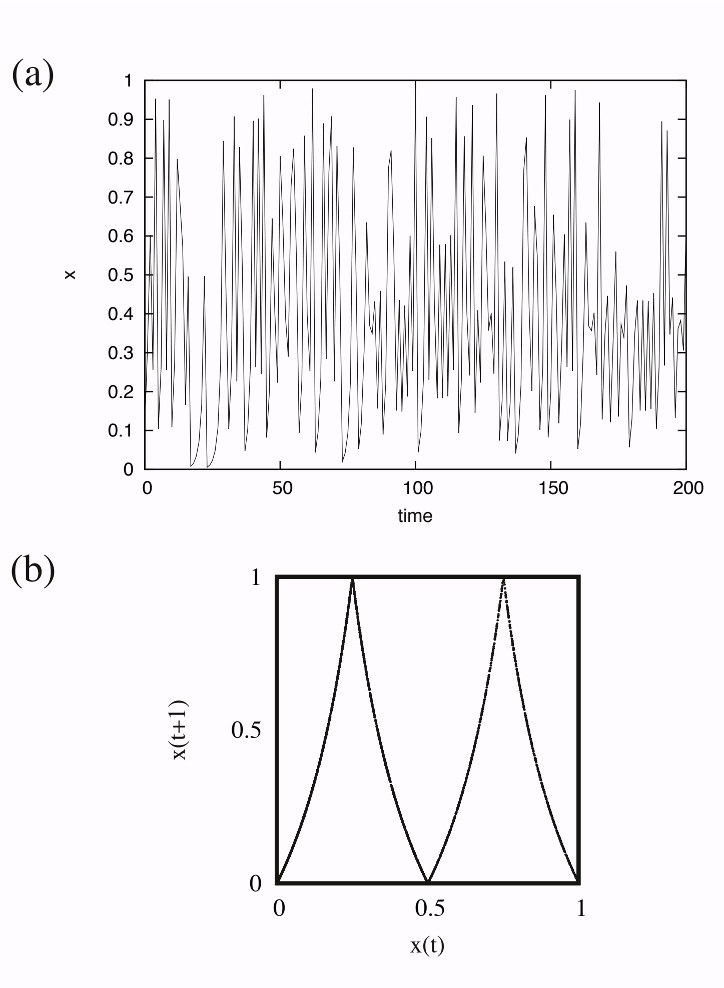

Figure 2.6: Behavior of the control experiment. (a) the intermittent time evolution of truth value. (b) the return map structure of the dynamics.

is defined by twisting sequence expressing real value. TSSRS shows intermittent pulse and self-similar map structure. From the comparison between TSSRS and the control experiment, this intermittency derives from not simple map composition but twisted orthogonality of the two forms. We conclude that self-referential approach to constructing heterarchy is capable of representing robustness in dynamical systems. Our aim of this research is to present the self-referential approach for the robustness and evolvability in dynamical models about uncertainty in real world. In the next chapter, we apply the time-state-scale re-entarnt form to a biological model as a example of effects from the heterarchical form in real world.

Chapter 3

Coupled Time-State-scale Re-entrant System

In this chapter, we show an example of time-state-scale re-entrant form related to the biological systems. The basis of the modeling biological systems is a logic-based dynamical system for the enzymatic reaction network. Tsuda et al. (Tsuda and Tadaki, 1997) presented the dynamical theory of a biological network using a self-referential description of functional manifestation of the kinase network. We assume the heterarchy in the model of biological network, here, that local hydrodynamical fluctuations and protein's interactions are not clear cut, and they interplay simultaneously. To reveal such interplay, we apply the schema of TSSRS to Tsuda's dynamical system.

3.1 A logic-based dynamical model of the enzymatic reaction system

In Tsuda's work, functional manifestation of the kinase network is represented with self-referential description of kinase and phosphatase activation. Although the functional manifestation of this network is based on the results of the chemical reactions between a complex and wide variety of enzymes, the most simplified model that treats the interaction between only two enzymes, namely kinase and phosphatase, is accepted in this theory. In fact, cellular signal processing depends heavily on the processes of phosphorylation and dephosphorylation derived from activations of kinase and phosphatase, respectively. To represent the reaction chain of the biological network, Tsuda introduced confusion between a protein and a protein's network, that is, a kinase and a kinase network.

Actual biological functional manifestation of enzymatic reaction system is the result of complex chemical process over the reaction network. The molecular cell biology express the complexity of chemical reaction network as that it consists not only of the local activation but of the feedback from the final product that realize the function of enzymes. Basically, enzymatic reaction is thought of one directed process from a enzyme to a target protein. However, there are some feedback process in this network processing, and it derives from the fact that a enzyme also has a property of a protein. Tsuda mentioned this point, and assume the identification between enzyme and protein as a self-reference. Basically, the self-reference relates to the observer's standing points, and that bases on the following five assumptions;

(1) The protein works as an observer for the k-p networks.

(2) The observation process is expressed as an inference process for the statement concerning the state of the k-p networks. The records are kept until the functional manifestation of the protein occurs.

(3) The protein calculates the truth values of statements and the process itself, according to a base like Łukasiewicz logic.

(4) For the protein, molecules are indistinguishable from others of the same kind.

(5) There is only one kination site for each kinase, phosphatase and protein.

Where the term ”kinase” and ”phosphatase” are two enzymes, and ”protein” is the properties as a protein of each enzymes, and ”k-p network” is the reaction network consisting of two enzymes. Here, the process of chemical reactions are replaced by inference processes on the observation of activation in each site of enzymes.

Let $< k >$ and $< p >$ be the statements, ’kinase is active’ and ’phosphatase is active’, respectively. The propositional description of the protein’s interaction is represented by,

$$\begin{aligned} < k > \quad &\Leftarrow < k > \leftrightarrow < k > \wedge < \neg p >, \\ < p > \quad &\Leftarrow < p > \leftrightarrow < k > \wedge < \neg p > . \end{aligned} \tag{3.1}$$

This propositional description creates a self-referential version of minimal expression of the kinase network. The confusion between a protein and a protein’s network is represented with $\leftrightarrow$ in both Eqs. (3.1) and (3.2). The implication $\Leftarrow$ represents time transition only. By introducing Łukaisewicz logic Eq. (2.2), these two statements are interpreted into the dynamical system on [0, 1] interval, and let

x and y be the truth value of the statements $< k >$ and $< p >$, respectively,

$$\begin{aligned} x_{t+1} &= x_t \leftrightarrow \min(x_t, 1 - y_t), \\ y_{t+1} &= y_t \leftrightarrow \min(x_t, 1 - y_t). \end{aligned} \tag{3.2}$$

The dynamical system of truth value of the statement about enzymatic activity is easily reduced to the network dynamics of the enzymatic reactions since the truth value of the statement corresponds to the degree of activity.

Applying the term $\min(x_t, 1 - y_t)$, the line $x + y = 1$ divides the unit square into two regions: region I ($x + y < 1$) and region II ($x + y > 1$). Therefore, the equations of the form of time are, in region I,

$$x_{t+1} = 1, \tag{3.3}$$

$$y_{t+1} = 1 - |x_t - y_t|, \tag{3.4}$$

, and in region II,

$$x_{t+1} = 2 - (x_t + y_t), \tag{3.5}$$

$$y_{t+1} = 1 - |1 - 2y_t|. \tag{3.6}$$

In order to avoid the self-reference, Tsuda introduced the assumption of the one-time-step delay in identifying consequence with premise. Ordinary, time transition in logic-dynamics is applied to the inference process, and deduction process substituting consequence to premise is momentarily executed. Let P and C be the premise and consequence of a statement, and let v_t be a map that assigns the truth value to the statemnt,

$$\begin{aligned} v_{t+1}(C) &= v_t(P), \\ v_t(P) &= v_t(C). \end{aligned} \tag{3.7}$$

Tsuda pointed out that one-time-step delay implies the action derived from the internal observer. Since the internal observer can not instantly indicate the whole system, deduction process spends time and inference process is momentarily executed.

$$\begin{aligned} v_{t+1}(C) &= v_{t+1}(P), \\ v_{t+1}(P) &= v_t(C). \end{aligned} \tag{3.8}$$

Applying Eq. (3.8) to region 1, the dynamical system consequently is obtained as follows, in region 1,

$$x_{t+1} = 1, \tag{3.9}$$

$$y_{t+1} = y_t, \tag{3.10}$$

, and in region II,

$$x_{t+1} = 2 - (x_t + y_t), \tag{3.11}$$

$$y_{t+1} = 1 - |1 - 2y_t|. \tag{3.12}$$

y_t in the right hand side of Eq. (3.10) implies one-time-step delay in region I. In this paper, we call this model the ”time-step-delay model”. If one-time-step delay is not assumed, the dynamical system shows periodic motion. On the other hand, the time-step-delay model shows chaotic behavior. Tsuda concludes that the difference between them implies invalidation of self-reference by introducing the perspective of the internal observer, and this reveals chaotic behavior. As mentioned earlier, our purpose is to construct heterarchy defined as dynamical hierarchy with respect to invalidation of the self-referential paradox. In this paper, we take an alternative way to invalidate the self-referential paradox, namely the time-state-scale re-entrant form.

3.2 Coupled TSSRS as the model of the enzymatic reaction system

In place of one-time-step delay, we introduce the time-state-scale re-entrant form to self-referential description of the kinase network. Feedback mechanism of kinase network is separated to two levels, local interaction and effects from final products. The former corresponds to sub-system level, and the latter corresponds to whole-system level. In actual biological enzymatic network, these feedback interactions of two levels are simultaneously occur. If we describe this situation as a model, the inconsistency between different levels is revealed. To realize the model of such a situation, we construct the time-state-scale re-entrant form in the self-referential model of kinase network.

First, we define a propositional description of the whole-system level as the form of time revealing logical self-reference. Since the kinase network that we suppose in this paper including only two enzymes, the statement of wole-system level, interaction with final products corresponds to interaction among two enzymes, namely inference process of the coupled two statement similar to Eq. (3.2). In the form of time, $\leftrightarrow$ is interpreted as classical equality $x \leftrightarrow y = \min(\max(x, 1-y), \max(1-x, y))$ with replacing min by MIN. Now let $\phi(x, y)$ and $\psi(x, y)$ be the temporal dynamics representing transition at the whole-system

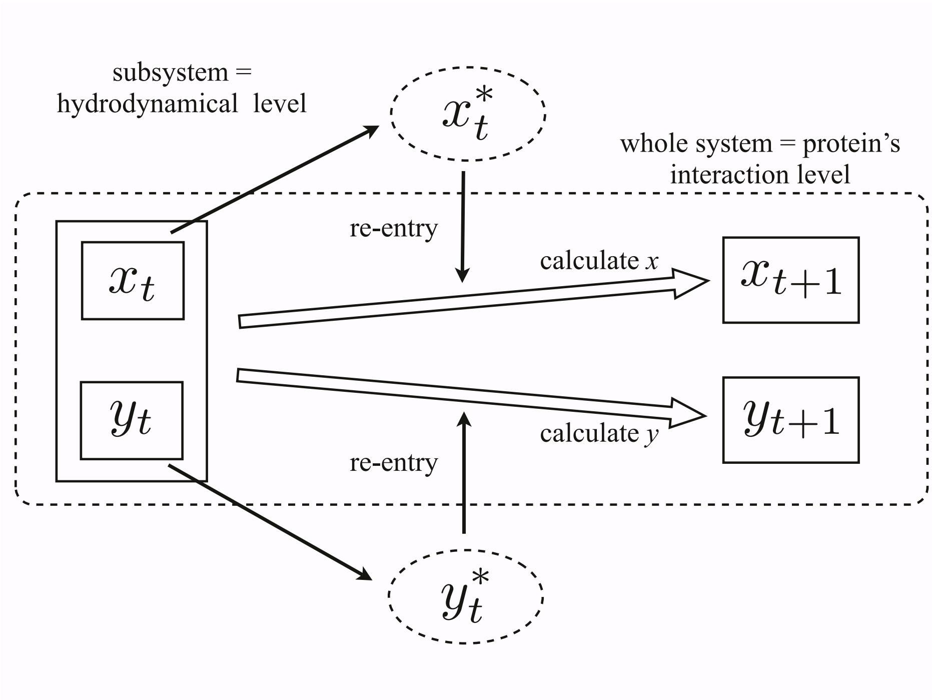

Figure 3.1: Schematic diagram of time development in the coupled TSSRS.

level, then,

$$\begin{aligned}\phi(x, y) &= x^t \leftrightarrow \min(x^t, 1-y^t) \\ &= \mathrm{MIN}(\max(x, 1-\min(x, 1-y)), \max(1-x, \min(x, 1-y))),\end{aligned} \tag{3.13}$$

$$\begin{aligned}\psi(x, y) &= y^t \leftrightarrow \min(x^t, 1-y^t) \\ &= \mathrm{MIN}(\max(y, 1-\min(x, 1-y)), \max(1-y, \min(x, 1-y))).\end{aligned} \tag{3.14}$$

Similar to Tsuda's dynamical system, applying the term $\min(x, 1-y)$, the line $x+y=1$ divides the unit square into region I ($x+y<1$) and region II ($x+y>1$).

Therefore, the equations of the temporal dynamics are, in region 1 ($x + y < 1$),

$$\begin{aligned} \phi(x, y) &= \mathrm{MIN}(\max(x, 1 - x), \max(1 - x, x)) \\ &= 1 \end{aligned} \tag{3.15}$$

$$\begin{aligned} \psi(x, y) &= \mathrm{MIN}(\max(y, 1 - x), \max(1 - y, x)) \\ &= \begin{cases} \dfrac{1 - y}{1 - x} & (x < y) \\ \dfrac{1 - x}{1 - y} & (\text{otherwise}) \end{cases}, \end{aligned} \tag{3.16}$$

and in region II ($x + y > 1$),

$$\begin{aligned} \phi(x, y) &= \mathrm{MIN}(\max(x, y), \max(1 - x, 1 - y)) \\ &= \begin{cases} \dfrac{1 - x}{y} & (x < y) \\ \dfrac{1 - y}{x} & (\text{otherwise}) \end{cases}, \end{aligned} \tag{3.17}$$

$$\begin{aligned} \psi(x, y) &= \mathrm{MIN}(\max(y, y), \max(1 - y, 1 - y)) \\ &= \begin{cases} \dfrac{y}{1 - y} & (y < \frac{1}{2}) \\ \dfrac{1 - y}{y} & (\text{otherwise}) \end{cases}, \end{aligned} \tag{3.18}$$

In addition, for the state-scale dynamics revealing frame-problem, we introduce another dynamical representation of two self-referential statements,

$$x_{s+1} = x_s \leftrightarrow 1 - x_s, \tag{3.19}$$

$$y_{s+1} = y_s \leftrightarrow 1 - y_s. \tag{3.20}$$

The reason why the statements are introduced is simply to define the local interaction level. A local interaction that derives from transformation of activation site in individual protein is also complex. We imply this complexity as the single self-referential statement. Since two statements is not coupled each other, we can define the state-scale transition function as $g(\alpha)$, where $\alpha \in [0,1]$. In the form of state-scale, $\leftrightarrow$ corresponds to classical equality $x \leftrightarrow y = \min(\max(x, 1-y), \max(1-x, y))$, and max is replaced with MAX. Hence, the state-scale transition function is defined as,

$$\begin{aligned} g(\alpha) &= \alpha \leftrightarrow 1-\alpha \\ &= \min(\mathrm{MAX}(\alpha, 1-\alpha), \mathrm{MAX}(1-\alpha, \alpha)) \\ &= 1 - |1 - 2\alpha|. \end{aligned} \tag{3.21}$$

This transition function is similar to Eq. (2.36). By introducing the statement of two levels, we can represent the two levels of complexity in kinase network, namely the complex feedback from final products of the network reaction and the complex transformation of activation site in individual enzymes.

Our aim of this study is very simple, as to construct the model representing the interaction between two levels in biological systems, here the local interaction and global feedback in kinase network as a complex chemical reaction network. As the way to represent the interaction between two levels, we construct the time-state-scale re-entrant form between the statements defined in this section. Here we define the time-state-scale re-entrant form between x_t and x_s, as y_t and y_s, respectively. Let $\square^*$ be the operation of re-entry defined by $x^* = U(g^N(V(x)))$, where U and V are defined same as single TSSRS (see chapter 2), then the time transition of time-state-scale re-entrant form is represented by,

$$x_{t+1} = \phi(x_t^*, y_t), \tag{3.22}$$

$$y_{t+1} = \psi(x_t, y_t^*). \tag{3.23}$$

In Fig. 3.1, we show the schematic diagram of time development in this dynamical system. The important property of heterarchy is that interplay between different levels is simultaneously executed. Thus, state-scale re-entry is executed just as the next state of directions is calculated, namely when the application of the form of time. In this schema, the unilateral transition $\square^*$ of each direction represents the simultaneity of interplay or interaction between two levels. Since this dynamical system provides the way to construct the application of TSSRS for the coupled statement, we call this the ”coupled TSSRS” in this paper.

3.3 Results of numerical experiments

In this section, we present some numerical results concerning heterarchy. The most important characterization of the heterarchical structural dynamical system is a particular intermittent motion (Kamiura and Gunji, 2006), a kind of on-off intermittency(Platt et al., 1993). The coupled TSSRS shows a complex intermittent motion as shown in Fig.3.2(a), and we also show the chaotic behavior of the time-step-delay model in Fig.3.2(b). The complex intermittent behavior of the coupled TSSRS includes two attracting phases: one is an attracting near fixed point $(x, y) = (1, 0)$ (Fig.3.2(c)), and the other is an attracting near periodic orbit (Fig.3.2(d)). Actually, the whole system level description without re-entry at the subsystem level simply oscillates. Periodic orbit in the coupled TSSRS is derived from this periodicity. On the other hand, intermittent motion of an attracting near fixed point is derived from the single TSSRS we presented in section 2. Broadly speaking, the coupled TSSRS chaotically moves in phase space, and is sometimes trapped attracters that are around fixed point and periodic orbit.

In conventional dynamical systems, chaotic, intermittent and periodic motions

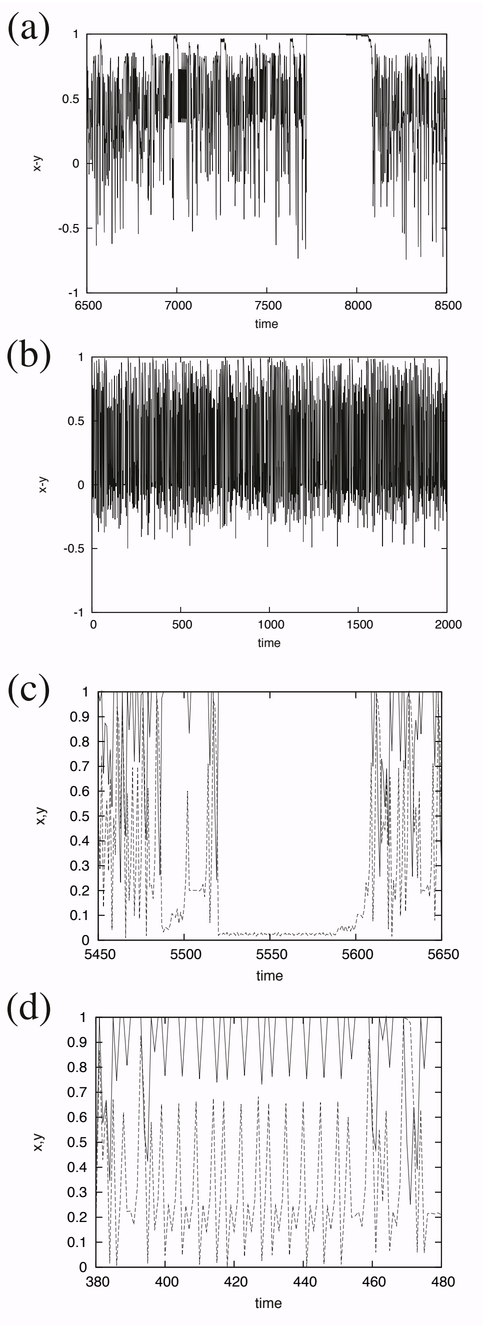

Figure 3.2: (a) Time evolution of $x - y$ when $6500 \leq t \leq 8500$ in the model based on the idea of TSSRS (coupled TSSRS). (b) Time evolution of $x - y$ when $0 \leq t \leq 2000$ in the model presented by Tsuda et al. (c) Typical behavior of an attracting near fixed point in the coupled TSSRS. (d) Typical behavior of an attracting near periodic orbit. In (c) and (d), solid line indicates the value of x_t and dashed line indicates the value of y_t.

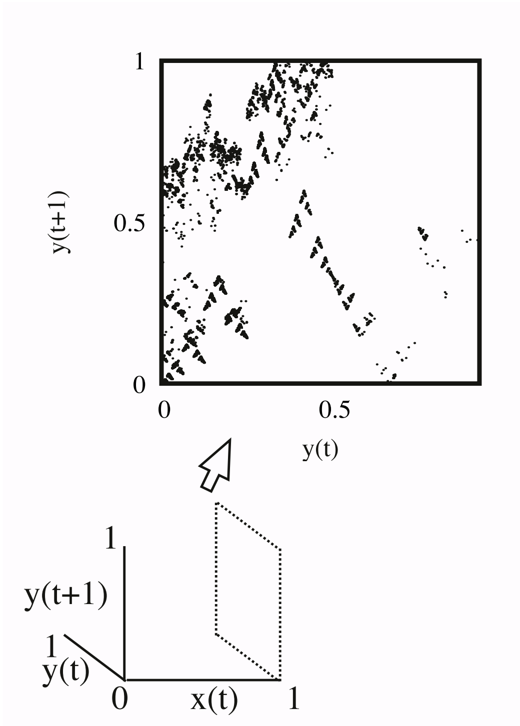

Figure 3.3: Recurrent plot on the plane $x = 1$ in phase space (x_t, y_t, y_{t+1}).

are separated by a bifurcation parameter. The coupled TSSRS, however, does not have such a parameter. Thus, we have to express the transition of attracting and chaotic phases another way. To investigate the transition of phases, we show a recurrent plot on the plane $x = 1$ in phase space (x_t, y_t, y_{t+1}). The regions of gasket structure in this plot indicate moving regions of attracting behavior, and those regions not having gasket structure but which are concentrated points correspond to behavior that escapes from the attractor. Single TSSRS moves among gasket structures by jumping between them (see, (Gunji et al., 2007)). When the motion jumps between gaskets, phase transition occurs at the landing sites of jumps, i.e. jumps land at either an attractor region or chaotic region. Thus, jumps among

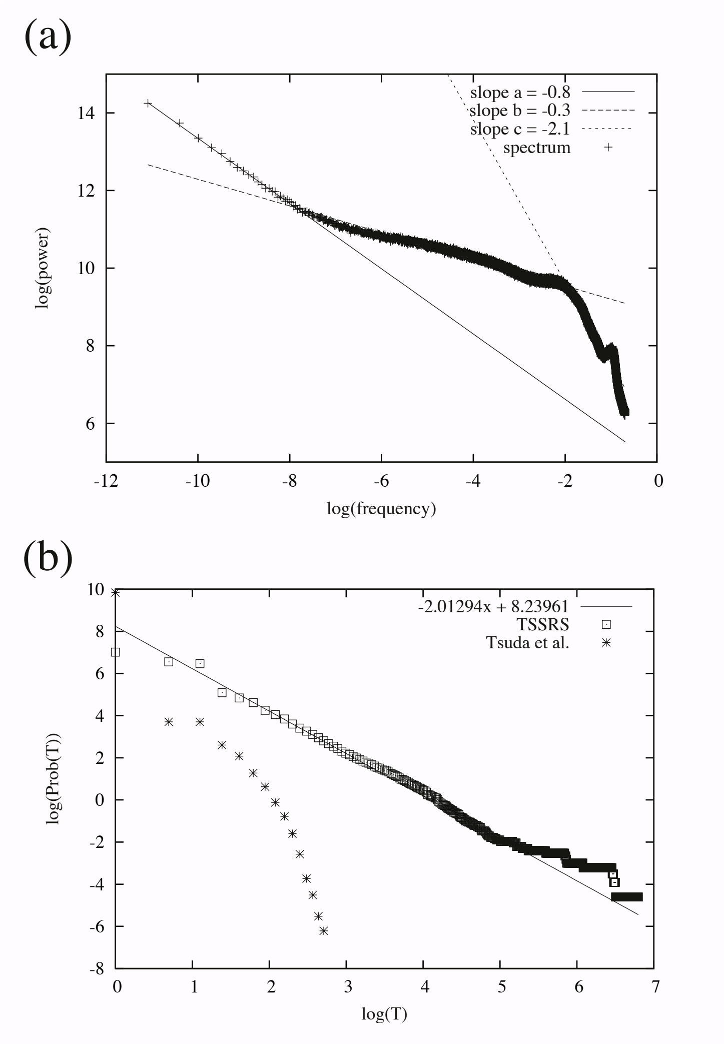

Figure 3.4: (a) Power spectrum of the value $x - y$. Power-law slopes are $a = -0.8, b = -0.3, and c = -2.1$. (b) Probability distribution of laminar phase of coupled TSSRS □ and time step delay model ∗. Power-law slope is $\alpha \approx -2$.

discrete fractal structures lead to bifurcation of the phase in the coupled TSSRS.

A useful method to characterize the phases in the coupled TSSRS is spectrum analysis. We show the power spectrum of the value $x-y$ of the coupled TSSRS in Fig. 3.4(a). The spectrum includes three regions of power-law distribution, where the slopes are $a = -0.8, b = -0.3$ and $c = -2.1$. A peak in slope c indicates periodic orbit. The power-law tail, slope a of this spectrum, corresponds to the motion around a fixed point. Ordinarily, power-law distribution of the spectrum implies some kind of fractal structure in the mechanism of dynamics. The most interesting point of this analytical result is that the motion between two attracting motions shows power-law slope b, indicating the existence of fractal structure in this dynamic. Note that each dynamic corresponding to the three different regions is a different time scale. Thus, the coupled TSSRS shows the dynamics including motions at different levels.

The laminar phase of intermittent behavior in our model indicates the stabilized truth value of proposition, in other words, functional manifestation of physiology. In this analogy, the time spent in stabilization of the truth value corresponds to the duration of a physiological or morphological property of an organism. A strategy for estimating the duration of time spent in the laminar phase has been proposed by Heagy et. al.(Heagy et al., 1994) The stability of laminar phase is estimated by probability distribution of time spent as follows:

$$\Lambda(T) = \mathrm{Prob}\left(\bigcap_{j=1}^{T} y_j \leq \tau \cap y_{n+1} > \tau \,\middle|\, y_1 \leq \tau\right). \tag{3.24}$$

In the x-direction, the value in the laminar phase is $x = 1$. Thus, our estimation of time spent in functional manifestation is represented as follows:

$$\Lambda(T) = \text{Prob}\left(\bigcap_{j=1}^{T} x_j = 1 \cap x_{n+1} < 1 \,\middle|\, x_1 = 1\right). \tag{3.25}$$

We show the probability distribution of time spent in the laminar phase in x-direction in Fig. 3.3(b). The log-log plot of probability distribution shows a power law with exponent $a = -2.01294(\pm - 0.06472) \approx -2$. This power-law distribution is observed in a large variety of biological systems, referred to as Lévy flight motion (Reynolds, 2005). The laminar phase that indicates $x = 1$, namely, the stabilized state in which ”kinase is active” is true and ”phosphatase is active” is false, corresponds to biological functional manifestation itself. This analogy is the starting point of the computational modeling of proteins(Szacilowski, 2007). This relationship proves the importance of a description of a logical propositional with respect to heterarchy.

Chapter 4

Concluding Remarks

A heterarchical property of a system cannot easily expressed computationally because the conceptual definition of heterarchy including paradoxical logical expression that is based on the inconsistency between whole-system and sub-system. To reveal this logical expression in the form of computational expression, we introduce the self-referential statement called ”chaotic liar” presented by Grim et al. (Grim et al., 1994). In the logical property of self-reference, we define the two reasoning of paradox called logical self-reference and frame-problem. The former logical self-reference represents the paradox as the consequence that sub-system cannot be completely indicated from the premise the whole-system is completely indicated. The latter frame-problem represents the incomplete indication of the whole-system from the premise that the sub-system is completely indicated. Two reasoning of the self-referential paradox are complementary and invalidating the premises of them. Hence, connecting the two directional reasoning, we can express the duality of whole-system and sub-system.

The definition of the directions for two reasonings is as in the abstract dynamical system with the state-value expressed as the binary sequence. One direction is the time that means an application of the function, and the other is the state-scale (space) that means an indication or an approximation of the state-scale by indicating the least significant bit. According to two reasoning of self-reference, chaotic liar is interpreted two dynamical systems called the form of time and state-scale. The connection between the two directions is defined by coding and twisting sequence, and that implies the system theoretic study of a brain as a computer. In the study of computational brain, the informations from the outside are coded and stored as a memory. The brain makes a decision with such a memorize information, and then the another information comes from outside as the result of the decision. The algorithmic definition of the information processing of computational brain. The time-state-scale re-entrant form is defined along this algorithmic definition and as connected two dynamics.

As a numerical results, time-state-scale re-entrant system shows an intermiitent pulse, and its dynamical structure forms self-similar gaskets. The self-similar gasket structure is well known fractal property of dynamical systems, and fractal property is considered to be important feature of multi-scale network or robust systems. The more indicated robustness is intermittent behavior of time series. Essentially, the truth value that is taken as the state value in TSSRS implies the reliability or possibility of realization of inconsistency between whole-system and sub-system. Intermittent behavior of the truth value implies the stabilized state that wholeness and parts are completely separated and formulate a consistency, and that also implies the perturbed state at pulse is executed when the inconsistency is realized. Hence, TSSRS indicates the abstract logical property of robust systems as the heterarchical systems.

Next, we present an example of TSSRS in biological systems. Heterarchical property of biological systems is thought to be realized in the network structure at the wide variety of scales. As similar to above mention, a computational modeling of heterarchicy in biological systems encounters a self-referential paradox between whole-system and subsystem. To address the difficulty of self-reference in computational expression of heterarchical systems, we introduce the time-state-scale re-entrant form to the self-referential description of the particular enzymatic reaction system called coupled TSSRS. The coupled TSSRS derives from the coupled self-referential statements indicating the signal propagation in the minimal biological network that have two nodes, activating and inactivating, excitatory and inhibitory. We assume the logical self-reference in such a network as the global feedback from final products as the results of interaction among nodes, and also assume the frame-problem as the self-feedback or self-reconfiguration of the interactive activation site in the individual molecular structure. The time-state-scale re-entrant form between the logical self-reference and frame-parobleem represents the heterarchical property of a biological network.

The coupled TSSRS shows a mixing behavior of intermittent phase, periodic phase and chaotic phase. Spectrum analysis clearly separates these three phases on the frequency distribution. Each of them shows the $1/f$ power-law distribution. Power-law distribution implies the existence of underlying multiple structure. It suggests that there is a heterarchical structure, namely, hierarchical structure with multiply overlapped dynamical property. This structure can be proved by fractal gasket structure in phase space. The fractal structure in phase space makes a jump from attractor to other regions. Overlapped hierarchical structure derives from such a jump among fractal structure. Furthermore, we estimate a probability distribution of time spent of protein's activation derived from the term "kinase is activate". The probability distribution of time spent shows a power-law distribution with exponent −2. −2 power-law distribution is closely relevant to the anomalous diffusion process, and its dynamical property of on-off intermittency. Most important feature of heterarchical structure is relevance to robustness and evolvability (Jen, 2003). In molecular cell biology, anomalous diffusion is one of representations of these notions (Bonci et al., 1996; Golding and Cox, 2006).[1]

From such a stream of biological study, our model significantly relates heterarchical property of biological systems.

An importance of constructing hierarchical property from the perspective of invalidation of self-reference is a possibility to realize the robust, evoluvable and emergent behavior in the closed systems. Hence, the framework of TSSRS is applicable to experimental designing, computational engineering and future study related to artificial life, computational biology. In addition, it is suggested that the possible methodology for artificial duplication of physiological, biological, cognitive and social function in the field of biocomputing and so on. For instance, slime mold computing(Tsuda et al., 2004)(Aono and Hara, 2007) is in this case. In the architecture, slime compute according to different levels of logic from control

[1]The anomalous diffusion with power law distribution is also found in animal behavior as the decision making (Viswanathan et al., 1996; Harmos et al., 2000). However, actually, the existence of this types of power law behaviors are the subject of controversy (Edwards et al., 2007).

mechanism. This paper assists the importance of such a methodology.

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CA representation of $U(g^N(V(x)))$

The function $U(g^N(V(x)))$ defined in chapter 2 (also defined in chapter 3 as x^*) corresponds almost to a rule of cellular automata (CA) (Wolfram, 1983). The elementary cellular automata (ECA) is defined as the surjective map $f : 2\times2\times2 \to 2$ and the configuration $\{x[k]\} \in 2^N, i = 0, 1, \ldots, N$, time evolution of this system is represented particularly by,

$$x^{t+1}[i] = f(x^t[i-1], x^t[i], x^t[i+1]). \tag{1}$$

This system has possible $2^8 = 256$ rules. Here, we show that the function $U(g^N(V(x)))$ equals to ECA with rule number 60.

The tent map g can be represented with the map of the binary sequence. Let $x = \{x[k]\} \in 2^N$, then, because the term 2, $x_{s+1} = g(x_s) = 1 - |1 - 2x_s|$ is interpreted to the binary function,

$$x_{s+1}[i] = x_s[0] + x_s[i+1], \quad (\text{mod } 2) \tag{2}$$

$$x_s := \sum_{k=0}^{N} 2^{-(k+1)} x_s[k]. \tag{3}$$

Taking the definition of U into consideration, target sequence of $U(g^N(x_0))$ is represented as,

$$U(g^N(x_0)) = \{x_{N+1}[0], x_N[0], x_{N-1}[0], \ldots, x_1[0]\}. \tag{4}$$

From Eq. (2) and (4), we have,

$$
\begin{aligned}
x_1[0] &= x_0[0] + x_1[0] \pmod 2, \\
&\vdots \\
x_{s+1}[0] &= x_s[0] + x_s[1] \pmod 2, \\
&\vdots \\
x_{N+1}[0] &= x_N[0] + x_N[1] \pmod 2.
\end{aligned}
\tag{5}
$$

Because the map $a + b (\mathrm{mod}\ 2)$ is symmetric to the inverse operation,

$$
x_s[i] + x_s[i+1] = 1 - x_s[i] + 1 - x_s[i+1] \pmod 2. \tag{6}
$$

Thus, we have the assumption of induction,

$$
x_{s+1}[i] + x_{s+1}[i+1] = x_s[i+1] = x_s[i+2] \pmod 2. \tag{7}
$$

By induction, we have,

$$
\begin{aligned}
x_s[0] &= x_{s-1}[0] + x_{s-1}[1] \\
&= x_{s-2}[1] + x_{s-2}[2] \\
&\vdots \\
&= x_0[s-1] + x_0[s] \qquad \pmod 2.
\end{aligned}
\tag{8}
$$

This equation leads to the ECA with the rule table as following,

$$
\begin{array}{cccc}
\multicolumn{4}{c}{x_0[s-1] \quad x_0[s] \quad x_0[s+1]} \\
\multicolumn{4}{c}{x_s[0]} \\
1\ 1\ 1 & 1\ 1\ 0 & 1\ 0\ 1 & 1\ 0\ 0 \\
0 & 0 & 1 & 1 \\
0\ 1\ 1 & 0\ 1\ 0 & 0\ 0\ 1 & 0\ 0\ 0 \\
1 & 1 & 0 & 0
\end{array}
\tag{9}
$$

The rule number is assigned as $2^5+2^4+2^3+2^2 = 60$. Hence the function $U(g^N(x)))$ corresponds to ECA rule 60.