Aspects of point interaction

Ohya, Satoshi

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氏　　名　　　　　　　大谷　聡

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【 学位論文題目 】

Aspects of point interaction　（点状相互作用の諸相）

審 査 委 員

主 査　教 授　林 青司

教 授　山田 泰彦

准教授　西野 友年

助 教　坂本 眞人

In this dissertation we study theoretical aspects of point interactions in quantum theory on a graph (quantum graph). We are mainly motivated by the following two different fields of quantum physics. The first is mesoscopic or nanoscale systems. Recent development of nanotechnology such as an electron-beam lithography enables us to fabricate mesoscopic networks of one-dimensional wires, whose simplest model is quantum graph. The second is perturbative quantum field theory with a single extra dimension in the presence of defects or boundaries, which can be viewed as point interactions along the direction of extra dimension. Apparently, these two fields of quantum physics have nothing in common. However, they actually have a single common point in respect that boundary condition plays a crucial role in their quantum dynamics.

Whatever the motivation, studies of point interactions all end up with the study of boundary conditions. The essential ingredient to treat boundary conditions in quantum system is the Kirchhoff's law of probability current at the vertex. In mathematical language, it is von Neumann's theory of self-adjoint extension of symmetric operator. These two methods are equivalent and reveal a much rich structure of point interaction available in quantum theory. More specifically, in a star graph with N bonds a point interaction at the vertex is specified by an $N \times N$ unitary matrix $U \in U(N)$. Therefore there exist in principle much variety of point interactions characterized by N^2 parameters. In other words, in such a system theory space spanned by all possible point interactions is just equivalent to the parameter space of unitary group $U(N)$.

As naively expected, any short ranged interaction, which may be a defect or impurity or brane, could be approximated by a point interaction in the long-wavelength limit. In this sense point interaction is essentially for low energy physics. According to the idea of renormalization group theory, such a low energy system will be well described by symmetries and relevant interactions regardless of microscopic details of a system. In this respect, after the introductory chapter we investigate symmetries and invariant subfamilies of point interactions and renormalization group flow in the parameter space of unitary group $U(N)$ in chapter 2 and 3, respectively.

In chapter 2, after the precise definition of point interaction on a star graph we review symmetry classes of $U(2)$ family of point interactions and then proceed to extend the analysis to $U(3)$ family of point interactions on a Y-junction of three half-lines. We reveal that the set of eight discrete transformations generated by the counterclockwise rotation through an angle $2\pi/3$ and $\mathbb{Z}_3$-reflection spans the Lie algebra $\mathfrak{su}(3)$ of the special unitary group $SU(3)$. One of the remarkable point to note is that these eight discrete transformations commute with the free Hamiltonian. Hence, under any elements of $\mathfrak{su}(3)$ spectrum of an original system is preserved. We further extend to a general star graph with N bonds and find that the set of N^2-1 discrete transformations, which is generated by the counterclockwise rotation through an angle $2\pi/N$ and $\mathbb{Z}_N$-reflection, consists of spectrum-preserving $\mathfrak{su}(N)$ algebra.

In chapter 3 we derive exact renormalization group flow of point interactions and found 2^N distinct fixed points in the parameter space spanned by eigenphases of unitary matrix $U \in U(N)$. In general the most infrared (IR) stable fixed point is the Dirichlet fixed point where the Dirichlet boundary conditions are realized. On the other hand, the most ultraviolet (UV) stable fixed point is the Neumann fixed point where the Neumann boundary conditions are realized. The other 2^N-2 fixed points have in general IR unstable directions. Because, in naively speaking, any short ranged interactions can be approximated by a point interaction in the long-wavelength limit, it is natural to expect that our analysis shows that there exists 2^N distinct universality classes of short ranged interactions that must flow into one of the fixed points in the IR limit.

Quantum theory on a graph is also responsible to perturbative quantum field theory. It becomes obvious when one considers a worldline formalism of perturbative quantum field theory, where internal lines of a given position-space Feynman diagram are identified as a single particle's worldline in configuration space. Then field theoretical S-matrix elements are obtained by one-particle quantum mechanical path integral with some background fields. There, Green's functions on a worldline such as a circle (for one-loop amplitudes) or an figure eight (for two-loop amplitudes of ϕ^4 theory) are needed for computations. Nowadays it is known that worldline formalism generally provides a powerful alternative method to compute effective action and/or S-matrix elements at least to one-loop order. Thanks to its nature of position-space oriented descriptions for quantum field theory, worldline formalism is also powerful enough to perturbative quantum field theory with an extra compact dimension for another reason: that is, for the clarity of winding number, that counts how many times an internal line winds around the direction of extra dimension. It plays an important role in extra dimensional models because any non-zero winding contributions are essentially UV finite thanks to its long-distance nature: If one computes a radiative correction of a quantity that naively diverges in UV as $\sim \Lambda^D$ with Λ being a cutoff scale, then such a loop induced correction from non-zero winding contributions must be of the form $\sim \sum_{w\in\mathbb{Z}} C^{(w)}(wL)^{-D}$, where w is the winding number, L is a circumference of extra dimension and $C^{(w)}$ is a possible (w-dependent) dimensionless coefficient. This estimation follows from the dimensional analysis and the fact that internal line winding around the extra dimension w times cannot be shorter than wL and hence it plays a role of a cutoff length scale for w-winding sector. The sum $\sum_{w\in\mathbb{Z}}$ ensures the summation of all possible physically distinct closed loops, which goes quite well with the philosophy of path integral. Thus, determination of whether given S-matrix elements are those from the contributions of zero- or non-zero winding is an essentially significant problem especially for the perturbative renormalization procedures and/or classification of UV finite quantities such as the Higgs mass in the gauge-Higgs unification scenario in a quantitative level. However, it has been a longstanding problem and not yet fully understood how to incorporate boundary conditions (which describes zero-thick branes in the context of extra dimension) into path integration on a compact space, where the Poisson summation formula is no longer available.

In this respect, chapter 4 is devoted to the study of path integral formulation of one-particle quantum mechanics on a rose graph. We find the most general path integral description of the Feynman kernel valid for $U(N)$ family of point interactions at the vertex of N-petal rose. We also discuss the topology change of figure eight and speculated that any compact graphs can be generated via topology change of an appropriate rose graph. This is the reason why we concern a rose graph.

In chapter 5 we study the worldline formalism of Abelian gauge theory with a single extra dimension compactified on a circle. We successfully derive the master formula for scalar one-loop amplitudes with arbitrary number of external gauge bosons in Abelian gauge theory.

In concluding chapter 6 we discuss a possibility that extra dimension may be a graph such as a Y-junction of three half-lines as a low energy description of domain wall junctions.

（別紙１）　　　　　　　　　　　　　　　論文審査の結果の要旨

氏名	大谷　聡		
論文題目	Aspects of point interaction （点状相互作用の諸相）		
審査委員	区　分	職　名	氏　　　名
	主　査	教授	林　青司
	副　査	助教	坂本　眞人
	副　査	教授	山田　泰彦
	副　査	准教授	西野　友年
	副　査		

要　　旨

物理学においては、物性系における欠陥や不純物の存在、素粒子物理学におけるブレーン等の存在に依る局所的、小さな領域における相互作用が重要な役割を果たす場合がしばしば見受けられるが、低エネルギーの長波長の極限では、これらの相互作用はいずれも実質的に点状の相互作用と見なして構わない。

この論文の主たる目的は、こうした点状相互作用の様々な理論的な側面を議論する事である。研究の背景としては、一つには一次元物性系のネットワークの製造技術の進歩がある。こうした系は量子グラフでモデル化される。もう一つは、素粒子物理学において良く議論される様な、欠陥やある種の境界条件を持った一次元的余剰次元のある量子場の理論の摂動論を、世界線を用いた定式化によって理解したいという観点である。一見これらには何ら共通点が無いが、境界条件が本質的に重要な役割を果たすという意味で統一的な議論が可能となる。

特に、一次元のコンパクト化した余剰次元を持った高次元理論に関しては、通常紫外領域のカットオフに強く依存した物理量と、理論の明確な予言となり得る赤外領域の物理量を明確に分離する手法としてポアソン（再）和公式が用いられるが、残念ながらこの公式は余剰次元に点状相互作用が存在する場合には、そのまま適用することが出来ないのである。この論文では、こうした点状相互が存在する場合にも適用可能な一般の N 点グリーン関数を導く公式を世界線を用いた定式化を用いて導いている。

１章の序論に続いて、２章では量子グラフ上の点状相互作用について議論している。確率の流れの保存則を基に、N 個のボンドを持った頂点における点状相互作用による境界条件が U(N) のパラメターを用いて記述可能な事を示し、また、こうした定式化を用いて点状相互作用をいくつかのカテゴリーに分類することに成功している。３章では U(N) で記述される点状相互作用のエネルギー依存性を議論するために、繰りこみ群を用いた解析が成されている。その結果、U(N) のパラメター空間において、2^N 個の異なる固定点がスケール不変な点状相互作用を表わすものとして存在することが示されている。４章では、N 個の花弁を持ったバラ形のグラフ上を伝搬する粒子についての径路積分における Feynman 核について議論が成されている。Feynman 核を、バラの中心における点状相互作用による反射あるいは透過の回数により分類された振幅の和の形で記述できる事が示されている。５章では、それまでの議論を量子場の理論の摂動論へ応用すべく、円状にコンパクト化した余剰次元のある D 次元時空上の可換ゲージ理論の枠組みにおけるゲージ場の任意の次数のグリーン関数（１ループのオーダー）を表わす公式を導いている。

以上みてきたように、本研究は、物理学のいくつかの分野に共通して現れる点状相互作用について、その様々な理論的側面を複数の角度から議論し明らかにしたもので、物性系や、最近素粒子論において話題となっている余剰次元を持った高次元理論におけるグリーン関数についても、任意の次数のグリーン関数を表わす公式を導く等、素粒子論、物性論の最近の話題にも関わる重要な知見を得たものとして価値ある集積であると認める。

よって、学位申請者の大谷　聡は、博士（理学）の学位を得る資格があると認める。